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Dynamic Resource Management under Weak Property Rights: A Tale of Thieves and Trespassers

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Dynamic Resource Management under Weak Property Rights: A Tale of Thieves and Trespassers

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Abstract

Using a dynamic framework with strategic interactions, we study the management of a non-renewable natural resource when property rights are generally weak. Under generally weak property rights both the resource stock and the revenues from exploiting it are imperfectly protected, due to trespassing and theft respectively. Trespassing and theft affect the legitimate owner's extraction decision: extracting the resource today protects the stock against trespassing but exposes the revenues to theft. Moreover, in an evolving institutional setting, the anticipation of a change in the strength of property rights further distorts the extraction decision: e.g., if the owner anticipates stronger property rights in the future, extraction is delayed. Our results indicate that the depletion of the resource is decreasing in the intensity of theft. In addition, when the owner and the trespassers are affected by theft, the depletion of the resource is below (above) the social optimal level if the intensity of theft is high (low).

JEL-Codes: K420, P480, Q320, Q380.

Keywords: depletion, institutions, non-renewable resources, regime shift, weak property rights.

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1 Introduction

Property rights ought to be adequately defined and secured for economic interactions to lead to efficient outcomes. However, in the imperfect world we live in, the problem of weak property rights is not an uncommon one. Such is the case of the management of non-renewable resources, where property rights have been for long central in the discussion of how to secure the optimal use of these resources. In this context, a weak protection of property rights is often associated with a problem of common access to the stock of the resource. That is, it is generally assumed that when property rights are weak, agents cannot be effectively excluded from accessing the pool of the resource, leading to it being over-exploited (tragedy of the commons).

Besides the problem of common access to the pool of the resource, the weakness of the property rights system can also have other manifestations. Hotte, McFerrin, and Wills (2013) rightly point out that the failure to fully appropriate the benefits from exploiting a resource is another form of weak property rights. When property rights are weak, the management of a non-renewable resource may not only be affected by insecure property rights over the stock of the resource, but also by imperfect property rights over the output generated from exploiting the resource.

Yergin's (2008) account of the first major oil discovery in the U.S. provides a good illustration of an environment with generally weak property rights. January 10, 1901 marked the beginning of the Texas oil boom at Spindletop hill in the south of the town of Beaumont. That day, the first successful drilling in the area caused a dramatically high oil gusher, and it did not take long for the news to spread across the country. In a short period of time a mass of workers flocked into Beaumont hoping to seize a share of its underground riches. As highlighted by Yergin (2008) "[w]ithin months, there were 214 wells jammed in on the hill, owned by at least a hundred different companies" (p. 70). This seemingly indiscriminate access to oil in the ground, would soon have its consequences, "by the middle of 1902 . . . the underground pressure gave out at Spindletop because of overproduction, and specially because of all those derricks on postage-stamp sized plots, and production on the Big Hill plummeted". While the oil was being rapaciously depleted, the "fortunes" made from the oil extraction were far from protected. Beaumont was not exactly a safe haven "there were two or three murders a night . . . and there were endless frauds to make sure that money changed hands quickly" (p. 69).

A similar situation occurred half a century before the Texas oil boom, when news of the gold discoveries in California were fast to spread across America. At the outset of the California gold rush in 1848, California was yet to be admitted to the Union, meaning that rush effectively took place in a "stateless" environment. In fact, a large fraction of first mineral discoveries in America occurred in a situation of statelessness. According to Couttenier, Grosjean, and Sangnier (2014) 35% of the counties where minerals were discovered between 1825 and WWII did not officially belong to a state or a colony at the time of the first discovery. The statelessness made it difficult to solve the coordination issue inherent to the public protection of property rights (Anderson & Libecap, 2014). The absence of a state, and its coordinating role, hindered the emergence of formal institutions of property rights protection. So, "not only were there no institutions to enforce the laws, there were no laws" (McDowell, 2002, p. 2). It has been argued that, during the rush, informal rules emerged to take the place of formal institutions regulating the access to private property, and that private efforts (partially) com-

pensated for the absence of publicly provided enforcement of property rights (e.g., Umbeck, 1977; McDowell, 2002). However, it is unclear whether the informal rules and the private efforts actually served to deter trespassing and other property and violent crimes. Clay and Wright (2005) contend that this set of informal rules and enforcement bodies rather “gave equal attention to the rights of claim-jumpers [i.e., trespassers] as to claim-holders [i.e., owners], a balance that in practice generated chronic insecurity” (p. 155). Clay and Wright (2005) go further and propose that despite the existence of these informal institutions and enforcement bodies, gold mining during the rush remained closer to an open access regime. Besides claim-jumping (trespassing), “robberies and assaults also seemed to be on the rise” (Rohrbough, 1997, p. 218), posing a direct threat on the miner’s output. Moreover, the absence of a governing body coordinating law enforcement, implied that individual efforts had to be diverted into the administration of protection and justice (Owens, 2002).

A present-day counterpart of the American gold rush of the mid 1800s is the case of illegal/informal mining in the developing world (Banchirigah, 2008; Hilson, 2002; Hilson & Potter, 2003). This activity, which is by no means marginal, is a modern example of the problem of generally weak property rights. Illegal miners are trespassers of the legitimate owner’s (the government’s) property rights over the stock of the resource. Next to this, activities related to the transformation of the illegal mineral output into cash typically occur outside the law. In practice, this means that illegal miners are especially unprotected against property and violent crime. Illegal miners cannot turn to the government for protection, for instance, to enforce contracts without threatening their own economic activity.

These pieces of anecdotal evidence share as common theme: the exploitation of a non-renewable natural resource in an environment of weak property rights, where both the stock in the ground and the output after extraction are at risk. With this as a background, this paper analyzes the dynamic management of a non-renewable resource when property rights are generally weak. In particular, we study whether under generally weak property rights (i.e., in the presence of theft and trespassing), the pace of depletion of the resource is too high or too low relative to the social optimum. Furthermore, we analyze how a dynamically institutional framework, that is an evolving protection of property rights, has an effect on the resource’s extraction path.

This paper fits into a long tradition of resource economics literature dealing with resource management under insecure property rights. This literature has largely focused on the “common access to the stock” side of the weak property rights story (e.g., Copeland & Taylor, 2009; Hardin, 1968; Van Long, 2011; Ostrom, 2008). The typical result is that the failure to internalize the effect own use of the resource on the rest of the users, leads to excessive use of the resource from the social perspective. In this sense, analyzing the effects of the interaction between two embodiments of a weak property rights system (i.e., weakly protected stock and flow) in a resource management problem is relevant in itself. From a general perspective, this type of analysis, combining a set of imperfections is an application of the second best theory (Lipsey & Lancaster, 1956). Essentially by assuming that, on top of the common access to the stock problem, the revenues from extraction are imperfectly protected, a new source of inefficiency is added to an already imperfect world. Our results indicate that a world with more imperfections may be preferred from the social perspective: agents may react in opposite ways to different imperfections and thus the aggregate effect is less damaging than that of the separate imperfections. Following the idea that the problem of weak property rights Ascan go beyond

the “common pool” problem, Hotte et al. (2013) study a static production problem in which both the input used to produce (“the common” in the resource literature) and the output are imperfectly protected. As an application of the second best theory, their results indicate that in the presence of both sources of imperfection, production can be too high or too low from the social perspective.

Our paper adds to the study of the inter-play between two manifestations of a generally weak property rights, by exploring its effects on the *dynamic* management of a resource. Specifically, we assume that access to the resource stock is not fully secured, and that the benefits from extracting it are imperfectly protected. Our aim is to understand the effect that the interaction between these two types of property rights imperfections, has on the rate of depletion of a non-renewable resource. More specifically, we study how the presence of these two imperfections and the dynamic evolution of the institutional quality, i.e., changes in the intensity of the imperfections, have an impact on the inter-temporal trade-offs governing the strategic interactions between the legitimate and illegitimate users of a non-renewable resource.

To the best of our knowledge, this is the first contribution studying the interaction between these two types of imperfections in a dynamic setup. Therefore, our main contribution is to explore the effects of an environment of generally weak property rights on the dynamic extraction path of a non-renewable resource. Furthermore, our analysis is based on a rich, yet tractable, dynamic framework in which institutional quality is allowed to evolve over time. Following Hotte et al. (2013), we refer to the illegitimate extraction of the resource as trespassing, and to the appropriation of someone else’s output as theft. The dynamic nature of the resource management problem creates a clear distinction between these two. Trespassing affects the stock that remains in the ground, while theft reduces the value of the extracted flow. Therefore, from the legitimate owner’s perspective faster depletion serves to protect the resource against trespassing, but increases its exposure to theft. On top of the clear distinction between the two imperfections, adopting a dynamic perspective allows us to explore the effect an evolving protection of property rights. In particular, when agents anticipate changes in the strength of property rights, the inter-temporal trade-offs governing their extraction decisions are further distorted.

The depletion of a non-renewable resource is in essence a consumption-saving problem, in which the benefits and costs from extracting today are weighed against the benefits of leaving the resource in the ground for future use. Adopting a dynamic perspective generates new insights on the interaction between the two types of inefficiencies. For instance, theft not only reduces the value of what is being currently extracted, but it also reduces the value of what remains in the ground, because it is eventually going to be extracted and will potentially be exposed to theft as well. So, from the inter-temporal point of view the effect of theft on the extraction path actually depends on whether the intensity of theft changes over time. If theft is expected to remain constantly intense over time, the legitimate owner has no motive to distort her extraction path. However, if theft is expected to decrease in intensity, say because thieves are expected to be captured, the owner would adopt a more conservative position towards the extraction of the resource. Therefore, not only the current property rights strength but also its expected evolution determine the current level of depletion. Although completely absent in a static analysis, this type of inter-temporal considerations remain central to understand the dynamic channels affecting the management of a non-renewable resource, specially

when the institutional framework is expected to change over time.

This document is organized in four sections including this introduction. In section 2 the theoretical model is set up and solved. In section 3 the main results are analyzed. Section 4 discusses some of the assumptions of the framework developed here. Finally, section 5 is devoted to the concluding remarks.

2 Model

2.1 Setup

The model presented here examines how the use of a non-renewable resource is affected by insecure property rights, where the imperfect protection is embodied by two types of distortions. First, the stock of the resource is imperfectly protected. That is, the rightful owner/user of the resource does not have exclusive access to the stock of the resource and other agents can trespass his property and exploit the remaining stock. Second, the proceeds from extraction are unprotected, and so other agents can appropriate a fraction of the owner's revenues from extraction.

To illustrate these two types of imperfections we build a continuous time infinite horizon model with three agents: owner (i), trespasser (j), and thief (h). The owner is endowed with a stock $S_0 > 0$ of a non-renewable resource; the trespasser (while active) also has access to this stock and can extract from it; and, the thief can put effort into appropriating a fraction of the owner's revenues from extraction. In the following we describe the exact interactions entailed by each type of distortion.

