

Egger, Hartmut; Udo, Kreickemeier; Jens, Wrona

**Conference Paper**

## Offshoring, Firm Selection, and Job Polarisation in General Equilibrium

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2016: Demographischer Wandel -  
Session: International Trade: Theory, No. C19-V1

**Provided in Cooperation with:**

Verein für Socialpolitik / German Economic Association

*Suggested Citation:* Egger, Hartmut; Udo, Kreickemeier; Jens, Wrona (2016) : Offshoring, Firm Selection, and Job Polarisation in General Equilibrium, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2016: Demographischer Wandel - Session: International Trade: Theory, No. C19-V1, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at:

<https://hdl.handle.net/10419/145514>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# Offshoring, Firm Selection, and Job Polarisation in General Equilibrium

Hartmut Egger\*

Udo Kreickemeier<sup>†</sup>

Jens Wrona<sup>‡</sup>

University of Bayreuth  
CESifo, GEP, and IfW

TU Dresden  
CESifo and GEP

University of Düsseldorf

February 15, 2016

## Abstract

We set up a general equilibrium model, in which offshoring to a low-income country can lead to job polarisation in the high-income country, with the number of jobs paying either very high or very low wages increasing, and jobs in the middle of the wage distribution disappearing. The firm population is heterogeneous with respect to firm productivity, and rent sharing leads to a positive link between wages and productivity at the firm level. Offshoring involves fixed and task-specific variable costs, and as a consequence it is chosen only by the most productive firms, and only for those tasks carrying the lowest variable offshoring costs. A reduction in those variable costs increases offshoring at the intensive and at the extensive margin, with domestic employment shifted from the newly offshoring firms in the middle of the productivity distribution to firms at the tails of this distribution, paying either very low or very high wages.

JEL-Classification: F12, F16, F23

Keywords: Offshoring, Job Polarisation, Heterogeneous Firms, Good Jobs, Bad Jobs

---

\*University of Bayreuth, Department of Law and Economics, Universitätsstr. 30, 95447 Bayreuth, Germany; Email: hartmut.egger@uni-bayreuth.de.

<sup>†</sup>TU Dresden, Faculty of Business and Economics, Helmholtzstr. 10, 01069 Dresden, Germany; Email: udo.kreickemeier@tu-dresden.de.

<sup>‡</sup>University of Düsseldorf, Düsseldorf Institute for Competition Economics (DICE), Universitätsstr. 1, 40225 Düsseldorf, Germany; Email: wrona@dice.hhu.de.

# 1 Introduction

The polarisation of jobs is an empirically well-documented phenomenon in many industrialised economies.<sup>1</sup> The literature dealing with job polarisation has pointed to biased technological change and offshoring as the two most important factors that have made jobs in the middle of the income distribution obsolete over the last two decades – with a significant role of offshoring mainly found for European countries which feature strong labour market imperfections (cf. [Goos et al., 2014](#)). Thereby, the “routinization hypothesis”, which postulates that technological change and offshoring have caused a movement of workers from jobs concentrated on routine tasks and paying mid-level wages to jobs highly intensive in abstract or service tasks at the upper and lower tail of the wage distribution, respectively, has become a widely accepted explanation for the observed polarisation pattern. Arguably, it has been instrumental for the success of the routinisation hypothesis that the according data on tasks and occupations are now available for many different countries. In contrast to the role played by tasks and occupations, the role of individual firms has not been looked at in this literature. But recent evidence from matched employer-employee data indicates that a significant part of the evolution of job polarization can be attributed to the movement of workers between firms rather than between jobs with differing task characteristics within a firm (cf. [Harrigan et al., 2015](#)).

Consistent with this evidence from employer-employee data, we argue in this paper that important complementary insights into the relationship between offshoring and job polarisation can be gained if we model selection into offshoring not only at the task level but also at the firm level, provided that wages have a firm-specific component. Recent evidence suggestive of the importance of such a firm-specific component comes from [Card et al. \(2013\)](#), who decompose changes in the German wage structure between 1985 and 2009 and find that an establishment-specific effect explains 25% of the increase in wage inequality over that period, not much short of the 40% explained by individual-specific effects, such as a worker’s occupation.<sup>2</sup> The story we have in mind is straightforward: Suppose that offshoring has fixed and variable costs, where the latter are task-specific as in [Grossman and Rossi-Hansberg \(2008\)](#). Furthermore, firms differ in their productivity, and – crucially – there is a rent-sharing mechanism such that more productive firms pay higher wages. This setup leads to an equilibrium, in which high-wage firms offshore some of

---

<sup>1</sup>For instance, [Autor et al. \(2006, 2008\)](#) and [Autor and Dorn \(2013\)](#) provide evidence for the US, [Goos and Manning \(2007\)](#) for UK, [Spitz-Oener \(2006\)](#) and [Dustmann et al. \(2009\)](#) for Germany, [Keller and Utar \(2015\)](#) for Denmark, and [Harrigan et al. \(2015\)](#) for France. [Goos et al. \(2009, 2014\)](#) document job polarisation for a sample of 16 European countries.

<sup>2</sup>It is a well documented fact that larger firms do pay higher wages (cf. [Idson and Oi, 1999](#); [Holzer et al., 2011](#)). In their seminal study, [Abowd et al. \(1999\)](#) find that firm-size wage differences in the US are largely explained by individual effects. However, in several studies for different European countries (cf. [Zweimüller and Winter-Ebmer, 1999](#); [Gruetter and Lalive, 2009](#); [Card et al., 2013](#)) the importance of firm-specific wage effects is emphasised.

their tasks, while low-wage firms do not offshore at all.<sup>3</sup> It is now natural for an across-the-board decrease in variable offshoring costs to lead to job polarisation: The newly offshoring firms, which pay intermediate wages, since they are less productive than the incumbent offshoring firms, but more productive than the non-offshoring firms, reduce the number of jobs in their home country. This reduction in labour demand leads to lower wages *ceteris paribus*, and therefore incumbent offshoring firms and non-offshoring firms – i.e. the firms paying either very high or very low wages – increase their domestic employment.

In order to formalize the story set out above, we develop a general equilibrium model, in which firms from an industrialised country have an incentive to offshore the production of some of their tasks to an emerging country, which has lower wages. As in [Grossman and Rossi-Hansberg \(2008\)](#), tasks can be ranked according to their variable offshoring costs, and in equilibrium there is an endogenous *extensive task margin* separating the tasks that are kept onshore from the tasks that are moved offshore. Firms are run by entrepreneurs, and as in [Lucas \(1978\)](#) the ability of the entrepreneur determines the productivity of the firm she is running. More productive firms make higher profits, and these profits are paid as remuneration to the entrepreneurs. Firm-level wages for production workers are higher in more productive firms, since workers have fair wage preferences as in [Akerlof and Yellen \(1990\)](#) and [Egger and Kreickemeier \(2012\)](#), and firms with higher operating profits have to pay more in order to elicit full effort from their employees. Following empirical evidence pointing towards international rent sharing in multinational firms (see [Dobbelaere, 2004](#); [Budd et al., 2005](#); [Martins and Yang, 2015](#)), we assume that the fair wage mechanism also applies to a firm's offshore operations. Hence, a more productive firm has to pay higher wages in its offshore location than a less productive competitor, but of course (since workers' outside options play a role as well) lower wages than in its home market.

The relative wage paid by offshoring firms in their two locations is the same across firms, and therefore all firms offshore the same set of tasks, provided they offshore at all. The gains from offshoring increase in firm productivity, and due to the fixed cost of offshoring our model features an *extensive firm margin* separating the offshoring firms from the national firms. The equilibrium features a continuous wage distribution, with well-paid workers employed by offshoring firms, workers with lower wages employed by national firms, and also a group of unemployed workers.<sup>4</sup> Our key result is to show formally that the intuition from our story above indeed holds true in a

---

<sup>3</sup>The self-selection of high-productivity firms into the importing of intermediates (a commonly used proxy for offshoring) has been documented for the U.S. (cf. [Bernard et al., 2007, 2012](#)), France (cf. [Biscourp and Kramarz, 2007](#)), Germany (cf. [Moser et al., 2015](#)), Belgium (cf. [Mion and Zhu, 2013](#)) and Denmark (cf. [Hummels et al., 2014](#)). [Paul and Yasar \(2009\)](#) provide explicit evidence on the self-selection into offshoring. Focussing on the Turkish textiles and apparel industry it is shown that firms which decided in favour of offshoring between 1990 and 1996 had an on average 36 percent higher labour productivity than those firms which did not so. See [Fariñas and Martín-Marcos \(2010\)](#), [Kohler and Smolka \(2013\)](#), and [Wagner \(2011\)](#) for similar evidence from Spain and Germany.

<sup>4</sup>The presence of involuntary unemployment ensures that less productive firms paying low wages are able to attract workers.

well specified general equilibrium model. This is not self evident, since it follows from the analysis in [Grossman and Rossi-Hansberg \(2008\)](#) that domestic employment in newly offshoring firms can increase or decrease: Whereas the direct effect of relocating task production abroad is of course to reduce domestic employment in the firms concerned, there is a counteracting effect since the offshoring firms become more competitive, can therefore increase their output, and hence hire more workers for those tasks that remain onshore.<sup>5</sup> We show that in our model at intermediate levels of transport costs the overall effect is indeed to reduce employment in newly offshoring firms, and to increase employment in firms at the extremes of the wage distribution. However, we also show that job polarisation is a feature of our model that does not survive at very high or very low levels of offshoring costs.

By developing a new theoretical framework linking job polarisation and offshoring our paper is related to the two large literatures that have recently examined both phenomena. Regarding the literature on job polarisation, our paper is obviously linked to the large empirical literature cited in [Fn. 1](#). [Acemoglu and Autor \(2011\)](#) develop the now-standard theoretical framework on job polarisation, which explains the phenomenon by negative shocks affecting the demand for routine tasks performed by middle skilled individuals. Our theoretical explanation is complementary to the framework by [Acemoglu and Autor \(2011\)](#) in that it emphasizes the role of high-wage and low-wage firms rather than high-wage and low-wage tasks in the explanation of job polarisation. Evidence in favor of firm-specific wages for workers that are observationally equivalent is provided by [Beaudry et al. \(2012\)](#), [Card et al. \(2013\)](#), [Green \(2015\)](#), and [Tschopp \(2015\)](#). Our paper is also related to the large literature that studies offshoring to low-wage countries, including [Jones and Kierzkowski \(1990\)](#), [Feenstra and Hanson \(1996\)](#), [Kohler \(2004\)](#), [Rodriguez-Clare \(2010\)](#), [Grossman and Rossi-Hansberg \(2008\)](#) and [Acemoglu et al. \(2015\)](#). By using the continuum-of-tasks of [Grossman and Rossi-Hansberg \(2008\)](#) in a framework with heterogeneous firms, our paper is related to the papers by [Antràs and Helpman \(2004\)](#), [Antràs et al. \(2006\)](#), [Davidson et al. \(2008\)](#), [Groizard et al. \(2014\)](#) and [Egger et al. \(2015\)](#) that also look at offshoring in models with firm heterogeneity. However, none of these papers considers the question of job polarisation.

The remainder of the paper is organized as follows. In [Section 2](#) we introduce the technologies for production and offshoring and show how the productivity gain from offshoring is determined as a function of – yet unspecified – relative wage rates in the two countries. [Section 3](#) develops the labor market model and shows how relative firm performance within and between the groups of national and offshoring firms is determined in the presence of firm-specific wage rates. In

---

<sup>5</sup>Unlike exporting, which always is associated with increasing firm-level employment (cf. [Biscourp and Kramarz, 2007](#); [Hummels et al., 2014](#)), the import of intermediates has been found to increase (cf. [Hakkala and Huttunen, 2015](#); [Moser et al., 2015](#)) or decrease (cf. [Biscourp and Kramarz, 2007](#); [Mion and Zhu, 2013](#); [Hummels et al., 2014](#)) employment in importing firms. See [Hummels et al. \(2015\)](#) for a synopsis of the literature.

Section 4 we close the model by introducing factor market constraints and the occupational choice mechanism. We provide a graphical solution for our key endogenous variables, namely the general equilibrium values of the external firm and task margins, the productivity gain from offshoring and the ratio of average wages in the two countries. In Section 5 we derive the effects of a change in the variable costs of offshoring on firm-level domestic employment and analyse under which conditions job polarisation can materialise. Section 6 determines the effects of offshoring on economy-wide unemployment. Section 7 concludes with a summary of the most important results.

## 2 The model: basics

We set up a two-country model of offshoring, in which a country called South, that in equilibrium has low wages, provides labour input for the offshore task production of firms that are based in a high-wage country, called North. The North has an endogenous mass  $M$  of monopolistically competitive firms that produce horizontally differentiated goods. They face the demand function

$$x(v) = Ap(v)^{-\sigma}, \quad (1)$$

where  $A$  is a variable capturing market size,  $v$  indexes the firm, and  $\sigma > 1$  is the constant price elasticity of demand. Following [Acemoglu and Autor \(2011\)](#) we model production by firm  $v$  as the mapping of a continuum of tasks, uniformly distributed over the unit interval, into an output good using a Cobb-Douglas technology. Firms use the same technology in all tasks, with one unit of labour required for the production of one unit of each task. Denoting the production and usage of task  $\eta$  in firm  $v$  by  $q(v, \eta)$ , we specify the production function as

$$q(v) = \varphi(v) \exp \left[ \int_0^1 \ln q(v, \eta) d\eta \right], \quad (2)$$

where  $\varphi(v)$  is firm-specific productivity. The tasks differ in the costs involved in moving them offshore, and following [Grossman and Rossi-Hansberg \(2008\)](#) we assume that tasks can be ranked according to their offshorability. Specifically, the variable cost of putting task  $\eta$  offshore is given by

$$\hat{t}(\eta) = \tau(1 - \eta)^{-t}, \quad (3)$$

with  $\tau > 1$  and  $t > 0$ . Hence,  $\tau$  represents the offshoring cost for the task with the lowest such cost, and with the elasticity of  $\hat{t}(\eta)$  given by  $t\eta/(1 - \eta)$ , higher values of  $t$  are associated with a more steeply increasing offshoring cost schedule.

A profit maximising firm in our model has to make three decisions: It must decide whether or

not to offshore part of its task production (this determines the *extensive firm margin*), and – in case it chooses to offshore at all – it needs to decide for which tasks it wants to use this option (this is the *extensive task margin*), and how much of each task to produce (this is the *intensive task margin*). We focus on the determination of the task margins first and postpone the determination of the extensive firm margin until Section 4.

Since the offshoring cost schedule is strictly increasing in  $\eta$ , the extensive task margin is determined by the indifference condition  $\hat{t}[\hat{\eta}(v)]w^*(v) = w(v)$ , where  $w(v)$  and  $w^*(v)$  denote the wage rates paid by firm  $v$  at home and abroad, respectively, and  $\hat{\eta}(v)$  denotes both the marginal task offshored and the share of tasks offshored by firm  $v$ . Substituting for  $\hat{t}(\eta)$  from Eq. (3), the indifference condition for the extensive task margin can be written as

$$\hat{\eta}(v) = 1 - \left[ \frac{\tau w^*(v)}{w(v)} \right]^{\frac{1}{t}}, \quad (4)$$

where general equilibrium constraints introduced below ensure  $\hat{\eta}(v) > 0$ . Turning to the intensive task margin, we show in the appendix that unit cost minimisation results in

$$c(v) = \frac{w(v)}{\varphi(v) \exp \{I(v) \ln \kappa[\hat{\eta}(v)]\}}, \quad (5)$$

where  $I(v)$  is an indicator function, taking the value of one in the case of offshoring and the value of zero otherwise, and

$$\kappa[\hat{\eta}(v)] \equiv \left( \frac{1}{[1 - \hat{\eta}(v)] \exp[\hat{\eta}(v)]} \right)^t. \quad (6)$$

Thereby,  $\kappa[\hat{\eta}(v)]$  denotes the *productivity effect* that offshoring has on firm  $v$ , provided it has chosen both task margins in order to minimise unit costs. Intuitively, we have  $\kappa[\hat{\eta}(v)] = 1$  if  $\hat{\eta}(v) = 0$  and  $d\kappa[\hat{\eta}(v)]/d\hat{\eta}(v) > 0$ .

### 3 A rent-sharing model of wages

To capture the empirical regularity that larger, more productive firms pay higher wages, we propose a model in which wages are determined by a rent-sharing mechanism. Thereby, we build upon the idea of gift exchange, whose main assumptions are rooted in insights from psychological research (see Akerlof, 1982; Akerlof and Yellen, 1990), and consider a fair-wage effort mechanism as a motive for rent sharing in our setting. The fair-wage effort mechanism postulates a positive link between a firm's wage payment and a worker's effort provision.<sup>6</sup> More specifically, Akerlof and Yellen (1990)

---

<sup>6</sup>The fair-wage-effort hypothesis is supported by empirical evidence from laboratory experiments (cf. Fehr et al., 1993, 1998b,a), field experiments (cf. Gneezy and List, 2006; Kube et al., 2013), and natural experiments (cf. Lee and Rupp, 2007). See Charness and Kuhn (2011) as well as List and Rasul (2011) for surveys on laboratory and field experiments in labour economics. Fehr et al. (2009) provide a detailed survey over the extensive experimental

assume that workers exert full effort, normalised to one, if and only if they are paid at least the wage they consider fair,  $\hat{w}$ , while they reduce their effort proportionally if the wage falls short of the fair wage. This establishes the following functional relationship between effort provision  $e$  and wage rate  $w$ :  $e = \min\{w/\hat{w}, 1\}$ . In this model, labour costs per efficiency unit are the same for all  $w \leq \hat{w}$ , so that firms do not have an incentive to pay less than the fair wage as this does not further increase their profits. Furthermore, as in other efficiency wage models, firms set  $w$  to minimise their labour costs per efficiency unit, and hence they are not willing to offer more than the fair wage, unless this were necessary in order to attract the required number of workers. Therefore, any equilibrium with unemployment will be characterised by  $w = \hat{w}$ .

