Arnold, Lutz Georg; Arnold, Lutz; Zelzner, Sebastian

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Abstract

We incorporate occupational choice between finance and entrepreneurship into the Grossman-Stiglitz (1980) noisy rational expectations equilibrium model. Sophisticated agents produce output and create jobs as entrepreneurs or contribute to informational efficiency in financial markets as informed traders. Finance possibly attracts too much talent, for instance if the amount of noise in the economy is small, so that the asset price at a rational expectations equilibrium is highly informative anyway. The main beneficiaries of the allocation of talent to entrepreneurial activity are workers, whose wage and employment prospects improve when more sophisticated agents choose to become entrepreneurs.

JEL classification: G14, J24

Key words: market efficiency, asymmetric information, allocation of talent, occupational choice
1 Introduction

The recent financial turbulence has sparked a discussion about the social benefits of financial trading. At the policy level, the discussion centers around the question of whether, in view of explicit safety nets and implicit state guarantees, financial institutions have incentives to take excessive risks. A different concern, which might be of equal importance for long-term growth and economic welfare, is that the financial sector attracts too much talent, which could produce larger social benefits in different occupations. This paper presents a general equilibrium model that addresses this issue. Asset prices fluctuate due to shocks to macroeconomic fundamentals and stochastic noise trader demand. A class of “sophisticated” agents become either traders or entrepreneurs. Entrepreneurs produce output and create jobs. Claims to the values they create are traded in a financial market. Traders contribute to informational efficiency by acquiring information about fundamentals. Trading possibly, though not necessarily, attracts too much talent in the model. For instance, agents are generally better-off without the opportunity to become traders if the volatility of noise trader demand is sufficiently low and they have a long position in the risky asset at equilibrium with occupational choice (OC). Probably the most important effect of measures which alter the incentives to become a trader or an entrepreneur is not on the agents who face this choice but on production sector workers: they benefit more strongly from the creation of jobs by entrepreneurs than from enhanced informational efficiency of asset prices. A production economy version of our model lends support to this view: depending on whether there is full employment or unemployment due to wage rigidity, an increase in the number of entrepreneurs benefits workers by raising their wage or aggregate employment, respectively.

Concerns that the financial sector attracts too much talent are fueled by empirical observations that high wages draw a large number of graduates into finance. Goldin and Katz (2008) observe that the proportion of male Harvard graduates from selected classes who work in the finance sector 15 years after graduation rose from 5 percent for early-1970s cohorts to 15 percent for early-1990s cohorts. According to the Harvard Magazine, the figure peaked at more than 20 percent in 2007, before labor demand collapsed with the onset of the subprime crisis.¹ Competition for talent does not stop when students have decided to specialize in science or engineering. Shortly before the financial crisis, serial entrepreneur and writer Vivek Wadhwa observed in his testimony to the the U.S. House of Representatives that “Thirty to forty percent of Duke Masters of Engineering Management students were accepting jobs outside of the engineering profession. They chose to

become investment bankers or management consultants rather than engineers”.\textsuperscript{2} Similarly, the \textit{Economist} reports that “[M]ost of the world’s top hedge funds prefer seasoned traders, engineers and mathematicians, people with insight and programming skills, to MBAs”.\textsuperscript{3} Célérié and Vallée (2013) remark in their empirical study of French graduate engineers that a sizeable portion of the post-2000 graduates worked in the City of London or on Wall street. Oyer (2008, p. 2622) finds “mixed evidence that initial jobs on Wall Street lead Stanford MBAs to start fewer businesses”. He adds that there is path dependence in OC: workers drawn into the financial by random events tend to stay there. While fierce competition for talent is undisputed, opinions diverge on whether this is a good thing. Esther Duflo replied to concerns that regulations would constrain the financial sector in the aftermath of the financial crisis: “Is there a risk of discouraging the most talented to work hard and innovate in finance? Probably. But it would almost certainly be a good thing.”\textsuperscript{4} At \textit{The Economist’s} 2013 Buttonwood Gathering, Robert Shiller (“When you study finance you are studying how to make things happen”) and Wadhwa (“Google – not Goldman Sachs – deserves our best minds”)\textsuperscript{5} exchanged opinions. Beck et al. (2014) find that in a broad cross section of countries financial intermediation (measured as the ratio of private credit to GDP) is positively correlated with economic growth, while the size of the financial sector (measured by its value added share in GDP) is insignificant if intermediation is controlled for.

Our model incorporates OC between finance and entrepreneurship into the noisy rational expectations equilibrium (REE) model of Grossman and Stiglitz (1980, henceforth: “GS”). The notion of informed trading is precisely the one mentioned, but not formalized, in Murphy et al.’s (1991, p. 506) classic paper on the allocation of talent:

“Trading probably raises efficiency since it brings security prices closer to their fundamental values ... But the main gains from trading come from the transfer of wealth to the smart traders ... Even though efficiency improves, transfers are the main source of returns in trading.”

Baumol (1990, p. 915) takes a similar position. The reason why the allocation of talent to finance is excessive when the amount of noise in the economy is small is as follows: The asset price in an REE is then highly informative, so that traders’ observation of macroeconomic fundamentals is of little

\textsuperscript{2}Quoted from Philippon (2010, p. 159).
value for them. At an equilibrium with positive fractions of agents in both occupations, the impact of output created as an entrepreneur on agents’ expected utility must also be low. If the opportunity to become a trader did not exist, agents would be able to capture greater rents as entrepreneurs.\(^6\) Notably, excessive allocation of talent to finance occurs despite the fact that there is no other financial market imperfection besides the standard REE information asymmetry and informed trading ameliorates this problem by conveying information on macroeconomic fundamentals.

Several models study the allocation of talent to finance versus entrepreneurship. The model most closely related to ours is Bolton et al. (2014). Agents become “dealers” or entrepreneurs in that model. Dealers have perfect information about the outcome of investment projects. They buy high quality assets created by entrepreneurs in an “opaque” OTC market. The (lower quality) assets not bought by dealers are traded at the appropriate discount by uninformed agents in a “transparent” organized exchange. Entrepreneurs have an incentive to provide effort for their projects in order to qualify for the OTC market. Whenever high effort is both socially efficient and part of an equilibrium, the mass of agents who become dealers is inefficiently high at equilibrium, i.e., the OTC market is too big. Bolton et al. (2014, p. 3) conjecture that in “the standard framework of trading in financial markets first developed by Grossman and Stiglitz (1980) . . . privately produced information leaks out in the process of trading, and as a result too little costly information may be produced by ‘insiders.’ Since many activities in the financial industry boil down to costly private ‘information acquisition’, the Grossman-Stiglitz model seems to suggest that the financial sector could be too small.” Our analysis shows that, on the contrary, a financial sector engaged in GS-type trading, which produces valuable information that leaks out to other traders, can be too big. That is, trading can be excessive, not only in the opaque OTC segment of the financial market, but also in organized exchanges, where both informed and less informed traders are active.\(^7\)

Phillipon (2010) and Cahuc and Challe (2012) present alternative models of the allocation of talent to finance. Other than Bolton et al. (2014) and our paper, they emphasize the financial intermediation role of the financial sector and the focus is not on the question of whether the financial sector is too big. Philippon (2010) embeds OC into an endogenous growth model with

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\(^6\)Following GS, noise trader demand is exogenous in our model. So welfare statement do not encompass noise traders. We argue, however, that a reallocation of talent to the real sector does not systematically worsen the terms at which they trade.

\(^7\)The limitations of our analysis are analogous as in Bolton et al. (2014). There is no moral hazard due to implicit or explicit state guarantees. There is no leverage, traders trade only on their own account. The only input required to set up a firm is entrepreneurial labor, so there is no financial intermediation. Entrepreneurs set up and run firms, no distinction is made between engineering and management tasks.
externalities emanating from investment. Once these externalities are internalized by means of an investment subsidy, there is no need for a preferential tax treatment of the financial sector: second-best can be achieved with a uniform income tax on income generated in the real and financial sectors. Cahuc and Challe (2012) integrate OC into the neoclassical overlapping-generations growth model. Only agents who specialize in finance are able to make loans to entrepreneurs. In the standard overlapping generations model without OC, asset bubbles can remove dynamic inefficiency due to over-investment by crowding out real investments. Cahuc and Challe (2012) consider a bubble on an intrinsically worthless asset that can only be traded by financiers. The bubble raises financiers’ profits, thereby crowding out employment in the real sector. If financial intermediaries are able to extract large rents, this effect outweighs the former crowding out effect, and bubbles lose their beneficial role.

The paper is organized as follows. Section 2 introduces the model. Section 3 derives the price function and agents’ expected utilities. Sections 4 and 5 characterize the equilibrium with and without noise trader shocks, respectively. Sections 6 and 7 investigate the welfare effects of trading activity and the question of whether the financial sector should be taxed. Section 8 embeds the model in a general equilibrium setup with a labor market. Section 9 concludes. Details of the algebra are delegated to the Appendix.

2 Model

Consider a CARA-Gaussian economy with three dates, “early”, “intermediate”, and “late”. There are two types of agents: a continuum of rational agents indexed by \(i \in [0, L]\) \((L > 0)\) and noise traders. We often call the rational agents simply “agents”. There is a single homogeneous consumption good. Prices are quoted in terms of this consumption good. Each agent is endowed with \(e (> 0)\) units of the good early. Rational agents consume only late. They are characterized by the CARA utility function \(U(\pi) = -\exp(-\rho \pi)\), where \(\pi\) is late consumption and \(\rho (> 0)\) is the coefficient of absolute risk aversion. Each agent has access to a storage technology that transforms endowments one-for-one into late consumption.

Rational agents face an OC decision: they become entrepreneurs or traders or stay inactive. There is no physical cost of becoming an entrepreneur or a trader. Agents choose the occupation whose payoff profile yields the highest expected utility.\(^8\) As a reference point for the investigation of

\(\footnote{As is well known from GS, in the absence of noise, if it is costly to become a trader, then no-one does so (since the equilibrium price is fully revealing). Our assumption that the only cost of becoming a trader is the opportunity cost of not becoming an entrepreneur allows us to obtain results on equilibria with no noise and with a positive mass}
whether trading is beneficial, we also consider the variant of the model without OC, in which agents do not have the opportunity to become traders.

$a (> 0)$ entrepreneurs jointly set up one firm, which produces $\theta$ units of output late, shared uniformly by the entrepreneurs. We regularly call the firm an entrepreneur helps set up “his” firm. $\theta$ is a macroeconomic shock, which is uniform across firms. It is the sum of two independent jointly normal random variables: $\theta = s + \varepsilon$, where $s \sim N(\bar{s}, \sigma_s^2)$ and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ (in Section 8, we consider a production economy, in which output $\theta$ depends on unskilled labor input). Since entrepreneurial labor is the only input required to start a business, entrepreneurs do not need finance, and there is no financial intermediation.

At the intermediate date, shares in the firms are traded in a competitive stock market. Following GS, noise traders inelastically demand $N \sim N(\bar{N}, \sigma_N^2)$ units of the risky asset (see also Grossman, 1976, and Hellwig, 1980). Traders observe $s$ and face residual uncertainty $\varepsilon$ about firms’ payoff. They are the only agents who acquire information about $s$, thereby contributing to the informational efficiency of the stock market. Entrepreneurs observe neither $s$ nor $\varepsilon$ nor the other agents’ trades. So they cannot tell if a high stock market value of the firms is due to large demand by noise traders or by rational traders, having favorable private information about profitability. Inactive agents, who decide to become neither an entrepreneur nor a trader, trade on the same limited information as entrepreneurs (like the uninformed traders in GS).

$M(\sigma_N^2, 1)$ and $M(\sigma_N^2, 0)$ denote the economies that obtain when the variance of noise trader demand is $\sigma_N^2$ and there is OC or not, respectively.

3 Price function and expected utilities

This section defines equilibrium and derives agents’ asset demands, the price function that relates the asset price to macroeconomic shocks, and agents’ expected utilities.

**Equilibrium**

Let $L_E$ denote the mass of agents who become entrepreneurs. The mass of firms and, hence, the supply of stocks is $L_E/a$. Let $P$ denote the stock market value of each firm and $I_E$, $I_0$, and $I_T$ entrepreneurs’, inactive agents’, and traders’ stock holdings, respectively. Rational agents make their OC and investment decisions so as to maximize expected utility conditional on available information. Consumption is

$\pi_E = e + P/a + (\theta - P)I_E$ for entrepreneurs,

$\pi_T = e + (\theta - P)I_T$ for traders, which provide a useful benchmark for the analysis of the model with noise.

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9Since there are no firm-specific shocks, this information structure does not entail that traders have information about individual firms that entrepreneurs do not have.
traders, and \( \pi_0 = e + (\theta - P)I_0 \) for inactive agents. While traders know \( s \) when they make their investment decision, entrepreneurs and inactive agents can only use the price level \( P \) they observe to infer information about \( s \). For the sake of brevity, we focus on equilibria at which the mass of entrepreneurs \( L_E \) is positive and no agent stays inactive, so that the mass of traders is \( L - L_E \).

