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Competitive pricing and quality disclosure to cursed consumers

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Abstract

We study the disclosure decision and price-setting behavior of competing firms in the presence of cursed consumers, who fail to be sufficiently skeptical about a firm’s quality upon observing non-disclosure of quality-relevant information. We show that neither competition nor the presence of sophisticated consumers necessarily offer protection to cursed consumers. Exploitation arises if markets are vertically differentiated, if there are many sophisticated consumers, and if it is more likely ex ante that product quality is high. Information campaigns that seek to educate consumers may encourage exploitation and decrease social welfare. Mandatory disclosure laws restore efficiency, but at the cost of redistributing rents from consumers to firms. Our simple model delivers a rich set of positive results, captures important markets, like those for food and consumer finance, and speaks to several recent policy initiatives aimed at consumer protection.

Keywords: naive, cursed, disclosure, unraveling, competition

JEL classification: C72, D03, D82, D83
1 Introduction

When firms are able to disclose verifiable information about product quality, non-disclosure should make consumers skeptical about the consumption value of a good, since only low-quality firms would want to hide information. However, evidence suggests that many consumers remain too confident about quality in the face of non-disclosure. These cursed consumers (Eyster and Rabin, 2005) fail to condition their quality perception on firms’ disclosure strategies, which makes them liable to being sold rip-offs, i.e. products whose price exceeds their value. To prevent firms from exploiting cursedness or naivete, policymakers have increasingly sought to educate consumers and have passed laws that force firms to disclose quality-related information about their products in a salient manner.

We provide an analytical framework to assess when rip-offs will be sold to cursed consumers and to evaluate viable consumer protection policies. We are particularly interested in whether competition between firms can protect consumers from being exploited and in understanding how policy measures interact with competition in driving market outcomes. Our main contribution lies in demonstrating that market forces are not always sufficient to prevent exploitation and that two prevalent consumer protection policies, namely information campaigns and mandatory disclosure laws, can sometimes harm consumers.

In a frictionless world, neoclassical theory predicts that full unraveling occurs (Grossman and Hart, 1980; Grossman, 1981; Milgrom, 1981). That is, all but the very worst firm should disclose quality information, leading to fully informed consumer choices and efficient market outcomes. However, empirical studies suggest that firms frequently fail to disclose private information about product quality.\textsuperscript{1} And while incomplete disclosure can be explained by information transmission costs, information unavailability, and strategic considerations that arise in richer theoretical settings (Matthews and Postle-

\textsuperscript{1}See Mathios (2000); Jin (2005); Fung, Graham and Weil (2007); Dranove and Jin (2010); Brown, Camerer and Lovallo (2012); Luca and Smith (2013).
Naivete on behalf of the uninformed party drives non-disclosure in experimental persuasion games, which are able to preclude rational explanations for nondisclosure. Figure 1a shows experimental results in Jin, Luca and Martin (2015), where senders choose whether to disclose a private quality signal and benefit from receivers believing that quality is high. Many senders with intermediate quality do not disclose. Mathios (2000) finds similar evidence in the field. Figure 1b depicts how food companies’ propensity to disclose the fat content of salad dressings varies with fat content. Although the information is known to firms and disclosure is virtually free, fat content is not only hidden for the fattiest of products.

Our framework applies to several other markets that feature disclosure decisions and opportunities for exploiting cursed consumers. Universities decide whether to disclose placement records of former students and independent education rankings. Restaurants and hotels decide whether to disclose customer reviews or hygiene ratings. Medical

2 See Forsythe, Isaac and Palfrey (1989); King and Wallin (1991); Forsythe, Lundholm and Rietz (1999); Jin, Luca and Martin (2015); Hagenbach and Perez-Richet (2015). Furthermore, our cursedness assumption finds support in other experimental games (see Eyster and Rabin (2005) for a summary of the evidence).

3 Jin, Luca and Martin (2015) refer to what we call quality as ‘secret number’.
facilities decide whether to participate in independent quality assessments. CEOs decide whether to communicate news about company performance to investors. Banks decide whether to disclose add-on costs associated with their loan contracts and services. Movie producers decide whether to make movies available to critics before they are released. In many of these markets, governments have sought to regulate disclosure and improve consumer responses to non-disclosure.

In our baseline model, two firms with homogeneous marginal costs and potentially heterogeneous quality levels compete. They simultaneously decide whether to disclose hard information about their product quality and what price to charge. Information cannot be misreported (e.g. for fear of litigation), only concealed. Consumers consist of both sophisticated and cursed types. Cursed or naive consumers maintain too favorable an expectation of non-disclosed qualities. Consumers are homogeneous in their tastes – though not in their subjective valuations conditional on non-disclosure – and efficiency demands that they all purchase the highest quality good.

If exploitation occurs in our model it takes the following intuitive form: a cursed consumer buys a low quality product at a price that is higher than her objective valuation. In competitive markets, exploitation occurs when firms are vertically differentiated, the fraction of naive consumers is small, and ex ante expected quality in the market is high.

In the absence of vertical differentiation, a Bertrand logic applies and firms price at marginal cost irrespective of quality level and sophistication of consumers. Now consider the case of vertical differentiation, i.e. heterogeneous quality. When there are many naive consumers, these represent a profitable segment of the market and the high quality firm prices aggressively to capture them. Hence, no exploitation takes

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4Our model not only applies to disclosure decisions along the extensive margin in these and other markets, but also to the intensive margin. Especially firms that are mandated to disclose information, may fail to do so in a salient manner (see Stango and Zinman (2011) for an example of non-salient disclosure and exploitation in consumer finance). In this case, naive consumers are those that require maximal salience of information to infer a good’s quality.

5For example, the Nutrition Labeling and Education Act in the US requires nutrition labeling of foods. The US Consumer Financial Protection Bureau enforces home mortgage disclosure laws and seeks to educate consumers to be more discerning. In LA County, hygiene quality grade cards have to be displayed in restaurant windows and New York reports risk-adjusted coronary artery bypass graft mortality rates.
place. If the prior is unfavorable, naive consumers’ beliefs about quality in the absence of disclosure is low, so the high quality product remains attractive even at relatively high prices. Again, naives buy from the high-quality firm and the market outcome is efficient. However, when these conditions are not met, a high quality firm primarily targets sophisticated consumers, leaving naives to sometimes buy from the low quality firm, who may charge excessive prices.

In competitive markets with vertical differentiation, a firm’s profits depend on consumers’ perceptions of its quality (dis)advantage. Because cursed consumers maintain too favorable an expectation of a silent (low-quality) firm, they strengthen competition by reducing the margins a high-quality firm can charge. Thus, their presence generates a positive externality both within the group of cursed consumers and for sophisticated ones. Expected prices and profits are lower than if firms’ qualities were commonly known, as in the case of only rational consumers.

Remedies against exploitation in the form of mandatory disclosure laws, which force firms to reveal the quality of their products, and information campaigns, which reduce the fraction of cursed consumers, have subtle and ambiguous effects on the distribution of rents and welfare. Mandatory disclosure ensures efficiency but also eliminates the competitive effect from naives’ inflated subjective outside options. Thus, it transfers welfare from consumers to firms and is always detrimental for rational consumers. For a cursed consumer, it is beneficial only when in its absence the average net value from a purchase under vertical differentiation is negative due to severe deception.

An information campaign is always beneficial to firms, harmful to rational consumers and harmful to consumers who remains cursed. Surprisingly, it may even be harmful to consumers who become educated provided the average net value from a purchase under vertical differentiation is positive absent the intervention. When the fraction of naive consumers is large, information campaigns may entail welfare losses. On the other hand, when the fraction of cursed consumers is small enough, an information campaign raises aggregate consumer surplus, both because exploitation is otherwise severe and because
the marginal loss due to a decrease in the inflated-outside-option effect vanishes.

