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Aging, Private Health Expenditures and Environment Quality

Fatma Safi

Abstract
This paper presents a simple two-period overlapping generation's model with uncertain lifetimes that contains environmental and health issues. It investigates an intergenerational conflict between old and young generations as regards two defensive expenditures, offsetting the influence of a worsening environment, represented here by health care and environmental investment. Workers support environmental maintenance while retirees prefer investing in healthcare. The author shows that an increasing support for private health expenditures in an aging economy leads to a higher level of capital accumulation and leads also to a higher level of environmental quality only if the maintenance efforts are larger than consumption externalities.

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Keywords Aging; environmental expenditures; environmental quality; overlapping generations, age-based conflict

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1. Introduction

In an aging economy, when the number of the elderly grow the age structure of the population differ. Therefore, age-based matters have a direct impact on population preferences. The old individuals don't profit from long-term expenditure choices. They have a preference for investments which are more useful in the short term, while the young prefer investments giving better effects over a longer period. This is the case of the investments in health status, such as health-care expenditures and environmental maintenance. Both of them improve health situations: the valuable effect of health expenditure is evident and the positive role of environmental quality on health is known as well (EEA, European Environmental Agency (2007)).

The workers support however environmental expenditures while the retirees prefer health-care investment. Aging heightens the number of old individuals in the population. Consequently, old individuals appreciate private health-care expenditures instead of the environment investments due to the fact that they take usually more time to be totally effective although, they can last for a longer time. They do not enjoy future environmental improvements. Aging simultaneously heightens the young generation's preference for environmental expenditures as they yield to results over a longer horizon. The benefit that the young generation receives when being old from the investment in environmental quality when young increases with aging, owing to a longer remaining lifespan in which to enjoy enhanced environmental quality [See Balestra; Davide (2012) and Magnani; Adeline (2007)]. It is worth mentioning that we are not claiming that old people are not interested at all in environmental maintenance, but that they are less concerned than the young people.

Both health and life expectancy are affected by environmental quality and health care expenditure. However, the weight of these two parameters might be age-based. With the

knowledge of this intergenerational conflict, we explore the question of how and whether a population of two groups: young and old; support two defensive expenditures represented here by healthcare and environmental maintenance and affecting capital accumulation and environment quality.

We set up a simple overlapping generations model OLG built on John and Pecchenino (1994)'s influential work such that the individual chooses his own level of health investment and not publicly through voting. We have shown the effects of the competitive mechanism in terms of health expenditures, environmental quality, capital accumulation and consumption possibilities.

This study goes along with diverse streams of the existing literature. In this respect, Chakraborty and Das (2005) postulate a positive relationship between the mortality risk and the private health investment and show that in the absence of annuities markets, health stocks can have persistent effects on income distribution. Chakraborty (2004) points out how development traps and persistent inequality may surge when survival probability is endogenous and depend negatively on health expenditure. Tubb (2011) supposes that agents are taxed and that taxation revenue can be spent on either environmental maintenance or on transfers to the old population. Aging enhances the proportion of elderly individuals and consequently enhances political pressure for the public planner to tilt the composition of public spending in favour of a transfer payment to the elderly. Since young population anticipates that higher longevity implies an increased return from such investment, ageing may simultaneously increase the young generation's demand for environmental investments.

This paper proceeds as follows. In Section 2 we introduce the theoretical model. Section 3 describes the competitive equilibrium model. Section 4 concludes.

2. Theoretical model

To formalize this model, consider a general equilibrium OLG closed and competitive economy populated by identical individuals. Each generation is alive for two periods such as life is divided between youth and old age. Each individual survives to the end of the first period t with certainty, while the second period $t+1$ length is uncertain. Let $p \in (0,1)$ be the probability that individual lives for two periods. The individual may die at the beginning of the retirement period with probability $1-p$ (probability of un-enjoying savings).

In the working period, individuals earn wage w_t by supplying inelastically one unit of labor. Individuals divide the income among consumption c_t^1, savings s_t for the retirement and payments for environmental maintenance m_t.[1] In the retirement period, individuals get the returns $(1+r_{t+1})$ on the savings and can be spent in consumption c_{t+1}^2 or in health expenditures z_{t+1}.[2]

Individuals face a tension between maintenance investment and health care. The individuals' constraints over the two periods can therefore be summarized as follows:

$$w_t = c_t^1 + s_t + m_t \tag{1}$$

$$c_{t+1}^2 = \left[(1+r_{t+1})s_t - z_{t+1}\right] \tag{2}$$

with $c_t^1, c_{t+1}^2, s_t, m_t \geq 0$.

Following John and Pecchenino (1994), the motion of the environmental quality is as follows:

$$E_{t+1} = (1-b)E_t - \beta\left(c_t^1 + c_t^2\right) + \delta m_t \tag{3}$$

[1] The superscript '1' refers to young individual. The subscript 't' represents period t

[2] The superscript '2' refers to an older individual. The subscript '$t+1$' represents period $t+1$

where E_t is the environment quality in period t, E_{t+1} is the environment quality in period $t+1$, $b \in (0.1)$ represents the natural rate of deterioration of the environment, $\beta > 0$ stands for the degradation of the environment and $\delta \rangle \mathbf{O}$ is the environmental improvement due to the actions of the young at t. The environment is supposed to be a public good which is affected by two economic actions: consumption and maintenance expenditure. On one hand, the environmental quality is negatively affected by the consumption activities. On the other hand the environment is positively affected by the payment of environmental maintenance m_t.