The worker's subjective evaluation of a fair wage depends on a firm-internal and a firm-external component. Following [Egger and Kreickemeier \(2012\)](#) and [Egger et al. \(2013\)](#), we associate the firm-external component with the income a worker can expect outside the job, which equals the average labour income in the economy. Denoting the unemployment rate of production workers by  $u$  and the average wage per production worker by  $\bar{w}$ , average labour income is given by  $(1-u)\bar{w}$ . The firm internal component is the operating profit of the firm,  $\pi(v)$ , which as shown below is directly linked to the income of the entrepreneur. It is this second component, which associates wage setting with a rent-sharing mechanism and makes the wage firm-specific in our model. Assuming that the fair wage is a weighted geometric average of the two components, in any equilibrium with unemployment, we obtain

$$w(v) \equiv \pi(v)^\theta [(1-u)\bar{w}]^{1-\theta}, \quad (7)$$

where  $\theta \in (0, 1)$  captures the strength of the rent-sharing motive in the fair-wage considerations of workers. We assume that workers in both countries have the same fairness considerations, implying that Eq. (7) holds analogously for the determination of  $w^*(v)$ , with the relevant average labour income given by  $(1-u^*)\bar{w}^*$ . Since total firm-level profits are a determinant of firm-level wages in both countries, our model features national as well as international rent-sharing.<sup>7</sup>

Using Eq. (7) to substitute for firm-level wages in Eq. (4), we can write the indifference condition for the extensive task margin as

$$\hat{\eta} = 1 - \tau^{\frac{1}{\iota}} \omega^{\frac{1-\theta}{\iota}} \quad \text{with} \quad \omega \equiv \frac{(1-u^*)\bar{w}^*}{(1-u)\bar{w}}. \quad (8)$$

Intuitively, since the international intra-firm wage differential  $\omega$  is the same for all firms, so are the evidence in favour of the fair-wage-effort hypothesis.

<sup>7</sup>Evidence supportive of international rent sharing within multinational firms is provided by [Dobbelaere \(2004\)](#), [Budd et al. \(2005\)](#), and [Martins and Yang \(2015\)](#). [Egger and Kreickemeier \(2013\)](#) develop a model with fair wage preferences as in Eq. (7) and show how international rent sharing can lead to a multinational wage premium in general equilibrium if countries are asymmetric.



share of offshored tasks  $\hat{\eta}$  and the productivity gain from offshoring  $\kappa(\hat{\eta})$ . Accounting for constant markup pricing,  $p(v) = [\sigma/(\sigma - 1)]c(v)$ , and considering  $\pi(v) = p(v)q(v)$ , we can then combine Eqs. (1) and (5) to derive the relative profit of two firms with differing productivities:

$$\frac{\pi(v')}{\pi(v'')} = \left\{ \frac{\varphi(v')}{\varphi(v'')} \frac{w(v'')}{w(v')} \frac{\exp [I(v') \ln \kappa(\hat{\eta})]}{\exp [I(v'') \ln \kappa(\hat{\eta})]} \right\}^{\sigma-1}. \quad (9)$$

A second link between relative profits and relative wages follows directly from the fair wage constraint (7):

$$\frac{w(v')}{w(v'')} = \left( \frac{\pi(v')}{\pi(v'')} \right)^{\theta}. \quad (10)$$

Eqs. (9) and (10) imply that relative wages and relative profits of two firms are fully described by these firms' productivity differential and their offshoring status. We therefore use this information to distinguish firms and can omit index  $v$  from now on. The relative (operating) profits and the relative wage of two firms with differing productivities but the same offshoring status  $i = \{d, o\}$  can be written as follows:

$$\frac{\pi_i(\varphi')}{\pi_i(\varphi'')} = \left( \frac{\varphi'}{\varphi''} \right)^{\xi} \quad \text{and} \quad \frac{w_i(\varphi')}{w_i(\varphi'')} = \left( \frac{\varphi'}{\varphi''} \right)^{\theta\xi}, \quad (11)$$

where  $\xi \equiv (\sigma - 1)/[1 + \theta(\sigma - 1)] > 0$ . As suggested by the empirical literature (cf. Blanchflower et al., 1996; Albaek et al., 1998; Idson and Oi, 1999; Dobbelaere, 2004; Lallemand et al., 2007; Frias et al., 2012), firms with higher productivity not only generate higher profits, but also pay higher wages. Moreover, the relative (operating) profits and relative wages of two firms with the same productivity but differing offshoring status can be computed as:

$$\frac{\pi_o(\varphi)}{\pi_d(\varphi)} = \kappa^{\xi} \quad \text{and} \quad \frac{w_o(\varphi)}{w_d(\varphi)} = \kappa^{\theta\xi}. \quad (12)$$

## 4 General equilibrium

Having derived links between the endogenous variables  $\kappa$ ,  $\hat{\eta}$  and the intra-firm wage differential  $\omega$ , we now introduce general equilibrium constraints to fully solve the model and analyse its comparative static properties. All goods produced under monopolistic competition serve as inputs into a homogeneous consumer good that is produced under perfect competition with a CES technology, as in Ethier (1982):  $Y = \left[ \int_{v \in V} q(v)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$ , with  $V$  being the set of available intermediates. We choose the final consumer good as the numeraire, and therefore the market size variable  $A$  in Eq. (1) is equal to  $Y$ , which denotes the total *value* and *volume* of the consumer good produced in the North. Trade is balanced, with North importing the task output produced in the South in exchange for the consumption good.

Following [Lucas \(1978\)](#), we assume that each monopolistically competitive firm is run by a single entrepreneur who acts as owner-manager and is the residual claimant of the firm, being paid the firm's operating profits net of fixed offshoring costs. The mass of owner-managers is endogenous, and it determines (and is equal to) the mass of firms  $M$ . In the North, being an owner-manager is one of three possible occupations. Alternatively, individuals could seek employment as production workers (with uncertain wage and employment prospects), or they could work in the perfectly competitive offshoring service sector.<sup>8</sup> The total supply of individuals in the North is exogenous and equal to  $N$ . Individuals are assumed to be risk-neutral, with each supplying one unit of labour inelastically. In the South, there are  $N^*$  individuals, and we assume that they have no choice but to seek employment as a production worker. Since the South has no entrepreneurs, all monopolistically competitive firms are headquartered in the North.<sup>9</sup> Agents in the North choose the occupation yielding the highest expected remuneration. As owner-manager of a firm, their remuneration depends on their entrepreneurial ability  $\varphi$ , which determines (and is equal to) the productivity of the firm they are running. Entrepreneurial abilities are distributed over interval  $[1, \infty)$  according to a Pareto function  $G(\varphi) = 1 - \varphi^{-k}$ , with shape parameter  $k > \max\{\xi, 1\}$ .<sup>10</sup> Individuals in the offshoring service sector are paid a wage  $s$ .

Assuming that the occupational choice is irreversible (for instance due to occupation-specific education), and that the least productive firm with productivity  $\varphi_d$  is a domestic producer (and hence does not have to pay the fixed cost of offshoring), the solution to the occupational choice problem in the North is given by the following indifference condition:

$$\pi_d(\varphi_d) = (1 - u)\bar{w} = s \tag{13}$$

The fixed cost of offshoring in our model is equal to  $s$ , and it results from the assumption that firms need the services of exactly one offshoring worker if they want to move part of their task production to the South. The profit gain from offshoring,  $\pi_o(\varphi) - \pi_d(\varphi) - s$ , is negative (by assumption) for the marginal firm with productivity  $\varphi = \varphi_d$ , strictly increasing in  $\varphi$ , according to [Eqs. \(11\)](#), and positive for firms with sufficiently high productivity. Hence, the marginal offshoring firm with productivity  $\varphi_o$  is characterized by indifference condition

$$\pi_o(\varphi_o) - \pi_d(\varphi_o) = s. \tag{14}$$

---

<sup>8</sup>Supportive evidence for the occupational choice mechanism in [Lucas \(1978\)](#) comes from [von Greiff \(2009\)](#), [Berglann et al. \(2011\)](#), who find that workers, who are displaced from their job or unemployed, are more likely to select into entrepreneurship. Focussing on active labour market policies (ALMP) in Germany, [Caliendo and Künn \(2011\)](#) find that subsidising start-ups for unemployed (unlike other ALMP) turned out to be quite successful.

<sup>9</sup>This extreme assumption is not necessary, but it provides the simplest possible means to ensure that the South is the low-wage country in equilibrium.

<sup>10</sup>This parameter constraint guarantees finite positive values of all variables of interest.

Together, Eqs. (11) to (14) imply a relationship between  $\kappa$  and the relative productivity  $\varphi_d/\varphi_o$ , which in turn is linked to the share of offshoring firms  $\chi$  according to  $\chi = (\varphi_d/\varphi_o)^k$ . Hence we get the indifference condition for the extensive firm margin

$$\kappa = \left(1 + \chi^{\frac{\xi}{k}}\right)^{\frac{1}{\xi}}. \quad (15)$$

As a last building block of our general equilibrium analysis, we now turn to the link between endowments and average wage rates of production workers, which eventually will give us a condition for the intra-firm wage differential  $\omega$ . For the South, the link between endowment  $N^*$  and the average wage rate is given by

$$(1 - u^*)\bar{w}^* = (1 - \gamma) \frac{\sigma - 1}{\sigma} \frac{Y}{N^*}, \quad (16)$$

where  $\gamma$  is the share of aggregate labour income accruing to the North, and we show in the appendix that it can be written as

$$\gamma \equiv \frac{1 + (1 - \hat{\eta})\chi - \hat{\eta}\chi^{\frac{k-\xi}{k}}}{1 + \chi}. \quad (17)$$

Intuitively, due to constant markup pricing the income of production workers is a fraction  $(\sigma - 1)/\sigma$  of total income, a fraction  $1 - \gamma$  of which accrues to workers in the South. For the North, the analogous equation is given by

$$(1 - u)\bar{w} = \gamma \frac{\sigma - 1}{\sigma} \frac{Y}{L}, \quad (18)$$

where the additional complication arises that the total supply of production workers in the North, denoted by  $L$ , is endogenous due to the occupational choice mechanism. In equilibrium  $L$  and  $N$  are linked via

$$L = \frac{\gamma k(\sigma - 1)}{k - \xi + \gamma k(\sigma - 1)} N, \quad (19)$$

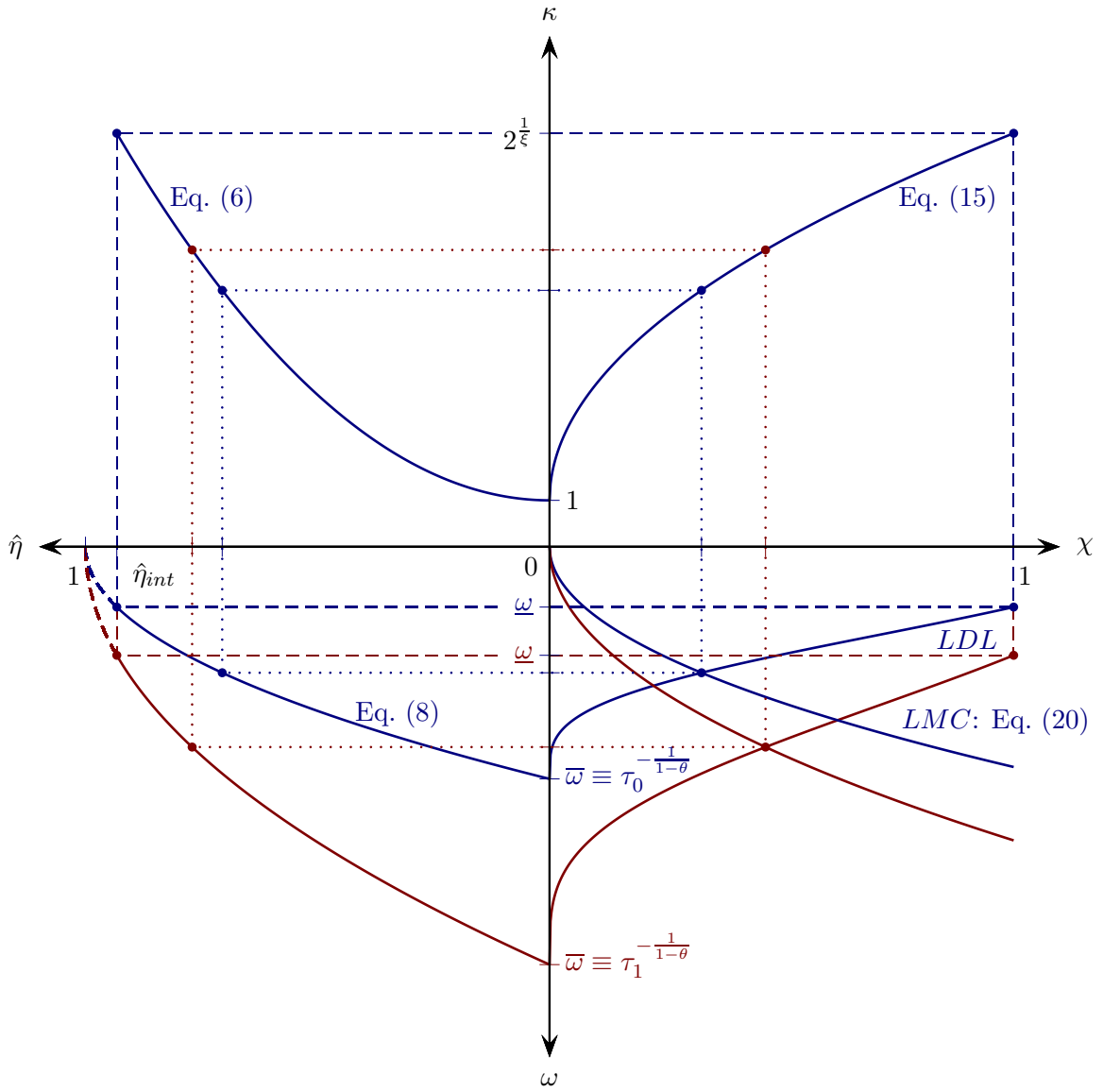
which follows, as we show in the appendix, from indifference condition (13) together with the resource constraint  $N = L + (1 + \chi)M$ . Combining Eqs. (16), (18), and (19) we get

$$\omega = \frac{(1 - \gamma)(\sigma - 1)k}{k - \xi + \gamma k(\sigma - 1)} \frac{N}{N^*}, \quad (20)$$

which we call the *labour market constraint (LMC)* since it links the three endogenous variables  $\hat{\eta}$ ,  $\chi$ , and  $\omega$  to the relative endowment  $N/N^*$ .

We can now solve for the four endogenous variables  $\omega$ ,  $\kappa$ ,  $\hat{\eta}$  and  $\chi$  using the condition for cost minimisation, Eq. (6), the indifference condition for the extensive task margin, Eq. (8), the indifference condition for the extensive firm margin, Eq. (15), and the labour market constraint, Eq. (20). It is convenient to illustrate the determination of equilibrium in Figure 1. The graphical

**Figure 1: General Equilibrium**



representations of Eqs. (6), (8), and (15) in Quadrants I to III are straightforward and do not require further elaboration. The upward sloping locus in Quadrant IV represents the *labour market constraint*, here drawn as a positive link between  $\chi$  and  $\omega$ , with  $\hat{\eta}$  adjusting endogenously according to the indifference condition for the extensive task margin given in Quadrant III. The downward sloping locus in Quadrant IV is implied by the loci in Quadrants I to III, giving combinations of  $\chi$  and  $\omega$  that are compatible with indifference along the extensive firm margin and the extensive task margin as well as with cost minimisation. Intuitively, a larger share of offshoring firms  $\chi$  must be the result of a higher productivity factor  $\kappa$ , which implies a higher share of offshored tasks  $\hat{\eta}$ , which in turn is compatible with indifference along the extensive task margin if the intra-firm wage differential  $\omega$  is smaller. Since it summarises demand conditions for host country labour, we label the downward sloping locus in Quadrant IV the *labour demand locus (LDL)*. Existence of an interior equilibrium requires that the two loci *LMC* and *LDL* intersect at  $\chi < 1$ . To characterize the parameter domain under which such an intersection point exists, we first define an exogenous upper bound of  $\hat{\eta}$ -values that are consistent with partitioning of firms by their offshoring status ( $\chi < 1$ ). This upper bound, which we denote by  $\hat{\eta}_{int}$ , is implicitly given by condition  $2 = (1 - \hat{\eta}_{int})^{-t\xi} \exp[-\hat{\eta}_{int}t\xi]$ , according to Eqs. (6) and (15), and lies in the unit interval. Under the parameter constraint

$$\frac{\hat{\eta}_{int}(\sigma - 1)k}{k - \xi + (1 - \hat{\eta}_{int})k(\sigma - 1)} \frac{N}{N^*} > (1 - \hat{\eta}_{int})^{\frac{t}{1-\theta}},$$

assumed in the following, it is guaranteed that *LMC* lies above *LDL* at  $\chi = 1$ , according to Eqs. (8) and (20). In this case, the two loci in Quadrant IV intersect in an interior equilibrium with  $\chi < 1$  for any finite value of  $\tau$ , and the implied values for  $\kappa$  and  $\hat{\eta}$  can be read off the loci in Quadrants I and III.

The effects of reduction in variable offshoring costs  $\tau$  can now be readily analysed using Figure 1. A decline in  $\tau$  from  $\tau_0$  to  $\tau_1$  implies that for a given intra-firm wage differential  $\omega$  the share of offshored tasks  $\hat{\eta}$  increases, and hence the indifference condition for the extensive task margin in Quadrant III of Figure 1 rotates outward. By implication the labour demand locus in Quadrant IV shifts outward as well: for a given intra-firm wage differential a larger share of the firm population chooses to offshore. The reduction in  $\tau$  also affects the labour market constraint: a higher value for  $\hat{\eta}$  implies that  $\omega$  stays constant if and only if  $\chi$  decreases, i.e. a larger share of offshored tasks must be counteracted by a smaller share of firms engaging in offshoring in order to be compatible with the labour market constraint at unchanged relative wages. Therefore the labour market constraint in Quadrant IV rotates leftward. Consequently, following a reduction in  $\tau$  the new labour market constraint and the new labour demand locus intersect at a higher level of  $\omega$ , and the implied

level of  $\chi$  is higher as well.<sup>11</sup> Since  $\chi$  goes up, so do  $\kappa$  and  $\hat{\eta}$ . A reduction in the variable costs of offshoring therefore leads to more offshoring along the extensive firm margin and along the extensive task margin. The productivity effect of offshoring increases, while at the same time the increased demand for host country workers leads to an increase in their relative average wage.