When low-quality firms follow the letter but not the spirit of a mandatory disclosure law and disclose their quality in a non-salient manner, some consumers may continue to misperceive low qualities. Then, disclosure laws, much like information campaigns, may be welfare decreasing. In recent years, the internet has given rise to many independent sources of information that may shed light on undisclosed qualities. Again, following the logic of information campaigns, this development may have led less tech-savvy individuals with a greater probability of being exploited.

These policy implications differ from the intuitions derived from a monopoly model of the firm-consumer relationship. A monopolist either attracts all consumers by disclosing and charging a price equal to their reservation value or withholds information and sells only to cursed consumers at an inflated price. Because the monopolist’s exploitative strategy entails negative utility for cursed consumers and becomes more attractive the larger their fraction, their presence exerts a negative externality within cursed consumers and no externality on sophisticated consumers, who derive zero utility from either strategy. Information campaigns and mandatory disclosure always weakly increase both welfare and consumer surplus.

In sum, our model suggests that mandatory disclosure laws and information campaigns may not make consumers better off in vertically differentiated, competitive markets. As we show in an extension to our model this case is highly relevant because vertical differentiation is the endogenous market outcome when firms can invest in quality. Our analysis casts doubt on undifferentiated policy measures built on the principle that transparency is always an effective remedy. Instead, policy interventions should be conditioned on evidence of rip-offs actually being sold in a market and coupled with the collection of data on customer satisfaction.

Related literature  The topic of imperfect skepticism on behalf of receivers and competition among senders in disclosure games has received little attention since it was first studied by Milgrom and Roberts (1986). In their model, senders only make disclosure
but no pricing decisions. Furthermore, there is only a single receiver and, hence, no heterogeneity. Therefore, as in our model with only cursed types, no exploitation occurs in equilibrium.

Nondisclosure in our model is driven by heterogeneity in consumers’ perceived valuations of one of the goods. This also occurs in horizontally differentiated competitive markets, as shown by Board (2009) and Levin, Peck and Ye (2009). However, because consumers in these models are rational, they cannot be exploited. Moreover, in our model heterogeneity disappears once firms disclose.

Nondisclosure may also obtain under competition when firms do not observe their rivals’ quality, disclosure decisions precede pricing decisions and marginal costs of production depend on quality, as firms can then convey quality information through prices (Janssen and Roy, 2015). Our model features opposite assumptions in all three dimensions and hence different strategic considerations.

The idea that unraveling breaks down due to some limitation of receivers’ rationality is present in Fishman and Hagerty (2003), who focus on a monopoly setting in which some consumers do not understand the content of disclosure (rather than having difficulties in interpreting non-disclosure, as in our model). Unlike in our model, mandatory disclosure can never harm consumers.

The implications of our cursedness assumption differ from those of other notions of irrationality. Salop and Stiglitz (1977) model irrationals as having higher costs of acquiring information about products. Contrary to our findings, irrationality eases competition and increases profit margins for every firm. In Armstrong and Chen (2009) inattentive consumers pay less attention to product quality. Unlike in our model, efficiency is decreasing in the number of inattentive consumers because these provide negative incentives for investments in quality.

In line with this paper, shrouded attributes models (Gabaix and Laibson, 2006; Armstrong and Vickers, 2012; Heidhues, Koszegi and Muooka, 2014) find that neither competition nor the presence of sophisticated consumers necessarily offers protection
to naive consumers. In these models, naive consumers may incur add-on costs that sophisticated consumers can avoid (e.g. by exerting effort). Shrouded attribute models therefore speak to products, like printers, that have add-ons, like ink cartridges, while our model also captures one-dimensional products, like food. However, our model may also fit settings in which firms can credibly disclose and commit to add-on costs or a lack thereof and sophisticated consumers can only avoid them by not buying the product in question.

Some key predictions of our model differ from those of the shrouded attributes literature. In Gabaix and Laibson (2006) and Heidhues, Koszegi and Murooka (2014) deceptive equilibria are more likely when there are fewer sophisticated consumers. In our model, the equilibrium with a small fraction of sophisticates is efficient and does not feature exploitation. Also, mandatory disclosure of add-on costs can never make naive consumers worse off in shrouded attributes models. Kosfeld and Schüwer (2014) show that educating consumers and thereby decreasing the proportion of naives in the population may increase the prevalence of exploitative practices and decrease welfare. But this is only the case if firms engage in third-degree price discrimination. We derive a similar result in a simpler setting and relying on a rather different mechanism. The mechanism we rely on has counterparts in the search literature. Anderson and Renault (2006) demonstrate how the presence of informed consumers may harm uninformed ones by making demand more inelastic.

The next section describes our setup, before section 3 derives results for two benchmarks of our model, the cases of monopoly and of competition with only rational consumers. Section 4 describes our main results and section 5 extends the model to allow for endogenous quality choices and more than two competitors. Section 6 concludes. All proofs are in the appendix.

6Unlike in our model, exploitation in these models can also arise when there is no vertical differentiation when this assumption holds. Another difference is that firms in our model can only disclose their own product quality, but not that of their competitors. Furthermore, our model predicts which firm exploits consumers in equilibrium: the low quality firm. And under some conditions, exploitative firms in our model can coexist alongside non-exploitative ones.
2 Model

Two firms producing substitute goods compete for a mass one of consumers with unit demand. Consumers have homogeneous preferences and derive utility \( q - p \) from a purchase, where \( p \) denote the price of the good and \( q \) its quality. The quality of the good of each firm is independently drawn from a common knowledge distribution with mean \( \mu \). We will refer to the firm with respectively the higher and the lower quality as firm \( h \) and \( \ell \). Furthermore, we will refer to a generic firm as \( i \) and to her competitor as \( j \). In most of the paper, we will focus on a binary distribution and suppose that each firm’s quality is equal to \( q_h > 0 \) with probability \( \theta \in (0,1) \) and to \( q_\ell \) with complementary probability, where \( 0 \leq q_\ell < q_h \).\(^7\) \( q_i \) and \( q_j \) are known by both firms and unknown to consumers. Each firm can credibly reveal the quality of her own product to consumers at no cost. Firms cannot disclose their rivals’ quality.\(^8\) Marginal costs of production are normalized to zero. We consider the following timing:

- \( t=0 \): Firms observe \( q_i \) and \( q_j \),

- \( t=1 \): Each firm simultaneously and independently chooses whether to disclose \((m_i = q_i)\) or not \( (m_i = \emptyset) \) the quality of her product and posts a price \( p_i \),

- \( t=2 \): Consumers observe firms’ disclosure and pricing decisions and chooses which product to buy, if any.

We depart from the rationality paradigm by assuming that a fraction \( \chi \in (0,1) \) of consumers are fully cursed (Eyster and Rabin, 2005), which, in this setting, implies that their perception of a silent firm’s quality is \( \mu \). The rest of consumers, whose proportion is \( 1 - \chi \), are rational. The relevant solution concept is sequential equilibrium (Kreps and Wilson, 1982), with the natural adaptation for this setting that cursed consumers must behave optimally given their possibly wrong beliefs. We will also restrict our attention

\(^7\)As long as \( \mu > 0 \), our results also apply to settings in which \( q_\ell < 0 \), i.e. in which a low quality good hurts consumers.

\(^8\)See Anderson and Renault (2009) for a model in which firms can disclose information on their rivals.
to equilibria in which firms charge non-negative prices\textsuperscript{9} and in which their disclosure decisions entail no randomization.

3 Benchmarks

3.1 Monopoly with cursed and rational consumers

Suppose that there is only one firm and denote by $q$ her realization of quality. The following proposition describes the unique equilibrium.\textsuperscript{10}

**Proposition 1** (Monopoly). When $q \geq \chi \mu$ the monopolist discloses, charges $p^* = q$ and attracts all consumers. When $q < \chi \mu$ the monopolist does not disclose, charges $p^* = \mu$ and attracts only cursed consumers.