The fact that there is no heterogeneity among rational agents ex ante and some of them become entrepreneurs means that expected utility is no higher for traders than for entrepreneurs, so we do not address the issue of excessive pay in finance.\(^{10} \) \((L_E, I_E, I_T, P)\) is such an equilibrium (an REE) of \( M(\sigma_N^2, 1) \) with \( \sigma_N^2 > 0 \) if \( I_E \) maximizes \( E[U(\pi_E)|P] \), \( I_T \) maximizes \( E[U(\pi_T)|s] \), the market for the risky asset clears (i.e., \( L_E/a = L_EI_E + (L - L_E)I_T + N \)), \( OC \) is optimal (i.e., \( E[U(\pi_E)] = E[U(\pi_T)] \)) and \( 0 < L_E \leq L \) or \( E[U(\pi_E)] \geq E[U(\pi_T)] \) and \( L_E = L \), and staying inactive is not preferred to becoming an entrepreneur (i.e., \( E[U(\pi_E)] \geq E[U(\pi_0)] \)).

**Demands and price function**

The optimal investment levels are

\[
I_E = \frac{E(\theta|P) - P}{\rho \text{var}(\theta|P)}, \quad I_T = \frac{s - P}{\rho \sigma_s^2},
\]

and \( I_0 = I_E \) (see the Appendix). Substitution into the market clearing condition for the risky asset yields

\[
P = \frac{L - L_E s}{\rho \sigma_s^2} + \frac{\frac{L_E}{\rho \text{var}(\theta|P)} E(\theta|P) - \frac{L_E}{\rho \text{var}(\theta|w)} E(\theta|w) - \frac{L_E}{\rho \text{var}(\theta|w)} }{\frac{L - L_E}{\rho \sigma_s^2} + \frac{L_E}{\rho \text{var}(\theta|P)} + \frac{L_E}{\rho \text{var}(\theta|w)}},
\]

where

\[
w = \frac{L - L_E}{\rho \sigma_s^2} s + N.
\]

From the updating rule for the mean of a normal random variable,

\[
E(\theta|w) = \bar{s} + \frac{\text{cov}(\theta, w)}{\text{var}(w)} [w - E(w)].
\]

Another important consequence of normality is that \( \text{var}(\theta|w) \) is non-random: from \( \text{var}(\theta|w) = \text{var}(s|w) + \sigma_s^2 \) and the updating rule \( \text{var}(s|w) = \sigma_s^2 - [\text{cov}(s, w)]^2/\text{var}(w) \), it follows that

\[
\text{var}(\theta|w) = \sigma_s^2 - \frac{[\text{cov}(s, w)]^2}{\text{var}(w)} + \sigma_s^2.
\]

---

\(^{10}\)Philippon and Reshef (2012, Section V) challenge the condition that expected utilities equalize at an equilibrium with incomplete specialization. They argue that the striking pay rise in the financial sector starting in the 1990s has been such that expected utility is higher in that sector. Bolton et al. (2014) provide an explanation for high pay in finance: agents differ with regard to the cost of becoming a dealer, so all dealers except the marginal one get higher expected utility than entrepreneurs. Oyer (2008, pp. 2620–2621), by contrast, holds that “the IB [investment banking] pay premium is a compensating differential for the type of work”. Recent headlines emphasize severe competition for engineers, which may cause a readjustment of high-talent agents’ compensation in finance and other sectors.
From (4) and (5), the equilibrium price $P$ in (2) is a linear function of $w$.

**Expected utilities**

An entrepreneur’s expected utility conditional on $P$ is

$$E[U(\pi_E)|P] = -\exp(-\rho e) \exp \left(-\frac{P}{\rho a} - z^2 \right),$$

where

$$z \equiv \frac{E(\theta|w) - P}{[2 \text{var}(\theta|w)]^{1/2}} \tag{6}$$

is the ratio of expected payoff to risk, conditional on $w$, for financial investment (see the Appendix). From (2)–(5), $z$ is a linear function of $w$. It can be shown that the linear dependence is negative (see the Appendix), so that $\text{cov}(P,z) = -[\text{var}(P) \text{var}(z)]^{1/2}$. Using the law of iterated expectations and Lemma 1 in Demange and Laroque (1995, p. 252), we obtain the following expression for an entrepreneur’s unconditional expected utility:

$$-\log\{-E[U(\pi_E)]\} = \rho e + \frac{\rho}{a} \left[ E(P) - \frac{\rho}{2a} \text{var}(P) \right]$$

$$+ \frac{\left[ E(z) - \frac{\rho}{2a} \text{cov}(P,z) \right]^2}{1 + 2 \text{var}(z)} + \frac{1}{2} \log \left[ 1 + 2 \text{var}(z) \right] \tag{7}$$

(see the Appendix). For the sake of convenience, we often call $-\log\{-E[U(\pi)]\}$ “expected utility” in what follows. If agents merely stored and consumed their endowment $e$, their expected utility would be given by $\rho e$. If entrepreneurs sold the $1/a$ initial ownership share of their firm and carried out no further financial transactions, they would get extra expected utility $-\log\{-E[U(e + P/a)]\} - \rho e = (\rho/a)[E(P) - \rho/(2a) \text{var}(P)] \equiv \text{GP}$. These “gains from production” are uniquely determined by the first two moments of the random asset price $P$. Define the additional terms in (7) as the “gains from trading” for entrepreneurs $\text{GT}_E$. $\text{GT}_E$ reflects the marginal impact of an entrepreneur’s trade in the stock market on his expected utility. $\text{GT}_E$ depends on the first two moments of $z$ and on its covariance with the $P$. This covariance matters because changes in $w$ (linearly) affect both the price $P$ at which entrepreneurs sell their firms and the expected payoff-risk ratio $z$. This effect is not present in GS, where agents are not engaged in entrepreneurial activity, and makes the application of the lemma from Demange and Laroque (1995) necessary.

An inactive agent’s unconditional expected utility is obtained analogously:

$$-\log\{-E[U(\pi_0)]\} = \rho e + \frac{[E(z)]^2}{1 + 2 \text{var}(z)} + \frac{1}{2} \log \left[ 1 + 2 \text{var}(z) \right].$$

This is (7) without the $P/a$ terms, which result from entrepreneurs’ sales of ownership shares in their firms. The final two terms in the sum on the right-hand, $\text{GT}_0$ say, give the inactive agent’s
gains from trading. As \( \text{cov}(P, z) \) is negative, \( GT_E > GT_0 \) whenever \( E(z) \geq 0 \) (as seen below, this condition is satisfied if \( N \leq L/a \), i.e., if noise trader demand does not exceed total asset supply). Under this condition, even though entrepreneurs trade on the same information as inactive agents, they derive greater benefits from their trades, since fluctuations in \( z \) provide a hedge against the entrepreneurial risk they carry.

A trader’s expected utility conditional on \( P \) is

\[
E[U(\pi_T)|P] = -\exp(-\rho \epsilon) \left[ \frac{\sigma^2}{\text{var}(\theta|w)} \right]^{\frac{1}{2}} \exp(-z^2).
\]

Using the law of iterated expectations, it follows that

\[
-\log(-E[U(\pi_T)]) = \rho \epsilon + \frac{1}{2} \log \left[ \frac{\text{var}(\theta|w)}{\sigma^2} \right] + \frac{[E(z)]^2}{1 + 2 \text{var}(z)} + \frac{1}{2} \log \left[ 1 + 2 \text{var}(z) \right]
\]

(see the Appendix). The sum on the right-hand side can be rewritten as \( \rho \epsilon + GI + GT_0 \), where \( GI = (1/2) \log[\text{var}(\theta|w)/\sigma^2] \) represents the “gains from being informed”, i.e., having information about \( s \) (\( GI \geq 0 \), since \( \text{var}(\theta|w) \geq \sigma^2 \)).

4 Equilibrium with noise

This section analyzes \( M(\sigma^2_N, 1) \) for \( \sigma^2_N > 0 \), i.e., the model with random noise trader demand and OC. At the end of the section we briefly consider the model without OC \( M(\sigma^2_N, 0) \).

Occupational choice

From (7) and (8), the unconditional expected utility of an entrepreneur is no less than the unconditional expected utility of a trader (i.e., \( E(U(\pi_E)) \geq E(U(\pi_T)) \)) exactly if

\[
d - \Delta(L_E) = \frac{\frac{\epsilon}{a} \text{cov}(P, z) \left[ \frac{\epsilon}{a} \text{cov}(P, z) - 2E(z) \right]}{1 + 2 \text{var}(z)} + \frac{\rho}{a} \left[ E(P) - \frac{\rho}{2a} \text{var}(P) \right] \geq \frac{1}{2} \log \left[ \frac{\text{var}(\theta|w)}{\sigma^2} \right].
\]

(9)

Equation (9) says that, compared to staying inactive, becoming an entrepreneur is no less attractive than becoming a trader: \( (GP + GT_E) - GT_0 \geq (GI + GT_0) - GT_0 \). \( GI \geq 0 \) implies that \( GP + GT_E \geq GT_0 \) if (9) holds. That is, if despite the benefits of being informed, agents are no better-off as traders than as entrepreneurs, then they are certainly no better-off by staying inactive. Equations (2)–(5) determine the moments and the covariance of \( P \) and \( z \) as continuous functions of \( L_E \) alone (closed-form solutions are in the Appendix). Denote the composite function obtained from substituting these functions into the left-hand side of (9) as \( \Delta(L_E) \). From (3) and (5), \( \text{var}(\theta|w) \) is also a
continuous function of \( L_E \) alone. Denote the function resulting from substituting this function into the right-hand side of (9) as \( \Gamma(L_E) \). Since \( L_E \) also uniquely determines \( I_E, I_T, \) and \( P \) via (1) and (2), we have:

**Proposition 4.1:** Let \( \sigma_N^2 > 0 \). Then \((L_E, I_E, I_T, P)\) is an equilibrium of \( M(\sigma_N^2, 1) \) if \( 0 < L_E \leq L, \Delta(L_E) = \Gamma(L_E), I_E \) and \( I_T \) satisfy (1), and \( P \) satisfies (2); or if \( L_E = L, \Delta(L) \geq \Gamma(L), I_E \) satisfies (1), and \( P \) satisfies (2).

There is no opportunity cost of becoming informed compared to staying inactive (corresponding to “c” in GS). Since \( \Delta(L_E) = (GP + GT_E) - GT_0 \) is the difference in the expected utilities of entrepreneurs and inactive agents, \((L_E, I_E, I_T, P)\) would also be an equilibrium if we introduced a cost of not being inactive no greater than \( \Delta(L_E) \) evaluated at the equilibrium of \( M(\sigma_N^2, 1) \).

Our focus on equilibria with a positive mass of entrepreneurs \( L_E \) requires that \((GP + GT_E) - GT_0 \) exceeds \( 0 \) for \( L_E = 0 \), i.e., \( \Delta(0) > \Gamma(0) \). Together with continuity of \( \Delta(L_E) \) and \( \Gamma(L_E) \), this condition implies that either there is \( L_E < L \) such that \( \Delta(L_E) = \Gamma(L_E) \) or \( \Delta(L) \geq \Gamma(L) \). Hence, from Proposition 4.1, it follows immediately that:

**Proposition 4.2:** Let \( \sigma_N^2 > 0 \). If \( \Delta(0) > \Gamma(0) \), an equilibrium of \( M(\sigma_N^2, 1) \) with \( L_E > 0 \) exists.

The two types of equilibria are illustrated in Figure 1. The filled circles indicate the equilibrium difference in the expected utilities of entrepreneurs and inactive agents \((GP + GT_E) - GT_0 \).

**No occupational choice**

In the absence of an OC decision (i.e., in model \( M(\sigma_N^2, 0) \) with \( \sigma_N^2 > 0 \)), \( L_E, I_T, \) and the condition...
that OC is made optimally (9) drop out of the definition of equilibrium \((I_E, P)\). Since no-one gathers information about \(s\), the price is uninformative: \(E(\theta|P) = \bar{s}\) and \(\text{var}(\theta|P) = \sigma_s^2 + \sigma_e^2\).

Entrepreneurs’ optimal investment level in (1) is

\[
I_E = \frac{\bar{s} - P}{\rho(\sigma_s^2 + \sigma_e^2)},
\]

(10)

and the price function (2) becomes

\[
P = \bar{s} - \frac{\rho(\sigma_s^2 + \sigma_e^2)}{L} \left( \frac{L}{a} - N \right).
\]

(11)

Since (9) is not a condition of equilibrium, we have to make sure that entrepreneurs are no worse-off than inactive agents: \(E[U(\pi_E)] \geq E[U(\pi_0)]\) or \(\text{GP} + \text{GT}_E \geq \text{GT}_0\). As \(\Delta(L_E)\) is defined as \((\text{GP} + \text{GT}_E) - \text{GT}_0\), this amounts to \(\Delta(L) \geq 0\).

**PROPOSITION 4.3:** Let \(\sigma_N^2 > 0\). \(I_E\) given by (10) and \(P\) given by (11) are the unique equilibrium of \(M(\sigma_N^2, 0)\) exactly if \(\Delta(L) \geq 0\).

Uniqueness follows from the fact that \(\Delta(L_E)\) is strictly decreasing (see the Appendix). The equilibrium difference in the expected utilities of entrepreneurs and inactive agents \((\text{GP} + \text{GT}_E) - \text{GT}_0\) is illustrated by the circles in Figure 1.

