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Buffer-stock savings and households' wealth response to income shocks

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Abstract

We structurally estimate a buffer-stock savings model using panel data from the Italian Survey of Household Income and Wealth that contains information not only about income and consumption but also wealth. We exploit the information about wealth and the responses of wealth and consumption to income shocks over different time horizons to infer the degree of insurance against permanent and transitory income shocks. The estimated model implies that Italian households can insure 5-10% of a permanent shock and 90-95% of a transitory shock. The degree of insurance against permanent shocks is at the low end of the range of existing estimates for the U.S.

Keywords: Consumption, Wealth, Permanent income, Precautionary savings, Incomplete markets, Insurance.

JEL Classification: D91, E21.

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1 Introduction

The ability of households to insure against income shocks determines the scope for tax and social insurance policies. The welfare gains from policies that reduce income fluctuations depend on the extent to which private risk sharing allows households to decouple consumption from income shocks.

A large literature has tried to estimate the amount of insurance available to households by analyzing the response of consumption to income shocks. Two polar benchmark models have provided the theoretical framework for this effort. On the one hand, the complete market model assumes that agents can insure *ex ante* against all contingencies and, therefore, implies that consumption should not respond to idiosyncratic income shocks be they permanent or transitory. The predictions of this model are typically strongly rejected (e.g. Cochrane 1991, Attanasio and Davis 1996). On the other hand, the permanent income model (PIH) assumes that unconstrained risk-free borrowing and lending is the only way to (self-)insure against income shocks and implies that consumption should respond fully to permanent shocks but only marginally to temporary ones. Contrary to the predictions of this theory, a common finding (e.g. Campbell and Deaton 1989, Blundell, Pistaferri and Preston 2008, Attanasio and Pavoni 2011) is that the marginal propensity to consume out of permanent income shocks is less than one. Equivalently, consumers partially insure against permanent income shocks (Blundell et al. 2008).

Carroll (2009) shows that a buffer-stock model with impatient consumers, constant-relative-risk-aversion (CRRA) preferences and a single, risk-free asset predicts a response of consumption to permanent income shocks that lies in between the above-mentioned lower and upper bounds implied by the complete-markets and PIH model, respectively. Carroll (2009) finds that the marginal propensity to consume out of permanent income shocks is strictly below, but close to, one, as long as income is subject to both permanent *and* transitory shocks: its average value varies between 0.75 and 0.92 for plausible degrees of patience and risk aversion.

If the marginal propensity to consume out of a permanent income shock is less than one, part of this shock is insured. Carroll's (2009) result thus implies that partial (self-)insurance against permanent income shocks may be consistent with an incomplete-markets structure that allows only for a risk-free asset. Conversely, any degree of consumption insurance beyond the self-insurance implied by the buffer-stock model suggests that additional insurance channels are at work (e.g. Attanasio and Pavoni 2011).

To sum up, the response of consumption to permanent income shocks provides a test of alternative incomplete-market models. Estimating such a response requires identifying the permanent and transitory component of the total income change observed in the data. Blundell et al. (2008) propose an identification strategy that relies on panel data for consumption and income and estimate a response of consumption to permanent income shocks of 0.64 based on data in the PSID for the U.S.¹

Krueger and Perri (2011) have proposed an alternative way to test the predictions of the buffer-stock and PIH incomplete-markets model that does not require identification of permanent and temporary income shocks, but that instead relies on panel information on consumption, income *and* wealth. They argue that the long run wealth response to income shocks is informative about the degree of partial insurance against permanent income shocks. Using the Survey

¹Blundell et al. (2008) define the partial insurance coefficients for (permanent and transitory) idiosyncratic income shocks as the fraction of the shock that translates into a consumption change. This is equivalent to the marginal propensity to consume out of the shock. We follow Kaplan and Violante (2010) in defining the insurance coefficient as the fraction of the shock that does *not* translate into a consumption change.

of Household Income and Wealth (SHIW) for Italy, a panel dataset containing information on consumption, income *and* wealth since 1987, they estimate a response of wealth to income changes that decreases with the time horizon. Krueger and Perri (2011) argue that this is consistent with the qualitative predictions of the PIH model but not with the predictions of the buffer-stock model, in which the partial insurance of permanent income shocks would generate a wealth response to income shocks that increases, rather than decreases, with the time horizon.

This result is somewhat surprising in the light of the findings by Blundell et al. (2008) and Kaplan and Violante (2010). One would expect that whether the buffer-stock model implies a response of wealth to income changes that decreases with the horizon depends on the implied degree of partial insurance of permanent income shocks. Carroll (2009) has shown that the buffer-stock model can generate a wide range of values for the response of consumption to permanent shocks for reasonable degrees of impatience and risk aversion. For some plausible parameter combinations, the response of consumption to permanent shocks is very close to one. By continuity one would expect that the predictions of the buffer-stock model may be then close to those of the PIH model. It thus appears crucial to estimate the degree of impatience and risk aversion by fitting moments of the same dataset for which the size of the response of consumption to income shocks have been estimated.

For this reason, we reassess the findings in Krueger and Perri (2011) by structurally estimating a buffer-stock model using the same SHIW dataset. In particular, we estimate the time-preference rate and the coefficient of relative risk aversion by the simulated methods of moments. Our target moments are the same moments that Krueger and Perri (2011) target: the response of consumption and wealth to income shocks at different horizons. We add mean or median wealth as an additional moment which helps to identify the rate of time preference.

The estimated model implies that Italian households can insure 5 to 10% of a permanent income shock and 90 to 95% of a transitory shock. The degree of insurance against permanent shocks in Italy is at the low end of the range of existing estimates for the U.S. reported in Blundell et al. (2008), Kaplan and Violante (2010) and Guvenen and Smith (2014). Our structural estimates are in line with existing estimates for Italy in Jappelli and Pistaferri (2006) and Jappelli and Pistaferri (2011) based on the consumption and income data of the SHIW.

Given our parameter estimates, we find that the buffer-stock model can match the pattern of both the consumption *and* wealth responses to income shocks at least as well as the PIH model. Namely, we find that partial insurance of permanent income shocks does not necessarily imply a response of wealth to income shocks that increases over time.

Our structural estimation is related to Gourinchas and Parker (2002) and Cagetti (2003) who estimate the rate of time preference and the coefficient of risk aversion in life-cycle versions of the buffer-stock model by matching, respectively, the consumption-age profile and the wealth-age profile. To the best of our knowledge, ours is the first paper that estimates a buffer-stock model by using panel information to match *profiles* of the consumption *and* wealth responses to income changes at different time horizons. Our estimates of the coefficient of relative risk aversion lie in between those of Gourinchas and Parker (2002) and Cagetti (2003) and are robust to whether mean or median wealth is included among the targeted moments.

The rest of the paper is structured as follows. Section 2 provides the theoretical background for the consumption and wealth responses to income shocks. It discusses why the precautionary-savings model may generate a wealth response to income shocks that declines over the considered time horizon. Section 3 describes the data. Section 4 lays out the empirical methodology and presents the results. We conclude in Section 5.

2 Theoretical background

This section introduces two canonical versions of the standard incomplete market model—the permanent-income and the buffer-stock model—and discusses the testable implications we are going to exploit in our empirical analysis. For both versions, we assume that consumers have an infinite horizon, derive time-separable utility from consumption, discount the future at rate δ and can borrow and lend at given interest rate r . In each period they face the dynamic budget constraint

$$c_t + a_{t+1} = (1 + r)a_t + y_t, \quad (1)$$

where c_t and y_t denote respectively the flows of consumption and labor income in period t and a_t the stock of wealth (net worth) at the end of period t .

2.1 The permanent-income hypothesis

In the permanent income model, consumers have a quadratic felicity function, borrowing is unconstrained—subject to solvency—and the interest rate r equals the discount rate δ .

The consumer’s stochastic labor income y_t follows the process

$$\begin{aligned} y_t &= z_t + \varepsilon_t, \\ z_t &= z_{t-1} + \eta_t, \end{aligned} \quad (2)$$

where $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon)$ is a transitory income shock and $\eta_t \sim \mathcal{N}(0, \sigma_\eta)$ is a permanent income shock. The shocks ε_t and η_t are assumed to be uncorrelated with each other in each period t and i.i.d. over time.² Since measurement error in income changes may be quantitatively important (Altonji and Siow 1987) and may affect the interpretation of the regression results presented in Section 4.1, we allow for the possibility that the econometrician does not observe the true income realization y_t but instead

$$\tilde{y} = y_t + \gamma_t,$$

where $\gamma_t \sim \mathcal{N}(0, \sigma_\gamma)$ is classical measurement error.

It is well known (e.g., Deaton 1992) that the changes in income, consumption and wealth in this model satisfy

$$\begin{aligned} \Delta \tilde{y}_t &= \eta_t + \Delta \varepsilon_t + \Delta \gamma_t \\ \Delta c_t &= \frac{r}{1+r} \varepsilon_t + \eta_t \\ \Delta a_t &= \frac{\varepsilon_t}{1+r}. \end{aligned} \quad (3)$$

The same changes can easily be expressed for an arbitrary number of periods N , by noticing that

$$\Delta^N x_t = \frac{x_t - x_{t-N}}{N} = \frac{1}{N} \sum_{\tau=t-N+1}^t \Delta x_\tau.$$

²At the estimation stage, the permanent-transitory decomposition in (2) is assumed to apply to the logarithm of labor income process, since the income distribution in the data is skewed. The specification of the process in levels is maintained here only for analytic tractability.