When $q_\ell < \chi \mu$, the monopolist exploits cursed consumers, whose expected utility is then negative. Provided that the low-quality good is of some value to consumers, total welfare is lower than expected gains from trade because after nondisclosure rational consumers refrain from buying.

**Corollary 1** (Welfare under monopoly). When $q_\ell < \chi \mu$, the monopolist’s profits exceed gains from trade and increase with $\chi$, while a cursed consumer’s utility is negative. If it is also the case that $q_\ell > 0$, then the equilibrium is inefficient.

The presence of rational consumers limits exploitation and, hence, it is beneficial to cursed ones. Conversely, naive consumers exert a negative externality on other cursed consumers and no externality on rational ones, whose utility is always zero because the monopolist either extracts all their surplus or excludes them.

We will repeatedly evaluate the impact of two intervention policies: mandatory disclosure laws that force firms to reveal their evidence and education campaigns that

\textsuperscript{9}While a firm who charges a negative price will never sell in equilibrium, she may force the active firm to charge a lower price.

\textsuperscript{10}Without loss of generality, we are adopting the convention that the monopolist elects to disclose when indifferent.
reduce the fraction of cursed consumers. The previous results directly imply the following.

**Corollary 2** (Policy interventions in monopoly). *Mandatory disclosure ensures efficiency, it is weakly beneficial to cursed consumers, neutral to rational consumers and weakly harmful to the monopolist. An education campaign weakly improves efficiency, is weakly beneficial to cursed consumers, neutral to rational consumers and weakly harmful to the monopolist.*

### 3.2 Competition when all consumers are rational

A combination of unraveling and Bertrand arguments implies that when all consumers are rational ($\chi = 0$) they perfectly infer firms’ qualities and buy from the firm with the better product, if any.

**Proposition 2** (Competition with only rational consumers). *In equilibrium, consumers perfectly learn firms’ qualities. When $q_i = q_j$, they buy form either firm at a zero price; when the two qualities differ, they buy from the high quality firm at $p^*_h = q_h - q_\ell$. As consumers perfectly infer firms’ qualities, mandatory disclosure is redundant.*

**Corollary 3** (Policy interventions with only rational consumers). *When all consumers are rational mandatory disclosure laws have no impact.*

### 4 Competition with cursed and rational consumers

#### 4.1 Deception and nondeception regions

In what follows, we normalize the low and high quality levels to zero and one, respectively.\(^{11}\) Thus, $\mu = \theta$.

\(^{11}\)While $q_h$ is simply a scale parameter, so that the normalization entails no loss of generality, $q_\ell = 0$ implies that consumers never buy from a firm they perceive as of low quality at a positive price. But as we demonstrate in section 5.1, vertical differentiation with minimal low quality arises endogenously when firms control quality.
Proposition 3. In equilibrium each firm discloses if and only if her quality is high. When the two firms have the same quality, they each charge a price of zero and make zero profits. When the two qualities differ

- if $\chi \geq \theta$, the high-quality firm charges $p_h^* = 1 - \theta$ and attracts all consumers;
- if $\chi < \theta$, the high-quality firm randomizes according to a distribution $g_h^*(p_h)$ with support $[1 - \chi, 1]$, the low-quality firm according to a distribution $g_l^*(p_l)$ with support $[\theta - \chi, \theta]$, rational consumers buy from the high-quality firm and cursed consumers from one firm or the other depending on the deal they find more attractive.

When firms’ quality levels are identical, competition implies zero profits regardless of consumers’ composition and no deception takes place. Competition is hence at least partially effective at protecting naives, who would always be exploited by a monopolist selling a zero-quality product (proposition 1). In the case of vertical differentiation, instead, the parameter space is partitioned into a nondeception ($\chi \geq \theta$) and a deception region ($\chi < \theta$).

Because rational consumers can never be fooled into buying a low quality product, whether deception arises ultimately depends on the incentives of the high-quality firm to attract cursed consumers. When cursed consumers are many or they hold sufficiently pessimistic beliefs about undisclosed quality, they represent a rather profitable segment of the market and the high-quality firm elects to attract them by charging a relatively low price. As their proportion decreases or their inflated assessment of a silent firm’s quality increases, they become less and less profitable, so that at some point the high-quality firm forgoes her aggressive pricing strategy. Firms then share the cursed segment of the market probabilistically. Pricing is in mixed strategies, with firms’ indifference obtaining from the usual trade-off that a higher price yields higher profits if a firm succeeds in capturing cursed consumers but also a lower probability of doing so. In particular, the high-quality firm is indifferent between attracting only rational consumers at their reservation value or also cursed ones with positive probability at a lower price.
Figure 2 illustrates equilibrium outcomes in the case of vertical differentiation as a function of the fraction of cursed consumers.

4.2 Players’ welfare and the effect of policy interventions

A distinctive feature of the equilibrium is that cursed consumers’ inflated perception of a silent firm’s quality strengthens competition relative to the full rationality benchmark. Thus, while a cursed consumer may well be hurt by her naivete, she exerts a positive externality on all others. Differently from a monopoly setting, interventions aimed at limiting consumers’ naivete have therefore double-edged effects. An assessment of these policies should focus on the case of vertical differentiation, as when firms’ products are homogeneous consumers always appropriate all gains from trade.

Figure 3a depicts firms’ profits. In the deception region, as the fraction of cursed consumers decreases, aggregate profits increase, due to an increase in the prices of both goods (figure 2a). While ex-post the profits of the high-quality firm always increase, the profits of the low-quality firm are inverse-U shaped, as if on the one hand both her sale price and her probability of attracting cursed consumers increase (figure 2b), on the other hand her pool of potential customers decreases. Interestingly, there is a region in which both types of firms would favor consumers’ education. Mandatory disclosure is outcome equivalent to a campaign that makes all consumers rational (figure 3b). The
high-quality firm is then de-facto a monopolist and she extracts all gains from trade.

Figure 4 considers instead consumers’ viewpoint. A rational consumer’s utility always decreases as the fraction of naives shrinks (figure 4a), because the price of the high-quality good increases. The same holds true for a cursed consumer, not only because the prices of both goods increase but also because so does her likelihood of purchasing the inferior product. Below some threshold $\hat{\chi}$ a cursed consumer’s net expected value from a purchase becomes negative, which is a necessary condition for policy interventions to be beneficial for consumers. In particular, when $\hat{\chi} > \chi$, an information campaign of large enough scale hurts even consumers who become rational. When the fraction of cursed consumers is small enough, instead, an information campaign is an effective measure to enhance consumers’ welfare, both because exploitation is severe and because competition is weak.\textsuperscript{12} A similar reasoning accounts for why mandatory disclosure enhances consumers’ welfare (figure 4b) only when most of them are rational.

The following propositions summarize and formalize these arguments.

**Proposition 4** (Mandatory disclosure in competition). \textit{When no deception occurs ($\chi \geq \theta$), mandatory disclosure preserves efficiency and simply transfers wealth from both types of consumers to firms. When deception occurs ($\chi < \theta$), mandatory disclosure restores

\textsuperscript{12}Indeed, when $\chi$ approaches zero, not only rational consumers are held almost at their reservation utility but also the marginal increase in prices due to the campaign is negligible. This can be seen in figure 2a from the zero slope of $E[p_h]$ when $\chi = 0$.}
efficiency, it is beneficial to firms, harmful to rational consumers and beneficial to cursed consumers if and only if $\chi$ is lower than some cutoff $\hat{\chi} \in (0, \theta)$; its effect on total consumers’ surplus is generally ambiguous but positive provided $\chi$ is sufficiently small.

**Proposition 5** (Information campaigns in competition). When no deception occurs ($\chi \geq \theta$), a reduction in the fraction of cursed consumers is either irrelevant or, if it causes deception to arise, it is beneficial to firms and harmful to all consumers and total welfare. When deception occurs ($\chi < \theta$), this reduction is beneficial to firms, harmful to a rational consumer and to a consumer who remains cursed; its effect on the welfare of a consumer who gets educated and total consumer surplus is generally ambiguous, but positive provided that $\chi$ is sufficiently small.

