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An Economic Theory of 'Destabilization War' '- Compromise for Peace versus Conventional, Guerilla, or Terrorist Warfare

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An Economic Theory of 'Destabilization War'

- Compromise for Peace versus Conventional, Guerilla, or Terrorist Warfare

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How can a small group of insurgents believe they can overcome the government by turning violent, even if the government is strongly superior? When does a conflict develop towards a peaceful conflict resolution, terrorism, a guerilla war, or a widely spread conventional civil war?

We develop a formal model for rebels and government and derive optimal choices. Further, we focus on three elements as important ingredients of a "destabilization war". All three of these - large random events, time preference (which we relate to ideology), and choice of duration of fight - are rarely considered in formal conflict theory. We can answer the above two questions with game theory analysis. First, insurgents rise up because they believe in the effect of destabilization through permanent challenging attacks. Large randomness is an important ally of rebels. While each single attack may have low impact, maybe at some time a large random event could lead to success. Hence, duration of activities is a constitutive element of this kind of violent conflict. Patience (low time preference), which may reflect the degree of ideological motivation of rebels, is crucial. Second, the mode of warfare or conflict resolutions that develops is generally path-dependent and conditioned on the full set of options (including compromise). Various conditions (level of funding, ease of recruitment, access to weapons) favor in a complex way different modes of warfare or a peaceful compromise.

Unlike in a "one battle war" with no time dimension, in a "continuing violent conflict" economic instruments become very powerful in terms of guiding conflict resolution in a certain direction.

JEL classifications: H56

Keywords: terrorism, civil war, conflict duration, game theory,
stochastic process, ideology

1 Introduction

In this paper we try to answer the following two questions using a formalized model of conflict and destabilizing war.

1) How can a small group of insurgents believe they can overcome the government, or cause a fundamental change, by turning violent even if the government is strongly superior? Why do insurgents believe that they have a chance?

2) Under what conditions are we more likely to observe one of the following outcomes of a conflict - (i) a political compromise and peaceful conflict resolution, (ii) a terrorist war, (iii) a guerilla war, (iv) a widespread civil war as conventional war, or (v) a resolution reached by rebels giving up their revolutionary project?

To the best of our knowledge there is no closed model that can answer these questions within one consistent framework. Therefore, we suggest a theory of "destabilization war". We introduce a rather general neoclassical model with optimal choices of rebels and government and derive optimal response functions for game theory analysis. We develop a two-person non-cooperative game involving sequential as well as simultaneous decisions. We define the game by specifying the order of decisions, possible actions, and payoffs resulting from action choices. As a solution concept, we use the subgame perfect equilibrium concept. While the modeling is rather complex we sometime need to suggest simplifying assumptions to keep the model treatable. However, we believe that these assumptions do not substantially limit our general findings.

2 Related literature

As this paper suggests an economic theory of destabilization war, we need to discuss the ingredients of such a conflict. Destabilization is a process in which observable events indicate a loss of control by government authorities, generating a diffuse physical or psychological threat to the government or to general security. Even if such events were peaceful in principle, this paper considers violent attacks against various targets or symbols. These attacks, however, provoke countermeasures by the government in an attempt to ensure security to reinforce its own position. A sequence of attacks and counter measures indicate a continuing conflict instead of a one-time battle.

Ideologies and time preference: Hence, destabilization is an ongoing process of longer duration. Time, time preferences, and time horizon of conflict parties seem to play an explicit role in this kind of conflict. Further, in the context of conflict time-horizons and time preferences can be related to underlying motivations of insurgents. E.g., if insurgents are motivated by pure greed, time preferences can be expected to be high and their time horizon so short that the return on investment in potential conflict should be realized during the insurgents' lifetime. If insurgents fight for "the one true belief" or identify them-

selves with religious or irrevocable ideological convictions, their time horizon may be much longer or may even be eternity. However, only a few papers have examined whether different time-horizons and -preferences resulting from religious beliefs or ideologies can lead to bargaining failure and eventually violent conflict. A notable exception is Duffy Toft's (2006) paper. She explores two bargaining problems, issue indivisibility and time horizons, as rationalist explanations for war, and applies these explanations to two case studies. Sandler et al. (1983) introduce a multidimensional utility function for terrorists, including elements describing their legal demands. Their idea is close to ours as it supports our argument that utility is not a one-dimensional construct solely relying on one dimensional income or payoffs, but also on potentially intangible factors. Wittman's often cited 1979 paper explores how the domestic and military costs, time preferences, and risk attitudes of two countries which are currently at war affect the timing and the outcome of peace. Streich and Levy (2009) review the Discounted Utility Model. They find that cooperation in an iterated Prisoner's Dilemma game is more difficult than Axelrod (1984) suggests, when accounting for the widely accepted fact that individuals highly discount the immediate future. Morrow (1989) analyses under what conditions an arms race results in war. One of the factors identified is incongruent time horizons. Specifically, the party that looks further into the future may want to take advantage of temporary superiority during an arms race. Garfinkel & Skaperdas (2000) develop a model showing that war becomes more probable the more the contending parties value the future. This is true even in the absence of information asymmetries and commitment problems. Voors et al. (2012) show that conflict is related to an increase in time preferences (when the time horizon shortens) of a population in a conflict zone. This is, however, the reverse relationship. In this contribution emphasis is given to how time preferences affect conflict, not on how conflict affects time preferences.

Our paper departs from Garfinkel & Skaperdas (2000). The effects of time preference and the decision to start a war are combined with the indivisibility of ideology and very unfavorable asymmetric military strength. Strong ideological beliefs combined with low time preference may motivate rebels to start a war even under very asymmetric and unfavorable conditions.

Duration of conflict and non conventional war: An ongoing process of conducting destabilizing attacks directly points towards longer duration as a constituent element of this kind of conflict. In most modeling we find the 'one battle' (without time dimension) that determines victory according to the relative military strength of both opponents in which one can outgun the other in a direct confrontation. However, wars can also be won through continuous violent fighting for a long duration. The much weaker conflict party typically opts for this strategy, as a direct confrontation would very likely result in their immediate defeat (Merari, 1993; Arreguín-Toft, 2001). Hence, wearing out the enemy, either physically, by employing guerilla tactics, or psychologically by employing terrorist tactics, seems essential in an economic theory of destabilization war.

It is striking that civil wars last on average much longer than interstate wars. The mean duration of the 78 interstate wars that took place between 1816 and 1985 is about 15 months. 17 percent of these wars lasted longer than three years and only seven percent lasted more than five years. Although this type of war is among the most destructive, protracted interstate conflict is relatively uncommon (Bennett and Stam, 1996). By contrast, civil wars usually last a very long time. As Fearon (2004) finds, 57 percent of the 123 civil wars which began between 1945 and 1999 lasted five years or more, while 32 percent lasted even ten years or more (data as presented by Powell, 2006). In a theoretical paper Fearon (2007) models that this often rather long conflict duration is due to information asymmetry. When facing a conflict against some emerging underground group, the government does not know enough about the enemy to reach an efficient outcome in pre-conflict bargaining. It is necessary to first fight in order to learn about the enemy's strength. The largely empirical literature on the resource curse suggests that duration or prolongation is rather a strategy of individual soldiers or their leaders that serves to enable them to gain a personal advantage from extracting primary resources (Collier, 2000; Collier et al., 2004). Balestri (2012) shows that the presence of exploitable gold resources tends to increase conflict intensity and that resources' lootability and accessibility in wartime can largely determine to what extent revenues can be appropriated and misused during conflict events. This is part of the greed argument originally brought forward by Collier and Hoeffler (2004). Akcinaroglu & Radziszewski (2007) show that an expectation of receiving military and other aid in the future can motivate rebels to sustain their fight. On the influence of third-party interventions on duration see also, e.g., Balch-Lindsay & Enterline (2000). Escriba-Folch (2010) shows that sanctions and their durations are statistically associated with shorter intrastate conflicts. The evidence suggests that maximizing costs (e.g., for buying new arms) via embargoes is more effective than other sanction types.

Arreguín-Toft develops a theory of asymmetric conflict and tests it empirically. Over time an increasing number of wars between unequally strong opponents has been won by the weaker side. While between 1800 and 1849 only roughly 12 percent of wars were won by the materially inferior side, this figure continuously grew to around 55 percent for the period 1950 to 1998. While traditional theory predicts that the stronger side should always win, Arreguín-Toft explains that the question of who wins depends on strategic interactions and their implications for conflict duration. He effectively combines his theory with Mack's (1975) relative interest asymmetry argument, stating that the weak win wars as they have a higher interest in winning, which implies relatively low political vulnerability, resulting in the stronger side losing because they lack the required support from their population. Arreguín-Toft shows that in order to win an asymmetric conflict, the weak must react asymmetrically to the strategy of the strong. This means that if the stronger side seeks direct confrontation, the weak side should pursue guerilla warfare, while if the strong side chooses indirect confrontation (termed barbarism), the inferior side should opt for a direct defense strategy. The reason why such asymmetric reactions are optimal for

the weak is that they lengthen the duration of a war. And as Mack (1975) has already found, a longer duration means that politically more vulnerable leaders, usually the leaders of stronger countries, come under pressure from their own population. Arreguín-Toft finds empirical support for his theory, with only 24 percent of wars won by the weak party if they choose the same approach to strategy as the strong side. By contrast, this figure rises to 63 percent when the weak choose an opposite strategic approach.

So far, the literature makes no mention of duration as an explicit strategy in itself. Departing from Arreguín-Toft (2001) who gives examples for strategic behavior and its implicit effects on duration, we introduce duration as a substantial element of a war strategy. Conflict duration is central to a militarily inferior group of insurgents. By explicitly choosing to prolong a fight it is possible to destabilize the government and ultimately overthrow it. This is in contrast to most models in which a fight (war) is a one-shot event without a time dimension.

Uncertainty in the course of time: Many papers model the onset, and the different outcomes, of violent conflict and war as a result of a stochastic probability (see, e.g., Carter, 2015; Rosendorf & Sandler, 2004; Fearon, 1995; Hirshleifer, 1988, 1989, 2001; Wittman, 1979; Powell, 2006; Garfinkel & Skaperdas, 2000). Also it is often assumed that both sides equally mutually know or estimate their respective probability of succeeding in violent conflict (Hirshleifer, 1988). If the two sides estimate their respective chance of success differently, a bargaining failure can result, as at least one side may demand too much (from an objective standpoint) in pre-war bargaining (Fearon, 1995; Wittman, 1979). As Fearon (1995) explains, this is a typical case where an information asymmetry causes a war.

We depart from this kind of probabilistic modeling. To model uncertainty in the course of time and determine the effects of longer-term duration of stochastic evolution of values, introducing stochastic processes is favorable. Further, the uncertainty of future developments can be illustrated using various perceptions of uncertainties, such as continuous stochastic developments or processes with strong non-continuous jumps. As in a theory of destabilization war we are interested in rare events with highly uncertain outcomes over a longer course of time, we introduce an Ito-Levy jump process as an adequate description of future uncertainty. We show how the combination of stochastic threats with a long duration of such actions is an effective way to overcome an apparently superior government.

Terrorism, guerrilla warfare, and conventional warfare, terms and definitions: There is no universal definition of terrorism. Schmid and Jongman (1988) identify at least 109 different academic and official definitions of terrorism. They find that 83.5 percent of these definitions use words such as violence and force. 65 percent use the word political, 51 percent emphasize fear and terror. 30.5 percent of authors argue that terrorism is a method, tactic, or strategy

of combat. Only 17.5 percent highlight that civilians, neutrals, non-combatants and outsiders are the victims (p. 5f.). Although there is some agreement, the authors provide more descriptive statistics on terrorism definitions to show that official and academics define terrorism differently. One way to reach consensus on a universal definition is proposed by Schmid and Jongman (1988, cited in Schmid [2004, p.382]). Their definition includes 16 elements (see appendix 6.1), which 200 scientists from the field of political terrorism were asked to rate. 81 percent of respondents agreed or partially agreed that this is how they would define terrorism.

A recent paper by Carter (2015, p.116) defines terrorism as a tactic which directs violence toward non-combatants, is intended for a much wider audience, and is meant to help achieve political goals. Carter (2015), referring to Hoffman (2006), Kydd and Walter (2006), and Richardson (2006), highlights that terrorism as a tactic is usually meant to spread fear among the population as a whole to indirectly impose a cost on the government and influence its politics. Comparing terrorism with guerilla tactics, Carter (2015) also is in line with Sandler and Hartley (1995) and Fearon and Laitin (2003), stating that guerrilla tactics are used to directly attack state forces to control territory and the population that resides within it. Carter (2015) also explains that the limits between terrorism and guerilla warfare are often blurred since many guerillas also commit terrorist attacks.

There is theoretical (Carter, 2015, Bueno de Mesquita, 2013) support for the claim that guerilla warfare is the strategy of choice for stronger rebel groups. It should be noted, however, that strength is a relative term. The stronger a group in relation to the government, the greater the chance it will opt for guerilla warfare or even, as we argue here, conventional warfare as opposed to terrorism. Several empirical studies (e.g., Fearon and Laitin, 2003; de la Calle and Sánchez-Cuenca 2012) support this claim. They state that there is a relationship between insurgency and poverty, political instability, and large populations. These factors are indicators of potential state and rebel strength. The result is that weaker states are more likely to face territorial guerrilla campaigns, while stronger states tend to be confronted with terrorism. The guerilla strategy needs another important ingredient, however: terrain in which it is easier to hide, such as, e.g., mountains or jungle. Guerilla warfare takes place in the countryside (Merari, 1993; Mao, 2007; Fearon and Laitin, 2003). If a country is heavily urbanized, it is difficult for guerillas to hide and control their territory from underground. Sometimes fighters in such an environment refer to themselves as “city guerillas”, following ideas by Marighella (1969). That said, Merari (1993) explains that the methods of city guerillas are rather the methods of terrorists (bombings, assassinations, etc.). Therefore their actions should be considered terrorism (not necessarily in the legal sense, but from a strategic perspective). In this paper we define terrorism, guerilla warfare, and conventional warfare in accordance with Merari (1993) (see appendix 6.1: Table A1).

Merari (1993) looks at terrorism from a rather pragmatic perspective, contextualizing it within a conflict model framework which also includes guerilla

warfare and conventional war.¹ Table A1 in the appendix shows that there is a whole vector of characteristics which distinguish terrorism, guerilla warfare, and conventional warfare from each other. Terrorism is an insurgency strategy used by a small group of citizens and directed towards the government.² The tactics of terrorism are rather associated with crime than with warfare - think, e.g., of hijacking, car bombs, and kidnapping. These tactics are typical for terrorists, but also for criminals with financial motivations. Further, the victims are often civilians and not enemy combatants. As can be seen in Table A1, terrorism is the simplest form of severe violent struggle. This is due to the fact that such actions can be conducted even if the number of fighters and supporters is very small and their weaponry is poor.

Once the number of fighters increase and/or their equipment improves, the rebels can afford to seize territory and organize themselves into a small army with platoons, companies, etc. Their tactics move towards regular military tactics (attacking military, police and administration staff instead of the public at large) although the magnitude of the attacks still remains relatively small. This is due to the material inferiority of the rebels. The guerillas' strategy is therefore typically to hit and run, operate from underground, wear out the enemy, but not try to destroy it in a direct confrontation. It is, just like terrorism, an asymmetrical strategy of violent struggle. Once a group consists of many fighters and has extensive military equipment and material, it may wage a direct confrontation of a kind that can be referred to as conventional warfare.

In the framework of our model we also look at these three basic forms of political violence (terrorism, guerilla warfare, and conventional warfare) against a ruling government. Each form is associated with a different strategy, whereby the tactics behind guerilla warfare and conventional warfare are similar (Merari, 1993). In our theory we extract from Merari the two dimensions, number of fighters N and capital intensity (military equipment per fighter) k to identify the mode of warfare.

Game theory analyzing the choice of warfare strategy: Only few theories attempt to explain warfare strategy. One of them is Merari (1993). The

¹Merari (1993) does not consider conventional warfare a mode of political violence that is used by insurgents in. Merari (as many others) understands conventional war as the type of political violence that states inflict upon states (interstate warfare). Interstate war is typically used as a synonym for conventional war. This is because most conventional wars, as Merari defines them, are between states. As various notorious examples show, it is possible that insurgent violence ultimately results in nearly or completely conventional warfare: think, e.g., of certain periods during the IS insurgency, which in fact is a civil war. On the contrary, some interstate wars are not pursued using conventional tactics but with guerrilla or even terrorist tactics (e.g., parts of the Vietnam War). In the same vein, guerilla warfare is often equated with civil war (see, e.g., Carter, 2015). What should be clear by now is that such synonyms are often but not always accurate, as the lines between them are blurring.

