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Partial cross ownership and explicit collusion

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Partial cross ownership and explicit collusion

February 27, 2016

Abstract

This article studies the unilateral and coordinated effects of non-controlling minority shareholdings (NCMS). It provides a comprehensive model by integrating the established models of Reynolds and Snapp (1986), Flath (1991), Malueg (1992), and Gilo *et al.* (2006). It is the first to add a competition authority. The model finds that NCMS lower the sustainability of collusion under a greater variety of situations than was indicated by earlier literature. The collusion destabilizing effect of NCMS is particularly prevalent in the presence of an effective antitrust authority.

JEL codes: G34, K21, L41

Keywords: Collusion, Coordinated Effects, Minority Shareholdings, Merger Control, Unilateral Effects

1 Introduction

This article shows in a formal model that non-controlling minority shareholdings (NCMS) among competitors *lower* the sustainability of explicitly collusive agreements under a great variety of circumstances and especially in the presence of an effective antitrust authority.

One speaks of non-controlling minority shareholdings when firm i buys a stake in a rival $-i$ that is lower than 50% and does not grant control rights, i.e., the buyer acquires a silent interest. The acquisition of non-controlling minority shareholdings is subject to merger control in some jurisdictions such as Austria, Germany, UK, US, and Japan but not in others like the European Union (EU). Therefore, in July 2014 the EU issued a White Paper discussing an amendment of the current EU Merger Regulation towards assessing non-controlling minority shareholdings. The present article contributes to this discussion by assessing the effects of NCMS on the sustainability of collusion.

More importantly, it makes a methodological contribution by integrating established models on NCMS into a more comprehensive one. This fills a gap in the literature because we analyze several sets of assumptions on, e.g., profit functions and models of competition that have not been studied by prior literature. (See the literature review in Section 2 for a more detailed discussion of this contribution.) We also provide analytic proofs for effects that have only been established numerically by prior literature. Moreover, our model is the first to study analytically how a competition authority impacts the effect of minority shareholdings on the sustainability of collusion. The model indicates that NCMS destabilize collusion under a wider set of assumption than has been suggested by earlier literature. These results appear to be in line with some case evidence.

The article is structured as follows. Section 2 details how the present study contributes to the existing literature. Section 3 provides the model and studies the unilateral effects of NCMS in the stage game. Section 4 analyzes the effects of NCMS on the sustainability of collusion. The robustness of our results is established in Section 5. We present some case evidence that appears to support our theoretical predictions in Section 6. Section 7

concludes the article. Proofs are stated in the Appendix.

2 Literature Review

Reynolds and Snapp (1986) provide one of the first economic studies of NCMS. They establish that – in a static Cournot model with symmetric firms and homogeneous goods – NCMS cause unilateral effects by softening competition and raising firms’ aggregate profits. Their analysis was extended to an infinitely repeated game by Malueg (1992) who shows that NCMS may have an ambiguous impact on the stability of tacit collusion. On the one hand, a colluding firm who holds shares of its co-conspirators receives a lower short-run gain when cheating on them. This is because the deviator receives lower dividends when deviating because being cheated depresses the profits of its former co-conspirators, whose shares the deviating firm holds. On the other hand, the unilateral effects of NCMS soften the long-run punishment that is imposed on the deviating firm. Hence, by lowering both the short-run gain from a deviation and the long-run punishment for such conduct NCMS have an ambiguous effect on the sustainability of collusion. Malueg (1992) shows that NCMS lower the sustainability of collusion when demand is convex.

We extend Malueg’s (1992) seminal contribution in several directions. While he studies symmetric shareholdings only we also allow for asymmetric NCMS. Besides Cournot competition with homogeneous goods, we also study Bertrand competition both with homogeneous or differentiated products. We present analytic proofs for effects that have been shown by Malueg (1992) only numerically.

Some of these extensions were inspired by Flath (1991, 1992) who presents a static model to study the unilateral effects of NCMS in Cournot competition with homogeneous goods, Bertrand competition with differentiated goods, and Bertrand competition with homogeneous goods. He points out that in a Cournot model, where quantities are strategic substitutes, an asymmetric increase of firm i ’s shareholdings of firm $-i$ only raises the

product-market profits of firm $-i$ but lowers the product-market profits of firm i . The profits of both firms only rise if they increase their shares in each other mutually. However, in a Bertrand model with differentiated products, where prices are strategic complements, even an asymmetric acquisition of NCMS may raise the profits of this firm.

Unfortunately, these results cannot be compared to those of Reynolds and Snapp (1986) and Malueg (1992) who assume that the firms maximize the sum of product-market profits plus dividends received minus dividends paid. Flath (1991) however assumes that the firms maximize product-market profits plus dividends received before subtracting dividends paid. We implement our model under both assumptions about firms' profit function and thus make these prior contributions comparable.

In doing so, we also contribute to Gilo *et al.* (2006). They use the profit function assumed by Flath (1991) and model a dynamic game to study the effects of NCMS on collusion. They analyze a Bertrand model with homogeneous goods and show that an increase in NCMS never hinders tacit collusion. This is because unilateral effects are absent in Bertrand competition with homogeneous goods. Therefore, the long-run punishment following a deviation is not softened by NCMS while the short-run gain falls because the deviator takes into account that a deviation lowers the dividends it receives from its former co-conspirators. We extend Gilo *et al.*'s (2006) model by also analyzing Cournot competition with homogeneous goods and Bertrand competition with differentiated goods.

We (i) enhance the comparability of Reynolds and Snapp (1986), Flath (1991), Malueg (1992), and Gilo *et al.* (2006), (ii) complete the analysis of assumptions that have not been studied jointly before, and (iii) provide formal proofs for effects that have only been established numerically by prior literature. This establishes a coherent framework for the analysis of the effects on NCMS on the stability of collusion. Moreover, we add a new element to this discussion by assuming an antitrust authority along the lines of Aubert *et al.* (2006), i.e., collusion may be detected with a certain probability and sanctioned thereafter. In the presence of an effective antitrust authority NCMS are quite likely to lower the sustainability

of collusion under a variety of conditions where the literature cited above suggested a stabilizing effect of NCMS on collusion. For example, under the assumption of a competition authority NCMS are found to destabilize collusion also for non-convex demand, which adds to Malueg (1992), and even for Bertrand competition with homogeneous products, which adds to Gilo *et al.* (2006).

The literature cited above studies how firm i 's shares of firm $-i$ affect the sustainability of collusion through the shareholdings' effect on firm i 's critical discount factor. We also study how firm $-i$'s shares of firm i affect firm i 's critical discount factor. As a central result, we find that minority shareholdings destabilize collusion in all variants of our model. This is because firm i 's discount factor never falls when firm $-i$ raises its shareholdings in firm i .

Note that the present article along with Reynolds and Snapp (1986), Flath (1991), Malueg (1992), and Gilo *et al.* (2006) concentrates on the most anticompetitive acquisition decisions, i.e., decisions that are purely driven by the rationale to receive a dividend and raise the acquirer's expected profits. Other authors (Karle *et al.* 2011) have pointed out that the acquisition of minority shareholdings may also be driven by efficiency considerations such as the generation of economies of scope in the production process. Our model indicates that even inherently anticompetitive NCMS, which were only acquired to raise the profits of the acquirer, often have pro-competitive effects by disrupting explicit collusion. To show this point most clearly we abstract from further efficiency considerations.

3 The Model

Subsection 3.1 presents the setup of the model. Subsection 3.2 establishes the unilateral effects of minority shareholdings in the stage game.

3.1 Setup

The timing of our game is based on the assumptions of Aubert *et al.* (2006). Two symmetric, risk-neutral firms $i \in \{1, 2\}$ play an infinitely repeated game where, in each period, they have the opportunity to communicate before interacting on the product market. In the first stage, communication takes place if both firms agree to communicate. In the second stage, firms can always choose the strategy 'compete'. If communication took place, they can choose the strategy 'collude' instead.¹ Both firms have the same discount factor $\delta_i \in (0, 1)$ and maximize the expected discounted sum of their profits. An antitrust authority detects a collusive agreement with probability ρ in every period and imposes a fine F on each firm. In each period, the product market profit of firm i is:

- $\pi_{i,c}$ if both firms compete,
- $\pi_{i,k} - \rho F$ if both firms collude,
- $\pi_{i,d} - \rho F$ if firm i competes and firm $-i$ colludes,
- $\pi_{i,-d} - \rho F$ if firm i colludes and firm $-i$ competes.

Further below we analyze three types of models, these are, Cournot competition with homogeneous products, Bertrand competition with homogeneous products, and Bertrand competition with differentiated products. We consider parameterizations where (1) applies.

$$\pi_{i,d} > \pi_{i,k} > \pi_{i,c} \geq \pi_{i,-d} \wedge \pi_{i,d} + \pi_{i,-d} < 2\pi_{i,k} \quad (1)$$

In addition to Aubert *et al.* (2006), we assume that firm i may hold a stake α_i in firm $-i$ while firm $-i$ may hold a stake α_{-i} in firm i . In line with Flath (1991), Gilo *et al.* (2006)

¹Along with Aubert *et al.* (2006) communication is not treated in a game theoretic context as a device to overcome problems arising from, e.g., incomplete information about each firm's type. Those considerations are absent from our model. Communication is rather treated in an antitrust context as a prerequisite for explicit collusion. It leaves traces that can be discovered by an antitrust authority, which would ultimately prosecute the firms. Hence, communication is merely needed to establish that firms violate antitrust laws.

and Shelegia and Spiegel (2012) each firm i is assumed to maximize its total payoff, i.e., the profit earned in the product market plus its share α_i in the profits of the other firm $-i$. This assumption is shown in (2).

$$\max \hat{\pi}_i = \pi_i + \alpha_i \hat{\pi}_{-i} \quad (2)$$

To be specific about notation, π_i denotes the reduced product-market profit / operating profit of firm i . It depends on the value of the shareholdings α_i and α_{-i} , i.e., $\pi_i(\alpha_i, \alpha_{-i})$, as well as exogenous variables such as demand parameters. For reasons of conciseness, we often write π_i instead of $\pi_i(\alpha_i, \alpha_{-i})$. In line with earlier literature (Reynolds and Snapp 1986, Malueg 1992, Gilo *et al.* 2006) the values of the shareholdings α_i and α_{-i} are assumed to have been chosen prior to the game analyzed here. The competitive, collusive, and deviant profits ($\hat{\pi}_{i,c}$, $\hat{\pi}_{i,k}$, $\hat{\pi}_{i,d}$) after dividends received can be expressed as in (3)-(5).

$$\hat{\pi}_{i,c} = \frac{\pi_{i,c} + \alpha_i \pi_{-i,c}}{1 - \alpha_i \alpha_{-i}} \quad (3)$$

$$\hat{\pi}_{i,k} = \frac{(\pi_{i,k} - \rho F) + \alpha_i (\pi_{-i,k} - \rho F)}{1 - \alpha_i \alpha_{-i}} \quad (4)$$

$$\hat{\pi}_{i,d} = \frac{(\pi_{i,d} - \rho F) + \alpha_i (\pi_{-i,d} - \rho F)}{1 - \alpha_i \alpha_{-i}} \quad (5)$$

3.2 The Stage Game: Unilateral Effects

This subsection establishes the unilateral effects of minority shareholdings. It analyzes how changes in α_i and α_{-i} affect the competitive, collusive, and deviant profits of firm i .