Trespassing Initially both the owner and the trespasser have access to the stock of the resource, and they simultaneously decide how much of the resource to extract at each point in time. Instantaneous extraction is denoted by R_i and R_j respectively; by extracting R_i units the owner gets $\theta (\theta - 1)^{-1} R_i^{1-\frac{1}{\theta}}$ while by extracting R_j the trespasser gets $(1 - \Omega) \theta (\theta - 1)^{-1} R_j^{1-\frac{1}{\theta}}$, with $\theta \in (1, 2)$.¹ $\Omega \in \{\omega, 1\}$ reflects the level of institutional strength against trespassing. If $\Omega = 1$, the trespasser has no incentives to deplete the resource and the resource is fully protected against trespassing. If $\Omega = \omega < 1$, the trespasser actively participates in the depletion of the resource. Extraction depletes the resource over time: $\dot{S}(t) = -R_i(t) - R_j(t)$; and cumulative extraction is constrained by the remaining stock of the resource $\int_t^\infty (R_i(v) + R_j(v)) dv \leq S(t)$. The assumption here is that the owner and the trespasser individually face an extraction technology constraint. That is, the interaction between the owner and the trespasser is purely of inter-temporal nature (it goes through the depletion of the stock) but, trespassing does not pose an intra-temporal externality on the owner (i.e., trespassing does not drive down the owner's marginal benefit from extraction). Instead of thinking of trespassing as a problem of a "common stock", one could in principle approach it as problem of access to a "common market". In that case, trespassing is equivalent to higher competition, which reduces the owner's marginal return to extraction. Then, the externality imposed by trespassing is fundamentally intra-temporal. Given that our main interest is to focus on the inter-temporal trade-offs, we abstract from the "common market" interpretation in order to preserve the transparency of the dynamic mechanisms.

¹The upper bound for θ guarantees the existence of an equilibrium in linear strategies in the trespassing game.

Theft Upon extraction, the owner gets a gross revenue flow of $\theta (\theta - 1)^{-1} R_i^{1-\frac{1}{\theta}}$.² However, a fraction τ of this flow can be appropriated by the thief. This fraction is endogenously determined by a ratio contest success function:

$$\tau(e_i, e_h) = \frac{(1 - \Lambda) e_h}{\Lambda e_i + (1 - \Lambda) e_h}$$

where e_h is the effort that the thief puts into appropriation and e_i is the protecting effort by owner, e_h and e_i have the same exogenous unit cost of w , and $\tau(0, 0) = 0$. The relative efficiency of the protective effort depends on the theft-specific dimension of institutional quality $\Lambda \in \{\lambda, 1\}$. This is a measure of the de facto protection against theft, with $\lambda = 1$ being perfect protection.

Institutional quality The institutional space in this economy is two-dimensional: Ω determines how strong is the institutional environment against trespassing, while Λ determines the institutional strength against theft. Along these two dimensions we can define four distinct regimes of general institutional quality: *i.* generally weak institutions, $\Omega = \omega$; $\Lambda = \lambda$ (i.e., a regime with theft and trespassing); *ii.* weak protection of income, $\Omega = 1$; $\Lambda = \lambda$ (i.e., a regime with only theft); *iii.* weak protection of wealth, $\Omega = 1$; $\Lambda = \lambda$ (i.e., a regime with only trespassing); *iv.* strong institutions $\Omega = 1$; $\Lambda = 1$ (i.e., a regime without theft and trespassing).

We assume that the initial state is one of generally weak property rights, and from there institutions improve at uncertain times. An institutional improvement in this context means Ω or Λ becoming equal to one. Moreover $\Omega = 1$ and $\Lambda = 1$ are absorptive states, i.e., once institutions become strong in one dimension they remain strong.³ The speed and direction of the institutional improvement is determined by two types of parameters: *i)* $\pi > 0$ determines the overall speed of change that is, how likely are institutions to improve; *ii)* the probabilities $p \in [0, 1]$ and $q \in [0, 1]$, determine whether this improvement occurs along the trespassing dimension or the theft dimension respectively. More specifically, the hazard of Ω shifting from ω to 1 is πp , while the hazard of Λ shifting from λ to 1 is πq . π is an economy-wide measure of how fast institutions are likely to improve, while p and q are crime-specific and can be related to the specific development path of institutions. For instance, the legal system may evolve in such a way that it initially has a bias towards the protection of wealth (property), and eventually shifts its attention to the protection of income.

We assume that all the co-movement in the institutional improvement runs through π (i.e., p and q are not related to each other). This way of connecting the likelihood of a regime shift when multiple shifts are possible follows from Sakamoto (2014). Note that regime shifts in this setup are always beneficial for the legitimate owner, as a regime shift translates into the once and for all elimination of a type of crime.

Objective and Equilibrium The three agents seek to maximize the Net Present Value of revenues, using the exogenous rate r as a discount. We look at Markovian strategies, and rely on the Feedback

²The implicit assumption here is that extracted output cannot be stored. In case output can be stored, nothing changes if stored output is as imperfectly protected as the stock in the ground. Otherwise, the possibility to store would lead the owner to speed-up extraction with the purpose of transforming the insecure stock in the ground into a secure stock above the ground.

³The assumption that institutions can only improve, is chosen to facilitate the exposition, and it is in line with the motivational anecdotes in section 1. However, one could think of empirical settings in which institutions were actually deteriorating over time, for instance after the collapse of the Soviet Union, or in which institutions follow more chaotic paths. As it is discussed in section 4, the modeling tools developed here serve to analyze these alternative institutional dynamics.

Nash Equilibrium as equilibrium concept. Moreover, in the trespassing game we focus on linear strategies. That is, the extraction strategy of each agent is set to be a linear function of the remaining stock.

2.2 Solution

As mentioned above, there are four distinct regimes that can be analyzed depending on the strength of each of the institutional dimensions. Initially institutions are generally weak and both types of criminals are active, and eventually institutions will become strong and both types of crime will vanish (provided that p and q are strictly positive). We do not assume any specific sequence for the path of institutional improvement, meaning that institutions may first improve in any of the two dimensions. As a benchmark we first present the case with strong institutions, then we analyze the “weak protection of income” regime (i.e., when only the thief is active), then the “weak protection of wealth” regime (i.e., when only the trespasser is active), and finally the regime with generally weak institutions (i.e., when there is trespassing and theft). With institutions improving over time the continuation value in the more insecure regimes is the expected net present value of the more secure regimes; therefore, we solve the problem in a backward-induction-style moving from most to least secure.

2.2.1 Strong institutions — No trespassing and no theft

The problem in the perfect protection regime is a standard one. Once both types of crime have been eliminated, no further regime shifts can occur. We use the solution of this institutional environment as the social benchmark. This benchmark entails assuming a social planner that is not constrained by the institutional quality and is free to distribute the rents between agents. Alternatively, one could think of an intermediate social benchmark in which the planner is constrained by the institutional environment (i.e., the different regimes and the hazards of a shift) and the weights of the individuals in the social welfare function, and that is only able to decide the level of extraction by the owner and the trespasser. Section 4.1.2 presents this alternative benchmark.

When institutions are strong, the Hamilton-Jacobi-Bellman (HJB) equation of the owner’s problem is:

$$rV_i(S(t)) = \max_{R_i} \left\{ \frac{R_i(t)^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} - \frac{\partial V_i(S(t))}{\partial S(t)} R_i(t) \right\}.$$

Using standard techniques to solve, depletion (defined as R/s) is

$$\frac{R_i(t)}{S(t)} = \theta r,$$

and the value of the remaining stock is

$$V_i(S(t)) = \frac{S(t)^{1-\frac{1}{\theta}}}{\left(1-\frac{1}{\theta}\right)(\theta r)^{\frac{1}{\theta}}}. \quad (2.1)$$

2.2.2 Weak protection of income — Only theft (th)

Under this regime, the owner's problem exhibits two main differences with respect to the perfect protection benchmark: first, the net flow of revenues needs to be adjusted by the total cost of theft, i.e., theft itself and protecting effort; second, it needs to account for the possibility of a regime shift. As mentioned above, the thief faces the risk of her activity becoming unprofitable; this means that the owner faces the "risk" of a regime shift from a theft only environment to one with strong institutions.

To address the first difference we assume that at every point in time the owner and the thief, after observing the flow of revenues that the former gets from extraction, engage in a contest over these revenues. Specifically, the owner keeps a fraction $1 - \tau$ of the flow of revenues, while τ goes to the thief. Remember that τ is endogenously determined by a ratio contest success function $\tau(e_i, e_h) = (1 - \Lambda) e_h (\Lambda e_i + (1 - \Lambda) e_h)^{-1}$. That is the fraction of revenues going to the owner and the thief depend on the contesting efforts (e_i and e_h) which are chosen simultaneously, after observing the flow of revenues. As for the second difference, one can introduce the effect of a regime shift taking into account that the effective hazard of a shift is constant and equal to πq , and that the continuation value for the owner is the stock's value under perfect protection and for the thief is 0. The HJB equations for the owner and the thief respectively are:⁴

$$rV_i^{th} = \max_{\{R_i, e_i\}} \left\{ (1 - \tau(e_i, e_h)) \frac{R_i^{1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}} - we_i - \frac{\partial V_i^{th}}{\partial S} R_i + \pi q (V_i - V_i^{th}) \right\},$$

$$rV_h^{th} = \max_{\{e_h\}} \left\{ \tau(e_i, e_h) \frac{R_i^{1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}} - we_h - \frac{\partial V_h^{th}}{\partial S} R_i - \pi q V_h^{th} \right\}.$$

Note that the superindex "th" in the value function stands for theft only, and the absence of it indicates perfect protection. The FOCs with respect to the contesting efforts (e_i and e_h) reveal that this is in essence a static problem (with dynamic consequences). If $\Lambda = 1$, there is no contest and the owner retains all the revenues from extraction. If $\Lambda = \lambda$, the optimal appropriation and protection efforts are determined by the following first-order conditions for the owner and the thief respectively:

$$\frac{(1 - \lambda) \lambda e_h}{(\lambda e_i + (1 - \lambda) e_h)^2} \frac{R_i^{1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}} - w = 0,$$

$$\frac{(1 - \lambda) \lambda e_i}{(\lambda e_i + (1 - \lambda) e_h)^2} \frac{R_i^{1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}} - w = 0.$$

In equilibrium

$$e_i^{th}(t) = e_h^{th}(t) = (1 - \lambda) \lambda \frac{R_i(t)^{1-\frac{1}{\theta}}}{\left(1 - \frac{1}{\theta}\right) w}.$$

Therefore $1 - \tau^{th} = \lambda$. With $\Lambda = \lambda$ the owner and the thief effectively engage in a contest over the revenues, and in equilibrium the owner retains a fraction λ of the revenues. The owner's revenues

⁴The time-dependence is omitted to simplify the notation.