### 5 Equilibrium with no noise

This section analyzes the case of non-random noise trader demand: \(N = \bar{N}\) and \(\sigma_N^2 = 0\). Again, we start with the version of the model with OC, i.e., model \(M(0, 1)\). As pointed out by GS, the subcases with and without informed traders have to be treated separately.

**Occupational choice**

We continue to focus on equilibria with a positive mass of entrepreneurs (i.e., \(L_E > 0\)). To begin with, suppose further that a subset of rational agents with positive mass decide to become traders (i.e., \(L_E < L\)). In this case, the results derived in the preceding section go through unchanged.

Given \(N = \bar{N}\), \(w\) defined in (3) fully reveals \(s\) to entrepreneurs. From (3) and (4), \(E(\theta|w) = s\). From (5), \(\text{var}(\theta|w) = \sigma_e^2\). So \(I_E\) equals \(I_T\), as given by (1). From the market clearing condition for the risky asset (2),

\[
P = s - \rho \frac{\sigma_e^2}{L} \left( \frac{L_E}{a} - \bar{N} \right).
\]

(12)

A higher price discount \(s - P = \rho \sigma_e^2 I_T\) is required to compensate agents for the risk of a larger investment position \(I_T\), so the equilibrium price is a decreasing function of the equilibrium amount
Figure 2: Equilibrium with no noise

of assets held by entrepreneurs and traders $L_E/a - N$ (the part of asset supply not held by noise traders). From (6), since $E(\theta|w) - P = s - P$ is non-random, $z$ is non-random, even though both $s$ and $P$ are risky. In fact, from (6) and (12),

$$z = \left(\frac{\rho^2}{2}\right)^\frac{1}{2} \frac{\rho}{L} \left(\frac{L_E}{a} - \bar{N}\right).$$

(13)

As cov$(P,z) = 0$, the gains from trade are identical for all agents (i.e., $G_T = G_T = 0$) and there is no benefit from being informed (i.e., $G_I = 0$). At an equilibrium with a positive mass of traders, (9) must hold with equality: $(GP + GT) - GT = G_I$. Hence, using $GP = (\rho/a)[E(P) - \rho/(2a) \text{var}(P)] = 0$, $G_T = G_T = 0$, $G_I = 0$, and (12),

$$\frac{\rho}{a} \left[ \tilde{s} - \frac{\rho \sigma^2}{L} \left(\frac{L_E}{a} - \bar{N}\right) - \frac{\rho \sigma^2}{2a} \right] = 0$$

(14)

at an equilibrium with $0 < L_E < L$. The left-hand side of (14) maps $L_E$ in the interval $[0, L]$ on the reals. Denote this mapping as $\Delta_0(L_E)$. Then we have:

**PROPOSITION 5.1:** If $\Delta_0(0) > 0$ and $\Delta_0(L) < 0$ for $L$ close to $L$, then $L_E$ determined by $\Delta_0(L_E) = 0$, $I_E = I_T$, $I_T$ given by (1), and $P$ given by (12) are the unique equilibrium of $M(0,1)$ with $L_E < L$. Otherwise an equilibrium of $M(0,1)$ with $L_E < L$ does not exist.

Equilibrium with no noise is illustrated in Figure 2. The downward-sloping function $\Delta_0(L_E)$ gives the expected utility of an entrepreneur relative to being inactive $GP$. The expected utility differential for a trader compared to an inactive agent $G_I$ is zero (see the filled circle in Figure 2).
The conditions $\Delta_0(0) > 0$ and $\Delta_0(L_E) < 0$ for $L_E$ close to $L$ are equivalent to $\bar{N}_1 < \bar{N}$ and $\bar{N} < \bar{N}_2$, respectively, where

$$\bar{N}_1 = -\frac{L}{\rho \sigma_z^2} \left( \bar{s} - \rho \sigma_s^2 \right), \quad \bar{N}_2 = \frac{L}{a} \left( \bar{s} - \frac{\rho \sigma_s^2}{2a} \right).$$

The fact that a fully revealing REE possibly exists, even though $G_l = 0$, is due to the fact that the only cost of becoming a trader is the opportunity cost of not becoming an entrepreneur. The GS non-existence result (non-existence of an equilibrium if there is no noise and the cost of becoming a trader is sufficiently small) would re-arise if there were a positive cost of not staying inactive.

The focus on equilibria at which no agent stays inactive is without loss of generality. Though each entrepreneur would be no worse-off if he chose to stay inactive, the equilibrium mass of entrepreneurs $L_E$ (and, hence, equilibrium $(L_E, P, I_E, I_T)$) is uniquely determined by condition (14) that $G_P = 0$. There is one indeterminacy: the mass of traders can be anywhere in the interval $(0, L - L_E]$ (the remaining agents staying inactive), since the price is fully informative for any positive value. The fact that (14) uniquely determines $L_E$ also implies that there cannot be an equilibrium with $0 < L_E < L$ and a positive mass of inactive agents for $\bar{N} \leq \bar{N}_1$ or $\bar{N} \geq \bar{N}_2$.

**No traders**

If no-one gathers information (i.e., $L_E = L$), the asset price is uninformative: $w = N$, $E(\theta \mid w) = \bar{s}$, and $\text{var}(\theta \mid w) = \sigma_s^2 + \sigma_z^2$. Agents’ asset demand is given by (10), and the equilibrium asset price is given by (11) with $N = \bar{N}$:

$$\frac{\rho}{a} P = \frac{\rho}{a} \left( \bar{s} - \frac{\rho (\sigma_s^2 + \sigma_z^2)}{L} \left( \frac{L}{a} - \bar{N} \right) \right).$$

As $P$ is non-random, an entrepreneur’s expected utility (7) is

$$- \log\{-E[U(\pi_E)]\} = \rho c + \rho P + z^2,$$

where, from (6),

$$z = \frac{\bar{s} - P}{\left[2(\sigma_s^2 + \sigma_z^2)\right]^{\frac{1}{2}}}. $$

An inactive agent’s expected utility is $- \log\{-E[U(\pi_0)]\} = \rho c + z^2$. The gains from trading are identical for entrepreneurs and inactive agents: $G_T = G_T = z^2$. Entrepreneurs’ gains from production $G_P$ are given by (16). For future reference, we denote the right-hand side of (16) as $\Delta_0(L)$.

In order for an equilibrium with no traders to prevail, a single agent must not have an incentive to become a trader. A single agent who decides to become a trader observes $s$ and invests $I_T =$
\( (s - P) / (\rho \sigma^2) \), where \( P \) is given by (16), since a single agent’s asset demand is negligible and, therefore, does not affect the asset price. His unconditional expected utility is given by

\[
-\log\{-E[U(\pi_T)]\} = \rho e + \frac{1}{2} \log \left( \frac{\sigma^2 + \alpha^2}{\sigma^2} \right) + z^2
\]

(see the Appendix). In order for equilibrium to prevail, it must not pay for an individual agent to become a trader. That is, \( GP = \Delta_0(L) \) must be no less than \( GI = (1/2) \log[(\sigma^2 + \alpha^2)/\sigma^2] \). Denote this latter expression as \( \Gamma_0(L) \). Then we have:

**Proposition 5.2:** If \( \Delta_0(L) \geq \Gamma_0(L) \), then \( L_E = L \), \( I_E \) given by (10), \( I_T = I_E \), and \( P \) given by (16) are the unique equilibrium of \( M(0,1) \) with \( L_E = L \). Otherwise an equilibrium of \( M(0,1) \) with \( L_E = L \) does not exist.

The assertion of Proposition 5.2 is similar as in GS, where an equilibrium without traders exists if the (exogenous) cost of information is sufficiently large. Here, the opportunity cost of trading, i.e., of setting up a firm, has to be sufficiently large in order for an equilibrium to exist. Evidently, this requires \( P > 0 \), so that entrepreneurs are strictly better-off than if they stayed inactive: \( GP = (\rho/a)P \). Other than an equilibrium with \( L_E < L \), this type of equilibrium would survive the introduction of a sufficiently small positive cost of not being inactive.

Again, the focus on equilibria without inactive agents is without loss of generality: whenever there are no traders, \( GI \) is positive, so inactive agents can do better by becoming traders.

**Multiple equilibria**

The conditions of Propositions 5.1 and 5.2 provide necessary and sufficient conditions for the existence of equilibrium with \( L_E < L \) and \( L_E = L \), respectively. The condition of Proposition 5.2 can be stated equivalently as \( \bar{N} \geq \bar{N}_3 \), where \( \bar{N}_3 \) satisfies \( \Delta_0(L) = \Gamma_0(L) \). An interesting corollary of Propositions 5.1 and 5.2 is that, as \( \bar{N}_2 \) and \( \bar{N}_3 \) do not generally coincide, multiplicity or non-existence of equilibrium arises for \( \bar{N} \) in the vicinity of \( \bar{N}_2 \):

**Proposition 5.3:** Let \( \bar{N}_1 < \bar{N} < \bar{N}_2 \). If \( \bar{N}_3 < \bar{N}_2 \), then an equilibrium with \( L_E < L \) and an equilibrium with \( L_E = L \) co-exist for \( \bar{N}_3 < \bar{N} < \bar{N}_2 \) in \( M(0,1) \). If \( \bar{N}_2 < \bar{N}_3 \), an equilibrium of \( M(0,1) \) does not exist for \( \bar{N}_2 < \bar{N} < \bar{N}_3 \).

The two cases are illustrated in Figure 3. The cause of multiplicity or non-existence is that an increase in \( \bar{N} \) from slightly below \( \bar{N}_2 \) to \( \bar{N} = \bar{N}_2 \) has discontinuous effects on entrepreneurs’ and traders’ expected utilities, each of which may be stronger than the other. To see this, recall that

\[
-\log\{-E[U(\pi_E)]\} + \log\{-E[U(\pi_T)]\} = GP - GI \text{ both for } L_E < L \text{ and for } L_E = L, \text{ since}
\]
cov\( (P, z) = 0 \) and, hence, \( GT_E = GT_0 \) in either case. For \( L_E < L \), both \( GP \) and \( Gl \) equal zero, so that agents are indifferent between entrepreneurship and finance: \( GP - Gl = 0 \). For \( \bar{N} = \bar{N}_2 \), \( GP \) jumps to the value \( \Delta_0(L) \) and \( Gl \) jumps to the value \( \Gamma_0(L) \). Whether multiple equilibria exist or no equilibrium at all depends on whether the difference \( GP - Gl \) is positive or negative, respectively.

No occupational choice

In the absence of an OC decision, i.e., in model \( M(0, 0), L_E, I_T \), and the condition that OC is made optimally drop out of the definition of equilibrium. We have to impose the condition that agents fare no worse as entrepreneurs than if they stayed inactive, i.e., that the (non-random) firm value \( P \) in (16) is non-negative. Let \( \bar{N}_4 \) be the \( \bar{N} \)-value that satisfies (16) for \( P = 0 \):

\[
\bar{N}_4 = \frac{L}{a} - \frac{L}{\rho(\sigma^2_d + \sigma^2_\varepsilon)^s} \quad \text{(19)}
\]

(\( \bar{N}_3 \)). From (16), \( \bar{N} \geq \bar{N}_4 \) is equivalent to \( \Delta_0(L) \geq 0 \).

**Proposition 5.4:** If \( \bar{N} \geq \bar{N}_4 \), then \( I_E \) given by (10) and \( P \) given by (16) are the unique equilibrium of \( M(0, 0) \). Otherwise an equilibrium with \( L_E = L \) does not exist.

### 6 Does finance attract too much talent?

We now turn to the question of whether finance attracts too much talent, at the expense of the non-financial sector. To begin with, we investigate whether rational agents are worse-off at an equilibrium with OC and \( L_E < L \) than at equilibrium without OC. If so, then policy measures which make trading unprofitable raise agents’ expected utility. One might object that rational
agents’ gains from not having the choice to become traders possibly come at noise traders’ expense. We discuss this issue later in this section. Anyway, it appears interesting to see that giving rational agents the opportunity to become traders makes them worse-off at equilibrium.

No noise

Consider first the case of no noise trader shocks. The virtue of this case is that it allows analytical results. We compare the equilibria of economies $M(0, 1)$ and $M(0, 0)$. Since eliminating the opportunity to become a trader is ineffective if no-one chooses to do so anyway, suppose there is an equilibrium of $M(0, 1)$ with $L_E < L$. From the remarks to Proposition 5.1, this requires $N_1 < N < N_2$. In order to address the question of whether agents are better-off with no trading, assume further that an equilibrium of $M(0, 0)$ with $L_E < L$ exists. From Proposition 5.4, this requires $\bar{N}_1 \leq \bar{N}_4$. Since there is no noise, $P$ reveals $s$, so there is no gain from being informed, and traders’ expected utility is $GT_0$. Since $z$ is safe, $GT_E$ also equals $GT_0$, and entrepreneurs’ expected utility is $GP + GT_0$. $L_E < L$ implies that this is no greater than traders’ expected utility, so $GP = 0$. Hence, from (13),

$$-\log\{-\mathbb{E}[U(\pi_E)]\} = \rho e + \frac{\sigma^2}{2} \left[ \frac{\rho}{L} \left( \frac{L_E}{a} - \bar{N} \right) \right]^2,$$

(20)

and the right-hand side also gives traders’ expected utility.