In **Cournot competition with homogeneous products**, firm i 's profit $\hat{\pi}(q_i, q_{-i})$ is a function of the outputs q_i and q_{-i} of the two firms. Appendix A shows that the best-response function $\hat{q}_i^R(q_{-i}, \alpha_i)$ of firm i depends on q_{-i} and α_i but not on α_{-i} . The firms compete if both play their best responses, making profits $\hat{\pi}_{i,c}(\alpha_i, \alpha_{-i}) = \hat{\pi}_i(\hat{q}_i^R(\alpha_i), \hat{q}_{-i}^R(\alpha_{-i}))$. The discussion

below relies on a result that has been established by Flath (1991) and is summarized in Lemma 1

Lemma 1. $\frac{\partial \pi_{i,c}}{\partial \alpha_i} < 0 \wedge \frac{\partial \pi_{i,c}}{\partial \alpha_{-i}} > 0$ in Cournot competition with homogeneous products

Proof. See Flath (1991) and Appendix A □

Lemma 1 implies that the competitive product-market profits $\pi_{i,c}$ of firm i (as opposed to the total payoff $\hat{\pi}_{i,c}$) rise in α_{-i} but fall in α_i . This is because quantities are strategic substitutes and firm i finds it optimal to reduce both its own output (i.e., $\partial \hat{q}_i^R / \partial \alpha_i < 0$) and its product-market profits in order to raise its total payoff $\hat{\pi}_{i,c}$ by receiving a higher dividend from firm $-i$.

In line with the related literature (Malueg 1992, Aubert *et al.* 2006, Gilo *et al.* 2006), the firms are assumed to collude in the product market by setting a 50%-share of the monopoly output (i.e., $q_{i,k} = q_{-i,k} = Q_k/2$), which is independent of α_i and α_{-i} . Therefore, the collusive profits are independent of the value of shareholdings (i.e., $\partial \pi_{i,k} / \partial \alpha_i = 0$, $\partial \pi_{i,k} / \partial \alpha_{-i} = 0$) by assumption.

Deviation profits are defined as $\hat{\pi}_{i,d}(\alpha_i) = \hat{\pi}_i(\hat{q}_i^R(\alpha_i), q_{-i,k})$ and $\hat{\pi}_{-i,-d}(\alpha_i) = \hat{\pi}_{-i}(\hat{q}_i^R(\alpha_i), q_{-i,k})$. Lemma 2 establishes the effect of α_i on the product market profits in a deviation period.

Lemma 2. $\frac{\partial \pi_{i,d}}{\partial \alpha_i} < 0 \wedge \frac{\partial \pi_{-i,-d}}{\partial \alpha_i} > 0$

Proof. See Appendix A □

When firm i deviates from collusion, it receives a lower dividend from firm $-i$ as compared to continued collusion ($\alpha_i \pi_{-i,-d} < \alpha_i \pi_{-i,k}$). The higher the value of α_i the stronger is this effect and the lower is the profit $\hat{\pi}_{i,d}$ that firm i earns after the payment of dividends. Accordingly, cross-shareholdings $\alpha_i > 0$ induce the deviating firm i to set a lower deviation quantity than with $\alpha_i = 0$ and, thus, earn lower deviation profits. This leaves higher profits for firm $-i$, i.e., $\partial \pi_{-i,-d} / \partial \alpha_i > 0$. Lemma 2 goes back to Malueg (1992) who, however, uses a profit function that is somewhat different from (2) (see Section 5).

Bertrand competition with differentiated products produces similar effects that, however, are still different in important respects. Firm i 's profit $\hat{\pi}(p_i, p_{-i})$ is a function of the prices p_i and p_{-i} of the two firms. Appendix A shows that the best-response function $\hat{p}_i^R(p_{-i}, \alpha_i)$ of firm i depends on p_{-i} and α_i but not on α_{-i} . The firms compete if both play their best responses, making profits $\hat{\pi}_{i,c}(\alpha_i, \alpha_{-i}) = \hat{\pi}_i(\hat{p}_i^R(\alpha_i), \hat{p}_{-i}^R(\alpha_{-i}))$. Flath (1991) establishes Lemma 3.

Lemma 3. $\partial\pi_{i,c}/\partial\alpha_i > 0$ if $|\alpha_i - \alpha_{-i}| < \Delta\alpha^*$ and $\partial\pi_{i,c}/\partial\alpha_{-i} > 0$ in Bertrand competition with differentiated products

Proof. See Flath (1991) and Appendix A □

Unlike in Cournot competition with homogeneous products, firm i 's competitive profit rises even for unilateral increases of its share α_i in firm $-i$ as long as α_i and α_{-i} are not too asymmetric. This is because in a Bertrand model with differentiated products prices are strategic complements. Shareholdings α_i induce firm i to raise its price, and firm $-i$ follows suit. Therefore, even an asymmetric increase in α_i may cause unilateral effects.

The firms are assumed to collude in the product market by setting the same prices $p_{i,k}$ and $p_{-i,k}$ that a jointly profit-maximizing monopolist would set. These prices are independent of α_i and α_{-i} , which implies $\partial\pi_{i,k}/\partial\alpha_i = 0$ and $\partial\pi_{i,k}/\partial\alpha_{-i} = 0$. The deviation profits are defined as $\hat{\pi}_{i,d}(\alpha_i) = \hat{\pi}_i(\hat{p}_i^R(\alpha_i), p_{-i,k})$ and $\hat{\pi}_{-i,d}(\alpha_i) = \hat{\pi}_{-i}(\hat{p}_i^R(\alpha_i), p_{-i,k})$. Appendix A shows that Lemma 2 (i.e., $\partial\pi_{i,d}/\partial\alpha_i < 0$ and $\partial\pi_{-i,d}/\partial\alpha_i > 0$) applies also in Bertrand competition with differentiated goods.

In **Bertrand competition with homogeneous products** – as was assumed by Gilo *et al.* (2006) – ”both firms set prices equal to marginal cost regardless of the state of any partial cross shareholding” (Flath 1991), i.e., the firms make zero profits ($\pi_{i,c} = 0$, $\partial\pi_{i,c}/\partial\alpha_i = 0$, and $\partial\pi_{-i,c}/\partial\alpha_i = 0$). Similarly, the collusive and the deviation profits are also independent of the value of minority shareholdings (i.e., $\partial\pi_{i,k}/\partial\alpha_i = 0$, $\partial\pi_{-i,k}/\partial\alpha_i = 0$, $\partial\pi_{i,d}/\partial\alpha_i = 0$, $\partial\pi_{-i,d}/\partial\alpha_i = 0$). A deviating firm would cut the collusive price marginally and earn

$\pi_{i,d} = 2\pi_{i,k}$ while the betrayed firm would earn $\pi_{-i,-d} = 0$. Under Bertrand competition with homogeneous products, minority shareholdings do not cause unilateral effects.

4 The Dynamic Game: Coordinated Effects

Using the framework introduced in Section 3, we study the effects of NCMS on the sustainability of collusion. Subsection 4.1 points out the forces that determine the effect of α_i on the critical discount factor δ_i^* in a general model. Subsection 4.2 applies this analysis to specific models of competition. Subsection 4.3 analyzes the effect of firm $-i$'s shareholdings α_{-i} on firm i 's critical discount factor δ_i^* . This effect has not been analyzed by any of the related papers.

4.1 The Critical Discount Factor

Collusion is profitable for the firms if inequality (6) is satisfied.

$$\pi_{i,k} - \pi_{i,c} > \rho F \quad \forall \quad i \in \{1, 2\} \quad (6)$$

Collusion is sustainable if inequality 7 applies.

$$\frac{\hat{\pi}_{i,k}}{1 - \delta_i} > \hat{\pi}_{i,d} + \frac{\delta_i}{1 - \delta_i} \hat{\pi}_{i,c} \quad \forall \quad i \in \{1, 2\} \quad (7)$$

The present value of deviation payoffs (i.e., the right-hand side of (7)) assumes a grim trigger strategy (Friedman 1971). This assumption is made to keep the model consistent with prior literature (Malueg 1992, Aubert *et al.* 2006, Gilo *et al.* 2006). Under the assumption of equation (2), the sustainability constraint 7 can be solved for the critical value δ^* of the discount factor as is shown in equation (8).

$$\delta_i > \frac{\hat{\pi}_{i,d} - \hat{\pi}_{i,k}}{\hat{\pi}_{i,d} - \hat{\pi}_{i,c}} = \frac{(\pi_{i,d} - \pi_{i,k}) + \alpha_i(\pi_{-i,-d} - \pi_{-i,k})}{(\pi_{i,d} - \rho F - \pi_{i,c}) + \alpha_i(\pi_{-i,-d} - \rho F - \pi_{-i,c})} \equiv \delta_i^* \quad (8)$$

For individual discount factors above this threshold, collusion is a stable outcome.

Proposition 1 establishes under which condition the critical discount factor rises in the value of minority shareholdings.