## 5 Extensions

### 5.1 Endogenous quality

In this section, we consider firms’ incentives to invest in product quality. The game is as before, except that, at the initial stage, it is each firm $i$ that chooses $q_i$ rather than nature. The cost of quality is given by $C(q_i) = q_i^2$.\(^{13}\) Moreover, we fix naive consumers’

\(^{13}\)Our results generalize to any continuous, differentiable and strictly convex cost function $C(q_i)$ satisfying $C(0) = 0$, $C'(0) = 0$ and $C'(\infty) > 1$. 

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belief about undisclosed quality at some exogenous level $\mu > 0$. For simplicity, we will consider investment decisions in pure strategies only.

**Proposition 6 (Endogenous quality).** One of the two firms chooses $q_h^* > 0$ and discloses while the other chooses $q_l^* = 0$ and does not disclose. A nondeceptive equilibrium prevails when $\mu \leq \hat{\mu}$ and a deceptive equilibrium when $\mu > \hat{\mu}(\chi)$, where $\hat{\mu}(\chi) \in (0, 1/4)$ is increasing in $\chi$. Investment in quality is socially efficient in the nondeceptive equilibrium and inefficiently low otherwise.

Because a silent firm has no incentives to invest in quality and at least one firm makes zero profits if both firms disclose, vertical differentiation with minimal low quality arises endogenously. Whether the high-quality firm serves the whole market or forgoes cursed consumers depends, as before, on their number and their optimism.

When the deceptive equilibrium prevails returns of investment fall short of the social optimum because the high-quality firm cannot recoup the cost by means of higher margins on all consumers. This disincentive effect on product quality amplifies the inefficiency associated with deception. Note also that the high-quality firm may earn less than the low-quality firm if naive consumers are sufficiently optimistic.

### 5.2 Several competitors

The analysis of section 4 naturally extends to markets with $n > 2$ firms. Analogous arguments imply that as soon as at least two firms have high-quality, they will disclose and attract all consumers at a zero price. Similarly, if all firms have low quality, the sale price will be zero and at least two firms will not disclose. Finally, in the case of vertical differentiation, i.e. when a single firm has an high-quality product, she will disclose and at least two low-quality firms will not. In such a case, the condition for the prevailing of an equilibrium with deception is the same as under vertical differentiation in duopoly ($\chi < \theta$). In the region without deception prices and market shares remain unaffected. In the region with deception, instead, $p_h^* = q_h$, at least two firms who do not disclose charge $p_l^* = 0$ and rational and cursed consumers buy respectively the high-quality and
low-quality good with probability one. Thus, relative to duopoly, deception has lower likelihood but also entails a higher level of inefficiency.

6 Conclusion

Our simple model generates positive predictions that may be tested empirically in the laboratory and the field. In particular, endowing consumers in competitive markets with information should lead to higher average prices and may lead to more exploitation (i.e. lower consumer satisfaction). These effects are reversed in monopolistic settings. The empirical exploration of non-disclosure to naives is thus likely to be aided by theoretical work that explicitly models horizontal differentiation in addition to the features of our model.

In this paper we assumed that the complexity of quality-relevant information is constant. Yet said complexity may vary widely across settings. We conjecture that the complexity of the information firms can disclose affects how non-disclosure is interpreted by naive consumers and hence the amount of information firms actually reveal. Exploring the link between informational complexity and naivete theoretically and in the lab is therefore a useful endeavor. It may inform how regulators can best design independent ratings to maximize information transmission.

It will furthermore be interesting to study the incentives faced by rating agencies and certifiers in the face of consumer naivete. In particular, what is the profit maximizing rating design, if firms pay agencies for certification? Finally, future work could explore the role of intermediaries, in the spirit of Murooka (2015), who studies competitive intermediation in a shrouded attribute model. It will be important to understand when intermediaries, such as search platforms, prevent and when they exacerbate exploitation in the context of our model.
References


A Appendix

A.1 Proof of proposition 1

After disclosure, the willingness to pay of all consumers is \( q \), so that the optimal price is \( p^*(q) = q \), yielding \( \pi^*(q) = q \). Besides, each type \( q \) can always make at least \( \chi\mu \) by not disclosing, charging \( p(\emptyset) = \mu \) and attracting cursed consumers. It is then easy to verify that the strategies described in the proposition represent an equilibrium. Indeed, given nondisclosure rational consumers infer that \( E[q|\emptyset] = q_\ell \) and at \( p^*(\emptyset) = \mu \) they refuse to buy. The most profitable price that attracts all consumers given nondisclosure is \( p(\emptyset) = q_\ell \), which yields \( \pi = q_\ell \). Disclosing is then optimal if and only if \( \pi^*(q) \geq \pi^*(\emptyset) \), namely whenever \( q \geq \chi\mu \). As for uniqueness, because the payoff from disclosing and not disclosing are respectively increasing and constant in \( q \), in any equilibrium in which type \( h \) does not disclose, nor does type \( \ell \), so that \( \pi(\emptyset) < q_h \) and an \( h \) type could always profitably deviate by disclosing, charging \( p = q_h \) and attracting all consumers.

A.2 Proof of corollary 1

Given the equilibrium strategies, the expected profits of the monopolist are:

\[
\Pi = \theta q_h + (1 - \theta)\chi\mu,
\]

which clearly exceed \( \mu \) whenever \( q_\ell < \chi\mu \) and are increasing in \( \chi \). The expected utility of a rational consumer is always \( U = 0 \), while the one of a cursed consumer is

\[
U_\chi = (1 - \theta)(q_\ell - \mu) < 0.
\]

As for total welfare,

\[
S = \Pi + \chi U_\chi = \theta q_h + (1 - \theta)\chi q_\ell < \mu.
\]  (1)
A.3 Proof of proposition 2

For given quality levels of the two firms, we will repeatedly use the Bertrand argument, which we summarize in the following lemma.

**Lemma 1** (Bertrand argument). Let $\tilde{q}_i$ and $\tilde{q}_j$ represent consumers’ perceptions of qualities of firm $i$ and $j$ and assume without loss of generality that $\tilde{q}_i \geq \tilde{q}_j \geq 0$. Then, in equilibrium:

- when $\tilde{q}_i = \tilde{q}_j$: consumers buy the product at a zero price; if $\tilde{q}_i > 0$, $p^*_i = 0 = p^*_j$, while if $\tilde{q}_i = 0$, one of the two prices, say $p^*_i$ is equal to zero and the other can be arbitrary;

- when $\tilde{q}_i > \tilde{q}_j$: consumers buy the product from firm $i$ at a positive price; if $\tilde{q}_j > 0$, $p^*_i = \tilde{q}_i - \tilde{q}_j$ and $p^*_j = 0$, while if $\tilde{q}_j = 0$, $p^*_i = \tilde{q}_i$ and $p^*_j$ can be arbitrary.

**Proof.** Define $u_i = \tilde{q}_i - p_i$ as consumers’ utility when buying from firm $i$ and $\pi_i = p_iD_i$ as firm $i$’s profits, where $D_i$ denotes her demand. Suppose first that $\tilde{q}_i = \tilde{q}_j$. Then, an equilibrium in which one firm, say firm $i$, makes positive profits cannot exist, as for any price pair such that $u_i \geq u_j$ and $\pi_i > 0$, $\pi_j < p_i$ and firm $j$ would profit from charging $p_j = p_i - \epsilon$ and attracting all consumers. Thus, if a firm is active, say firm $i$, $p^*_i = 0$. Unless $\tilde{q}_i = \tilde{q}_j = 0$, it must also be the case that $u_i = u_j$, i.e. that $p^*_j = 0$, or otherwise firm $i$ could make positive profits by charging $p^*_j - \epsilon$. When $\tilde{q}_i > \tilde{q}_j$, instead, firm $j$ cannot sell in equilibrium, as for any price pair such that $u_i \leq u_j$, $u_j \geq 0$ and $D_j > 0$, firm $i$ would profit from reducing $p_i$ and attracting all consumers. Unless $\tilde{q}_j = 0$, it must also be that $u_i = u_j$, i.e. that $p^*_i = \tilde{q}_i - \tilde{q}_j - p^*_j$, and that $p^*_j = 0$, as otherwise firm $i$ and $j$ could profitably deviating by respectively increasing and decreasing their prices. \[\square\]

Let $(q_i, q_j)$ represent a realization of firms’ qualities and $(m_i, m_j)$ firms’ messages in some on-the-equilibrium-path history. If consumers know firms’ qualities, lemma 1 characterizes equilibrium prices. Suppose there exists an equilibrium in which firms’ quality are not perfectly revealed, i.e., there exists an on-path history in which some
types are pooled. Such history requires that at least a firm, say firm \(i\), sends \(m_i = \emptyset\).