At an equilibrium of $M(0, 0)$, from (16)–(18),

$$-\log\{-\mathbb{E}[U(\pi_E)]\} = \rho e + \frac{\rho}{a} \left[ \tilde{s} - \rho \left( \frac{\sigma^2}{L} \right) \left( \frac{L}{a} - \bar{N} \right) \right] + \frac{\sigma^2}{2} \left[ \frac{\rho}{L} \left( \frac{L}{a} - \bar{N} \right) \right]^2.$$

(21)

The second and third terms in the sum on the right-hand side are $GP$ and $GT_E (= z^2)$, which, from (16) and (18), are both non-random.

**PROPOSITION 6.1:** Let $\bar{N}_1 < N < \bar{N}_2$, so that an equilibrium of $M(0, 1)$ with $L_E < L$ exists, and $\bar{N} \geq \bar{N}_4$, so that an equilibrium of $M(0, 0)$ with $L_E = L$ exists. If $L_E/a > \bar{N}$, then rational agents’ expected utility is lower at the equilibrium of $M(0, 1)$ than at the equilibrium of $M(0, 0)$.

**Proof:** If $L_E/a > \bar{N}$, then the final terms in brackets in (20) and (21) are positive, so the square is greater in (21) than in (20). $\bar{N} \geq \bar{N}_4$ implies that the second term in the sum on the right-hand side of (21) is non-negative. This proves the proposition. ||

Thus, in the case of no noise, the allocation of talent to trading is excessive whenever rational agents have a long position in the risky asset (i.e., if noise trader demand $\bar{N}$ falls short of supply $L_E/a$).
We have $\Delta \sigma$ value on the right-hand side of (20) for Figure 4. Since entrepreneurs' expected utility (7) is continuous in $L$, there is an equilibrium such that $\Delta \sigma L$ is a corresponding equilibrium for the value such that $\Delta \sigma L = 0$, i.e., the equilibrium value of $M(0, 1)$. From Proposition 4.1, there is a corresponding equilibrium $(L_E, I_E, I_T, P)$ with $L_E < L$ (illustrated by the lower left filled circle in Figure 4). Since entrepreneurs’ expected utility (7) is continuous in $L_E$, it is arbitrarily close to the value on the right-hand side of (20) for $\sigma_N^2$ sufficiently small.

We proceed to show that the presence of OC generally makes agents worse-off if $0 < L_E < L$ and they do not go short (i.e., $L_E/a > \bar{N}$) at equilibrium for $\sigma_N^2$ sufficiently small. This is because a small increase in $\sigma_N^2$ starting from $\sigma_N^2 = 0$ has a small impact on equilibrium, so the conclusion follows from Proposition 6.1.

**PROPOSITION 6.2:** Let $\bar{N}_1 < \bar{N}_2 < \bar{N}_3$, and $\bar{N}_3 > \bar{N}_4$. Then for $\sigma_N^2$ positive but sufficiently small, an equilibrium of $M(\sigma_N^2, 1)$ with $L_E$ close to the equilibrium value for $M(0, 1)$ and an equilibrium of $M(\sigma_N^2, 0)$ exist. If $L_E/a > \bar{N}$, then rational agents’ expected utility is lower at the equilibrium of $M(\sigma_N^2, 1)$ than at the equilibrium of $M(\sigma_N^2, 0)$.

**Proof:** For $L_E < L$, $\Delta(L_E)$ converges pointwise to $\Delta_0(L_E)$ and $\Gamma(L_E)$ converges pointwise to zero as $\sigma_N^2 \to 0$ (see the Appendix). Therefore, there is $L_E$ such that $\Delta(L_E) = \Gamma(L_E)$ in the vicinity of the value such that $\Delta_0(L_E) = 0$, i.e., the equilibrium value of $M(0, 1)$. From Proposition 4.1, there is a corresponding equilibrium $(L_E, I_E, I_T, P)$ with $L_E < L$ (illustrated by the lower left filled circle in Figure 4). Since entrepreneurs’ expected utility (7) is continuous in $L_E$, it is arbitrarily close to the value on the right-hand side of (20) for $\sigma_N^2$ sufficiently small.

We have $\Delta(L) \to \Delta_0(L)$ as $\sigma_N^2 \to 0$ (see the Appendix). The assumption $\bar{N} > \bar{N}_4$ implies $\Delta_0(L) >$
0. So $\Delta(L) > 0$ for $\sigma_N^2$ small enough. Existence of an equilibrium of $M(\sigma_N^2, 0)$ then follows from Proposition 4.3 (see the unfilled circle in Figure 4). Since entrepreneurs’ expected utility (7) is continuous in $L_E$, it is arbitrarily close to the value on the right-hand side of (21) for $\sigma_N^2$ sufficiently small.

The validity of the proposition then follows from Proposition 6.1.

**Multiple equilibria**

From Proposition 5.3, in the absence of noise, an equilibrium with $L_E < L$ and an equilibrium with $L_E = L$ coexist if $\bar{N}_1 < \bar{N} < \bar{N}_2$ and $\bar{N}_3 < \bar{N} < \bar{N}_2$. From Proposition 6.2, for $\sigma_N^2$ positive but small enough, there is an equilibrium with $L_E$ similar as at former the equilibrium. If in addition $\bar{N} > \bar{N}_4$, then expected utility is lower than without OC.

**PROPOSITION 6.3:** Let the conditions of Propositions 5.3 and 6.2 hold. Then for $\sigma_N^2$ positive but sufficiently small, there is an equilibrium with $L_E$ close to $L$, and rational agents’ expected utility at this equilibrium is lower than without OC.

**Proof:** Since $\Delta(L_E)$ converges pointwise to $\Delta_0(L_E)$ as $\sigma_N^2 \to 0$ for $L_E < L$, it becomes negative as $L_E$ rises beyond the equilibrium value in Proposition 6.2. For $L_E \to L$, it rises steeply towards $\Delta_0(L)$ (see the Appendix). $\Gamma(L_E)$ converges pointwise to zero as $\sigma_N^2 \to 0$ and then rises steeply to $\Gamma_0(L)$ (see the Appendix). $\bar{N} > \bar{N}_3$ implies $\Delta_0(L) > \Gamma_0(L) > 0$. So for $\sigma_N^2$ positive but sufficiently small, there is $L_E$ slightly below $L$ such that $\Delta(L_E) = \Gamma(L_E)$ (see Figure 4). From Proposition 4.1, there is a corresponding equilibrium $(L_E, I_E, I_T, P)$.

$\Delta'(L_E)$ grows arbitrarily large for $L_E$ slightly below $L$ as $\sigma_N^2 \to 0$ and dominates the other terms in $-\log\{-E[U(\pi_E)]\} = \Delta(L_E) + \rho c + [E(z)]^2/[1 + 2 \text{var}(z)] + (1/2) \log[1 + 2 \text{var}(z)]$. |
Large noise trader shocks

The closed-form solutions for all model variables as functions of \( L_E \) in the Appendix allow numerical analysis of the welfare effects of trading for large noise trader shocks. Numerical experimentation shows that the result that the financial sector is too big is not restricted to the analytically tractable case of small noise trader shocks. As seen in the preceding paragraph, the advantageousness of no OC is due to the fact that \(-\log\{-E[U(\pi_E)]\}\) is a (skewed) U-shaped function of \( L_E \) under the conditions of Proposition 6.2 and takes on a larger value at the right endpoint than at the equilibrium value(s) with OC. If expected utility at \( L_E = L \) exceeds expected utility at the \( L_E \)-value that solves (14) by a large amount, then the absence of OC remains favorable for sizable \( \sigma^2_N \)-values. Figure 5 below illustrates an example.\(^{11}\) The fact that the absence of OC is favorable for rational agents in this example can be seen immediately on the basis of the four curves in the first quadrant.

![Figure 5: Expected utilities with and without OC (numerical example)](image)

Noise trader consumption

So far we have confined attention to the impact of trading on the welfare of rational agents. So one can object that welfare gains for rational agents might come at the noise traders’ expense. To

\(^{11}\)Parameter values: \( \bar{s} = 10, \rho = 0.1, L = 10, \bar{M} = 11, \sigma^2_s = 100, \sigma^2_T = 16, \bar{N} = 2, \sigma^2_N = 9, a = 1. \) UE is the graph for the entrepreneurs, UT the graph for the traders, UI the graph for the inactive agents and UW the graph for the noise traders. With OC, equilibrium \( L_E \) is approximately 90% of \( L \), without OC it is \( L_E = L \). In the equilibrium with OC, it holds that \( E(\pi_E) \approx 9.3, E(V(\pi_E|P)) \approx 13.4 \) and \( E(\pi_T) \approx 13.7, E(V(\pi_T|\pi_E,P)) \approx 136.6 \). Ratios similar to the Sharpe ratio: \( \frac{E(\pi_E)}{\sqrt{E(V(\pi_E|P))}} \approx 2.54 \) and \( \frac{E(\pi_T)}{\sqrt{E(V(\pi_T|\pi_E,P))}} \approx 1.17.\)
discuss this issue, we have to say something about noise traders’ well-being, which contrasts with the maintained assumption that their trading behavior is not derived from maximization of a utility function. So the following remarks on noise trader welfare have to be interpreted with appropriate caution.

As a first shot, consider the impact of trading on the expected return and volatility of noise traders’ net payoff on their investments. Denote the mass of noise traders as $\bar{M}$, and suppose they are all alike, so that each invests $I_{\bar{M}} = \bar{N}/\bar{M}$ in stocks and makes a net payoff $(\theta - P)I_{\bar{M}}$. There is one simple special case in which noise traders are apparently better-off without OC, viz., when aggregate noise trader demand is the “sum” of a large mass $\bar{M}$ of small individual demands $I_{\bar{M}}$ and an increase in $L_E$ raises the expected net payoff per unit invested $E(\theta - P)I_{\bar{M}}$. This follows immediately from the fact that var$[(\theta - P)I_{\bar{M}}]$ is quadratic in $I_{\bar{M}}$. So as $(M$ grows large and) $I_{\bar{M}}$ goes to zero, the impact of a change in $L_E$ on the payoff variance vanishes, and noise traders benefit from the positive (linear) impact on $E(\theta - P)I_{\bar{M}}$. For instance, with non-stochastic noise trader demand (i.e., $\sigma_N^2 = 0$), the mean and variance of $E(\theta - P)I_{\bar{M}}$ go up by

$$\frac{\rho}{L} \left[ \sigma_n^2 \left( \frac{L}{a} - \bar{N} \right) + \sigma_z^2 \frac{L - L_E}{a} \right] I_{\bar{M}}$$

and $\sigma_z^2 I_{\bar{M}}^2$, respectively, when there is no OC (see the Appendix). The former effect is positive if both rational agents and noise traders have a long position in the asset (i.e., $L/a > \bar{N} > 0$). For $I_{\bar{M}} \to 0$, it dominates the variance effect.

Conversely, the quadratic variance effect grows faster than the linear effect of changes in $L_E$ on noise traders’ mean net payoff as $I_{\bar{M}}$ grows large. So if there are only “a few” noise traders with large investment positions $I_{\bar{M}} = \bar{N}/\bar{M}$, then they the increase in asset price volatility associated with no OC has a strong impact on the variance of their net payoff.

**Noise trader welfare**

In order to discuss aggregate welfare effects of financial trading, one has to impose a utility function for noise traders. Given that noise traders’ asset demand is exogenous, one has to assume further that noise traders are driven by “animal spirits”, rather than the objective to maximize utility, when they determine their demand for assets. Shleifer and Summers (1990, p. 23) advocate the significance of this approach:

> “not all demand changes appear to be so rational; some seem to be a response to changes in expectations or sentiment that are not fully justified by information. Such changes can be a response to pseudo-signals that investors believe convey information about future returns but that would not convey such information in a fully rational model
Changes in demand can also reflect investors’ use of inflexible trading strategies or of ‘popular models’.

The limitations of assuming that noise traders have a utility function, which they do not maximize early but which is used to evaluate their expected welfare, are evident. So the results in this paragraph have to be interpreted with appropriate caution (also bearing in mind that the most important externality of entrepreneurial activity is not the reduction of employment in finance but the creation of jobs; cf. Section 8).

Suppose noise traders have the same utility function $U = - \exp(-\rho \pi)$ as rational agents. The difference in the (transformed) expected utilities at an equilibrium without OC compared to an equilibrium with OC and $L_E < L$ is

$$\rho^2 \frac{N}{M} \left\{ \frac{1}{L} \left[ \sigma_s^2 \left( \frac{L}{a} - \bar{N} \right) + \sigma_e^2 \frac{L - L_E}{a} \right] - \frac{1}{2M} \sigma_s^2 \right\}$$

(see the Appendix).

One simple sufficient condition for gains: $L_E/a > \bar{N}$ and $\bar{M}$ large enough.

Another simple sufficient condition: $L_E/a > \bar{N}$, $L/a > (3/2)\bar{N}$, and $\bar{M} > L$.

$L_E/a > \bar{N}$ and $\sigma_s^2/\sigma_e^2$ small enough

**PROPOSITION 6.4**: Let $\bar{N}_1 < \bar{N} < \bar{N}_2$, so that an equilibrium of $M(0,1)$ with $L_E < L$ exists, and $\bar{N} \geq \bar{N}_4$, so that an equilibrium of $M(0,0)$ with $L_E = L$ exists. If $\frac{2L}{3\pi} > \bar{N}$, then noise traders’ expected utility is lower at the equilibrium of $M(0,1)$ than at the equilibrium of $M(0,0)$.