As shown in Krueger and Perri (2011), it follows from (3) that

$$\begin{aligned}\Delta^N y_t &= \frac{1}{N} \sum_{\tau=t-N+1}^t (\eta_t + \Delta \varepsilon_t + \Delta \gamma_t) \\ \Delta^N c_t &= \frac{1}{N} \sum_{\tau=t-N+1}^t \left(\frac{r}{1+r} \varepsilon_t + \eta_t \right) \\ \Delta^N a_t &= \frac{1}{N} \sum_{\tau=t-N+1}^t \frac{\varepsilon_t}{1+r}.\end{aligned}\tag{4}$$

This implies that the coefficients of the linear regressions

$$\Delta^N c_t = \beta_c^N \Delta^N y_t + u_t^N \tag{5}$$

$$\Delta^N a_t = \beta_a^N \Delta^N y_t + v_t^N \tag{6}$$

satisfy

$$\beta_c^N = \frac{\text{Cov}(\Delta^N c_t, \Delta^N y_t)}{\text{Var}(\Delta^N y_t)} = \frac{N\sigma_\eta^2 + r\sigma_\varepsilon^2/(1+r)}{N\sigma_\eta^2 + 2(\sigma_\varepsilon^2 + \sigma_\gamma^2)} = \frac{NQ + (1-M)\frac{r}{1+r}}{NQ + 2} \tag{7}$$

$$\beta_a^N = \frac{\text{Cov}(\Delta^N c_t, \Delta^N y_t)}{\text{Var}(\Delta^N y_t)} = \frac{\sigma_\varepsilon^2}{(1+r)[N\sigma_\eta^2 + 2(\sigma_\varepsilon^2 + \sigma_\gamma^2)]} = \frac{1-M}{(1+r)[NQ + 2]}, \tag{8}$$

where $Q = \sigma_\eta^2/(\sigma_\varepsilon^2 + \sigma_\gamma^2)$ measures the size of the variance of the permanent shock relative to the variance of transitory income (due to shocks and measurement error), and $M = \sigma_\gamma^2/(\sigma_\varepsilon^2 + \sigma_\gamma^2)$ measures how much of the variance of transitory income is due to measurement error.³

Equations (7) and (8) imply that the consumption response β_c^N is increasing and the wealth response β_a^N is decreasing in the horizon length N . Intuitively, transitory shocks average out, while permanent shocks cumulate, as the horizon increases. Since the PIH implies that consumption responds one-to-one and wealth not at all to permanent shocks, the response of consumption increases with N while that of wealth decreases. As pointed out by Krueger and Perri (2011), these two qualitative predictions of the PIH are testable with a panel data set on income, consumption *and* wealth.

2.2 The buffer-stock precautionary saving model

In this model, agents have a precautionary-saving motive due to the presence of either occasionally binding borrowing constraints or a utility function that displays prudence ($u'''(c) > 0$).

Carroll (1997) shows that if the felicity function has the CRRA form $u(c_t) = (c_t^{1-\alpha} - 1)/(1-\alpha)$, $\alpha > 0$ and agents are impatient—($\delta > r$)—saving displays buffer-stock behaviour. Agents accumulate assets—the precautionary saving motive more than offsets impatience—if wealth is below a target level and dissave—impatience dominates precautionary saving—if wealth is

³In Appendix A.1 we generalize the above results to the case in which the shock η_t is persistent but not necessarily permanent. We show that, for plausible parameter values, the PIH model can generate a wealth response to income changes which decreases in the time horizon, only if income shocks are very persistent and the variance of the measurement error is not too large. In other words, the measured wealth response to income changes imposes restrictions on the persistence of the shocks η_t and the size of measurement error in the permanent income model.

above that target level.⁴ If in addition log income is the sum of a permanent and transitory component

$$\log y_t = z_t + \varepsilon_t \quad (9)$$

with z_t and ε_t having the same properties as in Section 2.1, then agents target a wealth-to-permanent-income *ratio*. Buffer-stock saving behavior implies a response of consumption to permanent income shocks that is strictly less, though possibly close to, one (Carroll 2009). Intuitively, for given wealth a positive permanent income shock reduces the wealth-to-permanent-income ratio below its target, thus inducing an increase in saving that dampens the consumption response relative to the PIH. Symmetrically, a negative permanent income shock induces a fall in saving which again dampens the consumption response.

Therefore, depending on the magnitude of the saving response to permanent income shocks the buffer-stock model implies that the wealth response to income shocks β_a^N in equation (8) may be increasing in the time horizon N , rather than decreasing as in the PIH. Krueger and Perri (2011) argue that this is likely to be the case and they show that their conjecture is verified in a parameterized version of the buffer-stock model with $\sigma = 2$ and $r = \delta = 0.02$ that is simulated for 45 periods. On the basis of these findings they conclude that the empirical evidence on the wealth response to income shocks measured over different time horizons, presented in Section 4.1 below, supports the PIH but not the buffer-stock model.

Their findings are somewhat surprising in the light of, for example, Blundell et al. (2008) and Kaplan and Violante (2010) who estimate a marginal propensity to consume out of permanent shocks smaller than one which is consistent with the buffer-stock model but not with the PIH. Also according to Carroll's (2009) numerical simulations the buffer-stock model can generate a wide range of values for the marginal propensity to consume out of permanent income for reasonable combinations of the coefficient of relative risk aversion and the rate of time preference. Therefore one would expect that whether or not the wealth response to income changes in the buffer-stock model is consistent with the data, depends crucially on the value of those parameters. It thus seems important that the value of those parameters is estimated using the same dataset with which the responses are estimated. So, we structurally estimate a permanent-income model and a buffer-stock model by matching the profile of the consumption and wealth responses to income changes in the model to the profile of the responses in the data. In doing so, we build on Krueger and Perri's (2011) insight that panel data on income, consumption and wealth allow to exploit information in the time profile of the individual consumption and wealth responses to income changes, in order to estimate the importance of the precautionary saving motive.

3 Data

The Italian Survey of Households Income and Wealth (SHIW) is conducted by the Bank of Italy. Since 1987 the survey has been conducted every two years (with the exception of a three-year gap between 1995 and 1998) and covers a representative sample of around 8,000 households, a fraction of which are observed for a number of years. A rather unique feature of this data set is that it contains comprehensive panel information over a long time period about not only

⁴This would not be true if we assumed CARA utility and no borrowing constraints. In this case, the predictions for the consumption and wealth responses are identical to the permanent income model because the precautionary motive does not depend on the stock of wealth. Although analytically convenient, it is well known that CARA utility has counterfactual implications for how risk-taking behavior depends on wealth.

household income and consumption, but also wealth.⁵ As pointed out by Krueger and Perri (2011), the combination of the panel dimension together with the availability of wealth, in addition to consumption and income, helps to infer the response of household consumption to different type of income shocks. We use non-durable consumption, labor earnings after taxes and transfers, and net worth as the data counterpart of consumption c_t , non-capital income y_t and wealth a_t .

We focus on households with a head aged 25-55 so that labor earnings are not substantially influenced by labor force participation decisions related to education and retirement, which we do not model. Our benchmark sample also excludes entrepreneurs and self-employed for whom labor earnings are hard to measure. Finally, since our estimation strategy exploits the consumption and wealth responses to income changes at different horizons, we restrict our sample to only those households which we observe in sufficiently many consecutive waves. We stop at a time horizon of six years because extending the horizon for the income changes by another two years would reduce the sample size by 50%. This leaves us with an unbalanced sample of 520 households in the time period 1987 to 2012 for a total of 1,077 observations. All nominal variables are measured in units of Euro in 2000 and converted to adult-equivalents, using the OECD equivalence scale to control for differences in household size. Variables are then normalized by expressing them in units of average equivalized net labor earnings in the sample (approximately 10,000 Euro in 2000). Appendix A.4 provides further details on how we clean the data and construct our sample and Table 7 in the appendix presents summary statistics.

Since our aim is to infer the response of consumption and wealth to unanticipated, idiosyncratic income changes, we follow Krueger and Perri (2011) in purging the data from aggregate and predictable individual effects. We do so by regressing the observed changes on a quartic polynomial in the age of the household head, on education, time and regional dummies as well as age-education interaction dummies. We then use the residuals of these regressions in our empirical analysis.

4 Estimation

Since the buffer-stock model does not imply closed-form policy functions we estimate it by the method of simulated moments. In the spirit of indirect inference the moments we use as targets are generated by an auxiliary model which, in our application, provides useful economic insights to interpret the results.

Our discussion in Section 2.1 suggests that equations (5)-(6) are an ideal candidate for the auxiliary model as the regression coefficients are informative about the model parameters and thus the degree of insurance of permanent and transitory shocks. Therefore, we choose as target moments the regression coefficients β_c^N, β_a^N for $N = 2, 4, 6$. This yields a total of 6 reduced form parameters. The auxiliary model does not imply a target for the *level* of wealth. Yet, the ability to self-insure and the marginal propensity to consume out of transitory and permanent income shocks depend on the stock of assets available. For this reason, we add (the average or median) wealth as an additional target moment following Guvenen and Smith (2014).

The goal of our estimation procedure is to choose the parameters of the structural model so that the regression coefficients estimated on the data are close to those estimated on the model-simulated data. The metric we use is the the weighted sum of the squared deviations of the simulated model moments from the target data moments. The weighting matrix is

⁵The PSID in the U.S. only contains similarly rich data since 1999.

the variance-covariance matrix of the model moments, thus taking into account the model's predictions about the precision with which the data moments are estimated.

Column (1) in Table 1 reports the estimates for the responses of non-durable consumption and net worth to income changes over two, four and six years. Since there are some outlier wealth observations, we report the estimation results of median regressions which minimize absolute deviations and are thus robust to outliers. The responses of consumption and wealth to income shocks are positive, as one would expect. The responses of consumption are increasing in the length of the time horizon N whereas the wealth responses are decreasing and also less precisely estimated. Column (1) also reports the ratio of the mean and median values of net worth to average labour income in the sample. Agents in the sample hold an average net worth amounting to 2.5 times the size of average equivalized net labor earnings. The median is much smaller at 0.67, in line with the evidence from a large number of countries that the wealth distribution is right skewed.

We estimate only a subset of the model parameters in Section 2.2 and use external estimates for the others. In particular, we set the risk-free real interest rate to $r = 0.02$. We calibrate the relative importance of the variance of the permanent and transitory shock using estimates by Jappelli and Pistaferri (2006) and Jappelli and Pistaferri (2010) for Italy based on the SHIW. The variance of the transitory shock $\sigma_\varepsilon^2 = 0.0794$ and the variance of the permanent shock $\sigma_\eta^2 = 0.0267$.⁶ These parameters can be identified from income data alone and little would be gained from replicating them here.

Because we measure income changes with error and measurement error has a sizable effect on the wealth responses to income changes (see Appendix A.1.1), these responses also provide information about the extent of measurement error in the data. We thus estimate the measurement error together with the other parameters.