The individual's utility U is derived from consumption and environmental quality in first and second periods, where $U' > 0$ and $U'' < 0$. For simplification, preferences of each individual are defined by the log-linear lifetime utility U:

$$U = \ln c_t^1 + p \ln c_{t+1}^2 + \ln E_t + p \ln E_{t+1} \tag{4}$$

At each period t, the firms produce homogenous good in competitive markets using K the capital, L the labor according to a homogeneous of degree one production function. The production is described by an aggregate production function

$$Y_t = F(K_t, L_t) \tag{5}$$

Supposing that equation (5) fulfill constant returns to scale, the production function in intensive form becomes

$$y_t = f(k_t) \tag{6}$$

where Y_t is the output in period t, K_t the capital stock, L_t the labor supply, $k_t = K_t / L_t$ the capital-labor ratio, and y_t the output-labor ratio.

3. Optimisation problem

The individual takes as given the wage w_t, the return on the savings r_{t+1}, the stock of environment at the beginning of first period E_t, the probability of living p, and the environmental parameters b, β, δ. Therefore, the competitive life-cycle choice problem of the individual is to choose c_t^1, c_{t+1}^2, m_t, z_{t+1}, and s_t according to the maximisation program. Hence, the individual maximizes the utility function subject to the evolution of environmental quality (3) and the constraint (1)-(2).

By deriving (4) with respect to m_t and c_t^1, we get (7) and (8)

$$\frac{1}{c_t^1} = p(\beta + \delta)\frac{1}{E_{t+1}} \tag{7}$$

$$p(1 + r_{t+1})\frac{1}{c_{t+1}^2} = \delta \frac{1}{E_{t+1}} \tag{8}$$

Proof: See appendix A

The individual's maximization problem gives two first order conditions (FOCs) which are (7) and (8).

Equation (7) point out that young individuals choose consumption to equate the marginal rate of substitution between consumption when young and environmental quality in retirement period to the marginal rate of transformation $\beta + \delta$. At the intragenerationally efficient allocation, a decline in utility caused by a decrease in consumption by the young individuals is equal to a raise in utility thanks to the sum of the additional utility from declining consumption externalities β and from raising the environmental maintenance δ.

Equation (8) indicates that individuals choose savings to equate the marginal rate of substitution between consumption in retirement period and environmental quality in

retirement period to the marginal rate of transformation $\frac{p\delta}{(1+r_{t+1})}$.At the maximum of the utility, a lower utility due to falling consumption retirement period $(1+r_{t+1})$ is equal to a higher utility due to a rise of environmental maintenance effort $p\delta$.

The firm produces at time t profits:

$$\pi_t = F(K_t, L_t) - w_t L_t - (r_t + \sigma)K_t \tag{9}$$

where L_t indicates aggregate effective labour paid at a wage w_t , K_t aggregate physical capital and r_t denotes the return factor on savings from time $t-1$ to time t.

Supposing perfect competition in the factor markets, the profit maximization problem yields the following factor prices which are equal to their marginal productivities.

$$w_t = f(k_t) - k_t f'(k_t) \tag{10}$$

$$r_t = f'(k_t) - \sigma \tag{11}$$

where $\sigma \in (0,1)$ is the depreciation rate of capital and $f'(k_t) > \sigma$.

The first order conditions (FOCs) of the firm's maximization problem are (10) and (11).

4. Characterization of the equilibrium

A competitive equilibrium for the economy under analysis is a sequence, $\{c_t^1, c_{t+1}^2, m_t, w_t, r_t, s_t, z_{t+1}, p, k_t, E_t\}_{t=0}^{\infty}$ such that, given the initial conditions of the state parameters k_0 and E_0: firms maximize profits; old consumers maximize their utility function; and markets clear.

The first-order conditions of the utility maximization are (7)-(8) and the first-order conditions of profit maximization are (10) and (11). A market clearing condition for capital is $K_{t+1} = L_t s_t$ which point out that the total savings by young individuals in population $L_t s_t$

must equal their own addition to the future stock of capital K_{t+1}. Since there is no population growth, this condition is rewritten as

$$k_{t+1} = s_t \tag{12}$$

By plugging equations (7)-(10) and (11) into (1), it gives

$$m_t = f(k_t) - k_t f'(k_t) - \frac{1}{p(\beta+\delta)} E_{t+1} - k_{t+1} \tag{13}$$

Plugging equations (11)-(12) into (2) gives

$$c_{t+1}^2 = \left[(1 + f'(k_{t+1}) - \sigma)k_{t+1} - z_{t+1}\right] \tag{14}$$

Proof: See appendix A

For the sake of simplicity, we standardize the population of generation t as one. Therefore, by plugging equations (7)-(13) and (14) lagged once into (3) yields

$$\begin{aligned} E_{t+1} = (1-b)E_t - \beta\left[\frac{1}{p(\beta+\delta)} E_t + \left[(1 + f'(k_t) - \sigma)k_t - z_t\right]\right] \\ + \delta\left[f(k_t) - k_t f'(k_t) - \frac{1}{p(\beta+\delta)} E_t - k_{t+1}\right] \end{aligned} \tag{15}$$

Plugging as well equations (11) and (14) into (8) gives

$$E_{t+1} = \frac{1}{p}\left[\delta k_{t+1} - \frac{\delta z_{t+1}}{(1 + f'(k_{t+1}) - \sigma)}\right] \tag{16}$$

Equations (15) and (16) represent the law of motion for the environment.

Rewrite equations (15) as

$$\left[(1+p)/p\right]E_{t+1} - (1-b)E_t + \beta p\left[(1 + f'(k_t) - \sigma)k_t - z_t\right] - \delta\left[f(k_t) - k_t f'(k_t) - k_{t+1}\right] = 0 \tag{17}$$

Equation (16) is defining E_{t+1} as a function of k_{t+1}. Therefore, rewrite it as

$$E_{t+1} \equiv \phi(k_{t+1}) \tag{18}$$

5. The steady state

Since all parameters are constant in the steady state, time subscripts are eliminated. Let $\bar{k}$ and $\bar{E}$ indicate steady state values.