We will consider all possibilities.

- Suppose \(\langle \emptyset, q^*_\ell \rangle\). Then, lemma 1 implies that firm \(i\) makes at most \(p_i = \tilde{q}_i - q^*_\ell\). Unless \(\tilde{q}_i = q^*_h\), when \(q_i = q^*_h\) firm \(i\) could profitable deviate by disclosing and charging \(p_i = q^*_h - q^*_\ell - \epsilon\). Thus, either \(\tilde{q}_i = q^*_h\) or \(\tilde{q}_i = q^*_\ell\) and in either case, the two types perfectly separate.

- Suppose \(\langle \emptyset, q^*_h \rangle\). Then, lemma 1 implies that firm \(i\) makes zero profits and \(p_j \geq q^*_h - \tilde{q}_i\). Unless \(\tilde{q}_i = q^*_h\), when \(q_i = q^*_h\) firm \(i\) could profitable deviate by disclosing and charging \(q^*_h - \tilde{q}_i\). Thus, either \(\tilde{q}_i = q^*_h\) or \(\tilde{q}_i = q^*_\ell\) and in either case the two types perfectly separate.

- Suppose \(\langle \emptyset, \emptyset \rangle\) and assume without loss of generality that \(q_i \geq q_j\). Then, lemma 1 implies that firm \(i\) makes \(\pi_i = \tilde{q}_i - \tilde{q}_j\). Unless \(\tilde{q}_i = q^*_h\), when \(q_i = q^*_h\) firm \(i\) could profitable deviate by disclosing and charging \(q^*_h \tilde{q}_j\). Thus, either \(\tilde{q}_i = q^*_h\) or \(\tilde{q}_i = q^*_\ell\) and in either case the two types perfectly separate.

Thus, any equilibrium entails full-separation. To show that an equilibrium indeed exists, take one in which each firm \(i\) always discloses when she has high quality,\(^{14}\) does not disclose when she has low quality and consumer’s beliefs are such that \(\tilde{q}^*_i(\emptyset) = q^*_\ell\). When \(q_i = q^*_\ell\), firm \(i\)’s disclosure decision does not affect consumers’ valuation for her good, so that the outcome is the same regardless of whether she discloses. When \(q_i = q^*_h\), if firm \(i\) does not disclose, she will attract no consumers, so that disclosing is strictly optimal when \(q_j = q^*_\ell\) and weakly optimal when \(q_j = q^*_h\). As \(\mathbb{E}[q_i|m_i = \emptyset] = q^*_\ell\), consumers’ beliefs are correct and we hence identified an equilibrium.

\(^{14}\)While this policy looks natural, when \(q^*_\ell = 0\) there exist an outcome-equivalent equilibrium in which a firm does not always disclose when she has high quality. It prescribes that

- when realizations are \((q_h, q^*_\ell)\), \(m_i = \emptyset\) and \(m_j = q^*_\ell\);
- when realizations are \((q_h, q^*_h)\), \(m_i = q^*_h\) and \(m_j = q^*_h\);
- when realizations are \((q^*_\ell, q^*_h)\), \(m_i = \emptyset\) and \(m_j = \emptyset\);
- when realizations are \((q^*_\ell, q^*_h)\), \(m_i = q^*_\ell\) and \(m_j = \emptyset\).
A.4 Proof of proposition 3

Considering all cases, we will check that such a candidate equilibrium exists and characterize it. We will then prove uniqueness. In what follows, \( \tilde{q}_i \) and \( \tilde{q}_i^\chi \) will represent the beliefs respectively of rational and cursed consumers about firm \( i \)'s quality and \( u_i \) and \( u_i^\chi \) will represent their associated perceived utilities from buying from firm \( i \).

- Suppose both firms have low quality. Upon nondisclosure by both firms, \( \tilde{q}_i = \tilde{q}_j = 0 \), so that the willingness to pay of rational consumers is zero. The two firms hence compete only for cursed consumers (whose \( \tilde{q}_i^\chi = \tilde{q}_j^\chi = \theta \)) and, by lemma 1, they charge zero prices and make zero profits. If firm \( i \) deviates by disclosing, she attracts no consumers no matter the positive price she charges.

- Suppose both firms have high quality. When both firms disclose, lemma 1 implies they must charge a zero price and make zero profits. If firm \( i \) deviates by not disclosing, because \( \tilde{q}_i(\emptyset) = 0 \) and \( \tilde{q}_i^\chi(\emptyset) = \theta < 1 \), she attracts no consumers no matter the positive price she charges.

- Suppose the quality of the two firms differ. We will distinguish two sub-cases:
  - Suppose first that \( \chi \geq \theta \). Imagine the \( h \) firm discloses and the \( \ell \) firm does not, so that \( \tilde{q}_\ell = 0 \) and \( \tilde{q}_\ell^\chi = \theta \). Then, when \( p_h^* = 1 - \theta \) and \( p_\ell^* = 0 \), the \( h \) firm attracts all consumers. If firm \( \ell \) deviates by disclosing or by charging a positive price, she keeps attracting none. If firm \( h \) deviates by not disclosing, she attracts no consumers no matter the positive price she charges. If firm \( h \) deviates in prices, her best deviation is \( p_h^* = 1 \), which attracts only rational consumers (because \( u_\ell^\chi = \theta > u_h^\chi = 0 \)) and hence yields \( 1 - \chi \). This deviation is not profitable iff \( 1 - \theta \geq 1 - \chi \), that is, iff \( \chi \geq \theta \).
  - Suppose instead that \( \chi < \theta \). Imagine the \( h \) firm discloses and the \( \ell \) firm does not. We will construct mixed equilibrium pricing strategies such that firm \( h \) randomizes according to \( G_h(p_h) \) over \( [p_h^\ell, p_h^\bar{h}] \), firm \( \ell \) randomizes according to...
$G_\ell(p_\ell)$ over $[p_\ell, \bar{p}_\ell]$, rational consumers always buy from firm $h$ and cursed ones with positive probability from either firm. As supports, we guess $p_h = 1 - \chi$, $\bar{p}_h = 1$, $p_\ell = \theta - \chi$ and $\bar{p}_\ell = \theta$, so that $u_h^\chi(\bar{p}_h) = u_\ell^\chi(\bar{p}_\ell) = 0$ and $u_h^\chi(p_h) = u_\ell^\chi(p_\ell) = \chi$. Note that $p_\ell$ is positive if and only if indeed $\chi < \theta$. Given these supports, rational consumers never find it rational to buy from firm $\ell$. Fix $G_\ell(p_\ell)$ and assume it is atomless. The expected profits of firm $h$ for $p_h = \bar{p}_h$ are $\pi_h(\bar{p}_h) = 1 - \chi$, while for any other $p_h$ in the candidate support

$$\pi_h(p_h) = p_h \left( 1 - G_\ell(p_h - (1 - \theta)) \right) + (1 - \chi) p_h G_\ell(p_h - (1 - \theta)).$$

Solving $\pi_h(p_h) = \pi_h(\bar{p}_h)$ yields $G_\ell(p_h - (1 - \theta)) = \frac{p_h - (1 - \chi)}{\chi p_h}$ and, after the change of variable $p_h = p_\ell + 1 - \theta$, $G_\ell^*(p_\ell) = \frac{p_\ell - (\theta - \chi)}{\chi(p_\ell + 1 - \theta)}$. Note that $G_\ell^*(\bar{p}_\ell) = 0$ and $G_\ell^*(\bar{p}_\ell) = 1$. When firm $\ell$ randomizes according to $G_\ell^*(\cdot)$, firm $h$ is hence indifferent to any $p_h$ in the candidate support. Any $p_h$ above $\bar{p}_h$ would yield $\pi_h = 0$, while any $p_h < \bar{p}_h$ would yield $\pi_h = p_h < 1 - \chi$.