Finally, the assumption that labor income is lognormally distributed implies that the lowest income realization is zero. This implies that the natural borrowing limit is zero. We assume that this is the borrowing constraint that consumers face.

This leaves us with three parameters $\{\delta, \alpha, \sigma_\gamma^2\}$ to estimate: the rate of time preference, the coefficient of relative risk aversion and the variance of measurement error.

The estimation is conducted in the following way. We draw an initial distribution of wealth over 25,000 individuals according to the wealth distribution in the data and, for each individual, a 45-period long shock history. For each triplet of parameter on a grid, we solve and simulate the model and compute the targeted moments. The estimated parameter values are those that minimize the distance between the targeted moments in the model and in the data. After some experimentation with coarser grids, we specify the following finer grid with $1/(1 + \delta) \in [0.91, 0.98]$ with distance 0.0025 between adjacent gridpoints of the discount rate δ , $\alpha \in [1, 3]$ with distance 0.25 between adjacent gridpoints and $\sigma_\gamma^2 \in [0, 0.5]$ with distance 0.02 between adjacent gridpoints. See Appendix A.5.2 for further information on the model solution and estimation.

4.1 Results

Column (2) in Table 1 reports the results for our preferred specification in which our wealth target is the mean of net worth, expressed in units of average equivalized net labor earnings in the sample. The time preference rate δ is estimated to be 0.055, the estimated coefficient of relative risk aversion $\alpha = 2.8$ and the point estimate of the standard deviation of measurement error $\sigma_\gamma^2 = 0$. All three parameters are precisely estimated. In terms of the targeted moments,

⁶These estimates for Italy based on income differences are similar to estimates for the U.S. reported in Heathcote et al. (2010), Figure 18.

Table 1: Structural Estimation Results

	Data	Buffer-Stock Model		PIH	
	(1)	(2)	(3)	(4)	(5)
<i>Parameter estimates</i> ^a					
Discount Rate δ		0.055 (0.003)	0.060 (0.003)	0.033 (0.003)	– –
Risk Aversion α		2.800 (0.128)	2.600 (0.103)	2.800 (0.176)	– –
Measurement Error σ_γ^2		0.000 (0.004)	0.005 (0.002)	0.000 (0.021)	0.000 (0.000)
<i>Target moments</i> ^a					
β_c^2	0.266 (0.022)	0.254 (0.017)	0.280 (0.018)	0.202 (0.014)	0.254 (0.019)
β_c^4	0.279 (0.024)	0.390 (0.019)	0.423 (0.020)	0.314 (0.016)	0.399 (0.022)
β_c^6	0.384 (0.022)	0.482 (0.020)	0.518 (0.021)	0.390 (0.016)	0.496 (0.022)
β_a^2	0.433 (0.088)	0.395 (0.034)	0.319 (0.030)	0.409 (0.153)	0.367 (0.034)
β_a^4	0.380 (0.114)	0.362 (0.048)	0.306 (0.040)	0.410 (0.368)	0.294 (0.045)
β_a^6	0.395 (0.144)	0.353 (0.057)	0.301 (0.045)	0.445 (0.598)	0.246 (0.052)
Mean Wealth	2.638 (0.13)	2.764 (0.087)	1.714	6.810	2.644
Median Wealth	0.670	1.817	0.713 (0.031)	5.996	1.124

^a Standard errors in parentheses. Boldface indicates that the estimated moment is statistically different from its target at a 5% level. Framed moments are untargeted.

the model captures both the upward sloping profile of the consumption responses β_c^N and the downward sloping profile of the wealth response β_a^N though the regression coefficients β_c^4 and β_c^6 estimated on the simulated data are statistically different from their data counterpart. Effectively, the model implies a profile for the consumption responses β_c^N that is substantially steeper than in the data. Finally, the model also matches well the targeted mean wealth, though the untargeted median wealth is overestimated by a factor of three. As is well known the precautionary saving model cannot match both mean and median wealth in the absence of further assumptions, such as heterogeneity in time preference rates in Krusell and Smith (1998).

Column (3) in Table 1 reports results for the alternative specification in which the wealth moment targeted is the median value of 0.670 rather than substantially higher mean value of 2.638. Cagetti (2003) has shown that structural estimates of time preference and risk aversion can be very sensitive to whether one or the other target is used. In contrast to his findings, our results show that though targeting the median respectively increases and reduces the point estimates for the time preference and risk aversion parameters, the difference between the

estimates in column (2) and (3) is well within two standard errors. In terms of fitting the targeted moments, the model still captures qualitatively the profiles of the consumption and wealth responses in the data. Qualitatively, though, the specification in column (3) implies a substantially steeper profile for the consumption responses β_c^N and a worse fit of the wealth responses β_a^N .

It is insightful to compare the estimated responses for the buffer-stock model with those generated by the PIH model. In order to make the results comparable with the buffer-stock model, we assume the same log-normal income process.⁷ See Appendix A.5 for further information on consumption and asset accumulation in the permanent-income model with log-normally distributed income, as well as for further information on the methods used to simulate both the permanent-income and precautionary-savings model.

With an infinite horizon, PIH model requires the rate of time preference to equal the interest for the wealth distribution not to be degenerate. This leaves the standard deviation of measurement error as the only parameter to estimate. The targeted moments are only the consumption and wealth responses to income shocks, as wealth levels are uninformative given that agents' behavior, and thus the responses to income changes are independent of wealth in the PIH. The results are reported in column (4) in Table 1. Comparing the responses to those for the buffer-stock model in columns (2) and (3) reveals that qualitatively the three specifications are able reproduce the profile of consumption and wealth responses in the data. Quantitatively, the PIH model performs if anything marginally worse than the buffer-stock model. This result contrasts with the finding in Krueger and Perri (2011) of wealth responses to income changes that increase with the time horizon in the buffer-stock saving model. The main difference is that their parameterization with the interest rate equal to the rate of time preferences implies much larger wealth levels than observed in the data.⁸ Our estimation procedure instead ensures a degree of impatience for which average or median wealth predicted by the model are close to their data counterparts.

As an additional check, column (4) in Table 1 reports the results for the buffer-stock model when we target only the consumption and wealth responses but not wealth levels. The model still generates an upward sloping profile for the consumption responses, but the profile of the wealth responses implied by their point estimates becomes upward sloping. The counterfactually high values of mean and median wealth confirm our intuition above that matching wealth levels is crucial pin down the degree of impatience and the strength of the precautionary saving motive. It has to be noted, though, that the wealth responses for this case are not estimated precisely so that one cannot reject the hypothesis that their profile is negative. The consumption responses are still precisely estimated instead.

Turning to the estimated values of parameters, while the estimate of the time preference rate nearly halves, the estimate for the RRA coefficient is very similar to the two specifications in columns (2) and (3). This is somehow unexpected given the previous findings of Cagetti (2003), Gourinchas and Parker (2002) and Guvenen and Smith (2014) about the difficulty of identifying separately the time preference rate and the coefficient of relative risk aversion. Our results suggest that the risk aversion coefficient α is mainly identified by the profile of the consumption responses to income shocks while average or median wealth identifies the time preference rate.

⁷The consumption and wealth responses to income shocks in the PIH model with a log-normal income process (column (4) in Table 1) are quantitatively similar to the responses for the PIH model with a normally distributed income process in levels as reported in row 1 of Table 3, Appendix A.1.

⁸For an infinite horizon, the target level of wealth would be infinite for their parametrization of the buffer-stock model.

4.2 Insurance coefficients

As discussed in Section 2.2 there is a tight link between the profile of the wealth response to income changes and the response of consumption to permanent income shocks. The larger the saving response to permanent income shocks—i.e., the larger the fraction of the shock that is insured and thus does not translate into a consumption change—the more likely it is that the wealth response to income shocks increases rather than decreases with the time horizon.

Given the income process for household i at time t

$$\log y_{it} = z_{it} + \varepsilon_{it}$$

described in Section 2, one can define the insurance coefficient for shock $x = z, \varepsilon$ as

$$\phi^x = 1 - \frac{\text{Cov}(\Delta \log c_{it}, x_{it})}{\text{Var}(x_{it})}, \quad (10)$$

where the variances and covariances are taken cross-sectionally over the sample of individuals. The insurance coefficient in equation (10) measures the share of the variance of the x shock that does not translate into consumption growth.

It is straightforward to compute (10) on simulated model data since the shocks are observable. On the other hand, computing it on actual data requires identifying the x shocks, a non-trivial task. Blundell et al. (2008) (BPP) have proposed a strategy to estimate the insurance coefficients that amounts to effectively instrumenting the shocks with leads and lags of log income changes. Kaplan and Violante (2010) have argued that, while the BPP methodology produces unbiased estimates of the insurance coefficient for transitory shocks ϕ^ε , the estimate of the insurance coefficient for permanent shocks ϕ^z is downward-biased in the presence of occasionally-binding borrowing constraint.⁹ For this reason we report below both the true value of the insurance coefficient and the coefficients obtained by using BPP’s approach on the model data.

Columns (2)-(4) in Table 2 report the insurance coefficients for the three model specifications in columns (2)-(4) of Table 1. The insurance coefficient for permanent shocks is between 0.05 and 0.1 for specifications (2) and (3) in which we target either the mean or median wealth level in the estimation. The fact that the insurance coefficient is close to its value of zero in the PIH model explains why both specifications imply a profile of wealth responses to income changes in Table 1 that is qualitatively similar to that implied by the PIH model. For specification (4) the insurance coefficient is 0.2 instead. This is consistent with the fact that this specification implies a substantially stronger precautionary saving motive, resulting in an average wealth level twice as large as in the data. This explains that the wealth responses to income changes are increasing in the time horizon in Table 1, column (4).

Turning to the insurance coefficients for transitory shocks, they lie between 0.9 and 0.97 for the three specifications, indicating that households in the model can effectively smooth transitory shocks.

Note that the insurance coefficients for both permanent and temporary shocks are increasing in the mean wealth level the model generates. They are lowest for specification (3) which implies the lowest mean wealth level of 1.7 and highest for specification (4) which generates the highest mean wealth level of 6.8.