In steady state, equation (17) becomes

$$\bar{E} = \frac{p}{1+bp}\left\{-\beta p\left[\left(1+f'\left(\bar{k}\right)-\sigma\right)k-z\right]+\delta\left[f\left(\bar{k}\right)-\bar{k}f'\left(\bar{k}\right)-\bar{k}\right]\right\} \equiv \psi\left(\bar{k}\right) \tag{19}$$

In steady state, equation (18) becomes

$$\bar{E} = \frac{1}{p}\left[\delta\bar{k} - \frac{\delta z}{\left(1+f'\left(\bar{k}\right)-\sigma\right)}\right] \equiv \phi\left(\bar{k}\right) \tag{20}$$

The stable condition is given by the following equation,

$$k_{t+1} - \bar{k} = \left[\frac{(1-b)\phi' - (\delta+\beta p)kf''-\beta p\left(1+f'-\sigma\right)}{\delta+\left[(1+p)/p\right]\phi'}\right]\left(k_t - \bar{k}\right) \tag{21}$$

Proof: See appendix B

The coefficient on the right-hand side of this equation is less than one if and only if $\phi'\left(\bar{k}\right) > \psi'\left(\bar{k}\right)$ where

$$\phi'\left(\bar{k}\right) = \frac{\delta\left[\left(1+f'\left(\bar{k}\right)-\sigma\right)^2 + zf''\left(\bar{k}\right)\right]}{\left(1+f'\left(\bar{k}\right)-\sigma\right)^2}$$

The condition $0 < z < \frac{\left(1+f'-\sigma\right)^2}{-f''}$ is sufficient for $\phi' > 0$ .A greater capital stock is associated with greater environmental quality.

$$\psi'\left(\bar{k}\right) = \frac{p}{1+bp}\left[-\beta p\left(1+f'\left(\bar{k}\right)+\bar{k}f''\left(\bar{k}\right)-\sigma\right)-\delta\left(\bar{k}f''\left(\bar{k}\right)+1\right)\right]$$

Equations (15) and (16) can be rewritten as

$$\frac{1+bp}{p}\bar{E} = -\beta p\left[\left(1+f'\left(\bar{k}\right)-\sigma\right)k-z\right]+\delta\left[f\left(\bar{k}\right)-\bar{k}f'\left(\bar{k}\right)-\bar{k}\right] \tag{22}$$

$$\bar{E} = \frac{1}{p}\left[\delta\bar{k} - \frac{\delta z}{\left(1 + f'\left(\bar{k}\right) - \sigma\right)}\right] \tag{23}$$

The following analyse describe the comparative static behaviour of the steady state of this model.

The differentiation of (22)and (23) taking b, β, δ and σ as given yields

$$\begin{bmatrix} \frac{1+bp}{p} & \beta p\left(1 + f'\left(\bar{k}\right) - \sigma\right) + \left(\beta p + \delta\right)\bar{k}f''\left(\bar{k}\right) + \delta \\ p\left(1 + f'\left(\bar{k}\right) - \sigma\right) & f''\left(\bar{k}\right)\left(p\bar{E} - \delta\bar{k}\right) - \delta\left(1 + f'\left(\bar{k}\right) - \sigma\right) \end{bmatrix} \begin{bmatrix} \partial\bar{E} \\ \partial\bar{k} \end{bmatrix} = \begin{bmatrix} \beta p \\ -\delta \end{bmatrix} \partial z$$

The determinant of the left-hand-side matrix is

$$|D| = \begin{vmatrix} \frac{1+bp}{p} & \beta p\left(1 + f'\left(\bar{k}\right) - \sigma\right) + \left(\beta p + \delta\right)\bar{k}f''\left(\bar{k}\right) + \delta \\ p\left(1 + f'\left(\bar{k}\right) - \sigma\right) & f''\left(\bar{k}\right)\left(p\bar{E} - \delta\bar{k}\right) - \delta\left(1 + f'\left(\bar{k}\right) - \sigma\right) \end{vmatrix}$$

We set $\mathrm{X} = \beta p\left(1 + f'\left(\bar{k}\right) - \sigma\right) + \left(\beta p + \delta\right)\bar{k}f''\left(\bar{k}\right) + \delta$

and $\mathrm{Y} = f''\left(\bar{k}\right)\left(-\frac{\delta z}{\left(1 + f'\left(\bar{k}\right) - \sigma\right)}\right) - \delta\left(1 + f'\left(\bar{k}\right) - \sigma\right)$ with; $p\bar{E} - \delta\bar{k} = -\frac{\delta z}{\left(1 + f'\left(\bar{k}\right) - \sigma\right)}$

The determinant is $|D| = \frac{1+bp}{p}\mathrm{Y} - p\left(1 + f'\left(\bar{k}\right) - \sigma\right)\mathrm{X}$, where $0 < z < \frac{\left(1 + f' - \sigma\right)^2}{-f''}$ is sufficient for $\mathrm{Y} < 0$. $\mathrm{X} \geq 0$, this condition is not very restrictive. Thus, $|D|$ is negative.

Support mechanism

At an international level, there is an intergenerational conflict between young and old over two types of defensive expenditures due to their contradictory interests. Crucially, the young individuals support environmental care while retirees prefer investing in healthcare.