With (??) one can see that noise traders make higher expected profit without OC for two reasons: First, they are demanding assets ($\bar{N} > 0$) and a higher number of firms without OC increases asset supply and thus lowers the asset price. Second, because no more information about $s$ are known without OC, holding the asset becomes more risky and thus entrepreneurs want to sell shares of their firm more badly, which also lowers the asset price. However, because of the additional variance in $s$, profit variance also increases and this hurts noise traders’ expected utility. The netto-effect is clearly positive for $\bar{N} < \frac{2L}{3\pi}$ and might stay positive even for other cases.

**Small noise trader shocks**

Analogous to the analysis of the rational agents’ welfare, we now show that the assertion of Proposition 6.1* also holds for $\sigma_N^2 \rightarrow 0$. Therefor we first need to determine the noise traders’ expected utility.

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12 The implied limitations of the approach for welfare statements has motivated models in which noise trader demand is derived from stochastic liquidity needs or portfolio churning by asset managers (see Dow and Gorton, 2008).
utility for the general case of $\sigma_N^2 > 0$. Using the LIE, (??) and some algebra gets us (see the Appendix)

$$-\log [-E (u (\pi_M))] = \rho e + 0.5 \log (1 - 2V(b)) - \left[ E(c) + 0.5V(c) + \frac{(E(b) + Cov(b,c))^2}{1 - 2V(b)} \right],$$  \hspace{1cm} (22)

if $V(b) < 0.5$, with

$$b := \frac{N}{M} \sqrt{0.5\rho^2 \sigma_N^2 + 0.5\rho^2 \left[ \sigma_N^2 + V(P) - 2\alpha \sigma_N^2 \sqrt{V(P)\gamma} - V(P)\sigma_N^2 \gamma \right] + pM \sqrt{V(P)\gamma}},$$

$$c := \frac{N}{M} \left[ -\rho s + \rho E(P) - \rho \tilde{N} \sqrt{V(P)\gamma} \right].$$

We can show that for $\sigma_N^2 \to 0$, (22) converges to (??) for $L_E < L$ and to (??) for $L_E = L$ (see the Appendix). Applying Proposition 6.1*, we obtain:

**PROPOSITION 6.5:** Let $\tilde{N}_1 < \tilde{N} < \tilde{N}_2$, $\tilde{N} > \tilde{N}_4$ and $\frac{2L}{3a} > \tilde{N}$. Then for $\sigma_N^2$ positive but sufficiently small, the noise traders’ expected utility is lower at the equilibrium with $L_E < L$.

**Multiple equilibria**

As explained before, for $\sigma_N^2 \to 0$ and $\tilde{N}_3 < \tilde{N} < \tilde{N}_2$ there are two equilibria, of which one has the characteristic that its $L_E$ is close to $L$. If $\frac{2L}{3a} > \tilde{N}$, workers expected utility is still higher without OC. However, if this condition does not hold, one now has to suppose that in most cases expected utility at the equilibrium with $L_E$ close to $L$ is higher than without OC. This follows from the fact that the first term of (??) would then approximate zero and all depends on the second term.

**Large noise trader shocks**

The absence of OC can remain favorable for noise traders in the case of large noise trader shocks. An example is shown in figure 1, where the two curves in the fourth quadrant represent the transformed expected utilities of the noise traders in the cases of OC and No OC. Numerical experimentation leads us to assume that the welfare impacts of a switch from OC to No OC highly depend on $\tilde{N}$ and $\sigma_N^2$. For no OC to be favorable, we suspect that $\frac{N}{\sigma_N}$ has to be positive and sufficiently high. The explanation is similar to the one for the case of $\sigma_N^2 = 0$: A $\tilde{N}$ that is below 0 with high probability means that there is probably a positive aggregate noise trader supply of the asset. But if noise traders want to sell the asset, the higher number of firms (and thus a higher asset supply) in the case of No OC hurts their selling price. The higher volatility because of missing information about $s$ in the case of No OC again makes entrepreneurs want to sell assets more badly (to reduce risk exposure) and thus also hurts the selling price.
7 Taxes

One way to stop trading is to impose a prohibitive tax. Suppose a tax \( \tau \) is levied on agents who decide to become traders. The tax revenue is rebated either to the noise traders (in which case we let \( D = 0 \)) or to the entrepreneurs (in which case \( D = 1 \)). We first confine attention to the case \( \sigma^2_N > 0 \).

**Noise trader shocks**

The definition of equilibrium in Section 4 applies without modification for \( \tau \neq 0 \). The condition which states that agents are no worse-off as entrepreneurs than as traders becomes

\[
\Delta(L_E) + D \rho \tau \frac{L - L_E}{L_E} \geq \Gamma(L_E) - \rho \tau, \tag{23}
\]

where \( \Delta(L_E) \) and \( \Gamma(L_E) \) are defined as before. \( \tau(L - L_E)/L_E \) is the tax revenue per entrepreneur. From Proposition 4.1, we get:

**PROPOSITION 7.1:** Let \( \sigma^2_N > 0 \). \((L_E, I_E, I_T, P)\) is an equilibrium if \( 0 < L_E \leq L \), (23) holds with equality,

\[
\Delta(L_E) + D \rho \tau \frac{L - L_E}{L_E} \geq 0, \tag{24}
\]

\( I_E \) and \( I_T \) satisfy (1), and \( P \) satisfies (2); or if \( L_E = L \), (23) holds, \( \Delta(L) \geq 0 \), \( I_E \) satisfies (1), and \( P \) satisfies (2).

Conditions (24) and \( \Delta(L) \geq 0 \), respectively, ensure that agents are no worse-off as entrepreneurs than if they stay inactive. In the case of a positive tax on traders, other than in Section 4, this does not follow from the fact that \( GI \geq 0 \).

**PROPOSITION 6.5:** Let \((I_E, P)\) be an equilibrium with \( \tau = 0 \) and without OC. Then \((L, I_E, I_T, P)\) is an equilibrium with \( \tau \geq [\Gamma(L) - \Delta(L)]/\rho \) and with OC.

Suppose \( \tau \) satisfies the inequality in the proposition. Then (23) holds for \( L_E = L \) (irrespective of whether \( D = 0 \) or \( D = 1 \)). The existence of an equilibrium without taxes and OC implies \( \Delta(L) \geq 0 \) (see Proposition 4.3). It follows from Proposition 6.4 that an equilibrium with prohibitive taxation of trading activity exists (the value of \( I_T \) is arbitrary, since there are no traders). If equilibrium expected utility is higher without OC than with OC (for instance, since the conditions of Proposition 6.3 are satisfied), then agents prefer the imposition of the prohibitive tax.

Equation (23) determines the equilibrium mass of entrepreneurs \( L_E \) as a function of the tax rate on traders \( \tau \). Conversely, solving (23) holding with equality for \( \tau \) yields the tax rate required to achieve an equilibrium with a given mass of entrepreneurs \( L_E \). From Proposition 6.4:
PROPOSITION 6.6: Let \( \sigma^2_N > 0 \). Let \( L_E \) with \( 0 < L_E \leq L \) be given and

\[
\tau = \frac{\Gamma(L_E) - \Delta(L_E)}{\rho \left( D \frac{L-L_E}{L_E} + 1 \right)}.
\]

Then \( (L_E, I_E, I_T, P) \) is an equilibrium if (24) holds, \( I_E \) and \( I_T \) satisfy (1), and \( P \) satisfies (2).

No noise

We now turn to the case of \( \sigma^2_N = 0 \), for which enhanced analytical tractability allows us to derive the socially optimal tax and the resulting socially optimal \( L_E \). We show that with OC, in an equilibrium with \( 0 < L_E < L \), \( L_E \) is never too high, meaning there is never too little trading in equilibrium. The socially optimal level of \( L_E \) is either the equilibrium level of \( L_E \) or it is \( L_E = L \). We make two assumptions: First, the tax is levied on the entire non-productive sector, i.e. on traders as well as inactives, and it is uniformly redistributed to all agents. Second, we assume the social welfare function\(^\text{13}\)

\[
SWF := -L_E \log (-E[\pi_E]) + (L - L_E) [-\log (-E[\pi_T])] + \tilde{M} [-\log (-E[\pi_{\tilde{M}}])].
\]

If the government can set \( L_E \) directly, for a social optimum it sets \( L_E \) either according to \( \text{(??)} \) (see the Appendix) or \( L_E = L \). We now show that an optimal tax leads to the same \( L_E \). The condition for an equilibrium at \( 0 < L_E < L \) is \( GP + GT_E = GI + GT_0 - \rho \tau \), or \( GP + \rho \tau = 0 \). With \( P \) according to (12) this is

\[
\bar{s} - \frac{\rho \sigma^2_s \left( \frac{L_E}{a} - \bar{N} \right)}{L} - 0.5 \frac{\rho}{a} \sigma^2_s + a \tau = 0,
\]

from which follows

\[
L_E = L \left[ \frac{\bar{s} a}{\rho \sigma^2_s} - 0.5 \frac{\sigma^2_s}{\sigma^2_s} + \frac{\tau a^2}{\rho \sigma^2_s} \right] + a \bar{N}. \tag{26}
\]

Plugged in the SWF we get (with \( GP = -\rho \tau \) and \( GT_E = GT_0 \))

\(^{13}\)For technical reasons, we use the transformed expected utilities. Note that, when calculating the expected utilities, we take our CARA utility functions as cardinal (implied behaviour would be altered by non-affine transformations). The resulting expected utilities, however, are ordinal (implied behaviour is not altered by strictly increasing transformations). We thus assume that the expected utilities gain cardinal interpretation only after transformation.
\[ SWF = L \left( -\rho \tau + GT_E \right) + \tilde{M} \left[ \rho^2 \frac{\tilde{N} \sigma^2 \varepsilon \left( \frac{L_E}{a} - \tilde{N} \right)}{ML} - 0.5\rho^2 \left( \frac{\tilde{N}}{M} \right)^2 \sigma^2 \varepsilon + \rho\tau (L - L_E) = \right. \]

\[ = L \left[ -\rho \tau + \frac{\rho^2 \sigma^2 \varepsilon \left( \frac{L_E}{a} - \tilde{N} \right)^2}{2L^2} \right] + \rho\tau (L - L_E) + \]

\[ + \frac{\tilde{M}}{\rho^2} \left[ \frac{\tilde{N} \sigma^2 \varepsilon \left( \frac{L_E}{a} - \tilde{N} \right)}{ML} - 0.5\rho^2 \left( \frac{\tilde{N}}{M} \right)^2 \sigma^2 \varepsilon \right]. \tag{27} \]

Differentiating wrt \( \tau \) yields

\[
\frac{\partial SWF}{\partial \tau} = -\rho L + \frac{\rho^2 \sigma^2 \varepsilon}{L} \left( \frac{L_E}{a} - \tilde{N} \right) \frac{1}{a} L \frac{a^2}{\rho \sigma^2 \varepsilon} + \]

\[ + \frac{\rho^2 \tilde{N} \sigma^2 \varepsilon L a^2}{La \rho \sigma^2 \varepsilon} + \rho L - \left( \rho L_E + \frac{\rho a L}{\rho \sigma^2 \varepsilon} \right) = \]

\[ = \rho \left( L_E - a \tilde{N} \right) + \tilde{N} \rho a - \rho L_E - \frac{La}{\sigma^2 \varepsilon} = \]

\[ = -\frac{\tau}{\sigma^2 \varepsilon}. \]

FOC:

\[ \tau = 0. \tag{28} \]

There are two notes on this result: First, the socially optimal tax and the following \( L_E \) are not necessarily \( \tau = 0 \) and the equilibrium \( L_E \), but they might instead be a prohibitive tax and thus \( L_E = L \). Second, the result that trading is never too low in equilibrium here does not depend on the assumption \( \tilde{N} \geq \tilde{N}_4 \) (and thus not on \( \tilde{N} \geq \frac{L_E}{2a} > 0 \), not on \( \tilde{N} < \frac{L_E}{a} \) and it holds without any additional assumptions for the noise traders, i.e. without \( \tilde{M} > L \) and \( \tilde{N} < \frac{2L}{3a} \).

PROPOSITION 6.7: Assume there is the possibility to tax every individuum of the non-productive sector by an amount \( \tau \), which is then uniformly redistributed to all individuums. Let \( (25) \) describe the social welfare function. Then the socially optimal tax is either given by \( \tau = 0 \) or by a prohibitive tax. The socially optimal level of \( L_E \) is thus either the equilibrium \( L_E \) or \( L_E = L \).

8 Wages and employment

The analysis so far has been concerned with the effects of agents’ choice between entrepreneurship and finance on their welfare. This leaves out possibly the most important product of entrepreneurship: the creation of jobs for workers, paying wages which exceed what they could earn outside
the firm sector by far.\textsuperscript{14} To incorporate this into our model, this section considers a production economy with labor as an input in production. We analyze the model both with a perfect labor market and for several common wage setting regimes which give rise to unemployment.\textsuperscript{15}

**Production economy**

Denote the model with exogenous output of Section 2 as $M_0$ in what follows. We maintain the assumptions of $M_0$ unless stated otherwise. For the sake of brevity, we focus on the model variants with OC.