²Political violence can be classified by asking about the initiator of the violence (state, citizens) and the target (state, citizens) (Merari, 1993). This approach looks specifically at the interactions between the citizens as the initiator and the state as the target. Terrorism here refers therefore to the type of political violence that is initiated by citizens and directed towards the state and not vice versa (state terror).

mode of struggle adopted by insurgents could be the result of circumstances rather than a choice. However, there is literature which suggests an explicit choice of mode. Carter (2016) presents a model in which a group conditions its tactic of choice (guerilla, i.e., attack security forces, or terror, i.e., attack civilians) on the type of government response it expects. Terror provokes a soft response while a guerilla attack provokes a forceful response. Rebels' optimization problem is to maximize civilian casualties while limiting own damages. If rebels expect a forceful response of the government more damage is caused to the wider population than to the rebels themselves (as the government cannot properly discriminate between rebels and regular citizens). Hence, rebels are expected to try to provoke such a forceful response. The theoretical model is supported by an empirical analysis reconfirming that rebels are more likely to use terrorism in high-capacity states, while guerilla attacks are more likely chosen in states that are less capable of discriminating between rebels and the wider population. Carter (2015) develops a game-theoretical model in which the state first allocates limited resources across defensive and offensive security measures while choosing whether to attempt forceful elimination of the group or not. The rebels subsequently observe the government's actions and decide whether to use terrorism or whether to attempt to conquer (further) territory via guerrilla tactics. The government will always try to prevent the rebels from conquering further territory (which represents resources) and therefore will take security measures, so the rebels decide to use terrorism as a strategy. If a government lacks the resources to defend its territory, the rebels will rather decide in favor of territorial conflict. The model explains theoretically the empirical finding that very poor states (LDCs) more likely face territorial civil wars, MDCs middle income countries more likely face terrorist campaigns, and rich countries hardly experience either. Bueno de Mesquita (2013) develops a model in which a rebel group interacts with the general population. The rebels are equipped from the beginning with a certain capacity to conduct attacks, representing, e.g., funding, institutional design, or control over territory. Attacks on the government can be conducted either using conventional tactics (direct attacks) or irregular tactics (terrorism). The rebels may also decide not to attack at all. Which tactic is chosen depends on the number of available fighters in the group. While high capacity and success in fighting may ease recruitment, unsuccessful attacks have an ambiguous effect: on the one hand the rebel group becomes less attractive for potential recruits, but on the other even unsuccessful attacks cause negative externalities for the general economic environment. This further diminishes the opportunity costs of recruits, making it more attractive for them to join the rebels. So it is possible that a successful counterinsurgency leads to a situation where the rebels shift from conventional tactics (e.g., conquering territory) to irregular tactics, as many fighters are killed but at the same time sufficient new recruits are attracted in order to continue the struggle using guerilla tactics or terrorism. Bueno de Mesquita argues that including the option to potentially withdraw from conflict is important to show the non-monotonicity of the use of irregular tactics. He highlights the importance of jointly studying the causes of terrorism, insurgency, and civil war, not only in theoretical models, but also

empirically.

In this approach we take up Bueno de Mesquita's suggestion to jointly study all options including different modes of fighting as well as the option of compromise for peace, and to show how decisions change.

3 Modeling rebels and government

In general the model depicts a situation in which a rebel group has formed and considers fighting against the government. The government rules the country by a system of laws and distributes income to its supporters. The rebel group does not feel sufficiently included, neither concerning their income (distribution) nor with respect to their political and cultural values and the political process.

3.1 Rebels: utility, constraints, actions

Rebels' utility:

Generally, the rebel core consists of a group of rebel leaders and decision makers. The utility of the rebel core follows a preference order and can be described by a standard utility function. $u = u(x)$ where x is a vector characterizing the distinct elements important for the rebel.

What do we mean by distinct elements? One element can be purely income; however, income differs substantially from political or religious/ideological beliefs. Hence, if a rebel group has a strong belief in its mission there may be a high utility effect with respect to this element. Furthermore, the enforcement of ideology and belief may not be easily substituted by higher income and hence it is unlikely that a compromise through such substitution will be found. Therefore, it is important to be aware that such preferences and hence the respective properties of utility functions and their elements are crucial for decision-making and conflict resolution.³

Utility functions already describe whether rebels can compromise on certain elements and what the potential price may be. In addition, we introduce elements which are indivisible.⁴ Ideological beliefs may not only be difficult to

³Utility functions for insurgent groups have been frequently used, e.g., by Carter (2015, 2016), Bueno de Mesquita (2013), Enders and Sandler (2004), and Powell (2002). However, these papers look at utility from a one-dimensional perspective, where groups derive utility from income, payoffs, saved costs, etc. Bueno de Mesquita (2013) mentions that religion and ideology are important dimensions of a group's tactical choice (i.e., conventional vs. irregular warfare). However, he does not directly connect to a description of ideology and income in a utility function. An exceptional view is taken by Sandler et al. (1983) who introduce a multidimensional utility function for the terrorists including intangible elements.

⁴The concept of indivisible issues as a cause of rational conflicts is not new; see, e.g., Fearon (1995), Powell (2006), Goddard (2006), Hassner (2003), Duffy Toft (2002, 2006) and Brams & Taylor (1996). All these authors understand indivisible issues to be material objects, such as, e.g., Temple Mount in Jerusalem, a territory that is home to an ethnic group, or whatever is worth more in one piece than if divided. However, Duffy Toft (2002, 2006) explains that the reasons for the feeling of indivisibility are often religion or nationalism. In this paper we go one step further and say that a religious or nationalist ideology itself may be indivisible, i.e., it is not complementary to any other political system.

substitute by other elements, they may also be indivisible. E.g. for the ideology of the Extremist Political Islam, Sharia, as a full set of laws, seems indivisible. It is either introduced as a system or not at all. Similarly, a secular constitutional state based on human rights cannot let the rule of law coexist alongside Sharia rules. Both systems are indivisible as well as mutually incompatible. Therefore, as long as conflicts are driven by clear belief systems, it is important to discuss indivisible elements.

With this general discussion of rebels' potential preferences in mind, we suggest rebels' utility with such specific properties and elements. In our specification a group of insurgents has two dimensions of in their utility function, namely income distributed by the government and disposable to the rebel core Y_D and substantial political participation. Political ruling power is defined as a set of laws, which defines an indivisible and consistent regime L that motivated by beliefs and ideologies. This set of laws is indicated by index variable $L = \bar{L}, 0$. Regime 0 is a full set of laws that leads to a specific political and institutional system and defines the existing regime of the current government. Regime \bar{L} however, indicates the system rebels would like to establish, increasing the rebels' utility by $\bar{L} > 1$. As we assume these systems are incompatible with each other, there can be only one regime - either the existing legal system of the current government or a switch to the rebels' set of laws. The realization of the rebels' set of laws enters u together with rebels' income transfer from the government. Hence, utility each period is $u(t) = u(Y_D, L)$. Depending on the degree of substitutability of the two elements, rebels will have to insist on their own set of law or ideology, or they have to compromise. In the current example we assume that rebels are not completely dedicated to their ideology and can substitute even if $L = 0$. Neither of the two dimensions is necessary and income is substitutable for the introduction of an ideological system. The simplest functional form of such preferences is a linear additive utility function for each period described in (R1).

Rebels know that it is very unlikely they will succeed in achieving their goals immediately just with the first attack. However, rebels believe that a longer process of continuous attacks generates threat and destabilizes the government. Moreover, these attacks - even if it happens in a distant future - may eventually overthrow the government. Therefore, introducing an economic theory of destabilization war it is necessary to talk about time and time preferences. An ideology based on a "religious or ideological truth" is likely to have a long-term perspective. Even if such beliefs are currently not observable, eventually they will happen as they are "a truth". Therefore, we would expect that rebels' time preference rates ρ are low and the time horizon is even infinity, ∞ . By contrast, rebels who are more driven by greed would prefer a more prompt return. Hence this kind of group is likely to have a high time preference and a short time horizon. While there is some literature discussing time preferences as an important conflict-determining parameter,⁵ only few contributions directly connect time

⁵See, e.g., Wittmann (1979) or Garfinkel & Skaperdas (2000).

preference to ideology and believes.⁶ Hence, the present value of the rebels' utility stream defines rebels total welfare v ,

$$\begin{aligned} \text{general} \quad &: \quad u(t) = Y_D(t) + L(t), \quad \text{with} \quad L = 0, \bar{L}. \\ &: \quad v = \int_0^\infty u(t) e^{-\rho t} dt \end{aligned} \quad (\text{R1})$$

$$\begin{aligned} \text{fight} \quad &: \quad u(t) = Y_D(t) + L(t) \begin{cases} Y_D = 0, & L = 0 & \text{for } t < T \\ Y_D = Y & L = \bar{L} & \text{for } t > T \end{cases} \\ &: \quad Ev = E \int_0^\infty u(t) e^{-\rho t} dt = e^{-\rho ET} \frac{(Y_R + \bar{L})}{\rho} \end{aligned} \quad (\text{R}^f1)$$

$$\begin{aligned} \text{comp} \quad &: \quad u = Y_D(t) + (L = 0) \\ &: \quad Ev^c = P_R v^c + (1 - P_R) Ev^f \end{aligned} \quad (\text{R}^c1)$$

In the case of a fight, rebels' utility is described by get "all or nothing". During the period of fighting rebels obtain neither transfer payments nor their preferred set of laws. After overthrowing the government, however, they achieve everything. Hence, if T is the time of success, specified utilities and expected welfare is (R^f1)

In the case of a compromise, rebels' utility is determined by income that becomes disposable as result of the peace agreement Y_D . There is also an evaluation of the political agreement about the law system L . However, in the compromise model we assume that the government will not permit the introduction of the rebels' indivisible legal system, $L = 0$. Hence, each period's specified utility becomes Y_D . Yet the rebels are aware of the uncertainty of a lasting peace.⁷ They do not know if the government will keep its promises and estimate the probability of a sustainable peace agreement P_R , and that the peace agreement will turn into a fight with probability $1 - P_R$. Hence, the expected present value of a long-term compromise is (R^c1)

Rebels' finance constraint:

Generally, the rebel group faces several constraints. First, they have a financial constraint. Rebels have a given flow of financial income $F(t)$, and generally they may also receive government transfers Y_R . Access to finance is crucial for violent conflicts and hence is the first constraint we introduce.⁸

⁶Duffy Toft (2006) finds that (asymmetric) time horizons can lead to violence. If players discount the present, war becomes more likely (as current costs are valued less than the future benefits to be earned once the war is won). This is especially true if religious or nationalist beliefs influence bargaining. Tangible goods from the present can be exchanged for intangible goods in the afterlife (e.g., salvation or the knowledge that a nation's future generations will live a better life).

⁷This refers to one of the main reasons why violent conflict exists at all, namely bargaining failure. It has been extensively investigated within the framework of dyadic interactions (see, e.g., Fearon, 1995; Powell, 2006; or, for an extensive survey, Levy & Thompson, 2010.)

⁸See the discussion on "duration" in section 2.

Income is spent on recruiting more group members at cost p_N for each member, and on "wages" to cover the living expenses of existing members (fighters), $w_N N(t)$. Similarly, military capital $K(t)$ (equipment) must be purchased at price p_K and maintained at price w_K . If $R_N(t)$ is the recruitment rate of rebels each period and $R_K(t)$ is the purchasing rate of new military equipment, and $Y_D(t)$ is what is left as net disposable income for the rebel core, the rebels' budget constraint is

$$\begin{aligned} \text{general} \quad : \quad F(t) + Y_R(t) &= Y_D(t) + p_N R_N(t) + p_K R_K(t) \\ &+ w_N N(t) + w_K K(t) \end{aligned} \quad (\text{R}^2)$$

$$\text{fight} \quad : \quad F(t) = p_N R_N(t) + p_K R_K(t) + w_N N(t) + w_K K(t), \quad (\text{R}^f 2)$$

$$\text{comp} \quad : \quad Y_R(t) + F(t) = Y_D(t) + p_N R_N(t) + w_N N(t). \quad (\text{R}^c 2)$$

In the case of a fight, rebels receive no government transfers and use all their resources to win the fight $Y_D(t) = Y_R(t) = 0$. As we do not consider that rebels use war and looting as a convenient source of income,⁹ the finance constraint mutates to (R^f2)

In the case of a compromise, rebels receive an income transfer from the government $Y_R(t)$. Further, even in peacetime rebels have access to financial resources F . As there is no fighting these resources are not used for military purposes. In peacetime, the rebels' power base consists of political supporters. Hence, their financial resources are spent on recruiting political supporters $p_N R_N$ and to pay income to each supporter w_N . In reality this compensation may be paid in the shape of health insurance or social benefits for a supporter's family. However, the rebels' core would also like to earn income of their own as a peace dividend $Y_D(t)$. Hence the finance constraint for rebels is (R^c2),

Rebels' survival condition:

Generally, the number of fighters $N(t)$ is endogenous and represents the fundamental base for any rebel activity. Expanding this resource base is an important precondition of success. A group expands through recruitment. The importance of recruitment is broadly discussed in the literature.¹⁰ We define a

⁹ Collier (2000); Collier et al. (2004). Balestri (2012).

¹⁰ Rosendorff & Sandler (2004) develop a model in which a terrorist group tries to attract more recruits by committing successful and spectacular attacks, while hoping to provoke the government to react with force against the population, motivating even more moderates to rise up and support the terrorist group. The fact that state proactivity also causes negative externalities such as, e.g., the radicalization of the moderate population, is well established (see, e.g., Lake (2002), Merari (1993) and Kydd & Walter (2006)). Bueno de Mesquita (2005) also focuses on this peculiarity of terrorist recruitment. He models a game in which the government's countermeasures cause economic conditions to worsen, lowering the opportunity costs for recruits to join the group. However, these countermeasures also reduce the attractiveness of the group for recruits, as a more forceful response causes the group to weaken. Bueno de Mesquita & Dickson (2007) also model a game in the same style where a group attempts to provoke a government such that it reacts with so much force that the moderate population is eventually driven into the terrorist recruiters' arms. Bueno de Mesquita (2013)

dynamic process of recruitment with $R_N(t)$ being the number of newly recruited fighters per period. Also, group members may leave the group or - when fighting - may be killed or imprisoned. Hence, the loss of group members per period is denoted by the depreciation rate $D_N(t)$. If the net change in group size is $\dot{N}(t)$, the long-term survival of the rebel group requires $\dot{N}(t) \geq 0$. Further, the number of rebels must remain larger than one, $N > 1$, otherwise the group is liquidated and its ideology and goals disappear. We refer to this constraint as the *existence* or *survival condition*.

$$\text{general : } \dot{N}(t) = R_N(t) - D_N(t) \geq 0 \quad (\text{R3a})$$

$$\text{fight : } \dot{N}(t) = \underbrace{R_N(t) - G_S(t)N(t) + 2G_S(t)^2N(t)}_{\text{recruitment}} - \underbrace{N(t)G_S(t)}_{\text{depreciation}} \quad (\text{R}^f\text{3a})$$

$$\text{comp : } \dot{N}(t) = R_N(t) - N(t)d_N = 0 \quad (\text{R}^c\text{3a})$$

In the case of a fight, recruiting group members R_N follows three simple mechanisms. (i) Group members are actively recruited - at cost p_N for each member - by spending F_{RN} on recruitment. Hence, active recruitment is $R_N = \frac{1}{p_N}F_{RN}$. However, (ii) one government countermeasure is a campaign against rebels' recruitment activities. Hence with more G_S recruitment becomes more difficult and reduces R_N by $G_S N$. Furthermore (iii), not all fighters need to be recruited by offering them monetary incentives. Some fighters join the group in opposition to government actions. An effect of solidarity with rebels can be modeled by a second term. With an extension of oppressive actions, indicated by G_S sympathizers increasingly turn angry and join voluntary violent struggle $2G_S^2N$. Therefore, generally speaking recruitment is highly non-linear and the total recruitment mechanism adds up to $R_N - G_S N + 2G_S^2N$. Groups also lose (depreciate) existing fighters. The government's countermeasures determine the depreciation rate per rebel fighter. We assume that government actions have positive scale effects. At very high levels of spending, the government can increasingly control conditions and effectively eliminate insurgents, such that the total depreciation rate of rebels is $D_N = G_S^3N$. With this specification of group dynamics, the stationary (survival) condition turns into (R^f3a).

In the case of a compromise, rebels still attract new supporters by $F_{RN} = p_N R_N$ while others leave according to the depreciation rate of supporters $D_N = N d_N$. Here, depreciation is not a result of supporters being killed or arrested; it is more an effect describing how strong and active support for rebel groups can be maintained over time (R^c3a).

is a similar game that shows that moderates are more likely to join a terrorist organization the worse the outside option. Accordingly, the model predicts whether this results in a war with conventional (territorial conquest) or unconventional tactics (terrorism). Gates (2002) models how a rebel organization recruits fighters, arguing that the distance (e.g., cultural, religious, ethnic, geographic) between the leader and the rebels negatively influences recruitment.