⁹The borrowing constraint $a_{t+1} \geq 0$ is never binding in the theoretical model with log-normal income shocks in which the smallest income realization is zero: after this realization, consumption would be zero and marginal utility infinite if $a_{t+1} = 0$ so that agents always choose $a_{t+1} > 0$. The constraint may bind, however, in the simulated model in which the income process is discretized so that the lowest income value is strictly positive.

Table 2: Insurance Coefficients in the Buffer-Stock Model

Wealth level target:	Mean (2)	Median (3)	None (4)
<i>Insurance coefficient: true values</i>			
Permanent shock	0.10	0.07	0.20
Transitory shock	0.95	0.90	0.97
<i>Insurance coefficient: estimates based on BPP-methodology</i>			
Permanent shock	0.09	0.05	0.20
Transitory shock	0.95	0.93	0.97

Finally, our estimated coefficients are within the range of estimates by Jappelli and Pistaferri (2006) and Jappelli and Pistaferri (2011) based on income and consumption data of the SHIW. They report values between 0.01 and 0.13 for the insurance coefficient for permanent income shock and between 0.95 and 0.76 for transitory shocks.

Our estimate for the insurance coefficient for permanent shocks, though, is substantially smaller than the value of 0.36 estimated by Blundell et al. (2008) based on PSID data for the U.S. and the value of 0.22 in Kaplan and Violante's (2010) life-cycle model calibrated to U.S. data. The coefficient for transitory shock instead is very much in line with the findings in those two papers.

Appendix A.3 contains a robustness check of our results for our preferred specification (2) in which we target mean wealth. Krueger and Perri (2011) have argued that the residual income changes may be correlated with residual changes in real-estate wealth in the SHIW. For the subsample of renters, residual income changes thus better capture the effect of pure income shocks. We find that households in this subsample insure 93% of a temporary shock and 7% of a permanent shock. These insurance coefficients are slightly smaller than in our benchmark sample, possibly also because renters have less net worth to self insure.

5 Conclusions

We have used rather unique Italian panel data on consumption, income and wealth since 1987 to estimate the extent to which households self insure against income shocks. Building on insights in Krueger and Perri (2011), we focus on reduced-form estimates for the response of consumption and wealth to income shocks over different time horizons. These reduced-form estimates do not only provide important information about the persistence of income shocks but also, together with information on average wealth holdings, allow us to estimate the structural parameters of a standard buffer-stock precautionary savings model by indirect inference. The estimated model implies that Italian households can insure 5 – 10% of a permanent shock and 90 – 95% of a transitory shock. Compared with existing results for the U.S., this suggests that Italian households have substantially less insurance possibilities against permanent shocks than their

American counterparts who can insure 22 – 36% of a permanent shock according to recent estimates by Blundell et al. (2008) and Kaplan and Violante (2010). Understanding the reasons for this substantial difference is an important avenue for future research.

A Appendix

A.1 Responses to income shocks in the permanent-income model

We build on Krueger and Perri (2011) and derive analytic results for consumption and wealth responses to income shocks in the permanent-income model. This classic model is a useful starting point because it allows us to derive analytic solutions which are not available in the standard buffer-stock precautionary-savings model.

Compared with income process (2) in the main text, we assume that the persistent component of stochastic labor income z_t follows the process

$$z_t = \rho z_{t-1} + \eta_t,$$

with persistence $0 \leq \rho \leq 1$. That is, the shock η_t is allowed to be non-permanent.

To derive the predictions for the permanent-income model we assume a quadratic period utility function $u(c_t) = -\frac{1}{2}(\bar{c} - c_t)^2$. The intertemporal allocation of resources at an interior optimum is characterized by the standard Euler equation

$$u'(c_t) = \frac{1+r}{1+\delta} E_t u'(c_{t+1})$$

which, for the assumed quadratic utility function, simplifies to

$$c_t = \left(1 - \frac{1+r}{1+\delta}\right) \bar{c} + \beta(1+r) E_t c_{t+1}. \quad (11)$$

The purest version of the permanent-income model abstracts from tilting of the consumption profile and assumes $r = \delta$ so that $(1+r)/(1+\delta) = 1$. Under these assumptions, we now state how consumption, wealth and income change after transitory and persistent shocks. The proof, as for the other analytic results in this section, is provided in the next section of this appendix.

Remark 1 *In the permanent-income model with $r = \delta$ and income process (2), consumption, wealth and income changes are given by*

$$\begin{aligned} \Delta c_t &= \frac{r}{1+r} \varepsilon_t + \frac{r}{1+r-\rho} \eta_t, \\ \Delta a_{t+1} &= \frac{\varepsilon_t}{1+r} + \frac{1-\rho}{1+r-\rho} (\rho z_{t-1} + \eta_t), \\ \Delta y_t &= \Delta \varepsilon_t + \eta_t + (\rho - 1) z_{t-1}. \end{aligned}$$

The responses of consumption and wealth to the shocks in Remark 1 are well known so that we comment on them only briefly. Consumption changes by the annuity value of the transitory shock ε_t and wealth bears the remaining impact of that shock on resources. Consumption increases more after the persistent shock η_t and indeed that shock only affects consumption and not wealth if it is permanent ($\rho = 1$).

Because the panel data that we use do not contain direct information about transitory and persistent income shocks, we use the results in Remark 1 to derive predictions for changes

of wealth and consumption after unexpected changes in observed income. We compute the predictions of the model for changes over N periods since we exploit the SHIW data, as Krueger and Perri (2011), to compute consumption and wealth responses to changes in labor income over two, four and six years.

Remark 2 *If consumers behave according to the permanent-income model with $r = \delta$ and observed income follows the process (2), the response of consumption and wealth to changes in income over N periods is given by*

$$\beta_c^N = \frac{\text{cov}(\Delta^N c_t, \Delta^N y_t)}{\text{var}(\Delta^N y_t)} = \frac{\frac{1-\rho^N}{1-\rho} \frac{r}{1+r-\rho} Q + \frac{r}{1+r}}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho}\right) Q + 2}$$

$$\beta_a^N = \frac{\text{cov}(\Delta^N a_{t+1}, \Delta^N y_t)}{\text{var}(\Delta^N y_t)} = \frac{\frac{1-\rho^N}{1+\rho} \frac{1}{1+r-\rho} Q + \frac{1}{1+r}}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho}\right) Q + 2}.$$

with $Q \equiv \sigma_\eta^2 / \sigma_\varepsilon^2$.

Remark 2 makes explicit how the response of consumption and wealth to observed income changes depends on the relative importance of the persistent shock Q as well as on the periods N over which the change is measured. Note that Remark 2 nests the results of Krueger and Perri (2011) for $\rho = 1$ since by L'Hôpital's rule $\lim_{\rho \rightarrow 1} \frac{1-\rho^N}{1-\rho} = \lim_{\rho \rightarrow 1} \frac{-N\rho^{N-1}}{-1} = N$ and $\lim_{\rho \rightarrow 1} \frac{(\rho^N-1)^2}{1-\rho^2} = \lim_{\rho \rightarrow 1} \frac{1-2\rho^N+\rho^{2N}}{1-\rho^2} = \lim_{\rho \rightarrow 1} \frac{-2N\rho^{N-1}+2N\rho^{2N-1}}{-2\rho} = 0$.

The following corollary states how the responses change with N .

Corollary 1 *If consumers behave according to the permanent-income model with $r = \delta$ and $0 < Q < \infty$:*

- *the response of consumption to income shocks increases in the number of periods ($\partial \beta_c^N / \partial N > 0$) if $\rho = 1$ or $\rho < 1$ is sufficiently large and $Q < Q_c^*$;*
- *the response of wealth to income shocks decreases in the number of periods N over which the response is measured ($\partial \beta_a^N / \partial N < 0$) if $\rho = 1$ or $\rho < 1$ and $Q > Q_a^*$, where $Q_a^* < Q_c^*$.*

These results are intuitive. Consider first the case with a permanent income shock, $\rho = 1$. As the number of periods N increases, the wealth and consumption response to income changes depend more on the cumulated permanent shock rather than on the transitory shocks: the independently distributed transitory shocks offset each other over a longer horizon while the permanent shocks cumulate. Therefore the consumption response increases and the wealth response decreases in N .

If the component z_t in the labor income process (2) is not permanent but only persistent, the consumer changes his asset holdings to smooth consumption after changes in z_t . The effect of the change becomes weaker over time: the autocorrelation of the persistent shock ρ^N decreases in N for $0 < \rho < 1$. Thus, the effect of changes in the persistent income component z_t on consumption and wealth decreases in N ceteris paribus. The importance of this effect for the profile of the consumption and wealth response across N is smaller for high levels of persistence ρ (ρ^N then decreases less strongly in N). Corollary 1 shows that for a high enough persistence there exists $Q \in (Q_a^*, Q_c^*)$ so that the consumption response increases in N while the wealth response decreases in N .

Persistence $\rho \setminus$ Number of periods N	Consumption response β_c^N			Wealth response β_a^N		
	2	4	6	2	4	6
1	0.259	0.408	0.507	0.367	0.293	0.244
0.995	0.208	0.325	0.401	0.392	0.333	0.293
0.95	0.077	0.112	0.131	0.454	0.427	0.406
0.8	0.027	0.033	0.035	0.472	0.447	0.429
0.2	0.011	0.011	0.011	0.480	0.477	0.477

Table 3: Persistence and responses to income shocks over different number of periods N . Source: Authors' calculation. Note: The parameter values are $r = 0.02$, $Q = 0.34$.

Table 3 shows the behavior of the consumption and wealth response as a function of N for different ρ , using parameter values $r = 0.02$ and $Q = 0.34$ as in Krueger and Perri (2011), tables 6 and 7, where the value for Q is based on estimates for σ_η and σ_ε from Jappelli and Pistaferri (2006) for the Italian SHIW data. For these plausible parameter values, the consumption response to income shocks increases in N and the wealth response decreases in N , for all considered values of persistence ρ . For permanent shocks ($\rho = 1$) or very persistent shocks ($\rho = 0.995$) the wealth response falls more strongly and the consumption response increases more strongly in the number of periods N . The wealth and consumption response are flat, as one would expect, if the shock has very low persistence ($\rho = 0.2$).