net of theft and the cost of protection are given by

$$\left(1 - \tau^{th}\right) \frac{R_i(t)^{1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}} - w e_i^{th}(t) = \lambda \frac{R_i(t)^{1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}} - w e_i^{th}(t) = \lambda^2 \frac{R_i(t)^{1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}}.$$

The owner's extraction problem then leads to the following HJB equation:

$$rV_i^{th} = \max_{R_i} \left\{ \lambda^2 \frac{R_i^{1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}} - \frac{\partial V_i^{th}}{\partial S} R_i + \pi q (V_i - V_i^{th}) \right\}.$$

The associated first-order condition (FOC) is

$$\lambda^2 R_i^{-\frac{1}{\theta}} = \frac{\partial V_i^{th}}{\partial S},$$

which back into the HJB equation leads to

$$(r + \pi q) V_i^{th} = \frac{\lambda^{2\theta}}{\theta - 1} \left(\frac{\partial V_i^{th}}{\partial S} \right)^{1-\theta} + \pi q V_i.$$

Motivated by (2.1), we use

$$V_i^{th}(S(t)) = \frac{\lambda^2 S(t)^{1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}} \left(k_i^{th} \right)^{-\frac{1}{\theta}} \quad (2.2)$$

as a guess for the value function $V_i^{th}(S(t))$, while $V_i(S(t))$ directly comes from (2.1). Using this guess and the FOC in the HJB equation, it is obtained that the solution for the k_i^{th} constant, which is depletion rate in equilibrium (i.e., R/s), is implicitly given by

$$z_i^{th}(k_i^{th}) \equiv k_i^{th} + \frac{\theta \pi q}{\lambda^2 (\theta r)^{\frac{1}{\theta}}} \left(k_i^{th} \right)^{\frac{1}{\theta}} = \theta (r + \pi q), \quad (2.3)$$

with $z^{th}(\cdot)$ being increasing and concave in k^{th} . From this expression it becomes evident that (as expected) depletion is less rapacious in a more theft-prone environment: i.e., k_i^{th} is increasing in λ . This follows directly from z_i^{th} being strictly increasing in k_i^{th} and decreasing in λ . Intuitively, the lower λ the more harmful theft is, and thus the more is there to win from preserving the resource until after theft is eliminated ($\lambda = 1$). Note that with $\lambda = 1$, the depletion rate corresponds to the social optimum level θr .

Proposition 1. *Depletion in the theft only regime ($\Lambda = \lambda < 1$) is below the social optimum: $k_i^{th} \leq \theta r$.*

Proof: See Appendix A.1.

Proposition 2. *The more likely is protection against theft to improve, the higher the owner's incentives to preserve the resource: k_i^{th} is decreasing in πq .*

Proof: See Appendix A.1.

To interpret these two propositions one should bear in mind that a regime shift is favorable from the owner's viewpoint (i.e., shifting to a world without theft is good news for the owner); and that, the

problem of the owner is a typical consumption-savings trade-off. Saving the resource (not extracting today) comes at the cost of not consuming today but, has the potential advantage of leaving the resource to be extracted in a safer environment (theft will no longer be a threat at some point in the future). The intuition behind proposition 1 is that the owner slows down extraction while theft is a threat because it reduces marginal benefit of extraction but, this reduction is expected to have a finite end date (the thief is expected to be captured in finite time), thus the owner preserves the resource today with the objective of extracting a larger fraction of it in a potentially safer environment. As to proposition 2, the higher πq the less the owner expects to wait for the protection against theft to improve, and therefore the more willing the owner is to preserve the resource.

2.2.3 Weak protection of wealth — Only trespassing (TR)

In the presence of the trespasser two elements need to be accounted for: first, total extraction depends on how much both the owner and the trespasser extract; and second, as with the thief, the trespasser faces the risk her activity becoming unprofitable (i.e., Ω becoming 1).

The HJB equation of the owner's problem is:

$$rV_i^{TR} = \max_{R_i} \left\{ \frac{R_i^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} - \frac{\partial V_i^{TR}}{\partial S} (R_i + R_j) + \pi p (V_i - V_i^{TR}) \right\},$$

where πp is the effective hazard that Ω becomes 1, and $V_i(t)$ is the continuation value for the owner in case this occurs (see 2.1). The superscript "TR" stands for **T**Respassing only.

The trespasser's HJB equation, while $\Omega = \omega$ is:

$$rV_j^{TR} = \max_{R_j} \left\{ (1-\omega) \frac{R_j^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} - \frac{\partial V_j^{TR}}{\partial S} (R_i + R_j) - \pi p V_j^{TR} \right\}.$$

Note that the continuation value for the trespasser is 0 (i.e., the NPV of a shift to $\Omega = 1$ is 0). Both the owner and the trespasser choose their extraction simultaneously in a non-cooperative way, and base their extraction decisions on the remaining stock in the ground. The FOCs with respect to extraction for the owner and the trespasser respectively are

$$R_i^{-\frac{1}{\theta}} = \frac{\partial V_i^{TR}}{\partial S}; \quad (1-\omega) R_j^{-\frac{1}{\theta}} = \frac{\partial V_j^{TR}}{\partial S}.$$

Plugging this back in the owner's HJB equation,

$$(r + \pi p) V_i^{TR} = \frac{1}{\theta - 1} \left(\frac{\partial V_i^{TR}}{\partial S} \right)^{1-\theta} - (1-\omega)^\theta \frac{\partial V_i^{TR}}{\partial S} \left(\frac{\partial V_j^{TR}}{\partial S} \right)^{-\theta} + \pi p V_i,$$

and doing the same for the trespasser's HJB equation one gets

$$(r + \pi p) V_j^{TR} = \frac{(1-\omega)^\theta}{\theta - 1} \left(\frac{\partial V_j^{TR}}{\partial S} \right)^{1-\theta} - \frac{\partial V_j^{TR}}{\partial S} \left(\frac{\partial V_i^{TR}}{\partial S} \right)^{-\theta}.$$

Now, as in the previous case, we guess value functions for both agents of the form

$$V_i^{TR}(S(t)) = \frac{S(t)^{1-1/\theta}}{1-1/\theta} (k_i^{TR})^{-1/\theta}, \quad (2.4)$$

$$V_j^{TR}(S(t)) = (1-\omega) \frac{S(t)^{1-1/\theta}}{1-1/\theta} (k_j^{TR})^{-1/\theta}, \quad (2.5)$$

where k_n^{TR} is an agent-specific constant. This specification of the value function and the FOCs again imply that individual depletion (R_n/s) is given by k_n^{TR} . Using the guesses of the value functions and (2.1) we find:

$$\theta(r + \pi p) (k_i^{TR})^{-\frac{1}{\theta}} = (k_i^{TR})^{1-\frac{1}{\theta}} - (\theta - 1) (k_i^{TR})^{-\frac{1}{\theta}} k_j^{TR} + \frac{\theta \pi p}{(\theta r)^{\frac{1}{\theta}}}$$

and

$$\theta(r + \pi p) (k_j^{TR})^{-\frac{1}{\theta}} = (k_j^{TR})^{1-\frac{1}{\theta}} - (\theta - 1) (k_j^{TR})^{-\frac{1}{\theta}} k_i^{TR}.$$

Rearranging, the equilibrium values of k_i^{TR} and k_j^{TR} are implicitly given by a system of best response functions that can be rewritten as

$$z_i^{TR} (k_i^{TR}) \equiv (2 - \theta) k_i^{TR} + \frac{\pi p}{(\theta r)^{\frac{1}{\theta}}} (k_i^{TR})^{\frac{1}{\theta}} = \theta(r + \pi p), \quad (2.6)$$

and

$$k_j^{TR} - (\theta - 1) k_i^{TR} = \theta(r + \pi p). \quad (2.7)$$

Note that because the continuation value for the trespasser is 0, ω does not play a role in determining the speed of extraction by any of the two agents, and it only affects the trespasser's valuation of the resource.⁵ Moreover, $\theta > 1$ implies strategic complementarity in the extraction game. This follows from the fact that θ is a measure of the curvature of the revenue function. The higher θ the less concave the function, and thus the higher the substitution between extraction today and in the future (i.e., the lower the need to smooth individual extraction out). The presence of another agent with access to the stock of the resource lowers the "return" to preserve it, because part of what is left in the ground is going to be extracted by the other agent; in that sense extracting today protects the resource against future trespassing. When θ is relatively high the lower need for smooth extraction implies that the "return" motive dominates, therefore more rapacious depletion from one agent results also in more rapacious depletion by the other.

Lemma 1. *Given $\theta < 2$, there exists a unique pair of positive constants k_i^{TR} , k_j^{TR} that fulfills the FOCs of the TR problem.*

⁵This result is not only the outcome of the assumption that the continuation value for the trespasser is 0, but also derives from the specific modeling choice for the revenue function. Specifically, as the revenue function is iso-elastic in R , one can separate k_j and $(1 - \omega)$ in the guess for j 's value function.

Proof: See Appendix A.2.

Proposition 3. *i) The owner depletes the resource above the socially optimal depletion rate θr ; ii) before trespassing becomes unprofitable ($\Omega = 1$) the trespasser depletes the resource faster than the owner.*

Proof: See Appendix A.1.

A more rapacious extraction by the trespasser with respect to the owner (i.e., $k_j^{TR} > k_i^{TR}$) can be explained by the fact that, as opposed to the owner, the trespasser faces the risk of losing access to the resource. This on the one hand makes the trespasser effectively more impatient than the owner (i.e., the regime shift is costly for the trespasser). On the other hand, it means that the owner attaches a positive probability to the emergence of a regime free of trespassing in finite time; the scrap value of the resource once the trespasser is captured is an increasing function of the remaining stock, which creates incentives for the owner to preserve the resource.

Corollary 1. *The competition for the depletable stock exacerbates the over-extraction problem pushing the trespasser to deplete the resource even faster than the rate suggested by the “inflated” effective discount (i.e., $R_j/S > \theta(r + \pi p)$).*

Proposition 4. *The more likely it is that trespassing becomes unprofitable the slower the owner extracts: i.e., higher πp implies lower k_i^{TR} .*

Proof: See Appendix A.1.

At first glance higher πp is good news for the owner because of the better prospect of a future free of trespassing. This implies that the owner has stronger incentives to preserve the resource. However, a higher πp makes the trespasser effectively more impatient, because of the higher risk of losing access to the resource. As a result the trespasser becomes more rapacious, which reduces the return to savings for the owner (while active the trespasser extracts a larger fraction of what is left in the ground). Thus, there are two opposing forces determining what the owner should do. On the one hand, the owner wants to preserve the resource for the “trespassing-free” future; on the other hand, the owner does not want to leave the resource exposed to more rapacious trespassing. Which one of the two dominates depends on how concave the revenue function is, and thus on how feasible is the substitution between present and future extraction. With a moderately concave revenue function (i.e., $\theta > 1$), future extraction is a good substitute for extracting today, and delaying extraction is a good strategy: more patience triumphs over lower returns (i.e., the owner prefers to wait until after the trespasser is no longer around).