We assume that each noise trader supplies one unit of labor and use the terms “noise trader” and “worker” interchangeably. A worker’s initial wealth is $\tilde{e}_M + W$, where $W$ is the wage rate. The utility of leisure accruing to an unemployed worker is equivalent to $D(\ddot{a} > 0)$ units of consumption. The aggregate supply of labor is $\tilde{M}$ for $W \geq D$. As before, we assume that noise traders’ utility function is $U(\pi) = -\exp(-\rho\pi)$. Decisions made by workers or by unions representing the workers affect $U(\pi)$ only via initial wealth. So when we assume that labor market decisions are made with the objective of utility maximization, while trading behavior is not, we essentially assume that the workers-noise traders are myopic. A different interpretation treats noise traders and workers as separate types of agents. As shown below, the analysis is unchanged, since, given CARA utility, changes in initial wealth and changes in final wealth due to trading do not interfere with each other. This might be the preferred interpretation of the model if one wants to avoid to specify a utility function for noise traders.

As before, a entrepreneurs jointly set up one firm early, whose proceeds are shared uniformly by the entrepreneurs. The level of employment in the firm $M$ and the wage paid $W$ are also determined early. Firm output is $Y = \tilde{\theta} + F(M)$, and firm profit is $\theta = Y - WM$. $F$ is continuously differentiable, strictly increasing, and strictly concave. $\tilde{\theta}$ is the sum of two independent jointly normal random variables $\tilde{s} \sim N(\tilde{s}, \sigma_s^2)$ and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. As before, traders observe $\tilde{s}$ at the intermediate date, while entrepreneurs do not.

The fact that the impact of $\tilde{\theta}$ on firm profit $\theta$ is additive is crucial in order to preserve the single-asset framework of Section 2. If all other firms make profit $\theta$ with employment $M$, then a firm that chooses employment $M'$ makes profit $\theta' = \theta + \delta$, where $\delta \equiv F(M') - F(M) - W(M' - M)$. Since the firm’s profit differs from the other firms’ profit by the non-random amount $\delta$, buying a fraction $\lambda$ of the firm’s shares at cost $\lambda P'$ generates the same cash flow as buying a fraction $\lambda$ of one of

\textsuperscript{14}See Clark (2005) for a long-term perspective on the role of industrialization for workers’ wages.

\textsuperscript{15}Since all firms are alike, we do not address the issue of whether job creation is greater in small or in large firms (cf. Neumark et al., 2011; Haltiwanger et al., 2013).
the other firms at cost $\lambda P$ and storing $\lambda \delta$. Hence, arbitrage-freeness implies $P' - P = \delta$. The final wealth of the owners of the firm with employment $M'$ is $\pi_E' = e + (\theta - P)I_E = \pi_E + \delta/a$. Since the price differential $\delta$ is non-random, we have

$$E[U(\pi'_E)] = \exp \left( -\frac{\delta}{a} \right) E[U(\pi_E)],$$

and the entrepreneurs’ unanimous objective is to maximize $\delta$ or, equivalently, $F(M') - WM'$.

**Full employment**

Denote the production economy with a perfect labor market as model $M_1$. Let $\hat{M} = \bar{M}/(L_E/a)$ denote the number of workers per firm. Each firm employs $\hat{M}$ workers and pays them the wage rate $\hat{W} = F'(\hat{M})$ in $M_1$. Because of diminishing marginal returns, wages rise when the number of firms increases: $d\hat{W}/dL_E = -F''(\hat{M})\hat{M}/L_E > 0$. Thus, entrepreneurship benefits workers by raising their pay. $(L_E, M, I_E, I_T, P, W)$ is an equilibrium if, in addition to the conditions stated in Section 4, employment $M$ maximizes $\theta = Y - WM$ and the labor market clears (i.e., $M = \hat{M}$).

Let $\bar{M} = \hat{M}$ and

$$s = F(\hat{M}) - \hat{W}\bar{M} + \bar{s}. \quad (29)$$

The equilibrium analysis in Sections 4 and 5 goes through without modification, except that average firm profit now depends on the mass of entrepreneurs $L_E$:

$$\bar{s} = F\left( \frac{a\hat{M}}{L_E} \right) - F\left( \frac{a\hat{M}}{L_E} \right) \frac{a\hat{M}}{L_E} + \bar{s}. \quad (30)$$

**PROPOSITION 8.1:** Consider production economy $M_1$ and economy $M_0$ with $s$ given by (29). If $(L_E, I_E, I_T, P)$ is an equilibrium of $M_0$ and $\hat{W} > D$, then $(L_E, \hat{M}, I_E, I_T, P, \hat{W})$ is an equilibrium of $M_1$.

Equation (30) is an inverse relationship between the mass of entrepreneurs $L_E$ and expected profit $\bar{s}$; given full employment, an increase in the number of firms decreases the size and the profit of each firm. Condition (9) holding with equality is a second relationship between $L_E$ and $\bar{s}$. An increase in $\bar{s}$ raises $E(P)$ and does not affect the other variables in (9). So if $\Delta(L_E)$ intersects $\Gamma(L_E)$ from above, then an increase in $\bar{s}$ has a positive impact on $L_E$. These two relationships jointly determine $L_E$ and $\bar{s}$.

---

16 With non-additive shocks employment affects both expected return and risk, and one would have to check whether traders have positive demand for shares in firms with different levels of employment. See Biais et al. (2010) for a multi-asset version of GS.

17 Since $d[F(\hat{M}) - F'(\hat{M})\hat{M}] / dL_E = F''(\hat{M})\hat{M}^2 / L_E < 0$.

18 Since $dL_E / d\bar{s} = (\partial \Delta / \partial \bar{s}) / (\partial \Delta / \partial L_E) - (\partial \Delta / \partial L_E) > 0$. 

26
Wage setting

Turning to models with unemployment, we focus on specifications which give rise to complete wage rigidity: the wage rate is determined by the condition that \( M \) maximizes \( F(M) - WM \) and the respective wage setting rule alone. An increase in the mass of entrepreneurs then does not affect employment at the firm level (the intensive margin), but it increases aggregate employment by raising the mass of firms (the extensive margin). We consider two union and two efficiency wage models, labeled M2–M5.

M2: Workers are organized in decentralized firm-level unions. They are spread evenly across firms, so there are \( \hat{M} \) workers per firm. Unions monopolistically set the wage rate. Firms have the “right to manage” and choose the profit maximizing level of employment (cf. McDonald and Solow, 1981). If there is unemployment, the probability of being employed is \( \hat{M} \) for each worker. So unions maximize

\[
\frac{M}{\hat{M}} \left( -\exp \left\{ -\rho(e_M + W) \right\} \right) + \left( 1 - \frac{M}{\hat{M}} \right) \left( -\exp \left\{ -\rho(e_M + D) \right\} \right)
\]

(see the Appendix). For simplicity, \( F \) is Cobb-Douglas: \( F(M) = M^{1-b} \), where \( 0 < b < 1 \).

(L_E, M, I_E, I_T, P, W) is an equilibrium if, in addition to the conditions stated in Section 4, employment \( M \) maximizes \( F(M) - WM \), \( W \) maximizes (31) given the optimal choice of \( M \), and there is unemployment (i.e., \( M < \hat{M} \)).

M3: Employees can “work” or “shirk” at their workplace (cf. Shapiro and Stiglitz, 1984). A worker who “works” gets the wage rate \( W \). Effort is not perfectly observable: a shirker is caught shirking with probability \( q \) \((0 < q < 1)\). So he gets the utility of leisure \( D \) plus \( W \) with probability \( q \) and no payment otherwise.

If all workers work, firm output is \( Y = F(M) \). If everyone shirks, output is zero. So firms have to pay workers such that they choose not to shirk. \((L_E, M, I_E, I_T, P, W)\) is an equilibrium if, in addition to the conditions stated in Section 4, employment \( M \) maximizes \( F(M) - WM \), \( W \) is such that workers’ expected utility is as high if they work as if they shirk, and there is unemployment.

M4: Unions are organized as in M2. Firms have the right-to-manage. Rather than maximizing a utility function, firm-level unions set the wage rate \( W \) such that the wage bill \( WM \) is maximal (cf. Dunlop, 1944). \( F \) is CES with low substitutability: \( F(M) = [b + (1 - b)M^{(\eta-1)/\eta}]^{\eta/(\eta-1)} \), where \( 0 < b < 1 \) and the elasticity of substitution \( \eta \) obeys \( 0 < \eta < 1 \).

\((L_E, M, I_E, I_T, P, W)\) is an equilibrium if, in addition to the conditions stated in Section 4, employment \( M \) maximizes \( F(M) - WM \), \( W \) maximizes \( WM \) given the optimal choice of \( M \), and there is unemployment (i.e., \( M < \hat{M} \)).

M5: Firm output is \( Y = F[E(W)M] \), where \( E(W) \) is the effort provided by workers given the wage
they receive (cf. Solow, 1979). Workers provision of effort is determined by how fair they conceive the wage \( W \) they are paid. It is assumed that there is a unique “efficiency wage” \( \tilde{W} \) that maximizes \( E(W)/W \). Effort at the efficiency wage is normalized to \( E(\tilde{W}) = 1 \). \( (L_E, M, I_E, I_T, P, W) \) is an equilibrium if, in addition to the conditions stated in Section 4, employment \( M \) and the wage rate \( W \) jointly maximize \( F(M) - WM \), and there is unemployment.

**Unemployment**

The alternative wage setting regimes have in common that the real wage is rigid in the following sense:

**PROPOSITION 8.2:** In each of the production economies \( M2-M5 \), the condition that employment maximizes \( F(M) - WM \) and the respective assumption about wage setting jointly determine the wage rate \( \tilde{W} \) and employment per firm \( \tilde{M} \), independently of the other variables which make up an equilibrium \( (L_E, M, I_E, I_T, P, W) \).

The proof is in the Appendix. The proposition says that the model is block recursive and the labor market block can be solved first. As before, let \( s \) be defined by (29). From Proposition 8.2, contrary to the full employment case, \( \tilde{M} \) and \( \tilde{W} \) do not depend on \( L_E \). Hence, we obtain the model with exogenous production as a reduced form of the production economies with unemployment:

**PROPOSITION 8.3:** Consider any of the production economies \( M2-M5 \) and \( M0 \) with \( s \) defined by (29). If \( (L_E, I_E, I_T, P) \) is an equilibrium of \( M0 \), then \( (L_E, \tilde{M}, I_E, I_T, P, \tilde{W}) \) is an equilibrium of the production economy.

**Impacts on welfare**

As becoming a worker is not an OC decision and \( \tilde{M}, \tilde{W} \) do not depend on \( L_E \), sections 2-5.1 go through completely untouched (with the only exception that we exchange the "old" \( \tilde{s} \) with the "new" \( \tilde{s} \)). Section 5.2 confined attention to the noise traders. With our interpretation, the noise traders now are also the workers. As we assumed unemployment, an increasing number of entrepreneurs increases the probability for workers to find a job and thus benefits their welfare. If we assume an utility function for the noise traders/workers this can be shown analytically by calculating their expected utility, which now is

\[
-\log(-E(u(\pi_M))) = -\log \left( -E \left[ -\exp \left( -\rho \left( e + \frac{N}{M} (\theta - P) \right) \right) \right] + GW, \right.
\]

with

\[
GW := -\log \left[ \frac{\tilde{M} L_E}{M} \left( e^{-\rho(W-D)} - 1 \right) + 1 \right].
\]
GW describes the noise traders’ additional expected utility from working and with unemployment it is easily shown that \( \frac{\partial GW}{\partial L_E} > 0 \). So on top of the unaffected welfare effects already explained in section 5.2, with \( \frac{\partial GW}{\partial L_E} > 0 \) there is now an additional positive welfare effect, when moving from the case of OC and \( L_E < L \) to the case of no OC and \( L_E = L \).

If we include the labour market activity of noise traders in section 5.3, we can see that for \( \sigma_N^2 = 0 \), equilibrium \( L_E \) is always too low, meaning trading is always excessive in equilibrium. Specifically, the socially optimal non-prohibitive tax changes from \( \tau = 0 \) to \( \tau = \tilde{M}\frac{1-e^{-\rho(W-D)}}{\rho} > 0 \) and the optimal \( L_E(< L) \) changes to \( L_E = L + a\tilde{N} \), which is obviously higher than equilibrium \( L_E(< L) \) according to (??). Rational agents don’t take into account the positive effect of entrepreneurship for workers, when making their OC decisions. Thus, equilibrium is not socially optimal, as there is too little entrepreneurship.

**PROPOSITION 8.4:** Consider the models M2-M3, which add a production economy with unemployment to the basic model. Assume there is the possibility to tax every individual of the non-productive sector by an amount \( \tau \), which is then uniformly redistributed to all individuals. Let (25) describe the social welfare function and let \( GW \approx \tilde{M} \frac{L_E}{a} \left( 1 - e^{-\rho(W-D)} \right) \). Then the socially optimal tax is either a non-prohibitive but positive tax, or it is a prohibitive tax. The socially optimal level of \( L_E \) is thus always higher than the equilibrium level of \( L_E \) (or it is \( L_E = L \) in both cases).