A.1.1 Measurement error

As discussed in the data section, income changes observed by the econometrician are measured with error. We thus allow for measurement error in our estimation and derive consumption and wealth responses under the assumption that the econometrician observes the true income process y_t in equation (2) with error:

$$\tilde{y}_t = y_t + \gamma_t, \quad (12)$$

where $\gamma_t \sim \mathcal{N}(0, \sigma_\gamma^2)$ is classical measurement error and is assumed to be i.i.d. over time and uncorrelated with the income shocks ε_t and η_t .

Remark 3 *If consumers behave according to the permanent-income model with $r = \delta$ and observed income follows the process (12), the response of consumption and wealth to changes in income over N periods is given by*

$$\beta_c^N = \frac{\text{cov}(\Delta^N c_t, \Delta^N \tilde{y}_t)}{\text{var}(\Delta^N \tilde{y}_t)} = \frac{\frac{1-\rho^N}{1-\rho} \frac{r}{1+r-\rho} Q + \frac{r}{1+r} (1-M)}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho} \right) Q + 2} \quad (13)$$

$$\beta_a^N = \frac{\text{cov}(\Delta^N a_{t+1}, \Delta^N \tilde{y}_t)}{\text{var}(\Delta^N \tilde{y}_t)} = \frac{\frac{1-\rho^N}{1+\rho} \frac{1}{1+r-\rho} Q + \frac{1}{1+r} (1-M)}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho} \right) Q + 2}. \quad (14)$$

with

$$Q \equiv \frac{\sigma_\eta^2}{\sigma_\varepsilon^2 + \sigma_\gamma^2} \text{ and } M \equiv \frac{\sigma_\gamma^2}{\sigma_\varepsilon^2 + \sigma_\gamma^2}.$$

Persistence $\rho \setminus$ Number of periods N	Consumption response β_c^N			Wealth response β_a^N		
	2	4	6	2	4	6
1	0.254	0.404	0.503	0.092	0.073	0.061
0.995	0.203	0.321	0.398	0.117	0.113	0.111
0.95	0.072	0.107	0.128	0.181	0.210	0.223
0.8	0.022	0.029	0.031	0.202	0.225	0.233
0.2	0.005	0.006	0.006	0.210	0.212	0.212

Table 4: Measurement error and the responses to income shocks for different shock persistence. Source: Authors' calculation. Note: The parameter values are $r = 0.02$, $Q = 0.34$, $M = 0.75$.

Remark 3 makes explicit how the response of consumption and wealth to observed income changes depends on the relative importance of measurement error M . We summarize the effect of measurement error on the responses in the following corollary.

Corollary 2 *If consumers behave according to the permanent-income model with $r = \delta$ and income shocks are measured with error according to (12), measurement error affects the responses of wealth and consumption to observed income shocks in the following way:*

- *The response of wealth to income shocks is reduced more by measurement error than the response of consumption if the interest rate r is smaller than unity ($\partial\beta_a^N/\partial M < \partial\beta_c^N/\partial M < 0$).*
- *The effect of measurement error on the responses, in absolute terms, decreases in the number of periods N ($\partial^2\beta_a^N/\partial M\partial N > 0$, $\partial^2\beta_c^N/\partial M\partial N > 0$).*

Measurement error, as the transitory shock, matters more for smaller N since the measurement error is also independently distributed over time. Since the consumption response is *smaller* and the wealth response is *larger* if measured over a smaller number of periods N , the stronger attenuation bias for smaller N affects differently the profile of the wealth and consumption response over the number of periods. Measurement error reduces and may even reverse the negative sign of $\partial\beta_a^N/\partial N$: the derivative $\partial\beta_a^N/\partial N < 0$ becomes smaller in absolute terms and may even become positive. Measurement error instead increases the positive sign of the effect of the number of periods on the consumption response.

Table 4 displays results for the responses to income shocks if, for illustration purposes, measurement error accounts for 75% of the transitory variance in observed income data. Comparing the results in Table 4 and Table 3 shows that, for the plausible values of $r = 0.02$ and $Q = 0.34$, sizeable measurement error has only a very small effect on consumption responses but a large effect on wealth responses. In particular, the wealth response to income shocks is no longer always decreasing in the number of periods N . The wealth response falls in N only for very high levels of persistence ($\rho = 1$ or $\rho = 0.995$) and is nearly flat (as one would expect) if the shock has low persistence ($\rho = 0.2$). Yet, for intermediate values of $\rho = 0.8$ or $\rho = 0.95$ the wealth response is increasing in N .

These results show that, for plausible parameter values, the permanent-income model can generate a wealth response which decreases in N , $\partial\beta_a^N/\partial N < 0$, only if income shocks are very persistent and the size of (the variance of) the measurement error is not too large. In other words, a wealth response to income shocks which decreases in N imposes restrictions on ρ and the size of measurement error in the permanent-income model.

A.2 Proofs

Proof. Remark 1: We follow Deaton (1992), chapter 3, adapting the derivations to our assumptions about the income process (2). The intertemporal budget constraint

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} c_s = (1+r)a_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} y_s$$

holds for any realization of income and thus also in expectation:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} E_t c_s = (1+r)a_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} E_t y_s. \quad (15)$$

It follows from (11), applying the law of iterated expectations, that for $s > t$

$$c_t = E_t c_s,$$

so that

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} E_t c_s = c_t \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} = \frac{1+r}{r} c_t.$$

Thus (15) implies

$$c_t = r a_t + \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} E_t y_s. \quad (16)$$

Change of consumption over time

Using the lagged budget constraint (1) to substitute a_t , we get

$$c_t = r((1+r)a_{t-1} + y_{t-1} - c_{t-1}) + \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} E_t y_s. \quad (17)$$

Using (16) lagged one period and multiplying by $1+r$ yields

$$\begin{aligned} (1+r)c_{t-1} &= r(1+r)a_{t-1} + (1+r) \frac{r}{1+r} \sum_{s=t-1}^{\infty} \left(\frac{1}{1+r} \right)^{s-t+1} E_{t-1} y_s \\ &= r(1+r)a_{t-1} + r y_{t-1} + \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} E_{t-1} y_s. \end{aligned} \quad (18)$$

Subtracting (18) from (17) we find

$$\begin{aligned} \Delta c_t \equiv c_t - c_{t-1} &= \frac{r}{1+r} \left\{ \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (E_t y_s - E_{t-1} y_s) \right\} \\ &= \frac{r}{1+r} \left\{ \varepsilon_t + \sum_{s=t}^{\infty} \left(\frac{\rho}{1+r} \right)^{s-t} \eta_t \right\} \\ &= \frac{r}{1+r} \varepsilon_t + \frac{r}{1+r-\rho} \eta_t, \end{aligned}$$

where $E_t y_s = y_s$ for $s \leq t$ and the second equality follows from (2).

Change of wealth over time

Substituting (16) into the budget constraint, we have

$$\begin{aligned} a_{t+1} &= (1+r)a_t + y_t - c_t \\ &= a_t + y_t - \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} E_t y_s. \end{aligned}$$

Thus,

$$\begin{aligned} \Delta a_{t+1} &= y_t - \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} E_t y_s \\ &= y_t - \frac{r}{1+r} y_t - \frac{r}{1+r} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} E_t y_s \\ &= \frac{1}{1+r} \left(y_t - \frac{r\rho}{1+r} \sum_{s=t+1}^{\infty} \left(\frac{\rho}{1+r} \right)^{s-t-1} z_t \right), \end{aligned}$$

where equation (2) implies

$$E_t y_{t+s} = \rho E_t z_{t+s-1} + E_t \eta_{t+s} + E_t \varepsilon_{t+s} = \rho^s z_t.$$

Expanding, we get

$$\begin{aligned} \Delta a_{t+1} &= \frac{1}{1+r} \left(y_t - \frac{r\rho}{1+r-\rho} z_t \right) \\ &= \frac{\varepsilon_t}{1+r} + \frac{1-\rho}{1+r-\rho} (\rho z_{t-1} + \eta_t). \end{aligned}$$

Change of income over time

It follows immediately from the assumed income process (2) that

$$\begin{aligned} \Delta y_t &= \Delta z_t + \Delta \varepsilon_t \\ &= (\rho - 1) z_{t-1} + \eta_t + \Delta \varepsilon_t. \end{aligned}$$

■

Proof. Remark 2 and 3: We derive results for the general income process (12) with measurement error. The results of Remark 2 are easily obtained by setting $M = 0$ in equations (22) and (23) below.

Remark 1 implies that the N -period changes of consumption, wealth and income are

$$\Delta^N c_t = \sum_{\tau=t-N+1}^t \left(\frac{r}{1+r} \varepsilon_{\tau} + \frac{r}{1+r-\rho} \eta_{\tau} \right), \quad (19)$$

$$\Delta^N a_{t+1} = \frac{1-\rho^N}{1+r-\rho} \rho z_{t-N} + \sum_{\tau=t-N+1}^t \left(\frac{\varepsilon_{\tau}}{1+r} + \frac{1-\rho^{t-\tau+1}}{1+r-\rho} \eta_{\tau} \right), \quad (20)$$

$$\Delta^N \tilde{y}_t = (\rho^N - 1) z_{t-N} + \sum_{\tau=t-N+1}^t \rho^{t-\tau} \eta_{\tau} + \Delta^N \varepsilon_t + \Delta^N \gamma_t. \quad (21)$$

The coefficients of bivariate regressions of N -period consumption or wealth changes on N -period income changes are thus given by $cov(\Delta^N c_t, \Delta^N \tilde{y}_t) / var(\Delta^N \tilde{y}_t)$ and