2.2.4 Generally weak institutions — Trespassing and theft (TRth)

In the initial regime both trespassing and theft are active threats. This regime has three essential characteristics: first, the trespasser extracts from the owner’s stock, so total extraction is the sum of the owner’s and the trespasser’s extraction ($R_i + R_j$); second, the thief appropriates a fraction τ of the owner’s revenues, where revenues net of the total cost of theft are $\lambda^2 \theta (\theta - 1)^{-1} R_i^{1-\frac{1}{\theta}}$; third, the regime can shift in any of three directions namely, weak protection of revenues with hazard $\pi p (1 - q)$, weak protection of wealth with hazard $\pi q (1 - p)$, and strong institutions with hazard $\pi p q$. Taking these three features into account the HJB equation of the owner’s problem is:

$$rV_i^{TRth} = \max_{R_i} \left\{ \lambda^2 \frac{R_i^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} - \frac{\partial V_i^{TRth}}{\partial S} (R_i + R_j) + \pi \mathbf{E}^{TRth} [V_i] - \pi^{TRth} V_i^{TRth} \right\},$$

where $\mathbf{E}^{TRth} [V_i] = p(1-q)V_i^{th} + q(1-p)V_i^{TR} + pqV_i$, represents the expected continuation value after “a” regime shift. While $\pi^{TRth} \equiv \pi(p+q-pq)$ is the effective hazard of a regime shift occurring; that is, the risk of a shift in any of the three potential directions in which a shift can occur. Using equations (2.1)-(2.6) the owner’s HJB equation reduces to

$$(r + \pi^{TRth}) V_i^{TRth} = \max_{R_i} \left\{ \lambda^2 \frac{R_i^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} - \frac{\partial V_i^{TRth}}{\partial S} (R_i + R_j) + \pi \kappa_i \frac{S^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} \right\}. \quad (2.8)$$

With $\kappa_i \equiv p(1-q)\lambda^2 \left(k_i^{th}\right)^{-\frac{1}{\theta}} + q(1-p)\left(k_i^{TR}\right)^{-\frac{1}{\theta}} + pq(\theta r)^{-\frac{1}{\theta}}$ and $\partial \kappa_i / \partial \lambda > 0$. The sign of the derivative of κ_i with respect to λ follows from the equilibrium condition for k_i^{th} (2.3).

Similarly, the trespasser’s HJB equation is

$$rV_j^{TRth} = \max_{R_j} \left\{ (1-\omega) \frac{R_j^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} - \frac{\partial V_j^{TRth}}{\partial S} (R_i + R_j) + \pi \mathbf{E}^{TRth} [V_j] - \pi^{TRth} V_j^{TRth} \right\},$$

with $\mathbf{E}^{TRth} [V_j] = p(1-q)V_j^{th} + q(1-p)V_j^{TR} + pqV_j$ and π^{TRth} as defined above. From equations (2.4) and (2.7) and noting that the continuation value of trespassing becoming unprofitable (i.e., shifting to a regime only with theft or with perfect protection) is 0, the problem of the trespasser can be rewritten as

$$(r + \pi^{TRth}) V_j^{TRth} = \max_{R_j} \left\{ (1-\omega) \frac{R_j^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} - \frac{\partial V_j^{TRth}}{\partial S} (R_i + R_j) + \pi(1-\omega) \kappa_j \frac{S^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} \right\}, \quad (2.9)$$

with $\kappa_j \equiv q(1-p)\left(k_j^{TR}\right)^{-\frac{1}{\theta}}$ and $\pi^{TRth} \equiv \pi(p+q-pq)$ denoting the effective hazard of a regime shift (i.e., the hazard adjusted by the probability that institutions improve in at least one of the two dimensions). Using the system of HJB equations (2.8) and (2.9), it is obtained that the FOCs with respect to extraction are

$$\lambda^2 R_i^{-\frac{1}{\theta}} = \frac{\partial V_i^{TRth}}{\partial S}; \quad (1-\omega) R_j^{-\frac{1}{\theta}} = \frac{\partial V_j^{TRth}}{\partial S},$$

plugging this back into the value functions (2.8) and (2.9)

$$(r + \pi^{TRth}) V_i^{TRth} = \frac{\lambda^{2\theta}}{\theta-1} \left(\frac{\partial V_i^{TRth}}{\partial S} \right)^{1-\theta} - (1-\omega)^\theta \frac{\partial V_i^{TRth}}{\partial S} \left(\frac{\partial V_j^{TRth}}{\partial S} \right)^{-\theta} + \pi \kappa_i \frac{S^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}},$$

$$(r + \pi^{TRth}) V_j^{TRth} = \frac{(1 - \omega)^\theta}{\theta - 1} \left(\frac{\partial V_j^{TRth}}{\partial S} \right)^{1-\theta} - \lambda^{2\theta} \frac{\partial V_j^{TRth}}{\partial S} \left(\frac{\partial V_i^{TRth}}{\partial S} \right)^{-\theta} + \pi (1 - \omega) \kappa_j \frac{S^{1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}}.$$

Now, by using $V_i^{TRth} = \lambda^2 \theta (\theta - 1)^{-1} S^{1-\frac{1}{\theta}} (k_i^{TRth})^{-\frac{1}{\theta}}$ and $V_j^{TRth} = (1 - \omega) \theta (\theta - 1)^{-1} S^{1-\frac{1}{\theta}} (k_j^{TRth})^{-\frac{1}{\theta}}$ as guesses for each of the two value functions to solve this system of Differential Equations (DE), the following system is obtained:

$$z_i^{TRth} (k_i^{TRth}, k_j^{TRth}) \equiv k_i^{TRth} + \frac{\theta \pi \kappa_i}{\lambda^2} (k_i^{TRth})^{\frac{1}{\theta}} - (\theta - 1) k_j^{TRth} = \theta (r + \pi^{TRth}), \quad (2.10)$$

and

$$z_j^{TRth} (k_i^{TRth}, k_j^{TRth}) \equiv k_j^{TRth} + \theta \pi \kappa_j (k_j^{TRth})^{\frac{1}{\theta}} - (\theta - 1) k_i^{TRth} = \theta (r + \pi^{TRth}). \quad (2.11)$$

The solution to this system of non-linear equations will give the equilibrium depletion by the owner and the trespasser in the TRth regime. Given that $\theta > 1$ these equations clearly entail that k_i and k_j are strategic complements.

Lemma 2. *Given $\theta < 2$, there exists a unique pair $(k_i^{TRth}, k_j^{TRth}) \in \mathcal{R}_+^2$ solving the $z_i^{TRth} = z_j^{TRth} = \theta (r + \pi^{TRth})$ system. This means that the equilibrium extraction strategies exist and are unique.*

Proof: See Appendix A.2.

Lemma 3. *When only the owner is affected by theft k_i^{TRth} and k_j^{TRth} are increasing in λ .*

Proof: See Appendix A.2.

Equations (2.10) and (2.11) can be rewritten as

$$k_n^{TRth} = \theta (r - \pi g_n^{TRth}) + (\theta - 1) k_m^{TRth}, \quad (2.12)$$

with

$$g_n^{TRth} \equiv \sum_l P_l^{TRth} \left[\frac{\mu_n^{TRth}}{\mu_n^l} \left(\frac{k_n^{TRth}}{k_n^l} \right)^{\frac{1}{\theta}} - 1 \right],$$

and where $m, n \in \{i, j\}$ and $m \neq n$; πP_L^{TRth} corresponds to the effective hazard of a shift from regime TRth to regime L; $\mu_n^L = \lambda^{-2}$ if agent n is subject to theft in regime L, otherwise, $\mu_n^L = 1$; and $(k_n^L)^{-1/\theta} = 0$ if agent n is not active in regime L. Equation (2.12) shows the two main components distorting the depletion by agent n in the TRth regime (i.e., pushing depletion by n away from the socially optimal level θr). First, the anticipation of a regime shift affects the effective discount rate of n , an effect that is captured by g_n^{TRth} . If n considers the expected regime shift (i.e., the weighed composite of all potential future regimes) to be “good news” that is, if the agent anticipates that on average future institutions will be more favorable (than current institutions), the expectation of a regime shift

makes n effectively more patient, and consequently more conservative in the use of the resource; in this case $g_n^{TRth} > 0$. If on the contrary, from n 's perspective, future institutions are perceived as less favorable (than current institutions) so that a regime shift is "bad news", $g_n^{TRth} < 0$ and the anticipation of a shift makes the agent effectively more impatient, pushing depletion by n above θr . Finally, if future regimes are combination of "good" and "bad" regimes from the perspective n , it may be the case that the "bad" regimes are exactly compensated by the "good" regimes (i.e., $g_n^{TRth} = 0$), and therefore anticipation of a regime shift does not affect n 's effective impatience. Next to the effect on the effective discount imposed by the anticipation "good" and "bad" news, n 's depletion rate can also be distorted by the presence of another extracting agent, as captured by $(\theta - 1) k_m^{TRth}$ in (2.12); in the case of strategic complementarity (i.e., $\theta > 1$) the presence of another extracting agent increases n 's depletion.

3 Analysis

The depletion rates under different regimes k_i^{th} , k_i^{TR} , k_j^{TR} , k_i^{TRth} and k_j^{TRth} can be obtained by solving the non-linear system (2.3), (2.6), (2.7), (2.10), and (2.11). Figure 3.1, depicts a numerical example of the depletion rate under all the possible regimes for both the owner and the trespasser, as the theft intensity λ goes from 0 to 1 (i.e., as the distortion imposed by the imperfect protection of revenue flows decreases).⁶

As expected λ has no effect on the "TR" regime depletion rates (it does not enter the problem), the "TRth" depletion rates (k_i^{TRth} and k_j^{TRth}) are increasing in λ and converge to their "TR" counterparts (k_i^{TR} and k_j^{TR}) because theft becomes less distortive as $\lambda \rightarrow 1$. With respect to the social optimum level of depletion: k_i^{TR} , k_j^{TR} , and k_j^{TRth} are always above the social optimum θr ; k_i^{th} is always below; and k_i^{TRth} is below θr for low values of λ (i.e., when theft is very distortive) and it is above θr when λ is high (i.e., when theft is less distortive).

3.1 Owner and trespasser subject to theft

If both the owner and the trespasser face the threat of theft, and assuming that trespasser and thief engage in exactly the same type of contest over revenues as the owner and the thief, the system of DE for the trespasser in the TRth is simply going to be symmetric to that of the owner, where the one for the owner is still given by (2.10) and the trespasser's becomes:

$$z_j^{TRth} (k_i^{TRth}, k_j^{TRth}) \equiv k_j^{TRth} + \frac{\theta \pi \kappa_j}{\lambda^2} (k_j^{TRth})^{\frac{1}{\theta}} - (\theta - 1) k_i^{TRth} = \theta (r + \pi^{TRth}).$$

The fundamental difference between this case, and the one in which only the owner is affected by theft is that the de facto protection against theft (λ) has a direct effect on the trespassers depletion rate (instead of running solely through the effect on the owner's depletion).