Fix now $G_h(p_h)$ and assume it is atomless except possibly in $\bar{p}_h$. The expected profits of firm $\ell$ from $p_\ell = p_\ell$ are $\pi_\ell(p_\ell) = \chi(\theta - \chi)$, while for any other $p_\ell$ in the candidate support

$$\pi_\ell(p_\ell) = \chi p_\ell \left( 1 - G_h(p_\ell + 1 - \theta) \right).$$

Solving $\pi_\ell(p_\ell) = \pi_\ell(\bar{p}_\ell)$ yields $G_h(p_h - (1 - \theta)) = \frac{p_\ell - (\theta - \chi)}{\chi p_\ell}$ and, after the change of variable $p_\ell = p_h - (1 - \theta)$, $G_h^*(p_h) = \frac{p_h - (1 - \chi)}{\chi p_h - (1 - \theta)}$. Note that $G_h^*(\bar{p}_h) = 0$, while $G_h^*(\bar{p}_h) = \frac{\chi}{\theta} < 1$, which means that $G_h^*(\cdot)$ has an atom of size $\bar{p}_h \equiv \frac{\theta - \chi}{\theta}$ at $\bar{p}_h$. When firm $h$ randomizes according to $G_h^*(\cdot)$, firm $\ell$ is hence indifferent to any $p_\ell$ in the candidate support. Any $p_\ell$ above $\bar{p}_\ell$ would yield $\pi_\ell = 0$, while any $p_\ell \in (0, \bar{p}_\ell)$ would yield $\chi p_\ell < \chi p_\ell$. 

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Let us now consider deviations in the disclosure strategies. If firm $\ell$ deviates by disclosing, she attracts no consumers no matter the positive price she charges. If firm $h$ deviates by not disclosing, she never attracts rational consumers for any positive price and, as she lowers the valuation of cursed ones, she cannot increase her profits. Formally, any $p_h \geq \bar{p}_\ell$ will never attract cursed consumers, any $p_h \leq \bar{p}_\ell$ will always attract them and any $p_h \in [\bar{p}_\ell, \bar{p}_\ell)$ would yield:

$$\pi_h(p_h) = \chi(1 - G^*_\ell(p_h))p_h = \frac{(1 - \chi)p_h(\theta - p_h)}{p_h + (1 - \theta)}.$$  

Note that $\pi_h(p_h)$ is concave, i.e., $\pi_h''(p_h) = -\frac{2(1-\chi)(1-\theta)}{(p_h+(1-\theta))^3} < 0$, and clearly $\pi_h'(\bar{p}_\ell) < 0$. When $\chi(2 - \chi) \leq \theta$, $\pi_h'(p_\ell) \leq 0$, so that the optimal deviation is $p^d_h = \bar{p}_\ell$, yielding $\pi_h(p_\ell) = \chi(\theta - \chi)$. As this is less than firm $h$’s equilibrium payoff (namely, $1 - \chi$), deviating is not profitable. When $\chi(2 - \chi) > \theta$, the optimal is interior and given by the stationary point of $\pi_h(p_h)$, namely $p^d_h = (\sqrt{1 - \theta} - (1 - \theta))$. This yields $\pi_h(p^d_h) = (1 - \chi)(2 - \theta) - 2\sqrt{1 - \theta}$, which once again is lower than firm $h$’s original payoff.

As for uniqueness, note first of all that there cannot exist an equilibrium in which a firm discloses when her quality is $q_j = q_\ell$. Indeed, if this was the case, we would have $\tilde{q}_j = \tilde{q}_j^\chi = 0 = \pi_j = 0$ and firm $i$’s would behave as a monopolist (proposition 1). As cursed consumers would obtain their perceived reservation utility, firm $j$ could profitably deviate by not disclosing and attracting them with a $p_j > 0$. Suppose the realization of qualities is $(q_h, q_\ell)$, so that firms’ messages are $\langle m_i, \emptyset \rangle$. Because in equilibrium a firm whose $q_i = q_\ell$ never discloses, the valuation of rational consumers when firm $i$ is silent must satisfy $\tilde{q}_i(\emptyset) < 1$. Thus, if $m^*_i = \emptyset$, given any candidate equilibrium prices firm $i$ can profit by disclosing because by doing so she strictly raises the valuation of all consumers for her good. The same argument applies when the realization of qualities is $(q_h, q_\ell)$ regardless of whether only one firm or both disclose, because in either case
\( \tilde{q}_h(\theta) < 1 \). To conclude, note that once the disclosure decisions are as in the proposition, the equilibrium pricing strategies characterized above are unique. Indeed, when \( \chi \geq \theta \), for any candidate equilibrium of the pricing game in which firm \( h \) does not attract cursed consumers with probability one, she would have an incentive to decrease \( p_h \) to ensures that she does so. When \( \chi < \theta \), instead, no pure strategy equilibrium can exist. Indeed, as shown above, given the unique candidate equilibrium prices for which firm \( h \) attracts cursed consumers with probability one, she would want to deviate. Similarly, there cannot exist a candidate equilibrium in which firm \( \ell \) attracts cursed consumers with probability one, as at candidate equilibrium prices it should be that

\[
\begin{align*}
\chi_h &= 1 - p_h^* = \theta - p_h^* \\
u_h^\chi &= \chi_h = \theta - p_h^* \\
u_h^\chi &= \chi_h = \theta - p_h^*
\end{align*}
\]

and firm \( h \) would profit from slightly decreasing \( p_h \). Uniqueness of the mixed-strategy equilibrium follows from noting that, because of standard arguments, price supports cannot have interior atoms or holes and that, unless \( \bar{p}_h = 1 \), in such candidate equilibrium firm \( h \) would profit from charging \( p_h > \bar{p}_h \).

### A.5 Proof of proposition 4

Under mandatory disclosure, the equilibrium outcome is the same as when consumers are all rational (proposition 2). The equilibrium outcomes of proposition 2 and 3 do not coincide only when the two firms’ qualities differ, which is also the unique case in which consumers pay a positive price and firms make positive profits. In this event, in the equilibrium of proposition 2 all consumers buy from firm \( h \) at \( p_h^* = 1 \) and experience zero utility. In the equilibrium at proposition 3, instead, when \( \chi \geq \theta \), they all buy from firm \( h \) at \( p_h^* = 1 - \theta < 1 \) and obtain \( \theta \). When \( \chi < \theta \), firms’ profits are \( \pi_h = 1 - \chi \) and \( \pi_\ell = \chi(\theta - \chi) \), so that \( \pi = \pi_h + \pi_\ell = (1 - \chi(1 - \theta + \chi)) < 1 \). The expected utility of a
rational consumer is \( U = 1 - \mathbb{E}[p_h] \), where

\[
\mathbb{E}[p_h] = \int_{1-\chi}^{1} g_h^*(p_h)p_h \, dp_h + \frac{\theta - \chi}{\theta} = 1 - \chi - (\theta - \chi) \log \left( 1 - \frac{\chi}{\theta} \right) < 1. \tag{2}
\]