$cov(\Delta^N a_{t+1}, \Delta^N \tilde{y}_t) / var(\Delta^N \tilde{y}_t)$. Equations (19), (20) and (21) allow to compute these variance and covariances as

$$\begin{aligned} var(\Delta^N \tilde{y}_t) &= \left(\frac{(\rho^N - 1)^2}{1 - \rho^2} + \sum_{\tau=t-N+1}^t \rho^{t-\tau} \right) \sigma_\eta^2 + 2\sigma_\varepsilon^2 + 2\sigma_\gamma^2 \\ &= \left(\frac{(\rho^N - 1)^2}{1 - \rho^2} + \frac{1 - \rho^N}{1 - \rho} \right) \sigma_\eta^2 + 2\sigma_\varepsilon^2 + 2\sigma_\gamma^2, \\ cov(\Delta^N c_t, \Delta^N \tilde{y}_t) &= \frac{1 - \rho^N}{1 - \rho} \frac{r}{1 + r - \rho} \sigma_\eta^2 + \frac{r}{1 + r} \sigma_\varepsilon^2 \end{aligned}$$

and

$$\begin{aligned} cov(\Delta^N a_{t+1}, \Delta^N \tilde{y}_t) &= \left(-\frac{\rho(1 - \rho^N)^2}{(1 + r - \rho)(1 - \rho^2)} + \sum_{\tau=t-N+1}^t \rho^{t-\tau} \frac{1 - \rho^{t-\tau+1}}{1 + r - \rho} \right) \sigma_\eta^2 + \frac{\sigma_\varepsilon^2}{1 + r} \\ &= \left[-\frac{\rho(1 - \rho^N)^2}{1 - \rho^2} + \sum_{\tau=t-N+1}^t \left(\rho^{t-\tau} - \rho^{2(t-\tau)+1} \right) \right] \frac{\sigma_\eta^2}{1 + r - \rho} + \frac{\sigma_\varepsilon^2}{1 + r} \\ &= \left(-\frac{\rho(1 - \rho^N)^2}{1 - \rho^2} + \frac{1 - \rho^N}{1 - \rho} - \frac{1 - \rho^{2N}}{1 - \rho^2} \rho \right) \frac{\sigma_\eta^2}{1 + r - \rho} + \frac{\sigma_\varepsilon^2}{1 + r} \\ &= \frac{1 - \rho^N}{1 - \rho^2} \frac{1 - \rho}{1 + r - \rho} \sigma_\eta^2 + \frac{\sigma_\varepsilon^2}{1 + r} = \\ &= \frac{1 - \rho^N}{1 + \rho} \frac{\sigma_\eta^2}{1 + r - \rho} + \frac{\sigma_\varepsilon^2}{1 + r}. \end{aligned}$$

Using the definitions $Q \equiv \sigma_\eta^2 / (\sigma_\varepsilon^2 + \sigma_\gamma^2)$ and $M \equiv \sigma_\gamma^2 / (\sigma_\varepsilon^2 + \sigma_\gamma^2)$,

$$\begin{aligned} \beta_c^N &= \frac{cov(\Delta^N c_t, \Delta^N \tilde{y}_t)}{var(\Delta^N \tilde{y}_t)} = r \frac{\frac{1}{1-\rho} \frac{1-\rho^N}{1+r-\rho} \sigma_\eta^2 + \frac{1}{1+r} \sigma_\varepsilon^2}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho} \right) \sigma_\eta^2 + 2\sigma_\varepsilon^2 + 2\sigma_\gamma^2} \\ &= r \frac{\frac{1}{1-\rho} \frac{1-\rho^N}{1+r-\rho} Q + \frac{1}{1+r} (1-M)}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho} \right) Q + 2} \\ &= r \frac{\frac{1}{1-\rho} A(\rho, N) + \frac{1}{1+r} (1-M)}{B(\rho, N)} \end{aligned} \tag{22}$$

and

$$\begin{aligned} \beta_a^N &= \frac{cov(\Delta^N a_{t+1}, \Delta^N \tilde{y}_t)}{var(\Delta^N \tilde{y}_t)} = \frac{\frac{1-\rho^N}{1+\rho} \frac{\sigma_\eta^2}{1+r-\rho} + \frac{1}{1+r} \sigma_\varepsilon^2}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho} \right) \sigma_\eta^2 + 2\sigma_\varepsilon^2 + 2\sigma_\gamma^2} \\ &= \frac{\frac{1}{1+\rho} \frac{1-\rho^N}{1+r-\rho} Q + \frac{1}{1+r} (1-M)}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho} \right) Q + 2} \\ &= \frac{\frac{1}{1+\rho} A(\rho, N) + \frac{1}{1+r} (1-M)}{B(\rho, N)} \end{aligned} \tag{23}$$

with

$$A(\rho, N) \equiv \frac{1 - \rho^N}{1 + r - \rho} Q \text{ and } B(\rho, N) \equiv \left(\frac{(\rho^N - 1)^2}{1 - \rho^2} + \frac{1 - \rho^N}{1 - \rho} \right) Q + 2.$$

■

Proof. Corollary 1: We derive the results for the general case with measurement error where the results of Corollary 1 are easily obtained setting $M = 0$.

It follows from Remark 3 that

$$\frac{\partial \beta_a^N}{\partial N} = \frac{\frac{1}{1+\rho} \frac{\partial A(\rho, N)}{\partial N} B(\rho, N) - \frac{\partial B(\rho, N)}{\partial N} \left(\frac{1}{1+\rho} A(\rho, N) + \frac{1}{1+r} (1-M) \right)}{B(\rho, N)^2} \quad (24)$$

and

$$\frac{\partial \beta_c^N}{\partial N} = r \frac{\frac{1}{1-\rho} \frac{\partial A(\rho, N)}{\partial N} B(\rho, N) - \frac{\partial B(\rho, N)}{\partial N} \left(\frac{1}{1-\rho} A(\rho, N) + \frac{1}{1+r} (1-M) \right)}{B(\rho, N)^2}. \quad (25)$$

The sign of $\partial \beta_a^N / \partial N$ and $\partial \beta_c^N / \partial N$ depends on the sign of the respective numerator in (24) and (25):

$$\text{sign} \left(\frac{\partial \beta_a^N}{\partial N} \right) = \frac{1}{1+\rho} \text{sign} \left[\frac{\partial A(\rho, N)}{\partial N} B(\rho, N) - \frac{\partial B(\rho, N)}{\partial N} \left(A(\rho, N) + \frac{1+\rho}{1+r} (1-M) \right) \right]$$

and

$$\text{sign} \left(\frac{\partial \beta_c^N}{\partial N} \right) = \frac{1}{1-\rho} \text{sign} \left[\frac{\partial A(\rho, N)}{\partial N} B(\rho, N) - \frac{\partial B(\rho, N)}{\partial N} \left(A(\rho, N) + \frac{1-\rho}{1+r} (1-M) \right) \right].$$

Note that

$$\frac{\partial A(\rho, N)}{\partial N} = \begin{cases} -\frac{\rho^N \ln \rho}{1+r-\rho} Q > 0 & \text{if } 0 < \rho < 1 \text{ and } Q > 0 \\ 0 & \text{if } \rho = 0 \text{ or } \rho = 1. \end{cases}$$

and

$$\frac{\partial B(\rho, N)}{\partial N} = \begin{cases} \frac{\rho^N \ln \rho}{1-\rho} \left(\frac{2(\rho^N-1)}{1+\rho} - 1 \right) Q > 0 & \text{if } 0 < \rho < 1 \text{ and } Q > 0 \\ 0 & \text{if } \rho = 0 \\ Q > 0 & \text{if } \rho = 1 \text{ and } Q > 0, \end{cases}$$

where L'Hôpital's rule implies $B(1, N) = NQ + 2$.

The consumption response as a function of N

Substituting in the expressions for $A(\rho, N)$, $B(\rho, N)$, and their respective derivatives with respect to N ,

$$\begin{aligned} \text{sign} \left(\frac{\partial \beta_c^N}{\partial N} \right) &= \frac{1}{1-\rho} \text{sign} \left[-\frac{\rho^N \ln \rho}{1+r-\rho} Q \left(\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho} \right) Q + 2 \right) \right. \\ &\quad \left. - \frac{\rho^N \ln \rho}{1-\rho} \left(\frac{2(\rho^N-1)}{1+\rho} - 1 \right) Q \left(\frac{1-\rho^N}{1+r-\rho} Q + \frac{1-\rho}{1+r} (1-M) \right) \right] \\ &= \frac{1}{1-\rho} \text{sign} \left[-\frac{\rho^N \ln \rho}{(1-\rho^2)(1+r-\rho)} Q \left(2(1-\rho^2) - Q(\rho^N-1)^2 \right) \right. \\ &\quad \left. - \frac{\rho^N \ln \rho}{1+r} \left(\frac{2(\rho^N-1)}{1+\rho} - 1 \right) Q(1-M) \right] \\ &= \frac{1}{1-\rho} \text{sign} \left[-\frac{\rho^N \ln \rho}{(1-\rho^2)(1+r-\rho)} Q \times \right. \\ &\quad \left(2(1-\rho^2) - \frac{(1-\rho^2)(1+r-\rho)}{1+r} \left(\frac{2(1-\rho^N)}{1+\rho} + 1 \right) - Q(\rho^N-1)^2 \right) \\ &\quad \left. - \frac{\rho^N \ln \rho}{1+r} \left(\frac{2(1-\rho^N)}{1+\rho} + 1 \right) QM \right] \end{aligned}$$

For $0 < \rho < 1$, the second term in square brackets is positive for $0 < M \leq 1$. The first term is positive if Q is sufficiently small so that

$$2(1 - \rho^2) - \frac{(1 - \rho^2)(1 + r - \rho)}{1 + r} \left(\frac{2(1 - \rho^N)}{1 + \rho} + 1 \right) - Q(\rho^N - 1)^2 > 0$$

or

$$Q < \frac{2(1 - \rho^2) - \frac{(1 - \rho^2)(1 + r - \rho)}{1 + r} \left(\frac{2(1 - \rho^N)}{1 + \rho} + 1 \right)}{(\rho^N - 1)^2} \equiv Q_c^*.$$

Note that $Q_c^* \geq 0$ only for $\rho > 0$.

Let us now consider two special cases. If $\sigma_\varepsilon^2 = 0$ and $\sigma_\gamma^2 = 0$ so that $Q = \infty$,

$$\beta_c^N = r \frac{1 + \rho}{(2 + \rho - \rho^N)(1 + r - \rho)}$$

which is decreasing in N for $0 < \rho < 1$, constant at $1/(1 + r)$ for $\rho = 0$ and constant at zero for $\rho = 1$.