Proposition 1. *The inequality $\frac{\partial \delta_i^*}{\partial \alpha_i} > 0$ applies if inequality (9) is satisfied.*

$$\frac{(\pi_{i,k} - \rho F - \pi_{i,c})(\pi_{i,d} - \pi_{-i,-d})}{(\pi_{i,d} + \alpha_i \pi_{-i,-d}) - (\pi_{i,k} + \alpha_i \pi_{-i,k})} < \frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i} \quad (9)$$

Proof. See the Appendix. □

The terms in equation (9) take the following signs: Collusion must be profitable to be established ($\pi_{i,k} - \rho F - \pi_{i,c} > 0$, see equation (6)). The deviating firm earns a higher profit than the betrayed firm ($\pi_{i,d} - \pi_{-i,-d} > 0$). Given the prisoner dilemma structure of the game a firm earns a higher profit (including dividends) by deviating from a collusive agreement as compared to adhering to it ($(\pi_{i,d} + \alpha_i \pi_{-i,-d}) - (\pi_{i,k} + \alpha_i \pi_{-i,k}) > 0$). The term $\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i}$ captures the unilateral effects of the minority shareholdings as were established in Lemmas 1 and 3.

Higher shareholdings α_i have two effects on the sustainability of collusion. Effect 1: Malueg (1992) argues that higher shareholdings lower the sustainability of collusion by causing unilateral effects and softening the punishment following a deviation. This can also be seen in our model where one finds $\partial \delta_i^* / \partial \alpha_i > 0$ when the right-hand side of inequality (9) is sufficiently high. Effect 2: NCMS have a stabilizing effect on collusion because a higher value of α_i causes a greater loss of dividend income when firm i deviates from the collusive agreement. This can be seen from inequality (9) because $\alpha_i(\pi_{-i,k} - \pi_{-i,-d})$ measures the loss of dividends received from firm $-i$ when firm i deviates. A higher value of α_i raises the left-hand side of (9) and, thus, contributes to situations with $\partial \delta_i^* / \partial \alpha_i < 0$.

This analysis contributes to Malueg (1992) who concentrates on the study of Cournot competition with homogeneous goods and symmetric shareholdings. Our model is more general because in Subsection 4.2 it also analyzes Bertrand competition with either differentiated

or homogeneous products. It allows for asymmetric shareholdings, too. More importantly, we prove these effects in a formal and fairly general model while Malueg (1992) often relies on numeric examples.

To the best of our knowledge, we are the first to study the effect of an antitrust authority on the stability of collusion when firms hold minority shares. This is done by solving inequality (9) for ρF , which yields (10).

$$\rho F > \pi_{i,k} - \pi_{i,c} - \frac{\left[\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i} \right] [(\pi_{i,d} + \alpha_i \pi_{-i,d}) - (\pi_{i,k} + \alpha_i \pi_{-i,k})]}{\pi_{i,d} - \pi_{-i,-d}} \equiv (\rho F)^* \quad (10)$$

One finds $\partial \delta_i^* / \partial \alpha_i > 0$ when $\rho F > (\rho F)^*$ applies. A high value of ρF causes the expected dividend income of firm i to be low even when the firm adheres to the collusive agreement. Therefore, a high value of ρF weakens the stabilizing effect 2 from above, which stated that α_i makes firm i more hesitant to deviate as this would lower its dividend income. Technically, a higher value of ρF lowers the left-hand side of (9), which makes it more likely to find $\partial \delta_i^* / \partial \alpha_i > 0$. As effect 2 is weakened for higher values of ρF , effect 1 gains importance, which stated that shareholdings α_i stabilize collusion especially when they cause unilateral effects and soften punishments.

4.2 Coordinated Effects in Different Models of Competition

Our model confirms the results of Gilo *et al.* (2006) who study the stability of collusion in an infinitely repeated **Bertrand model with homogeneous goods** and cost-symmetric firms. They suggest that an increase in α_i never hinders collusion. This is because their model is characterized by the absence of unilateral effects ($\partial \pi_{i,c} / \partial \alpha_i = 0, \partial \pi_{-i,c} / \partial \alpha_{-i} = 0$). Therefore, the right-hand side of inequality (9) takes a value of zero, which implies $\partial \delta_i^* / \partial \alpha_i < 0 \quad \forall \quad \alpha_i$. In the absence of unilateral effects the collusion-destabilizing effect 1 from Proposition 1 vanishes because NCMS cannot destabilize collusion by softening punishments. This only leaves the second effect from Proposition 1: Higher shareholdings α_i stabilize collusion because firm i would receive a lower dividend income otherwise, i.e., when deviating from a collusive

agreement.

Our model also contributes to Malueg (1992) who studies the effect of minority shareholdings on collusion in a model of **Cournot competition with homogeneous goods**. He assumes a duopoly with two cost-symmetric firms, too, but studies the effect of symmetric shareholdings ($\alpha_i = \alpha_{-i}$) only and proves his results by recourse to numeric models. We add to his analysis by, first, allowing for asymmetric shareholdings, second, proving most of our results analytically and, third, introducing a competition authority. Note that Malueg (1992) uses a profit function that is somewhat different from equation (2). This difference is minor as will be discussed in Section 5.

In a Cournot model, an increase in shareholdings causes unilateral effects ($\partial\pi_{i,c}/\partial\alpha_i > 0, \partial\pi_{-i,c}/\partial\alpha_i > 0$; see Lemma 1) such that the right-hand side of equation (9) is positive. It depends on the absolute value of the unilateral effects whether one finds $\partial\delta_i^*/\partial\alpha_i > 0$ or $\partial\delta_i^*/\partial\alpha_i < 0$. Malueg (1992) shows that a symmetric increase of shareholdings α_i raises δ_i^* for convex demand but not for linear or concave demand. We complement Malueg's (1992) findings by proving Proposition 2.

Proposition 2. *In a Cournot model and in the presence of an effective antitrust authority with $\rho F > (\rho F)^*$, a symmetric increase of shareholdings α_i raises δ_i^* even for non-convex demand.*

Proof. To be added. □

Proposition 2 shows one of our main results: In the presence of an effective antitrust authority, an increase in shareholdings α_i has pro-competitive effects by raising the critical discount factor δ_i^* in even more situations than was predicted by Malueg (1992).

Lemma 3 established that in **Bertrand competition with differentiated goods** NCMS cause unilateral effects for both firms ($\partial\pi_{i,c}/\partial\alpha_i > 0 \wedge \partial\pi_{i,c}/\partial\alpha_{-i} > 0$) even for asymmetric increases of the shareholdings α_i and α_{-i} , whereas Lemma 1 established that in the Cournot model with homogeneous goods the inequalities $\partial\pi_{i,c}/\partial\alpha_i > 0 \wedge \partial\pi_{i,c}/\partial\alpha_{-i} > 0$

are only found for symmetric increases of the shareholdings. This suggests that the right-hand side of inequality (9) (i.e., $\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i}$) may especially for asymmetric values of α_i and α_{-i} and sufficiently differentiated goods be greater in Bertrand-competition with differentiated goods than in Cournot-competition with homogeneous goods. This finding supports our suggestion from above that pro-competitive effects of NCMS may be much more prevalent than is suggested by Malueg's (1992) seminal contribution alone.

4.3 The Effect of α_{-i} on δ_i^*

Our previous analysis was concerned with the effects of firm i 's stake in firm $-i$ on the critical discount factor δ_i^* of firm i only. Proposition 3 establishes an important result when studying the effect of firm $-i$'s stake α_{-i} on the critical discount factor δ_i^* of firm i . To the best of our knowledge, none of the related papers has ever studied $\partial \delta_i^* / \partial \alpha_{-i}$.

Proposition 3. $\frac{\partial \delta_i^*}{\partial \alpha_{-i}} \geq 0 \quad \forall \quad \alpha_{-i}$

Proof. See the Appendix. □

Proposition 3 suggests an important result, i.e., the critical discount factor δ_i^* is non-negative in the value of firm $-i$'s shareholdings. This is even true when α_{-i} lowers firm $-i$'s product market profits ($\partial \pi_{-i,c} / \partial \alpha_{-i} < 0$). In Bertrand competition with homogeneous products (with $\partial \pi_{i,c} / \partial \alpha_i = 0$ and $\partial \pi_{-i,c} / \partial \alpha_i = 0$) one finds $\partial \delta_i^* / \partial \alpha_{-i} = 0$. This leaves Gilo *et al.*'s (2006) conclusion intact, i.e., NCMS never hinder collusion. However, in Cournot competition with homogeneous goods and Bertrand competition with differentiated goods firm i 's critical discount factor δ_i^* always rises when firm $-i$ expands its shareholdings α_{-i} . This supports our hypothesis that pro-competitive effects of NCMS are rather prevalent.

4.4 Endogenous Detection Probability

Reynolds and Snapp (1986, p. 149) hypothesize that partial "ownership [...]" could actually be counter-productive if such an involvement (however small) drew the attention of antitrust

agencies.” None of the related models of NCMS has formally modeled a competition authority. Therefore, we are the first to study how an increase in the detection probability ρ that is caused by an increase in the shareholdings α_i affects the effect of α_i on the critical discount factor δ_i^* . Our study suggests Proposition 4.

Proposition 4. *If $\frac{\partial \rho}{\partial \alpha_i} > \left(\frac{\partial \rho}{\partial \alpha_i}\right)^*$ then $\frac{\partial \delta_i^*}{\partial \alpha_i} > 0$ even in Bertrand competition with homogeneous goods.*

$$\left(\frac{\partial \rho}{\partial \alpha_i}\right)^* = \frac{2(\pi_{i,k} - \rho F)}{(1 - \alpha_i^2)F} \quad (11)$$

Proof. See Appendix 4 □

When the value of the detection probability rises strongly enough in α_i , cross-shareholdings α_i make collusion harder to sustain. Proposition 4 implies that higher shareholdings α_i may raise the critical discount factor δ_i^* even in a Bertrand model with homogeneous goods. This reverses some of the results of Gilo *et al.* (2006) who argued that in their model NCMS never hinder collusion. Again, our results imply that minority shareholdings have a more detrimental effect on collusion than is commonly thought.