In this paper we translate non-linear recruitment mechanisms by discussing equation (Rf3) which describes recruitment specifications under "fight" conditions.

Rebels' military feasibility condition:

Generally, a second dynamic constraint is given with respect to the military ability of rebels. A rebel group can endogenously purchase military equipment at rate $R_K(t)$ leading to an increasing military capital stock $K(t)$. Similarly, equipment can depreciate or be destroyed at depreciation rate $D_K(t)$. Maintaining continuous sustainable military capacity requires non-shrinking capacity at whatever level. Hence, to maintain the continuous ability to conduct military actions, military capacity $K(t)$ may not permanently decrease. This condition is referred to as the *military feasibility condition*.

$$\text{general : } \dot{K}(t) = R_K(t) - D_K(t) \geq 0 \quad (\text{R3b})$$

$$\text{fight : } \dot{K}(t) = \underbrace{R_K(t)}_{\text{purchases}} - \underbrace{K(t) d_K G_S(t)}_{\text{depreciation}} = 0 \quad (\text{R}^f\text{3b})$$

$$\text{comp : } \text{not existing}$$

In the case of a fight, rebels spend financial resources F_{RK} on military goods at price p_K . Hence, the stock of military equipment will change by $R_K = \frac{1}{p_K} F_{RK}$. Similarly, equipment can depreciate or be destroyed if the government fights the insurgents and spends resources G_S to reduce their military capability. A given stock $K(t)$ is partly destroyed subject to the government's military efficiency d_K , hence $D_K = K d_K G_S$. **In the case of a compromise**, rebels have no military equipment.

Rebels' creation of threat:

Generally, in this model destabilizing actions are violent attacks which generate minor or major **threats** $\theta(t)$ to the government or government supporters. Rebels believe that the government will only survive if security does not fall below a minimum level S^{\min} . Likewise, the level of threat must exceed a bearable maximum θ^{\max} . As the government reduces the threat by introducing countermeasures through spending G_S , the maximum threat necessarily depends on the government's security expenditure $\theta^{\max} = \theta^{\max}(G_S)$. Threat and generation of threat is different in case of fight or compromise. In case of fighting it is a violent physical or psychological threat, in case of compromise it is diffuse political menace. Thus, specification for the mechanism that generates threat for the case of fighting is (R^f4) and (R^f5), and the specification for compromise is (R^c4).

In the case of a fight, rebels randomly conduct violent attacks in order to destabilize. Even if each attack is a singular local event, it is meant to cause fear and terror on a larger scale. Hence, the impact on society and the threat to the survival of the government is highly uncertain. Due to unforeseeable circumstances a singular local event can trigger a process that spreads and grows to become a massive threat to the government. Therefore, the threat is modeled as stochastic shocks that directly generate insecurity. While the frequency or level of intensity of these attacks are determined by the rebels, the

extent of the resulting threat is highly stochastic. The idea is simple. Imagine a terrorist attack on the presidential palace, whose result may range from minor damage to the killing of the president; even if the president is killed, he or she will be followed by a successor who may be even worse. So the extent of the threat resulting from attacks is highly uncertain. This is why uncertainty is an extremely important element in violent conflicts which so far has not been considered sufficiently. The standard way to include uncertainty in conflict decision is to assume probabilities of well-defined outcomes.¹¹ However, here we go much further. In our model we cannot give probabilities of a specific outcome of a potential event in a particular period. We model the random element by a stochastic process. In this process we cannot determine how large the threat of an attack will turn out to be and when it will happen. The process we introduce is an Ito-Levy jump diffusion process

$$d\theta(t) = \alpha\theta dt + \sigma\theta dW + \int_{\mathcal{U}} zN(t, dz) \quad (\text{R}^f 4)$$

with $\sigma > 0$ the volatility and W the Wiener process. N is a Poisson process with intensity λ and the step heights z taken from a Borel set \mathcal{U} , and assuming no deterministic drift, $\alpha = 0$. Hence, $d\theta$ describes the sum of shocks of random size that occur up to t . Furthermore, rebels expect that the government will collapse if security falls below a certain threshold, or in other words, if for a society a certain level of maximum bearable threat θ^{\max} is reached. The rebels' perception of the effects of their attacks is that they create a threat that can randomly have very large effects. The collapse occurs when this threshold is randomly hit for the first time. Hitting the threshold can happen at any time since the stochastic process generates random occurrence and random magnitude of effects. It could be tomorrow, it could be in a hundred years' time. However, for a particular random process there is an expected time when this threshold is hit the first time. This time can be derived as the expected first hitting time, ET . Figure A 3 illustrates the idea. In order to derive the expected time of collapse we have to find out when θ , described by the stochastic differential equation $d\theta$, can be expected to hit the given boundary θ^{\max} . We follow Kou and Wang (2003) and obtain the result stated in Lemma 1 part (i).¹²

If rebels believe in a process of continuing destabilization which is expected to end with the collapse of the government, they may try to accelerate this process. Two components can be identified that may be influenced by rebels, namely the frequency of violent assault λ and the mean impact of attacks resulting in a mean change in security $\frac{1}{\eta}$. In this model we assume that the frequency of attacks is an increasing function of the number of rebels N available for conducting attacks. As we assume that all rebels conduct attacks, N is also the total size of the rebel group. Hence, the frequency of attacks is described as $\lambda = \lambda(N) = N^\varepsilon$, with $\varepsilon < 1$ describing the percentage rate of increase in frequency of attacks if

¹¹E.g., Carter, 2015; Bueno de Mesquita, 2013, Fearon, 1995; Hirshleifer, 1988; Garfinkel & Skaperdas, 2000.

¹²See appendix 6.2.

the size of the group increases by one percent. Hence, the number of rebels has decreasing marginal effect on frequency.

The second component which is also related to rebels' activities is the mean of the exponential distribution, $\frac{1}{\eta}$ which indicates the average magnitude of impacts. We assume that this mean magnitude is determined by the military equipment available for rebels, here specified as $\frac{1}{\eta} = K^{1-\varepsilon}$. As a result, the overall drift of threat $\lambda \frac{1}{\eta}$ is determined by the two major endogenous resources which are at the rebels' disposal $\lambda \frac{1}{\eta} = N^\varepsilon K^{1-\varepsilon}$. This increasing threat due to rebels' military actions may be more easy to discuss if we rewrite $\lambda \frac{1}{\eta} = Nk^{1-\varepsilon}$. Hence, the overall drift in threat level is now determined by the size of the group and the capital intensity of arming them. With simple light weapons and hence a low capital intensity, the threat level increases only slowly. However, if the number of insurgents rises, even these attacks become an increasing threat. Further and in contrast, if a group remains small insurgents still may generate a high level of threat if it is well equipped.¹³ As a result the first hitting time as determined by rebels' characteristics can be described by Lemma 1 1 part (ii), (see also appendix 6.2).

Lemma 1 (*Expected time of governments collapse*) part (i): Let θ be an Levy process, θ^{\max} a constant threshold expected by the rebels to be required for a collapse and $h(z) = \eta e^{-\eta z}$ be the density function of the exponential distribution. Then the first hitting time of θ^{\max} is

$$ET = \frac{1}{\lambda \frac{1}{\eta}} \left[\theta^{\max} + \left(\frac{1}{\eta} - \frac{1}{\mu_2^*} \right) (1 - e^{-\theta^{\max} \mu_2^*}) \right] \quad (R^f 5)$$

part (ii):

$$ET = \frac{1}{Nk^{(1-\varepsilon)}} \left[\theta^{\max} + \left(K^{1-\varepsilon} - \frac{1}{\mu_2^*} \right) (1 - e^{-\theta^{\max} \mu_2^*}) \right], \quad \frac{dET}{dN} < 0, \quad \frac{dET}{dK} < 0 \quad (R^f 6)$$

Therefore, a rationally behaving group minimizes the expected first hitting time $E(T)$ by an optimal choice for the number of fighters N and equipment k . The military strategy here is to generate random destabilizing attacks that may have such strong effects that eventually (at the time of collapse) the government will be pushed out. When the rebels attack, they do not expect that this

¹³The idea of discussing a rebel group's military impact as a kind of military production function including optimal choices of military elements such as fighters and weapon types and equipment is not new, although the Contest Success Function (CSF) has been applied far more often by economists to model conflict (see, e.g., Hausken, 2005; Hishleifer, 1988, 1989, 1991, 1994, 1995, 1998, 2000, 2001; Garfinkel & Skaperdas, 2007, Skaperdas 1996). The seminal contribution that introduces military production functions to the field of conflict research is Hitch & McKean (1961). More recent publications that employ typical military production functions based on neo-classical economic theory are Lichbach (1987), Hurley (2005) and Das and Lahiri (2006) with theoretical contributions, and Hildebrandt (1999) and Rotte & Schmidt (2003) who test the military production function empirically.

particular single shock will suffice, however they hope that the randomness of events combined with such shocks may at some random condition push the government out. *The randomness of potentially large impacts of an attack is the rebels' most important ally.*

In the case of a compromise, no explicit physical threat is generated either by rebels or the government. However, the rebels believe that the government could break the compromise agreement. Hence there is a subjective probability of sustainability of the agreement. This probability is a subjective perception since the rebels believe that peace can be assured more reliably, the larger the number of their followers N . Hence, with $1 > \varphi_R > 0$, P_R is positively related to N ,

$$P_R(N) = N^{\varphi_R}, \quad (\text{R}^c4)$$

3.2 Government: utility, constraints, actions

In the original state of the model society, a government and its supporters have established a regime of social organization and laws that is designed by the ruling group only. This ruling group has full political control. The appearance of an opposing group challenges the government. The opposing group wants to switch to another political regime and aims for income participation. In response, the government evaluates the threat and provides security accordingly.

Government's utility:

Generally, the first element in contributing to a government's utility is to provide income transfers to its supporters, Y_G . This keeps the government in power, which is a major objective. The second element is to provide security S . Supporters do not just like to obtain income, they also want to live in a safe environment. Third, we introduce the law system as element of the government's utility function. The government's current law system enters utility as a necessary element specifically as $1/(1 + L^\infty)$. In the original state of society the government and its supporters stand for a set of laws that is designed completely according to their own preferences. The ruling group has full political control. Hence, with time preference γ government's utility $U(t)$ and welfare V can be specified as (G1). Often governments are considered myopic. Here we follow this suggestion, assuming γ to be large.

$$\begin{aligned} \text{general} \quad : \quad U(t) &= [(Y_G(t))^v + S(t)] \frac{1}{(1 + L^\infty)} \\ : \quad V &= \int_0^\infty U(t) e^{-\gamma t} dt \end{aligned} \quad (\text{G1})$$

$$\begin{aligned} \text{fight} \quad : \quad U(t) &= Y_G(t)^v + S(t) \\ : \quad EV &= \int_0^\infty [(Y_G(t))^v + S(t)] e^{-\gamma t} dt \end{aligned} \quad (\text{G}^f1)$$

$$\begin{aligned} \text{com} \quad : \quad U(t) &= Y_G(t)^v + S(t) \\ : \quad EV^c &= P_G V^c + (1 - P_G) V^f. \end{aligned} \quad (\text{G}^c1)$$

In the case of a fight, the government's indivisible set of laws is necessary and cannot be substituted by another legal system. Using the above definition of the legal system and the fact that both systems are not compatible, mixing the systems is not feasible. Hence if the current set of laws is maintained, $L = 0$, the government's utility function turns into the standard form. However, if society switched to the rebels' set of laws, $L = \bar{L}$ the government's utilities would turn to zero ($G^f 1$). After the government is overthrown, the utility streams turn to zero.¹⁴

In the case of a compromise, the utility function G1 indicates that $L = 0$ is indivisible and necessary for the government. Hence, again, the government cannot compromise on the legal system. However, it could compromise on the distribution of income. According to the utility function, security and income for its own supporters are easy to substitute. Since the government's utility is $U(t) = Y_G(t)^v + S(t)$. Hence welfare of peace is $V^c = \int_0^\infty U(t) e^{-\gamma t} dt$. However, a peace compromise is not necessarily sustainable. Rebels may break the agreement. P_G is the probability of a sustainable peace compromise according to the government's perception. We assume P_G to depend on the number of rebel supporters N . With more rebel supporters, the government believes less in the rebels' commitment to the peace agreement. Hence, the probability of sustainable peace decreases with the number of rebel supporters. In particular $P_G = N^{-\omega_G}$, $0 < \varphi_G < 1$. Hence, the expected welfare of a sustainable compromise is ($G^c 1$)

Government's finance constraint:

Generally, the government has a given income $Y(t)$ defining government capacity.¹⁵ Given income is used to spend on security $G_S(t)$ as countermeasure against rebel activities and for income transfers. Transfers are given to the government's supporters $Y_G(t)$ and - when appropriate - to rebels $Y_R(t)$.

$$\text{general : } Y(t) = G_S(t) + Y_G(t) + Y_R(t) \quad (G2)$$

¹⁴ γ being large allows for further simplifications: precisely, in the case of a fight we look at two integrals, $\int_0^{ET} U(t) e^{-\gamma t} dt$ and $\int_{ET}^\infty (U(t) = 0) e^{-\gamma t} dt = 0$. However, even if U is a positive constant $\int_{ET}^\infty U(t) e^{-\gamma t} dt = \frac{U}{\gamma} e^{-\gamma ET} \approx 0$ for large γ and distant ET . Therefore, for a myopic government we can simplify the calculations without a serious loss of generality by the assumption $\int_0^{ET} U(t) e^{-\gamma t} dt \approx \int_0^\infty U(t) e^{-\gamma t} dt$.

¹⁵The fact that the government's financial resources are limited and that its financial power is directly related to conflict onset and development is well established. See, e.g., Carter (2015), Powell (2007), Enders et al. (2014) Bueno de Mesquita (2013), Fearon & Laitin (2003). Meierrieks & Krieger (2011) provide an extensive survey of empirical evidence on the determinants of terrorism including the strength of political and governmental institutions. They highlight that more liberal and democratic nations are significantly less likely to produce transnational terrorism (Krueger and Maleckova 2003; Burgoon 2006; Kurrild-Klitgaard et al. 2006; Plümper and Neumayer 2010; Krueger and Laitin 2008; Piazza 2008). Basuchoudhary and Shughart (2010) and Lai (2007) find that higher levels of economic freedom reduce production against terrorism. Regardless of the exact type of institutional order, government activity may also have effects on terrorism. Burgoon (2006) finds that welfare policies significantly discourage terrorist activity. Freytag et al. (2011) establish that government expenditures are to a certain degree also associated with a reduction in terrorism.

$$\text{fight} : Y(t) = Y_G(t) + G_S(t). \quad (\text{G}^f2)$$

$$\text{com} : Y(t) = Y_G(t) + Y_R(t) \quad (\text{G}^c2)$$

In the case of a fight, the government gives no transfers to rebels, $Y_R(t) = 0$. Hence we obtain (G^f2) . **In the case of a compromise**, government does not spend on countermeasure against rebels ($G_S(t) = 0$) but just let rebels participate in transfers, $Y_R(t) > 0$ (G^c2) .

Government's security production and survival condition:

Generally, there is a given security level \bar{S} which the government can maintain through standard actions. In addition there is a threat from rebels which has negative effects on security $\theta(t)$. By taking countermeasures $G_S(t)$ the government can reduce this threat

$$\text{general} : S(t) = S(\bar{S}, \theta, G_S) \quad (\text{G3})$$

$$\text{fight} : S(t) = \bar{S} - \theta(t) G_S(t)^{-\beta} > S^{\min} > 0, \beta < 1 \quad (\text{G}^f3)$$

$$: \theta^{\max} > (S^{\min} - \bar{S}) G_S(t)^{\beta} \quad (\text{G}^f3b)$$

$$\text{com} : S = \bar{S} - \theta(t) Y_R^{-\beta}. \quad (\text{G}^c3)$$

In the case of a fight, the general security level \bar{S} is reduced by threat $\theta(t)$. By spending on a protection good the government can reduce this threat with diminishing returns according to $\frac{\theta(t)}{G_S(t)^{\beta}}$. Hence, each period's level of security is (G^f3) .

In the case of a compromise, a no-attack agreement is signed with the insurgent group. Further, military threat turns into political competition such that there is no specific, concrete, or explicit threat from rebels. However, the government does not feel completely safe and still perceives some diffuse threat $\theta(t)$. Moreover, the government knows that rebels would like to gain from the peace agreement and expects rebels to be calmer, more cooperative and better behaved if they benefit more from the peace compromise. Hence, the government's transfers to the rebels $Y_R(t)$ promote their commitment to good conduct and reduce the diffuse threat by $Y_R^{-\beta} \theta(t)$. If \bar{S} is again the general non-rebel-specific security level, total security once the compromise is reached can be described as (G^c3) .