If $\rho = 1$,

$$\beta_c^N = \frac{NQ + \frac{r}{1+r}(1 - M)}{NQ + 2},$$

so that

$$\begin{aligned} \frac{\partial \beta_c^N}{\partial N} &= \frac{NQ^2 + 2Q - NQ^2 - \frac{r}{1+r}(1 - M)Q}{(NQ + 2)^2} \\ &= \frac{2Q - \frac{r}{1+r}(1 - M)Q}{(NQ + 2)^2}. \end{aligned}$$

Since $0 \leq M \leq 1$, the consumption response depends positively on N if $\sigma_\varepsilon^2 > 0$ and $\sigma_\eta^2 > 0$ so that $0 < Q < \infty$.

The wealth response as a function of N

We have

$$\begin{aligned} \text{sign} \left(\frac{\partial \beta_a^N}{\partial N} \right) &= \frac{1}{1 + \rho} \quad \text{sign} \left[-\frac{\rho^N \ln \rho}{1 + r - \rho} Q \left(\left(\frac{(\rho^N - 1)^2}{1 - \rho^2} + \frac{1 - \rho^N}{1 - \rho} \right) Q + 2 \right) \right. \\ &\quad \left. - \frac{\rho^N \ln \rho}{1 - \rho} \left(\frac{2(\rho^N - 1)}{1 + \rho} - 1 \right) Q \left(\frac{1 - \rho^N}{1 + r - \rho} Q + \frac{1 + \rho}{1 + r}(1 - M) \right) \right] \\ &= \frac{1}{1 + \rho} \quad \text{sign} \left[-\frac{\rho^N \ln \rho}{(1 - \rho^2)(1 + r - \rho)} Q \left(2(1 - \rho^2) - Q(\rho^N - 1)^2 \right) \right. \\ &\quad \left. - \frac{\rho^N \ln \rho}{1 + r} \frac{1 + \rho}{1 - \rho} \left(\frac{2(\rho^N - 1)}{1 + \rho} - 1 \right) Q(1 - M) \right] \\ &= \frac{1}{1 + \rho} \quad \text{sign} \left[-\frac{\rho^N \ln \rho}{(1 - \rho^2)(1 + r - \rho)} Q \times \right. \\ &\quad \left(2(1 - \rho^2) - \frac{(1 + \rho)^2(1 + r - \rho)}{1 + r} \left(\frac{2(1 - \rho^N)}{1 + \rho} + 1 \right) - Q(\rho^N - 1)^2 \right) \\ &\quad \left. - \frac{\rho^N \ln \rho}{1 + r} \frac{1 + \rho}{1 - \rho} \left(\frac{2(1 - \rho^N)}{1 + \rho} + 1 \right) Q M \right] \end{aligned}$$

For $0 < \rho < 1$, the second term in square brackets is positive for $0 < M \leq 1$. The first term is negative if Q is sufficiently large so that

$$2(1 - \rho^2) - \frac{(1 + \rho)^2(1 + r - \rho)}{1 + r} \left(\frac{2(1 - \rho^N)}{1 + \rho} + 1 \right) - Q(\rho^N - 1)^2 < 0$$

or

$$Q > \frac{2(1 - \rho^2) - \frac{(1 + \rho)^2(1 + r - \rho)}{1 + r} \left(\frac{2(1 - \rho^N)}{1 + \rho} + 1 \right)}{(\rho^N - 1)^2} \equiv Q_a^*.$$

Note that $Q_a^* < Q_c^*$ for $0 < \rho < 1$, so that there exists $Q \in [Q_a^*, Q_c^*]$ for which the wealth response negatively depends on N while the consumption response positively depends on N .

Let us now consider again two special cases. If $\sigma_\varepsilon^2 = 0$ and $\sigma_\gamma^2 = 0$ so that $Q = \infty$,

$$\beta_a^N = \frac{1 - \rho}{(2 + \rho - \rho^N)(1 + r - \rho)},$$

which is decreasing in N for $0 < \rho < 1$, constant at $1/(1 + r)$ for $\rho = 0$ and constant at zero for $\rho = 1$.

If $\rho = 1$,

$$\beta_a^N = \frac{1}{1 + r} \frac{1 - M}{NQ + 2}$$

which is unambiguously decreasing in N for $0 < Q < \infty$. ■

Proof. Corollary 2:

Using the results of Remark 2, we find that the effect of measurement error on the wealth and consumption response is

$$\frac{\partial \beta_a^N}{\partial M} = -\frac{\frac{1}{1+r}}{B(\rho, N)}$$

and

$$\frac{\partial \beta_c^N}{\partial M} = r \frac{\partial \beta_a^N}{\partial M}$$

so that $\partial \beta_a^N / \partial M < \partial \beta_c^N / \partial M < 0$ for $-1 < r < 1$. The effect of measurement error on the responses of wealth and consumption increases in the number of periods N since

$$\frac{\partial^2 \beta_a^N}{\partial M \partial N} = \frac{\frac{1}{1+r} \frac{\partial B(\rho, N)}{\partial N}}{B(\rho, N)^2} > 0.$$

where the inequality follows from $\partial B(\rho, N) / \partial N > 0$ for $0 < \rho \leq 1$, as established in Corollary 1. ■

A.3 Estimation results for the subsample of renters

Krueger and Perri (2011) have argued that the residual income changes may be correlated with residual changes in real-estate wealth. Thus, we perform a robustness check in this appendix and exclude homeowners from our sample as in Krueger and Perri (2011). This sample selection is not innocuous because housing tenure may respond to income shocks and renters have less net worth to smooth out income shocks. The advantage of considering only renters is that the residual income changes are not correlated with changes in housing wealth for this subsample

Number of years N	Consumption response β_c^N			Wealth response β_a^N		
	2	4	6	2	4	6
Responses	0.197	0.269	0.355	0.092	0.208	0.162
Standard error	(0.043)	(0.037)	(0.036)	(0.071)	(0.095)	(0.069)
	Median			Mean		
Net worth	0.26			0.59		
Standard deviation				(0.05)		

Table 5: Consumption and wealth responses to income shocks and net worth for the subsample of renters in the SHIW. Source: Authors' calculation. Standard errors are clustered at the household level.

by construction. This makes it more likely that the responses to these income changes capture the effect of pure income shocks.

Our subsample of renters in the SHIW consists of 322 households in the time period 1987 to 2012 accounting for 609 observations. The summary statistics in column (3) of Table 7 in appendix A.4 show that these households are fairly similar to our benchmark sample in column (2) but for their smaller net worth. This implies that the data targets for wealth in the estimation are smaller: the median and mean of net worth in Table 5 are much lower than in our benchmark sample (see Table 1 in the main text). The consumption responses to income shocks are only slightly smaller whereas the wealth responses are less than half the size of those in the benchmark sample.

Given these differences in the data targets, it is not obvious whether a buffer-stock model that matches these targets (allowing for measurement error) implies similar insurance coefficients as the estimated model for the benchmark sample. Focussing on our preferred specification where we target mean net worth, Table 6 shows that the smaller net worth target requires a smaller buffer-stock saving motive which the model achieves with a higher discount rate and a lower level of relative risk aversion: the discount rate increases to 0.06 (from 0.055 in the benchmark) and the estimate for the coefficient of relative risk aversion decreases to 2.7 (from 2.8 in the benchmark). The lower wealth responses can be replicated by the model by allowing for some measurement error which is estimated to have variance $\sigma_\gamma^2 = 0.015$. For the permanent-income model without a precautionary saving motive, we provide analytic results in corollary 2 of appendix A.1.1 to give some intuition for why measurement error reduces the wealth responses.

The subsample of renters has less net worth and thus a smaller buffer-stock of savings to self-insure against shocks. It is thus intuitive that the estimated model implies smaller insurance coefficients for this subsample than for our benchmark: agents insure 93% of a temporary shock and 7% of a permanent shock compared with 95% and 10%, respectively, in the benchmark. The model thus shows that the focus on the subsample of renters is not innocuous for the conclusions about the degree of insurance of shocks. Future research may further investigate this issue by modeling the choice of becoming a renter or homeowner, possibly taking into account that housing wealth is less liquid than we have assumed in our benchmark analysis and thus may not be as effective for self insurance against income shocks.

The results reported in this appendix are robust if we target the estimates for the responses to income shocks reported in Krueger and Perri (2011), Table 5, which slightly differ from our estimates. We find that the different targets only slightly affect the parameter estimates of the model. For our preferred specification, in which we target the mean of net worth, the estimate

Parameters (std.err.)	$\delta = 0.06$ (??), $\alpha = 2.7$ (??), $\sigma_\gamma^2 = 0.015$ (??)						
	Net worth	Consumption response			Wealth response		
	Mean	N=2	N=4	N=6	N=2	N=4	N=6
Model moments	0.61	0.258	0.402	0.498	0.152	0.147	0.147
Standard error	(0.03)	(0.024)	(0.027)	(0.027)	(0.027)	(0.033)	(0.038)
SHIW data moments	0.59	0.197	0.269	0.355	0.092	0.208	0.162

Table 6: Estimation results for the precautionary-savings model targeting moments for the subsample of renters. Source: Authors' calculation. Notes: The other parameter values are $\rho = 1$, $\sigma_\varepsilon^2 = 0.0794$, $\sigma_\eta^2 = 0.0267$ and $\underline{a} = 0$.

for the discount rate is 0.0625 and the risk aversion is 2.8 in this case. The insurance coefficients are unchanged at the level of precision we report, still implying that agents insure 93% of a temporary shock and 7% of a permanent shock. Further details on these results are available on request.

A.4 Data appendix

The variables used in the analysis are defined as (see also the definitions in the SHIW and Krueger and Perri (2011)):

Non-durable consumption: all expenditures but for expenditures on transport equipment, valuables, household equipment, home improvement, insurance premia and contributions to pension funds. The measure includes the effectively paid or the imputed rent.

After-tax and transfer labor income: after-tax wages and salaries, fringe benefits and transfers (pensions, arrears and other transfers).

Net-financial assets: sum of deposits, checked deposits, repos, postal savings certificates, government securities and other securities (bonds, mutual funds, equity, shares in private limited companies and partnerships, foreign securities, loans to cooperatives) net of financial liabilities (liabilities to banks and financial companies, trade debt and liabilities to other households).

Net worth: sum of net-financial assets and real estate wealth.