5 Robustness Analysis

Up to this point, the article relied on profit function (12), i.e., each firm was assumed to maximize its total payoff, i.e., the profit earned in the product market plus its share α_i in the product market profits of the other firm. A different profit function has been used by Reynolds and Snapp (1986) and Malueg (1992), who assumed that the firms maximize the sum of product-market profits after the payment of dividends plus the dividends received as is shown in (12).

$$\max \tilde{\pi}_i = (1 - \alpha_{-i})\pi_i + \alpha_i\pi_{-i} \quad (12)$$

The present section demonstrates that the same qualitative results are found when assuming profit functions (2) or (12). We show in Appendix C that Lemma 1 (i.e.,

$\partial\pi_{i,c}/\partial\alpha_i < 0 \wedge \partial\pi_{i,c}/\partial\alpha_{-i} > 0$ in Cournot competition with homogeneous products), Lemma 3 (i.e., $\partial\pi_{i,c}/\partial\alpha_i > 0$ if $|\alpha_i - \alpha_{-i}| < \Delta\alpha^{**}$ and $\partial\pi_{i,c}/\partial\alpha_{-i} > 0$ in Bertrand competition with differentiated products), and Lemma 2 (i.e., $\partial\pi_{i,d}/\partial\alpha_i < 0 \wedge \partial\pi_{-i,-d}/\partial\alpha_i > 0$) also apply when assuming profit function (12).

Moreover, let $\tilde{\delta}_i^*$ denote the critical discount factor under the assumption of profit function (12). Proposition 5 establishes that the qualitative results of our model are no different when assuming profit function (12).

Proposition 5. *The inequality $\frac{\partial\tilde{\delta}_i^*}{\partial\alpha_i} > 0$ applies under the following condition:*

$$\frac{(1 - \alpha_{-i})[(\pi_{i,k} - \rho F - \pi_{i,c})\pi_{-i,-d} - (\pi_{-i,k} - \rho F - \pi_{-i,c})\pi_{i,d} - (\pi_{i,c} - \pi_{-i,c})\pi_{i,k}]}{(1 - \alpha_{-i})(\pi_{i,d} - \pi_{i,k}) + \alpha_i(\pi_{-i,-d} - \pi_{-i,k})} < (1 - \alpha_{-i})\frac{\partial\pi_{i,c}}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{-i,c}}{\partial\alpha_i} \quad (13)$$

Proof. See the Appendix □

Proposition 5 establishes the condition under which an increase in α_i raises the critical discount factor $\tilde{\delta}_i^*$ and compares to Proposition 1. The interpretation of both propositions is the same. NCMS contribute to raising the critical discount factor $\tilde{\delta}_i^*$ by softening competition, i.e., they cause unilateral effects that are sufficiently strong (effect 1). NCMS have a depressing effect on $\tilde{\delta}_i^*$ because a deviation by firm i results in lower dividends received from firm $-i$ (effect 2). A high value of ρF facilitates situations with $\partial\tilde{\delta}_i^*/\partial\alpha_i > 0$. These are the same effects that were derived above under Gilo *et al.*'s (2006) profit function (2). Therefore Proposition 5 shows that our results are equally valid when assuming Malueg's (1992) profit function (12).

6 Evidence

The contribution of this article is mainly theoretical. Yet, this section presents some anecdotal evidence that appears to be in line with the predictions of the model.

Proposition 1 in combination with equation (10) suggests that in the presence of an antitrust authority pursuing an effective anti-cartel policy minority shareholdings contribute

to destabilizing collusion. This suggests the following hypotheses for empirical work: Explicit collusion among firms holding NCMS in each other could be observed in past times when antitrust enforcement had not been as effective as today (Hypothesis 1), in jurisdictions where antitrust enforcement is still relatively weak (Hypothesis 2), and in situations where firms are not aware of or do not pay attention to antitrust enforcement (Hypothesis 3).

Some evidence supporting Hypothesis 1 is provided by Leslie (2004) showing that cross-shareholdings among cartel firms were mainly a phenomenon of the first half of the 20th century. Leslie (2004, p. 581-583) names examples of cartels from this era such as aluminum or lamps: For example, Alcoa purchased an interest in Norsk Aluminium Company, Det Norske Nitrid, and Societa dell'Alluminio Italiano. Similarly, by 1935 General Electric possessed stocks of several other lamp producers such as Osram and Philips. When cartel enforcement became more effective in the second half of the 20th century, the colluding firms apparently refrained from acquiring such shares. Therefore, more recent examples of cartels among firms holding minority shares in each other are harder to find and may possibly be explained along the lines of Hypotheses 2 and 3.

The European Needles cartel of the 1990s may serve as an example for Hypothesis 3, i.e., the firms did not pay much attention to antitrust enforcement.² Three firms took part in the conspiracy: William Prym GmbH & Co. KG, Coats Holdings Ltd, and Entaco Ltd. In 1994 William Prym acquired a minority share of 10.1% in Entaco, which might serve as an example of minority shareholdings among colluding firms. However, rather than keeping the conspiracy secret the firms entered into a series of *written* market sharing agreements to partition both product and geographic markets. Interestingly, the lawyers of the firms had been involved in making at least some of these agreements. This may suggest that the firms were not aware of the illegality of their conduct.

Some evidence for Hypothesis 2 was provided by the OECD (2009): In Turkey, minority shareholdings played a role in the cartels among aerated concrete producers and

²The description of the Needles case F-1/38.338 is based on the Commission Decision as of October 10, 2004: <http://goo.gl/ZcUWv1>, accessed on February 5, 2015

scheduled maritime transportation by roll-on/roll-off vessels. In Chinese Taipei (Taiwan) cross-shareholdings could be observed among two cable TV service providers who engaged in market allocation in the years prior to 2003. Note that the Turkish competition authority was established in 1997, and the FTC of Taiwan in 1992. It may be hypothesized that competition enforcement in these countries is not as strong, yet, as, for example, in the United States or Europe and that firms may not always take antitrust laws into account when making business decisions. This was suggested by Hypothesis 2.

7 Conclusion

This article presents a coherent framework for the analysis of the unilateral and the coordinated effects of minority shareholdings. This is done by combining the established models of Reynolds and Snapp (1986), Flath (1991), Malueg (1992), and Gilo *et al.* (2006). These earlier contributions are not always readily comparable, first, because they use different profit functions. Second, some combinations of assumptions have – so far – remained unexplored. Third, they sometimes rely on numerical proofs. We fill these gaps in the literature by studying the ‘missing’ combinations of assumptions and thus making the earlier papers better comparable. The properties of the model are proven analytically. Additionally, we extend this literature by – to the best of our knowledge – being the first to add an antitrust authority (Aubert *et al.* 2006) to a model of NCMS.

Malueg (1992) pointed out a trade-off: The existence of unilateral effects may facilitate situations where the critical discount factor rises in the level of NCMS, which helps to prevent coordinated effects. Our study indicates that NCMS lower the sustainability of collusion under an even greater variety of circumstances than was acknowledged by this earlier literature. NCMS have a particularly detrimental effect on collusion in the presence of an antitrust authority that pursues an effective anti-cartel policy. Case evidence seems to support the prediction that in jurisdictions with effective anti-cartel enforcement minority

shareholdings among cartel firms are rare even today.

Our analysis raises the question whether antitrust authorities should not be concerned much with the coordinated effects of minority shareholdings as long as they maintain an effective enforcement of cartels? And – more provocatively – should antitrust authorities accept some acquisitions of NCMS even if they cause unilateral effects? This is because coordinated effects may be reduced in the presence of unilateral effects, and the net effect on consumer surplus may be positive. We are somewhat reluctant to answer these questions based on the present model only. This is because several extensions of the model should ideally be studied before giving such policy advice.

For example, it will be interesting to add an acquisition stage to the present model that endogenizes firms' decision to acquire NCMS. The model should also be augmented by several assumptions that have been studied in models on collusion without NCMS but have not been considered in the context of minority shareholdings, yet. For example, assuming optimal penal codes instead of a grim trigger strategy might prevent situations where unilateral effects of NCMS lower the stability of collusion. The firms might also decide about splitting the collusive profits unequally or make side payments, which would be especially important when assuming cost-asymmetric firms. In order to give policy advice it will also be helpful to explicitly model the antitrust authority's function of enforcement costs. Moreover, one needs to consider certain efficiency justifications for acquiring NCMS.

The methodological value of the present article lies in presenting a comprehensive model that integrates established models on NCMS and is ready to be extended in a variety of directions. We explore one of these directions by adding a competition authority.

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A Appendix to Section 3

Appendix A starts by proving Lemmas 1 and 2 along with Propositions 1 and 3 while assuming objective function (2) and Cournot competition with homogeneous goods and constant marginal costs. More specifically, we assume marginal costs of zero which is innocuous to our proofs.

Proof of Lemma 1 for Cournot competition with homogeneous goods and profit fn (2).

Lemma 1 proposes $\frac{\partial \pi_{i,c}}{\partial \alpha_i} < 0 \wedge \frac{\partial \pi_{i,c}}{\partial \alpha_{-i}} > 0$. To see this, reaction function (16) is determined by maximizing profit function (14) with respect to q_i .

$$\hat{\pi}_i = \frac{1}{1 - \alpha_i \alpha_{-i}} [p(q_i, q_{-i})q_i + \alpha_i p(q_i, q_{-i})q_{-i}] \quad (14)$$

$$\frac{\partial \hat{\pi}_i}{\partial q_i} = \frac{1}{1 - \alpha_i \alpha_{-i}} \left[\frac{\partial p}{\partial q_i} q_i + p(q_i, q_{-i}) + \alpha_i \frac{\partial p}{\partial q_i} q_{-i} \right] \stackrel{!}{=} 0 \quad (15)$$

$$\hat{q}_i^R(q_{-i}) = q_i^R(q_{-i}) - \alpha_i \theta q_{-i} \quad (16)$$

The term $q_i^R(q_{-i})$ denotes firm i 's reaction function when it maximizes product market profits π_i only. For specific demand curves (such as $p = (1 - q_i - q_{-i})^x$ with $x > 0$ (Malueg 1992)) one can prove $-1 < \frac{\partial q_i^R(q_{-i})}{\partial q_{-i}} < 0$. The term $\hat{q}_i^R(q_{-i})$ denotes firm i 's reaction function when it maximizes accounting profits $\hat{\pi}_i$. The variable θ denotes a scaling factor with $0 < \theta < 1$. For $p = (1 - q_i - q_{-i})^x$ this parameter takes the form $\theta = \frac{x}{1+x}$. For $\alpha_i > 0$, one finds $\hat{q}_i^R(q_{-i}) < q_i^R(q_{-i})$ which implies $\frac{\partial \hat{q}_i^R}{\partial \alpha_i} < 0$. For a given value of q_{-i} this implies $\frac{\partial \pi_i(q_{-i})}{\partial \alpha_i} < 0$, since q_i^R maximizes π_i . Using $\frac{\partial \hat{q}_{-i}^R(q_i)}{\partial q_i} < 0$ and $\hat{q}_i^R(q_{-i}) < q_i^R(q_{-i})$ one finds $\hat{q}_{-i}^R(\hat{q}_i^R) > \hat{q}_{-i}^R(q_i^R)$,

