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Supply Chain Reliability and the Role of Individual Suppliers

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Abstract

We study a one-period supply chain problem consisting of numerous suppliers delivering a homogenous good. Individual supply is uncertain and may exhibit dependencies with other suppliers as well as with the stochastic demand. Assuming that reliability of supply represents an economic value for the customer that shall be paid accordingly, we first derive an analytical solution for the contribution of an individual supplier to supply chain reliability. Second, applying concepts from cooperative game-theory, we propose a payoff scheme based on marginal contributions that explicitly accounts for the statistical properties of the problem. A number of desirable properties is thus achieved, including static efficiency as well as efficient investment incentives. Lastly, in order to demonstrate the relevance and applicability of the concepts developed, we consider the example of payoffs for reliability in power systems that are increasingly penetrated by interdependent variable renewable energies. We investigate empirical data on wind power in Germany, thereby confirming our analytical findings. In practice, our approach could be applied to design and organize supply chains and their reliability more efficiently. For instance, in the field of power systems, the approach could improve designs of capacity or renewable support mechanisms.

Keywords: supply chain reliability, capacity uncertainty, individual contribution, cooperative game theory, shapley value, power system

JEL: C44, C71, D47, Q42, Q48

1. Introduction

While most supply chains face the requirement to provide high levels of reliability, the consequences of supply shortages in power systems can be particularly dramatic. For instance, in the early 2000s insufficient supply capacities caused a series of blackouts in the Californian power system affecting several hundred thousand customers. The State of California was forced to initiate short-term countermeasures to alleviate the crisis, amounting to an estimated 40 bn.\$ in additional energy costs from 2001 to 2003 (Weare (2003)). The economy was estimated to slow down by 0.7-1.5%, entailing an increase in unemployment by 1.1% (Cambridge Energy Research Associates

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(2001)). But also other supply chains, such as manufacturing or food production industries, may suffer severe economic losses from supply shortages.

To manage and foster supply chain reliability, sourcing from multiple suppliers is an effective and often applied practice.¹ While the principle idea of risk diversification is rather simple, organizing and coordinating multiple individual suppliers to ensure envisaged levels of supply chain reliability at reasonable costs is often challenging. An economic approach to tackle this challenge consists in defining reliability as a good that can be provided by individual suppliers. The provision is then paid according to the individual supplier's contribution to supply chain reliability, with a price determined in a competitive auction, for instance. In principle, such an approach ensures and incentivizes an efficient short- and long-term management of supply chain reliability.

An example for such explicit reliability-related payments can be found in so-called capacity mechanisms, nowadays established in many power markets.² Key ingredient to all such mechanisms is the determination of an individual supplier's contribution to supply chain reliability, commonly known as prequalification, which is used as a basis for subsequent payments. Despite the apparent relevance of this measure, it appears that existing approaches for its determination lack generality and consistency.³ The reason may lie in the difficulty to assess an individual supplier's contribution, especially when accounting for the full complexity of the problem which is inherently stochastic. Indeed, a contribution not only depends on the stochastic nature of the individual supplier itself, but also on all other stochastic suppliers that are present.

It is hence the goal of this paper to comprehensively investigate supply chain reliability and the role of individual suppliers therein. For this purpose, we consider a one-period supply chain problem consisting of numerous suppliers delivering a homogenous good.⁴ Individual supply is uncertain and may exhibit dependencies with other suppliers as well as with the stochastic demand. Due to the stochastic nature of the problem, demand can be served at an intended reliability level, possibly including supply disruptions.⁵ Assuming that reliability of supply represents an economic value for the customer and shall be paid accordingly, we are particularly interested in answering the following two questions: First, what is the contribution of individual suppliers to supply chain

¹For a general overview of mitigation strategies, see, e.g., Tang (2006), Tomlin (2006) or Snyder et al. (2014).

²In fact, the Californian electricity crisis has triggered much of the worldwide debate and development around capacity mechanisms (for a good overview, see, e.g., Joskow (2008) or Cramton et al. (2013)). Recently, the discussion about the need and design of capacity mechanisms has been regaining momentum due to the large-scale deployment of variable renewable energies (such as wind or solar power), whose impact on power system reliability is considered a *crucial issue of common interest* (Council of European Energy Regulators (2014)).