Education: the categories are elementary school, middle school, high school, college degree and postgraduate education.

Regions: regions are Northern, Centre and Southern regions (including islands), respectively.

Sample construction:

The SHIW data between 1987 and 2012 includes 87,629 observations for 54,070 households. We express all nominal variables in units of Euro in the year 2000. We trim the top 2.5% of the wealth distribution (dropping 2,190 observations) and we clean the sample of those households who report zero food consumption (dropping 24 observations). As Krueger and Perri (2011) we construct the sample with households which appear in consecutive waves (resulting in 43,482 observations). We select the prime-age households whose head has an age between 25 and 55 (21,835 observations) and whose members are not in self-employment or employed in entrepreneurial activities (16,796 observations). In the Krueger-Perri sample we only consider households without real estate (5,487 observations).

Following Krueger and Perri (2011) we construct measures for shocks to labor income, consumption and net worth by purging the changes observed in the data from those changes

Variables	Prime-age sample (aged 25-55)	Benchmark sample (aged 25-55 & not self-employed & obs. in 4 consecutive waves)	Krueger-Perri sample (benchmark sample & no real estate)
Age of household head	42.90 (7.91)	45.64 (6.28)	45.40 (6.49)
Household size	3.30 (1.28)	3.41 (1.22)	3.37 (1.27)
Labor earnings (after tax/transfer)	10,347 (8,794)	9,255 (4,359)	8,699 (4,242)
Standard deviation of changes in residual earnings:			
2-period change	-	(1,308)	(1,330)
4-period change	-	(744)	(708)
6-period change	-	(538)	(522)
Net worth	70,590 (159,900)	24,409 (40,724)	5,098 (11,442)
Non-durable consumption	8,758 (5009)	7,625 (3,453)	7,114 (3,391)
Education: none	0.01 (0.12)	0.01 (0.09)	0.01 (0.10)
Education: elementary school	0.15 (0.36)	0.12 (0.33)	0.15 (0.36)
Education: middle school	0.36 (0.48)	0.47 (0.50)	0.51 (0.50)
Education: high school	0.35 (0.48)	0.33 (0.47)	0.28 (0.45)
Education: college degree	0.11 (0.31)	0.06 (0.25)	0.04 (0.20)
Education: postgraduate	0.007 (0.09)	0.003 (0.05)	0 (-)
Region: North	0.44 (0.50)	0.47 (0.50)	0.45 (0.50)
Region: Center	0.20 (0.40)	0.17 (0.38)	0.17 (0.38)
Region: South (incl. islands)	0.36 (0.48)	0.36 (0.48)	0.37 (0.48)
Number of households	33,505	520	322

Table 7: Summary statistics for the SHIW sample of households with a head aged 25 – 55, for our benchmark sample of households observed in at least three consecutive waves excluding households in self-employment or entrepreneurial activities, and for the sample of Krueger and Perri (2011) in which also households with real estate wealth are excluded. Sources: Authors' calculation based on SHIW data 1987–2012. Note: Standard deviation in brackets. Monetary variables are converted to Euro in 2000 and expressed in adult equivalent units.

which are predictable. We thus regress the respective observed changes on a quartic polynomial of the age of the household head, on education, time and regional dummies as well as the age-education interaction dummies. We then use the residuals of these regressions as our measure of shocks, where we take into account that income shocks are measured with error. These changes of variables are annualized because the SHIW is a biannual survey with the exception of the three-year difference between the wave of 1995 and 1998.

We then select the households for which we observe at least the second, fourth and sixth difference of income (resulting in 554 households). We further clean the sample from outliers by regressing the changes of wealth and consumption observed over two, four and six years on our constructed measure for income shocks. After these regressions we consider as outliers those observations for which either of the studentized residuals are larger than three. We also remove observations with so-called “high leverage,” i.e. an observation with an extreme value in the regressor. An observation has high leverage if its value in the diagonal of the projection matrix is greater than $(2k + 2)/n$ where k is the number of predictors and n the number of observations. This leaves us with a sample of 1077 observations for 520 households.

Table 7 provides summary statistics for (i) the full prime-age sample, (ii) the benchmark sample of households observed in at least four consecutive waves (to compute changes over a time horizon up to six years) excluding households with members in self-employment or entrepreneurial activities, and (iii) the Krueger-Perri sample in which also households with real estate wealth are excluded. The statistics in column (2) show that households in the benchmark sample are less wealthy and less educated than in the full prime-age sample in column (1). This is partly because attrition in the panel is correlated with these characteristics illustrating the trade-off between exploiting the panel dimension of the data and maintaining the representativeness of the sample. Table 7 further shows that the standard deviation of the changes in residual earnings decreases with the time horizon as transitory changes and measurement error wash out over longer time horizons.

A.5 Calibration and model estimation

A.5.1 The permanent-income model with log-normally distributed income

We derive how we can simulate consumption and wealth in the permanent-income model if income is log-normally distributed. Although the assumption of normally distributed income levels in the text allowed us to obtain analytically simpler results, the assumption of log-normality in the simulation makes the estimation results of the permanent-income and precautionary-savings model comparable. We assume that income consists of permanent and transitory component:

$$y_t = y_t^p \varepsilon_t$$

with

$$y_t^p = y_{t-1}^p \eta_t .$$

If log-income is normally distributed as $\ln y \sim N(\mu, \sigma^2)$, then $E(e^{\ln y}) = e^{\mu + \frac{\sigma^2}{2}}$. Thus, if the level of income has a mean normalized to one, the permanent shock η and transitory shock ε have to be distributed as follows:

$$\log \eta \sim N\left(-\frac{\sigma_\eta^2}{2}, \sigma_\eta^2\right) ,$$

$$\log \varepsilon \sim N\left(-\frac{\sigma_\varepsilon^2}{2}, \sigma_\varepsilon^2\right) .$$

Under these assumptions, $\ln y_t$ is distributed as

$$\ln(y_t) \sim N(\ln(y_{t-1}) - \frac{\sigma_\eta^2}{2} - \frac{\sigma_\varepsilon^2}{2}, \sigma_\eta^2 + \sigma_\varepsilon^2) .$$

Thus,

$$E_{t-1} \left(e^{\ln y_t} \right) = e^{\ln y_{t-1}^p - \frac{\sigma_\eta^2}{2} - \frac{\sigma_\varepsilon^2}{2} + \frac{\sigma_\eta^2 + \sigma_\varepsilon^2}{2}} = y_{t-1}^p$$

Analogous to equation (3.3) in Deaton (1992),

$$c_t = ra_t + \frac{r}{1+r} \sum_{s=t}^{\infty} \frac{E_t(y_s)}{(1+r)^{s-t}} .$$

We use this equation to derive consumption c_t as function of income and current assets:

$$\begin{aligned} c_t &= ra_t + \frac{r}{1+r} \left[\sum_{s=t+1}^{\infty} \frac{E_t(e^{\ln y_s})}{(1+r)^{s-t}} + y_t^p \varepsilon_t \right] , \\ &= ra_t + \frac{r}{1+r} \left[\sum_{s=t+1}^{\infty} \frac{y_t^p}{(1+r)^{s-t}} + y_t^p \varepsilon_t \right] , \\ &= ra_t + \frac{r}{1+r} \left[y_t^p \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} + y_t^p \varepsilon_t \right] , \\ &= ra_t + \frac{r}{1+r} \left[y_t^p \left(\frac{1}{1+r} \right) \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} + y_t^p \varepsilon_t \right] , \\ &= ra_t + \frac{r}{1+r} \left[\frac{y_t^p}{r} + y_t^p \varepsilon_t \right] , \\ &= ra_t + \frac{y_t^p}{1+r} + \frac{r}{1+r} y_t^p \varepsilon_t , \\ &= ra_t + y_t^p + r y_t^p \left(\frac{\varepsilon_t - 1}{1+r} \right) \end{aligned}$$

Using the budget constraint, we solve for a_{t+1} :

$$\begin{aligned} a_{t+1} &= a_t(1+r) + y_t - c_t , \\ &= a_t(1+r) + y_t^p \varepsilon_t - ra_t - \frac{y_t^p}{1+r} - \frac{r}{1+r} y_t^p \varepsilon_t , \\ &= a_t + y_t^p \left(\frac{\varepsilon_t - 1}{1+r} \right) . \end{aligned}$$

These are the expressions for c_t and a_{t+1} used in the simulations which we explain further in the next subsection.

A.5.2 Solution and estimation of the precautionary-savings model

The solution and estimations follow Hintermaier and Koeniger (2010) so that we only mention computational issues which are not discussed in that paper. We use 31 states to approximate the

permanent part of the income process. The additional transitory shock ε_t is discretized with the quadrature method using 12 points. The grid for wealth is triple-exponential with 1,600 points (being much finer where the policies have more curvature). We employ the endogenous-grid method (EGM) proposed by Carroll (2006) to solve the model.

As we vary the variance of the measurement error σ_γ^2 across model cases, we adjust the variance of the transitory shock σ_ε^2 so that (i) the relative importance of permanent shocks $Q \equiv \sigma_\eta^2/(\sigma_\varepsilon^2 + \sigma_\gamma^2)$ remains constant at 0.34 and (ii) the cross-sectional variance of log-income $\sigma_{\log(Y)}^2 = \sigma_\varepsilon^2 + T\sigma_\eta^2 + \sigma_\gamma^2$ matches the observed variance of 0.32 in the SHIW for our simulations with $T = 45$ periods.

In the simulations, we draw initial wealth from the empirical wealth distribution of our benchmark sample and set initial income to the mean. We simulate the model economy for 45 periods for 25,000 consumers, drawing both the transitory and the permanent shock with the normal random number generator and interpolating the policy functions to obtain consumption and savings for the simulated values of income and wealth. We clean the simulated data from outliers following the same steps as for the SHIW data (described in the data appendix), add the measurement error for income and then estimate the responses of consumption and wealth to income changes.

We estimate the model using the simulated methods of moments, as described in Hintermaier and Koeniger (2010). To compute the variance-covariance matrix we draw, with replacement, 10,000 random samples of the sample size constructed from the SHIW. We compute the data moments for each of these finite samples and their variance/covariance across the 10,000 samples.

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