## 5 Good jobs, bad jobs, and the vanishing middle

Having characterized the general equilibrium in our offshoring model, we now look at the effect that offshoring has on firm-level domestic employment, acknowledging that this effect has two components: a *direct effect*, capturing the consequences that the decision of a firm to put a share  $\hat{\eta}$  of its tasks offshore has on its own domestic employment; an *indirect effect*, capturing adjustments in firm-level employment due to changes in general equilibrium variables, such as the occupational choice of agents. To determine the direct effect, we first note that due to the marginal cost saving, offshoring generates an output stimulus in the amount of  $q^o(\varphi)/q^n(\varphi) = \kappa^{\xi\sigma/(\sigma-1)}$ . This output stimulus raises the demand for domestic labour ceteris paribus and is commonly referred to by the term *firm-level productivity effect* in the literature. The productivity effect is counteracted by a negative *international relocation effect* as in the process of offshoring tasks previously manufactured at home are shifted abroad to be produced by foreign workers. This effect is reinforced by the firm's incentive to partially substitute tasks expensively produced at home with tasks performed abroad. The combined international relocation effect amounts to  $(1 - \hat{\eta})/\kappa$ . Accounting for Eq. (6), the direct effect of a firm's offshoring decision on domestic employment at the firm level can then be expressed in logarithmic form as follows:

$$\ln l_o(\varphi) - \ln l_d(\varphi) = [1 - t(1 - \theta)\xi] \ln(1 - \hat{\eta}) - t(1 - \theta)\xi\hat{\eta} \equiv \lambda(\hat{\eta}) \quad (21)$$

The firm-level employment effect is the same for all producers choosing to offshore and it depends on the share of offshored tasks. Low levels of  $\hat{\eta}$  are associated with a high offshoring cost parameter, and for a high  $\tau$  the marginal cost saving from offshoring is small, making offshoring an attractive choice only for the most productive producers. Since more productive firms employ more workers than their less productive competitors, the international relocation effect is, for a given  $\hat{\eta}$ , relatively strong if  $\tau$  is high. At the same time, the firm-level productivity effects is small in this case, because the cost saving from offshoring is small. Hence, the international relocation effect dominates the firm-level productivity effect, so that domestic employment is reduced in offshoring firms if  $\hat{\eta}$  is

---

<sup>11</sup>The curves drawn in Quadrants I and II establish a positive link between  $\chi$  and  $\hat{\eta}$ , and this is sufficient for  $d\chi/d\tau < 0$ . To see this we can recollect from above that the rightward shift of the labour demand locus is induced by an increase in  $\hat{\eta}$ , and making use of the positive link between  $\hat{\eta}$  and  $\chi$  established by the curves in Quadrants I and II, we can thus safely conclude that  $\chi$  must increase along with  $\omega$ , when  $\tau$  declines.

small. At lower levels of  $\tau$  the firm-level productivity effects becomes stronger, because the marginal cost of previously offshored tasks falls if  $\tau$  decreases and because offshoring becomes attractive for a wider span of tasks. At the same time the international relocation effect becomes less pronounced because newly offshoring firms at lower levels of  $\tau$  have a smaller workforce to begin with and because newly offshored tasks of incumbent offshoring firms lead to less job losses *ceteris paribus*, as these firms have already substituted cheap foreign tasks for expensive domestic ones, leaving relatively few workers in the remaining domestic activities. The larger is  $t$  the stronger is the cost saving involved in an increase of  $\hat{\eta}$  (see above) and the stronger the firm-level productivity gain therefore gets for a given increase in  $\hat{\eta}$ .

Under the parameter constraint

$$t > \frac{\ln 2}{\xi [(1 - \theta) \ln 2 - 1 + 2^{-(1-\theta)}]} \equiv t_1(\theta), \quad \text{with } t_1'(\theta) > 0, \quad (22)$$

$\lambda(\hat{\eta})$  is u-shaped and we can define two critical values of  $\hat{\eta}$ , namely  $\hat{\eta}_{tm}$ , and  $\hat{\eta}_{fm}$ , with  $0 < \hat{\eta}_{tm} < \hat{\eta}_{fm} < \hat{\eta}_{int}$  (see the appendix). Let us first look at  $\hat{\eta}_{tm}$ , which gives the value of  $\hat{\eta}$  that minimises  $\lambda(\hat{\eta})$ . For all  $\hat{\eta} < \hat{\eta}_{tm}$ , incumbent offshoring firms reduce their domestic employment if the offshoring cost parameter falls and thus the share of offshored tasks increases. The opposite is true if  $\hat{\eta} > \hat{\eta}_{tm}$ . In this case, incumbent offshoring firms increase their domestic employment if the offshoring cost parameter falls and thus the share of offshored tasks increases. These firm-level domestic employment effects refer to adjustments at the extensive task margin and are non-monotonic. The second adjustment margin important for understanding employment effects of offshoring at the firm level is the extensive firm margin, which captures the domestic employment effect associated with entry of newly offshoring firms if  $\tau$  declines. We find that newly offshoring firms lower their domestic employment if  $\hat{\eta} < \hat{\eta}_{fm}$ , whereas they increase their domestic employment if  $\hat{\eta} > \hat{\eta}_{fm}$ .

The employment adjustments at the two margins add up to the direct firm-level domestic employment effect of offshoring, which are instrumental to understand the indirect effects materialising in general equilibrium. At high levels of  $\tau$  and hence low levels of  $\hat{\eta}$ , offshoring destroys domestic jobs at both the extensive task and the extensive firm margin. This lowers expected domestic wage income  $(1 - u)\bar{w}$  relative to the profit income of the marginal entrepreneur  $\pi_d(\varphi_d)$  and prompts some agents to leave the pool of production workers and become owner-managers or offshoring service workers. Hence, labour supply  $L$  falls, whereas the service sector expands and the mass of entrepreneurs increases. At lower levels of  $\tau$  and intermediate levels of  $\hat{\eta}$  – in particular for  $\hat{\eta} \in (\hat{\eta}_{tm}, \hat{\eta}_{fm})$  – job destruction of newly offshoring firms is counteracted by job creation in incumbent offshoring firms, with the latter counteracting the incentives of agents to leave the pool

of production workers. Whereas labour supply still shrinks and the service sector further expands in this case, the effects on the mass of entrepreneurs is not clearcut any longer. Finally, at low levels of  $\tau$  and thus a large share of offshorable tasks  $\hat{\eta} > \hat{\eta}_{fm}$ , offshoring leads to creation of new domestic jobs at the extensive and the intensive margin. In this case, both labour supply and the mass of entrepreneurs must decrease, whereas the service sector expands in response to a decline in  $\tau$  and thus increase in  $\hat{\eta}$ .<sup>12</sup>

The indirect firm-level domestic employment effect of offshoring is directly linked to the change in the mass of entrepreneurs and thus the domestic labour demand effect triggered by changes in the occupational choice of agents. We can determine this effect by looking at equation

$$\mu(\hat{\eta}) \equiv -\ln \left\{ (k - \xi)[1 + \chi(\hat{\eta})] + k(\sigma - 1) \left[ 1 + (1 - \hat{\eta})\chi(\hat{\eta}) - \hat{\eta}\chi(\hat{\eta})^{\frac{k-\xi}{k}} \right] \right\} \quad (23)$$

with  $\chi(\hat{\eta}) = \{(1 - \hat{\eta})^{-t\xi} \exp[-\hat{\eta}t\xi] - 1\}^{k/\xi}$ , according to Eqs. (6) and (15). Thereby,  $\mu(\hat{\eta})$  represents a monotonic transformation of  $\ln(M/N)$ :  $\mu(\hat{\eta}) = \ln(M/N) - \ln(k - \xi)$ . In the appendix, we present derivation details and formally show that, with  $t > t_1(\theta)$ ,  $\mu(\hat{\eta})$  is hump-shaped and has a unique maximum in interval  $(0, \hat{\eta}_{fm})$ , which we denote by  $\hat{\eta}_{pol}^1$ . This lends support to our intuitive discussion above, regarding the direct effects of offshoring on firm-level domestic employment and its consequences for firm entry and exit.

With the direct and indirect firm-level domestic employment effects at hand, we are now equipped to analyse in detail how these two effects interact in establishing job polarisation. For this purpose, we first recollect from Section 3 that rent sharing generates firm-level wage differences, implying that our model gives a clear distinction between good and bad jobs. Good jobs are jobs in high-productivity firms, as these are jobs that offer high wages. Bad jobs, on the other hand, are jobs in low-productivity firms, because these firms pay low wages. Intuitively, job polarisation therefore evolves in our model if for some intermediate levels of  $\hat{\eta}$  offshoring creates new jobs in low-productivity domestic firms as well as in high-productivity incumbent offshoring firms, whereas it destroys at the same time jobs in newly offshoring firms featuring medium levels of productivity.

---

<sup>12</sup>Firm exit and the associated decline in the mass of entrepreneurs is enforced by the resource constraint. Combining the fair wage constraint in Eq. (7) with the indifference condition in Eq. (13), we find that  $w_d(\varphi_d) = \pi_d(\varphi_d)$  and, in view of constant markup pricing,  $l_d(\varphi_d) = \sigma - 1$ . Hence, an expansion of  $M$  and thus a decline in  $\varphi_d$  increases employment in domestic and even more so in offshoring firms, implying that labour supply would have to increase in order to restore  $(1 - u)\bar{w} = \pi_d(\varphi_d)$ . However, since for a given share of offshoring firms, a larger  $M$  requires an expansion of the service sector, this is inconsistent with a binding resource constraint. A formal proof for the effects of offshoring on labour supply, the size of the service sector, and the mass of entrepreneurs can be found in Appendix A.5



Computing the employment levels of domestic producers and offshoring firms (in logs), we obtain:<sup>13</sup>

$$\ln l_d(\varphi) = \sigma - 1 + \frac{(1 - \theta)\xi}{k} \ln(k - \xi) + \frac{(1 - \theta)\xi}{k} \mu(\hat{\eta}) + (1 - \theta)\xi \ln \varphi, \quad (24)$$

$$\ln l_o(\varphi) = \sigma - 1 + \frac{(1 - \theta)\xi}{k} \ln(k - \xi) + \frac{(1 - \theta)\xi}{k} \mu(\hat{\eta}) + \lambda(\hat{\eta}) + (1 - \theta)\xi \ln \varphi, \quad (25)$$

respectively. From Eq. (24), we can infer that offshoring induces domestic producers to create new jobs if and only if  $\mu'(\hat{\eta}) > 0$ , which is equivalent to  $\hat{\eta} < \hat{\eta}_{pol}^1$ . This establishes an upper bound for the  $\hat{\eta}$ -interval that allows for job polarisation. To determine the lower bound of this interval, we have to look at Eq. (25). In the appendix, we show that for very low levels of  $\hat{\eta}$  the direct domestic employment effect dominates the indirect domestic employment effect, leading to job destruction in incumbent offshoring firms. We further know from Eq. (21) that – provided  $t > t_1(\theta)$  – the direct domestic employment effect leads to a creation of new domestic jobs in incumbent offshoring firms if  $\hat{\eta} > \hat{\eta}_{tm}$  and from Eq. (23) we can infer that this employment stimulus is augmented by the indirect firm-level employment effect if at the same time  $\hat{\eta} < \hat{\eta}_{pol}^1$ . In the appendix, we study in detail whether these two conditions are mutually excluding. We find that under the additional parameter constraint

$$1 - \frac{(\sigma - 1)[k(1 - 2\theta) - \xi(1 - \theta)]}{k - \xi} < 0, \quad (26)$$

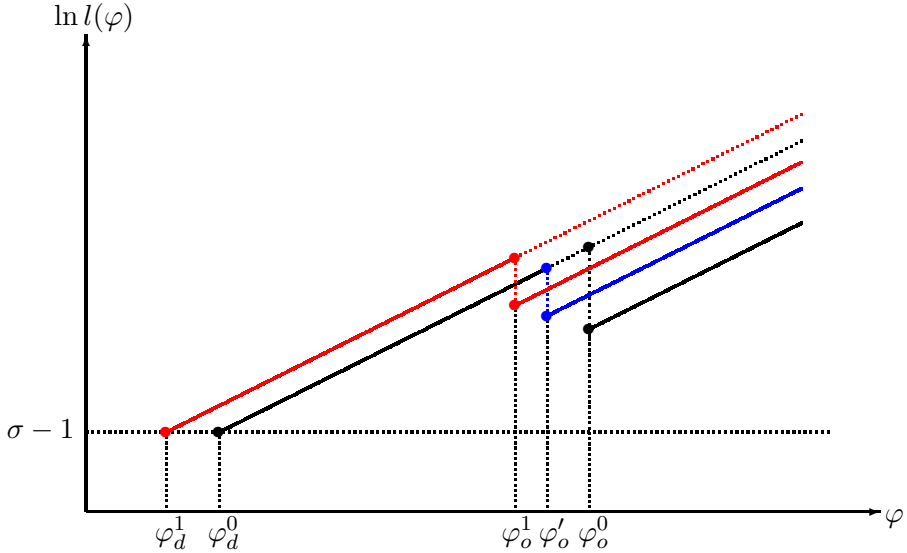
there exists a unique  $t_2(\theta)$ , with  $t_2'(\theta) > 0$ , which is implicitly defined by  $\hat{\mu}'(\hat{\eta}_{tm}) = 0$  and has the following interpretation:  $\hat{\eta}_{pol}^1 > (=)\hat{\eta}_{tm}$  if  $t > (=)t_2(\theta)$  and  $\hat{\eta}_{pol}^1 < \hat{\eta}_{tm}$  otherwise. This determines the parameter domain, we are looking at in the subsequent analysis. More specifically, we focus on the parameter domain confined by (26) and consider  $t > \max\{t_1(\theta), t_2(\theta)\}$ .<sup>14</sup> In the appendix, we show that in this case there exists a unique  $\hat{\eta}_{pol}^0 < \hat{\eta}_{tm}$ , such that  $dl_o(\varphi)/d\hat{\eta} = 0$  if  $\hat{\eta} = \hat{\eta}_{pol}^0$  and  $dl_o(\varphi)/d\hat{\eta} > 0$  for all  $\hat{\eta} \in (\hat{\eta}_{pol}^0, \hat{\eta}_{pol}^1)$ . Hence,  $(\hat{\eta}_{pol}^0, \hat{\eta}_{pol}^1)$  characterises the interval, in which a (small) increase in the share of offshored tasks leads to job polarisation. Offshoring in this case increases employment in both low-productivity domestic firms and high-productivity incumbent offshoring firms, whereas it destroys jobs in newly offshoring firms, since the direct discrete job destruction effect dominates the indirect marginal job creation effect.

Figure 2 depicts the case of job polarisation, i.e. the effects that a small increase in  $\hat{\eta}$  within interval  $(\hat{\eta}_{pol}^0, \hat{\eta}_{pol}^1)$  has on firm-level domestic employment, graphically. Holding general equilibrium

<sup>13</sup>Combining Eqs. (1), (5) and (11), and accounting for  $p_i(\varphi) = [\sigma/(\sigma - 1)]c_i(\varphi)$ ,  $q_d(\varphi) = \varphi l_d(\varphi)$ , we can compute  $l_d(\varphi) = (\varphi/\varphi_d)^{(1-\theta)\xi} l_d(\varphi_d)$ . Combining the indifference condition in (13) with the fair-wage constraint in (7) – applied for the marginal firm – gives  $\pi_d(\varphi_d) = w_d(\varphi_d)$ . Noting that constant markup pricing establishes  $(\sigma - 1)\pi_d(\varphi_d) = w_d(\varphi_d)l_d(\varphi_d)$ , we then obtain  $l_d(\varphi_d) = \sigma - 1$ . Substituting Eq. (19) for  $L$  in the resource constraint  $N = L + (1 + \chi)M$ , solving the resulting expression for  $M$ , and using this solution in  $\varphi_d = (M/N)^{-1/k}$  then allows us to compute Eq. (24), whereas Eq. (25) follows from combining Eqs. (21) and (24).

<sup>14</sup>The ranking of  $t_1(\theta)$  and  $t_2(\theta)$  depends on the size of the other model parameters and can change in  $\theta$ . For instance, setting  $k = \sigma = 4$ , we find that  $t_2(\theta) < t_1(\theta)$  for  $\theta = 0.1$  and  $t_1(\theta) < t_2(\theta)$  for  $\theta = 0.2$ .

variables and the occupational choice of agents constant, newly offshoring firms from interval  $(\varphi'_o, \varphi_o^0]$  reduce their domestic employment, because the international relocation effect dominates the firm-level productivity effect as long as  $\hat{\eta} < \hat{\eta}_{fm}$ . However, incumbent offshoring firms with productivity  $\varphi > \varphi_o^0$  expand their domestic workforce due to  $\hat{\eta} > \hat{\eta}_{tm}$  and they do so, despite an increasing fraction of tasks they put offshore. This direct firm-level domestic employment effects are captured by the shift of the employment profile from the black schedule to the blue schedule in Figure 2. Since  $\hat{\eta} < \hat{\eta}_{pol}^1$  has been assumed, new firms enter at the lower bound of the productivity distribution and all domestic and offshoring firms will expand their workforce, leading to the creation of new domestic jobs. These indirect (general equilibrium) effects are captured by the shift of the employment profile to the red schedule in Figure 2. Offshoring leads to job polarisation in this case, because jobs are destroyed in interval  $(\varphi_o^1, \varphi_o^0)$ , whereas new jobs are created in the other productivity domains.



**Figure 2:** *The case of job polarisation*

We summarize the insights from our analysis in this section in the following proposition.

**Proposition 1** *Under the parameter constraint in (26) and the additional assumption that  $t > \max\{t_1(\theta), t_2(\theta)\}$ , offshoring in our model generates job polarisation if  $\hat{\eta}$  increases within interval  $(\hat{\eta}_{pol}^0, \hat{\eta}_{pol}^1)$ .*

**Proof** Analysis in the text and formal proof in the appendix.

A key insight from the analysis above is that offshoring does not have the same effects on job creation and job destruction in early and late stages of globalisation. More specifically, the domestic jobs that are destroyed, when offshoring is an exception and only attractive for firms with the highest productivity levels, are good jobs that offer high wages. However, when offshoring becomes

common practice among producers with high and low productivity, the domestic jobs destroyed by offshoring are ones from lower ranks of the wage distribution. Our analysis therefore indicates that job polarisation is not a technical peculiarity, but rather a natural phenomenon that can be associated with a medium stage in the offshoring era.