Government's perception of threat:

Generally, the government's threat perception is $\theta(t)$. This depends on the specific type of fight or a compromise with peaceful coexistence. Hence, we state the perception of threat as equation number (G^i4) and (G^i5) .

In the case of a fight, the government is challenged with a number of attacks. These attacks and the effects for the government are random. The

government regards the evolution of the threat as a stochastic process, namely a Brownian motion¹⁶

$$d\theta(t) = \alpha\theta dt + \sigma\theta dW \quad (\text{G}^f 4)$$

with α' being the government's perception of the evolution of the threat level due to rebels' activities. That is, α is taken as an overall indicator of the perception of a threatening impact of rebels' military actions. The government relates the shift in threat to the number of rebels N , $\alpha = \alpha'N$. The expected threat and its impact on security can be reduced by spending on countermeasures and the protection good $G_S(t)$. Accordingly, the government can generate an optimal spending on security which we discuss in the next section. However, the government perceives a total magnitude of this threat as determined in lemma 2 (see also appendix 6.2.).

Lemma 2 (*perceived total threat*) *Expected perceived present value of threat at time τ is*

$$E \int_0^\infty \theta(t) e^{-\gamma t} dt = \frac{\theta(\tau)}{\gamma - \alpha} \quad (\text{G}^f 5)$$

In the case of a compromise, the government's perception of threat is driven by the pure number of rebels, now as an opposition group, even if there is no expectation of attacks. Hence the perceived and expected total threat is

$$\theta(t) = N^\beta \quad (\text{G}^c 4)$$

$$E \int_0^\infty \theta(t) e^{-\gamma t} dt = \frac{1}{\gamma} N^\beta \quad (\text{G}^c 5)$$

4 Game theoretic Analysis

In this section we study the strategic interaction between the government and the rebel group and set up a model of a two-person non-cooperative game involving sequential as well as simultaneous decisions. We define the game by specifying the order of decisions, possible actions and payoffs resulting from action choices. As a solution concept, we use the subgame perfect equilibrium concept which requires that with any decision, players choose actions that constitute a Nash equilibrium in the remaining game.

A key quantity for the analysis and evaluation of outcomes of the game is the rebel group's initial size N_0 .

The structure of the proposed game is as follows. In the beginning, fundamental decisions on whether to compromise or to fight are taken sequentially.

¹⁶It seems we assume asymmetries between the generation of threats by rebels using an Ito Levy process and the perception of threat by the government. However, even if there is a formal difference we believe that there is no serious loss of generality by doing so. Due to a myopic government ET does not substantially change the decision problem as $\int_0^{ET} U(t) e^{-\gamma t} dt \approx \int_0^\infty U(t) e^{-\gamma t} dt$ (see discussion of (G^f1)). Therefore, to derive qualitative results it is sufficient to use the much simpler Brownian motion to describe threat evolution and determine the expected utility effects.

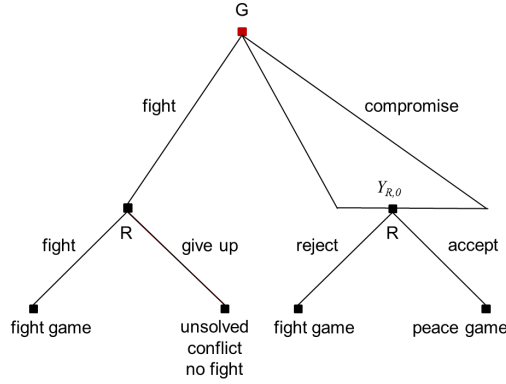


Figure 1: Gametree

Depending on the chosen actions the two players play a simultaneous move game, which is played in either a fight or a peace scenario. The government's strategic variables are the expenses required either to fight the group or to maintain peace, while the rebel group chooses its size and capital intensity. We assume that the (final) payoffs are present values of the stochastic payoff processes discussed in the previous section. Note that according to proposition 4 the governmental choice problems for payment flows $G_S(t)$ and $Y_R(t)$ admit an optimal control that is constant *across time*, while proposition 3 guarantees this stationarity for the group size. Hence we look only at stationary solutions. This allows us to summarize the dynamics and restrict actions to constant payment streams represented by real numbers G_S and Y_R .

The game tree depicted in Figure 1 illustrates the order of sequential decisions. In the beginning, the government and the rebel group move sequentially which eventually results in one of the following three outcomes: 1) they do not settle the conflict and play a fight game, 2) they agree on a compromise and play a peace game, or 3) the rebel group decides to disband and the active conflict is over.

The goal of our game theoretic analysis is to identify conditions under which a peaceful resolution of the conflict is possible or more likely. Formally speaking, this means under which conditions the peace game/peace outcome or the fight game will be played in equilibrium.

We start with a more detailed discussion on the sequential moves section, then turn to the simultaneous moves games and their equilibria, then bring both together to analyze the "complete game".

4.1 Sequential decisions

Stage 1 The government moves first and decides on whether to fight the rebels or to compromise by making an offer for a transfer payment $Y_R \in [0, Y]$

to the group.

Stage 2a in the latter case we have a sustainable peace situation without transfers from the government to the rebel group. However, in that situation the group may enjoy some outside option payoff, denoted by \bar{v} .

Stage 2b If the government has chosen to compromise at stage 1, then the rebel group may either accept the compromise and play a *peace game* in which a payment to the group will be fixed, or it may reject the offer and instead fight, i.e., to play the same *fight game* as above.

Therefore, the rebels' decision at stage 2 determines whether the fight game is played or not. Formally, a two-player simultaneous move game (or a game in strategic form) is given by a tuple (S^1, S^2, π^1, π^2) collecting the sets of possible strategies S^1, S^2 for the two players and payoff functions $\pi^i : S^1 \times S^2 \rightarrow \mathbb{R}$ ($i = 1, 2$). Both players take their actions without observing the other player's choice. The number $\pi^i(s^1, s^2)$ is understood as the payoff to player i , when player 1 chooses strategy $s^1 \in S^1$ and player 2 chooses $s^2 \in S^2$. As solution concept for simultaneous moves games we discuss the undisputably most prominent concept, known as *Nash equilibrium*. A Nash equilibrium for a two-person game is a pair of strategies $\bar{s} = (\bar{s}^1, \bar{s}^2)$ such that for each player i , \bar{s}^i maximizes player i 's payoff, given that the other player j uses his equilibrium strategy \bar{s}^j . That means that in a Nash equilibrium each player plays a best response (strategy) against their opponent's equilibrium strategy, meaning that a unilateral deviation to a non-equilibrium strategy s'^i does not increase i 's payoff.

As a solution for the complete game we employ the subgame perfect equilibrium concept. In our setup, a subgame perfect equilibrium is a collection of the strategies of both players, specifying actions at each decision node of the tree, such that in each simultaneous move game a Nash equilibrium is played and optimal decisions are taken in the sequential moves section, anticipating that Nash equilibrium payoffs result. To determine subgame perfect equilibria, we move backwards in the game tree. Starting with the analysis of Nash equilibria in the fight and the peace game, we discuss optimal choices in stage 1 and their resulting outcome. We will argue that, e.g., time preference in the sense of an indicator of ideological impact influence the equilibrium outcome.

As a Nash equilibrium consists of strategies that are mutually best responses, we devote the next two subsections to an intensive study of best responses in the fight and peace game. This will allow us to discuss different scenarios and their equilibrium outcomes in terms of group size, warfare strategy, or transfer payments.

4.2 Simultaneous move games: fight game

In order to describe the fight game we need to specify the decision problems and optimal choices for the two players (rebels and government) in more detail and determine best responses and evaluate the outcomes. The basic decision

variables are the government's expenses G_S in the conflict and for the rebel group, its size N and capital intensity k .

4.2.1 Rebels' optimal choices in case of fight

From the above discussion we can summarize that a group that chooses to turn to violent fighting will maximize its expected present value. As Ev is monotonic decreasing in ET , obviously rebels can also minimize the expected time until the government collapses. Hence, rebels' minimize the expected first hitting time which is the expected time of overthrowing the government under the finance constraint, the survival constraint, and the military feasibility constraint. Hence when fighting, the rebels' optimization problem turns into the problem

$$\max_{N,K} : Ev, \text{ s.t. } \begin{cases} \text{financial constraint} & (R^f 2) \\ \text{survival condition} & (R^f 3a) \\ \text{military feasibility condition} & (R^f 3b) \end{cases} \quad (R^f 6)$$

and is fully stated in appendix 6.3. Results of optimal choices are given in proposition 1.

Proposition 3 (*rebels' best responses*) *A rebel group with a resource constraint, survival constraint, and military feasibility constraint will minimize the expected time until defeating the government by choosing the optimal stationary group size in terms of the number of fighters N^f , and capital intensity k^f conditional on the government's spending on countermeasures, G_S .*

$$\begin{aligned} N^f &= N^f(G_S, F, p_N, w_N), \\ k^f &= k^f(G_S, p_N, w_N, p_K, w_K), \end{aligned} \quad (R^f 7)$$

Hence, the expected time of overthrowing the government is ET^f , leading to rebels' expected welfare from fighting Ev^f

$$\begin{aligned} ET^f &= ET^f(N^f, k^f, \theta^{\max}) \\ Ev^f &= Ev^f(N^f, k^f, G_S, F, \rho, Y, \bar{L}), \quad \frac{dEv^f}{dG_S} < 0, \quad \frac{dEv^f}{d\rho} < 0 \end{aligned} \quad (R^f 8)$$

For a proof see appendix 6.3. The best responses of rebels under fight conditions can be summarized qualitatively by the R_R^f - curve in the fight game of figure 2. In figure 2 we see both the optimal number of rebels and the optimal capital intensity as a response to the government's countermeasures.

4.2.2 Government's optimal choices in case of fight

The government has two options: to defend itself and hope that it will eventually defeat the rebels, or to compromise and sign a sustainable peace agreement. In this section we consider the option of fighting a war against rebels.

$$\max_{G_S} : EV, \text{ s.t. } \begin{cases} \text{financial constraint} & (G^f 2) \\ \text{security production \& survival condition:} & (G^f 3) \end{cases} \quad (G^f 6)$$

The government maximizes expected welfare by providing security through spending to achieve optimal level on security G_S .¹⁷ Hence the government's security policy is a function of a set of parameters and the threat generated by the rebel group.

Proposition 4 (*government's best responses in case of a fight*) *If the government produces protection and generates security based on the expected path of perceived threat level it will choose an optimal stationary expenditure on security G_S^f conditional on the rebel's group size N . The corresponding expected welfare level is EV^f .*

$$\begin{aligned} G_S^f &= G_S^f(N), \quad \frac{dG_S}{dN} > 0 & (G^f 7) \\ EV^f &= (G_S^f, N, \theta(\tau)), \quad \text{with: } \frac{dEV^f}{dN} < 0, \quad \frac{dEV^f}{d\gamma} < 0 & (G^f 8) \end{aligned}$$

For a proof see appendix 6.4. The government's best responses under fight conditions can be summarized qualitatively by the R_G^f -curve in the fight game in figure 2.

4.2.3 Nash equilibria when fighting the "Destabilization War"

In the fight game, the government takes a decision on how much to spend on security $G_S \in [0, Y] = S^1$, while the rebel group decides on its group size $N \in \mathfrak{R}$ and capital intensity $k \in \mathfrak{R}$, thus $S^2 := \mathfrak{R} \times \mathfrak{R}$. The payoffs depend on the combination of chosen actions, specifically the chosen action profile (G_S, N, k) . More precisely, they are given by the present values of utility flows as calculated in the previous section (see (G^f8) and (R^f8)), i.e.,

$$\pi^1(G_S, N, k) = EV^f(G_S, N, k) \quad \pi^2(G_S, N, k) = Ev^f(G_S, N, k)$$

Figure 2 sums up the best responses of rebels and government. As Nash equilibria are pairs of best responses, the intersection points are precisely the Nash equilibria. With the help of this figure we can discuss the outcome of the conflict when fighting ensues. As we conduct a graphic qualitative analysis we broadly discuss the most interesting outcomes. A large variety of constellations

¹⁷At this point, a full dynamic analysis would require the solution of an optimal control problem to determine the optimal path $\{G_S(t)\}$. This optimal path would be summed up to determine total wealth from security and total wealth from consumption. However, as we are only interested in these two total wealth effects and not in the exact particular time path we approximate the average spending derived from an optimal control path by a stationary average spending. This procedure allows to approximate the spending decisions between consumption and security at an accumulated stock level in a very simple manner

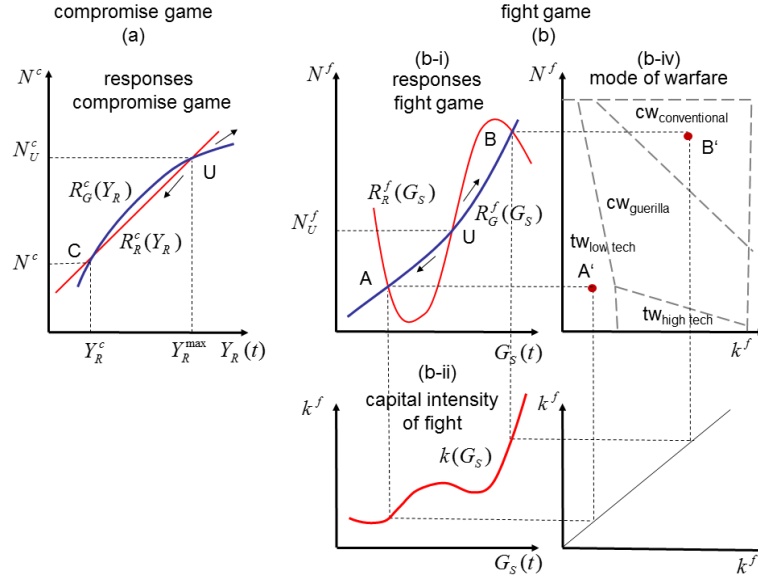


Figure 2: Potential equilibria: (a) compromise for peace, (b) fight game

may lead to more and various shapes of the two curves including potentially more (or fewer) equilibria. However, we have shown that the shape of the curves described in figure 2 prevail under standard conditions (see appendices 6.3 and 6.4). Hence we take this shape as an example for our qualitative discussion.

Multiple stationary equilibria and path dependence: The first interesting result is that we potentially obtain more than one stationary equilibrium. In our example we have two stable Nash equilibria, A and B , and an unstable stationary point U . The two stable equilibria show rather different characteristics. A is characterized by a smaller number of fighters and a low capital intensity of the violent actions. B however, is characterized by a large number of fighters and high capital intensity of military activities. Comparing these characteristics with the categories discussed in figure 1 and the broader characteristics described in table A1 we can now identify the kind of warfare which ensues in case the conflict turns violent. At stationary equilibrium A the warfare activities are likely to be dominated by terrorism. Fighters look like civilians and hide among the civilian population, attack soft targets or state symbols with guns, explosive belts and car bombs without occupying their own territory. B however, can be expected to involve a violent conflict that is more a conventional civil war with modern military equipment, a centralized military command structure and conventional battles, and a group that may occupy some territory.

War and warfare strategy. How does a conflict turn into a terrorist war, a guerilla war or even a conventional civil war? Is warfare strategy an explicit once-and-for-all determined optimal strategic choice as suggested by Carter (2015) or Bueno de Mesquita (2013)? Or is it the result of conflict evolution? Our answers to these questions are more complex. In our framework, the strategy of warfare the rebels conduct is determined by two components: the number of rebel fighters N and the military equipment per fighter indicated by capital intensity of fight k (see Figure 2). Capital intensity is the value (number and quality) of weapons available per fighter. Hence, a small group of insurgents fighting with low capital intensity (guns, hand grenades) are likely to build a terrorist group; see the south-west area in figure 2. They are most effective in generating a threat through terrorist warfare strategy. For a guerilla war we can expect the rebel group to consist of a larger number of insurgents. Its capital intensity is sufficient to attack not only soft civilian targets but also government forces (see the center area in figure 2). If the group of insurgents becomes sufficiently large and is equipped sufficiently, it may attack government forces in a conventional way. If N and k are large we are in the north east corner of figure 2. Obviously, we can easily connect these two dimensions with other characteristics such as land, as suggested by Carter (2015) and Bueno de Mesquita (2013). If a group has large amounts of military equipment at its disposal it is likely to control land. Guerilla and conventional warfare seem related to the control of some land. Hence land is regarded here as a complementary factor for military capital and not as a fundamental source of finance. In this model we suggest that the type of warfare chosen depends on the path of best-response reactions and the particular starting condition of the violent conflict (see figure 2b). If the rebel group starts fighting with a number of fighters larger than N_U best response reactions would lead to stationary equilibrium B and hence a conventional civil war. If the number of fighters is below N_U best responses would lead to a conflict that can be described as terrorism in stationary equilibrium A . Further, even if the conflict moves towards a conventional civil war, a change in the relative location of the curves could redirect conflict evolution towards terrorism and vice versa.

