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Straight-time and Overtime: A Sequential-Lottery Approach

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Abstract

This note explores the problem of aggregation with non-convex labor supply decisions in an economy with both straight time and overtime. In contrast to Hansen and Sargent (1988), the paper models this as a sequential decision. Instead of changing from one to infinity, with a sequential non-convexity, the aggregate elasticity of labor supply for overtime work is a function of overall participation rate, and the aggregate elasticity of labor supply for full-time work depends on the share of workers doing overtime.

JEL classification: E1, J2, J4

Keywords: Indivisible Labor, Straight time, Overtime, Sequential Lotteries

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1 Introduction and Model Description

This note explores the problem of non-convex labor supply decision in an economy with both straight time and overtime. In contrast to Hansen and Sargent (1988), the paper models this as a sequential decision. The aggregate representation features dis-utilities of both regular and overtime hours that are dependent on the other types of hours.

2 Model Description

2.1 Households' problem

The theoretical setup is a static economy without capital. There is a unit mass of ex-ante identical households. Preferences are defined over consumption (c) and leisure (l), and utility function $u(c, l)$ as follows:

$$u = \ln(c) + \alpha \ln(l), \quad (1)$$

where parameter $\alpha > 0$ measures the relative preference for leisure, and each household has one unit of time. The household faces a sequential labor-supply decision: In stage 1, each household must decide whether to work or not. In stage 2, conditional on working, the household decides whether to work straight-time (\bar{h}), or overtime (h^o).¹ The wage rate is w for straight-time hours and w^o for over-time hours, with $w^o > w$. In addition to the labor income, each household claims an equal share of profits π in the economy, $\int_0^1 \pi di = \Pi$ (hence $\pi = \Pi$).²

Household's utility maximization problem of choosing $\{c, h\}$ optimally by taking $\{w, w^o, \pi\}$ as given, can be split into three sub-cases: c^u will denote consumption of households that do not work, with $c^u = \pi$ and $l^u = 1$. Similarly, full-time workers³ enjoy $c^{e,f} = w\bar{h} + \pi$ and $l^{e,f} = 1 - \bar{h}$, and overtime workers enjoy $c^{e,o} = w\bar{h} + w^o h^o + \pi$ and $l^{e,o} = 1 - \bar{h} - h^o$ (where

¹Those are taken as given, e.g. $\bar{h} = 40$ hours per week, and $h^o = 8$ hours of overtime work.

²This is more of a technical assumption, which is imposed to guarantee that even if a household does not supply any labor, it will enjoy a positive consumption.

³In what follows, "full-time" and "straight-time" will be used interchangeably.

superscript e denotes workers, f refers to the full-time workers, and o denotes overtime workers).

2.2 Firms

There is a representative firm producing a homogeneous final consumption good (its price is normalized to unity). The production function is given by

$$Y = F(\bar{H}, H^o), F_1 > 0, F_2 > 0, F_{11} < 0, F_{22} < 0, F_{12} = 0. \quad (2)$$

There are two capacity constraint: (i) If all households work straight-time only, the marginal product of a regular hour of work is zero, i.e. $F_1(\bar{h}) = 0$; (ii) If every employee works overtime, the marginal productivity of overtime labor also becomes equal to zero, i.e. $F_2(h^o) = 0$.⁴ As in Hansen and Sargent (1988), the firm treats straight-time labor and overtime labor differently.⁵

The firm acts competitively by taking wages $\{w, w^o\}$ as given, and chooses hours $\{\bar{H}\}$ and $\{H^o\}$ to maximize profit:

$$\max_{\bar{H}, H^o} F(\bar{H}, H^o) - w\bar{H} - w^o H^o \quad \text{s.t.} \quad \bar{H} \geq 0, H^o \geq 0. \quad (3)$$

2.3 Decentralized Competitive Equilibrium

A Decentralized Competitive Equilibrium (DCE) is defined by allocations $\{c^u, c^{e,f}, c^{e,o}, \bar{H}, H^o\}$, wage rates $\{w, w^o\}$, and aggregate profit Π s.t. (i) all households maximize utility; (ii) the stand-in firm maximizes profit; (iii) all markets clear.

2.4 Characterizing the DCE

It will be shown that in the DCE, if it exists, only some of the households will be employed. Indeed, if nobody works, nothing is produced, so $\pi = 0$ and $u = -\infty$. In addition, given that $\lim_{\bar{H} \rightarrow 0} F_1(\bar{H}) = \infty$, the firm would pay a very high wage rate to hire a bit of labor.

⁴This is a technical assumption to guarantee that in equilibrium there will be positive economic profits.

⁵This is due to the presence of some labor frictions, which are not explicitly modelled here. To simplify the analysis we assume the two types of hours are not substitutes.

Accepting such a wage would improve substantially household's utility, so nobody working is not an equilibrium (market-clearing) outcome.