Given that utility from environmental quality is logarithmic, *E* cannot be negative in equilibrium thus

$$\bar{E} > 0 \quad \Leftrightarrow \quad z < \left(1 + f'\left(\bar{k}\right) - \sigma\right)\bar{k}$$

On one hand, elderly cannot enjoy improvements in the quality of future environment. They prefer spending in private health-care expenditures in the detriment of environment investment though this spending is relatively low since they live in a clean environment; their health is in a good state. They continue to invest in this curative option until a critic value which is the total return of capital. By choosing the curative option, they also choose to invest more of their wage for the next period (negative consumption effect in the period t).

Therefore, they have more precautionary savings which lead to capital accumulation and to worsening the environment quality by increasing their consumption possibilities (positive consumption effect in the period $t+1$). On the other hand, risk-aversion is also important in shaping the combination between health-care spending and environmental quality.

Thus, if old population expresses a higher aversion to the risk with respect to consumption, they have more precautionary savings which lead to capital accumulation and to worsening the environment quality by increasing the young's consumption in the period $t+1$. Since elderly are living in healthy environment, they might be less risk averse. Accordingly, they prefer to consume more in spite of the higher risk (positive consumption effect in t) which lessens the capital and deteriorates s the environment.

For young generation now, environmental expenditure is supported over health-care in an aging economy. Since aging means a higher return from such spending, young people perceive increased aging as a good motive to spend in maintaining the environment healthy – as they are going to live longer to benefit from it. So, a higher environmental maintenance at young age forces them to lower their savings and consumption in first period t. Thus, environmental investment has a negative effect on capital accumulation and a positive effect on environment.

On the other hand, the young generation consumption possibilities in second period $t+1$ are reduced since their precautionary saving is low due to maintenance effort in t. Then, this is a positive effect on the environment quality. It is however important to note that the young individuals are in this case living in a relatively clean environment.

Thus, in an aging economy, they may have a stronger incentive to save and accumulate capital for the next period (in order to increase their consumption when old) than to invest in abatement expenditures seeing as the environment is not much of a problem for the time being. As a result, they reach the second period with a quite high capital stock, thus a high production as well, which enhances their second period consumption (positive consumption effect in $t+1$) and worsens the environment.

However, young people will need then to invest much more in healthcare since the environment has been severely damaged due to the lack of expenditures in maintenance in period t by previous generation. Thus, a disincentive by young generation towards abatement expenditures in an aging economy has a negative effect on environment and a positive effect on capital accumulation.

Under aging societies, the intergenerational conflict that arises from different attitude of young and old towards environment and health spending leads to contradictory effects on capital accumulation and on environment quality. In order to recognize whether the positive effects overcome the negative effects or vice versa, we study the impact of a higher healthcare support by aging population on capital stock and the environment.

Equation (23) can be rewritten as

$$\bar{E} = \frac{\delta}{p}\left[\bar{k} - \frac{z}{\left(1 + f'\left(\bar{k}\right) - \sigma\right)}\right]$$

Private health expenditures support effect:

$$D=\begin{bmatrix} \dfrac{1+bp}{p} & \beta p\left(1+f'\left(\bar{k}\right)-\sigma\right)+\left(\beta p+\delta\right)\bar{k}f''\left(\bar{k}\right)+\delta \\ p\left(1+f'\left(\bar{k}\right)-\sigma\right) & f''\left(\bar{k}\right)\left(p\bar{E}-\delta\bar{k}\right)-\delta\left(1+f'\left(\bar{k}\right)-\sigma\right) \end{bmatrix}$$

and $\quad H=\begin{bmatrix} \beta p\partial z \\ -\delta\partial z \end{bmatrix}$

This gives $\quad D_1=\begin{bmatrix} \beta p\partial z & \beta p\left(1+f'\left(\bar{k}\right)-\sigma\right)+\left(\beta p+\delta\right)\bar{k}f''\left(\bar{k}\right)+\delta \\ -\delta\partial z & f''\left(\bar{k}\right)\left(p\bar{E}-\delta\bar{k}\right)-\delta\left(1+f'\left(\bar{k}\right)-\sigma\right) \end{bmatrix}$

$$D_2=\begin{bmatrix} \dfrac{1+bp}{p} & \beta p\partial z \\ p\left(1+f'\left(\bar{k}\right)-\sigma\right) & -\delta\partial z \end{bmatrix}$$

$$\partial\bar{k}=\frac{\left|D_2\right|}{\left|D\right|}=\frac{-\dfrac{1+bp}{p}\delta\partial z-p\left(1+f'\left(\bar{k}\right)-\sigma\right)\beta p\partial z}{\left|D\right|}$$

$$\frac{\partial\bar{k}}{\partial z}=\frac{1}{\left|D\right|}\left\{-\frac{1+bp}{p}\delta-\beta p^2\left(1+f'\left(\bar{k}\right)-\sigma\right)\right\}>0$$

Since $\left|D\right|<0$ then $\partial\bar{k}/\partial z>0$

In aging population, the positive effects of supporting healthcare on capital accumulation overcome the negative ones.

This result goes in the same path of that achieved for example by Gutierrez (2008) work where environmental degradation made individuals acquire health costs when old, but diverges from that obtained by John and Pecchenino (1995) who find that economies in which consumption causes greater environmental degradation accumulate less capital.