Hence many elements determine the direction in which a conflict and warfare strategy will develop. With comparative statics we can identify some of these elements. We do this by focusing on rebels' responses. As expected, we can unambiguously identify that higher external finance, $d\omega > 0$, will shift the R_R^f - curve upwards and increases the stationary size of the group N , the sustainable value of military equipment K , and drives expected welfare of fighting, Ev^f . Interesting enough, the volume of external finance does not determine the capital intensity of fighting. Which kind of warfare will evolve is not directly affected. The extent of external finance purely determines the magnitude of violent activities. The warfare characteristics of the conflict are rather determined by relative costs and structural parameters. For instance, if the procurement costs of military equipment become relatively lower compared to the cost of recruiting fighters $\frac{p_N}{p_K}$ (easy access to weapons) fighting becomes more capital-intensive and is more likely to turn into a guerilla or conventional war.

Similarly, if military equipment is not very effective in generating a threat to the government $(1 - \varepsilon)/\varepsilon$ compared to the frequently conducted attacks by rebels, rebels will choose a lower capital intensity of their fight and be more likely to opt for a high frequency of terrorist attacks as their warfare strategy. This is also the case if the government is fairly efficient in fighting and destroying regular military targets d_K instead of looking for insurgents and finding and catching rebel fighters. The same is true if recruiting is rather easy for rebels due to broad support by the population, the strong attractiveness of their ideology, or their proclaimed reason for violent insurgency. Furthermore, it is interesting to note that time preferences or ideology parameters do not primarily affect the choice of warfare. Neither ρ nor L affect the optimal choice of k or N and hence the k -curve. Time preferences affect only the level of welfare in A or B . Since either A or B will be reached depending on starting conditions, the mode of violent conflict is not an explicit strategic choice. The kind of warfare that evolves is an implication of the choice between peace and fight and the resulting dynamic adjustment starting from a particular point of departure.

Can rebels be defeated? Rebels are defeated if their survival constraint and/or military feasibility condition cannot be fulfilled and the group size steadily shrinks to fewer than one fighter, $\dot{N} = R_N - D_N < 0$. As long as the group has financial resources and stationary equilibrium is at $N > 1$, the number of fighters will always remain positive, even if the group is rather small due to government countermeasures. However, if stationary equilibrium drops to below one the last fighter cannot be replaced by a new recruit when he or she captured or killed, $1 > N > \frac{\frac{1}{p_N} F_{RN}}{(G_S - 2G_S^2 + G_S^3)}$. Hence the group is continuously reduced and eventually defeated if for each period

$$G_S - 2G_S^2 + G_S^3 > \frac{1}{p_N} F_{RN}$$

Would rebels give up? To answer that question, we already foreshadow the equilibrium analysis of the complete game. Should the government choose to face the conflict at stage 1, it would be optimal for rebels to give up if their payoff from fighting becomes lower than the given fallback position \bar{v} .

4.2.4 Summarized values of fight game

In sum, $\pi^{f,1}(G_S, N, k) = EV^f(G_S, N, k)$, and $\pi^{f,2}(G_S, N, k) = Ev^f(G_S, N, k)$ denote the government's and the rebels' payoffs in the fight game, respectively. As the result of best response dynamics fight game will either move into stationary fight equilibrium A or B ; hence, we need to consider both outcomes. A could be terrorism and B could be conventional civil war. However, both outcomes have different payoffs. From derivatives in (G^f8) and (R^f8) we conclude

for the relation of equilibrium payoffs

$$\begin{aligned}\pi^{A,1} &:= EV^f(A) > EV^f(B) =: \pi^{B,1} \\ \pi^{A,2} &:= Ev^f(B) > Ev^f(B) =: \pi^{B,2}\end{aligned}$$

Hence, we can think of a compatible example by choosing pairs of payoffs for the various outcomes we would like to consider. However, whether equilibrium A or B is reached is not a matter of choice but a path-dependent result of the process. Furthermore, these examples are arbitrary and remain useless unless we investigate the effects of changes in interesting parameters. Therefore, we check how changes in time preferences affect the value of certain outcomes. As discussed in a previous section rebels' time preference is associated with the degree of belief and ideology. Low time preference is associated with a strong conviction that the true belief will eventually prevail. From derivatives in (G^c8) and (R^c8) we know that $\frac{dEV^f}{d\gamma} < 0$ and $\frac{dEv^f}{d\rho} < 0$.

4.3 Simultaneous move games: compromise game

In analogy to our analysis of the fight game, we start by specifying both players' decision problems in the compromise game in greater detail, determine best responses, and evaluate the outcomes. To compromise means to move from violence and fight to a political struggle. Note, that under peace conditions all parameters are generally different than under fight conditions. However, to keep the notation simple we do not indicate this explicitly. In the compromise game, the government chooses to make a payment to the rebel group $Y_R \in [0, Y]$, while the group adjusts its group size N .

4.3.1 Rebels' optimal choices in case of compromise:

From the above discussion we can summarize that a rebel group that agrees to compromise will maximize the expected present value of the related income stream Y_D by choosing an optimal number of supporters. Hence, the rebels' problem is

$$\max_N : Ev \quad \text{s.t.} \quad \begin{cases} \text{financial constraint} & (\text{R}^c2) \\ \text{security prod. \& survival cond.} & (\text{R}^c3a) \end{cases} \quad (\text{R}^c6)$$

In case of compromise rebels will behave as stated in proposition 3.

Proposition 5 (*best responses in case of a compromise*) *A rebel group will maintain an optimal group size of supporters to signal credibility and strength for a lasting peace by choosing N^c . By compromising rebels obtain expected welfare Ev^c*

$$\begin{aligned}N^c &= (Y_R, F, v^f, \rho, p_N, w_N), & \frac{dN^c}{dY_R} > 0 & \quad (\text{R}^c7) \\ Ev^c &= (N^c, Y_R, F, v^f, \rho, p_N, w_N), & \text{with } \frac{dEv^c}{dY_R} > 0, \quad \frac{dEv^c}{d\rho} < 0 & \quad (\text{R}^c8)\end{aligned}$$

For a proof see appendix 6.5. The best responses under compromise conditions can be summarized qualitatively by the R_R^c - curve in figure 2a.

4.3.2 Government's optimal choices in case of compromise

The second option for the government is to compromise. In such a case the government looks at the potential threat and then estimates how much it is worth to offer income to the rebels to compromise and hence prevent a fight. The goal of the government is to maximize expected utility by allocating tax resources to the general public good and to a protection good producing security subject to the budget constraint. As V is linear connected to U it is sufficient to maximize U .

$$\max_{Y_R} : EV^c \quad \text{s.t.} \quad \begin{cases} \text{financial constraint} & (G^c 2) \\ \text{security production \& survival condition:} & (G^c 3) \end{cases} \quad (G^c 6)$$

The optimal governmental choices for a peace period are given in proposition 4.

Proposition 6 *In compromising, the government attempts to establish a lasting peace agreement with rebels by transferring an optimal amount of income Y_R^c to the rebels and their supporters and giving income to government supporters Y_G^c . Hence the maximum expected PV of a compromise and peace is EV^c ,*

$$\begin{aligned} Y_R^c &= Y_R^c(N), \quad \frac{dY_R}{dN} > 0, \quad \frac{d^2Y_R}{dN^2} > 0 & (G^c 7) \\ Y_G^c &= Y - Y_R^c \\ EV^c &= (Y_R^c, N, Y, EV^f) \quad \text{with} \quad \frac{dEV^c}{dN^c} < 0, \quad \frac{dEV^c}{d\gamma} < 0 & (G^c 8) \end{aligned}$$

Best responses under compromise conditions can be summarized qualitatively in figure 2. For formal discussion see appendix 6.6.

4.3.3 Best response outcomes of finding a compromise

Prior to playing the peace game, the rebel group has accepted the compromise offer by the government and with that the associated payment of $Y_{R,0} \in [0, Y]$, while the rebel group chooses its size $N \in \mathfrak{R}$. We assume that capital intensity in a peace state is zero. Payoffs are denoted by $\pi^{c,1}(Y_R, N)$ for the government and $\pi^{c,2}(Y_R, N)$ for the rebel group, i.e.,

$$\pi^{c,1}(Y_R, N) = EV^c(Y_R, N), \quad \pi^{c,2}(Y_R, N) = Ev^c(Y_R, N)$$

Figure 2a sums up the best responses of rebels and government. Again, each intersection point is a Nash equilibrium in the peace game. With the help of this figure we can discuss the outcome of conflict when both opponents decide to strike a compromise and hold the peace.

Stationary equilibrium and path dependence: The first interesting result is that we again potentially obtain more than one equilibrium. C is the stable compromise for a peace equilibrium. Whether C can be reached at all depends on the starting condition, namely the offer $Y_{R,0}$ the government makes when choosing to compromise at stage 1. Another stationary but unstable point is U . If the the starting point is in the unstable area (north east of U), a stable peace compromise solution cannot be reached in the simultaneous move game.

Maximum transfer income It is also interesting to note that the stable process to a peace compromise will only be chosen if the original transfer payment offered by the government is not too high (more than $Y_{R,0} > Y_{R,0}^{\max}$), and/or if the initial number of supporters of the rebel group is not too large (more than N_0^{\max}). Therefore, as in the fight game the initial conditions are crucial for the outcome of the conflict.

4.3.4 Summarized values of compromise game

If we denote the equilibrium payoffs in the Nash equilibrium C , the government's and rebels' payoff in the compromise game are

$$\pi^{C,1} := EV^{\text{government}}(C), \quad \pi^{C,2} := Ev^{\text{rebels}}(C)$$

As the compromise game is an alternative to fight and war, we have to consider the respective payoffs in the Nash equilibrium.

We can think of a consistent example by choosing two payoffs for the compromise outcomes: However, these examples are arbitrary and remain useless unless we investigate the effects of changes in interesting parameters. We hence verify how changes in time preferences affect the value of certain outcomes. From the derivatives in (G^c8) and (R^c8) we know that an increase in the government's and the rebels' time preference rate will reduce welfare of peace.

4.4 Subgame perfection: Compromise for peace or fight

Start a "Destabilization War" or compromise for peace: To answer the first question in this paper we illustrate how and under what conditions conflict manifests as a destabilization war, or when we observe a peace compromise. We give examples for outcomes under various initial conditions.

Having discussed the Nash equilibria in the fight game and the compromise game, we are ready to see what the subgame perfect equilibria of the complete game look like. As noted above, in our model we determine such equilibria by considering Nash equilibria in the simultaneous moves games and using them as the basis for optimal decisions in stage 1. To this end, we summarize the fight and compromise games by looking at their equilibrium payoffs. Figure 3 depicts the reduced game tree with terminal payoffs that result from equilibrium behavior. The superscript e in the numbers $\pi^{e,1}, \pi^{e,2}$ stands for fight game equilibrium A or B . In our discussion we assume that the best outcome for

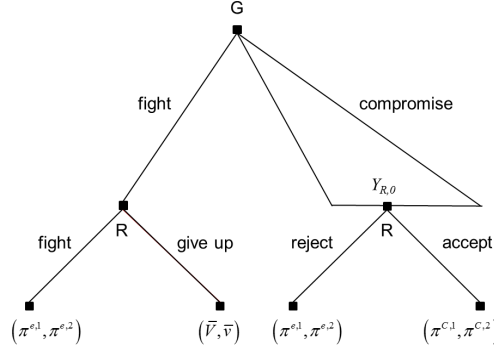


Figure 3: Reduced game

the government is the dissolution of the rebel group, which results in a payoff of \bar{V} , which is higher than any other equilibrium payoff for the government. Furthermore, in the degenerate case that two payoffs are equal, we assume that decision makers opt for peace.

Considering rebels' optimal choices at stages 2a and 2b, they will choose to fight at stage 2a if $\pi^{e,2} > \bar{v}$ and at stage 2b if $\pi^{e,2} > \pi^{C,2}$. Recall that the resulting equilibrium ($e = A$ or $e = B$) depends on the initial number of fighters. Consequently, the government's decision at stage 1 can take subsequent optimal behavior into account.

We ask when a peaceful resolution prevails. First, we obtain a compromise as subgame perfect equilibrium outcome if the government's offer at stage 1 is such that the equilibrium in the peace game can be reached and if

$$\pi^{C,2} \geq \pi^{e,2}, \quad \pi^{e,2} > \bar{v}, \quad \text{and} \quad \pi^{C,1} \geq \pi^{e,1} \quad (9)$$

hold. Hence, from an analytical point of view, it is the relation of equilibrium payoffs and outside options that has to be taken into account. A close inspection reveals that compromise can only prevail if payoffs in the Nash equilibrium C of the peace game Pareto-dominate Nash equilibrium payoffs in the fight game (depending on whether initial group size leads to equilibrium A or B).

Second, we obtain dissolution as the subgame perfect equilibrium outcome if the payoff \bar{v} is attractive enough for the rebels compared to their Nash equilibrium payoff in the fight game, i.e., if

$$\bar{v} \geq \pi^{e,2}. \quad (10)$$

Then, together with the assumption that \bar{V} is maximal for the government, the government will choose to fight at stage 1 and the rebels will give up.

As for the measures that can be taken to increase the likelihood of peace, we demonstrate the above by considering the following relation of payoffs:

$$\bar{V} > \pi^{C,1} > \pi^{A,1} > \pi^{B,1} \quad (11)$$

$$\pi^{A,2} > \pi^{C,2} > \pi^{B,2} \geq \bar{v}. \quad (12)$$

Since in the Nash equilibrium A the rebels' payoff is larger than their payoff in Nash equilibrium C , the fight game will be played in subgame perfect equilibrium when the initial size N_0 is low enough. If N_0 is large enough so that B results as Nash equilibrium of the fight game, then the conditions in (11 and 12) are met and the compromise game will be played in equilibrium.

It is also important to note that in this setting the simultaneous view on potential outcomes also suggests that the decision in favor of either warfare strategy cannot be easily separated from the war or compromise decision. The outcomes of each of these strategies always need to be compared to the compromise for peace outcome.

Ideology and time preferences Next, time preferences indicate some aspects of belief or ideology. If rebels are motivated by an ideology or "the one true belief" they can be expected to apply long-term thinking with a long-term time horizon and low time preferences. Although such groups actively fight for their beliefs, they can wait for their irrevocable convictions to become reality, even if this may only happen in the distant future. They regard their ideology as a historical prediction that will eventually come to pass due to divine revelation or a predetermined destiny. Therefore, ideological motivation can be related to lower time preferences ρ . We hence discuss the effects of ideology on the relationship between payoffs and thus on the subgame perfect equilibrium outcome. The government's time preference γ reflects the length of its planning horizon. In the following example, we consider γ_0, γ_1 with $\gamma_0 < \gamma_1$ and ρ_0, ρ_1 with $\rho_0 < \rho_1$ are considered. With slight misuse in notation, denote by $\pi^{e,1}(\gamma_i)$ and $\pi^{e,2}(\rho_i)$ the Nash equilibrium payoffs $e = A, B, C$ depending on time preferences.

Collecting results on derivatives of payoffs with regard to time preferences, we need to ensure that (i) payoff relations in case of fight hold, i.e., $\pi^{A,1}(\rho_i) > \pi^{B,1}(\rho_i)$, $\pi^{A,1}(\gamma_i) > \pi^{B,1}(\gamma_i)$, (ii) for the effects of an increase in γ , $[\gamma_1 > \gamma_0]$, the welfare effect under peace conditions is larger than under fight conditions, $\pi^{C,1}(\gamma_0) - \pi^{C,1}(\gamma_1) > \pi^{e,1}(\gamma_0) - \pi^{e,1}(\gamma_1)$ ($e = A, B$), and (iii) for an increase in ρ , $[\rho_1 > \rho_0]$, the welfare effect under peace conditions is smaller than under fight conditions, $\pi^{C,2}(\rho_0) - \pi^{C,2}(\rho_1) < \pi^{e,1}(\rho_0) - \pi^{e,1}(\rho_1)$ ($e = A, B$). Hence the following table is consistent with these conditions:

| welfare of | government | | rebels | |
|------------------------|-------------------|------------------|-------------------|------------------|
| | low γ_0 | high γ_1 | low ρ_0 | high ρ_1 |
| terrorism | $\pi^{A,1} = 80$ | $\pi^{A,1} = 70$ | $\pi^{A,2} = 110$ | $\pi^{A,2} = 70$ |
| conventional civil war | $\pi^{B,1} = 30$ | $\pi^{B,1} = 20$ | $\pi^{B,2} = 50$ | $\pi^{B,2} = 10$ |
| compromise for peace | $\pi^{C,1} = 100$ | $\pi^{C,1} = 80$ | $\pi^{C,2} = 100$ | $\pi^{C,2} = 90$ |

The relation of governmental equilibrium payoffs for γ_0 and γ_1 is the same as in (11), while for the rebels (12) gives the payoff relations at ρ_0 . At higher time preferences ρ_1 , the payoff in the compromise equilibrium becomes more attractive and even more attractive than the fight equilibrium A . This demonstrates that a higher time preference, i.e., a higher preference for more current gains, on the rebels' side makes a peace solution more likely.