## 6 Unemployment of production workers

Whereas the focus of the previous section was on the domestic employment effects at the firm level, we now look at the implications for economy-wide employment of production workers. This is of interest, because rent sharing between entrepreneurs and workers makes the labour market in the manufacturing sector imperfectly competitive and thereby leads to involuntary unemployment of these workers. To see this, we can note that the fair-wage constraint implies  $w_d(\varphi_d) = \pi_d(\varphi_d)$ , which together with the indifference condition for the marginal entrepreneur leads to

$$u = 1 - \frac{w_d(\varphi_d)}{\bar{w}}. \quad (27)$$

Whenever the average wage of employed production workers is higher than the wage paid by the marginal firm, which in turn is the case whenever there is firm-level rent sharing, this is accompanied in equilibrium by a strictly positive rate of unemployment of production workers,  $u > 0$ . In the appendix, we show that using Eq. (27), unemployment rate  $u$  can be expressed as a function of  $\hat{\eta}$ :

$$u = \frac{\theta\xi + (k - \xi) [1 - \delta(\hat{\eta})]}{\theta\xi + (k - \xi)}, \quad (28)$$

with

$$\delta(\hat{\eta}) = \frac{1 + [(1 - \hat{\eta})^{-t\xi} \exp[-\hat{\eta}t\xi] - 1]^{\frac{k-(1-\theta)\xi}{\xi}} [(1 - \hat{\eta})^{1-t(1-\theta)\xi} \exp[-\hat{\eta}t(1-\theta)\xi] - 1]}{1 + [(1 - \hat{\eta})^{-t\xi} \exp[-\hat{\eta}t\xi] - 1]^{\frac{k-\xi}{\xi}} [(1 - \hat{\eta})^{1-t\xi} \exp[-\hat{\eta}t\xi] - 1]}. \quad (29)$$

It is easily confirmed that  $\delta(0) = 1$ , and therefore the unemployment rate of production workers  $u$  is lower in an equilibrium with offshoring than in autarky if  $\delta(\hat{\eta}) > 1$  and higher than in autarky if  $\delta(\hat{\eta}) < 1$ . In the appendix, we formally show that there exists a unique  $\hat{\eta}_u < \hat{\eta}_{int}$ , such that  $\delta(\hat{\eta}) >, =, < 1$  if  $\hat{\eta}_u >, =, < \hat{\eta}$ . This implies that offshoring decreases the unemployment rate of production workers at early stages (high levels of  $\tau$ ), whereas it increases the unemployment rate of production workers at later stages (low levels of  $\tau$ ).

An intuition for this result can be inferred from Eq. (27) and our insights from the previous subsection regarding the effects of offshoring on the creation/destruction of good and bad jobs. For an increase in the share of offshorable tasks  $\hat{\eta}$  and the share of offshoring firms  $\chi$  from zero

to a small positive level the international relocation effect dominates the firm-level productivity effect in Eq. (21), and since the direct dominates the indirect firm-level domestic employment effect offshoring destroys good jobs in high-productivity firms, offering high wages. This together with the fact that new low-productivity firms enter leads to a decrease of the average wage paid by firms relative to the wage paid by the marginal firm. However, this is only compatible with indifference of the marginal entrepreneur between occupations if unemployment of production workers decreases as well. The effect of a marginal increase in offshoring on  $u$  is reversed at later stages of offshoring (and thus high levels of  $\hat{\eta}$  and  $\chi$ ), since now incumbent offshoring firms create additional high-wage jobs (whereas domestic producers destroy low-wage jobs) pushing up the average wage relative to the wage paid by the marginal firm. This is only compatible with indifference of the marginal entrepreneur between the available occupations if unemployment of production workers increases as well.

Whereas changes in the unemployment rate of production workers  $u$  shed light on how the employment prospects in the manufacturing sector change due to offshoring, they capture only part of the effect that offshoring exerts on the overall employment of resources in the North. More specifically, adjustments in  $u$  do not account for changes in the endogenous supply of production workers  $L$  triggered by adjustments in the occupational choice of agents.<sup>15</sup> From our analysis in the previous section we know that the relationship between offshoring cost parameter  $\tau$  and labour supply  $L$  is monotonically positive, implying that  $L$  falls when  $\hat{\eta}$  and  $\chi$  increase. To obtain a measure of economy-wide unemployment, we multiply the unemployment rate of production workers by the share of agents seeking employment as production workers:  $U = uL/N$ . Accounting for Eqs. (19) and (27), we can express the economy-wide rate of unemployment under offshoring as  $U = \Lambda(\hat{\eta})U^a$ , where

$$\Lambda(\hat{\eta}) \equiv \frac{\theta\xi + (k - \xi) [1 - \delta(\hat{\eta})]}{\theta\xi} \frac{[k - \xi + k(\sigma - 1)]\gamma}{k - \xi + k(\sigma - 1)\gamma}, \quad (30)$$

and  $U^a$  is the economy-wide rate of unemployment under autarky (superscript  $a$ ). The first fraction of  $\Lambda(\hat{\eta})$  is equal to  $u/u^a$  and the second fraction is equal to  $L/L^a$ . Unemployment rate  $U$  is lower with  $\hat{\eta} > 0$  than with  $\hat{\eta} = 0$  if  $\Lambda(\hat{\eta}) < 1$ , while the opposite is true if  $\Lambda(\hat{\eta}) > 1$ . Since the labour supply effect works unambiguously in favour of a reduction in overall unemployment, and the unemployment rate of production workers decreases when the offshoring cost parameter is high, and hence only few firms find it attractive to offshore, economy-wide unemployment unambiguously decreases at early stages of offshoring.

---

<sup>15</sup>The importance of occupational choice for understanding how a country's labour market absorbs the consequences of trade and offshoring has recently been pointed out by [Liu and Trefler \(2011\)](#) and [Artuç and McLaren \(2012\)](#).

At later stages, the positive effect of a decline in the supply of production workers is counteracted by an increase in the unemployment rate of production workers  $u$ , and offshoring may lead to a higher economy-wide rate of unemployment  $U$ . Intuitively, if the shape parameter  $t$  of the offshoring cost function is large, the firm-level productivity effect arising from the cost saving for all inframarginal tasks put offshore is large, whereas the international relocation effect arising from the replacement of domestic by foreign workers in offshored tasks is small, because only a small fraction of tasks are put offshore at low levels of  $\tau$  in this case. Therefore, the domestic job creation of incumbent offshoring firms (relative to domestic producers) at later stages of offshoring is sizable, and since offshoring firms are high-productivity firms that offer good jobs with high wage payments to their workforce, this leads to a strong increase in the average wage paid in the manufacturing industry relative to the wage paid by the marginal producer. To restore indifference of the marginal entrepreneur between the different occupations available in the North, the increase in the unemployment rate of production workers in the North must also be pronounced and therefore can dominate the decline in  $L$  at later stages of offshoring (low levels of  $\tau$ ) in this case.

The following proposition summarises our insights regarding the effects of offshoring on economy-wide unemployment.

**Proposition 2** *Unemployment in the North decreases with the share of offshoring firms and the share of offshored tasks at high levels of the offshoring cost parameter  $\tau$  (and thus low levels of  $\hat{\eta}$  and  $\chi$ ). Under the sufficient condition*

$$t > t_U \equiv \frac{\ln 2}{\xi(1 - \zeta - \ln \zeta)}, \quad \zeta \equiv \frac{\theta\xi}{\theta\xi + (1 - 2^{-\theta})[k\sigma - \xi]} \quad (31)$$

*the effect is reversed as more firms offshore a larger share of their tasks, and unemployment surpasses its autarky level if offshoring becomes common practice among high- as well as low-productivity producers (i.e. if  $\chi$  close to one and thus  $\hat{\eta}$  close to  $\hat{\eta}_{int}$ ).*

**Proof** See the appendix.

Condition  $t > \max\{t_1(\theta), t_2(\theta)\}$  is not sufficient for the parameter constraint in (31) – even though numerical exercises indicate that  $t > \max\{t_1(\theta), t_2(\theta)\}$  is only consistent with  $t < t_U$  if both  $k$  and  $\sigma$  are small. For the empirically relevant parameter domain with  $k \geq 2$  (cf. Corcos et al., 2012),  $\hat{\eta}_{int} < 1/2$  is sufficient for the economy-wide rate of unemployment in the neighbourhood of  $\chi = 1$  to be higher than under autarky.<sup>16</sup> Noting from the studies of Blinder (2006) and Blinder and Krueger (2013) that only about 25 percent of the jobs in the US are vulnerable to offshoring – as

---

<sup>16</sup>We show this result in a footnote of the proof of Proposition 2.

they perform routine tasks and do not require face-to-face communications – the non-monotonicity in the effect of offshoring on economy-wide unemployment described in Proposition 2 is not just a technical peculiarity but provides relevant insights regarding the relationship between offshoring and unemployment in the real world.

## 7 Conclusion

Empirical studies have found job polarisation to be pervasive across many developed economies over the past two decades. The leading theoretical explanation focuses on wage differences between jobs that differ in their composition of tasks in combination with negative shocks to medium-wage jobs intensively using routine tasks, which could result from biased technological change or from offshoring. In this paper, we have suggested that wage differences between firms that are linked to firm productivity can be the key ingredient in a mechanism complementary to the established one that links increasing job polarisation to the increased offshoring of tasks to low-wage countries. As a consequence of a decrease in variable offshoring costs, more firms choose offshoring in our model, and they choose it for a wider range of tasks. We show that at intermediate levels of offshoring costs this leads to job polarisation, since the following three things happen: newly offshoring producers, which have intermediate productivity, reduce the number of domestic jobs, while incumbent offshoring producers with high productivity and low-productivity non-offshoring firms increase the number of domestic jobs. At least one of these three effects fails to materialize if offshoring costs are very high or very low, and hence our model also specifies conditions under which job polarisation can be expected to occur.

Wage differences between firms in our model are a result of a labour market imperfection that also leads to involuntary unemployment among production workers, and we show that changes in offshoring costs lead to a non-monotonic general equilibrium change in involuntary unemployment. Accounting for the endogenous adjustment in the supply of production workers due to an occupational choice mechanism, we show that lower offshoring costs lead to lower economy-wide unemployment when offshoring costs are high, whereas the opposite happens when offshoring costs are already low.

To the best of our knowledge, this paper provides the first attempt to highlight movements of workers between firms that offer different wages as an important channel through which offshoring can generate job polarisation. We hope that our model can provide guidance for future empirical research to shed light on the importance of this new channel relative to the routinisation hypothesis that points to the movement of workers between jobs with differing task compositions as the main driver for job polarisation.

## References

- ABOWD, J. M., F. KRAMARZ, AND D. N. MARGOLIS (1999): “High Wage Workers and High Wage Firms,” *Econometrica*, 67, 251–334. 1
- ACEMOGLU, D. AND D. AUTOR (2011): “Skills, Tasks and Technologies: Implications for Employment and Earnings,” in *Handbook of Labor Economics*, ed. by O. Ashenfelter and D. Card, North Holland/NL: Elsevier, vol. 4, 1043–1171. 3, 4
- ACEMOGLU, D., G. GANCIA, AND F. ZILIBOTTI (2015): “Offshoring and Directed Technical Change,” *American Economic Journal: Macroeconomics*, 7, 84–112. 3
- AKERLOF, G. A. (1982): “Labor Contracts as Partial Gift Exchange,” *Quarterly Journal of Economics*, 97, 543–569. 5
- AKERLOF, G. A. AND J. L. YELLEN (1990): “The Fair Wage-Effort Hypothesis and Unemployment,” *Quarterly Journal of Economics*, 105, 255–283. 2, 5
- ALBAEK, K., M. ARAI, R. ASPLUND, E. BARTH, AND E. STROJER MADSEN (1998): “Measuring Wage Effects of Plant Size,” *Labour Economics*, 5, 425–448. 7
- ANTRÀS, P., L. GARICANO, AND E. ROSSI-HANSBERG (2006): “Offshoring in a Knowledge Economy,” *Quarterly Journal of Economics*, 121, 31–77. 3
- ANTRÀS, P. AND E. HELPMAN (2004): “Global Sourcing,” *Journal of Political Economy*, 112, 552–580. 3
- ARTUÇ, E. AND J. MCLAREN (2012): “Trade Policy and Wage Inequality: A Structural Analysis with Occupational and Sectoral Mobility,” Unpublished Manuscript, University of Virginia. 18
- AUTOR, D. H. AND D. DORN (2013): “The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market,” *American Economic Review*, 103, 1553–97. 1
- AUTOR, D. H., L. F. KATZ, AND M. S. KEARNEY (2006): “The Polarization of the U.S. Labor Market,” *American Economic Review*, 96, 189–194. 1
- (2008): “Trends in U.S. Wage Inequality: Revising the Revisionists,” *The Review of Economics and Statistics*, 90, 300–323. 1
- BEAUDRY, P., D. A. GREEN, AND B. SAND (2012): “Does industrial composition matter for wages? A test of search and bargaining theory,” *Econometrica*, 80, 1063–1104. 3
- BERGLANN, H., E. R. MOEN, K. RØED, AND J. F. SKOGSTRØM (2011): “Entrepreneurship: Origins and Returns,” *Labour Economics*, 18, 180–193. 8
- BERNARD, A. B., J. B. JENSEN, S. J. REDDING, AND P. K. SCHOTT (2007): “Firms in International Trade,” *Journal of Economic Perspectives*, 21, 105–130. 2
- (2012): “The Empirics of Firm Heterogeneity and International Trade,” *Annual Review of Economics*, 4, 283–313. 2
- BISCOURP, P. AND F. KRAMARZ (2007): “Employment, Skill Structure and International Trade: Firm-level Evidence for France,” *Journal of International Economics*, 72, 22–51. 2, 3
- BLANCHFLOWER, D. G., A. J. OSWALD, AND P. SANFEY (1996): “Wages, Profits, and Rent-Sharing,” *Quarterly Journal of Economics*, 111, 227–251. 7
- BLINDER, A. S. (2006): “Offshoring: The Next Industrial Revolution?” *Foreign Affairs*, 85, 113–128. 19



- BLINDER, A. S. AND A. B. KRUEGER (2013): “Alternative Measures of Offshorability: A Survey Approach,” *Journal of Labor Economics*, 31, S97–S128. 19
- BUDD, J. W., J. KONINGS, AND M. J. SLAUGHTER (2005): “Wages and International Rent Sharing in Multinational Firms,” *Review of Economics and Statistics*, 87, 73–84. 2, 6
- CALIENDO, M. AND S. KÜNN (2011): “Start-up Subsidies for the Unemployed: Long-term Evidence and Effect Heterogeneity,” *Journal of Public Economics*, 95, 311–331. 8
- CARD, D., J. HEINING, AND P. KLINE (2013): “Workplace Heterogeneity and the Rise of West German Wage Inequality,” *The Quarterly Journal of Economics*, 128, 967–1015. 1, 3
- CHARNESS, G. AND P. KUHN (2011): “Chapter 3 - Lab Labor: What Can Labor Economists Learn from the Lab?” Elsevier, vol. 4, Part A of *Handbook of Labor Economics*, 229 – 330. 5
- CORCOS, G., M. D. GATTO, G. MION, AND G. I. OTTAVIANO (2012): “Productivity and Firm Selection: Quantifying the ‘New’ Gains from Trade,” *Economic Journal*, 122, 754–798. 19
- DAVIDSON, C., S. J. MATUSZ, AND A. SHEVCHENKO (2008): “Outsourcing Peter to Pay Paul: High-Skill Expectations and Low-Skill Wages with Imperfect Labor Markets,” *Macroeconomic Dynamics*, 12, 463–479. 3
- DOBBELAERE, S. (2004): “Ownership, Firm Size and Rent Sharing in Bulgaria,” *Labour Economics*, 11, 165 – 189. 2, 6, 7
- DUSTMANN, C., J. LUDSTECK, AND U. SCHÖNBERG (2009): “Revisiting the German Wage Structure,” *The Quarterly Journal of Economics*, 124, 843–881. 1
- EGGER, H., P. EGGER, AND U. KREICKEMEIER (2013): “Trade, Wages, and Profits,” *European Economic Review*, 64, 332 – 350. 6
- EGGER, H. AND U. KREICKEMEIER (2012): “Fairness, Trade, and Inequality,” *Journal of International Economics*, 86, 184–196. 2, 6
- (2013): “Why Foreign Ownership May be Good for You,” *International Economic Review*, 54, 693–716. 6
- EGGER, H., U. KREICKEMEIER, AND J. WRONA (2015): “Offshoring Domestic Jobs,” *Journal of International Economics*, 97, 112–125. 3
- ETHIER, W. J. (1982): “National and International Returns to Scale in the Modern Theory of International Trade,” *American Economic Review*, 72, 389–405. 7
- FARIÑAS, J. C. AND A. MARTÍN-MARCOS (2010): “Foreign Sourcing and Productivity: Evidence at the Firm Level,” *World Economy*, 33, 482–506. 2
- FEENSTRA, R. C. AND G. H. HANSON (1996): “Foreign Investment, Outsourcing, and Relative Wages,” in *Political Economy of Trade Policy: Essays in Honor of Jagdish Bhagwati*, ed. by R. C. Feenstra, G. M. Grossman, and D. A. Irwin, MIT Press, Cambridge/MA, 89–127. 3
- FEHR, E., L. GOETTE, AND C. ZEHNDER (2009): “A Behavioral Account of the Labor Market: The Role of Fairness Concerns,” *Annual Review of Economics*, 1, 355–384. 5
- FEHR, E., A. KIRCHLER, ERICH WEICHBOLD, AND S. GÄCHTER (1998a): “When Social Norms Overpower Competition: Gift Exchange in Experimental Labor Markets,” *Journal of Labor Economics*, 16, pp. 324–351. 5
- FEHR, E., G. KIRCHSTEIGER, AND A. RIEDL (1993): “Does Fairness Prevent Market Clearing? An Experimental Investigation,” *The Quarterly Journal of Economics*, 108, 437–59. 5



- (1998b): “Gift Exchange and Reciprocity in Competitive Experimental Markets,” *European Economic Review*, 42, 1–34. 5
- FRIAS, J. A., D. S. KAPLAN, AND E. VERHOOGEN (2012): “Exports and Within-Plant Wage Distributions: Evidence from Mexico,” *American Economic Review*, 102, 435–40. 7
- GNEEZY, U. AND J. A. LIST (2006): “Putting Behavioral Economics to Work: Testing for Gift Exchange in Labor Markets Using Field Experiments,” *Econometrica*, 74, 1365–1384. 5
- GOOS, M. AND A. MANNING (2007): “Lousy and Lovely Jobs: The Rising Polarization of Work in Britain,” *The Review of Economics and Statistics*, 89, 118–133. 1
- GOOS, M., A. MANNING, AND A. SALOMONS (2009): “Job Polarization in Europe,” *American Economic Review*, 99, 58–63. 1
- (2014): “Explaining Job Polarization: Routine-Biased Technological Change and Offshoring,” *American Economic Review*, 104, 2509–26. 1
- GREEN, D. A. (2015): “Chasing After ‘Good Jobs’: Do They Exist and Does it Matter if They Do?” *Canadian Journal of Economics*, forthcoming. 3
- GROIZARD, J. L., P. RANJAN, AND A. RODRIGUEZ-LOPEZ (2014): “Offshoring and Jobs: The Myriad Channels of Influence,” *European Economic Review*, 72. 3
- GROSSMAN, G. M. AND E. ROSSI-HANSBERG (2008): “Trading Tasks: A Simple Theory of Offshoring,” *American Economic Review*, 98, 1978–1997. 1, 2, 3, 4
- GRUETTER, M. AND R. LALIVE (2009): “The Importance of Firms in Wage Determination,” *Labour Economics*, 16, 149 – 160. 1
- HAKKALA, K. N. AND K. HUTTUNEN (2015): “Displacing Tasks: Understanding the Employment Effects of Offshoring,” mimeo. 3
- HARRIGAN, J., A. RESHEF, AND F. TOUBAL (2015): “The March of the Techies: Technology, Trade, and Job Polarization in France, 1994–2007,” *Unpublished Manuscript, University of Virginia*. 1
- HOLZER, H. J., J. I. LANE, D. B. ROSENBLUM, AND F. ANDERSSON (2011): *Where Are All The Good Jobs Going? What National and Local Job Quality and Dynamics Mean for U.S. Workers*, New York: Russell Sage Press. 1
- HUMMELS, D., R. JØRGENSEN, J. MUNCH, AND C. XIANG (2014): “The Wage Effects of Offshoring: Evidence from Danish Matched Worker-Firm Data,” *American Economic Review*, 104, 1597–1629. 2, 3
- HUMMELS, D., J. R. MUNCH, AND C. XIANG (2015): “Offshoring and Labor Markets,” *under revision at the Journal of Economic Literature*. 3
- IDSON, T. L. AND W. Y. OI (1999): “Chapter 33 Firm Size and Wages,” Elsevier, vol. 3, Part B of *Handbook of Labor Economics*, 2165 – 2214. 1, 7
- JONES, R. W. AND H. KIERZKOWSKI (1990): “The Role of Services in Production and International Trade: A Theoretical Framework,” in *The Political Economy of International Trade: Essays in Honor of Robert E. Baldwin*, ed. by R. W. Jones and A. O. Krueger, Cambridge/UK: Basil Blackwell, 31–48. 3
- KELLER, W. AND H. UTAR (2015): “International Trade and Job Polarization: Evidence at the Worker Level,” mimeo, University of Colorado. 1