Figure 3.2 depicts k_i^{TRth} , k_j^{TRth} and the sum of the two, when theft affects both the owner and the trespasser. Evidently, when the distortion induced by theft is large (i.e., λ is low), the depletion by both the owner and the trespasser is low, in fact total depletion of the resource is below the social

⁶The rest of the parameters are set to: $\theta = 3/2$, $p = q = 1/2$, $r = 1/5$, and $\pi = 1/10$.

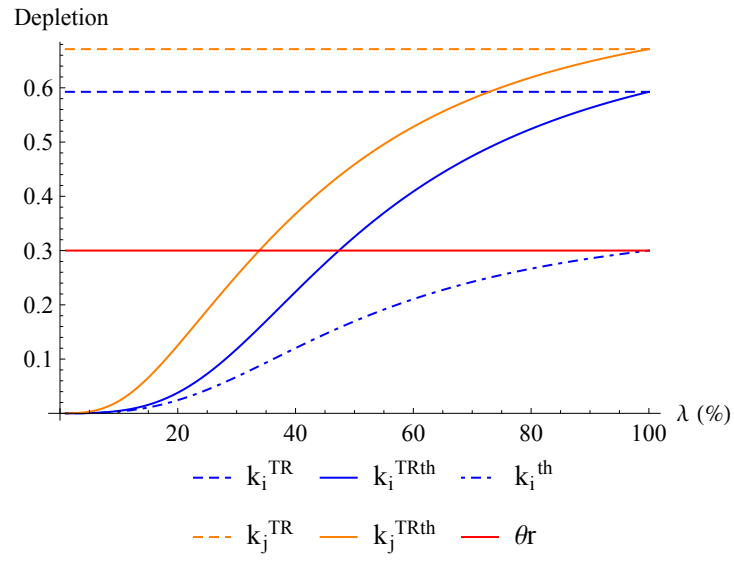


Figure 3.1: Depletion for different λ s. Both owner and trespasser affected by theft

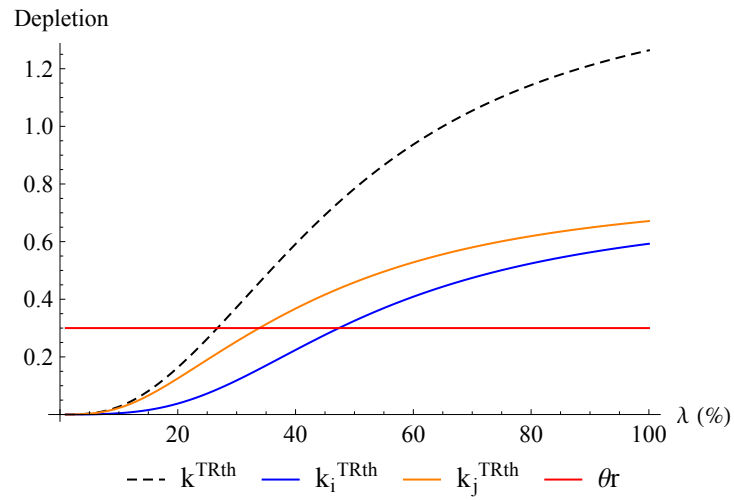


Figure 3.2: Depletion for different λ s. Both owner and trespasser affected by theft

optimum (θr). On the contrary, if the theft distortion is mild, then the total depletion is too high from the social perspective ($> \theta r$). This means that if theft affects both the owner and the trespasser, in the presence of theft and trespassing the socially optimum level of depletion is attainable if theft occurs in the right measure (i.e., the effects of the two distortions exactly cancel out).

3.2 The illegal mining model: only the trespasser is subject to theft

The case of illegal mining seems to be better portrayed by a case in which only the trespasser is directly affected by theft. In this case λ affects only the net revenues from extraction for the trespasser. Again we end up with a system of equations from which one can solve the equilibrium levels of depletion. From the owner's HJB equation one gets

$$z_i^{TRth} \left(k_i^{TRth}, k_j^{TRth} \right) \equiv k_i^{TRth} + \theta \pi \kappa_i \left(k_i^{TRth} \right)^{\frac{1}{\theta}} - (\theta - 1) k_j^{TRth} = \theta \left(r + \pi^{TRth} \right),$$

where $\kappa_i \equiv q(1-p)k_i^{TR-\frac{1}{\theta}} + p(\theta r)^{-\frac{1}{\theta}}$. From this expression it is clear that once the trespasser is captured (which conditional on a regime shift occurs with probability p) the fate of the thief is irrelevant for the owner's problem. While from the trespasser's HJB it is obtained that

$$z_j^{TRth} \left(k_i^{TRth}, k_j^{TRth} \right) \equiv k_j^{TRth} + \frac{\theta \pi \kappa_j}{\lambda^2} \left(k_j^{TRth} \right)^{\frac{1}{\theta}} - (\theta - 1) k_i^{TRth} = \theta \left(r + \pi^{TRth} \right),$$

where $\kappa_j \equiv q(1-p)k_j^{TR-\frac{1}{\theta}}$ remains the unchanged. Evidently, once the thief is captured, the trespasser has the same valuation for the future as in the case when theft only affects the owner.

Lemma 4. *When only the trespasser is affected by theft, k_i^{TRth} and k_j^{TRth} are increasing in λ irrespective of θ .*

Proof: See Appendix A.2.

Interestingly, even if theft is strong enough to completely discourage trespassing (i.e., $\lambda = 0$) the owner's extraction path in the TRth regime would still be distorted. More precisely, if $\lambda = 0$ and thus $k_j^{TRth} = 0$, k_i^{TRth} is above θr because k_i^{TR} is above θr . From the owner's perspective, the scenario with $\lambda = 0$ can be described solely in terms of trespassing: initially during the TRth regime there is no trespassing; then if there is no theft trespassing becomes again an active threat; finally the economy would move back to a regime with inactive trespassing. Even if in the TRth regime theft is such that trespassing plays no immediate role on the owner's problem, the potential for a future regime in which trespassing is active affects current extraction. The mechanism for the distortion in the TRth regime is therefore purely dynamic, and it is actually driven by the same forces that delineate the extraction path in the TR regime. That is, as the owner responds to trespassing by engaging in over-extraction, the owner also over-extracts if there is no current trespassing but there is a potential shift towards a regime with active trespassing.

3.3 Total extraction and institutional quality

Irrespective of whether the owner, the trespasser, or both are affected by theft, total extraction is increasing in the de facto protection against theft λ . The agent(s) directly affected by theft has incentives

to deplete the resource faster if theft is weaker; that is the case because the net marginal return to extraction is higher the weaker theft is; this tilts the inter-temporal trade-off towards current depletion. Due to the strategic interaction between the owner and the trespasser, the agent that is not directly affected by theft also reacts to changes in λ . Specifically, if λ increases the agent that is not directly affected by theft also accelerates depletion, therefore total depletion increases with λ .

Proposition 5. *Whenever the trespasser and the thief are active: i) total depletion is increasing in λ irrespective of which agent (the owner, the trespasser, or both) is subject to theft; ii) if only the owner is directly affected by theft total depletion is always above the social optimum level θr ; iii) if both the owner and the trespasser are subject to theft there exists a $\lambda^* \in (0, 1)$ such that if $\lambda \gtrless \lambda^*$ total depletion is $\gtrless \theta r$; iv) if only the trespasser is subject to theft total depletion exceeds θr for every $\lambda > 0$.*

Proof: See Appendix A.1.

Thus, an improvement in terms of how prone to theft the environment is always leads to more rapacious depletion. This means that in case that only the owner is affected by theft, an improvement in the institutional environment (in terms of a higher λ) exacerbates the over-extraction problem. Actually, a necessary condition for there to be an imperfect level of λ such that depletion is optimal from the social perspective is that the trespassers is directly affected by theft. However, this does not mean that the outcome is a first best because resources are inefficiently diverted into protection and theft.

4 Discussion

4.1 Social Benchmark

4.1.1 Inter-temporal and intra-temporal distortions

As mentioned above the existence of λ^* does not imply that the first best is achievable, only that the total depletion is at its first best level if $\lambda = \lambda^*$. Because of the second best nature of the problem under the TRth regime even if total extraction is optimal, costly effort is diverted in the theft game. Taking into account the total net gain from the protection-theft contest (i.e., sum of revenues net of total effort), the contest is the least efficient when $\lambda = 1/2$ and the most efficient when λ is either 0 or 1.⁷ This is the case because the more symmetric the contest the more effort each agent puts into it (i.e., the higher the stakes), but when the de facto protection is clearly favorable to one agent, both agents face little incentives to engage in the contest.

The total distortion imposed by trespassing and theft comes both in the form of a potentially distorted depletion path and of diverted effort theft game, the former being inter-temporal in nature and the latter intra-temporal. We know that the closer we are to λ^* the less intense the inter-temporal distortion becomes (i.e., the closer is total depletion to the optimal level θr), and the further away we are from $\lambda = 1/2$ the less intense the intra-temporal distortion is. This means that if $\lambda^* < 1/2$ ($> 1/2$), and $\lambda \in (\lambda^*, 1/2)$ a reduction (an increase) in λ is efficiency improving.

⁷If one does not take the revenues of the thief into account the contest is the least distortive when $\lambda = 1$ and the most distortive when $\lambda = 0$.