This is so since in their model, individuals pay taxes to sustain environmental quality when they are young and consequently an increase in degradation reduces their savings for the futures.

$$\partial\bar{E} = \frac{|D_1|}{|D|} = \frac{\beta p \partial z\left[f''(\bar{k})(p\bar{E} - \delta\bar{k}) - \delta(1 + f'(\bar{k}) - \sigma)\right] + \delta\partial z\left[\beta p(1 + f'(\bar{k}) - \sigma) + (\beta p + \delta)\bar{k}f''(\bar{k}) + \delta\right]}{|D|}$$

$$\frac{\partial\bar{E}}{\partial z} = \frac{\delta}{|D|}\left\{\delta - f''(\bar{k})\left[\beta p\left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - \bar{k}\right) - \delta\bar{k}\right]\right\}$$

Since $|D| < 0$ then $\frac{\partial\bar{E}}{\partial z} > 0 \qquad \forall \beta p < \delta$

Proof: See Appendix C

As to the environmental quality, supporting health care is harmful to the environment once the maintenance efforts are less than the consumption externalities $\delta < \beta p$. If consumption externalities are lower than the maintenance efforts $\delta > \beta p$, healthcare support is beneficial to the environment.

Proposition:

Under the stable condition, an increase in private health expenditures (higher z) leads to a higher level of capital accumulation and leads to a higher level of environmental quality if $\delta > \beta p$.

5. Conclusion

We have developed a two period overlapping generations model with uncertain lifetimes where agents are affected by environmental quality. To offset this inconvenience, they can invest in defensive expenditures, either in maintenance or healthcare (the preventive versus the curative option). Individuals face tension between those two options. We explore the question of how and whether a population of two groups young and old support and affecting capital accumulation and environment quality. We have shown that an increase of the support to private health expenditures in an aging economy leads to a higher level of

capital accumulation and leads to a higher level of environmental quality if the maintenance efforts are bigger than the consumption externalities.

6. References

Balestra, Carlotta and Davide Dottori. (2012). Aging society, health and the environment. *Journal of Population Economics*, 25, 1045–1076. http://link.springer.com/article/10.1007/s00148-011-0380-x

Chakraborty, S., and M. Das. (2005). Mortality, human capital and persistent inequality. *Journal of Economic Growth*, 10,159–119. http://www.jstor.org/stable/40216101

Chakraborty, S. (2004). Endogenous lifetime and economic growth. *Journal of Economic Theory* , 116(1), 119–137. http://www.sciencedirect.com/science/article/pii/S0022053103002965

Gutierrez M.-J., (2008). Dynamic inefficiency in an overlapping generation economy with pollution and health costs. *Journal of Public Economic Theory*, 10(4), 563-594. http://onlinelibrary.wiley.com/doi/10.1111/j.1467-9779.2008.00377.x/abstract;jsessionid=8545CE47CF648040CA574BE312477BDE.f02t03

Magnani, E.., and Adeline Tubb (2007). The link between economic growth and environmental quality: Does population ageing matter? *School of Economics Discussion Paper*, 2007/12. https://www.researchgate.net/publication/5172281_The_link_between_economic_growth_and_environmental_quality_What_is_role_of_demographic_change

EEA, European Environmental Agency. (2007). "Europe's environment- The fourth assessment." Technical Report 1. http://www.eea.europa.eu/publications/state_of_environment_report_2007_1

John A., and R. Pecchenino. (1994). An overlapping generation model of growth and the environment. *The Economic Journal*, 104, 1393-1410.

Tubb,A.(2011). Does Population Ageing Affect Government Environmental Expenditure? *Committee for Economic Development of Australia*.

Appendices

Appendix A

Proof of equations (7) - (8) *and* (16)

To type the objective function of the individual, we substitute the constraints (1)-(2) and (3) into the utility function (4)

$$U = \ln c_t^1 + p\ln\left[(1+r_{t+1})(w_t - c_t^1 - m_t) - z_{t+1}\right] + \ln E_t + p\ln\left[(1-b)E_t - \beta(c_t^1 + c_t^2)\right]$$

Deriving U with respect to c_t^1 gives

$$\frac{1}{C_t^1} + p\left[-(1+r_{t+1})\frac{1}{c_{t+1}^2} - \beta\frac{1}{E_{t+1}}\right] = 0$$

$$\frac{1}{C_t^1} = p\left[(1+r_{t+1})\frac{1}{c_{t+1}^2} + \beta\frac{1}{E_{t+1}}\right]$$

Deriving U with respect to m_t gives

$$p\left[-(1+r_{t+1})\frac{1}{c_{t+1}^2} + \delta\frac{1}{E_{t+1}}\right] = 0$$

$$\left[-(1+r_{t+1})\frac{1}{c_{t+1}^2} + \delta\frac{1}{E_{t+1}}\right] = 0$$

$$p(1+r_{t+1})\frac{1}{c_{t+1}^2} = \delta\frac{1}{E_{t+1}}$$

Thus;

$$\frac{1}{C_t^1} = p\left[(1+r_{t+1})\frac{1}{c_{t+1}^2} + \beta\frac{1}{E_{t+1}}\right]$$

$$\frac{1}{C_t^1} = p\left[\delta\frac{1}{E_{t+1}} + \beta\frac{1}{E_{t+1}}\right]$$

$$\frac{1}{c_t^1} = p(\beta+\delta)\frac{1}{E_{t+1}}$$

and;

$(8) \Rightarrow E_{t+1} = \dfrac{\delta c_{t+1}^2}{p(1+r_{t+1})}$

Plug (2) and (11) into (8)

$$E_{t+1} = \frac{\delta[(1+f'(k_{t+1})-\sigma)k_{t+1} - z_{t+1}]}{p(1+f'(k_{t+1})-\sigma)}$$

$$E_{t+1} = \frac{\delta[(1+f'(k_{t+1})-\sigma)k_{t+1} - z_{t+1}]}{p(1+f'(k_{t+1})-\sigma)}$$

$$E_{t+1} = \frac{1}{p}\left[\delta k_{t+1} - \frac{\delta z_{t+1}}{(1+f'(k_{t+1})-\sigma)}\right]$$