- KOHLER, W. (2004): “International Outsourcing and Factor Prices with Multistage Production,” *Economic Journal*, 114, C166–C185. 3
- KOHLER, W. AND M. SMOLKA (2013): “Global Sourcing and Firm Selection,” University of Tuebingen Working Papers in Economics and Finance 63. 2
- KUBE, S., M. A. MARÉCHAL, AND C. PUPPE (2013): “Do Wage Cuts Damage Work Morale? Evidence From A Natural Field Experiment,” *Journal of the European Economic Association*, 11, 853–870. 5
- LALLEMAND, T., R. PLASMAN, AND F. RYCX (2007): “The Establishment-size Wage Premium: Evidence from European Countries,” *Empirica*, 34, 427–451. 7
- LEE, D. AND N. G. RUPP (2007): “Retracting a Gift: How Does Employee Effort Respond to Wage Reductions?” *Journal of Labor Economics*, 25, 725–761. 5
- LIST, J. A. AND I. RASUL (2011): “Chapter 2 - Field Experiments in Labor Economics,” Elsevier, vol. 4, Part A of *Handbook of Labor Economics*, 103 – 228. 5
- LIU, R. AND D. TREFLER (2011): “A Sorted Tale of Globalization: White Collar Jobs and the Rise of Service Offshoring,” NBER Working Paper 17559. 18
- LUCAS, R. E. J. (1978): “On the Size Distribution of Business Firms,” *Bell Journal of Economics*, 9, 508–523. 2, 8
- MARTINS, P. S. AND Y. YANG (2015): “Globalised Labour Markets? International Rent Sharing across 47 Countries,” *British Journal of Industrial Relations*, 53, 664–691. 2, 6
- MION, G. AND L. ZHU (2013): “Import competition from and offshoring to China: A curse or blessing for firms?” *Journal of International Economics*, 89, 202–215. 2, 3
- MOSER, C., D. M. URBAN, AND B. WEDER DI MAURO (2015): “On the Heterogeneous Employment Effects of Offshoring: Identifying Productivity and Downsizing Channels,” *Economic Inquiry*, 53, 220–239. 2, 3
- PAUL, C. J. M. AND M. YASAR (2009): “Outsourcing, Productivity, and Input Composition at the Plant Level,” *Canadian Journal of Economics*, 42, 422–439. 2
- RODRIGUEZ-CLARE, A. (2010): “Offshoring in a Ricardian World,” *American Economic Journal: Macroeconomics*, 2, 227–258. 3
- SPITZ-OENER, A. (2006): “Technical Change, Job Tasks, and Rising Educational Demands: Looking Outside the Wage Structure.” *Journal of Labor Economics*, 24, 235 – 270. 1
- TSCHOPP, J. (2015): “Wage Formation: Towards Isolating Search and Bargaining Effects from the Marginal Product,” *Economic Journal*, forthcoming. 3
- VON GREIFF, J. (2009): “Displacement and Self-employment Entry,” *Labour Economics*, 16, 556–565. 8
- WAGNER, J. (2011): “Offshoring and Firm Performance: Self-selection, Effects on Performance, or Both?” *Review of World Economics*, 147, 217–247. 2
- ZWEIMÜLLER, J. AND R. WINTER-EBMER (1999): “Firm-Size Wage Differentials in Switzerland: Evidence from Job-Changers,” *American Economic Review*, 89, 89–93. 1

## A Appendix

### A.1 Derivation of Eq. (5)

In view of the Cobb-Douglas technology in Eq. (2) – which puts equal weights on all tasks – cost minimisation over  $q(v, \eta)$  gives the well-known result that expenditures of a firm are the same for all of its tasks. This links the production of task  $\eta < 1$  to the production of task 1. We have

$$q(v, \eta) = \begin{cases} \left(\frac{1-\eta}{1-\hat{\eta}(v)}\right)^{-t} q(v, 1) & \text{for all } \eta \leq \hat{\eta}(v) \\ q(v, 1) & \text{for all } \eta > \hat{\eta}(v) \end{cases}, \quad (\text{A.1})$$

where  $\hat{\eta}(v) = 0$  if firm  $v$  performs all of its tasks domestically and  $\hat{\eta}(v) > 0$ , otherwise. The total variable costs of producing intermediate goods output in quantity of  $q(v)$  can then be expressed as  $C(v) = q(v, 1)w(v)$ . According to Eq. (2), output of firm  $v$  can be written as  $q(v) = \varphi(v)q(v, 1) \exp\{I(v) \ln \kappa[\hat{\eta}(v)]\}$ , where  $\kappa[\hat{\eta}(v)]$  is given by Eq. (6) and  $I(v)$  is an indicator function with value 1 if the firm is engaged in offshoring and value 0, otherwise. The marginal cost in Eq. (5) is finally obtained when dividing  $C(v)$  by  $q(v)$ .

### A.2 Derivation of Eq. (17)

The global wage bill is the aggregate wage income of production workers in the North and the South and, in view of constant markup pricing, proportional to global operating profits:

$$(1-u)L\bar{w} + (1-u^*)N^*\bar{w}^* = N(\sigma-1) \int_{\varphi_d}^{\varphi_o} \pi_d(\varphi) dG(\varphi) + N(\sigma-1) \int_{\varphi_o}^{\infty} \pi_o(\varphi) dG(\varphi). \quad (\text{A.2})$$

Making use of Eqs. (11), (12), and (15), we can compute

$$(1-u)L\bar{w} + (1-u^*)N^*\bar{w}^* = M(\sigma-1)\pi_d(\varphi_d) \frac{k}{k-\xi} (1+\chi). \quad (\text{A.3})$$

In a similar vein, we can compute the wage bill accruing to domestic workers, according to

$$\begin{aligned} (1-u)L\bar{w} &= N(\sigma-1) \int_{\varphi_d}^{\varphi_o} \pi_d(\varphi) dG(\varphi) + \hat{\eta}N(\sigma-1) \int_{\varphi_o}^{\infty} \pi_o(\varphi) dG(\varphi) \\ &= M(\sigma-1)\pi_d(\varphi_d) \frac{k}{k-\xi} \left[1 + (1-\hat{\eta})\chi - \hat{\eta}\chi^{\frac{k-\xi}{k}}\right]. \end{aligned} \quad (\text{A.4})$$

Dividing Eq. (A.4) by Eq. (A.3) establishes  $\gamma$  in Eq. (17), which completes the proof.

### A.3 Derivation of Eq. (19)

Combining Eqs. (17) and (A.4) gives

$$M(1+\chi)\pi_d(\varphi_d) \frac{k(\sigma-1)\gamma}{k-\xi} = (1-u)L\bar{w}. \quad (\text{A.5})$$

Substituting  $\pi_d(\varphi_d) = (1-u)L\bar{w}$  from Eq. (13) and  $(1+\chi)M = N - L$  from the resource constraint, we obtain

$$(N - L)\frac{k(\sigma - 1)\gamma}{k - \xi} = L. \quad (\text{A.6})$$

Solving the latter for  $L$ , then establishes Eq. (19). This completes the proof.

#### A.4 The properties of $\lambda(\hat{\eta})$ and a characterisation of $t_1(\theta)$

Note first that  $\lambda(\hat{\eta}) \geq 0$  requires  $\ln(1 - \hat{\eta}) \geq t(1 - \theta)\xi [\ln(1 - \hat{\eta}) + \hat{\eta}]$ , according to Eq. (21). Moreover, combining Eqs. (6) and (15) gives  $-t\xi [\ln(1 - \hat{\eta}_{int}) + \hat{\eta}_{int}] = \ln 2$ . This establishes  $\ln(1 - \hat{\eta}_{int}) > -(1 - \theta)\ln 2$  as sufficient condition for  $\hat{\eta}_{fm} < \hat{\eta}_{int}$ . Combining  $\ln(1 - \hat{\eta}_{int}) > -(1 - \theta)\ln 2$  with  $-t\xi [\ln(1 - \hat{\eta}_{int}) + \hat{\eta}_{int}] = \ln 2$ , the sufficient condition for  $\lambda(\hat{\eta}_{int}) > 0$  can be reformulated as condition (22). Hence, if  $t > t_1(\theta)$ , there exists a unique  $\hat{\eta}_{fm} \in (0, \hat{\eta}_{int})$ , such that  $\lambda(\hat{\eta}) > 0$  if  $\hat{\eta} > \hat{\eta}_{int}$ ,  $\lambda(\hat{\eta}) = 0$  if  $\hat{\eta} = \hat{\eta}_{int}$ , and  $\lambda(\hat{\eta}) < 0$  if  $\hat{\eta} \in (0, \hat{\eta}_{int})$ . Furthermore, it is easily confirmed by differentiating  $\lambda(\hat{\eta})$ , that there exists a unique  $\hat{\eta}_{tm} = [t(1 - \theta)\xi]^{-1}$ , such that  $\lambda'(\hat{\eta}) >, =, < 0$  if  $\hat{\eta} >, =, < \hat{\eta}_{tm}$ . Evaluating  $\lambda(\hat{\eta})$  at  $\hat{\eta}_{tm} = 1/a$ , with  $a \equiv t(1 - \theta)\xi$ , gives

$$\lambda(\hat{\eta}_{tm}) = (1 - a) \ln \left( \frac{a - 1}{a} \right) - 1 \equiv \bar{\lambda}(a), \quad (\text{A.7})$$

which, in view of  $\bar{\lambda}'(a) > 0$  and  $\lim_{a \rightarrow \infty} \bar{\lambda}(a) = 0$ , is negative. This establishes  $\hat{\eta}_{tm} < \hat{\eta}_{fm}$  and completes the proof.

#### A.5 Derivation of Eq. (23) and characterisation of $\hat{\eta}_{pol}^1$

To determine  $M/N$ , we can substitute Eq. (19) for  $L$  in resource constraint  $N = L + (1 + \chi)M$ . Solving the resulting expression for  $M/N$  gives

$$\frac{M}{N} = \frac{k - \xi}{(1 + \chi)[k - \xi + \gamma k(\sigma - 1)]}. \quad (\text{A.8})$$

Taking logs and substituting into  $\mu(\hat{\eta}) = \ln(M/N) - \ln(k - \xi)$ , then gives  $\mu(\hat{\eta}) = -\ln[(k - \xi)(1 + \chi) + (1 + \chi)\gamma k(\sigma - 1)]$ . Substituting  $\gamma$  from Eq. (17), then establishes Eq. (23), with  $\chi(\hat{\eta})$  determined by Eqs. (6) and (15).

Regarding the effect of changes in  $\hat{\eta}$  on  $L$ ,  $\chi M$  and  $M$ , we can first note from Eq. (19) that  $dL/d\gamma > 0$ . Accounting for  $\gamma'_{\hat{\eta}} < 0$ ,  $\gamma'_{\chi} < 0$ , and  $\chi'(\hat{\eta}) > 0$  (see Figure 1),  $dL/d\hat{\eta} < 0$  is immediate. In a second step, we can note from Eq. (A.8) that

$$\frac{d(\chi M)}{d\hat{\eta}} = \frac{M}{1 + \chi(\hat{\eta})} \chi'(\hat{\eta}) - \frac{\chi(\hat{\eta})Mk(\sigma - 1)}{k - \xi + \gamma k(\sigma - 1)} \left[ \gamma'_{\hat{\eta}} + \gamma'_{\chi} \chi'(\hat{\eta}) \right]. \quad (\text{A.9})$$

Accounting for  $\gamma'_{\hat{\eta}} < 0$ ,  $\gamma'_{\chi} < 0$  and  $\chi'(\hat{\eta}) > 0$ , we can safely conclude that  $d(\chi M)/d\hat{\eta} > 0$ . To determine the relationship between  $\hat{\eta}$  and  $M$ , we can look at the properties of  $\mu(\hat{\eta})$ , which captures this link. In the interest of tractability, we define  $f(\hat{\eta}) \equiv \exp[-\mu(\hat{\eta})]$ , implying that  $M = (k - \xi)N/f(\hat{\eta})$  holds, according to Eqs. (23) and (A.8). Hence there is a monotonic negative relationship between  $f(\hat{\eta})$  and  $\mu(\hat{\eta})$ , so that  $\mu'(\hat{\eta}) >, =, < 0$  if  $0 >, =, < f'(\hat{\eta})$ . Differentiation of

$f(\hat{\eta})$  gives  $f'(\hat{\eta}) = \chi'(\hat{\eta})\hat{f}(\hat{\eta})$ , with

$$\hat{f}(\hat{\eta}) \equiv k - \xi + k(\sigma - 1) \left[ 1 - \hat{\eta} - \frac{1}{tk} \frac{1 - \hat{\eta}}{\hat{\eta}} - \frac{k - \xi}{k} \hat{\eta} \chi(\hat{\eta})^{-\frac{\xi}{k}} \right]. \quad (\text{A.10})$$

Accounting for  $\lim_{\hat{\eta} \rightarrow 0} \hat{f}(\hat{\eta}) = -\infty$ , we can conclude that  $f(\hat{\eta})$  declines at low levels of  $\hat{\eta}$ . Furthermore, substituting  $\chi(\hat{\eta}_{int}) = 1$  and thus  $(1 - \hat{\eta}_{int})^{-t\xi} \exp[-\hat{\eta}_{int}t\xi] = 2$  into Eq. (21) and noting that  $t > t_1(\theta)$  establishes  $\lambda(\hat{\eta}_{int}) > 0$ , we additionally obtain  $1 - \hat{\eta}_{int} > 2^{-(1-\theta)}$ . Noting further that  $\chi(0) = 0$  and thus  $f(0) = (k - \xi) + k(\sigma - 1)$  as well as  $f(\hat{\eta}_{int}) = 2(k - \xi) + 2(1 - \hat{\eta}_{int})k(\sigma - 1)$ , it follows that  $t > t_1(\theta)$  is sufficient for  $f(\hat{\eta}_{int}) > f(0)$ , which implies that  $f(\hat{\eta})$  must have a minimum on interval  $(0, \hat{\eta}_{int})$ . We now show that the minimum is unique and the only extremum of  $f(\hat{\eta})$  on the respective interval. This is equivalent to showing that  $\hat{f}(\hat{\eta}) = 0$  has a unique interior solution on the unit interval.

Differentiation of  $\hat{f}(\hat{\eta})$  gives

$$\hat{f}'(\hat{\eta}) = -\frac{k(\sigma - 1)}{1 - \hat{\eta}} \left\{ 1 - \hat{\eta} - \frac{1}{tk} \frac{1 - \hat{\eta}}{\hat{\eta}} - \frac{k - \xi}{k} \hat{\eta} \chi(\hat{\eta})^{-\frac{\xi}{k}} - \frac{1}{tk} \left( \frac{1 - \hat{\eta}}{\hat{\eta}} \right)^2 + \frac{k - \xi}{k} \chi^{-\frac{\xi}{k}} g(\hat{\eta}) \right\} \quad (\text{A.11})$$

with  $g(\hat{\eta}) \equiv 1 - t\xi \hat{\eta}^2 \chi(\hat{\eta})^{-\xi/k} [1 + \chi(\hat{\eta})^{\xi/k}]$ . Evaluated at  $\hat{f}(\hat{\eta}) = 0$ , we find that the first three terms in the bracket expression add up to  $[\hat{f}(\hat{\eta}) - (k - \xi)]/[k(\sigma - 1)] = -(k - \xi)/[k(\sigma - 1)] < 0$ . Accordingly,  $g(\hat{\eta}) \leq 0$  is sufficient for an extremum to be a minimum and, if  $g(\hat{\eta}) \leq 0$  holds for all possible  $\hat{\eta}$ , this minimum must be unique. Substituting  $\chi(\hat{\eta})$  from the main text we can compute  $\lim_{\hat{\eta} \rightarrow 0} g(\hat{\eta}) = -1$  and  $g(1) = 1 - t\xi < 0$ , where the negative sign of  $g(1)$  follows from assumption  $t > t_1(\theta)$  and the observation that  $t_1(\theta)\xi > 1$ , according to Eq. (22). Furthermore, we can compute

$$\begin{aligned} g'(\hat{\eta}) &= -t\xi \left[ 2\hat{\eta} \frac{1 + \chi(\hat{\eta})^{\frac{\xi}{k}}}{\chi(\hat{\eta})^{\frac{\xi}{k}}} - \frac{\xi}{k} \hat{\eta}^2 \chi(\hat{\eta})^{-\frac{\xi}{k}-1} \chi'(\hat{\eta}) \right] \\ &= \frac{g(\hat{\eta}) - 1}{\hat{\eta}} \left\{ 2 + [g(\hat{\eta}) - 1](1 - \hat{\eta})^{t\xi-1} \exp[\hat{\eta}t\xi] \right\}. \end{aligned} \quad (\text{A.12})$$

Hence,  $g'(\hat{\eta}) = 0$  requires either  $g(\hat{\eta}) = 1$ , which is impossible for any  $\hat{\eta} > 0$ , or  $[1 - g(\hat{\eta})](1 - \hat{\eta})^{t\xi-1} \exp[\hat{\eta}t\xi] = 2$ , which is only possible if  $g(\hat{\eta}) < 0$ . To see this, note that  $(1 - \eta)^{t\xi-1} \exp[\hat{\eta}t\xi]$  has a maximum at  $\hat{\eta} = 1/(t\xi)$ , implying that  $(1 - \eta)^{t\xi-1} \exp[\hat{\eta}t\xi] \leq [(t\xi - 1)/t\xi]^{t\xi-1} \exp[1]$ . Since the latter declines in  $t\xi$  (provided that  $t\xi > 1$ ) and since  $\lim_{t\xi \rightarrow \infty} [(t\xi - 1)/t\xi]^{t\xi-1} \exp[1] = 1$ , it follows that  $g'(\hat{\eta}) = 0$  requires  $g(\hat{\eta}) < 0$ . Taking stock, the previous analysis establishes that  $g(\hat{\eta})$  is either monotonic in  $\hat{\eta}$  or has a (not necessarily unique) extremum with a negative function value. In both cases,  $g(0) = -1$  and  $g(1) = 1 - t\xi < 0$  are sufficient for  $g(\hat{\eta})$  to have a negative sign for all possible  $\hat{\eta}$ . Together with our insights from above, we can thus safely conclude that  $f(\hat{\eta})$  has unique interior minimum on interval  $(1, \hat{\eta}_{int})$ , which we denote by  $\hat{\eta}_{pol}^1$ . This establishes the hump shape of  $\mu(\hat{\eta})$ .