Following the same argument, everyone working full-time only is not an equilibrium, as then $w = F_1(\bar{h}) = 0$, and nobody would choose to work for free. Similarly, nobody working overtime does not constitute an equilibrium, since $\lim_{H^o \rightarrow 0} F_1(H^o) = \infty$. That is, the firm would be willing to offer an extremely high premium to hire a bit of overtime labor, and it would not be optimal to refuse such an offer. Lastly, everyone working overtime does not constitute an equilibrium outcome either as $w^o = F_2(h^o) = 0$, and nobody would choose to work overtime for free.

Denote the mass of workers by $q \in (0, 1)$, and the unemployed by $1 - q$. Out of those working, $\lambda \in (0, 1)$ will work overtime, while the rest, $1 - \lambda$, will work only straight-time. Thus, a total of $q\lambda + q(1 - \lambda) = q$ would at least work full-time, while $q\lambda$ will work overtime. Thus, in equilibrium, $\bar{H} = q\bar{h}$, and $H^o = q\lambda h^o$. Also, $c^{e,o} > c^{e,f} > c^u$ and $l^u > l^{e,f} > l^{e,u}$. Lastly, all three groups of households enjoy the same utility level.

From the firm's problem, the wage rate is

$$w = F_1(q\bar{h}), \quad (4)$$

and the overtime wage equals

$$w^o = F_2(q\lambda h^o). \quad (5)$$

Economic profits equal

$$\pi = \Pi = F(q\bar{h}, q\lambda h^o) - F_1(q\bar{h})q\bar{h} - F_2(q\lambda h^o)q\lambda h^o > 0, \quad (6)$$

which follows from the assumption that the production function features decreasing returns to scale.

To show that the DCE actually exists, it is sufficient to show the existence of a unique pair $(q, \lambda) \in (0, 1) \times (0, 1)$ by analyzing a system of two non-linear equations using the

fact that in equilibrium utility is the same for all households: In stage 2, it must be that households are indifferent between working straight time and working overtime:

$$u(c^{e,o}, l^{e,o}) = u(c^{e,f}, l^{e,f}), \quad (7)$$

or

$$\ln(w\bar{h} + w^o h^o + \pi) + \alpha \ln(1 - \bar{h} - h^o) = \ln(w\bar{h} + \pi) + \alpha \ln(1 - \bar{h}). \quad (8)$$

This equation determines the split between full-time $(1 - \lambda)$ and overtime (λ) workers, conditional on the fact that only a certain share q of the population has decided to work. That is, $\lambda = \lambda(q)$.

Similarly, in stage 1, households deciding not to work should be indifferent to households who work full time:

$$u(c^u, l^u) = u(c^{e,f}, l^{e,f}), \quad (9)$$

or

$$\ln(\pi) = \ln(w\bar{h} + \pi) + \alpha \ln(1 - \bar{h}). \quad (10)$$

This equation implicitly defines the proportion q of households deciding to work, conditional on the $q\lambda$ share of those who will work overtime. Note that the two wage rates and profit will all be functions of q, λ (and the values of straight and overtime work, \bar{h} and h^o , respectively, which are assumed to be given). Plugging those back into the utility functions, we obtain two non-linear equations in two unknowns. Proving existence and uniqueness of optimal $(q, \lambda) \in (0, 1) \times (0, 1)$ follows trivially from the Intermediate Value Theorem and the assumptions imposed on the functional forms of utility and the production function.

In addition, given the indivisible labor, the First Welfare Theorem does not hold, so this equilibrium is not Pareto efficient. By giving each household the same consumption (independent of the fact whether they worked or not, or whether they worked full-time or not), the equilibrium allocation can be improved upon. This is demonstrated in the claim below.

Claim: The consumption bundle $\{c^u, c^{e,f}, c^{e,o}\}$ obtained from the DCE above is not efficient, i.e., there is an alternative feasible allocation that will make everyone better off.⁶

Proof: The proof involves solving the problem backwards. In stage 2, a λ fraction of the already employed workers from stage 1 is chosen to supply overtime labor services. Regardless of total hours worked, all workers are given the same consumption c^w , where:

$$c^w = (1 - \lambda)c^{e,f} + \lambda c^{e,o}. \quad (11)$$

In stage 1, a fraction q of the population is chosen to work, but all households are given \tilde{c} , where

$$\tilde{c} = qc^w + (1 - q)c^u = q[(1 - \lambda)c^{e,f} + \lambda c^{e,o}] + (1 - q)c^u. \quad (12)$$

Note that the bundles offered need to be feasible and constitute a Pareto improvement. Showing feasibility is trivial:

$$qc^w = q[(1 - \lambda)c^{e,f} + \lambda c^{e,o}] = q(1 - \lambda)c^{e,f} + q\lambda c^{e,o}. \quad (13)$$

Similarly,

$$\begin{aligned} qc^w + (1 - q)c^u &= q[(1 - \lambda)c^{e,f} + \lambda c^{e,o}] + (1 - q)c^u \\ &= q(1 - \lambda)c^{e,f} + q\lambda c^{e,o} + (1 - q)c^u. \end{aligned} \quad (14)$$