Instead,

\[
\mathbb{E}[p'] = \int_{\theta - \chi}^{\theta} g'_h(\theta)p \, dp = \frac{-\chi(1-\theta) - (1-\chi)\log(1-\chi)}{\chi},
\]

and the expected utility of a cursed consumer is

\[
U_\chi = -\mathbb{P}(\bar{u}_l^\chi \geq u_h^\chi)\mathbb{E}[p|u_h^\chi \geq \bar{u}_l^\chi] + \mathbb{P}(\bar{u}_h^\chi > \bar{u}_l^\chi) \left( 1 - \mathbb{E}[p_h|u_h^\chi > \bar{u}_l^\chi] \right)
\]

\[
= -\frac{\theta - \chi}{\theta} \mathbb{E}[p] - \frac{\chi}{\theta} \mathbb{P}(p \leq p_h - (1-\theta)\, p_h < 1) \mathbb{E}[p|p \leq p-h - (1-\theta), p_h < 1] + \frac{\chi}{\theta} \mathbb{P}(p_h \leq p + (1-\theta)\, p_h < 1) \left( 1 - \mathbb{E}[p_h|p_h \leq p + (1-\theta), p_h < 1] \right)
\]

\[
= -\frac{(\theta - 1)\chi(\theta - 2\chi + \chi) + (\chi - 1)(\chi - \theta) ((\theta - 2)\theta \log(1 - \frac{\chi}{\theta}) + \log(1 - \chi)))}{(\theta - 1)^2}\chi. \tag{15}
\]

Naturally, \( \lim_{\chi \to \theta} U_\chi = \theta \) and \( \lim_{\chi \to 0} U_\chi = -\theta \). Besides, differentiating \( U_\chi \) with respect to \( \chi \) yields:

\[
U'_\chi(\chi) = \frac{-(\theta - 1)\theta \chi(1 + \chi) + (\theta - \chi^2) \left( \log(1 - \chi) - (2 - \theta)\theta \log(1 - \frac{\chi}{\theta}) \right)}{(1 - \theta)^2}\chi^2.
\]

---

\[15\] We used:

\[
P^-_\chi \equiv \mathbb{P}(p \leq p_h - (1-\theta)|p_h < 1) = \int_{\theta - \chi}^{\theta} \int_{1-\theta + p}^{1} g_h^*(p) \frac{\theta}{\chi} g_h^*(p) \, dp \, dp_h = \frac{(\chi - \theta)\chi(1 - \chi)\log \left( \frac{(1 - \chi)\theta}{\chi} \right)}{\chi^2(1 - \theta)^2},
\]

\[
P^- \equiv \mathbb{P}(p \leq \theta - (1-\theta)\, p_h < 1) = \int_{\theta - \chi}^{\theta} \int_{1-\theta + p}^{1} p g_h^*(p) \frac{\theta}{\chi} g_h^*(p) \, dp \, dp_h = \frac{(\theta - \chi)(1 - \chi)\log(1 - \chi)}{\chi^2},
\]

\[
P^-_h \equiv \mathbb{P}(p_h < p + (1-\theta)\, p_h < 1) = 1 - P^-_\chi, \text{ and}
\]

\[
P^-_h \equiv \mathbb{P}(p_h < p + (1-\theta)\, p_h < 1) = \int_{\theta - \chi}^{\theta} \int_{1-\theta + p}^{1} p \frac{\theta}{\chi} g_h^*(p) \, dp \, dp_h = \frac{(1 - \chi)\theta \left( \chi + (\chi - \theta) \log \left( \frac{\theta}{\chi} \right) \right)}{\chi^2}.
\]
The sign of $U_\chi'(\chi)$ coincides with the sign of its numerator $N$, which is positive.\footnote{We have \[ \frac{\partial N(\chi, \theta)}{\partial \chi} = \chi \left( \theta^2 - 1 + \frac{(2 - \theta)(1 - \theta)\theta}{\theta - \chi} + \frac{1 - \theta}{1 - \chi} - 2\log(1 - \chi) + 2(2 - \theta)\theta \log \left( \frac{1 - \chi}{\theta} \right) \right), \] and \[ \frac{\partial^2 N(\chi, \theta)}{\partial \chi \partial \theta} = \frac{(1 - \theta)\chi (-4\theta^2 (1 - \chi) + \theta(10 - 9\chi)\chi - \chi(2 - \chi(4\chi - 3)) + 4(\theta - \chi)^2 (1 - \chi) \log \left( \frac{1 - \chi}{\theta} \right))}{(\theta - \chi)^2 (1 - \chi)}. \] As each term in the numerator of $\frac{\partial^2 N(\chi, \theta)}{\partial \chi \partial \theta}$ is negative (unless $\theta = \chi = 1$, in which case $\frac{\partial^2 N(\chi, \theta)}{\partial \chi \partial \theta} = 0$ ) and the denominator is positive, $\frac{\partial^2 N(\chi, \theta)}{\partial \chi \partial \theta} < 0$. Moreover, $\frac{\partial N(\chi, 1)}{\partial \chi} = 0$, which implies that $\frac{\partial N(\chi, \theta)}{\partial \chi} > 0$, because \[ \frac{\partial N(\chi, 1)}{\partial \chi} - \frac{\partial N(\chi, \theta)}{\partial \chi} = \int_0^1 \frac{\partial^2 N(\chi, t)}{\partial \chi \partial \theta} dt. \] As $N(0, \theta) = 0$ and $\frac{\partial N(\chi, \theta)}{\partial \chi} > 0$, it follows that $N(\chi, \theta) > 0$.} Thus, $U_\chi$ is increasing, and there exists a unique cutoff $\hat{\chi} \in (0, \theta)$ such that $U_\chi < 0$ when $\chi < \hat{\chi}$, $U_\chi = 0$ when $\chi = \hat{\chi}$ and $U_\chi > 0$ when $\chi > \hat{\chi}$.

Total consumers’ surplus is

$$S \equiv \chi U_\chi + (1 - \chi)U.$$ (4)

When $\chi \geq \hat{\chi}$, $U_\chi \geq 0$ and $U > 0$, so that the effect of mandatory disclosure on $S$ is negative. When, $\chi < \hat{\chi}$, the effect is a priori ambiguous but positive if $\chi$ is small enough, because in such case $S < 0$. Indeed,

$$S'(\chi) = \chi U_\chi'(\chi) + U_\chi(\chi) - U(\chi) + (1 - \chi)U''(\chi),$$

where $U = 1 - E[p_h]$ and $U'(\chi) = \log \left( \frac{1 - \chi}{\theta} \right)$. As $\chi$ goes to zero, all terms go to zero except $U_\chi(\chi) < 0$. Thus, because $S(0) = 0$ and $S'(0) < 0$, $S(\chi) < 0$ for small enough $\chi$.