We next show that  $\hat{\eta}_{pol}^1 < \hat{\eta}_{fm}$ . This is equivalent to showing  $f'(\hat{\eta}_{fm}) > 0$  or  $\hat{f}'(\hat{\eta}_{fm}) > 0$  (see above). Noting from the main text that  $\hat{\eta}_{fm}$  is implicitly determined by  $\lambda(\hat{\eta}_{fm}) = 0$ , we can infer from Eq. (21) that  $(1 - \hat{\eta}_{fm})^{-t\xi} \exp[-\hat{\eta}_{fm}t\xi] = (1 - \hat{\eta}_{fm})^{-\frac{1}{1-\theta}}$ . This allows us to compute

$\hat{f}(\hat{\eta}_{fm}) = k - \xi + k(\sigma - 1)F(\hat{\eta}_{fm})$ , with

$$F(\hat{\eta}) \equiv 1 - \hat{\eta} - \frac{1}{tk} \frac{1 - \hat{\eta}}{\hat{\eta}} - \frac{k - \xi}{k} \hat{\eta} \frac{(1 - \hat{\eta})^{\frac{1}{1-\theta}}}{1 - (1 - \hat{\eta})^{\frac{1}{1-\theta}}}. \quad (\text{A.13})$$

Differentiation of  $F(\hat{\eta})$  establishes

$$\begin{aligned} (1 - \hat{\eta})F'(\hat{\eta}) &= - \left\{ 1 - \hat{\eta} - \frac{1}{tk} \frac{1 - \hat{\eta}}{\hat{\eta}^2} + \frac{k - \xi}{k} \frac{(1 - \hat{\eta})^{\frac{1}{1-\theta}}}{1 - (1 - \hat{\eta})^{\frac{1}{1-\theta}}} \left[ 1 - \hat{\eta} - \frac{1}{1 - \theta} \frac{\hat{\eta}}{1 - (1 - \hat{\eta})^{\frac{1}{1-\theta}}} \right] \right\} \\ &= - \left\{ 1 - \hat{\eta} - \frac{1}{tk} \frac{1 - \hat{\eta}}{\hat{\eta}} - \frac{k - \xi}{k} \hat{\eta} \frac{(1 - \hat{\eta})^{\frac{1}{1-\theta}}}{1 - (1 - \hat{\eta})^{\frac{1}{1-\theta}}} \right. \\ &\quad \left. + \frac{k - \xi}{k} \frac{(1 - \hat{\eta})^{\frac{1}{1-\theta}}}{1 - (1 - \hat{\eta})^{\frac{1}{1-\theta}}} \left[ 1 - \frac{1}{1 - \theta} \frac{\hat{\eta}}{1 - (1 - \hat{\eta})^{\frac{1}{1-\theta}}} \right] - \frac{1}{tk} \left( \frac{1 - \hat{\eta}}{\hat{\eta}} \right)^2 \right\}. \quad (\text{A.14}) \end{aligned}$$

Two remarks are in order here. First,  $k - \xi + k(\sigma - 1)F(\hat{\eta}) \leq 0$  requires

$$1 - \hat{\eta} - \frac{1}{tk} \frac{1 - \hat{\eta}}{\hat{\eta}} - \frac{k - \xi}{k} \hat{\eta} \frac{(1 - \hat{\eta})^{\frac{1}{1-\theta}}}{1 - (1 - \hat{\eta})^{\frac{1}{1-\theta}}} < 0. \quad (\text{A.15})$$

and thus the second line in Eq. (A.14) to be negative. Second, regarding the bracketed term in the third line of Eq. (A.14) we can note that  $1 - (1 - \theta)^{-1} \hat{\eta} [1 - (1 - \hat{\eta})^{1/(1-\theta)}]^{-1} >, =, < 0$  is equivalent to  $\hat{F}(\hat{\eta}) \equiv (1 - \theta)[1 - (1 - \hat{\eta})^{1/(1-\theta)}] - \hat{\eta} >, =, < 0$ . Accounting for  $\hat{F}'(\hat{\eta}) = (1 - \hat{\eta})^{\theta/(1-\theta)} - 1 \leq 0$  – which holds with strict inequality if  $\hat{\eta} > 0$  – and  $\hat{F}(0) = 0$ , it follows that  $1 - (1 - \theta)^{-1} \hat{\eta} [1 - (1 - \hat{\eta})^{1/(1-\theta)}]^{-1}$  is negative. We can therefore conclude that if  $k - \xi + k(\sigma - 1)F(\hat{\eta}) = 0$  has a solution at  $\hat{\eta}'$ , then  $F'(\hat{\eta}') > 0$  is guaranteed. Accordingly, if we find a  $\hat{\eta}'_0$ , for which  $k - \xi + k(\sigma - 1)F(\hat{\eta}) \geq 0$ , it follows that  $k - \xi + k(\sigma - 1)F(\hat{\eta}) > 0$  must hold for all  $\hat{\eta} > \hat{\eta}'_0$ . Let us now evaluate  $F(\hat{\eta})$  at  $\hat{\eta}_{tm} = [a(t, \theta)]^{-1}$ , with  $a(t, \theta) \equiv t(1 - \theta)\xi > 1$ . This gives

$$F(\hat{\eta}_{tm}) = \frac{1}{a} \left[ \frac{k - (1 - \theta)\xi}{k} (a - 1) - \frac{k - \xi}{k} \frac{(a - 1)^{\frac{1}{1-\theta}}}{a^{\frac{1}{1-\theta}} - (a - 1)^{\frac{1}{1-\theta}}} \right], \quad (\text{A.16})$$

where the arguments of  $a(t, \theta)$  have been neglected in the interest of a parsimonious notation. Since

$$\frac{(a - 1)^{\frac{1}{1-\theta}}}{a^{\frac{1}{1-\theta}} - (a - 1)^{\frac{1}{1-\theta}}} = \frac{1}{[a/(a - 1)]^{\frac{1}{1-\theta}} - 1} < \frac{1}{a/(a - 1) - 1} = a - 1, \quad (\text{A.17})$$

it follows for  $\theta > 0$  that  $F(\hat{\eta}_{tm}) > (\theta\xi/k)(a - 1)/a > 0$ . However, from above we know that  $F(\hat{\eta}_{tm}) > 0$  establishes  $k - \xi + k(\sigma - 1)F(\hat{\eta}) > 0$  for all  $\hat{\eta} > \hat{\eta}_{tm}$ . Since we know from Appendix A.4 that  $\hat{\eta}_{fm} > \hat{\eta}_{tm}$  we can safely conclude that  $F(\hat{\eta}_{fm}) > 0$  and thus  $\hat{f}(\hat{\eta}_{fm}) > 0$  must hold. This confirms  $\hat{\eta}_{pol}^1 < \hat{\eta}_{fm}$ .

## A.6 The impact of an increase in $\hat{\eta}$ from zero to a small positive value on $l_o(\varphi)$

To determine the marginal impact of an increase in  $\hat{\eta}$  on  $l_o(\varphi)$ , we can differentiate

$$\nu(\hat{\eta}) \equiv \exp \left[ \frac{(1-\theta)\xi}{k} \ln \mu(\hat{\eta}) + \lambda(\hat{\eta}) \right] = f(\hat{\eta})^{\frac{(1-\theta)\xi}{k}} (1-\hat{\eta})^{1-t(1-\theta)\xi} \exp[-t(1-\theta)\xi\hat{\eta}]. \quad (\text{A.18})$$

This gives

$$\nu'(\hat{\eta}) = -[1-t(1-\theta)\xi] \frac{\nu(\hat{\eta})}{1-\hat{\eta}} - t(1-\theta)\xi\nu(\hat{\eta}) + \frac{(1-\theta)\xi}{k} \frac{f'(\hat{\eta})}{f(\hat{\eta})} \nu(\hat{\eta}), \quad (\text{A.19})$$

which, using the insights from Appendix A.5, can be reformulated to

$$\nu'(\hat{\eta}) = -\nu(\hat{\eta}) \left\{ [1-t(1-\theta)\xi] \frac{1}{1-\hat{\eta}} + t(1-\theta)\xi - \frac{(1-\theta)\xi}{k} \frac{\chi'(\hat{\eta})\hat{f}(\hat{\eta})}{f(\hat{\eta})} \right\} = -\nu(\hat{\eta})m(\hat{\eta}), \quad (\text{A.20})$$

with

$$m(\hat{\eta}) \equiv [1-t(1-\theta)\xi] \frac{1}{1-\hat{\eta}} + t(1-\theta)\xi \left[ 1 - \chi(\hat{\eta})^{1-\frac{\xi}{k}} \left( 1 + \chi(\hat{\eta})^{\frac{\xi}{k}} \right) \frac{\hat{\eta}}{1-\hat{\eta}} \frac{\hat{f}(\hat{\eta})}{f(\hat{\eta})} \right]. \quad (\text{A.21})$$

We can note that  $\nu(0) = f(0)^{\frac{(1-\theta)\xi}{k}} = [k-\xi+k(\sigma-1)]^{\frac{(1-\theta)\xi}{k}} > 0$  and that

$$\begin{aligned} \lim_{\hat{\eta} \rightarrow 0} m(\hat{\eta}) &= 1 + \frac{t(1-\theta)\xi(\sigma-1)(k-\xi)}{k-\xi+k(\sigma-1)} \lim_{\hat{\eta} \rightarrow 0} \frac{\hat{\eta}^2}{1-\hat{\eta}} \chi(\hat{\eta})^{1-2\frac{\xi}{k}}, \\ &= 1 + \frac{t(1-\theta)\xi(\sigma-1)(k-\xi)}{k-\xi+k(\sigma-1)} \lim_{\hat{\eta} \rightarrow 0} \frac{\hat{\eta}^2}{(1-\hat{\eta}) \left\{ (1-\hat{\eta})^{-t\xi} \exp[-\hat{\eta}t\xi] - 1 \right\}} \\ &\quad \times \lim_{\hat{\eta} \rightarrow 0} \left\{ (1-\hat{\eta})^{-t\xi} \exp[-\hat{\eta}t\xi] - 1 \right\}^{\frac{k-\xi}{\xi}} \\ &= 1 + \frac{2(1-\theta)(\sigma-1)(k-\xi)}{k-\xi+k(\sigma-1)} \lim_{\hat{\eta} \rightarrow 0} \left\{ (1-\hat{\eta})^{-t\xi} \exp[-\hat{\eta}t\xi] - 1 \right\}^{\frac{k-\xi}{\xi}} = 1. \end{aligned} \quad (\text{A.22})$$

This establishes  $\nu'(0) < 0$  and completes the proof.

## A.7 Characterising $t_2(\theta)$ and establishing the ranking $\hat{\eta}_{pol}^1 < \hat{\eta}_{tm}$

In the subsequent, we show that under condition (26), there exists a unique  $t_2(\theta)$ , with  $t_2'(\theta) > 0$ , such that  $t > t_2(\theta)$  establishes  $f'(\hat{\eta}_{tm}) < 0$ . Evaluating  $f'(\hat{\eta}) = \chi'(\hat{\eta})\hat{f}(\hat{\eta})$  at  $\hat{\eta}_{tm} = [t(1-\theta)\xi]^{-1}$  gives

$$f'(\hat{\eta}_{tm}) = \chi(\hat{\eta}_{tm})^{\frac{k-\xi}{k}} \left[ 1 + \chi(\hat{\eta}_{tm})^{\frac{\xi}{k}} \right] \frac{tk(k-\xi) [1 + (\sigma-1)f_0(t, \theta)]}{t(1-\theta)\xi - 1}, \quad (\text{A.23})$$

with

$$f_0(t, \theta) \equiv \left( \frac{k - (1-\theta)\xi}{k - \xi} - \frac{[a(t, \theta) - 1]^{-1}}{\{[a(t, \theta) - 1]/a(t, \theta)\}^{-\frac{a(t, \theta)}{1-\theta}} \exp[-1/(1-\theta)] - 1} \right) \frac{a(t, \theta) - 1}{a(t, \theta)} \quad (\text{A.24})$$

and  $a(t, \theta) \equiv t(1-\theta)\xi$ . Hence,  $f'(\hat{\eta}_{tm}) < 0$  requires  $1 + (\sigma-1)f_0(t, \theta) < 0$ . We can show the following auxiliary result.

**Lemma A.1** *Let us define*

$$n(a) \equiv (a-1) \left\{ \left( \frac{a-1}{a} \right)^{-\frac{a}{1-\theta}} \exp \left[ -\frac{1}{1-\theta} \right] - 1 \right\}. \quad (\text{A.25})$$

Then, if  $\theta < 1/4$   $n(a)$  has a unique minimum at  $\hat{a} > 1$ , such that  $n'(a) < 0$  if  $a < \hat{a}$  and  $n'(a) > 0$  if  $a > \hat{a}$ . If  $\theta \geq 1/4$ ,  $n(a)$  decreases for all possible  $a$ . Furthermore, we have  $\lim_{a \rightarrow 1} n(a) = \infty$  and, since  $\theta < 1/2$  is established by condition (26),  $\lim_{a \rightarrow \infty} n(a) = [2(1-\theta)]^{-1} < 1$ , implying that  $n(a) = 1$  has a unique solution, which lies in interval  $(1, \hat{a})$ .

**Proof** Whereas the limits of  $n(a)$  are straightforward to compute, it is more demanding to show existence of a unique minimum of  $n(a)$  on interval  $(1, \infty)$ . Twice differentiating  $n(a)$  establishes

$$n'(a) = - \left( \frac{a-1}{a} \right)^{-\frac{a}{1-\theta}} \exp \left[ -\frac{1}{1-\theta} \right] \left[ (a-1) \ln \left( \frac{a-1}{a} \right) + \theta \right] \frac{1}{1-\theta} - 1 \quad (\text{A.26})$$

$$n''(a) = \left( \frac{a-1}{a} \right)^{-\frac{a}{1-\theta}} \exp \left[ -\frac{1}{1-\theta} \right] \frac{1}{1-\theta} \hat{n}(a), \quad (\text{A.27})$$

with

$$\begin{aligned} \hat{n}(a) &\equiv \left[ \ln \left( \frac{a-1}{a} \right) + \frac{1}{a-1} \right] \left[ (a-1) \ln \left( \frac{a-1}{a} \right) + \theta \right] \frac{1}{1-\theta} - \left[ \ln \left( \frac{a-1}{a} \right) + \frac{1}{a} \right], \\ &= \left[ \ln \left( \frac{a-1}{a} \right) + \frac{1}{a-1} \right]^2 \frac{a-1}{1-\theta} - 2 \ln \left( \frac{a-1}{a} \right) - \frac{2a-1}{a(a-1)} \end{aligned} \quad (\text{A.28})$$

and

$$\hat{n}'(a) = \frac{n_0(a) - \theta}{(a-1)^2 a^2 (1-\theta)}, \quad n_0(a) \equiv (a-1)^2 \left\{ a \ln \left( \frac{a-1}{a} \right) \left[ 2 + a \ln \left( \frac{a-1}{a} \right) \right] + 1 \right\}. \quad (\text{A.29})$$

We can further compute  $n'_0(a) = 2a(a-1)(2a-1)\tilde{n}_0(a)$ , with

$$\tilde{n}_0(a) \equiv \frac{2}{a(2a-1)} + \frac{4a-1}{a(2a-1)} \ln \left( \frac{a-1}{a} \right) + \ln \left( \frac{a-1}{a} \right)^2. \quad (\text{A.30})$$

Differentiating  $\tilde{n}_0(a)$  gives

$$\tilde{n}'_0(a) = \frac{4a^2 - 3a + 1}{(a-1)[a(2a-1)]^2} \bar{n}_0(a), \quad \bar{n}_0(a) \equiv \frac{4a-1}{4a^2 - 3a + 1} + \ln \left( \frac{a-1}{a} \right). \quad (\text{A.31})$$

Accounting for

$$\bar{n}'_0(a) = \frac{10a^2 - 7a + 1}{a(a-1)(4a^2 - 3a + 1)^2} > 0, \quad \lim_{a \rightarrow \infty} \bar{n}_0(a) = 0, \quad (\text{A.32})$$

it follows that  $\bar{n}_0(a) < 0$  and thus  $\tilde{n}'(a) < 0$ . Noting further that  $\lim_{a \rightarrow \infty} \tilde{n}_0(a) = 0$ , we can further conclude that  $\tilde{n}_0(a) > 0$  and thus  $n'_0(a) > 0$ . Accounting for  $\lim_{a \rightarrow 1} n_0(a) = 0$  and  $\lim_{a \rightarrow \infty} n_0(a) = 1/4$ , we can then distinguish two cases, regarding the sign of  $\hat{n}'(a)$ . If  $\theta < 1/4$ , there exists a unique  $\bar{a} > 1$ , such that  $\hat{n}'(a) >, =, < 0$  if  $a >, =, < \bar{a}$ . If  $\theta \geq 1/4$ , then  $\hat{n}'(a) < 0$  holds for all possible  $a > 1$ . Computing  $\lim_{a \rightarrow 1} \hat{n}(a) = \infty$  and  $\lim_{a \rightarrow \infty} \hat{n}(a) = 0$ , it follows that if  $\theta < 1/4$ ,  $\hat{n}(a) = 0$  has a unique solution at  $a_n \in (1, \bar{a})$ , such that  $\hat{n}(a) >, =, < 0$  if  $a_n >, =, < a$ .