Appendix B

Proof of equations (21)

To find the stable condition, plug (18) and (18) lagged once into (17) to have the following first-order nonlinear difference equation in k:

$$[(1+p)/p]\phi(k_{t+1}) - (1-b)\phi(k_t) + \beta p[(1+f'(k_t)-\sigma)k_t - z_t] - \delta[f(k_t) - k_t f'(k_t) - k_t$$

$$\Rightarrow [(1+p)/p]\phi(k_{t+1}) - (1-b)\phi(k_t) + \beta p(1+f'(k_t)-\sigma)k_t - \beta p z_t - \delta f(k_t) + \delta k_t f'(k_t) + \delta k_{t+1} = 0$$

$$\Rightarrow -[(1+p)/p]\phi(k_{t+1}) + (1-b)\phi(k_t) - \beta p(1+f'(k_t)-\sigma)k_t + \beta p z_t + \delta f(k_t) - \delta k_t f'(k_t) = \delta k_{t+1}$$

$$\Rightarrow \frac{-[(1+p)/p]\phi(k_{t+1}) + (1-b)\phi(k_t) - \beta p(1+f'(k_t)-\sigma)k_t + \beta p z_t + \delta f(k_t) - \delta k_t f'(k_t)}{\delta} = k_{t+1}$$

$$\Rightarrow \frac{-[(1+p)/p]\phi(k_{t+1}) + (1-b)\phi(k_t) + \delta f(k_t) - \delta k_t f'(k_t) + \beta p z_t}{\delta} = k_{t+1} + \frac{\beta p(1+f'(k_t)-\sigma)k_t}{\delta}$$

$$\Rightarrow \frac{-[(1+p)/p]\phi(k_{t+1}) + (1-b)\phi(k_t) + \delta f(k_t) - \delta k_t f'(k_t) + \beta p z_t}{\delta} = k_{t+1} + \left(1 - 1 + \frac{\beta p(1+f'(k_t)-\sigma)}{\delta}\right)k_t$$

$$\Rightarrow \frac{-\left[(1+p)/p\right]\phi(k_{t+1})+(1-b)\phi(k_t)+\delta f(k_t)-\delta k_t f'(k_t)+\left[-\delta-\beta p(1+f'(k_t)-\sigma)\right]k_t+\beta p z_t}{\delta}=k_{t+1}-k_t$$

This equation can be linearized thanks to Taylor's rule that said:

$$f(x)=f(\bar{x})+\frac{\partial f}{\partial x/\bar{x}}(x-\bar{x})$$

$$\Rightarrow \frac{-\left[(1+p)/p\right]\phi(k_{t+1})+(1-b)\phi(k_t)+\delta f(k_t)-\delta k_t f'(k_t)+\left[-\delta-\beta p(1+f'(k_t)-\sigma)\right]k_t+\beta p z_t}{\delta}$$

$$=\frac{-\left[(1+p)/p\right]\phi+(1-b)\phi+\delta f-\delta k f'+\left[-\delta-\beta p(1+f'-\sigma)\right]k+\beta p z}{\delta}$$

$$+\frac{-\left[(1+p)/p\right]\phi'+(1-b)\phi'-\delta k f''-\delta-\beta p(1+f'-\sigma)-\beta p k f''}{\delta}k_{t+1}-k_t$$

$$\Rightarrow k_{t+1}-k_t=0+\left[\frac{-\left[(1+p)/p\right]\phi'+(1-b)\phi'-\delta k f''-\delta-\beta p(1+f'-\sigma)-\beta p k f''}{\delta}\right](k_t-\bar{k})$$

$$\Rightarrow k_{t+1}-\bar{k}=\left[\frac{-\left[(1+p)/p\right]\phi'+(1-b)\phi'-\delta k f''-\beta p(1+f'-\sigma)-\beta p k f''}{\delta}\right](k_t-\bar{k})$$

$$\Rightarrow k_{t+1}-k_t=-k_t+\bar{k}+\left[\frac{\delta-\left[(1+p)/p\right]\phi'+(1-b)\phi'-\delta k f''-\delta-\beta p(1+f'-\sigma)-\beta p k f''}{\delta}\right](k_t-\bar{k})$$

$$\Rightarrow k_{t+1}-\bar{k}+\frac{\left[(1+p)/p\right]\phi'}{\delta}=\left[\frac{(1-b)\phi'-(\delta+\beta p)k f''-\beta p(1+f'-\sigma)}{\delta}\right](k_t-\bar{k})$$

$$\Rightarrow (k_{t+1}-\bar{k})\left[1+\frac{\left[(1+p)/p\right]\phi'}{\delta}\right]=\left[\frac{(1-b)\phi'-(\delta+\beta p)k f''-\beta p(1+f'-\sigma)}{\delta}\right](k_t-\bar{k})$$

$$\Rightarrow (k_{t+1}-\bar{k})\left[\frac{\delta+\left[(1+p)/p\right]\phi'}{\delta}\right]=\left[\frac{(1-b)\phi'-(\delta+\beta p)k f''-\beta p(1+f'-\sigma)}{\delta}\right](k_t-\bar{k})$$

$$\Rightarrow k_{t+1}-\bar{k}=\left[\frac{(1-b)\phi'-(\delta+\beta p)k f''-\beta p(1+f'-\sigma)}{\delta}\right]\frac{\delta}{\delta+\left[(1+p)/p\right]\phi'}(k_t-\bar{k})$$

$$k_{t+1}-\bar{k}=\left[\frac{(1-b)\phi'-(\delta+\beta p)k f''-\beta p(1+f'-\sigma)}{\delta+\left[(1+p)/p\right]\phi'}\right](k_t-\bar{k})$$