Ideology as indivisible objective Peaceful conflict resolution is only possible if the set of objectives of the rebels or the government are sufficient substitutes, and if indivisible objectives are not simultaneously essential/necessary in both preference systems. If strong beliefs are the dominant motivation of the rebel core, such groups would fight even until eternity. In this case either the rebel core and the belief must be completely eliminated, or rebels' resources and recruitment mechanisms must be reduced to a level optimal with respect to society's security interests and respective resource absorption. This can be easily illustrated by looking at utility functions. Again, the law system is indivisible and can take the values $L = 0, \bar{L}$. $L = \bar{L} > 1$ indicates that the set of laws of the rebels is introduced and $L = 0$ indicates that the law system of the government is maintained. If for both parties their preferred law system is necessary, the utility functions may look like $U(t) = [Y_G(t) + S(t)] \frac{1}{(1+L^\infty)}$ (government) and $u = [Y_R(t) + L] L$ (rebels). In this case we cannot find a positive utility for both parties simultaneously. If $L = 0$ and the government's legal system is maintained, the government will keep a positive utility, however the rebels will obtain a utility level of zero, $u = [Y_R(t) + L] L = 0$. Hence, if fighting gives any positive level of welfare rebels would always prefer to fight. Correspondingly, if the rebels' system is introduced, $L = \bar{L} > 1$ the government will tend to obtain no utilities, $U(t) = [Y_G(t) + S(t)] \frac{1}{(1+\bar{L}^\infty)}$. Hence the government would always prefer to fight as long as this leads to any positive level of utility. Formally, if the rebels' ideology is indivisible, the rebels' payoff in the compromise equilibrium is zero. Similarly, the outside option \bar{v} is zero, since the dissolution of the group will not render the law system, i.e., $L = 0$ and hence $\bar{v} = 0$. As a consequence, neither (11) nor (12) are satisfied, so that in the subgame perfect equilibrium no peaceful solution can prevail. In other words, if ideology and beliefs are an indivisible and essential element for both parties, fighting is the only possible outcome of the conflict.

Random chance and duration as constitutive element of "Destabilization War". The randomness of potentially large impacts of an attack is the rebels' most important ally. Even if they know that a certain attack may be not sufficient to overthrow the government, there may be randomly success. This event can be soon or in the distant future. However, rebels have an idea of when this can be expected to happen, as long as they continue fighting. Therefore, for rebels there is an expectation of the duration of their war, and of how long it will take until the process of destabilization generates an event that eventually overthrows the government and leads to victory. The randomness of outcomes

of attacks and the duration of a destabilizing process are constitutive elements of a destabilization war. This becomes clear if we look at the expected time of overthrowing the government ET^f , leading to rebels' expected present value of benefits from fighting $Ev^f = e^{-\rho ET^f} \frac{(Y+\bar{L})}{\rho}$. While it is obvious that a short duration is preferable, a long fight is possible and may be more beneficial than a compromise. Furthermore, if the time preference is very low a long fight may not strongly reduce the value of outcome such that even a long duration is not a strong obstacle to a fight. If a group is motivated by strong beliefs and has low time preferences, it may expect a long struggle and still see large benefits in their activities.

5 Summary and conclusions

To answer the two questions from the beginning of the paper we develop a formal model of a destabilization war. The model we develop has two sections. In the first section we describe the conditions and actions of the two parties (rebels and the government) and derive optimal choices with respect to the expected welfare of each group. We suggest there are three major ingredients of this unconventional type of violent conflict. The first element of a theory of destabilization war is that insurgents generate a continuous threat through random attacks with random impact. We model these random events by a stochastic Ito-Levy jump process. The second element to consider is the duration of the war. The explicit choice of the duration of the fight is rarely considered in formal conflict theory. The third element is the importance of time preferences, which we relate to ideology. In the second section, we go through a game theory analysis. We develop a two-person non-cooperative game involving sequential as well as simultaneous decisions. We define the game by specifying the order of decisions, possible actions, and payoffs resulting from action choices. As a solution concept, we use the subgame perfect equilibrium concept which requires that at any decision, the players choose actions that constitute a Nash equilibrium in the remaining game.

Answers to question to 1 from the begining is: Under conditions of asymmetric military capacity, insurgents do not opt for the one-battle war in which they will likely be defeated. Creating a continuous threat through random attacks with random impacts may have such strong effects that eventually the government will be pushed out. When rebels attack they do not expect that a particular singular action may achieve this. However, they do hope that an attack may eventually turn randomly into such a large event that the government is overthrown. The randomness of potentially large impacts of an attack is the rebels' most important ally. Duration matters, too. Wearing out the enemy and demoralize their supporters, either physically by employing guerilla tactics, or psychologically by employing terrorist tactics, seems an effective strategy in such violent conflicts. Choices relating to duration are closely linked to time preferences, so time preferences play an important role. In this model we relate time preferences to ideology and belief. Time horizon or time preference reflect

the motivation of a group. Ideology or "the one true belief" are connected to a long-term horizon such that these groups are happy to wait for realization of their vision to take place sometime in the distant future. They consider their utopias to be a historical prediction that will eventually come true due to a divine revelation or a true historically predetermined destiny. If strong beliefs are the dominant motivation of the rebel core, such groups would go opt for an eternal fight. In this case either the rebel core and the belief must be completely eliminated or rebels' resources and recruitment mechanisms weakened to a level that is optimal with respect to a society's security interests and resource absorption. If ideology is not the core motivation, compromises through economic participation are possible, but may be dominated by war strategies.

Answers to question 2 from the beginging is: The answer to the second question is more complex. However, we can identify four major elements. (i) Conflict resolution is generally path-dependent. This holds for the decision "peace or fight?" but also for the choice of the mode of violent conflict. (ii) Peaceful conflict resolution is only possible if the set of objectives of the rebels or the government are sufficient substitutes, and indivisible objectives are not simultaneously "necessary elements" in both preference systems. Further, rebels' high time preferences as well as a lower original number of rebels would also promote peaceful conflict resolution. (iii) A terrorist war is more likely if a) the original size of the group is relatively small when the conflict commences, b) insurgents have weak external support for violent fighting, and c) the group is rather isolated with low support from the population and high costs of recruitment. (iv) Conventional warfare and larger civil wars are connected with larger financial resources on the part of both rebels and the government plus rebels' easy access to military equipment.

Implications: Unlike in a short (no time dimension) one-shot battle, in a continuing violent conflict economic instruments are very powerful in terms of directing conflict resolution. Managing a continuous dynamic fighting process is an economic challenge concerning permanent flows of income and resources. If such economic flows are cut off, difficult and expensive military abilities on either side dry up. Hence economic instruments can be very powerful pacification methods. However, they can also be used to fuel violent conflicts and civil wars, as has been seen in many proxy wars.

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6 Appendix

6.1 Terms and definitions

Definition of terrorism: *Terrorism is an [1] anxiety-inspiring method of repeated [2] violent action, employed by (semi-) [3] clandestine individual, group, or state actors, for [4] idiosyncratic, criminal, or political reasons, whereby – in contrast to assassination – the direct targets of violence are not the main targets. The [5] immediate human victims of violence are generally chosen [6] randomly (targets of opportunity) or [7] selectively (representative or symbolic targets) from a target population, and serve as message generators. [8] Threat- and violence-based [9] communication processes between terrorist (organization), (imperiled) victims, and main targets are used to [10] manipulate the main target (audience(s)), turning it into a [11] target of terror, a [12] target of demands, or a [13] target of attention, depending on whether [14] intimidation, [15] coercion, or [16] propaganda is primarily sought. (1988, cited in Schmid [2004, p.382]).*

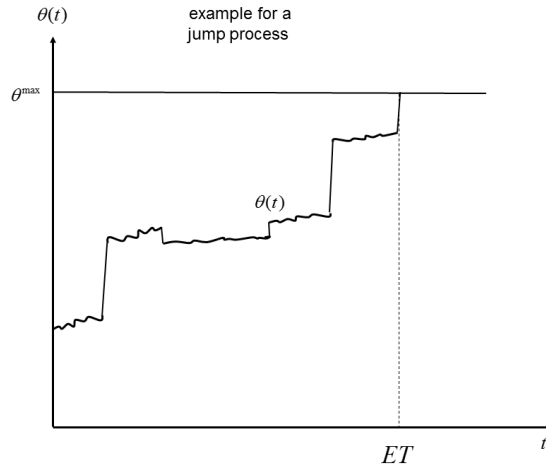


Figure 4: Example for a jum process

Table A1: Characteristics of Terrorism, Guerilla, and Conventional Warfare as Modes of Violent Struggle

| | Conventional Warfare | Guerilla Warfare | Terrorism |
|--------------------------|---|--|--|
| Unit Size in Battle | Large (armies, corps, divisions) | Medium (platoons, companies, battalions) | Small (usually less than ten persons) |
| Weapons | Full range of military hardware (air force, armor, artillery, navy) | Mostly infantry-type light weapons but sometimes heavier equipment as well | Hand guns, hand grenades, rifles, car bombs, remote control bombs, barometric pressure bombs |
| Tactics | Usually joint operations involving several military branches | Commando-type tactics | Specialized tactics: Kidnapping, assassinations, car-bombing, hijacking, barricade-hostage, etc. |
| Targets | Mostly military units, industrial and transportation infrastructure | Mostly military, police and administration staff, as well as political opponents | State symbols, political opponents, and the public at large |
| Intended Impact | Physical destruction | Mainly physical attrition of the enemy | Psychological coercion |
| Control of Territory | Yes | Yes (although there may be no clear front lines) | No |
| Uniform | Wear uniform | Often wear uniform | Do not wear uniform |
| Recognition of War Zones | War limited to recognized geographical area | War limited to country in strife | No recognized war zones. Operations carried out worldwide |
| International Legality | Yes, if conducted by rules | Yes, if conducted by rules | No |
| Domestic Legality | Yes | No | No |

Source: In accordance with Merari (1993)

6.2 Appendix: expected first hitting time and EPV of threat process

6.2.1 Expected First hitting time:

We assume a particular probability distribution for the step heights, and follow Kou and Wang (2003) by assuming an exponential distribution with the density function $h(z) = \eta e^{-\eta z}$ so that the waiting period until a jump occurs is exponentially distributed with mean $\frac{1}{\eta}$. By using the Girsanov theorem we can derive the probability density function of T ¹⁸ which is referred to as the *Inverse Gaussian Distribution*.¹⁹ With the threshold θ^{\max} we obtain, according to the procedure described by Kou and Wang (2003):

$$ET = \frac{1}{\lambda \frac{1}{\eta}} \left[\theta^{\max} + \left(\frac{1}{\eta} - \frac{1}{\mu_2^*} \right) (1 - e^{-\theta^{\max} \mu_2^*}) \right]$$

with $\frac{1}{\eta}$ being the mean of the exponential distribution, $\frac{\lambda}{\eta}$ denotes the overall drift and μ_2^* is defined as the unique root of $H(\mu_2^*) = 0$ with $H(x) := \frac{1}{2}x^2\sigma^2 + \lambda \left(\frac{\eta}{\eta+x} - 1 \right)$ and $0 < \eta < \mu_2^* < \infty$, or $\theta^{\max} > \left(\frac{1}{\eta} - \frac{1}{\mu_2^*} \right) (1 - e^{-\theta^{\max} \mu_2^*})$

$$ET = N^{-\varepsilon} K^{-(1-\varepsilon)} \left[\theta^{\max} + \left(K^{1-\varepsilon} - \frac{1}{\mu_2^*} \right) (1 - e^{-\theta^{\max} \mu_2^*}) \right]$$

$$\frac{dET}{dN} = -\varepsilon k^{-(1-\varepsilon)} \left[\theta^{\max} + \left(K^{1-\varepsilon} - \frac{1}{\mu_2^*} \right) (1 - e^{-\theta^{\max} \mu_2^*}) \right] < 0$$

$$\begin{aligned} \frac{dET}{dK} &= -(1-\varepsilon) N^{-\varepsilon} K^{-(1-\varepsilon)-1} [\dots] + (1-\varepsilon) N^{-\varepsilon} K^{-(1-\varepsilon)} K^{-\varepsilon} (1 - e^{-\theta^{\max} \mu_2^*}) \\ &= -(1-\varepsilon) N^{-\varepsilon} K^{-(1-\varepsilon)} K^{-1} \left[\theta^{\max} - \frac{1}{\mu_2^*} (1 - e^{-\theta^{\max} \mu_2^*}) \right] < 0 \end{aligned}$$

$$\text{with } C = \left[\theta^{\max} + \left(K^{1-\varepsilon} - \frac{1}{\mu_2^*} \right) (1 - e^{-\theta^{\max} \mu_2^*}) \right], \text{ and } C' = \left[\theta^{\max} - \frac{1}{\mu_2^*} (1 - e^{-\theta^{\max} \mu_2^*}) \right].$$

Derivative $\frac{dET}{dG_S}$ The threshold θ^{\max} depends on government protection. The maximum bearable threat is determined by G_S ($\theta^{\max} = (S^{\min} - \bar{S}) (G_S)^{\beta}$)

$$\begin{aligned} \frac{dET}{dG_S} &= N^{-\varepsilon} K^{-(1-\varepsilon)} \left[\frac{d\theta^{\max}}{dG_S} - \mu_2^* \left(K^{1-\varepsilon} - \frac{1}{\mu_2^*} \right) e^{-\theta^{\max} \mu_2^*} \frac{d\theta^{\max}}{dG_S} \right] \\ &= N^{-\varepsilon} K^{-(1-\varepsilon)} \left[1 - \mu_2^* \left(K^{1-\varepsilon} - \frac{1}{\mu_2^*} \right) e^{-\theta^{\max} \mu_2^*} \right] \frac{d\theta^{\max}}{dG_S} > 0 \end{aligned}$$

¹⁸ An extensive discussion is offered by Karatzas and Shreve (1991, p.196) and Karlin and Taylor (1975, p.363).

¹⁹ The term "inverse Gaussian distribution" stems from the inverse relationship between the cumulant generating functions of these distributions and those of the Gaussian distributions. For a detailed discussion of the inverse Gaussian distribution, see Johnson, Kotz, and Balakrishnan (1995) or Dixit (1993).

θ^{\max}

Expected present value of threat The expected total magnitude of the threat is indicated by the present value of θ obtained by accumulating the discounted threats with a time preference rate ρ

$$\begin{aligned} EV\{\theta\} &= E \int_{\tau}^{\infty} e^{-\rho(t-\tau)} \theta(t) dt \\ &= \frac{\theta(\tau)}{(\rho - \lambda\delta)} \quad \text{with} \quad \lambda\delta < \rho \end{aligned}$$

with λ being the frequency of attacks. δ sums up all single impacts $\delta = \int_{f^{-1}(U)} zh(dz) + \int_U zh(dz)$ where z is an indicator for the magnitude of the threat generated by attacks.

6.3 Appendix: The rebels' optimal choice when fighting

Decision problem: To determine optimal factor inputs N, K of warfare
The rebels' objective is to maximize Ev by minimizing ET , as Ev is a monotone downward sloping function in ET .