4.1.2 Restricted social benchmark

Up to this point θr has been used as the benchmark depletion rate. As mentioned above, this is the socially optimal rate of depletion in the absence of any imperfection (i.e., the rate that maximizes the NPV of total revenues under perfectly protected property rights). Alternatively, one could define an intermediate (second-best) benchmark. Suppose that the planner can only determine the level of extraction by the owner and the trespasser, taking the weights of the three agents in the social welfare function, the institutional path, and the distributional rule of the theft game as given. That is, this “constrained planner” cannot change the institutional environment, and, for instance, cannot distribute rents to the trespasser if there is a strong protection of the wealth in the ground. Similarly, this planner cannot choose the effort levels of the theft game, and therefore cannot eliminate the waste of rents generated by theft. The objective of the planner is to maximize the NPV of the expected (weighed) sum of revenues of the three agents. In the TRth regime, the problem of the constrained planner is given by

$$rV_s^{TRth} = \max_{R_i, R_j} \left\{ \phi_{ih} \frac{R_i^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \phi_{jh} \frac{R_j^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} - \frac{\partial V_s^{TRth}}{\partial S} (R_i + R_j) + \pi \mathbf{E}^{TRth} [V_s] - \pi^{TRth} V_s^{TRth} \right\},$$

where ϕ_m is the social weight of agent $m \in \{i, j, h\}$, and $\phi_{nh} \equiv \lambda^2 \phi_n + (1-\lambda)^2 \phi_h$ is the effective social weight attached to the extraction by agent $n \in \{i, j\}$. Moreover, $\mathbf{E}^{TRth} [V_s] = p(1-q)V_s^{th} + q(1-p)V_s^{TR} + pqV_s$ is the expected continuation (social) value after a regime shift. By solving this problem, it is obtained that the total (social) rate of depletion $k_s^{TRth} \equiv (R_i^{TRth} + R_j^{TRth})/s$ is implicitly given by

$$z_s^{TRth} (k_s^{TRth}) \equiv k_s^{TRth} + \theta \pi \kappa_s (k_s^{TRth})^{\frac{1}{\theta}} = \theta (r + \pi^{TRth}), \quad (4.1)$$

with κ_s representing the expected value of the depletion rate after a regime shift (to the power of $-1/\theta$) weighed by the effective social weight of each potential future regime ϕ_s as a proportion of the current regime’s effective social weight:

$$\kappa_s \equiv \frac{p(1-q)\phi_{ih}(k_s^{th})^{-\frac{1}{\theta}} + q(1-p)\phi_{ij}(k_s^{TR})^{-\frac{1}{\theta}} + pq\phi_i(\theta r)^{-\frac{1}{\theta}}}{\phi_{ijh}}.$$

Following the same logic of the decentralized problem, the depletion rates k_s^{TR} and k_s^{th} come from solving the planner’s problem under the TR and th regimes respectively. The effective social weights of the future regimes are given by ϕ_i under perfect protection, ϕ_{ih} under the th regime, and $\phi_{ij} \equiv (\phi_i^\theta + \phi_j^\theta)^{\frac{1}{\theta}}$ under the TR regime. The effective social weight in the TRth regime is $\phi_{ijh} \equiv (\phi_{ih}^\theta + \phi_{jh}^\theta)^{\frac{1}{\theta}}$.

One can compare the solution of the constrained planner to that of the unconstrained planner by rewriting (4.1) in a similar fashion to (2.12):

$$k_s^{TRth} = \theta (r - \pi g_s^{TRth}), \quad (4.2)$$

with

$$g_s^{TRth} \equiv \sum_l P_l^{TRth} \left[\frac{\phi_l}{\phi_{ijh}} \left(\frac{k_s^{TRth}}{k_s^l} \right)^{\frac{1}{\theta}} - 1 \right],$$

where again πP_L^{TRth} corresponds to the effective hazard of a shift from regime $TRth$ to regime L ; and ϕ_L is the relevant social weight in regime L : ϕ_i , ϕ_{ij} , ϕ_{ih} in the perfect protection, the TR, and the th regime respectively. As can be seen from (4.2) the source of distortion for the constrained planner, with respect to the unconstrained planner's (i.e., first best) depletion θr , is the anticipation of a regime. This is the case because, as opposed to the decentralized solution, from the perspective of the constrained planner there is no competition for the extraction of the resource (i.e., there is no "common pool" distortion). The effect of anticipating a regime shift on the effective discount rate of the constrained planner is captured by g_s^{TRth} . If $g_s^{TRth} > 0$ (< 0) [$= 0$], the constrained planner considers the expected regime shift (i.e., the weighed composite of all potential future regimes) to be good news (bad news) [no news]. Therefore the effective discount rate of the constrained planner is lower than (higher than) [equal to] the discount rate of the unconstrained planner, and total depletion by the constrained planner in the $TRth$ regime is $< \theta r$ ($> \theta r$) [$= \theta r$].

Comparing the decentralized solution ($k_i^{TRth} + k_j^{TRth}$) to the constrained planner's solution (4.1), and assuming $\phi_i, \phi_j, \phi_h > 0$,

Proposition 6. *If both the owner and the trespasser are subject to theft, there exist a $\underline{\lambda} \in (0, 1)$ and a $\bar{\lambda} \in [\underline{\lambda}, 1)$ such that if $\lambda < \underline{\lambda}$ ($\lambda > \bar{\lambda}$) total depletion in the decentralized solution, $k_i^{TRth} + k_j^{TRth}$, is below (above) the social level of depletion chosen by the constrained planner (4.1).*

Proof: See Appendix A.1.

This means that, if the constrained planner's solution is used as reference and if both extracting agents are affected by theft, the depletion of the resource in the decentralized solution is above (below) the planner's level if the intensity of theft is low (high); therefore, a low theft intensity leads to a depletion that is too high from the social perspective irrespective of whether the social benchmark is determined by constrained or the unconstrained planner's solution.

Figure 4.1 presents a numerical example of the comparison between the depletion by the constrained planner and the total depletion under the decentralized solution as a function of the theft intensity.⁸ Each of the three U-shaped curves represents the level of depletion k_s^{TRth} for a given set of social weights (ϕ_i, ϕ_j, ϕ_h) with $\phi_j = \phi_h = 1/2 (1 - \phi_i)$. From this illustration it becomes evident that, under the alternative social benchmark provided by the constrained planner's solution, the decentralized level of depletion is above (below) the social level when the intensity of theft is low (high).

In the figure, the shape of the curves representing k_s^{TRth} reveals the effect of attaching a positive social weight to the thief. In the extremes (i.e., when λ is equal to 0 or 1) the rents from extraction are fully distributed (either to the extractors or to the thief); the planner, as opposed to the extractors, attaches value to the rents captured by the thief, and therefore extraction in the planner's solution is positive in either extreme, while in the decentralized solution there is no extraction when $\lambda = 0$. If λ moves away from these extreme values, the theft game inefficiently distributes rents from extraction;

⁸The rest of the parameters are set to: $\theta = 3/2$, $p = q = 1/2$, $r = 1/5$, and $\pi = 1/10$. Therefore the level of depletion under the unconstrained planner's solution θr is $3/10$.

this is, the rents are partially dissipated in the form of protective and grabbing efforts. Remember that the constrained planner takes the distribution of the theft game as given and can only choose the level of extraction. The more inefficient the distribution generated by the theft game, the more patient the constrained planner effectively is: the planner prefers to delay extraction for until after property is protected against theft. Therefore, the higher the distortion generated by the theft game (which from the social perspective is maximal at some intermediate λ), the lower the level of depletion. The lower the social weight of the thief relative to the owner's, the lower the value of λ at which the inefficiency of the theft game is maximal.⁹

The figure also shows that, for a given level of λ , increasing the social weight of the owner (ϕ_i) at the expense of the weights of the trespasser and the thief, leads to a lower depletion (i.e., the k_s^{TRth} curve shifts downwards as ϕ_i increases). This is the case because, the higher the weight attached to the owner, the higher the degree of “good news” embedded in the anticipation of a regime shift, and consequently the more patient the planner is. If the planner cares about the thief and the trespasser, anticipating that these agents may not be active in the future is a mix of good and bad news. It is good news because the owner is better off, and the planner attaches a weight to this, but it is bad news because the planner also values the loss of revenues by the thief and trespasser. Whether an institutional improvement is “in net” anticipated as good or bad news by the constrained planner depends on the social weights of the three agents. In figure 4.1 one observes that for a low ϕ_i , k_s^{TRth} maybe above θr (in the figure $\theta r = 0.3$), meaning that $g_s^{TRth} < 0$ and therefore the unconstrained planner perceives the institutional change as bad news.

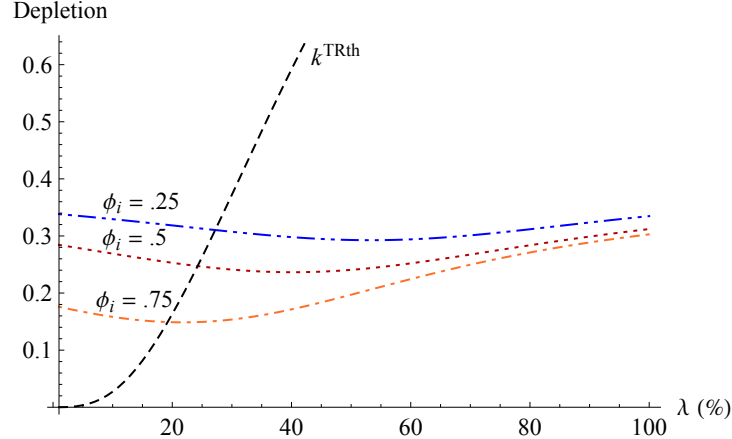


Figure 4.1: Constrained's planner depletion for different λ s

4.2 Alternative institutional paths

The model assumes that institutions improve over time, which is arguably what happened in the context of the California Gold Rush and the Texas Oil Boom. However, a path of improving institutions may not always be a good reflection of reality; think for instance of the collapse of the Soviet

⁹From the figure one can observe that the λ minimizing k_s^{TRth} shifts to the left as ϕ_i/ϕ_h increases. With $\phi_i = \phi_h$, the inefficiency is maximal (k_s^{TRth} is minimal) at $\lambda = 1/2$.

Union, Mobutu's dictatorship, or the Libyan power vacuum after Gaddafi's ouster. Nevertheless, the modeling apparatus developed in this paper is equipped to analyze alternative institutional dynamics. The effect of alternative institutional dynamics on the results depends on one of the fundamental mechanisms unveiled by the model: the effective discount rate applied by individuals depends on how they expect to be affected by institutional changes; that is, whether they anticipate institutional changes as "good news" or as "bad news" (in expected value). For instance, in the current setup an institutional improvement in the theft dimension is internalized as "good news" by both the owner and the trespasser if they are both affected by theft; both the owner and the trespasser are better off in the absence of theft. Therefore the prospect of this institutional improvement makes them effectively more patient. How much more patient they become depends on how much of an improvement the elimination of theft actually is, i.e., the intensity of theft. Now, suppose that the economy is facing an institutional collapse. This means, for instance, that from the TR regime the economy eventually shifts into the TRth regime. During the TR regime, the owner and the trespasser anticipate institutional changes to be "bad news". The prospect of theft in the future makes both i and j effectively more impatient today, when theft is not yet occurring. How much more impatient depends on how intense they anticipate theft to be in the future: the more intense the more impatient. Interestingly, despite already playing a role in the TR regime, through the anticipation of the regime shift, the intensity of theft will not play a role in the TRth regime, provided that this is an absorptive regime. Furthermore, if institutions are expected to deteriorate over time, i 's depletion will be distorted even in case institutions are strong. Specifically, in the absence of theft and trespassing, i will over-extract the resource in anticipation of the institutional collapse.

4.3 Common Stock Vs. Common Market

Going back to the original model with imperfect institutions, we approach trespassing as a problem of common access to the stock in the ground. However, one could also study it as a "common market" (e.g., Boyce & Vojtassak, 2008; Datta & Mirman, 1999; Salo & Tahvonen, 2001; Sandal & Steinshamn, 2004) problem. However, as opposed to the inter-temporal nature of the "common stock" externality, the "common market" one is essentially intra-temporal. Arguably, the "common market" externality mechanically implies that the owner slows down extraction because of the lower instantaneous marginal return. In such case, the resource "over-use" arises as consequence of a larger number of suppliers; that is, there is no individual "over-use" but, there is "over-use" in the aggregate because of the coordination failure.