³For instance, there seems to be no best practice on how to deal with the reliability of intermittent and interdependent renewable resources in power systems, reflected in a variety of different existing approaches. Moreover, all of them incorporate inefficient design features, as we will show in the course of this paper.

⁴The need to source a homogenous good from multiple suppliers may stem, e.g., from capacity constraints or the requirement for risk diversification (e.g., Minner (2003) or Tang (2006)).

⁵Note that instead of *reliability*, one could also consider the generalized case of supply *quality*. However, for the sake of clarity, we will stick to the term reliability throughout the paper. Nevertheless, the concepts and results derived could also be applied to other dimensions of supply quality, such as time-to-respond, etc.

reliability? And second, how should individual suppliers be paid for what they contribute? In addition, we shall demonstrate practical relevance and applicability of our analytical findings in a novel and pertinent empirical case study based on wind energy in the German power system.

By solving these problems, our paper contributes a general, consistent and applicative framework to address the role of individual suppliers for supply chain reliability. In practice, it could be employed to design and manage supply chain reliability more efficiently. It may prove to be a useful tool for allocation problems in monopsonistic markets or public procurements, for designing bonus payments, for accounting issues in integrated supply chains, or for the assessment of existing or new contract conditions. In the field of power systems, the approach could improve the design of capacity mechanisms which so far appears to be incomplete.

The rest of the paper is structured as follows: Section 2 reviews the related literature. Section 3 introduces and solves for the contribution of individual suppliers to supply chain reliability. In Section 4, we characterize our problem as a cooperative game to develop and investigate a suitable payoff scheme. The empirical case study is presented in Section 5. Section 6 concludes.

2. Related literature

Our paper is closely related to the literature dealing with supply chain reliability and the problem of strategic sourcing of a homogenous good from multiple suppliers, facing either supply disruptions (i.e., a binomial distribution of uncertainty) or capacity uncertainty (i.e., an uncertain exogenous upper bound on the actual quantity supplied).⁶

Supply disruptions in a supply chain consisting of multiple suppliers have been studied by Parlar and Perry (1996), Gürlér and Parlar (1997), Li et al. (2004) and Tomlin (2006), however, without considering stochastic dependencies among the suppliers. As a natural extension, later papers allow for dependencies between binomial supply disruptions (i.e., Babich et al. (2007), Wagner et al. (2009) and Li et al. (2010)).

Continuous distributions of the suppliers' uncertainties – as used in our paper – were presented by Dada et al. (2007) and Masih-Tehrani et al. (2011). The former paper proves that when selecting among a set of possible suppliers that are different in reliability and costs, cost generally takes precedence over reliability. While this result is derived under the assumption of supply distributions being independent, Masih-Tehrani et al. (2011) captures capacity uncertainty including multivariate dependencies, finding that the buyer's best strategy is risk diversification by choosing suppliers with independent distributions in order to avoid simultaneous supply disruptions.

In contrast to the above literature on supply chain reliability, our paper deviates in several important aspects. First and foremost, instead of analyzing the *costs* of supply chain reliability

⁶Note that the former is an extreme case of the latter. Also note that both are different from *yield uncertainty* which incorporates a dependency of the uncertainty on the order quantity. As we consider yield uncertainty as a different problem class, we do not review the related literature here. For a broader review of the literature dealing with supply chain risks, the reader is referred to Tang (2006) and Snyder et al. (2014).

under *exogenous prices*,⁷ we take a different perspective on the problem by *endogenously* determining the individual supplier's *value* (i.e., contribution and corresponding payoff) for supply chain reliability.⁸ Note that this is a fundamentally different view on the problem that becomes relevant, e.g., for managing supply chains in public procurements or bonus payments where prices are not fixed ex-ante.

In addition, and in contrast to most papers mentioned, we consider arbitrary and interdependent distributions of (un)availability and demand. Moreover, we study implications for the supply chain organization as well as investment incentives that are, to the best of our knowledge, hardly considered in the existing literature.