i.e., $\frac{\partial \hat{q}_i^R}{\partial \alpha_i} > 0$. From $-1 < \frac{\partial \hat{q}_i^R(q_i)}{\partial q_i}$ it follows $\frac{\partial \hat{q}_i^R}{\partial \alpha_i} = \frac{\partial \hat{q}_i^R}{\partial q_i} \cdot \frac{\partial \hat{q}_i^R}{\partial \alpha_i} < -\frac{\partial \hat{q}_i^R}{\partial \alpha_i}$, such that $\frac{\partial(\hat{q}_i^R + \hat{q}_{-i}^R)}{\partial \alpha_i} < 0$ and $\frac{\partial p(\hat{q}_i^R + \hat{q}_{-i}^R)}{\partial \alpha_i} > 0$. Bringing together $\frac{\partial p(\hat{q}_i^R + \hat{q}_{-i}^R)}{\partial \alpha_i} > 0$ and $\frac{\partial \hat{q}_i^R}{\partial \alpha_i} > 0$ proves $\frac{\partial \pi_{-i,c}}{\partial \alpha_i} > 0$. Combining $\frac{\partial \pi_i(q_{-i})}{\partial \alpha_i} < 0$ (for a given q_{-i}), $\frac{\partial \pi_i(q_{-i})}{\partial q_{-i}} < 0$, and $\frac{\partial \hat{q}_i^R}{\partial \alpha_i} > 0$ proves $\frac{\partial \pi_{i,c}}{\partial \alpha_i} < 0$. \square

Proof of Lemma 2 for Cournot competition with homogeneous goods. Lemma 2 proposes $\frac{\partial \pi_{i,d}}{\partial \alpha_i} < 0 \wedge \frac{\partial \pi_{-i,-d}}{\partial \alpha_i} > 0$. The assumption $q_{-i,-d} = q_{-i,k}$ implies $\frac{\partial q_{-i,-d}}{\partial \alpha_i} = 0$. Reaction function (16) with $\frac{\partial \hat{q}_i^R(q_{-i,-d})}{\partial \alpha_i} < 0$ causes $\frac{\partial(q_{-i,-d} + \hat{q}_i^R(q_{-i,-d}))}{\partial \alpha_i} < 0$ and thus $\frac{\partial p_d}{\partial \alpha_i} > 0$. This proves $\frac{\partial \pi_{-i,-d}}{\partial \alpha_i} = \frac{\partial p_d}{\partial \alpha_i} \cdot q_{-i,-d} > 0$. Finding $\frac{\partial \hat{q}_i^R(q_{-i,-d})}{\partial \alpha_i} < 0$ also implies that with α_i firm i sets a lower than the profit-maximizing output $q_i^R(q_{-i,-d}) > \hat{q}_i^R(q_{-i,-d})$ which causes $\pi_i(q_i^R, q_{-i,-d}) > \pi_i(\hat{q}_i^R, q_{-i,-d})$. This proves $\frac{\partial \pi_{i,d}}{\partial \alpha_i} < 0$. \square

We turn to Bertrand competition with differentiated goods and constant marginal costs.

Proof of Lemma 3. Lemma 3 proposes that $\partial \pi_{i,c} / \partial \alpha_i \lesseqgtr 0 \wedge \partial \pi_{i,c} / \partial \alpha_{-i} > 0$. To see this, we derive the reaction function (19) by maximizing profit function 17 with respect to p_i

$$\hat{\pi}_i = \frac{1}{1 - \alpha_i \alpha_{-i}} [q_i(p_i, p_{-i})p_i + \alpha_i q_{-i}(p_i, p_{-i})p_{-i}] \quad (17)$$

$$\frac{\partial \hat{\pi}_i}{\partial p_i} = \frac{1}{1 - \alpha_i \alpha_{-i}} \left[\frac{\partial q_i}{\partial p_i} p_i + q_i(p_i, p_{-i}) + \alpha_i \frac{\partial q_{-i}}{\partial p_i} p_{-i} \right] \stackrel{!}{=} 0 \quad (18)$$

$$\hat{p}_i^R = p_i^R(p_{-i}) + \alpha_i B p_{-i} \quad (19)$$

The term $p_i^R(p_{-i})$ denotes firm i 's reaction function when it maximizes product market profits π_i only. The term $\hat{p}_i^R(p_{-i})$ denotes firm i 's reaction function when it maximizes accounting profits $\hat{\pi}_i$. For specific demand curves (such as $q_i = 1 - p_i + \beta p_{-i}$ with $0 < \beta < 1$) one can prove $0 < \frac{\partial p_i^R(p_{-i})}{\partial p_{-i}} < 1$ and finds $B = \beta/2 > 0$. For $\alpha_i > 0$, one finds

$\hat{p}_i^R(p_{-i}) > p_i^R(p_{-i})$ which implies $\frac{\partial \hat{p}_i^R}{\partial \alpha_i} > 0$. Given $\frac{\partial \hat{p}_i^R}{\partial \alpha_{-i}} = \left(\frac{\partial p_i^R}{\partial p_{-i}} \frac{\partial \hat{p}_{-i}^R}{\partial \alpha_{-i}} + \alpha_i B \frac{\partial \hat{p}_{-i}^R}{\partial \alpha_{-i}} \right)$, $\frac{\partial p_i^R}{\partial p_{-i}} > 0$, and $\frac{\partial \hat{p}_{-i}^R}{\partial \alpha_{-i}} > 0$ one finds $\frac{\partial \hat{p}_i^R}{\partial \alpha_{-i}} > 0$. For $q_i = 1 - p_i + \beta p_{-i}$ one finds $\frac{\partial \hat{p}_i^R}{\partial \alpha_{-i}} < \frac{\partial \hat{p}_{-i}^R}{\partial \alpha_{-i}}$ and equivalently $\frac{\partial \hat{p}_{-i}^R}{\partial \alpha_i} < \frac{\partial \hat{p}_i^R}{\partial \alpha_i}$. Using this inequality and $0 < \beta < 1$ one can show that $\frac{\partial q_i(\hat{p}_i^R, \hat{p}_{-i}^R)}{\partial \alpha_i} < 0$ applies. Finding that an increase in α_i raises the equilibrium price p_i and lowers the equilibrium quantity q_i suggests $\partial \pi_{i,c} / \partial \alpha_i \leq 0$. Firm $-i$ would only choose $\frac{\partial \hat{p}_{-i}^R}{\partial \alpha_{-i}} > 0$ if this causes $\frac{\partial \hat{\pi}_{-i}}{\partial \alpha_{-i}} > 0$. Therefore, one necessarily finds $\frac{\partial \pi_{i,c}}{\partial \alpha_{-i}} > 0$ if $\frac{\partial \pi_{-i,c}}{\partial \alpha_{-i}} \leq 0$. The inequality $\frac{\partial \pi_{-i,c}}{\partial \alpha_{-i}} \leq 0$ only applies for sufficiently pronounced increases of p_{-i} . For weaker increases of p_{-i} one still finds $\frac{\partial \pi_{i,c}}{\partial \alpha_{-i}} > 0$. □

Proof of Lemma 2 for Bertrand competition with differentiated goods. Lemma 2 proposes $\frac{\partial \pi_{i,d}}{\partial \alpha_i} < 0 \wedge \frac{\partial \pi_{-i,-d}}{\partial \alpha_i} > 0$. The assumption $p_{-i,-d} = p_{-i,k}$ implies $\frac{\partial p_{-i,-d}}{\partial \alpha_i} = 0$. Reaction function (19) with $\frac{\partial \hat{p}_i^R(p_{-i,-d})}{\partial \alpha_i} > 0$ causes $\frac{\partial q_{-i,-d}}{\partial \alpha_i} > 0$. This proves $\frac{\partial \pi_{-i,-d}}{\partial \alpha_i} = \frac{\partial q_{-i,-d}}{\partial \alpha_i} \cdot p_{-i,-d} > 0$. Finding $\frac{\partial \hat{p}_i^R(p_{-i,-d})}{\partial \alpha_i} > 0$ also implies that with α_i firm i sets a higher than the profit-maximizing price $p_i^R(p_{-i,-d}) < \hat{p}_i^R(q_{-i,-d})$ which causes $\pi_i(p_i^R, p_{-i,-d}) > \pi_i(\hat{p}_i^R, p_{-i,-d})$. This proves $\frac{\partial \pi_{i,d}}{\partial \alpha_i} < 0$. □

For Cournot competition with homogeneous goods, Lemma 1 proposes $\frac{\partial \pi_{i,c}}{\partial \alpha_i} < 0 \wedge \frac{\partial \pi_{i,c}}{\partial \alpha_{-i}} > 0$ such that the sign of $\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i}$ can, in principle, be ambiguous. The same is true for Bertrand competition with differentiated goods when the shareholdings α_i and α_{-i} are sufficiently asymmetric. Lemma 4 however proposes that this term is positive, which is needed to prove Propositions 1 and 3.

Lemma 4. $\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i} \geq 0$

Proof. Assume Cournot competition with homogeneous goods. We derive the accounting

profits $\hat{\pi}_{i,c}$ for α_i , as is shown in equation (20).

$$\begin{aligned}\frac{\partial \hat{\pi}_{i,c}}{\partial \alpha_i} &= \frac{\left(\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \pi_{-i,c} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i} \right) (1 - \alpha_i \alpha_{-i}) + \alpha_{-i} (\pi_{i,c} + \alpha_i \pi_{-i,c})}{(1 - \alpha_i \alpha_{-i})^2} \\ &= \frac{\hat{\pi}_{-i,c}}{1 - \alpha_i \alpha_{-i}} + \frac{\frac{\partial \pi_{i,c}}{\partial q_i} \frac{\partial \hat{q}_i^R}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial q_{-i}} \frac{\partial \hat{q}_{-i}^R}{\partial q_i} \frac{\partial \hat{q}_i^R}{\partial \alpha_i}}{1 - \alpha_i \alpha_{-i}}\end{aligned}\quad (20)$$

The shareholdings α_i affect $\pi_{i,c}$ directly via its effect on \hat{q}_i^R . The shareholdings α_i affect $\pi_{-i,c}$ only indirectly because the value of \hat{q}_{-i}^R is a function of $\hat{q}_i^R(\alpha_i)$. The option to keep its output constant $\left(\frac{\partial q_{i,c}}{\partial \alpha_i} = 0 \right)$ allows firm i to ensure that the second summand of (20), which is the same as $\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i}$, is not lower than zero. This proves Lemma 4 for Cournot competition with homogeneous goods.