In contrast, if  $\theta \geq 1/4$ ,  $\hat{n}(a) > 0$  holds for all possible  $a > 1$ . We can thus safely conclude that  $\theta \geq 1/4$  establishes  $n''(a) > 0$ , whereas  $\theta < 1/4$  establishes  $n''(a) >, =, < 0$  if  $a_n >, =, < a$ . Since  $\lim_{a \rightarrow 1} n'(a) = -\infty$  and  $\lim_{a \rightarrow \infty} n'(a) = 0$ , it follows from the properties of  $n''(a)$  that  $n'(a)$  is negative for all possible  $a > 1$  if  $\theta \geq 1/4$ , whereas  $n'(a)$  is negative for small  $a$  and positive for high ones if  $\theta < 1/4$ . In the former case,  $n(a)$  has no interior extremum, whereas in the latter case, it has a minimum, which we denote by  $\hat{a}$ , and, since it is not possible that a minimum follows a minimum, according to the properties of  $n''(a)$ , the minimum must be unique. In view of  $\lim_{a \rightarrow 1} n(a) = \infty$  and  $\lim_{a \rightarrow \infty} n(a) = [2(1 - \theta)]^{-1} < 1$ , it must be true that  $n(a) = 1$  has a unique solution, which lies in the interval  $(1, \hat{a})$ . This completes the proof of auxiliary result Lemma A.1.

We next show the following auxiliary result.

**Lemma A.2** *Let us define*

$$\hat{f}_0(a) \equiv \left[ \frac{k - (1 - \theta)\xi}{k - \xi} - \frac{1}{n(a)} \right] \frac{a - 1}{a}, \quad (\text{A.33})$$

which has the following properties. First, we have  $\hat{f}_0(a) = f_0(t, \theta)$  if  $a = a(t, \theta)$ . Second, we have  $\hat{f}_0(1) = 0$  and  $\hat{f}'_0(1) > 0$ . Third, there exists a unique  $\underline{a} \in (1, \infty)$  such that  $\hat{f}(a) >, =, < 0$  if  $\underline{a} >, =, < a$ . Fourth, if  $\hat{f}_0(a)$  has an extremum at  $a > \underline{a}$ , this must be a minimum. Fifth, we have

$$\lim_{a \rightarrow \infty} \hat{f}_0(a) = -\frac{k(1 - 2\theta) - (1 - \theta)\xi}{k - \xi} \quad (\text{A.34})$$

which is negative under condition (26), and  $\lim_{a \rightarrow \infty} \hat{f}'_0(a) = 0$ .

**Proof** Computing  $\hat{f}_0(a)$  for  $a = 1$  and  $a \rightarrow \infty$  and showing that  $\hat{f}_0(a) = f_0(t, \theta)$  if  $a = a(t, \theta)$  are straightforward tasks. Twice differentiating  $\hat{f}_0(a)$  gives

$$\hat{f}'_0(a) = \left[ \frac{k - (1 - \theta)\xi}{k - \xi} - \frac{1}{n(a)} \right] \left( \frac{1}{a} \right)^2 + \frac{n'(a)}{n(a)^2} \frac{a - 1}{a}, \quad (\text{A.35})$$

$$\hat{f}''_0(a) = -2 \left[ \frac{k - (1 - \theta)\xi}{k - \xi} - \frac{1}{n(a)} \right] \left( \frac{1}{a} \right)^3 + 2 \frac{n'(a)}{n(a)^2} \left( \frac{1}{a} \right)^2 - 2 \frac{[n'(a)]^2}{n(a)^3} \frac{a - 1}{a} + \frac{n''(a)}{n(a)^2} \frac{a - 1}{a}, \quad (\text{A.36})$$

with  $\lim_{a \rightarrow 1} \hat{f}'_0(a) = [k - (1 - \theta)\xi]/(k - \xi) > 0$  and  $\lim_{a \rightarrow \infty} \hat{f}'_0(a) = 0$ . Combining  $\hat{f}_0(1) = 0$  and  $\hat{f}'_0(1) > 0$  with  $\lim_{a \rightarrow \infty} \hat{f}_0(a) < 0$  has the following two implications. On the one hand, there must be an  $\underline{a} \in (0, 1)$  for which  $\hat{f}_0(\underline{a}) = 0$  and  $\hat{f}'_0(\underline{a}) < 0$  and, on the other hand,  $\hat{f}_0(a)$  must have a maximum on interval  $(0, \underline{a})$ . In the following, we show that  $\hat{f}_0(a)$  has a unique maximum on interval  $(0, \infty)$ , implying that  $\underline{a}$  must be unique as well and that any extremum in interval  $(\underline{a}, \infty)$  must be a minimum.

For this purpose, we note that an extremum of  $\hat{f}_0(a)$  requires  $\hat{f}'_0(a) = 0$  and denote by  $\hat{a}_0$  those values of  $a$  that fulfill this condition. We then compute  $\hat{f}''_0(\hat{a}_0) = \hat{f}_1(\hat{a}_0)[n(\hat{a}_0)]^{-3}\hat{a}_0^{-1}$ , with

$$\hat{f}_1(a) = 2n'(a)n(a) - (a - 1)[n'(a)]^2 + (a - 1)n''(a)n(a). \quad (\text{A.37})$$

Substitution of  $n(a)$ ,  $n'(a)$  and  $n''(a)$  from Lemma A.1, then gives  $\hat{f}_1(a) = \hat{f}_2(a)[n(a) + (a -$

1)]/[a(1 -  $\theta$ )], with

$$\hat{f}_2(a) \equiv - \left[ \left( \frac{a-1}{a} \right)^{-\frac{a}{1-\theta}} \exp \left[ -\frac{1}{1-\theta} \right] + 1 \right] \left\{ \left[ (a-1) \ln \left( \frac{a-1}{a} \right) + 1 \right]^2 \frac{a}{1-\theta} - 1 \right\} - 2. \quad (\text{A.38})$$

For the subsequent analysis, it is useful to introduce

$$v_1(a) \equiv (a-1) \ln \left( \frac{a-1}{a} \right) + 1 \geq 0, \quad v_2(a) \equiv 3 \ln \left( \frac{a-1}{a} \right) + \frac{3a-1}{a(a-1)} \geq 0, \quad (\text{A.39})$$

$$v_3(a) \equiv (3a-1) \ln \left( \frac{a-1}{a} \right) + 3 \leq 0, \quad v_4(a) \equiv (4a-1) \ln \left( \frac{a-1}{a} \right) + 4 \leq 0, \quad (\text{A.40})$$

with  $v_2(a) = v_3'(a)$ . We can compute  $\lim_{a \rightarrow 1} \hat{f}_2(a) = -\infty$ ,  $\lim_{a \rightarrow \infty} \hat{f}_2(a) = 0$ , and

$$\begin{aligned} \hat{f}_2'(a) = \frac{1}{1-\theta} \frac{v_1(a)}{a-1} & \left\{ \left( \frac{a-1}{a} \right)^{-\frac{a}{1-\theta}} \exp \left[ -\frac{1}{1-\theta} \right] \left[ v_1(a)^2 \frac{a}{1-\theta} - 1 \right] \right. \\ & \left. - \left[ \left( \frac{a-1}{a} \right)^{-\frac{a}{1-\theta}} \exp \left[ -\frac{1}{1-\theta} \right] + 1 \right] (a-1)v_3(a) \right\}. \end{aligned}$$

Let us now hypothesize that  $\hat{f}_2(a) = 0$  has a solution  $a_0(\theta) > 1$  ( $a_0$ , in short). Then,  $\hat{f}_2'(a_0) >, =, < 0$  is equivalent to  $\hat{\Psi}(a_0, \theta) >, =, < 0$ , with

$$\hat{\Psi}(a, \theta) \equiv \left[ v_1(a)^2 \frac{a}{1-\theta} \right]^2 - 1 - 2(a-1)v_3(a). \quad (\text{A.41})$$

Differentiation of  $\hat{\Psi}(a)$  establishes

$$\hat{\Psi}'_a(a, \theta) = 2 \left\{ \left[ v_1(a)^3 \frac{a}{(1-\theta)^2} - 1 \right] v_3(a) - (a-1)v_2(a) \right\}, \quad (\text{A.42})$$

$$\hat{\Psi}''_{aa}(a, \theta) = 2 \left\{ \left[ v_1(a)^3 \frac{a}{(1-\theta)^2} - 2 \right] v_2(a) + \frac{1}{(1-\theta)^2} v_1(a)^2 v_3(a) v_4(a) + \frac{a+1}{a^2(a-1)} \right\}. \quad (\text{A.43})$$

If  $\hat{\Psi}'_a(a, \theta) = 0$  has a solution at  $\underline{a}_1(\theta) > 1$  ( $\underline{a}_1$ , in short). Then,  $\hat{\Psi}''(\underline{a}_1, \theta) >, =, < 0$  is equivalent to  $\hat{\Psi}_0(\underline{a}_1) >, =, < 0$ , with

$$\begin{aligned} \hat{\Psi}_0(a) \equiv -\frac{2}{av_1(a)v_3(a)} & \left\{ \left[ \left( 2a \ln \left( \frac{a-1}{a} \right) + 1 \right) v_1(a) - (a-1)v_3(a)v_4(a) \right] v_2(a) \right. \\ & \left. - \left[ v_3(a)v_4(a) + \frac{a+1}{a(a-1)} v_1(a) \right] v_3(a) \right\} \quad (\text{A.44}) \end{aligned}$$

being independent of  $\theta$ . It is tedious but straightforward to show that  $\lim_{a \rightarrow 1} \hat{\Psi}_0(a) = -\infty$ ,  $\lim_{a \rightarrow \infty} \hat{\Psi}_0(a) = 0$ , and that  $\hat{\Psi}_0(a) = 0$  has a unique solution at  $\underline{a}_2 > 1$ , such that  $\hat{\Psi}_0(a) >, =, < 0$  if  $a >, =, < \underline{a}_2$ . The correctness of this result can be verified with any standard mathematics software (and details of the proof are available upon request).

We can further note that  $\lim_{a \rightarrow 1} \hat{\Psi}'_a(a, \theta) = -\infty$ , and  $\lim_{a \rightarrow \infty} \hat{\Psi}'_a(a, \theta) = 0$  hold for all  $\theta \in (0, 1)$ . Let us assume that  $\hat{\Psi}'_a(a, \theta) = 0$  has a solution at  $\underline{a}_1(\theta) \in [1, \infty)$ . For instance, this is the case if  $\theta \rightarrow 0$ . Accounting for  $\lim_{a \rightarrow 1} \hat{\Psi}'_a(a, \theta) = -\infty$  and  $\hat{\Psi}_0(a) >, =, < 0$  if  $a >, =, < \underline{a}_2$ , it follows

that  $\underline{a}_1(\theta)$  is unique and larger than  $\underline{a}_2$ , i.e.  $\underline{a}_1(\theta) > \underline{a}_2 > 1$ . However, this insight builds on the presumption that  $\hat{\Psi}'_a(a, \theta) = 0$  has in fact a solution on interval  $[1, \infty)$ , which we have to check. For this purpose, we solve  $\hat{\Psi}'_a(a, \theta) = 0$  for  $\theta$  and denote the resulting expression by  $\hat{\theta}(a)$ . In view of Eq. (A.42), this gives

$$\hat{\theta}(a) \equiv 1 - \sqrt{\frac{av_1(a)^3v_3(a)}{(a-1)v_2(a) + v_3(a)}}, \quad (\text{A.45})$$

with  $\lim_{a \rightarrow 1} \hat{\theta}(a) = 0$ ,  $\lim_{a \rightarrow \infty} \hat{\theta}(a) = 1 - \sqrt{6}/4 \equiv \hat{\theta}_1$ , and  $\hat{\theta}'(a) > 0$ .<sup>17</sup> This has the following implications. If  $\theta < \hat{\theta}_1$ , then  $\hat{\Psi}'_a(a, \theta)$  has a unique solution in  $a$  on interval  $(1, \infty)$ , which we denote  $\underline{a}_1(\theta)$  (see above). In this case,  $\hat{\Psi}'_a(a, \theta) >, =, < 0$  if  $a >, =, < \underline{a}_1(\theta)$ . In contrast, if  $\theta \geq \hat{\theta}_1$ ,  $\hat{\Psi}'_a(a, \theta) < 0$  holds for all possible  $a > 1$ .

With these insights at hand, we are now prepared to determine the sign of  $\hat{\Psi}(a, \theta)$ . For this purpose, we can note that  $\lim_{a \rightarrow 1} \hat{\Psi}(a, \theta) = \theta(2-\theta)/(1-\theta)^2 > 0$  and  $\lim_{a \rightarrow \infty} \hat{\Psi}(a, \theta) = 0$ , implying that  $\hat{\Psi}(a, \theta) > 0$  holds for all possible finite values of  $a$  if  $\theta \geq \hat{\theta}_1$ . In contrast, if  $\theta < \hat{\theta}_1$ ,  $\hat{\Psi}(a, \theta)$  has an extremum at  $\underline{a}_1(\theta)$ , which, in view of  $\underline{a}_1(\theta) > \underline{a}_2$ , must be a minimum. In this case, there exists a unique  $\underline{a}_0(\theta) < \underline{a}_1(\theta)$  ( $\underline{a}_0$ , in short), such that  $\hat{\Psi}(a, \theta) >, =, < 0$  if  $\underline{a}_0(\theta) >, =, < a$ . Since we know from above that  $\hat{f}'_2(a) >, =, < 0$  is equivalent to  $\hat{\Psi}(a, \theta) >, =, < 0$  and that  $\lim_{a \rightarrow 1} \hat{f}_2(a) = -\infty$ ,  $\lim_{a \rightarrow \infty} \hat{f}_2(a) = 0$ , it is immediate that  $\hat{f}_2(a) < 0$  holds for all possible finite values of  $a$  if  $\theta \geq \hat{\theta}_1$ . In contrast, if  $\theta < \hat{\theta}_1$ ,  $\hat{f}_2(a)$  has an extremum at  $\underline{a}_0(\theta)$ , which – recollecting from above that  $\hat{f}'_2(a) >, =, < 0$  if  $\underline{a}_0(\theta) >, =, < a$  – must be a maximum. In this case, there exists a unique  $a_0(\theta) < \underline{a}_0(\theta)$ , such that  $\hat{f}_2(a) >, =, < 0$  if  $a >, =, < a_0(\theta)$ . Our previous insights  $\hat{f}_0(1) = 0$ ,  $\hat{f}'_0(1) > 0$ , and  $\lim_{a \rightarrow \infty} \hat{f}_0(a) < 0$  imply that  $\hat{f}_0(a)$  must have at least one extremum at  $\hat{a}_0 > 1$ , which must be a maximum (and thus requires  $\hat{f}_2(\hat{a}_0) < 0$ ). If  $\theta \geq \hat{\theta}_1$ , this extremum must be unique, because in this case  $\hat{f}_2(a) < 0$  holds for all possible finite values of  $a$ . If  $\theta < \hat{\theta}_1$ ,  $\hat{f}_0(a)$  may have a second extremum on interval  $a \in (1, \infty)$ , which then must be a minimum. Both the conclusion that  $\hat{f}_0(a)$  cannot have more than two interior extrema and the conclusion that  $\hat{f}_0(a)$  has exactly one maximum follow from the properties of  $\hat{f}_2(a)$  and the insight that in the case of multiple extrema, a minimum must follow a maximum and vice versa. As an immediate consequence of this, it follows that there must exist a unique  $\underline{a} \in (1, \infty)$ , such that  $\hat{f}_0(a) >, =, < 0$  if  $\underline{a} >, =, < a$ . This completes the proof of Lemma A.2.

Setting  $a = a(t, \tau) = t(1-\theta)\xi$ , condition (26) is equivalent to  $1 + (\sigma - 1) \lim_{a \rightarrow \infty} \hat{f}_0(a) < 0$ . From Lemma A.2 we can infer that either  $\hat{f}_0(a)$  is negatively sloped over the whole interval  $(\underline{a}, \infty)$  or it has a unique minimum at  $\hat{a}_0 > \underline{a}$ , such that  $\hat{f}_0(a)$  decreases over subinterval  $(\underline{a}, \hat{a}_0)$  and increases over subinterval  $(\hat{a}_0, \infty)$ . In both cases, there exists a unique  $a_2 \in (\underline{a}, \infty)$ , implicitly defined by  $1 + (\sigma - 1) \lim_{a \rightarrow \infty} \hat{f}_0(a_2) = 0$ , with  $\hat{f}'_0(a_2) < 0$  and  $1 + (\sigma - 1) \hat{f}_0(a) < 0$  for all  $a > a_2$ . In view of  $a = a(t, \tau)$  and  $a'_t(t, \tau) > 0$ , this characterises a unique  $t_2(\theta)$ , such that  $1 + (\sigma - 1) \hat{f}(t, \theta) >, =, < 0$  if  $t_2(\theta) >, =, < t$ . To determine the impact of  $\theta$  on  $t_2(\theta)$ , we can apply the implicit function theorem

<sup>17</sup>Provided that  $\hat{\Psi}'_a(a, \theta) = 0$  has a solution in  $\theta$  on interval  $(0, 1)$ , which is given by  $\hat{\theta}(a)$ , we have  $\hat{\Psi}''_{a\theta}(a, \hat{\theta}(a)) > 0$  and  $\hat{\Psi}''_{aa}(a, \hat{\theta}(a)) < 0$  – due to  $\underline{a}_1(\theta) > \underline{a}_2$  and  $\lim_{a \rightarrow 1} \hat{\Psi}'_a(a, \theta) = -\infty$  (see above). Applying the implicit function theorem to  $\hat{\Psi}'_a(a, \theta) = 0$  then establishes the positive sign of  $\hat{\theta}'(a)$ .

to  $1 + (\sigma - 1)\hat{f}(t, \theta) = 0$ . Accounting for

$$\hat{f}'_t(t, \theta) = \hat{f}'_0(a_2)(1 - \theta)\xi, \quad \hat{f}'_\theta(t, \theta) = \frac{\xi}{k - \xi} \frac{k - \sigma + 1}{k - \sigma + 1 + \theta k(\sigma - 1)} - \hat{f}'_0(a_2) \frac{t_2 \xi^2 \sigma}{\sigma - 1}, \quad (\text{A.46})$$

and noting from above that  $\hat{f}'_0(a_2) < 0$ , it follows that  $t'_2(\theta) > 0$ . This completes the proof.