Our paper is also related to the more specific field of supply chain reliability in power systems, which has been investigated either from a technical or economic perspective. From a technical perspective, the goal has been to develop methodologies to assess the technical ability to provide reliability of supply (e.g., Garver (1966), Billinton (1970) or Billinton and Allan (1996)). The role of individual units for supply reliability, often referred to as capacity credit or capacity value, has also been discussed (for recent surveys, see, e.g., Amelin (2009) or Keane et al. (2011)). However, it appears that these analyses remain very technical and have never been transferred to a broader and generalized supply chain context. Moreover, they almost exclusively deal with the problem numerically rather than in a consistent analytical framework. In contrast, our paper contributes a comprehensive and consistent analytical framework along with generalized implications for managing supply chain reliability.

Economically, power system reliability has been studied with respect to the (in)ability and potential failures of power markets to provide reliability as a market outcome (e.g., Joskow (2008) or Cramton et al. (2013)). However, even though various designs of capacity-related payoffs have been proposed, the role and implications of stochastic and interdependent suppliers have so far been disregarded. Our paper fills this gap by suggesting suitable approaches to incorporate those suppliers into reliability-related mechanisms in order to ensure economically efficient outcomes.

3. Supply chain reliability and the contribution of individual suppliers

3.1. Supply chain reliability

We consider a one-period supply chain \mathcal{S} consisting of numerous suppliers delivering a homogeneous good. Suppliers are characterized by their joint stochastic availability of supply capacity C .⁹ Demand D is also assumed stochastic. Due to considering only one period without additional

⁷In the reviewed literature, suppliers' prices are either given as a parameter, or result from some supplier interaction (such as a Cournot game) without the buyer being able to have an influence.

⁸Technically, the difference stems from the fact that we determine the individual supplier's contribution based on an endogenous demand adjustment, while the literature's objective is to serve an exogenous (though, often stochastic) demand level.

⁹Here and in the following – unless indicated differently – capital letters are used for random variables.

backup (such as, e.g., inventory storage, emergency service, etc.), supply shortages occur whenever C is unable to cover D . Consequently, we can define supply chain reliability in probabilistic terms as follows:¹⁰

Definition 1. *Supply chain reliability:* Probability of a supply chain \mathcal{S} , characterized by the stochastic overall availability of supply capacity C , to be able to cover a stochastic demand D , i.e.:

$$R^{\mathcal{S}} = \Pr(D \leq C) = \Pr(D - C \leq 0) = \Pr(X \leq 0) = F_X(0). \quad (1)$$

In Equation (1), we have used F_X for the cumulative distribution function (cdf) of the capacity shortage $X = D - C$. Note that stochastic dependencies between the random variables are so far unspecified. For instance, the overall availability of supply capacity C may result from multiple interdependent supply capacities of numerous individual suppliers. Furthermore, C may exhibit dependencies with the stochastic demand D .

It is worthwhile to mention structural correspondence of our Definition 1 with another well-known risk measure, i.e., Value-at-Risk (VaR), defined as $\Pr(X \leq VaR) = R^{\mathcal{S}}$ (e.g., Jorion (2007)). The measures coincide for VaR being normalized to zero, i.e., when $R^{\mathcal{S}}$ corresponds to the probability that X is lower than zero.

3.2. The contribution of individual suppliers

We now investigate the contribution of an individual supplier with random production capacity $Y < C$ to the reliability of supply chain \mathcal{S} . To this end, we remove it from the existing system \mathcal{S} , and subsequently determine the reliability of supply of the new diminished supply chain \mathcal{T} as:

$$R^{\mathcal{T}} = \Pr(D \leq C - Y) \quad (2)$$

Note that as Y is positive, $R^{\mathcal{T}} \leq R^{\mathcal{S}}$ always holds. With the goal to capture the contribution of Y to supply chain reliability, we follow the concept of incremental VaR. I.e., we capture the change in risk exposure induced by the adjustment of X by Y while requiring the corresponding probability to remain at the original level (e.g., Tasche and Tibiletti (2003) or citeJorion2007). In other words, we measure the demand reduction necessary to bring $R^{\mathcal{T}}$ back to the original reliability level $R^{\mathcal{S}}$.^{11,12} Analytically, we define the contribution of an individual supplier as follows:

¹⁰Note that we implicitly assume an inelastic demand with no reaction as capacity becomes scarce. If the good is marketed, this implies that market clearing cannot be guaranteed, e.g., because of the lack of real time pricing. Consequently, there is a risk of situations with all available capacities producing, but still being unable to fully serve demand – irrespective of the price level.