In the following, we prove (21) for Bertrand competition with differentiated goods.

$$\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i} > 0 \quad (21)$$

Therefore, we derive $\hat{\pi}_{i,c}$ (see equation (17)) for α_i , as is shown in equation (22).

$$\begin{aligned}\frac{\partial \hat{\pi}_{i,c}}{\partial \alpha_i} &= \frac{\left(\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \pi_{-i,c} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i} \right) (1 - \alpha_i \alpha_{-i}) + \alpha_{-i} (\pi_{i,c} + \alpha_i \pi_{-i,c})}{(1 - \alpha_i \alpha_{-i})^2} \\ &= \frac{\hat{\pi}_{-i,c}}{1 - \alpha_i \alpha_{-i}} + \frac{\frac{\partial \pi_{i,c}}{\partial p_i} \frac{\partial \hat{p}_i^R}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial p_{-i}} \frac{\partial \hat{p}_{-i}^R}{\partial p_i} \frac{\partial \hat{p}_i^R}{\partial \alpha_i}}{1 - \alpha_i \alpha_{-i}}\end{aligned}\quad (22)$$

The shareholdings α_i affect $\pi_{i,c}$ directly via its effect on \hat{p}_i^R . The shareholdings α_i affect $\pi_{-i,c}$ only indirectly because the value of \hat{p}_{-i}^R is a function of $\hat{p}_i^R(\alpha_i)$. The option to keep its price constant $\left(\frac{\partial p_{i,c}}{\partial \alpha_i} = 0 \right)$ allows firm i to ensure that the second summand of (22), which is the same as $\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i}$, is not lower than zero. This proves Lemma 4 for Bertrand competition with differentiated goods.

□

Lemma 2 proposes $\frac{\partial \pi_{i,d}}{\partial \alpha_i} < 0 \wedge \frac{\partial \pi_{-i,-d}}{\partial \alpha_{-i}} > 0$ such that the sign of $\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,-d}}{\partial \alpha_i}$ can, in principle, be ambiguous. Lemma 5 however proposes that this term takes a value of zero, which is needed to prove Propositions 1 and 3.

Lemma 5. $\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,-d}}{\partial \alpha_i} = 0$

Proof. For Cournot competition with homogeneous goods, the equality stated in Lemma 5 can be restated as in (23).

$$\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,-d}}{\partial \alpha_i} = \frac{\partial q_i^R}{\partial \alpha_i} \left[\left(\frac{\partial p}{\partial q_i} q_i + p \right) + \alpha_i \left(\frac{\partial p}{\partial q_i} q_{-i} \right) \right] \quad (23)$$

The bracketed term in the right-hand side of equation (23) is the same as the bracketed term in first order condition 15 that takes a value of zero in the optimum.

For Bertrand competition with differentiated goods, the equality stated in Lemma 5 can be restated as in (24).

$$\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,-d}}{\partial \alpha_i} = \frac{\partial p_i^R}{\partial \alpha_i} \left[\left(\frac{\partial q_i}{\partial p_i} p_i + q_i \right) + \alpha_i \left(\frac{\partial q_{-i}}{\partial p_i} q_{-i} \right) \right] = 0 \quad (24)$$

The bracketed term in the right-hand side of equation (24) is the same as the bracketed term in first order condition 18 that takes a value of zero in the optimum. This proves Lemma 5.

□

B Appendix to Section 4

Proof of Proposition 1. To determine $\partial \delta_i^* / \partial \alpha_i$, re-write δ_i^* as follows.

$$\delta_i^* = \frac{u(\alpha_i, \alpha_{-i})}{v(\alpha_i, \alpha_{-i})} \quad (25)$$

$$\text{with } u(\alpha_i, \alpha_{-i}) = (\pi_{i,d} - \pi_{i,k}) + \alpha_i (\pi_{-i,-d} - \pi_{-i,k}) > 0$$

$$\text{and } v(\alpha_i, \alpha_{-i}) = (\pi_{i,d} - \rho F - \pi_{i,c}) + \alpha_i (\pi_{-i,-d} - \rho F - \pi_{-i,c}) > 0$$

Using $\frac{\partial \pi_{i,k}}{\partial \alpha_i} = \frac{\partial \pi_{-i,k}}{\partial \alpha_i} = 0$ and $\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,-d}}{\partial \alpha_i} = 0$ from Lemma 5, $\partial \delta_i^* / \partial \alpha_i$ can be written as in (26).

$$\begin{aligned} \frac{\partial \delta_i^*}{\partial \alpha_i} &= \frac{\frac{\partial u(\alpha_i, \alpha_{-i})}{\partial \alpha_i} \cdot v(\alpha_i, \alpha_{-i}) - \frac{\partial v(\alpha_i, \alpha_{-i})}{\partial \alpha_i} \cdot u(\alpha_i)}{v(\alpha_i, \alpha_{-i})^2} \\ \text{with } \frac{\partial u(\alpha_i, \alpha_{-i})}{\partial \alpha_i} &= \pi_{-i,-d} - \pi_{-i,k} \\ \text{and } \frac{\partial v(\alpha_i, \alpha_{-i})}{\partial \alpha_i} &= \pi_{-i,-d} - \rho F - \pi_{-i,c} - \left(\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i} \right) \end{aligned} \quad (26)$$

Given $v(\alpha_i, \alpha_{-i}) > 0$ the sign of $\frac{\partial \delta_i^*}{\partial \alpha_i}$ is the same as that of its numerator as is shown in (27).

$$\begin{aligned} \frac{\partial u(\alpha_i, \alpha_{-i})}{\partial \alpha_i} \cdot v(\alpha_i, \alpha_{-i}) - \frac{\partial v(\alpha_i, \alpha_{-i})}{\partial \alpha_i} \cdot u(\alpha_i, \alpha_{-i}) &= \dots \\ (\pi_{i,k} - \rho F - \pi_{i,c}) (\pi_{-i,-d} - \pi_{i,d}) + \left(\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i} \right) u(\alpha_i, \alpha_{-i}) \end{aligned} \quad (27)$$

Equation (27), and thus $\frac{\partial \delta_i^*}{\partial \alpha_i}$, is positive when inequality (9) applies. This proves Proposition 1. \square

Proof of Proposition 3. Proposition 3 suggests $\frac{\partial \delta_i^*}{\partial \alpha_{-i}} \geq 0$ even for $\frac{\partial \pi_{-i,c}}{\partial \alpha_{-i}} < 0$. Equation (28) provides $\frac{\partial \delta_i^*}{\partial \alpha_{-i}}$.

$$\begin{aligned} \frac{\partial \delta_i^*}{\partial \alpha_{-i}} &= \frac{\frac{\partial u(\alpha_i, \alpha_{-i})}{\partial \alpha_{-i}} \cdot v(\alpha_i, \alpha_{-i}) - \frac{\partial v(\alpha_i, \alpha_{-i})}{\partial \alpha_{-i}} \cdot u(\alpha_i, \alpha_{-i})}{v(\alpha_i, \alpha_{-i})^2} \\ \text{with } \frac{\partial u(\alpha_i, \alpha_{-i})}{\partial \alpha_{-i}} &= \left(\frac{\partial \pi_{i,d}}{\partial \alpha_{-i}} - \frac{\partial \pi_{i,k}}{\partial \alpha_{-i}} \right) + \alpha_i \left(\frac{\partial \pi_{-i,-d}}{\partial \alpha_{-i}} - \frac{\partial \pi_{-i,k}}{\partial \alpha_{-i}} \right), \\ \text{and } \frac{\partial v(\alpha_i, \alpha_{-i})}{\partial \alpha_{-i}} &= \left(\frac{\partial \pi_{i,d}}{\partial \alpha_{-i}} - \frac{\partial \pi_{i,c}}{\partial \alpha_{-i}} \right) + \alpha_i \left(\frac{\partial \pi_{-i,-d}}{\partial \alpha_{-i}} - \frac{\partial \pi_{-i,c}}{\partial \alpha_{-i}} \right) \end{aligned} \quad (28)$$

Using $\frac{\partial \pi_{i,k}}{\partial \alpha_i} = \frac{\partial \pi_{-i,k}}{\partial \alpha_i} = 0$ and $\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,-d}}{\partial \alpha_i} = 0$ from Lemma 5 equation (28) can be simplified as is shown in equation (29).

$$\frac{\partial \delta_i^*}{\partial \alpha_{-i}} = \frac{\left(\frac{\partial \pi_{i,c}}{\partial \alpha_{-i}} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_{-i}} \right) \cdot u(\alpha_i, \alpha_{-i})}{v(\alpha_{-i})^2} \quad (29)$$

Equation (29) was derived making also use of $\frac{\partial \pi_{i,d}}{\partial \alpha_{-i}} = 0 \wedge \frac{\partial \pi_{-i,-d}}{\partial \alpha_{-i}} = 0$. These equalities apply because of $q_{-i,-d} = q_{-i,k}$, which implies $\frac{\partial q_{i,d}}{\partial \alpha_{-i}} = 0$ and $\frac{\partial q_i^R}{\partial q_{-i}} \cdot \frac{\partial q_{i,d}}{\partial \alpha_{-i}} = 0$. Given $u(\alpha_i, \alpha_{-i}) > 0 \wedge v(\alpha_i, \alpha_{-i}) > 0$, the inequality $\frac{\partial \delta_i^*}{\partial \alpha_{-i}} > 0$ applies when inequality 30 is satisfied.

$$\frac{\partial \pi_{i,c}}{\partial \alpha_{-i}} \geq -\alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_{-i}} \quad (30)$$

The weak inequality (30) is always satisfied: Lemma 4 implies $\frac{\partial \pi_{-i,c}}{\partial \alpha_{-i}} + \alpha_{-i} \frac{\partial \pi_{i,c}}{\partial \alpha_{-i}} \geq 0$, which can be combined with (30) as is shown in (31).