Appendix C

$$\partial\bar{E} = \frac{|D_1|}{|D|} = \frac{\beta p \partial z\left[f''(\bar{k})(p\bar{E} - \delta\bar{k}) - \delta(1 + f'(\bar{k}) - \sigma)\right] + \delta\partial z\left[\beta p(1 + f'(\bar{k}) - \sigma) + (\beta p + \delta)\bar{k}f''(\bar{k}) + \delta\right]}{|D|}$$

$$\frac{\partial\bar{E}}{\partial z} = \frac{1}{|D|}\left\{\beta p\left[f''(\bar{k})(p\bar{E} - \delta\bar{k}) - \delta(1 + f'(\bar{k}) - \sigma)\right] + \delta\left[\beta p(1 + f'(\bar{k}) - \sigma) + (\beta p + \delta)\bar{k}f''(\bar{k}) + \delta\right]\right\}$$

$$\frac{\partial\bar{E}}{\partial z} = \frac{1}{|D|}\left[-\frac{\beta p \delta z}{(1 + f'(\bar{k}) - \sigma)} f''(k) + \delta(\beta p + \delta)kf''(k) + \delta^2\right]$$

$$\frac{\partial\bar{E}}{\partial z} = \frac{1}{|D|}\left[-\frac{\beta p \delta z}{(1 + f'(\bar{k}) - \sigma)} f''(k) + \delta\beta pkf''(k) + \delta^2 kf''(k) + \delta^2\right]$$

$$\frac{\partial\bar{E}}{\partial z} = \frac{1}{|D|}\left[\delta^2 - f''(k)\left(\frac{\beta p \delta z}{(1 + f'(\bar{k}) - \sigma)} - \delta\beta pk - \delta^2 k\right)\right]$$

$$\frac{\partial\bar{E}}{\partial z} = \frac{1}{|D|}\left\{\delta^2 - f''(k)\left[\beta p\delta\left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - k\right) - \delta^2 k\right]\right\}$$

$$\frac{\partial\bar{E}}{\partial z} = \frac{\delta}{|D|}\left\{\delta - f''(\bar{k})\left[\beta p\left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - \bar{k}\right) - \delta\bar{k}\right]\right\}$$

Determination of the sign of: $\delta - f''(k)\left[\beta p\left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - k\right) - \delta k\right]$

- $when: \delta - f''(k)\left[\beta p\left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - k\right) - \delta k\right] > 0$

$$\Rightarrow \delta > f''(k)\left[\beta p\left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - k\right) - \delta k\right]$$

$$\Rightarrow \frac{\delta}{f''(k)} < -\left[\beta p\left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - k\right) - \delta k\right]$$

$$\Rightarrow \frac{\delta}{f''(k)} < -\beta p\left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - k\right) + \delta k$$

$$\Rightarrow \frac{\delta}{f''(k)} - \delta k < -\beta p\left(\frac{z}{(1+f'(\bar{k})-\sigma)} - k\right)$$

$$\Rightarrow -\beta p\left(\frac{z}{(1+f'(\bar{k})-\sigma)} - k\right) > \frac{\delta}{f''(k)} - \delta k$$

$$\Rightarrow \left(\frac{z}{(1+f'(\bar{k})-\sigma)} - k\right) < -\frac{\delta}{\beta p}\left(\frac{1}{f''(k)} - k\right) < 0 \quad \Leftrightarrow \bar{E} > 0$$

$$\Rightarrow k - \frac{z}{(1+f'(\bar{k})-\sigma)} > \frac{\delta}{\beta p}\left(\frac{1}{f''(k)} - k\right)$$

$$\Rightarrow \delta\left(k - \frac{z}{(1+f'(\bar{k})-\sigma)}\right) > \frac{\delta^2}{\beta p}\left(\frac{1}{f''(k)} - k\right)$$

$$\Rightarrow E > \frac{\delta^2}{\beta p}\left(\frac{1}{f''(k)} - k\right)$$

$$\Rightarrow \delta\left(-\frac{z}{(1+f'(\bar{k})-\sigma)} + k\right) > \frac{\delta^2}{\beta p}\left(\frac{1-kf''(k)}{f''(k)}\right)$$

$$\Rightarrow \left(-\frac{z}{(1+f'(\bar{k})-\sigma)} + k\right) > \frac{\delta}{\beta p}\left(\frac{1-kf''(k)}{f''(k)}\right)$$

$$\Rightarrow -\frac{z}{(1+f'(\bar{k})-\sigma)} > \frac{\delta}{\beta p}\left(\frac{1-kf''(k)}{f''(k)}\right) - k$$

$$\Rightarrow -z > \frac{\delta(1+f'(\bar{k})-\sigma)}{\beta p}\left(\frac{1-kf''(k)}{f''(k)}\right) - k(1+f'(\bar{k})-\sigma)$$

$$\Rightarrow -z > \frac{\delta(1+f'(\bar{k})-\sigma) - \delta k(1+f'(\bar{k})-\sigma)f''(k) - \beta p k(1+f'(\bar{k})-\sigma)f''(k)}{\beta p f''(k)}$$

$$\Rightarrow z < \frac{-\delta(1+f'(\bar{k})-\sigma) + \delta k(1+f'(\bar{k})-\sigma)f''(k) + \beta p k(1+f'(\bar{k})-\sigma)f''(k)}{\beta p f''(k)}$$

$$\begin{cases} z < \dfrac{\left(1+f'\left(\bar{k}\right)-\sigma\right)\left[\bar{k}f''(k)(\delta+\beta p)-\delta\right]}{\beta p f''(k)} \\ z < \dfrac{\left(1+f'-\sigma\right)^2}{-f''} \end{cases}$$