In contrast, in the "common stock" case the inter-temporal externality creates two opposing forces delineating the owner's behavior. On the one hand, it reduces the incentives to preserve the resource because the stock left in the ground would be shared with the trespasser in the future. On the other hand, for extraction-smoothing purposes the owner may act more conservatively to counter the excessively high depletion induced by trespassing. In terms of the consumption-saving trade-off, the former means that the return to savings is lower, reducing the incentives to save, while the latter implies that the owner cannot fully appropriate her own "savings" in the future, hence there is an increased need for "savings" to finance future "consumption".

5 Concluding remarks

Weak property rights in the management of non-renewable resources can go beyond the “common access” (or “trespassing”) problem typically explored in the resource economics literature. The history of resource rushes (e.g., oil and gold) provides prominent examples of cases in which the legitimate owners of a resource not only had to deal with trespassing but also with the risk of theft. The interaction of these two types of property rights imperfections, one affecting the stock (wealth) in the ground and the other affecting the stream of revenues (income) from extracting it, has a significant effect on the inter-temporal trade-off governing the choice of an extraction path.

In principle, the legitimate owner of the resource needs to take into account that extracting the resource protects it from trespassing, but exposes it to theft. The model that we develop highlights that the dynamic implications of an environment with generally weak property rights are rich and go beyond this intuitive trade-off. These implications are rooted in the dynamic strategic interactions between agents, as well as in the possibility of shifts towards regimes with stronger property rights (i.e., regimes with no theft or no trespassing). Among the results we find that an improvement in the institutional quality in terms of a higher probability of eliminating trespassing, exacerbates the over-extraction imposed by trespassing itself. Moreover, an improvement in the institutional quality in terms of a reduction of the theft intensity, leads to a more rapacious depletion of the resource whenever income above the ground is imperfectly protected. This is the case because the inter-temporal distortion imposed by the current existence of theft and the prospect of it disappearing in the future, leads to slow down current depletion. The lower the intensity of current theft, the lesser its effect in slowing down extraction.

In terms of efficiency, if the trespasser is affected by theft the first-best level of extraction may be achieved. However, the waste inherent to the protection-theft contest implies that the net present value of extraction revenues is still below the first-best level. Efficiency unambiguously improves as the parameter determining the theft intensity in equilibrium moves away from the level that maximizes wasteful effort and towards the level that allows for optimal depletion. Thus, in some cases a more theft-prone environment may be more desirable in terms of efficiency.

As a potential avenue for future research using this type of dynamic framework, with a broader source of imperfect property rights and multiple regime shifts, one could think of the interaction between governments and private extraction companies, and how this interaction is affected by the different alternatives that the former can use to capture a share of the latter’s revenues. In such a framework instead of trespassing, expropriation would be the source of insecure stocks; and instead of theft, revenues would be subject to taxation. An interesting feature of a setup along those lines is the dual role of the government as both “trespasser” and “thief”. Such a model may shed light on the type of tools that a government should use when trying to maximize the net present revenues that it gets from the riches in the ground, once the strategic response of the firm is taken into account. This framework would serve to answer when should the government be more or less aggressive in taxing the resource sector? when (if ever) is expropriation a better choice? how strong should the government’s commitment against expropriation be?

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Appendix

A Proofs

A.1 Propositions

Proposition 1

Proof. Evaluating $z_i^{th}(\cdot)$ at θr

$$z_i^{th}(\theta r) = \theta \left(r + \frac{\pi q}{\lambda^2} \right) \geq \theta (r + \pi q) = z_i^{th}(k_i^{th}).$$

With z_i^{th} being strictly increasing, it follows that $k_i^{th} \leq \theta r$, with strict inequality whenever theft is a relevant ($\lambda < 1$) regime with finite expected end date ($q > 0$). \square

Proposition 2

Proof. From (2.3), we find that in equilibrium

$$\left(1 + \frac{\pi q (k_i^{th})^{\frac{1}{\theta}-1}}{\lambda^2 (\theta r)^{\frac{1}{\theta}}} \right) \frac{\partial k_i^{th}}{\partial \pi q} = \theta \left(1 - \frac{(k_i^{th})^{\frac{1}{\theta}}}{\lambda^2 (\theta r)^{\frac{1}{\theta}}} \right).$$

Thus, $\text{sign}(\partial k_i^{th} / \partial \pi q) = \text{sign}(\lambda^{2\theta} \theta r - k_i^{th})$. Evaluating $z_i^{th}(\cdot)$ at $\lambda^{2\theta} \theta r$ it is obtained that

$$z_i^{th}(\lambda^{2\theta} \theta r) = \theta (\lambda^{2\theta} \theta r + \pi q) < \theta (r + \pi q) = z_i^{th}(k_i^{th}).$$

Given that z_i^{th} is strictly increasing, it follows that $k_i^{th} > \lambda^{2\theta} \theta r$ so that $\partial k_i^{th} / \partial \pi q < 0$. \square

Proposition 3

Proof. i) Evaluating z_i^{TR} at θr : $z_i^{TR}(k_i^{TR}) \equiv \theta (r + \pi p) \geq (2 - \theta) \theta r + \pi p = z_i^{TR}(\theta r) \leftrightarrow \theta \geq 1$. Given that $z' > 0, \theta > 1$ then implies $k_i^{TR} > \theta r$.

ii) $k_j^{TR}(k_i^{TR}) > k_i^{TR}$ requires $k_i^{TR} < \theta (2 - \theta)^{-1} (r + \pi p)$. Evaluating z_i^{TR} at $\theta (2 - \theta)^{-1} (r + \pi p)$ it is obtained that $z_i^{TR}(\theta (2 - \theta)^{-1} (r + \pi p)) = \theta (r + \pi p) + (\theta (2 - \theta)^{-1} (r + \pi p))^{\frac{1}{\theta}} > \theta (r + \pi p) \equiv z_i^{TR}(k_i^{TR})$. Following the same argument as above $k_i^{TR} < \theta (2 - \theta)^{-1} (r + \pi p)$ and so $k_j^{TR} > k_i^{TR}$. \square

Proposition 4

Proof. From $z_i^{TR}(k_i^{TR}) = \theta (r + \pi p)$ one can obtain:

$$\frac{\partial k_i^{TR}}{\partial \pi p} = \frac{\theta - \left(\frac{k_i^{TR}}{\theta r}\right)^{\frac{1}{\theta}}}{(2 - \theta) + \frac{\pi p}{k_i^{TR}} \left(\frac{k_i^{TR}}{\theta r}\right)^{\frac{1}{\theta}}}.$$

Therefore, $\text{sign}(\partial k_i^{TR} / \partial \pi p) = \text{sign}\left(\theta - \left(k_i^{TR} / \theta r\right)^{\frac{1}{\theta}}\right)$. Using $z(k_i^{TR})$:

$$\theta \geq \left(\frac{k_i^{TR}}{\theta r}\right)^{\frac{1}{\theta}} \longleftrightarrow k_i^{TR} \geq \frac{\theta r}{2 - \theta}.$$

From $z_i^{TR}(\cdot)$ and $z' > 0$, the last expression is equivalent to

$$\theta(r + \pi) \geq z_i^{TR}\left(\frac{\theta r}{2 - \theta}\right) \longleftrightarrow \theta \geq \frac{1}{(2 - \theta)^{\frac{1}{\theta}}},$$

where $\theta < (2 - \theta)^{-\frac{1}{\theta}}$ for any $\theta \in (1, 2)$; therefore, $k_i^{TR} < \theta r (2 - \theta)^{-1}$ and $\partial k_i^{TR} / \partial \pi p < 0$. \square

Proposition 5

Proof. i) Expressing the equilibrium conditions of the z_i^{TRth}, z_j^{TRth} system more generally as [and assuming $\omega = 0$ to save notation]:

$$z_i^{TRth}(k_i^{TRth}, k_j^{TRth}) \equiv k_i^{TRth} + \theta \pi \mu_i \kappa_i \left(k_i^{TRth}\right)^{\frac{1}{\theta}} - (\theta - 1) k_j^{TRth} = \theta(r + \pi^{TRth}),$$

and

$$z_j^{TRth}(k_i^{TRth}, k_j^{TRth}) \equiv k_j^{TRth} + \theta \pi \mu_j \kappa_j \left(k_j^{TRth}\right)^{\frac{1}{\theta}} - (\theta - 1) k_i^{TRth} = \theta(r + \pi^{TRth}).$$

Where $\mu_n = \lambda^{-2}$ if agent n is subject to theft, and $\mu_n = 1$ otherwise.

Taking the derivatives of z_i^{TRth} and z_j^{TRth} with respect to λ and rearranging terms one respectively gets:

$$a_i \frac{\partial k_i^{TRth}}{\partial \lambda} - (\theta - 1) \frac{\partial k_j^{TRth}}{\partial \lambda} = -\theta \pi \left(k_i^{TRth}\right)^{\frac{1}{\theta}} \frac{\partial(\mu_i \kappa_i)}{\partial \lambda},$$

and

$$a_j \frac{\partial k_j^{TRth}}{\partial \lambda} - (\theta - 1) \frac{\partial k_i^{TRth}}{\partial \lambda} = -\theta \pi \left(k_j^{TRth}\right)^{\frac{1}{\theta}} \frac{\partial(\mu_j \kappa_j)}{\partial \lambda}$$

Where $a_n \equiv 1 + \pi \mu_n \kappa_n \left(k_n^{TRth}\right)^{\frac{1}{\theta} - 1} > 1$. Using the former in the latter and rearranging terms:

$$\frac{\partial k_i^{TRth}}{\partial \lambda} = \frac{-\theta \pi \left(\left(k_i^{TRth}\right)^{\frac{1}{\theta}} \frac{\partial(\mu_i \kappa_i)}{\partial \lambda} + \frac{\theta - 1}{a_j} \left(k_j^{TRth}\right)^{\frac{1}{\theta}} \frac{\partial(\mu_j \kappa_j)}{\partial \lambda} \right)}{a_i - \frac{(\theta - 1)^2}{a_j}},$$

by symmetry

$$\frac{\partial k_j^{TRth}}{\partial \lambda} = \frac{-\theta \pi \left(k_j^{TRth \frac{1}{\theta}} \frac{\partial(\mu_j \kappa_j)}{\partial \lambda} + \frac{\theta-1}{a_i} k_i^{TRth \frac{1}{\theta}} \frac{\partial(\mu_i \kappa_i)}{\partial \lambda} \right)}{a_j - \frac{(\theta-1)^2}{a_i}};$$

adding the last two up

$$\frac{\partial (k_i^{TRth} + k_j^{TRth})}{\partial \lambda} = \frac{-\theta \pi \left(\left((a_j + \theta - 1) \left(k_i^{TRth} \right)^{\frac{1}{\theta}} \frac{\partial(\mu_i \kappa_i)}{\partial \lambda} + (a_i + \theta - 1) \left(k_j^{TRth} \right)^{\frac{1}{\theta}} \frac{\partial(\mu_j \kappa_j)}{\partial \lambda} \right) \right)}{a_i a_j - (\theta - 1)^2} > 0.$$