9 Conclusion

We incorporate occupational choice between finance and entrepreneurship into the Grossman-Stiglitz (1980) noisy rational expectations equilibrium model. Sophisticated agents produce output and create jobs as entrepreneurs or contribute to informational efficiency in financial markets as informed traders. Finance possibly attracts too much talent, for instance if the amount of noise in the economy is small, so that the asset price at a rational expectations equilibrium is highly informative anyway. The main beneficiaries of the allocation of talent to entrepreneurial activity are workers, whose wage and employment prospects improve when more entrepreneurial agents choose to become entrepreneurs.

\(^{19}\) For this result we use the approximation \( \log(1+x) \approx x \), for \( x \) small, which in our case implies \( GW \approx \frac{\tilde{M} L_E}{a} \left( 1 - e^{-\rho(W-D)} \right) \). This approximation is made for technical reasons. As \( \frac{\tilde{M} L_E}{a} \left( 1 - e^{-\rho(W-D)} \right) \) is a product of two variables, which are both \( > 0 \) and \( < 1 \), we expect it to be quite small.
References


Appendix

The derivations below make use of Lemma 1 in Demange and Laroque (1995, p. 252), which says that for normal random variables \( x \) and \( y \),

\[
E \left[ \exp \left( x - y^2 \right) \right] = \frac{\exp \left\{ \mathbb{E}(x) + \frac{1}{2} \text{var}(x) - \frac{[\mathbb{E}(y) + \text{cov}(x,y)]^2}{1 + 2 \text{var}(y)} \right\}}{[1 + 2 \text{var}(y)]^{\frac{1}{2}}}. \tag{A.1}
\]

**Equation (1):**

Making use of (A.1) with \( y \) identically equal to zero, we have

\[
E[U(\pi_E)|P] = -\exp \left\{ -\rho \left\{ e + \frac{P}{a} + \mathbb{E}(\theta|P) - P \right\} I_E \right\} + \frac{\rho^2}{2} \mathbb{E}(\theta|P) I_E^2 \right\}. \tag{A.2}
\]

Maximizing with respect to \( I_E \) yields the first equation in (1). Given that inactive agents trade on the same information as entrepreneurs, \( I_0 = I_E \) follows from the fact that optimum investment does not depend on initial wealth. Similarly, using \( \mathbb{E}(\theta|s) = s \) and \( \text{var}(\theta|s) = \sigma_{\theta s}^2 \),

\[
E[U(\pi_T)|s] = -\exp \left\{ -\rho \left\{ e + (s - P)I_T \right\} \right\} + \frac{\rho^2}{2} \sigma_{\theta s}^2 I_T^2 \right\}, \tag{A.3}
\]

and maximization with respect to \( I_T \) yields the second equation in (1).

**Equations (7) and (8):**

Substituting for \( I_E \) from (1) into (A.2) yields

\[
E[U(\pi_E)|P] = -\exp \left\{ -\rho e - \frac{P}{a} - \frac{[\mathbb{E}(\theta|P) - P]^2}{\text{var}(\theta|P)} + \frac{1}{2} \frac{[\mathbb{E}(\theta|P) - P]^2}{\text{var}(\theta|P)} \right\}. \tag{A.2}
\]

The expression in the main text follows from collecting terms and the definition of \( z \). Taking expectations, using the law of iterated expectations, we obtain

\[
E[U(\pi_E)] = -\exp(-\rho e) \mathbb{E} \left[ \exp \left( -\rho \frac{P}{a} - z^2 \right) \right]. \tag{A.4}
\]

Since \( P \) and \( z \) are normal, we can apply (A.1) to get

\[
E \left[ \exp \left( -\rho \frac{P}{a} - z^2 \right) \right] = \frac{\exp \left\{ \mathbb{E} \left( -\rho \frac{P}{a} \right) + \frac{1}{2} \text{var} \left( -\rho \frac{P}{a} \right) - \frac{[\mathbb{E}(z) + \text{cov}(-\rho \frac{P}{a}, z)]^2}{1 + 2 \text{var}(z)} \right\}}{[1 + 2 \text{var}(z)]^{\frac{1}{2}}}. \tag{A.4}
\]

Substituting this into (A.4) and rearranging terms gives

\[
E[U(\pi_E)] = -\exp(-\rho e) \frac{\exp \left\{ -\rho \frac{a}{P} \mathbb{E}(P) + \frac{1}{2} \left( \frac{a}{P} \right)^2 \text{var}(P) - \frac{[\mathbb{E}(z) - \frac{a}{P} \text{cov}(P,z)]^2}{1 + 2 \text{var}(z)} \right\}}{[1 + 2 \text{var}(z)]^{\frac{1}{2}}},
\]
which can be rewritten as (7). An inactive agent’s expected utility is obtained analogously; the terms containing $P/a$ drop out.

Similarly, substituting for $I_T$ from (1) into (A.3) yields

$$E[U(\pi_T)|s] = -\exp \left\{ -\rho e - \left[ \frac{s - P}{(2\sigma_e^2)^{1/2}} \right]^2 \right\}.$$  \hfill (A.5)

Set $y = (s - P)/(2\sigma_e^2)^{1/2}$. Notice that $E(s|P) = E(\theta|P)$ and $\text{var}(s|P) = \text{var}(\theta|P) - \sigma_e^2$, so that $E(y|P) = [E(\theta|P) - P]/(2\sigma_e^2)^{1/2}$ and $\text{var}(y|P) = [\text{var}(\theta|P) - \sigma_e^2]/(2\sigma_e^2)$. Applying the law of iterated expectations to (A.5) and using (A.1), we obtain

$$E[U(\pi_T)|P] = -\exp(-\rho e) \cdot \exp \left\{ -\frac{[E(\theta|P) - P]^2}{2\sigma_e^2 \text{var}(\theta|P)} \right\} \left[ \frac{\text{var}(\theta|P)}{\sigma_e^2} \right]^{1/2}.$$  \hfill (A.10)

The expression in the main text follows upon rearranging terms and using the definition of $z$. Taking expectations, again making use of the law of iterated expectations and (A.1), yields

$$E[U(\pi_T)] = -\exp(-\rho e) \cdot \frac{\sigma_e^2}{\text{var}(\theta|w)} \cdot \frac{1}{[1 + 2 \text{var}(z)]^{1/2}},$$

which can be rewritten as (8).

**The functions $\Delta(L_E)$ and $\Gamma(L_E)$:**

Let

$$\alpha = \frac{L - L_E}{\rho \sigma_e^2}, \quad \beta = \frac{L_E}{\rho \text{var}(\theta|w)}, \quad \gamma = \frac{1}{\alpha^2 \sigma_e^2 + \sigma_N^2}.$$  \hfill (A.6)

Then,

$$\begin{align*}
\text{var}(\theta|w) &= \gamma \sigma_e^2 \sigma_N^2 + \sigma_e^2, \quad \text{(A.7)} \\
E(P) &= \bar{s} - \frac{L_E}{\alpha + \beta}, \quad \text{(A.8)} \\
\text{var}(P) &= \frac{1}{\gamma} \left[ \frac{1 + \alpha \beta \gamma \sigma_e^2}{\alpha + \beta} \right]^2, \quad \text{(A.9)} \\
E(z) &= \frac{L_E}{\alpha} \left( \frac{1}{\alpha + \beta} \right) \left[ 2 \left( \gamma \sigma_e^2 \sigma_N^2 + \sigma_e^2 \right) \right]^{1/2}, \quad \text{(A.10)} \\
\text{var}(z) &= \frac{\gamma \sigma_N^2}{(\alpha + \beta)^2 \left( \gamma \sigma_e^2 \sigma_N^2 + \sigma_e^2 \right)}, \quad \text{(A.11)} \\
\text{cov}(P,z) &= -\frac{(1 + \alpha \beta \gamma \sigma_e^2) \sigma_N^2}{(\alpha + \beta)^2 \left[ 2 \left( \gamma \sigma_e^2 \sigma_N^2 + \sigma_e^2 \right) \right]^{1/2}}. \quad \text{(A.12)}
\end{align*}$$
Note that $\bar{s}$ affects only $E(P)$. By definition, $w = \alpha s + N$, so $\text{var}(w) = \alpha^2 \sigma_s^2 + \sigma_N^2$ and $\text{cov}(s, w) = \alpha \sigma_s^2$. Substituting this into (5) yields

$$\text{var}(\theta|w) = \sigma_s^2 \left( 1 - \frac{\alpha^2 \sigma_s^2}{\alpha^2 \sigma_s^2 + \sigma_N^2} \right) + \sigma_{\varepsilon}^2.$$  

Equation (A.7) follows from the definition of $\gamma$ in (A.6). $\text{var}(\theta|w)$ converges to $\sigma_{\varepsilon}^2$ as $\sigma_N^2$ goes to zero.

According to the updating rule for the mean of a normal random variable, $E(\theta|w) = E(\theta) + [\text{cov}(\theta, w)]/\text{var}(w)\left[w - E(w)\right]$. Using $E(\theta) = \bar{s}$, $\text{var}(w) = \alpha^2 \sigma_s^2 + \sigma_N^2$, $\text{cov}(\theta, w) = \alpha \sigma_s^2$, and the definitions of $w$, $\alpha$, and $\gamma$,

$$E(\theta|w) = \bar{s} + \alpha \gamma \sigma_s^2 \left[ \alpha (s - \bar{s}) + N - \bar{N} \right]. \quad (A.13)$$

This can be used to rewrite (2) as

$$P = \frac{\alpha s + N + \beta \left( \bar{s} + \alpha \gamma \sigma_s^2 \left[ \alpha (s - \bar{s}) + N - \bar{N} \right] - \frac{L_E}{\alpha} \right)}{\alpha + \beta}$$

or, rearranging terms,

$$P = \bar{s} + \frac{(1 + \alpha \beta \gamma \sigma_s^2) \left[ \alpha (s - \bar{s}) + N - \bar{N} \right] - \frac{L_E}{\alpha} - \bar{N}}{\alpha + \beta}. \quad (A.14)$$

Equation (A.8) follows upon taking expectations.

The variance of $P$ is

$$\text{var}(P) = \frac{(1 + \alpha \beta \gamma \sigma_s^2)^2 \left( \alpha^2 \sigma_s^2 + \sigma_N^2 \right)}{(\alpha + \beta)^2}.$$  

Using the definition of $\gamma$, we obtain (A.9).

Substituting $E(\theta|w)$ from (A.13) and $P$ from (A.14) into the definition of $z$ yields

$$z = \bar{s} + \alpha \gamma \sigma_s^2 \left[ \alpha (s - \bar{s}) + N - \bar{N} \right] - \frac{(1 + \alpha \beta \gamma \sigma_s^2) \left[ \alpha (s - \bar{s}) + N - \bar{N} \right] - \frac{L_E}{\alpha} - \bar{N}}{\alpha + \beta} \frac{1}{\left[ 2 \left( \gamma \sigma_s^2 \sigma_N^2 + \sigma_{\varepsilon}^2 \right) \right]^2}.$$  

Simplifying terms, using $1 - \alpha^2 \gamma \sigma_s^2 = \gamma \sigma_N^2$, we get

$$z = \frac{-\gamma \sigma_N^2 \left[ \alpha (s - \bar{s}) + N - \bar{N} \right] + \frac{L_E}{\alpha} - \bar{N}}{(\alpha + \beta) \left[ 2 \left( \gamma \sigma_N^2 + \sigma_{\varepsilon}^2 \right) \right]^2}. \quad (A.15)$$

Taking expectations yields (A.10).

The variance of $z$ is

$$\text{var}(z) = \frac{\gamma^2 \left( \alpha \sigma_N^2 \right)^2 (\alpha^2 \sigma_s^2 + \sigma_N^2)}{(\alpha + \beta)^2} \frac{2 \left( \gamma^2 \sigma_s^2 \sigma_N^2 + \sigma_{\varepsilon}^2 \right)}{\left( \gamma \sigma_s^2 \sigma_N^2 + \sigma_{\varepsilon}^2 \right)}. \quad (A.16)$$
Equation (A.11) follows from the definition of $\gamma$.
From (A.14) and (A.15),
\[
\text{cov}(P, z) = \frac{1 + \alpha \beta \gamma \sigma_s^2}{\alpha + \beta} \cdot \frac{-\gamma \sigma_N^2}{(\alpha + \beta)\left[2(\gamma \sigma_s^2 \sigma_N^2 + \sigma_s^2)\right]^{\frac{1}{2}}} \left(\alpha^2 \sigma_s^2 + \sigma_N^2\right).
\]
Equation (A.12) follows from the definition of $\gamma$. Using (A.9) and (A.11), (A.12) can be rewritten as $\text{cov}(P, z) = -[\text{var}(P) \text{var}(z)]^{\frac{1}{2}}$, which proves that $P$ and $z$ are perfectly negatively correlated.