### A.6 Proof of proposition 5

Consider an information campaign that reduces the fraction of cursed consumers from $\chi$ to $\chi_I < \chi$. As in the case of mandatory disclosure, this intervention may have
an impact only when firms’ qualities differ. Suppose initially that $\chi > \theta$. If $\chi_I$ is such that $\chi_I \geq \theta$ still holds, this intervention has no impact. If the decrease is such that $\chi_I < \theta$, instead, firms profits $\pi = \pi_h + \pi_\ell$ increase from $1 - \theta$ to $\pi = (1 - \chi_I (1 - \theta + \chi_I))$, while the utilities $U$ and $U_\chi$ of a rational and a cursed consumer decrease compared to $\theta$. Indeed, not only prices increase, $p_h^*$ from $(1 - \theta)$ to $\mathbb{E}[p_h] \in (1 - \chi_I, 1)$ and $p_\ell^*$ from $0$ to $\mathbb{E}[p_\ell] \in (\theta - \chi_I, \theta)$, but a cursed consumer also purchases the $q_\ell$ product with positive probability. Therefore, total consumers’ surplus $S$, which is defined in equation (4), decreases. Let us now consider the effect of a decrease from $\chi$ to $\chi_I$ starting from a situation in which $\chi < \theta$. As $\pi'(\chi) < 0$, firms’ overall profits increase. While those of the $h$ firm always increase, i.e. $\pi_h'(\chi) < 0$, those of the $\ell$ firm increase only when the fraction of cursed consumers before and after the campaign is large enough. Indeed, we have that $\pi_\ell'(\chi) < 0$ if and only if $\chi > \frac{\theta}{2}$ and, as $\pi_\ell(0) = \pi_\ell(\theta) = 0$, $\pi_\ell$ has a unique global maximum in $\frac{\theta}{2}$. $U = 1 - \mathbb{E}[p_h]$ decreases because $\mathbb{E}[p_h]$ increases: differentiating equation (2) yields $\frac{d\mathbb{E}[p_h]}{d\chi} = \log \left(1 - \frac{\chi}{\theta}\right) < 0$. $\mathbb{E}[p_\ell]$ also increases: differentiating equation (3) yields $\frac{d\mathbb{E}[p_\ell]}{d\chi} = \frac{\chi + \log(1 - \chi)}{\chi^2} < 0$, which is negative because for any $y \in (0, 1)$, $y < -\log(1 - y) < \frac{y}{1-y}$. As shown in the proof of proposition 4, $U'_\chi(\chi) > 0$ and there is a unique $\hat{\chi}$ such that $U_\chi < 0$ if and only if $\chi < \hat{\chi}$. Thus, $U_\chi$ always decreases due to the campaign. If we consider total surplus of cursed consumers $S_\chi = \chi U_\chi$, we have

$$S'_\chi(\chi) = \chi U'_\chi(\chi) + U_\chi(\chi).$$

When $\chi_I \geq \hat{\chi}$ the effect of the campaign is negative, because $U'_\chi(\chi) > 0$ and $U_\chi(\chi) \geq 0$ imply $S'_\chi(\chi) > 0$. When $\chi_I < \hat{\chi}$ the effect is ambiguous, even though for $\chi$ sufficiently small it must be positive because the first term of $S'_\chi(\chi)$ goes to zero and the second term is negative, so that $S'_\chi(\chi) < 0$. As for total consumers’ surplus $S$, the effect is a ambiguous but positive for $\chi$ sufficiently small, because as shown in the proof of proposition 4 in such case $S'(\chi) < 0$. 

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A.7 Proof of proposition 6

In equilibrium at most one firm can choose a positive quality level. Indeed, suppose by contradiction that in some equilibrium \( q^*_h > 0 \) and \( q^*_\ell > 0 \), where \( q^*_h \geq q^*_\ell \). If both firms disclose, lemma 1 implies that firm \( \ell \) makes zero sale profits and hence obtains a negative payoff. It must then be the case that one firm, say firm \( i \), does not disclose. But then, given rational and naive consumers’ beliefs about \( q^*_i \) in this candidate equilibrium, firm \( i \) could profitably deviate at the initial stage by choosing \( q_i = 0 \). Thus, in what follows we can take \( q^*_\ell = 0 \) and, as demonstrated in the proof of proposition 3, firm \( \ell \) does not disclose. By a similar argument, it must be that firm \( h \) discloses \( q_h \) whenever positive. Indeed, in any candidate equilibrium in which firm \( h \) remains silent, rational consumers must infer that \( q^*_h = 0 \). Then, by lemma 1, firms would then compete a la Bertrand for naive consumers and make zero sale profits and firm \( h \) would make a loss.

Given firms’ quality choices, with in particular \( q_\ell = 0 \), at the second stage results on the two possible equilibrium configurations of proposition 3 under vertical differentiation apply and naturally generalize to any \( q_h \) and \( \mu \). In a candidate equilibrium without deception the \( h \) firm charges \( p_h = q_h - \mu \) and the \( \ell \) firm \( p_\ell = 0 \), while in a candidate equilibrium with deception firm \( h \) randomizes over \([ (1 - \chi)q_h, q_h ] \) and firm \( \ell \) over \([ \mu - \chi q_h, \mu ] \). Besides, if we denote firm \( h \)'s profits in the two scenarios respectively as \( \pi_{h,nd} \equiv q_h - \mu - C(q_h) \) and \( \pi_{h,d} \equiv (1 - \chi)q_h - C(q_h) \), the former or the latter configuration prevails depending on whether \( \pi_{h,nd} \geq \pi_{h,d} \), that is, on whether \( q_h \geq \frac{\mu}{\chi} \).

Thus, at the first stage \( q^*_h \) must solve

\[
\max \left\{ \max_{q_h \geq \frac{\mu}{\chi}} \pi_{h,nd}, \max_{q_h \in [0, \frac{\mu}{\chi}]} \pi_{h,d} \right\},
\]

where we ignored that \( \max \pi_{h,nd} \) may be negative because \( \max \pi_{h,d} \geq 0 \). But because at the second stage the prevailing of the equilibrium configuration with or without deception is determined exclusively by a comparison of firm \( h \)'s profits in the two scenarios, whenever the solution of one of the two maximization problems is \( q^*_h = \frac{\mu}{\chi} \), the maximum
of the other problem is at least as high.\footnote{Formally, note first of all that \( \pi_{h,nd}(\frac{\mu}{\chi}) = \pi_{h,d}(\frac{\mu}{\chi}) \equiv \pi = \frac{\mu(\chi^2 - \mu)}{\chi^2} \) by construction. The solution of the second problem is \( q_{h,d}^* = \frac{\mu}{\chi} \) if \( \mu \leq \bar{\mu} = \frac{1}{2}(\chi - \chi^2) \), yielding \( \pi^*_{h,d} = \frac{1}{4}(1 - \chi)^2 \geq \pi \). \( \pi(\mu) \) is zero in \( \mu = 0 \), it is increasing in \( \mu \) if and only if \( \mu \leq \tilde{\mu} \) and, by construction, when \( \mu = \tilde{\mu} \) it holds that \( \pi(\mu) = \pi^*_{h,d} \). The solution of the first problem is \( q_{h,d}^* = \frac{1}{2} \) if \( \mu \leq \tilde{\mu} = \frac{1}{2} \), yielding \( \pi^*_{h,nd} = \frac{1}{4} - \mu \), and \( q_{h,d}^* = \frac{\mu}{\chi} \) otherwise, yielding \( \pi \leq \pi^*_{h,nd} \). \( \pi^*_{h,nd} \) is decreasing in \( \mu \) and negative if and only if \( \mu \geq \frac{1}{4} \). Besides, \( \bar{\mu} > \tilde{\mu} \). Thus, when the constraint binds in the first maximization problem the maximum of the second problem is higher: if \( \tilde{\mu} \geq \frac{1}{4} \) because \( \pi < 0 \) and if \( \bar{\mu} < \frac{1}{4} \) because we are in the decreasing part of \( \pi \). Similarly, when the constraint binds in the second maximization problem the maximum of the first problem is higher because \( \pi < \pi^*_{h,nd} \).} Hence firm’s \( h \) problem simplifies to

\[
\max \left\{ \max_{q_n \in \mathbb{R}^+} \pi_{h,nd}, \max_{q_d \in \mathbb{R}^+} \pi_{h,d} \right\}.
\]

The two maximizers of \( \pi_{h,nd} \) and \( \pi_{h,d} \) are respectively \( q^*_{h,nd} = \frac{1}{2} \) and \( q^*_{h,d} = \frac{1}{2}\chi \), yielding \( \pi^*_{h,nd} = \frac{1}{4} - \mu \) and \( \pi^*_{h,d} = \frac{1}{4}(1 - \chi)^2 \). We have that \( \pi^*_{h,nd} \geq \pi^*_{h,d} \) if and only if \( \mu \leq \hat{\mu}(\chi) \equiv \frac{1}{4}(2\chi - \chi^2) \), where \( \hat{\mu}'(\chi) > 0 \).