$$\frac{\partial \pi_{i,c}}{\partial \alpha_{-i}} \geq -\frac{1}{\alpha_{-i}} \frac{\partial \pi_{-i,c}}{\partial \alpha_{-i}} \geq -\alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_{-i}} \quad (31)$$

Using $\frac{1}{\alpha_{-i}} \geq 1 \geq \alpha_i$ shows that (31) is always satisfied. This proves Proposition 3. \square

Proof of Proposition 4. Under the assumption of Bertrand competition with homogeneous goods ($\pi_{i,c} = 0$ and $\pi_{i,d} = 2\pi_{i,k}$) the critical discount factor δ_i^* can be written as in (32).

$$\delta_i^* = \frac{(1 - \alpha_i)\pi_{i,k}}{2\pi_{i,k} - (1 + \alpha_i)\rho F} \quad (32)$$

Assuming $\frac{\partial \rho}{\partial \alpha_i}$ yields $\frac{\partial \delta_i^*}{\partial \alpha_i}$ as is shown in (33).

$$\frac{\partial \delta_i^*}{\partial \alpha_i} = \frac{-2\pi_{i,k}^2 + 2\rho F\pi_{i,k} + (1 - \alpha_i^2)\frac{\partial \rho}{\partial \alpha_i}F\pi_{i,k}}{(2\pi_{i,k} - (1 + \alpha_i)\rho F)^2} \quad (33)$$

If collusion is profitable ($\pi_{i,k} > \rho F$) the denominator of (33) is positive. The numerator is positive if inequality (11) is satisfied. This proves Proposition 4. \square

C Appendix to Section 5

This appendix proves Lemmas 1-3 and Proposition 5 for profit function (12). As in Appendix A, we assume marginal costs of zero. Note that the unilateral effects of NCMS in Cournot competition with homogeneous goods when assuming profit function (12) have been explored by Reynolds and Snapp (1986). They show that duopolists would produce the monopoly level of output for $\alpha_i = \alpha_{-i} = 0.5$. Therefore, we assume $\alpha_i \in [0, 0.5]$ whenever using profit function (12).

Proof of Lemma 1 for Cournot competition with homogeneous goods and profit fn (12).

Lemma 1 proposes $\frac{\partial \pi_{i,c}}{\partial \alpha_i} < 0 \wedge \frac{\partial \pi_{i,c}}{\partial \alpha_{-i}} > 0$. To see this, reaction function (36) is determined by maximizing profit function (34) with respect to q_i .

$$\tilde{\pi}_i = (1 - \alpha_{-i})p(q_i, q_{-i})q_i + \alpha_i p(q_i, q_{-i})q_{-i} \quad (34)$$

$$\frac{\partial \tilde{\pi}_i}{\partial q_i} = (1 - \alpha_{-i}) \left[\frac{\partial p}{\partial q_i} q_i + p(q_i, q_{-i}) \right] + \alpha_i \frac{\partial p}{\partial q_i} q_{-i} \stackrel{!}{=} 0 \quad (35)$$

$$\tilde{q}_i^R(q_{-i}) = q_i^R(q_{-i}) - \frac{\alpha_i}{1 - \alpha_{-i}} \theta q_{-i} \quad (36)$$

Reaction function (36), which was derived for profit function (12), resembles reaction function (16), which was derived for profit function (2). The definition of $q_i^R(q_{-i})$ and θ is the same in both cases. The signs of the partial derivatives are identical, and the proof follows the same lines. This proves $\frac{\partial \pi_{i,c}}{\partial \alpha_i} < 0 \wedge \frac{\partial \pi_{i,c}}{\partial \alpha_{-i}} > 0$. \square

Proof of Lemma 2 for Cournot competition with homogeneous goods and profit fn (12).

Lemma 2 proposes $\frac{\partial \pi_{i,d}}{\partial \alpha_i} < 0 \wedge \frac{\partial \pi_{-i,-d}}{\partial \alpha_i} > 0$. Again, given the similarity of best-response functions (36) and (16) the proof is the same as the one conducted above when assuming profit function (2). \square

Proof of Lemma2: Lemma 2 suggests that in a Bertrand model with differentiated goods, higher shareholdings α_i reduce the optimal deviation profit $\pi_{i,d}$, i.e., $\partial\pi_{i,d}/\partial\alpha_i < 0$ but increase the profits of the betrayed firm, i.e., $\partial\pi_{-i,-d}/\partial\alpha_i > 0$. This can directly be inferred from firm i 's FOC (35): Given that firm $-i$ sets the agreed-upon quantity (i.e. 50% of the monopoly quantity) $q_{-i,k}$, that is independent of α_i (and thus the same with and without shareholdings, i.e., $q_{-i,k} = \bar{q}_{-i,k}$), the best response $q_{i,d}^R$ of firm i is the lower the higher the value of α_i . This keeps both the price p_d and the profits $\pi_{-i,-d}$ of firm $-i$ at a higher level than without shareholdings, i.e., $\partial\pi_{-i,-d}/\partial\alpha_i > 0$. On the other hand, the best response function \bar{q}_i^R ensures by definition the combination of price and outputs, $\bar{q}_{i,d}$ and \bar{p}_d , that maximize the value of product market profits without shareholdings, $\bar{\pi}_{i,d}$. Given the fact, that the output of firm $-i$ is the same with and without shareholdings, an one-sided deviation of that strategy (caused by NCMS), i.e. $q_{i,d} < \bar{q}_{i,d}$, must lower the product market profits of firm i , i.e., $\partial\pi_{i,d}/\partial\alpha_i < 0$.

Proof of Proposition 1: Given profit function (12), the profits in competition, collusion, and in case of a deviation can be written as in (37) to (39). The critical discount factor is given by (40)

$$\hat{\pi}_{i,c} = (1 - \alpha_{-i})\pi_{i,c} + \alpha_i\pi_{-i,c} \quad (37)$$

$$\hat{\pi}_{i,k} = (1 - \alpha_{-i} + \alpha_i)(\pi_{i,k} - \rho F) \quad (38)$$

$$\hat{\pi}_{i,d} = (1 - \alpha_{-i})(\pi_{i,d} - \rho F) + \alpha_i(\pi_{-i,d} - \rho F) \quad (39)$$

$$\delta_i > \frac{(1 - \alpha_{-i})(\pi_{i,d} - \pi_{i,k}) + \alpha_i(\pi_{-i,d} - \pi_{-i,k})}{(1 - \alpha_{-i})(\pi_{i,d} - \rho F - \pi_{i,c}) + \alpha_i(\pi_{-i,d} - \rho F - \pi_{-i,c})} \equiv \delta_i^* \quad (40)$$

Using $\partial\pi_{i,k}/\partial\alpha_i = 0$ and $\partial\pi_{-i,k}/\partial\alpha_i = 0$ the first derivative of (40) with respect to α_i yields

(41).

$$\frac{\partial \delta_i^*}{\partial \alpha_i} = \frac{\frac{\partial u(\alpha_i)}{\partial \alpha_i} \cdot v(\alpha_i) - u(\alpha_i) \cdot \frac{\partial v(\alpha_i)}{\partial \alpha_i}}{v(\alpha_i)^2}$$

with $u(\alpha_i) = (1 - \alpha_{-i})(\pi_{i,d} - \pi_{i,k}) + \alpha_i(\pi_{-i,-d} - \pi_{-i,k}) > 0$,

$$v(\alpha_i) = (1 - \alpha_{-i})(\pi_{i,d} - \rho F - \pi_{i,c}) + \alpha_i(\pi_{-i,-d} - \rho F - \pi_{-i,c}) > 0 \quad (41)$$

$$\frac{\partial u(\alpha_i)}{\partial \alpha_i} = (1 - \alpha_{-i}) \frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,-d}}{\partial \alpha_i} + \pi_{-i,-d} - \pi_{-i,k},$$

$$\text{and } \frac{\partial v(\alpha_i)}{\partial \alpha_i} = (1 - \alpha_{-i}) \left(\frac{\partial \pi_{i,d}}{\partial \alpha_i} - \frac{\partial \pi_{i,c}}{\partial \alpha_i} \right) + \alpha_i \left(\frac{\partial \pi_{-i,-d}}{\partial \alpha_i} - \frac{\partial \pi_{-i,c}}{\partial \alpha_i} \right) + \pi_{-i,-d} - \rho F - \pi_{-i,c}$$

In the following, we show that equality 42 applies.

$$(1 - \alpha_i) \frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,-d}}{\partial \alpha_i} = 0 \quad (42)$$

The product market profits of the deviating firm i and the betrayed firm $-i$ can be written as follows.

$$\pi_{i,d} = p(q_i(\alpha_i, \alpha_{-i}), q_{-i}) q_i \quad (43)$$

$$\pi_{-i,-d} = p(q_i(\alpha_i, \alpha_{-i}), q_{-i}) q_{-i} \quad (44)$$

As discussed above the betrayed firm $-i$ sticks to the agreed-upon quantity (i.e., 50% of monopoly output), which is independent of α_i and q_i . This implies $\partial q_{-i} / \partial \alpha_i = 0$. However, the optimal deviation output of firm i is a function of α_i (i.e., $\partial q_i / \partial \alpha_i < 0$). To see this, we use (12) and write firm i 's deviation profit as in (45). Profit maximization yields the first-order condition (46), which proves $\partial q_i / \partial \alpha_i < 0$.

$$\hat{\pi}_{i,d} = (1 - \alpha_{-i}) \cdot [pq_i - \rho F + \alpha_i(pq_{-i} - \rho F)] \quad (45)$$

$$\frac{\partial \hat{\pi}_{i,d}}{\partial q_i} = (1 - \alpha_{-i}) \left(\frac{\partial p}{\partial q_i} q_i + p \right) + \alpha_i \frac{\partial p}{\partial q_i} q_{-i} \stackrel{!}{=} 0 \quad (46)$$

Given $\partial q_{-i}/\partial \alpha_i = 0$ and $\partial q_i/\partial \alpha_i < 0$ the product market profits of firm i and $-i$ (see (43) and (44)) depend on α_i as is shown in (47) and (48).

$$\frac{\partial \pi_{i,d}}{\partial \alpha_i} = \frac{\partial p}{\partial q_i} \frac{\partial q_i}{\partial \alpha_i} q_i + p \frac{\partial q_i}{\partial \alpha_i} \quad (47)$$

$$\frac{\partial \pi_{-i,-d}}{\partial \alpha_i} = \frac{\partial p}{\partial q_i} \frac{\partial q_i}{\partial \alpha_i} q_{-i} \quad (48)$$

Plugging (47) and (48) in equality 42 yields (49) that must equal zero given first-order condition (46).

$$(1 - \alpha_{-i}) \frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,-d}}{\partial \alpha_i} = \frac{\partial q_i}{\partial \alpha_i} \left[(1 - \alpha_{-i}) \left(\frac{\partial p}{\partial q_i} q_i + p \right) + \alpha_i \frac{\partial p}{\partial q_i} q_{-i} \right] = 0 \quad (49)$$

This proves equality 42.