Next, the new allocations makes households better off in expected terms:

$$\begin{aligned} & q \left\{ (1 - \lambda) \left[\ln \left(q[(1 - \lambda)c^{e,f} + \lambda c^{e,o}] + (1 - q)c^u \right) + \alpha \ln(1 - \bar{h}) \right] \right. \\ & \quad \left. + \lambda \left[\ln \left(q[(1 - \lambda)c^{e,f} + \lambda c^{e,o}] + (1 - q)c^u \right) + \alpha \ln(1 - \bar{h} - h^o) \right] \right\} \\ & \quad + (1 - q) \ln \left(q[(1 - \lambda)c^{e,f} + \lambda c^{e,o}] + (1 - q)c^u \right) = \\ & \ln \left(q[(1 - \lambda)c^{e,f} + \lambda c^{e,o}] + (1 - q)c^u \right) + q(1 - \lambda)\alpha(1 - \bar{h}) + \lambda q\alpha \ln(1 - \bar{h} - h^o) > \\ & q(1 - \lambda) \ln(c^{e,f}) + q\lambda \ln(c^{e,o}) + (1 - q) \ln(c^u) + q(1 - \lambda)\alpha(1 - \bar{h}) + \lambda q\alpha \ln(1 - \bar{h} - h^o) = \\ & q \left[(1 - \lambda) \left(\ln(c^{e,f}) + (1 - \alpha) \ln(1 - \bar{h}) \right) + \lambda \left(\ln(c^{e,o}) + \alpha \ln(1 - \bar{h} - h^o) \right) \right] + (1 - q) \ln(c^u). \end{aligned} \quad (15)$$

⁶The DCE not being Pareto efficient is a result of the sequential discrete labor choice. A similar result has been established in Rogerson (1988) for an economy where each household faces a single discrete labor choice.

Thus, the initial DCE allocation can be improved if we allow for employment lotteries, or randomization. In particular, the SP runs a lottery, where q share of the population is chosen to work at least full-time, and then out of those q , λ proportion is selected to work over time, while at the same time everyone receives the same consumption bundle.⁷ In that equilibrium every household receives the same consumption level, which is independent of the employment status. Next, re-define $C = \tilde{c}$ to obtain an aggregate utility function of the form

$$U = \ln(C) + q\lambda\alpha \ln(1 - \bar{h} - h^o) + q(1 - \lambda)\alpha \ln(1 - \bar{h}), \quad (16)$$

or

$$U = \ln(C) + q\lambda\alpha \ln(1 - \bar{h} - h^o) + q\alpha \ln(1 - \bar{h}) - q\lambda\alpha \ln(1 - \bar{h}). \quad (17)$$

Letting

$$A \equiv -\alpha \ln(1 - \bar{h} - h^o) + \alpha \ln(1 - \bar{h}) > 0, B \equiv -\alpha \ln(1 - \bar{h}) > 0, \quad (18)$$

the resulting utility function becomes

$$U = \ln(C) - Aq\lambda - Bq, \quad (19)$$

which differs from Hansen and Sargent's (1988) formulation with simultaneous labor decision (not work, work straight time, or work overtime):

$$U = \ln(C) - \tilde{A}\lambda - \tilde{B}q, \quad (20)$$

where \tilde{A} and \tilde{B} are coefficients that come out as a result of the aggregation under Hansen and Sargent's (1988) one-stage, three-realization lottery.

Due to the non-convexities in both cases, the elasticity of labor supply at micro level differs from the aggregate one. Instead of changing from one to infinity (and the two types of hours becoming separable), as in Hansen and Sargent (1988), with a sequential non-convexity the

⁷Alternatively, imagine there is an insurance market, where each household can buy employment insurance that pays out the full-time labor income $w\bar{h}$ with probability q , and conditional on the payout, with probability λ the household receives also a second payout equal to the overtime labor income w^oh^o .

elasticity of labor supply for overtime work is a function of overall participation rate, and the elasticity of labor supply for full-time work depends on the share of workers doing overtime. Such a non-linear interdependence of the two types of hours would affect the first-order conditions for hours and affect model dynamics when the propagation of technology shocks is considered. However, incorporating such a utility function in a real-business-cycle model is left as a venue for future research.

3 Conclusions

This note explored the problem of two-stage non-convex labor supply decisions in an economy where agents first decide whether to participate in the labor market or stay unemployed, and then, conditional on being hired, need to decide whether they will work only the full-time equivalent, or engage in overtime hours. The novelty was that the aggregated utility function produced interesting non-linearities that were not present at individual level. Instead of changing from one to infinity, as in Hansen and Sargent (1988), with a sequential non-convexity, the elasticity of labor supply for overtime work is a function of overall participation rate, and the elasticity of labor supply for full-time work depends on the share of workers doing overtime.

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