Problem:

$$\begin{aligned} \min_{N,K} : \quad ET &= \frac{1}{N^\varepsilon K^{(1-\varepsilon)}} \left[\theta^{\max} + \left(K^{1-\varepsilon} - \frac{1}{\mu_2^*} \right) (1 - e^{-\theta^{\max} \mu_2^*}) \right] \\ \text{s.t. (i)} : \quad F\omega &= p_N R_N + p_K R_K + w_N N + w_K K. \quad \text{finance constraint} \\ \text{(ii)} : \quad \dot{N} &= R_N - G_S N + 2G_S^2 N - NG_S^3 = 0 \quad \text{survival condition} \\ \text{(iii)} : \quad \dot{K} &= R_K - K d_K G_S = 0 \quad \text{military feasibility condition} \end{aligned}$$

from specifications (R^f1) to (R^f3b) we can plug in (ii) and (iii)

$$R_N = N (G_S^3 - 2G_S^2 + G_S)$$

$$\begin{aligned} \text{(ii)} : \quad \frac{1}{p_N} F_{RN} &= NG_S^3 - (2G_S^2 N - G_S N) \iff F_{RN} = p_N N (G_S - 2G_S^2 + G_S^3) \\ \text{(iii)} : \quad \frac{1}{p_K} F_{RK} &= K d_K G_S \iff F_{RK} = p_K K d_K G_S \end{aligned}$$

Lagrange function:

$$\begin{aligned} \min_{N,K,\psi} : \quad \mathcal{L} &= N^{-\varepsilon} K^{-(1-\varepsilon)} \left[\theta^{\max} + \left(K^{1-\varepsilon} - \frac{1}{\mu_2^*} \right) (1 - e^{-\theta^{\max} \mu_2^*}) \right] \\ &\quad - \psi (p_N (G_S - 2G_S^2 + G_S^3) N + p_K d_K G_S K + w_N N + w_K K - F\omega) \end{aligned}$$

FOC, We can now state the first order conditions for the above Lagrange problem

$$\begin{aligned}
(1) \quad \frac{d\mathcal{L}}{dK} &= -(1-\varepsilon)k^\varepsilon C' - \psi(p_K d_K G_S + w_K) = 0 \\
(2) \quad \frac{d\mathcal{L}}{dN} &= -\varepsilon N^{-\varepsilon-1} K^{-(1-\varepsilon)} C - \psi(p_N (G_S - 2G_S^2 + G_S^3) + w_N) = 0 \\
(3) \quad \frac{d\mathcal{L}}{d\psi} &= p_N (G_S - 2G_S^2 + G_S^3) N + p_K K d_K G_S + w_N N + w_K K - F\omega = 0
\end{aligned}$$

Optimal behavior, capital intensity k of fighting from (1) and (2) we obtain: k :

$$\begin{aligned}
\frac{-(1-\varepsilon)N^{-\varepsilon}K^{-(1-\varepsilon)-1}C'}{-\varepsilon N^{-\varepsilon-1}K^{-(1-\varepsilon)}C} &= \frac{(p_K d_K G_S + w_K)}{(p_N (G_S - 2G_S^2 + G_S^3) + w_N)} \\
\frac{(1-\varepsilon)C'N}{\varepsilon C} \frac{N}{K} &= \frac{(p_K d_K G_S + w_K)}{(p_N (G_S - 2G_S^2 + G_S^3) + w_N)}
\end{aligned}$$

With $C' = \left[\theta^{\max} - \frac{1}{\mu_2^*} (1 - e^{-\theta^{\max} \mu_2^*}) \right]$ and $C = \left[\theta^{\max} + \left(K^{1-\varepsilon} - \frac{1}{\mu_2^*} \right) (1 - e^{-\theta^{\max} \mu_2^*}) \right]$ the difference between the two terms is rather small ($C' \approx C$) and reduces with smaller K . In this model K is a variable open to any scaling. Hence, for the sake of a massive simplification of the algebra and without serious loss of generality, we assume that K is scaled sufficiently small such that we can regard $C' = C$.

$$\begin{aligned}
k &= \frac{K}{N} = \frac{(1-\varepsilon)p_N}{\varepsilon p_K} \frac{\left(G_S - 2G_S^2 + G_S^3 + \frac{w_N}{p_N} \right)}{\left(d_K G_S + \frac{w_K}{p_K} \right)} \\
\lim_{G_S \rightarrow 0} k &= \frac{(1-\varepsilon)p_N}{\varepsilon w_K} > 0, \quad \lim_{G_S \rightarrow \infty} k = \infty
\end{aligned}$$

First derivative $\frac{dk}{dG_S}$:

$$\frac{dk}{dG_S} = \frac{(1-\varepsilon)p_N}{\varepsilon p_K} \frac{1}{\left(d_K G_S + \frac{w_K}{p_K} \right)^2} \left[\begin{aligned} &-4 \frac{w_K}{p_K} G_S + 3 \frac{w_K}{p_K} G_S^2 - 2d_K G_S^2 \\ &+ 2d_K G_S^3 + \left(\frac{w_K}{p_K} - \frac{w_N}{p_N} d_K \right) \end{aligned} \right]$$

Extrema:

$$\frac{dk}{dG_S} = \frac{(1-\varepsilon)p_N}{\varepsilon p_K} \frac{1}{\left(d_K G_S + \frac{w_K}{p_K} \right)^2} \left[\begin{aligned} &-4 \frac{w_K}{p_K} G_S + 3 \frac{w_K}{p_K} G_S^2 - 2d_K G_S^2 \\ &+ 2d_K G_S^3 + \left(\frac{w_K}{p_K} - \frac{w_N}{p_N} d_K \right) \end{aligned} \right] = 0$$

$$-4\frac{w_K}{p_K}G_S + 3\frac{w_K}{p_K}G_S^2 - 2d_KG_S^2 + 2d_KG_S^3 + \left(\frac{w_K}{p_K} - \frac{w_N}{p_N}d_K\right) = 0$$

There are potentially three extrema

Shape of capital intensity curve:

$$\begin{aligned} \lim_{G_S \rightarrow 0} \frac{dk}{dG_S} &= \frac{(1-\varepsilon)}{\varepsilon} \frac{p_N}{p_K} \frac{1}{\left(\frac{w_K}{p_K}\right)^2} \left(\frac{w_K}{p_K} - \frac{w_N}{p_N}d_K\right) \begin{matrix} \leq \\ \geq \end{matrix} 0 \\ \lim_{G_S \rightarrow 0} \frac{dk}{dG_S} &= \infty \end{aligned}$$

Optimal behavior, size of the rebel group N : With (2') and (3) we get N :

$$K = N \frac{(1-\varepsilon)}{\varepsilon} \frac{G_S - 2G_S^2 + G_S^3 + \frac{w_N}{p_N}p_N}{d_KG_S + \frac{w_K}{p_K}p_K}$$

$$\begin{aligned} F\omega &= p_N (d_N G_S (1 + G_S^2) - G_S^2) N \\ &+ p_K d_K G_S N \frac{(1-\varepsilon)}{\varepsilon} \frac{p_N (G_S - 2G_S^2 + G_S^3) + w_N}{p_K d_K G_S + w_K} \\ &+ w_N N + w_K N \frac{(1-\varepsilon)}{\varepsilon} \frac{p_N (G_S - 2G_S^2 + G_S^3) + w_N}{p_K d_K G_S + w_K} \\ F\omega &= N \left[\frac{1}{\varepsilon} (p_N (G_S - 2G_S^2 + G_S^3) + w_N) \right] \end{aligned}$$

$$\begin{aligned} N &= \frac{\varepsilon F\omega}{p_N (G_S - 2G_S^2 + G_S^3) + w_N} \\ \lim_{G_S \rightarrow 0} N &= \frac{\varepsilon F\omega}{w_N}, \lim_{G_S \rightarrow \infty} N = 0 \end{aligned}$$

First derivative $\frac{dN}{dG_S}$:

$$\frac{dN}{dG_S} = - \frac{\varepsilon F\omega}{(G_S - 2G_S^2 + G_S^3) + \frac{w_N}{p_N}} (1 - 4G_S + 3G_S^2)$$

Second derivative $\frac{d^2 N}{dG_S^2}$:

$$\frac{d^2 N}{dG_S^2} = -\varepsilon F\omega \left[\frac{(-4 + 6G_S)}{(G_S - 2G_S^2 + G_S^3) + \frac{w_N}{p_N}} - \frac{(1 - 4G_S + 3G_S^2)(1 - 4G_S + 3G_S^2)}{\left((G_S - 2G_S^2 + G_S^3) + \frac{w_N}{p_N}\right)^2} \right]$$

Extrema:

$$\frac{dN}{dG_S} = -\frac{\varepsilon F\omega}{(G_S - 2G_S^2 + G_S^3) + \frac{w_N}{p_N}} (1 - 4G_S + 3G_S^2)$$

There are two extrema:

$$G_S^2 - \frac{4}{3}G_S + \frac{1}{3} = 0, \quad G_{S,1} = \frac{2}{3} - \left[\frac{4}{9} - \frac{3}{9}\right]^{\frac{1}{2}} = \frac{1}{3}$$

$$G_{S,2} = \frac{2}{3} + \left[\frac{4}{9} - \frac{3}{9}\right]^{\frac{1}{2}} = 1$$

Shape of the curve

$$\lim_{G_S \rightarrow 0} \frac{dN}{dG_S} = -\frac{\varepsilon F\omega}{w_N} p_N < 0$$

$$\lim_{G_S \rightarrow \infty} \frac{dN}{dG_S} = -\varepsilon F\omega \frac{p_N \left(\frac{1}{G_S^2} - \frac{4}{G_S} + 3\right) G_S^2}{\left[p_N \left(\frac{1}{G_S^3} - \frac{2}{G_S^2} + 1\right) + \frac{1}{G_S^3}\right] G_S^3} = 0$$

Expected value of welfare: The expected value of fight is

$$Ev^f = e^{-\rho ET^f} \frac{(Y + \bar{L})}{\rho}$$

Derivative of indirect utility function: Using envelop theorem:

$$\mathcal{L} = e^{-\rho ET^f} \frac{(Y + \bar{L})}{\rho} - \psi \left(\frac{p_N (G_S - 2G_S^2 + G_S^3) N + p_K d_K G_S K}{+w_N N + w_K K - F\omega} \right)$$

$$\frac{dEv^f}{dG_S} = \frac{d\mathcal{L}}{dG_S} = -\rho e^{-\rho ET^f} \frac{(Y + \bar{L})}{\rho} \frac{dET^f}{dG_S} - \psi (p_N (1 - 4G_S + 3G_S^2) N + p_K d_K K) < 0$$

for sufficient large $\frac{p_K}{p_N} d_K \frac{K}{N}$

$$\frac{dEv^f}{d\rho} = \frac{d\mathcal{L}}{d\rho} = -(\rho ET^f + \bar{L}) e^{-\rho ET^f} \frac{(Y + \bar{L})}{\rho^2} < 0.$$

6.4 Appendix: The government's optimal choice when fighting

Decision problem: To determine the optimal countermeasure expenditure G_S^f :

Problem:

$$\begin{aligned}
\max_{G_S} : \quad & EV = \int_0^\infty [E(Y_G(t))^v + S(t)] e^{-\gamma t} dt \\
s.t : \quad & (i) \quad Y(t) = Y_G(t) + G_S(t) \\
& : \quad \text{with } S(t) = \bar{S}^f - (G_S)^{-\beta} \theta(t), \quad \text{and } S \geq S^{\min}; \beta, v < 1
\end{aligned}$$

$$\begin{aligned}
\max_{G_S} : \quad & EV = E \int_0^\infty [(Y - G_S)^v + \bar{S}^f - (G_S)^{-\beta} \theta(t)] e^{-\gamma t} dt \\
& = \frac{1}{\gamma} (Y - G_S)^v + \frac{1}{\gamma} \bar{S}^f - G_S^{-\beta} \int_0^\infty \theta(t) e^{-\gamma t} dt \\
& = \frac{1}{\gamma} (Y - G_S)^v + \frac{1}{\gamma} \bar{S}^f - G_S^{-\beta} \frac{\theta(\tau)}{(\gamma - \alpha)}
\end{aligned}$$

Derivatives:

$$\begin{aligned}
\frac{dEV}{dG_S} &= -\frac{1}{\gamma} v (Y - G_S)^{v-1} + \beta G_S^{-\beta-1} \frac{\theta(\tau)}{(\gamma - \alpha)} \\
\frac{d^2 EV}{d(G_S)^2} &= -\left[(1-v) \frac{1}{\gamma} v (Y - G_S)^{v-2} + (1+\beta) \beta G_S^{-\beta-2} \frac{\theta(\tau)}{(\gamma - \alpha)} \right] < 0
\end{aligned}$$

FOC.

$$0 = F = \frac{dEV}{dG_S} - \frac{1}{\gamma} v (Y - G_S)^{v-1} + \beta G_S^{-\beta-1} \frac{\theta(\tau)}{(\gamma - \alpha' N)}$$

Optimal behavior: Take a countermeasure to protect G_S : From this first order condition we can derive the optimal expenditure on security G_S^f when fighting using the IFT with $\alpha = \alpha' N$. G_S^f will be a function of the size of the rebel group.

Implicit function theorem:

$$G_S^f = G_S^f(N)$$

Proof:

$$\begin{aligned}
\frac{dF}{dG_S} &= -\left[(1-v) \frac{1}{\gamma} v (Y - G_S)^{v-2} + (1+\beta) \beta G_S^{-\beta-2} \frac{\theta(\tau)}{(\gamma - \alpha)} \right] < 0 \\
\frac{dF}{dN} &= G_S^{-2} \frac{\theta(\tau)}{(\gamma - \alpha' N)^2} \alpha' > 0
\end{aligned}$$

As $\left[(1-v) \frac{1}{\gamma} v (Y - G_S)^{v-2} + 2G_S^{-3} \frac{\theta(\tau)}{(\gamma - \alpha)} \right] \neq 0$, the determinant of the Jacobian-Matrix is not zero and the implicit function theorem can be applied. Hence, it follows for the derivative:

$$\frac{dG_S}{dN} = -\frac{\frac{dF}{dN}}{\frac{dF}{dY_R}}$$

Derivative:

$$\frac{dG_S}{dN} = \frac{G_S^{-2} \frac{\theta(\tau)}{(\gamma - \alpha' N)^2} \alpha'}{\left[(1 - v) \frac{1}{\gamma} v (Y - G_S)^{v-2} + (1 + \beta) \beta G_S^{-\beta-2} \frac{\theta(\tau)}{(\gamma - \alpha)} \right]} > 0$$

Government's expected value of welfare when fighting, EV^f and derivatives:

$$EV^f = \frac{1}{\gamma} (Y - G_S)^v + \frac{1}{\gamma} \bar{S}^f - \left(G_S^f \right)^{-\beta} \frac{\theta(\tau)}{\gamma - \alpha' N} = \frac{1}{\gamma} \left[(Y - G_S)^v + \bar{S}^f - \left(G_S^f \right)^{-\beta} \frac{\theta(\tau)}{1 - \frac{\alpha' N}{\gamma}} \right]$$

Derivative of indirect welfare function $\frac{dEV^f}{dN} < 0$: Using implicit function theorem:

$$\mathcal{L} = \frac{1}{\gamma} (Y_G)^v + \frac{1}{\gamma} \bar{S}^f - G_S^{-\beta} \frac{\theta(\tau)}{(\gamma - \alpha' N)} - \psi(Y_G + G_S - Y)$$

$$\frac{dEV^f}{dN} = \frac{d\mathcal{L}}{dN} = - \left(G_S^f \right)^{-\beta} \frac{\theta(\tau)}{(\gamma - \alpha' N)^2} \alpha' < 0$$

Derivative of indirect welfare function $\frac{dEV^f}{d\gamma} < 0$: Using implicit function theorem:

$$\begin{aligned} \frac{dEV^f}{d\gamma} &= \frac{d\mathcal{L}}{d\gamma} = -\frac{1}{(\gamma)^2} \left[(Y - G_S)^v + \bar{S}^f - \left(G_S^f \right)^{-\beta} \frac{\theta(\tau)}{1 - \frac{\alpha' N}{\gamma}} \right] \\ &\quad + \frac{1}{\gamma} \left(G_S^f \right)^{-\beta} \frac{\theta(\tau)}{\left(1 - \frac{\alpha' N}{\gamma} \right)^2} \left(1 - \frac{\alpha' N}{\gamma} \right) \frac{\alpha' N}{\gamma^2} \\ &= - (Y - G_S)^v - \bar{S}^f + \left(1 + \frac{\alpha' N}{\gamma} \right) \frac{\theta(\tau)}{\left(1 - \frac{\alpha' N}{\gamma} \right)} \left(G_S^f \right)^{-\beta} < 0 \\ \theta(\tau) &< \frac{\left(1 - \frac{\alpha' N}{\gamma} \right)}{\left(1 + \frac{\alpha' N}{\gamma} \right)} \left(G_S^f \right)^\beta [(Y - G_S)^v + \bar{S}] \quad \text{for } \theta(\tau) \text{ sufficiently small} \end{aligned}$$

6.5 Appendix: Rebels, optimal choice when compromise

Decision problem:

Problem:

$$\begin{aligned}
\max_N Ev^c &= P_R v^c + (1 - P_R) Ev^f = \frac{N^{\varphi_R}}{\rho} [Y_R + F\omega - (p_N d_N + w_N) N] + (1 - N^{\varphi_R}) Ev^f \\
s.t.(i) &: Y_R + F\omega = Y_D + p_N R_N + w_N N \quad \text{finance constraint} \\
(ii) &: \dot{N} = R_N - N d_N = 0 \quad \text{survival condition}
\end{aligned}$$

Derivatives:

$$\begin{aligned}
\frac{dEv^c}{dN} &= \frac{1}{\rho} \varphi N^{\varphi-1} [Y_R + F\omega - (p_N d_N + w_N) N] - \frac{1}{\rho} N^{\varphi_R} (p_N d_N + w_N) - \varphi N^{\varphi-1} Ev^f \\
\frac{d^2 Ev^c}{dN^2} &= \left[-\frac{1}{\rho} \varphi ((1 - \varphi)) N^{\varphi-2} [Y_R + F\omega - (p_N d_N + w_N) N] - \frac{1}{\rho} \varphi N^{\varphi-1} (p_N d_N + w_N) + (1 - \varphi) \varphi N^{\varphi-2} Ev^f \right] < 0
\end{aligned}$$