Given equality 42 and assuming $\pi_{i,k} = \pi_{-i,k}$, (41) can be simplified as is shown in (50).

$$\begin{aligned} \frac{\partial \delta_i^*}{\partial \alpha_i} &= \frac{\frac{\partial u(\alpha_i)}{\partial \alpha_i} \cdot v(\alpha_i) - \frac{\partial v(\alpha_i)}{\partial \alpha_i} \cdot u(\alpha_i)}{v(\alpha_i)^2} \\ \text{with } \frac{\partial u(\alpha_i)}{\partial \alpha_i} &= \pi_{-i,-d} - \pi_{i,k} \\ \text{and } \frac{\partial v(\alpha_i)}{\partial \alpha_i} &= \pi_{-i,-d} - \rho F - \pi_{-i,c} - \left((1 - \alpha_{-i}) \frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i} \right) \end{aligned} \quad (50)$$

As it can be shown that the inequalities $u(\alpha_i) > 0$ and $v(\alpha_i) > 0$ apply, the sign of $\partial \delta^*/\partial \alpha_i$ is the same as that of its numerator. This numerator can be written as is shown in (51).

$$\begin{aligned} &\frac{\partial u(\alpha_i)}{\partial \alpha_i} \cdot v(\alpha_i) - \frac{\partial v(\alpha_i)}{\partial \alpha_i} \cdot u(\alpha_i) = \dots \\ &(1 - \alpha_{-i}) [(\pi_{i,k} - \rho F - \pi_{i,c})\pi_{-i,-d} - (\pi_{-i,k} - \rho F - \pi_{-i,c})\pi_{i,d} - (\pi_{i,c} - \pi_{-i,c})\pi_{i,k}] \\ &+ \left((1 - \alpha_{-i}) \frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_i} \right) u(\alpha_i) \end{aligned} \quad (51)$$

Equation (51), and thus $\partial\delta^*/\partial\alpha_i$, is positive when inequality (13) applies, which proves Proposition 5.

In the following, we show that the right-hand side of inequality (13) is weakly positive, i.e., we prove (52).

$$(1 - \alpha_{-i}) \frac{\partial\pi_{i,c}}{\partial\alpha_i} + \alpha_i \frac{\partial\pi_{-i,c}}{\partial\alpha_i} \geq 0 \quad (52)$$

To prove (52), we derive the profit $\hat{\pi}_{i,c}$ (see equation (34)) for α_i , as is shown in equation (53).

$$\begin{aligned} \frac{\partial\hat{\pi}_{i,c}}{\partial\alpha_i} &= (1 - \alpha_{-i}) \frac{\partial\pi_{i,c}}{\partial\alpha_i} + \pi_{-i,c} + \alpha_i \frac{\partial\pi_{-i,c}}{\partial\alpha_i} \\ &= \pi_{-i,c} + (1 - \alpha_{-i}) \left(\frac{\partial\pi_{i,c}}{\partial q_i} \frac{\partial q_i}{\partial\alpha_i} + \frac{\partial\pi_{i,c}}{\partial q_{-i}} \frac{\partial q_{-i}}{\partial\alpha_i} \right) + \alpha_i \left(\frac{\partial\pi_{-i,c}}{\partial q_i} \frac{\partial q_i}{\partial\alpha_i} + \frac{\partial\pi_{-i,c}}{\partial q_{-i}} \frac{\partial q_{-i}}{\partial\alpha_i} \right) \end{aligned} \quad (53)$$

The first summand in the second line shows the effect of the higher dividend income when firm i expands its shareholdings α_i of firm $-i$. The second summand shows the effect of α_i on the product market profits of the two firms. Note that we re-write the first derivatives slightly because α_i affects $\pi_{i,c}$ only through its effect on q_i and q_{-i} . Similarly, α_i affects $\pi_{-i,c}$ only because of the effects on q_i and q_{-i} . If, however, firm i would keep its output constant, only firm $-i$ would vary its output and $\partial\pi_{i,c}/\partial\alpha_i > 0 \wedge \partial\pi_{-i,c}/\partial\alpha_{-i} < 0$ would apply. Given, $\partial q_i/\partial\alpha_i = 0$, the best response function of firm $-i$ would ensure, that $(1 - \alpha_i)(\partial\pi_{-i,c}/\partial\alpha_i) + \alpha_{-i}\partial\pi_{i,c}/\partial\alpha_i = 0$.³ Given, that $\alpha_i < 0.5 \wedge \alpha_{-i} < 0.5$ and $\partial\pi_{i,c}/\partial\alpha_i > 0 \wedge \partial\pi_{-i,c}/\partial\alpha_{-i} < 0$, the option to keep its output constant ($\partial q_i/\partial\alpha_i = 0$) allows firm i to ensure that the second summand of (53) is not lower than zero. In fact, firm i would only vary its output when this has an additional, positive effect on $\hat{\pi}_{i,c}$. This proves the weak inequality (52).

³The prove for this is equivalent to the prove of equality (42), $(1 - \alpha_i)(\partial\pi_{i,d}/\partial\alpha_i) + \alpha_i(\partial\pi_{-i,-d}/\partial\alpha_i) = 0$.

Proof of Proposition 3 Proposition 3 suggests $\partial\delta_i^*/\partial\alpha_{-i} \geq 0$ even for $\partial\pi_{-i,c}/\partial\alpha_{-i} < 0$.

Equation (54) provides the first derivative of δ_i^* when assuming target function (12).

$$\frac{\partial\delta_i^*}{\partial\alpha_{-i}} = \frac{\frac{\partial u(\alpha_{-i})}{\partial\alpha_{-i}} \cdot v(\alpha_{-i}) - \frac{\partial v(\alpha_{-i})}{\partial\alpha_{-i}} \cdot u(\alpha_{-i})}{v(\alpha_{-i})^2}$$

with $u(\alpha_{-i}) = (1 - \alpha_i)(\pi_{i,d} - \pi_{i,k}) + \alpha_i(\pi_{-i,-d} - \pi_{-i,k})$,

$$v(\alpha_{-i}) = (1 - \alpha_i)(\pi_{i,d} - \rho F - \pi_{i,c}) + \alpha_i(\pi_{-i,-d} - \rho F - \pi_{-i,c}), \quad (54)$$

$$\frac{\partial u(\alpha_{-i})}{\partial\alpha_{-i}} = (1 - \alpha_i) \left(\frac{\partial\pi_{i,d}}{\partial\alpha_{-i}} - \frac{\partial\pi_{i,k}}{\partial\alpha_{-i}} \right) + \alpha_i \left(\frac{\partial\pi_{-i,-d}}{\partial\alpha_{-i}} - \frac{\partial\pi_{-i,k}}{\partial\alpha_{-i}} \right),$$

and $\frac{\partial v(\alpha_{-i})}{\partial\alpha_{-i}} = (1 - \alpha_i) \left(\frac{\partial\pi_{i,d}}{\partial\alpha_{-i}} - \frac{\partial\pi_{i,c}}{\partial\alpha_{-i}} \right) + \alpha_i \left(\frac{\partial\pi_{-i,-d}}{\partial\alpha_{-i}} - \frac{\partial\pi_{-i,c}}{\partial\alpha_{-i}} \right)$

Lemma ?? implies $\partial\pi_{i,k}/\partial\alpha_{-i} = 0$ and $\partial\pi_{-i,k}/\partial\alpha_{-i} = 0$. Moreover, it can be shown that $(1 - \alpha_{-i})(\partial\pi_{i,d}/\partial\alpha_{-i}) + \alpha_i(\partial\pi_{-i,-d}/\partial\alpha_{-i}) = 0$ apply. The prove for this is equivalent to the prove of equality (42).

Therefore, equation (54) can be simplified as is shown in equation (55).

$$\frac{\partial\delta_i^*}{\partial\alpha_{-i}} = \frac{\left((1 - \alpha_{-i}) \frac{\partial\pi_{i,c}}{\partial\alpha_{-i}} + \alpha_i \frac{\partial\pi_{-i,c}}{\partial\alpha_{-i}} \right) \cdot u(\alpha_{-i})}{v(\alpha_{-i})^2} \quad (55)$$

Given $u(\alpha_{-i}) > 0$ and $v(\alpha_{-i}) > 0$, the weak inequality $\partial\delta_i^*/\partial\alpha_{-i} \geq 0$ applies when inequality 56 is satisfied.

$$(1 - \alpha_{-i}) \frac{\partial\pi_{i,c}}{\partial\alpha_{-i}} \geq -\alpha_i \frac{\partial\pi_{-i,c}}{\partial\alpha_{-i}} \quad (56)$$

The weak inequality (56) is always satisfied: Consider that inequality (52), which was proven above, can be stated for firm $-i$ as follows.

$$(1 - \alpha_i) \frac{\partial\pi_{-i,c}}{\partial\alpha_{-i}} + \alpha_{-i} \frac{\partial\pi_{i,c}}{\partial\alpha_{-i}} \geq 0 \quad (57)$$

The weak inequalities (56) and (57) can be combined as is shown in (58).

$$(1 - \alpha_{-i}) \frac{\partial \pi_{i,c}}{\partial \alpha_{-i}} \geq \alpha_{-i} \frac{\partial \pi_{i,c}}{\partial \alpha_{-i}} \geq -(1 - \alpha_{-i}) \frac{\partial \pi_{-i,c}}{\partial \alpha_{-i}} \geq -\alpha_i \frac{\partial \pi_{-i,c}}{\partial \alpha_{-i}} \quad (58)$$

Using $(1 - \alpha_{-i}) \geq 0.5 \geq \alpha_{-i} \wedge 0.5 \geq \alpha_i$ shows that (58) is always satisfied. Therefore, an increase in α_{-i} always raises δ_i^* .