**Proof of Proposition 4.3:**
The existence part is obvious. To prove uniqueness, we have to show that $\Delta'(L_E) < 0$. We have:
\[
\begin{align*}
E(P) &= \bar{s} - \frac{\rho}{L} (\sigma_s^2 + \sigma_z^2) \left(\frac{L_E}{a} - \bar{N}\right) \\
\text{var}(P) &= \left(\frac{\rho}{L}\right)^2 (\sigma_s^2 + \sigma_z^2)^2 \sigma_N^2 \\
E(z) &= \frac{\rho}{L} \left(\frac{\sigma_s^2 + \sigma_z^2}{2}\right)^\frac{1}{2} \left(\frac{L_E}{a} - \bar{N}\right) \\
\text{var}(z) &= \left(\frac{\rho}{L}\right)^2 \frac{(\sigma_s^2 + \sigma_z^2)^2}{2} \sigma_N^2 \\
\text{cov}(P, z) &= -\left(\frac{\rho}{L}\right)^2 \frac{(\sigma_s^2 + \sigma_z^2)^2}{2} \sigma_N^2.
\end{align*}
\]
Equations (A.16) and (A.17) follow immediately from (11).
Inserting $E(\theta|w) = \bar{s}$ and $\text{var}(\theta|w) = \sigma_s^2 + \sigma_z^2$ into the definition of $z$ in (6) yields
\[
z = \frac{\rho}{L} \left(\frac{\sigma_s^2 + \sigma_z^2}{2}\right)^\frac{1}{2} \left(\frac{L_E}{a} - \bar{N}\right).
\]
Equations (A.18) and (A.19) follow immediately.
Equations (11) and (A.21) yield (A.20).
Differentiating (9), using (A.16) and (A.18)–(A.20) proves
\[
\Delta'(L_E) = \left(\frac{\rho}{L}\right)^2 \frac{L}{a} (\sigma_s^2 + \sigma_z^2) \left[\frac{(\xi)^2 (\sigma_s^2 + \sigma_z^2) \sigma_N^2}{1 + (\xi)^2 (\sigma_s^2 + \sigma_z^2) \sigma_N^2} - 1\right] < 0.
\]

**Traders’ utility in $M(0, 1)$:**
A trader’s expected utility conditional on $s$ is given by (A.5). Taking expectations, using (A.1) with $x$ identically equal to zero, the fact that $P$ is safe, and (18) yields the expression in the main text.

**Welfare for $\sigma_N^2 \to 0$:**
For $L_E < L$, we obtain the following limits of the functions defined in (A.6)–(A.12) as $\sigma_N^2 \to 0$:

\[
\begin{align*}
\alpha \gamma \sigma_s^2 & \to \frac{1}{\alpha} \\
\text{var}(\theta|w) & \to \sigma_v^2 \\
\beta & \to \frac{L_E}{\rho \sigma_\xi^2} \\
\alpha + \beta & \to \frac{L}{\rho \sigma_\xi^2} \\
E(P) & \to \bar{s} - \frac{\rho \sigma_\xi^2}{L} \left( \frac{L_E}{a} - \bar{N} \right) \\
\text{var}(P) & \to \sigma_s^2 \\
E(z) & \to \frac{\rho}{L} \left( \frac{\sigma_s^2}{2} \right) \left( \frac{L_E}{a} - \bar{N} \right) \\
\text{var}(z) & \to 0 \\
\text{cov}(P,z) & \to 0.
\end{align*}
\]

(A.22)

Inserting these expressions into the left-hand side of (9) yields $\Delta_0(L_E)$. This proves that $\Delta(L_E) \to \Delta_0(L_E)$ pointwise. From (A.22), it follows that the right-hand side of (9) goes to zero, i.e., $\Gamma(L_E) \to 0$ pointwise.

For $L_E = L$, from (A.6)–(A.12),

\[
\begin{align*}
\alpha & = 0 \\
\gamma \sigma_N^2 & = 1 \\
\text{var}(\theta|w) & = \sigma_s^2 + \sigma_v^2 \\
\beta & = \frac{L}{\rho (\sigma_s^2 + \sigma_v^2)} \\
E(P) & = \bar{s} - \frac{\rho (\sigma_s^2 + \sigma_v^2)}{L} \left( \frac{L}{a} - \bar{N} \right) \\
\text{var}(P) & = \left[ \frac{\rho (\sigma_s^2 + \sigma_v^2)}{L} \right]^2 \sigma_N^2 \\
E(z) & = \frac{\frac{L}{a} - \bar{N}}{\rho \sigma_s^2 + \rho \sigma_v^2} \frac{L}{ \left( 2(\sigma_s^2 + \sigma_v^2) \right)^2} \\
\text{var}(z) & = \frac{\sigma_N^2}{\left[ \frac{L}{\rho \sigma_s^2 + \rho \sigma_v^2} \right]^2 2(\sigma_s^2 + \sigma_v^2)} \\
\text{cov}(P,z) & = \frac{-\sigma_N^2}{\left[ \frac{L}{\rho \sigma_s^2 + \rho \sigma_v^2} \right]^2 2(\sigma_s^2 + \sigma_v^2)}.\end{align*}
\]

Inserting these expressions into the left-hand side of (9) and taking the limit as $\sigma_N^2 \to 0$ yields
\(\Delta_0(L)\) (i.e., the right-hand side of (16)). Substitution into the right-hand side of (9) and taking the limit \(\sigma_N^2 \to 0\) yields \(\Gamma_0(L)\).

**Noise trader consumption:**

From (12) and (16), respectively,

\[
\theta - P = \frac{\rho \sigma_e^2}{L} \left( \frac{L_E}{a} - \bar{N} \right) + \varepsilon \tag{A.23}
\]

at an equilibrium with \(L_E < L\) and

\[
\theta - P = s - \bar{s} + \frac{\rho \left( \sigma_s^2 + \sigma_e^2 \right)}{L} \left( \frac{L_E}{a} - \bar{N} \right) + \varepsilon \tag{A.24}
\]

at an equilibrium without OC. The expressions for the differences in mean and variance in the main text follow immediately.

**Noise trader utility**

Let \(\rho e_M\) denote noise traders’ endowment, so that \(\pi_M = e + (\theta - P)I_M\) is their final wealth. Let \(\sigma_N^2 = 0\). Consider first an equilibrium with \(L_E < L\). Using \(\pi_M = \hat{e}_M + (\theta - P)\bar{N}/\bar{M}\) and (A.23), noise traders’ expected utility can be written as

\[
E[U(\pi_M)] = - \exp(-\rho e_M) E \left\{ \exp \left[ \frac{\rho \sigma_e^2}{L} \left( \frac{L_E}{a} - \bar{N} \right) + \varepsilon \right] \frac{\bar{N}}{\bar{M}} \right\}.
\]

As final wealth is normal, we can apply (A.1) to get

\[
- \log \{-E[U(\pi_M)]\} = \rho e_M + \frac{\bar{N}}{\bar{M}} \frac{\rho \sigma_e^2}{L} \left( \frac{L_E}{a} - \bar{N} \right) - \frac{1}{2} \left( \frac{\rho \bar{N}}{\bar{M}} \right)^2 \sigma_e^2. \tag{A.25}
\]

Following the same steps, using (A.24) instead of (A.23), we get noise traders’ expected utility without OC:

\[
- \log \{-E[U(\pi_M)]\} = \rho e_M + \frac{\bar{N}}{\bar{M}} \frac{\rho \sigma_s^2 + \sigma_e^2}{L} \left( \frac{L_E}{a} - \bar{N} \right) - \frac{1}{2} \left( \frac{\rho \bar{N}}{\bar{M}} \right)^2 (\sigma_s^2 + \sigma_e^2).
\]

Subtracting the former expression from the latter yields the increase in noise traders’ expected utility in the main text.

**Optimum \(L_E\) for \(\sigma_N^2 = 0\):**

Social welfare is the sum of \(L_E(- \log \{-E[U(\pi_E)]\})\), \((L - L_E)(- \log \{-E[U(\pi_T)]\})\), and \(\bar{M}(- \log \{-E[U(\pi_M)]\})\). Let \(\sigma_N^2 = 0\) and \(0 < L_E < L\). As shown in Section 5, \(- \log \{-E[U(\pi_E)]\} = \text{GP} + \text{GT}_E\), where \(\text{GP}\) is given by the left-hand side of (14), and \(\text{GT}_E = z^2\) with \(z\) given by (13).
\[-\log\{-E[U(\pi_T)]\} = z^2, \text{ as } GI = 0. \text{ Noise traders' welfare } - \log\{-E[U(\pi_M)]\} \text{ is given by (A.25).} \]

Using these results, social welfare can be expressed as a function of \(L_E\) alone:

\[
S(L_E) = \rho (Le + \bar{M} e_M) + L_E \beta \left[ \frac{\rho \sigma^2}{\bar{a}} \left( \frac{L_E}{a} - \bar{N} \right) + \frac{\beta^2 \sigma^2}{2L} \left( \frac{L_E}{a} - \bar{N} \right)^2 + \bar{N} \rho \sigma^2 \right] \frac{L_E}{a} - \bar{N} \right) - \frac{\rho \sigma^2}{2a} \]

Taking the derivative yields

\[
S'(L_E) = \Delta_0(L_E)
\]

and \(S''(L_E) < 0.\)

**Proof of Proposition 8.1:**

**M2:** Unions maximize workers’ expected utility \(E[U(\pi_M)]\). The reason why this reduces to maximizing (31) is the following: If all other unions set wage \(W\), which leads to profit \(\theta\), then a union that sets wage \(W'\) makes profit \(\theta'\), with \(\theta' = \theta + \delta\), where \(\delta = F\left(M(W')\right) - F\left(M(W)\right) - M\left(W' - W\right).\)

Since profits differ by the non-random amount \(\delta\), arbitrage freeness implies \(P' = P + \delta\). The final wealth of a worker who gets wage \(W'\) is \(
\pi'_M = \frac{N}{\bar{N}} (\theta - P) + W' - D + e, \text{ whereas } W' - D = 0 \text{ if he is unemployed. We see that final wealth that results from trading, } \frac{N}{\bar{N}} (\theta - P), \text{ is independent of } W', \text{ since } \theta' - P' = P + \delta - \theta - \delta = \theta - P. \text{ As with CARA-utility initial endowments don’t matter for utility maximization, unions just maximize } \frac{M}{\bar{M}} \left\{ - \exp \left[ - \rho(e_M + W - D) \right] \right\} + \left( 1 - \frac{M}{\bar{M}} \right) \left\{ - \exp \left[ - \rho(e_M - D - GT_M) \right] \right\} \text{ in order to maximize } E[U(\pi_M)]. \)

A noise trader’s expected utility is

\[
\frac{M}{\bar{M}} \left\{ - \exp \left[ - \rho(e_M + W - GT_M) \right] \right\} + \left( 1 - \frac{M}{\bar{M}} \right) \left\{ - \exp \left[ - \rho(e_M + D - GT_M) \right] \right\},
\]

where \(GT_M\) are noise traders’ gains from trading, given by the two final terms on the right-hand side of (A.25). Maximization is equivalent to maximization of (31).

The firm’s labor demand curve is

\[
M = \left( \frac{1 - b}{W} \right)^{\frac{1}{2}}.
\]

Maximization of (31) subject to this constraint is equivalent to maximization of

\[
bW^{\frac{1}{2}} \left\{ - \exp \left[ - \rho(W - D) \right] + 1 \right\}.
\]

Setting the derivative equal to zero yields

\[
W^{-\frac{1}{2}} \exp \left[ - \rho(W - D) \right] \{ 1 + \rho b W - \exp [\rho (W - D)] \} = 0.
\]
There is a unique positive $\bar{W} (> D)$ such that the condition holds for $W = \bar{W}$, and the derivative changes from positive to negative at $\bar{W}$, so that $\bar{W}$ maximizes expected utility. Employment is $\tilde{M} = [(1 - b)/\bar{W}]^{1/b}$. There is unemployment if $\tilde{M} < \hat{M}$. A simple sufficient condition is

M3: The expected utility from working is

$$-\exp[-\rho(e_{\hat{M}} + W) - GT_{\hat{M}}].$$

A shirker gets expected utility

$$(1 - q) \{-\exp[-\rho(e_{\hat{M}} + W + D) - GT_{\hat{M}}]\} + q \{-\exp[-\rho(e_{\hat{M}} + D) - GT_{\hat{M}}]\}.$$

Equalizing these expected utilities yields the efficiency wage, necessary to prevent shirking:

$$\bar{W} = \frac{1}{\rho} \log \left[ \frac{\exp(\rho D) - (1 - q)}{q} \right].$$

Employment is $\tilde{M} = (F')^{-1}(\bar{W})$. There is always unemployment for $\tilde{M}$ sufficiently large.

M4: Equating the marginal product of labor to the wage rate yields the following expression for each firm’s wage bill:

$$WM = \left[ b + (1 - b)M^{\frac{n-1}{n}} \right] \frac{1}{\eta} M^{\frac{n-1}{n}} (1 - b).$$

Setting the derivative equal to zero gives:

$$\frac{1 - \eta}{\eta} W \left[ \frac{1 - b}{1 - \eta b} + (1 - b)M^{\frac{n-1}{n}} - 1 \right] = 0.$$

Optimum employment is given by the value $M = \tilde{M}$ at which the derivative changes from positive to negative:

$$\tilde{M} = \left( \frac{\eta}{1 - \eta} \frac{1 - b}{b} \right)^{\frac{n}{\eta - 1}}.$$

The corresponding wage rate is

$$\bar{W} = \left[ \frac{1 - \eta}{(1 - b)^{\eta}} \right]^{\frac{1}{1 - \eta}}.$$

M5: Firm profit can be expressed as

$$F \left[ \frac{E(W)}{W} WM \right] - WM.$$