Where the sign follows from $\theta < 2$, $a_n > 1$, and $\partial(\mu_n \kappa_n)/\partial \lambda < 0$ for $n \in \{i, j\}$.

ii) When **only the owner** is subject to theft:

If $\lambda = 0$ and therefore $k_i^{TRth} = 0$, k_j^{TRth} in equilibrium is given by

$$z_j^{TRth}(0, k_j^{TRth}) = k_j^{TRth} + \theta \pi \kappa_j \left(k_j^{TRth} \right)^{\frac{1}{\theta}} = \theta \left(r + \pi^{TRth} \right).$$

Evaluating z_j^{TRth} in $(0, \theta r)$ and comparing with $z_j^{TRth}(0, k_j^{TRth})$:

$$z_j^{TRth}(0, \theta r) = \theta r + \theta \pi \kappa_j (\theta r)^{\frac{1}{\theta}} \geq \theta \left(r + \pi^{TRth} \right) = z_j^{TRth}(0, k_j^{TRth}),$$

using the definitions of κ_j and π^{TRth}

$$\longleftrightarrow q(1-p) \left(\frac{\theta r}{k_j^{TR}} \right) \geq q(1-p) + p.$$

From proposition 3, $k_j^{TR} > \theta r$, hence $q(1-p)\theta r \left(k_j^{TR} \right)^{-1} < q(1-p) + p$. Given that z_j is increasing in k_j , it is immediate that $k_j^{TRth}|_{\lambda=0} > \theta r$. Given that $k_j^{TRth}|_{\lambda=1} = k_j^{TR}$, it also follows from proposition 3 that $k_j^{TRth}|_{\lambda=1} > \theta r$. As k_j^{TRth} is continuous and monotonically increasing in λ (see lemma 3) and $k_j^{TRth}|_{\lambda=0}, k_j^{TRth}|_{\lambda=1} > \theta r$, the intermediate value theorem implies that $k_j^{TRth} > \theta r$ for any $\lambda \in [0, 1]$. Hence total depletion $(k_i^{TRth} + k_j^{TRth})$ is always above the social optimum level.

iii) When **the owner and the trespasser** are subject to theft:

If $\lambda = 0$, $k_i^{TRth} = k_j^{TRth} = 0$; if $\lambda = 1$, $k_i^{TRth} = k_j^{TRth} > \theta r$, which follows from proposition 3. This means that total depletion in the TRth regime is below θr when $\lambda \rightarrow 0$ (it goes to 0) and it is above θr when $\lambda \rightarrow 1$. As total depletion is continuous and monotonically increasing in λ (see i), from the intermediate value theorem there is a unique $\lambda^* \in (0, 1)$ such that total depletion is $\geq \theta r$ if $\lambda \geq \lambda^*$.

iv) When **only the trespasser** is subject to theft:

Assuming $\lambda = 0$ (and thus $k_j^{TRth} = 0$), and evaluating z_i in θr when only the trespasser is affected by theft:

$$z_i^{TRth}(\theta r, 0) = k_i^{TRth} + \theta \pi \kappa_i \left(k_i^{TRth} \right)^{\frac{1}{\theta}} \geq \theta \left(r + \pi^{TRth} \right) \equiv z_i^{TRth}(k_i^{TRth}, 0) \longleftrightarrow k_i^{TR} \geq \theta r.$$

From proposition 3 we know that $k_i^{TR} > \theta r$. Using the fact that z_i is increasing in k_i^{TRth} , $k_i^{TRth} > \theta r$, and thus $k_i^{TRth} + k_j^{TRth} > \theta r$, for $\lambda = 0$. Moreover, as k_i^{TRth} is increasing in λ (see lemma 4) both k_i^{TRth} and $k_i^{TRth} + k_j^{TRth}$ are above θr for any value of $\lambda > 0$. \square

Proposition 6

Proof. This proposition follows from $k_i^{TRth} + k_j^{TRth} \Big|_{\lambda=0} = 0 < k_s^{TRth} \Big|_{\lambda=0}$ and $k_s^{TRth} \Big|_{\lambda=1} < \theta(r + \pi p) < k_i^{TRth} + k_j^{TRth} \Big|_{\lambda=1}$ and the continuity of k_s^{TRth} in λ . \square

A.2 Lemmas

Lemma 1

Proof. Note that the $z_i^{TR}(k_i)$ is strictly increasing in k_i ; furthermore, $z_i^{TR}(0) = 0$. Thus, there is a unique value $k_i^{TR} > 0$ such that $z_i^{TR}(k_i^{TR}) = \theta(r + \pi p)$. Moreover, for each k_i^{TR} there is a unique k_j^{TR} : $k_j^{TR} = (\theta - 1)k_i^{TR} + \theta(r + \pi p)$. With $\theta > 1$, $k_i^{TR} > 0$ implies $k_j^{TR} > 0$. \square

Lemma 2

Proof. First note that, given that given $\theta > 1$, $\partial k_j / \partial k_i|_{z_i} > 0$ and $\partial k_j / \partial k_i|_{z_j} > 0$. Moreover, $z_i^{TRth}(0, k_j)$ implies $k_j < 0$ and $z_j^{TRth}(0, k_j)$ implies $k_j > 0$. Then, to get a single crossing (equilibrium) it suffices to show that $\partial k_j / \partial k_i|_{z_i} > \partial k_j / \partial k_i|_{z_j}$ holds for any $k_i > 0$. Differentiating z_i^{TRth} with respect to k_i :

$$1 + \frac{\pi \kappa_i}{\lambda^2} k_i^{\frac{1}{\theta}-1} - (\theta - 1) \frac{\partial k_j}{\partial k_i} = 0,$$

therefore

$$\frac{\partial k_j}{\partial k_i} \Big|_{z_i} = \frac{1 + \frac{\pi \kappa_i}{\lambda^2} k_i^{\frac{1}{\theta}-1}}{\theta - 1} > \frac{1}{\theta - 1}; \forall k_i > 0.$$

Differentiating z_j^{TRth} with respect to k_i :

$$\frac{\partial k_j}{\partial k_i} \Big|_{z_j} = \frac{\theta - 1}{1 + \frac{\pi \kappa_j k_j^{\frac{1}{\theta}-1}}{1-\omega}} < \theta - 1; \forall k_i > 0.$$

Then, given that $\theta < 2$, $\partial k_j / \partial k_i|_{z_i} > (\theta - 1)^{-1} > \theta - 1 > \partial k_j / \partial k_i|_{z_j}$, which implies that z_i and z_j have a single crossing (k_i^{TRth}, k_j^{TRth}) in \mathcal{R}_+^2 . \square

Lemma 3

Proof. Taking the derivatives of z_i^{TRth} and z_j^{TRth} with respect to λ and rearranging terms on respectively gets:

$$\frac{\partial k_i^{TRth}}{\partial \lambda} \left(1 + \frac{\pi \kappa_i}{\lambda^2} k_i^{TRth \frac{1}{\theta}-1} \right) - (\theta - 1) \frac{\partial k_j^{TRth}}{\partial \lambda} = -\theta \pi \left(k_i^{TRth \frac{1}{\theta}} \right) \frac{\partial (\kappa_i / \lambda^2)}{\partial \lambda},$$

and

$$\frac{\partial k_j^{TRth}}{\partial \lambda} = \frac{\theta - 1}{1 + \pi \kappa_j (k_j^{TRth})^{\frac{1}{\theta} - 1}} \frac{\partial k_i^{TRth}}{\partial \lambda};$$

using the latter in the former and rewriting

$$\frac{\partial k_i^{TRth}}{\partial \lambda} = \frac{-\theta \pi (k_i^{TRth})^{\frac{1}{\theta}} \frac{\partial (\kappa_i / \lambda^2)}{\partial \lambda}}{1 + \frac{\pi \kappa_i}{\lambda^2} (k_i^{TRth})^{\frac{1}{\theta} - 1} - \frac{(\theta - 1)^2}{1 + \pi \kappa_j (k_j^{TRth})^{\frac{1}{\theta} - 1}}} > 0.$$

The sign follows from both the numerator and the denominator being positive. From the definition of κ_i , $\frac{\kappa_i}{\lambda^2}$ is decreasing in λ , while $1 > (\theta - 1)^2 \left(1 + \pi \kappa_i (k_i^{TRth})^{\frac{1}{\theta} - 1}\right)^{-1}$ for any $\theta \in (1, 2)$. Using this and $\partial k_j^{TRth} / \partial \lambda, \theta > 1$ implies $\partial k_j^{TRth} / \partial \lambda > 0$. That is, as λ increases k_i^{TRth} and k_j^{TRth} increase. \square

Lemma 4

Proof. Taking the derivatives of z_i^{TRth} and z_j^{TRth} with respect to λ and rearranging terms one respectively gets:

$$\frac{\partial k_i^{TRth}}{\partial \lambda} = \frac{\theta - 1}{1 + \pi \kappa_i (k_i^{TRth})^{\frac{1}{\theta} - 1}} \frac{\partial k_j^{TRth}}{\partial \lambda},$$

and

$$\frac{\partial k_j^{TRth}}{\partial \lambda} \left(1 + \frac{\pi \kappa_j}{\lambda^2 (1 - \omega)} (k_j^{TRth})^{\frac{1}{\theta} - 1}\right) - (\theta - 1) \frac{\partial k_i^{TRth}}{\partial \lambda} = 2 \frac{\theta \pi \kappa_j}{\lambda^3 (1 - \omega)} (k_j^{TRth})^{\frac{1}{\theta}};$$

using the former in the latter and rewriting

$$\frac{\partial k_j^{TRth}}{\partial \lambda} = \frac{2 \frac{\theta \pi \kappa_j}{\lambda^3 (1 - \omega)} (k_j^{TRth})^{\frac{1}{\theta}}}{1 + \frac{\pi \kappa_i}{\lambda^2 (1 - \omega)} (k_j^{TRth})^{\frac{1}{\theta} - 1} - \frac{(\theta - 1)^2}{1 + \pi \kappa_i (k_i^{TRth})^{\frac{1}{\theta} - 1}}} > 0.$$

where the sign of $\partial k_j^{TRth} / \partial \lambda$ follows from $\theta \in (1, 2)$; $\partial k_j^{TRth} / \partial \lambda > 0$, together with $\theta > 1$, implies $\partial k_i^{TRth} / \partial \lambda > 0$. That is, as λ increases both k_i^{TRth} and k_j^{TRth} increase. \square