### A.8 Characterisation of $\hat{\eta}_{pol}^0$

From Appendices A.4, A.5 and A.7, we know that parameter constraint (26) together with Assumption  $t < \max\{t_1(\theta), t_2(\theta)\}$  establish  $\hat{\eta}_{pol}^1 \in (\hat{\eta}_{tm}, \hat{\eta}_{fm})$  and thus  $\lambda'(\hat{\eta}) > 0$ ,  $\mu'(\hat{\eta}) > 0$  for all  $\hat{\eta} \in (\hat{\eta}_{int}, \hat{\eta}_{pol}^1)$ . In view of Eq. (25), we then have  $d \ln l_o(\varphi)/d\hat{\eta} > 0$  in the respective  $\hat{\eta}$ -interval. Noting further from Appendix A.6 that  $dl_o(\varphi)/d\hat{\eta} < 0$  holds in the neighbourhood of  $\hat{\eta} = 0$ , it follows from the differentiability of  $\lambda(\hat{\eta})$  and  $\mu(\hat{\eta})$  that there must exist a unique  $\hat{\eta}_{pol}^0$  in interval  $(0, \hat{\eta}_{tm})$  such that  $dl_o(\varphi)/d\hat{\eta} = 0$  at  $\hat{\eta} = \hat{\eta}_{pol}^0$  and  $dl_o(\varphi)/d\hat{\eta} > 0$  for all  $\hat{\eta} \in (\hat{\eta}_{pol}^0, \hat{\eta}_{pol}^1)$ . This completes the proof.

### A.9 Derivation of Eq. (28)

Adding up domestic employment over all purely domestic and offshoring firms in the source country gives

$$(1 - u)L = N \int_{\varphi_d}^{\varphi_o} l_d(\varphi) dG(\varphi) + N \int_{\varphi_o}^{\infty} l_o(\varphi) dG(\varphi). \quad (\text{A.47})$$

Substituting Eqs. (24) and (25) and using  $l_d(\varphi_d) = \sigma - 1$ , we can compute

$$(1 - u)L = M l_d(\varphi_d) \beta(\hat{\eta}) \frac{k}{k - (1 - \theta)\xi}, \quad (\text{A.48})$$

with

$$\beta(\hat{\eta}) \equiv 1 + [(1 - \hat{\eta})^{-t\xi} e^{-\hat{\eta}t\xi} - 1]^{[k - (1 - \theta)\xi]/\xi} [(1 - \hat{\eta})^{1 - t(1 - \theta)\xi} e^{-t(1 - \theta)\xi\hat{\eta}} - 1]. \quad (\text{A.49})$$

Furthermore, making use of Eq. (A.4) and noting that constant markup pricing implies  $(\sigma - 1)\pi(\varphi_d) = l_d(\varphi_d)w(\varphi_d)$ , we can express the total wage bill in the North as follows:

$$(1 - u)L\bar{w} = M l_d(\varphi_d) w(\varphi_d) \alpha(\hat{\eta}) \frac{k}{k - \xi}, \quad (\text{A.50})$$

with  $\alpha(\hat{\eta}) \equiv (1 + \chi)\gamma = 1 + [(1 - \hat{\eta})^{-t\xi} e^{-\hat{\eta}t\xi} - 1]^{(k - \xi)/\xi} [(1 - \hat{\eta})^{1 - t\xi} e^{-\hat{\eta}t\xi} - 1]$ . Together Eqs. (A.48) and (A.50) determine the wage ratio  $w(\varphi_d)/\bar{w} = \delta(\hat{\eta})(k - \xi)/[k - (1 - \theta)\xi]$ , where  $\delta(\hat{\eta}) = \beta(\hat{\eta})/\alpha(\hat{\eta})$  has been considered. Substitution into Eq. (27), then establishes Eq. (28).

## A.10 Offshoring and the unemployment rate of production workers $u$

From Eq. (28), we can infer that offshoring decreases (increases) unemployment rate  $u$  relative to autarky, if  $\delta(\hat{\eta}) > (<)1$ . Let us define

$$\psi(\hat{\eta}) \equiv \{(1 - \hat{\eta})^{-t\xi} e^{-\hat{\eta}t\xi} - 1\}^\theta, \quad \psi_1(\hat{\eta}) \equiv (1 - \hat{\eta})^{1-t\xi} e^{-\hat{\eta}t\xi} - 1, \quad (\text{A.51})$$

$$\psi_2(\hat{\eta}) \equiv (1 - \hat{\eta})^{1-t(1-\theta)\xi} e^{-\hat{\eta}t(1-\theta)\xi} - 1. \quad (\text{A.52})$$

Then, it follows from Eq. (29) that for  $\hat{\eta} < \hat{\eta}_{fm}$ ,  $\delta(\hat{\eta}) >, =, < 1$  is equivalent to  $\hat{\psi}(\hat{\eta}) >, =, < \psi(\hat{\eta})$ , with  $\hat{\psi}(\hat{\eta}) \equiv \psi_1(\hat{\eta})/\psi_2(\hat{\eta})$ , whereas for  $\hat{\eta} \in (\hat{\eta}_{fm}, \hat{\eta}_{int}]$ ,  $\delta(\hat{\eta}) >, =, < 1$  is equivalent to  $\psi(\hat{\eta}) >, =, < \hat{\psi}(\hat{\eta})$ . It is notable that  $\psi(\hat{\eta})$  increases monotonically in  $\hat{\eta}$ , from a minimum value of 0 at  $\hat{\eta} = 0$  to a maximum value of 1 at  $\hat{\eta} = \hat{\eta}_{int}$ . From the main text, we know that  $\psi_2(\hat{\eta})$  has a unique minimum at  $\hat{\eta}_{tm} = 1/[t(1-\theta)\xi]$  and that  $\psi_2(\hat{\eta}) = 0$  has a unique solution in interval  $(0, \hat{\eta}_{int})$ , denoted by  $\hat{\eta}_{fm}$ , such that  $\psi_2(\hat{\eta}) >, =, < 0$  if  $\hat{\eta} >, =, < \hat{\eta}_{fm}$ . In a similar vein,  $\psi_1(\hat{\eta})$  has a unique minimum at  $\hat{\eta}_{tm}^0 = 1/(t\xi)$  and  $\psi_1(\hat{\eta}) = 0$  has a unique solution in interval  $(0, \hat{\eta}_{int})$ , denoted  $\hat{\eta}_{fm}^0$ , such that  $\psi_1(\hat{\eta}) >, =, < 0$  if  $\hat{\eta} >, =, < \hat{\eta}_{fm}^0$ . Thereby,  $\hat{\eta}_{tm}^0 < \hat{\eta}_{tm}$  and, in view of  $\psi_1(\hat{\eta}) > \psi_2(\hat{\eta})$ ,  $\hat{\eta}_{fm}^0 < \hat{\eta}_{fm}$ . With these insights at hand, we can distinguish the following cases, regarding the sign and size of  $\hat{\psi}(\hat{\eta})$ : (i)  $0 > \psi_1(\hat{\eta}) > \psi_2(\hat{\eta})$  and thus  $\hat{\psi}(\hat{\eta}) \in (0, 1)$  if  $\hat{\eta} < \hat{\eta}_{fm}^0$ ; (ii)  $\psi_1(\hat{\eta}) > 0 > \psi_2(\hat{\eta})$  and thus  $\hat{\psi}(\hat{\eta}) < 0$  if  $\hat{\eta} \in (\hat{\eta}_{fm}^0, \hat{\eta}_{fm})$ ; (iii)  $\psi_1(\hat{\eta}) > \psi_2(\hat{\eta}) > 0$  and thus  $\hat{\psi}(\hat{\eta}) > 1$  if  $\hat{\eta} > \hat{\eta}_{fm}$ . For completeness, we also have  $\lim_{\hat{\eta} \rightarrow 0} \hat{\psi}(\hat{\eta}) = 1$ ,  $\hat{\psi}(\hat{\eta}_{fm}^0) = 0$ ,  $\lim_{\hat{\eta} \rightarrow \hat{\eta}_{fm}^-} \hat{\psi}(\hat{\eta}) = -\infty$ , and  $\lim_{\hat{\eta} \rightarrow \hat{\eta}_{fm}^+} \hat{\psi}(\hat{\eta}) = \infty$ . We can thus safely conclude that  $\psi(\hat{\eta}) > \hat{\psi}(\hat{\eta})$  and thus  $\delta(\hat{\eta}) < 1$  if  $\hat{\eta} \in (\hat{\eta}_{fm}^0, \hat{\eta}_{fm})$  and  $\psi(\hat{\eta}) < \hat{\psi}(\hat{\eta})$  and thus  $\delta(\hat{\eta}) < 1$  if  $\hat{\eta} > \hat{\eta}_{fm}$ . In both cases (and hence for all  $\hat{\eta} \geq \hat{\eta}_{fm}^0$ ), the unemployment rate of production workers is therefore higher under offshoring than in autarky.

We now look at the remaining domain  $\hat{\eta} \in (0, \hat{\eta}_{fm}^0)$  and show that  $\hat{\psi}(\hat{\eta})$  falls in this domain from a maximum level of  $\hat{\psi}(0) = 1 > \psi(0) = 0$  to a minimum level of  $\hat{\psi}(\hat{\eta}_{fm}^0) = 0 < \psi(\hat{\eta}_{fm}^0)$ . If this is the case, there must be a unique  $\hat{\eta}_u \in (0, \hat{\eta}_{fm}^0)$ , such that  $\psi(\hat{\eta}) >, =, < \hat{\psi}(\hat{\eta})$  if  $\hat{\eta} >, =, < \hat{\eta}_u$ . Differentiation of  $\hat{\psi}(\hat{\eta})$  establishes

$$\begin{aligned} \hat{\psi}'(\hat{\eta}) &= \frac{[1 - \hat{\eta}t(1-\theta)\xi][1 + \psi_2(\hat{\eta})]\psi_1(\hat{\eta}) - (1 - \hat{\eta}t\xi)[1 + \psi_1(\hat{\eta})]\psi_2(\hat{\eta})}{(1 - \hat{\eta})[\psi_2(\hat{\eta})]^2}, \\ &= \frac{1 + \psi_2(\hat{\eta})}{(1 - \hat{\eta})[\psi_2(\hat{\eta})]^2} \Psi(\hat{\eta}; \theta, \xi), \end{aligned} \quad (\text{A.53})$$

with

$$\Psi(\hat{\eta}; \theta, \xi) \equiv (1 - \hat{\eta}t\xi)[(1 - \hat{\eta})^{-t\theta\xi} e^{-\hat{\eta}t\theta\xi} - 1] + \hat{\eta}t\theta\xi[(1 - \hat{\eta})^{1-t\xi} e^{-\hat{\eta}t\xi} - 1], \quad (\text{A.54})$$

and  $\hat{\psi}(\hat{\eta}) >, =, < 0$  if  $\Psi(\hat{\eta}; \theta, \xi) >, =, < 0$ . Twice differentiating  $\Psi(\cdot)$  with respect to  $\theta$  (keeping  $\xi$  constant) gives

$$\begin{aligned} \Psi'_\theta(\hat{\eta}; \theta, \xi) &= -t\xi(1 - \hat{\eta}t\xi)[\ln(1 - \hat{\eta}) + \hat{\eta}](1 - \hat{\eta})^{-t\theta\xi} \exp[-\hat{\eta}t\xi] \\ &\quad + \hat{\eta}t\xi \left\{ (1 - \hat{\eta})^{1-t\xi} \exp[-\hat{\eta}t\xi] - 1 \right\} \end{aligned} \quad (\text{A.55})$$

$$\Psi''_{\theta\theta}(\hat{\eta}; \theta, \xi) = (t\xi)^2(1 - \hat{\eta}t\xi)[\ln(1 - \hat{\eta}) + \hat{\eta}]^2(1 - \hat{\eta})^{-t\theta\xi} \exp[-\hat{\eta}t\xi] > 0. \quad (\text{A.56})$$

Hence, if  $\Psi(\hat{\eta}; \theta, \xi)$  has an extremum in  $\theta$ , it must be a minimum. We can now compute

$$\Psi(\hat{\eta}; 1, \xi) = (1 - \hat{\eta}t\xi) \left\{ (1 - \hat{\eta})^{-t\xi} \exp[-\hat{\eta}t\xi] - 1 \right\} + \hat{\eta}t\xi \left\{ (1 - \hat{\eta})^{1-t\xi} \exp[-\hat{\eta}t\xi] - 1 \right\} \quad (\text{A.57})$$

Differentiating the latter with respect to  $\hat{\eta}$  gives

$$\Psi_{\hat{\eta}}(\hat{\eta}; 1, \xi) = t\xi\hat{\eta}(1 - \hat{\eta})^{-t\xi} \exp[-\hat{\eta}t\xi] \left\{ (1 - \hat{\eta}t\xi) \frac{\hat{\eta}}{1 - \hat{\eta}} - 1 \right\}, \quad (\text{A.58})$$

which, in view of  $t > 1/\xi$  (which is always fulfilled if  $t > \max\{t_1(\theta), t_2(\theta)\}$ ), is unambiguously negative. Since  $\Psi(0; 1, \xi) = 0$ , we can therefore safely conclude that  $\Psi(\hat{\eta}; 1, \xi) < 0$  holds for all  $\hat{\eta} > 0$ . Combining this with  $\Psi(\hat{\eta}; 0, \xi) = 0$ , it follows from  $\Psi''_{\theta\theta}(\hat{\eta}; \theta, \xi) > 0$  that  $\Psi(\hat{\eta}; \theta, \xi) < 0$  holds for all possible  $\hat{\theta}, \hat{\eta} > 0$ , proving that  $\hat{\psi}'(\hat{\eta}) < 0$ . Hence, there exists indeed a unique  $\hat{\eta}_u \in (0, \hat{\eta}_{fm}^0)$ , for which  $\psi(\hat{\eta}) = \hat{\psi}(\hat{\eta})$ . Putting together the insights for the three different cases, we can thus safely conclude that  $\delta(\hat{\eta}) >, =, < 1$  if  $\hat{\eta}_u >, =, < \hat{\eta}$ . This completes the proof.

## A.11 Proof of Proposition 2

From the main text we know that  $\Lambda(\hat{\eta}) = (u/u^a)(L/L^a)$ , with superscript  $a$  referring to autarky. That offshoring increases the economy-wide rate of unemployment at low levels of  $\hat{\eta}$  follows then from the observation that  $\delta(\hat{\eta}) > 1$  and thus  $u < u^a$  if  $\hat{\eta} < \hat{\eta}_u$  (see above) and the fact that labour supply increases monotonically in  $\hat{\eta}$  (and thus  $\chi$ ). Furthermore, the economy-wide rate of unemployment  $U$  is larger (smaller) at  $\hat{\eta} = \hat{\eta}_{int}$  and thus  $\chi = 1$  than under autarky if  $\Lambda(\hat{\eta}_{int}) > (<)1$ , which is equivalent to<sup>18</sup>

$$\hat{\eta}_U \equiv 1 - \frac{\theta\xi}{\theta\xi + (1 - 2^{-\theta})(k\sigma - \xi)} > (<)\hat{\eta}_{int}. \quad (\text{A.59})$$

Taking into account that  $\hat{\eta}_{int}$  is implicitly defined as function of  $t$  by  $\chi = 1$  and thus by  $\{(1 - \hat{\eta}_{int})^{-t\xi} \exp[-\hat{\eta}_{int}t\xi] - 1\}^{k/\xi} = 1$ , it follows that  $\lim_{t \rightarrow 0} \hat{\eta}_{int} = 1$ ,  $\lim_{t \rightarrow \infty} \hat{\eta}_{int} = 0$ , and  $d\hat{\eta}_{int}/dt < 0$ . Hence, there exists a unique  $t$  for which  $\hat{\eta}_U = \hat{\eta}_{int}$ . This critical  $t$  is denoted  $t_U$  and given by (31). For  $t > (<)t_U$ , we have  $\Lambda(\hat{\eta}_{int}) > (<)1$ . This completes the proof.

<sup>18</sup>It is clear that  $\hat{\eta}_U \in (0, 1)$ . Furthermore, one can show that  $\hat{\eta}_U > 1/2$  if  $k \geq 2$ . To see this, note that  $\hat{\eta}_U >, =, < 1/2$  is equivalent to  $0 >, =, < \Gamma(\theta, k, \xi) \equiv 2^\theta\theta\xi - (2^\theta - 1)(k\sigma - \xi)$ . To determine the sign of  $\Gamma(\theta, k, \xi)$ , let us first consider a parameter domain with  $k \geq \xi \geq 2$ . In this case, we have  $\Gamma(\theta, k, \xi) \leq \Gamma(\theta, \xi, \xi) = z(\theta)\xi$ , with  $z(\theta) \equiv (\sigma - 1) - 2^\theta(\sigma - 1 - \theta)$ . Since  $\xi \geq 2$  implies  $\sigma - 1 \geq 2$  and  $z(\theta)$  decreases in  $\sigma - 1$ , we can further conclude that  $z(\theta) \leq 2 - 2^\theta(2 - \theta) \equiv \underline{z}(\theta)$ . Differentiation of  $\underline{z}(\theta)$  gives  $\underline{z}'(\theta) = 2^\theta[1 - \ln 2(2 - \theta)]$  and  $\underline{z}''(\theta) = 2^\theta \ln 2[2 - \ln 2(2 - \theta)] > 0$ . From inspection of these derivatives, it follows that  $\underline{z}(\theta)$  has a unique extremum, which is a minimum. Noting further that  $\underline{z}(0) = \underline{z}(1) = 0$ , it is clear that  $\underline{z}(\theta) < 0$  must hold for all  $\theta \in (0, 1)$ . This proves that  $\Gamma(\theta, k, \xi) < 0$  and thus  $\hat{\eta}_U < 1/2$  if  $k \geq \xi \geq 2$ . Let us now consider a parameter domain with  $k \geq 2 > \xi$ . In this case, we have  $\Gamma(\theta, k, \xi) \leq \Gamma(\theta, 2, \xi) = \hat{z}(\theta)\xi$ , with  $\hat{z}(\theta) \equiv 2^\theta\theta - (2^\theta - 1)[(\sigma + 1)/(\sigma - 1) + 2\sigma\theta]$ . Noting that  $\sigma + 1 + 2\sigma\theta(\sigma - 1) = (\sigma - 1)[1 + 2/\xi + 2\theta(\sigma - 1)]$ , it follows from  $\xi < 2$  that  $\sigma + 1 + 2\sigma\theta(\sigma - 1) > 2(\sigma - 1)[1 + \theta(\sigma - 1)]$  and thus  $\hat{z}(\theta) < 2[1 + \theta(\sigma - 1)] - 2^\theta\{2[1 + \theta(\sigma - 1)] - \theta\} < \underline{z}(\theta)$ . From above, we know that  $\underline{z}(\theta) < 0$  holds for all  $\theta \in (0, 1)$ . This confirms that  $\Gamma(\theta, k, \xi) < 0$  and thus  $\hat{\eta}_U > 1/2$  if  $k \geq 2 > \xi$ .