FOC:

$$\begin{aligned}
\frac{dEv^c}{dN} &= \frac{1}{\rho} \varphi N^{\varphi-1} [Y_R + F\omega - (p_N d_N + w_N) N] - \frac{1}{\rho} N^{\varphi_R} (p_N d_N + w_N) - \varphi N^{\varphi-1} Ev^f = 0 \\
0 &= \varphi (Y_R + F\omega) N^{-1} - \varphi (p_N d_N + w_N) - (p_N d_N + w_N) - \rho \varphi N^{-1} Ev^f
\end{aligned}$$

Optimal behavior, number of supporters N^c : Optimal group size N^c can be explicitly determined:

$$\begin{aligned}
(\varphi + 1) (p_N d_N + w_N) &= \varphi (Y_R + F\omega) N^{-1} - \rho \varphi N^{-1} Ev^f \\
N^c &= \frac{\varphi}{(\varphi + 1)} \frac{(Y_R + F\omega - \rho Ev^f)}{(p_N d_N + w_N)}
\end{aligned}$$

Derivative $\frac{dN^c}{dY_R}$:

$$\frac{dN^c}{dY_R} = \frac{\varphi_R}{(1 + \varphi_R) (p_N d_N + w_N)} > 0$$

Rebels' expected value of peace, Ev^c and derivatives:

$$Ev^c = (Y_R + F\omega) \frac{(N^c)^{\varphi_R}}{\rho} - (p_N d_N + w_N) \frac{(N^c)^{1+\varphi_R}}{\rho} + (1 - (N^c)^{\varphi_R}) Ev^f$$

Derivative of indirect welfare function $\frac{dEv^c}{dY_R}$: Using envelop theorem:

$$\begin{aligned}
\mathcal{L} &= \frac{N^{\varphi_R}}{\rho} Y_D + (1 - N^{\varphi_R}) Ev^f - \psi (Y_D + p_N R_N + w_N N - Y_R - F) \\
\frac{dEv^c}{dY_R} &= \frac{d\mathcal{L}}{dY_R} = \psi > 0
\end{aligned}$$

Derivative of indirect welfare function $\frac{dEv^c}{d\rho}$: Using envelop theorem:

$$\begin{aligned}\mathcal{L} &= \frac{N^{\varphi_R}}{\rho} Y_D + (1 - N^{\varphi_R}) E e^{-\rho ET^f} \frac{(Y_R + \bar{L})}{\rho} - \psi(Y_D + p_N R_N + w_N N - Y_R - F) \\ \frac{dEv^c}{d\rho} &= \frac{d\mathcal{L}}{d\rho} = -\frac{N^{\varphi_R}}{\rho^2} Y_D - (1 - N^{\varphi_R}) (\rho ET^f + 1) e^{-\rho ET^f} \frac{(Y + \bar{L})}{\rho^2} < 0\end{aligned}$$

Comparing effects of ρ on Ev^c for peace and fight: Is the welfare effect of a changing time preference stronger under peace or fight conditions? We expect the effect to be stronger in fight conditions:

$$\begin{aligned}\left[\begin{array}{l} -\frac{N^{\varphi_R}}{\rho^2} [Y_R + F\omega - (p_N d_N + w_N) N] \\ - (1 - N^{\varphi_R}) (\rho ET^f + 1) e^{-\rho ET^f} \frac{(Y + \bar{L})}{\rho^2} \end{array} \right] &> - (\rho ET^f + 1) e^{-\rho ET^f} \frac{(Y + \bar{L})}{\rho^2} \\ \left[\begin{array}{l} -\frac{N^{\varphi_R}}{\rho^2} [Y_R + F\omega - (p_N d_N + w_N) N] \\ + N^{\varphi_R} (\rho ET^f + 1) e^{-\rho ET^f} \frac{(Y + \bar{L})}{\rho^2} \end{array} \right] &> 0 \\ (\rho ET^f + 1) e^{-\rho ET^f} (Y + \bar{L}) &> [Y_R + F\omega - (p_N d_N + w_N) N]\end{aligned}$$

holds for $Y + 1$ being sufficiently large compared to $F\omega$. If the flow of rewards derived from fighting $Y + \bar{L}$ is sufficiently large compared to the flow of rewards derived from peace and in particular the exogenous component $F\omega$, an increase in time preference rate ρ (less ideology) will reduce welfare of fighting much more than welfare of peace and compromise.

6.6 Appendix: Government's optimal choice: compromise

Optimization problem:

Decision problem:

$$\begin{aligned}\max_{Y_R} : EV^c &= N^{-\varphi_G} \frac{1}{\gamma} [(Y_G(t))^v + S(t)] + (1 - N^{-\varphi_G}) V^f \\ s.t. : Y(t) &= Y_G(t) + Y_R(t), \\ with : S(t) &= \bar{S}^c - (Y_R)^{-\beta} N^\beta \geq \bar{S}, \beta > 1, P_G = Y_R^{\varphi_G}\end{aligned}$$

$$\max_{Y_R} : EV^c = N^{-\varphi_G} \frac{1}{\gamma} \left[(Y(t) - Y_R(t))^v + \bar{S}^c - (Y_R)^{-\beta} N^\beta \right] + (1 - N^{-\varphi_G}) V^f$$

Derivatives:

$$\begin{aligned}
\frac{dEV^c}{dY_R} &= N^{-\varphi_G} \frac{1}{\gamma} \left[-v(Y(t) - Y_R(t))^{v-1} + \beta(Y_R)^{-\beta-1} N^\beta \right] \gtrless 0 \\
\frac{d^2 EV^c}{dY_R^2} &= N^{-\varphi_G} \frac{1}{\gamma} \left[(1-v)v(Y(t) - Y_R(t))^{v-2} - (1+\beta)\beta(Y_R)^{-\beta-2} N^\beta \right] < 0 \\
&: \text{ for (i) } \frac{(1-v)v(Y(t) - Y_R(t))^{v-2}}{(1+\beta)\beta(Y_R)^{-\beta-2} N^\beta} < N^\beta
\end{aligned}$$

FOC:

$$0 = F = \frac{dEV^c}{dY_R} = N^{-\varphi_G} \frac{1}{\gamma} \left[-v(Y(t) - Y_R(t))^{v-1} + \beta(Y_R)^{-\beta-1} N^\beta \right]$$

Optimal behavior, transfer to rebels Y_R : Optimal Y_R cannot be determined explicitly hence we use the IFT.

Implicit function theorem (IFT) $Y_R = Y_R(N)$: Using the implicit function theorem we obtain

$$Y_R = Y_R(N)$$

Proof:

$$\begin{aligned}
\frac{dF}{dY_R} &= N^{-\varphi_G} \frac{1}{\gamma} \left[\frac{(1-v)v(Y(t) - Y_R(t))^{v-2}}{-(1+\beta)\beta(Y_R)^{-\beta-2} N^\beta} \right] < 0, \quad \text{for (i) to hold.} \\
\frac{dF}{dN} &= -\varphi_G N^{-\varphi_G-1} \frac{1}{\gamma} \left[\frac{-v(Y(t) - Y_R(t))^{v-1}}{+\beta(Y_R)^{-\beta-1} N^\beta} \right] + (N)^{-\varphi_G} \frac{1}{\gamma} \beta^2 (Y_R)^{-\beta-1} N^{\beta-1}
\end{aligned}$$

As $N^{-\varphi_G} \frac{1}{\gamma} \left[(1-v)v(Y(t) - Y_R(t))^{v-2} - (1+\beta)\beta(Y_R)^{-\beta-2} N^\beta \right] \neq 0$, the determinant of the Jacobian-Matrix is not zero and the implicit function theorem can be applied. Hence, it follows for the derivative:

$$\frac{dY_R}{dN} = -\frac{\frac{dF}{dN}}{\frac{dF}{dY_R}}$$

Derivative $\frac{dY_R}{dN}$:

$$\begin{aligned}
\frac{dY_R}{dN} &= -\frac{\left[-\varphi_G N^{-\varphi_G-1} \frac{1}{\gamma} \left[-v(Y(t) - Y_R(t))^{v-1} + \beta(Y_R)^{-\beta-1} N^\beta \right] \right]}{(N)^{-\varphi_G} \frac{1}{\gamma} \left[(1-v)v(Y(t) - Y_R(t))^{v-2} - (1+\beta)\beta(Y_R)^{-\beta-2} N^\beta \right]} \\
&= \frac{\varphi_G \left[-v(Y(t) - Y_R(t))^{v-1} + \beta(Y_R)^{-\beta-1} N^\beta \right] - N^{\beta-\varphi_G} \frac{1}{\gamma} \beta^2 (Y_R)^{-\beta-1} N^{\beta-1}}{\left[(1-v)v(Y(t) - Y_R(t))^{v-2} - (1+\beta)\beta(Y_R)^{-\beta-2} N^\beta \right] N} > 0
\end{aligned}$$

show that first derivative, $\frac{dY_R}{dN} > 0$: (a) is negative a if (i) holds and (b) is negative if (ii) holds. Hence, if both hold simultaneously, $\frac{dY_R}{dN} > 0$.

$$(a) : (1-v) v (Y(t) - Y_R(t))^{v-2} - (1+\beta) \beta (Y_R)^{-\beta-2} N^\beta < 0$$

$$(b) : -v (Y(t) - Y_R(t))^{v-1} + \beta (Y_R)^{-\beta-1} N^\beta < 0$$

$$(ii) : \frac{(1-v) v (Y(t) - Y_R(t))^{v-2}}{(1+\beta) \beta (Y_R)^{-\beta-2}} < N^\beta$$

consistence of both conditions:

$$\underbrace{\frac{(1-v) v (Y(t) - Y_R(t))^{v-2}}{(1+\beta) \beta (Y_R)^{-\beta-2}}}_{(ii)} < N^\beta = N^\beta < \underbrace{\frac{v (Y(t) - Y_R(t))^{v-1}}{\beta (Y_R)^{-\beta-1}}}_{(i)}$$

$$\frac{(1-v) v (Y(t) - Y_R(t))^{-1}}{(1+\beta) \beta (Y_R)^{-1}} < 1$$

$$\frac{(1-v) v (Y(t) - Y_R(t))^{-1}}{(1+\beta) \beta (Y_R)^{-1}} = \frac{(1+\beta) \beta (Y_R)}{(1-v) v (Y(t) - Y_R(t))} < 1$$

$$\frac{(1+\beta) \beta}{(1-v) v} < \frac{Y(t) - Y_R(t)}{Y_R} \text{ which we assume.}$$

show that second derivative $\frac{d^2 Y_R}{dN^2} > 0$:

$$\begin{aligned} \frac{d^2 Y_R}{dN^2} &= \frac{\varphi_G \beta^2 (Y_R)^{-\beta-1} N^{\beta-1} - (\beta - \varphi_G) N^{\beta-\varphi_G-1} \frac{1}{\gamma} \beta^2 (Y_R)^{-\beta-1}}{\left[\frac{(1-v) v (Y(t) - Y_R(t))^{v-2} - (1+\beta) \beta (Y_R)^{-\beta-2} N^\beta}{(1+\beta) \beta (Y_R)^{-\beta-2} N^\beta} \right] N} \\ &\quad - \frac{\left(\frac{\varphi_G \beta^2 (Y_R)^{-\beta-1} N^{\beta-1}}{-(\beta - \varphi_G) N^{\beta-\varphi_G-1} \frac{1}{\gamma} \beta^2 (Y_R)^{-\beta-1}} \right)}{\left(\left[\frac{(1-v) v (Y(t) - Y_R(t))^{v-2}}{-(1+\beta) \beta (Y_R)^{-\beta-2} N^\beta} \right] N \right)^2} \left[\underbrace{\left(\frac{(1-v) v (Y(t) - Y_R(t))^{v-2}}{-(1+\beta) \beta (Y_R)^{-\beta-2} N^\beta} \right)}_{<0} \right. \\ &\quad \left. - (1+\beta) \beta^2 (Y_R)^{-\beta-2} N^{\beta-1} \right] \\ \frac{d^2 Y_R}{dN^2} &> 0 \end{aligned}$$

Government's expected value of peace, EV^c and derivatives:

$$EV^c = N^{-\varphi_G} \frac{1}{\gamma} [(Y_G(t))^v + S(t)] + (1 - N^{-\varphi_G}) EV^f$$

$$EV^c = N^{-\varphi_G} \frac{1}{\gamma} [(Y_G(t))^v + \bar{S}^c - (Y_R)^{-\beta} N^\beta] + (1 - N^{-\varphi_G}) EV^f$$

Derivatives of indirect welfare function $\frac{dEV^c}{dN} < 0$: Using envelop theorem:

$$\mathcal{L} = N^{-\varphi_G} \frac{1}{\gamma} \left[(Y_G(t))^v + \bar{S}^c - (Y_R)^{-\beta} N^\beta \right] + (1 - N^{-\varphi_G}) EV^f - \psi(Y_G + Y_R - Y)$$

$$\frac{dEV^c}{dN} = \frac{d\mathcal{L}}{dN} = \left[\begin{array}{c} -\varphi_G \frac{1}{\gamma} (N)^{-\varphi_G-1} \left[(Y(t) - Y_R(t))^v + \bar{S}^c - (Y_R)^{-\beta} N^\beta \right] \\ -N^{-\varphi_G} \frac{1}{\gamma} \beta (Y_R)^{-\beta} N^{\beta-1} + \varphi_G (N)^{-\varphi_G-1} EV^f \end{array} \right] < 0$$

$$\begin{aligned} (Y - G_S)^v + \bar{S}^f - \left(G_S^f \right)^{-\beta} \frac{\theta(\tau)}{1 - \frac{\alpha' N}{\gamma}} &< (Y(t) - Y_R(t))^v + \bar{S}^c - (Y_R)^{-\beta} N^\beta + \frac{1}{\varphi_G} \beta (Y_R)^{-\beta} N^\beta \\ 0 &< \left[\begin{array}{c} ((Y(t) - Y_R(t))^v - (Y - G_S)^v) + (\bar{S}^c - \bar{S}^f) \\ + \left(\left(G_S^f \right)^{-\beta} \frac{\theta(\tau)}{1 - \frac{\alpha' N}{\gamma}} - Y_R^{-\beta} N^\beta - \frac{1}{\varphi_G} \beta Y_R^{-\beta} N^\beta \right) \end{array} \right] \end{aligned}$$

Derivatives of indirect welfare function $\frac{dEV^c}{d\gamma} < 0$: Using envelop theorem:

$$\begin{aligned} \frac{dEV^c}{d\gamma} &= \frac{d\mathcal{L}}{d\gamma} = -P_G \frac{1}{\gamma^2} [(Y_G(t))^v + S(t)] + (1 - P_G) \frac{dEV^f}{d\gamma} < 0 \\ &= -P_G \frac{1}{\gamma^2} [(Y_G(t))^v + S(t)] - (1 - P_G) \left(G_S^f \right)^{-\beta} \frac{\theta(\tau)}{(\gamma - \alpha' N)^2} \alpha' < 0 \end{aligned}$$

Comparing effects of γ on EV^c for peace and fight: Relative change under fight and compromise conditions. Expect: increase in time preference reduces peace welfare more than welfare stemming from fighting

$$\begin{aligned} \underbrace{-\frac{1}{\gamma^2} [(Y_G(t))^v + S(t)]}_{\text{compromise effect}} &< \underbrace{-\frac{dEV^f}{d\gamma}}_{\text{fight effect}} \\ -[(Y_G(t))^v + S(t)] &< -(Y - G_S)^v - \bar{S}^f + \left(1 + \frac{\alpha' N}{\gamma} \right) \frac{\theta(\tau)}{\left(1 - \frac{\alpha' N}{\gamma} \right)} \left(G_S^f \right)^{-\beta} \\ -(Y - Y_R)^v - \bar{S}^c + N^\beta Y_R^{-\beta} &< -(Y - G_S)^v - \bar{S}^f + \left(1 + \frac{\alpha' N}{\gamma} \right) \frac{\theta(\tau)}{\left(1 - \frac{\alpha' N}{\gamma} \right)} \left(G_S^f \right)^{-\beta} \\ 0 &< [(Y - Y_R)^v - (Y - G_S)^v] + [\bar{S}^c - \bar{S}^f] \\ &\quad + \left[\left(1 + \frac{\alpha' N}{\gamma} \right) \frac{\theta(\tau)}{\left(1 - \frac{\alpha' N}{\gamma} \right)} \left(G_S^f \right)^{-\beta} - N^\beta Y_R^{-\beta} \right] \end{aligned}$$