To resolve this equations system, we proceed by subtraction

$$\Rightarrow \frac{\left(1+f'-\sigma\right)\left[\bar{k}f''(\delta+\beta p)-\delta\right]}{\beta p f''} + \frac{\left(1+f'-\sigma\right)^2}{f''} > 0$$

$$\Rightarrow \frac{\left(1+f'-\sigma\right)f''\left[\bar{k}f''(\delta+\beta p)-\delta\right]+\beta p\left(1+f'-\sigma\right)^2 f''}{\beta p f''^2} > 0$$

$$\Rightarrow \frac{\left(1+f'-\sigma\right)f''\left[\bar{k}f''(\delta+\beta p)-\delta+\beta p\left(1+f'-\sigma\right)\right]}{\beta p f''^2} > 0$$

$$\Rightarrow \left(1+f'-\sigma\right)f''\left[\bar{k}f''(\delta+\beta p)-\delta+\beta p\left(1+f'-\sigma\right)\right] > 0$$

$$\Rightarrow \bar{k}f''(\delta+\beta p)-\delta+\beta p\left(1+f'-\sigma\right) < 0$$

$$\Rightarrow \beta p\left(1-\sigma+f'+\bar{k}f''\right)+\delta\left(\bar{k}f''-1\right) < 0$$

$$\Rightarrow \beta p\left(1-\sigma+f'+\bar{k}f''\right) < -\delta\left(\bar{k}f''-1\right)$$

$$\Rightarrow p < \frac{\delta\left(1-\bar{k}f''\right)}{\beta\left(1-\sigma+f'+\bar{k}f''\right)}$$

Under our hypothesis;

$$\begin{cases} 1-\bar{k}f'' > 1 \\ f'-\sigma+1+\bar{k}f'' > 1+\bar{k}f'' \end{cases} \Leftrightarrow \begin{cases} \dfrac{1-\bar{k}f''}{f'-\sigma+1+\bar{k}f''} > \dfrac{1}{1+\bar{k}f''} \\ \dfrac{1-\bar{k}f''}{f'-\sigma+1+\bar{k}f''} > 0 \end{cases}$$

$$\Leftrightarrow \frac{1}{1+\bar{k}f''} > 0 \Leftrightarrow 0 < -\bar{k}f'' < 1$$

$$\Rightarrow \frac{1}{1+\bar{k}f''} > 1 \Rightarrow \begin{cases} \dfrac{1-\bar{k}f''}{f'-\sigma+1+\bar{k}f''} > 1 \\ \dfrac{\delta}{\beta}\dfrac{1-\bar{k}f''}{f'-\sigma+1+\bar{k}f''} > p \end{cases}$$

$$\Rightarrow \frac{1-\bar{k}f''}{f'-\sigma+1+\bar{k}f''}\frac{\beta}{\delta}\frac{f'-\sigma+1+\bar{k}f''}{1-\bar{k}f''} > \frac{1}{p} \quad \Rightarrow \frac{\beta}{\delta} > \frac{1}{p}$$

$$\Rightarrow \frac{\beta p}{\delta} > 1 \qquad \Rightarrow \beta p > \delta$$

$$\Rightarrow \delta - f''(k)\left[\beta p\left(\frac{z}{(1+f'(\bar{k})-\sigma)}-k\right)-\delta k\right] > 0 \qquad \forall \beta p > \delta$$

$$\text{Since} |D| < 0,\ \frac{\partial \bar{E}}{\partial z} < 0 \ \forall \beta p > \delta$$

$$\bullet\ if; \delta - f''(k)\left[\beta p\left(\frac{z}{(1+f'(\bar{k})-\sigma)}-k\right)-\delta k\right] < 0$$

$$\Rightarrow \delta < f''(k)\left[\beta p\left(\frac{z}{(1+f'(\bar{k})-\sigma)}-k\right)-\delta k\right]$$

$$\Rightarrow \frac{\delta}{f''(k)} > -\left[\beta p\left(\frac{z}{(1+f'(\bar{k})-\sigma)}-k\right)-\delta k\right]$$

$$\Rightarrow \frac{\delta}{f''(k)} > -\beta p\left(\frac{z}{(1+f'(\bar{k})-\sigma)}-k\right)+\delta k$$

$$\Rightarrow \frac{\delta}{f''(k)} - \delta k > -\beta p\left(\frac{z}{(1+f'(\bar{k})-\sigma)}-k\right)$$

$$\Rightarrow -\beta p\left(\frac{z}{(1+f'(\bar{k})-\sigma)}-k\right) < \frac{\delta}{f''(k)} - \delta k$$

$$\Rightarrow \left(\frac{z}{(1+f'(\bar{k})-\sigma)}-k\right) > -\frac{\delta}{\beta p}\left(\frac{1}{f''(k)}-k\right) > 0$$

$$\Rightarrow k - \frac{z}{(1+f'(\bar{k})-\sigma)} < \frac{\delta}{\beta p}\left(\frac{1}{f''(k)}-k\right)$$

$$\Rightarrow \delta\left(k - \frac{z}{(1+f'(\bar{k})-\sigma)}\right) < \frac{\delta^2}{\beta p}\left(\frac{1}{f''(k)}-k\right)$$

$$\Rightarrow \overline{E} < \frac{\delta^2}{\beta p}\left(\frac{1}{f''(k)} - k\right) < 0$$

Given that utility from environmental quality is logarithmic, E cannot be negative. Thus, this case cannot be a steady state because the utility function is not defined.