¹¹Noticeably, this approach has for long been used in the context of power systems under the name of effective load carrying capability, which was originally developed by Garver (1966).

¹²As an alternative measure, one could also consider the change in reliability induced by removing supply capacity Y , i.e., $\Delta R = R^{\mathcal{S}} - R^{\mathcal{T}}$, however, without altering the principle results derived hereafter.

Definition 2. Contribution of an individual supplier to supply chain reliability: Level of demand v by which D needs to be reduced in order to maintain the original level of reliability, i.e.:

$$\Pr(D - v \leq C - Y) = R^S \quad (3)$$

Due to its complexity, Equation (3) has commonly been solved for v numerically by means of iteration (see, e.g., Wang (2002) or Keane et al. (2011)). Advantages of the numerical solution include its straightforward implementability as well as the implicit coverage of statistical dependencies when using concurrent observations of demand and supply. However, numerical solutions – even if conducted for a wide range of parameter constellations and application examples – do not allow for generalizations of the results obtained. Moreover, the numerical solution often entails a high computational burden.

In contrast, analytical solutions allow to readily calculate the desired results and provide further general insights. Nevertheless, only very few authors have engaged with the analytical analysis of Equation (3). In the literature related to portfolio risks, the incremental VaR is conveniently solved analytically up to a first order approximation (e.g., Tasche and Tibiletti (2003) or Jorion (2007)). In contrast, Dragoon and Dvortsov (2006) propose the z-method, considering higher order terms and the special case of a normally distributed capacity shortage over demand and independence with the individual supplier (i.e., $X \sim \mathcal{N}$ and $X \perp Y$). Extending this approach, Zachary and Dent (2011) present a closed-form solution with an arbitrary distribution of X , but for independent X and Y . Even though the latter paper discusses the natural extension to the case of dependent distributions, the formal proof is not included. Hence, in the following proposition we present the generalized solution of Equation (3) for v , with arbitrary dependence between the distributions of X and Y , and $\sigma_{(\cdot)}$ being their standard deviation.

Proposition 1. For $\sigma_Y \ll \sigma_X$, the contribution v of an individual capacity Y to supply chain reliability is approximated by

$$v = \mu_{Y|X \approx 0} - \frac{\sigma_{Y|X \approx 0}^2}{2} \frac{f'_X(0)}{f_X(0)}. \quad (4)$$

Proof. From Equations (3) and (1) it follows that the equation to be solved for v is

$$\Pr(D - v \leq C - Y) = \Pr(X + Y \leq v) = \Pr(X \leq 0) = F_X(0). \quad (5)$$

The cumulative distribution function $\Pr(X + Y \leq v)$ can be expressed as integrals over the joint probability density function $f_{X,Y}$, with X, Y being arbitrary dependent distributions. Refor-

mulating using conditional distributions and integrating over x , we obtain

$$\Pr(X + Y \leq v) = \int_{-\infty}^{\infty} \int_{-\infty}^{x=v-y} f_{X,Y}(x, y) dx dy \quad (6)$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{x=v-y} f_{Y|X}(y|x) f_X(x) dx dy \quad (7)$$

$$= \int_{-\infty}^{\infty} f_{Y|X \leq v-y}(y|X \leq v-y) F_X(v-y) dy = F_X(0). \quad (8)$$

Instead of continuing with the explicit form of the cumulative distribution function F_X , we approximate it via Taylor expansion around the critical point $v - y = 0$ up to the second order polynomial degree, i.e.,

$$F_X(v-y) \approx F_X(0) + f_X(0)(v-y) + \frac{f'_X(0)}{2}(v-y)^2. \quad (9)$$

Noticeably, in the above equation we have induced and accepted an approximation error of $o((v-y)^2)$, which occurs if derivatives of order two or higher are non-zero.

Next, note that if $\sigma_Y \ll \sigma_X$, it follows that $\Pr(X + Y \leq v) \approx \Pr(X + \mu_Y \leq v)$ and hence, that $v \approx \mu_Y$ in Equation (5). We now insert (9) in (8), and reformulate using the concept of conditional expectations:

$$\int_{-\infty}^{\infty} f_{Y|X \leq v-y}(y|X \leq v-y) \left[F_X(0) + f_X(0)(v-y) + \frac{f'_X(0)}{2}(v-y)^2 \right] dy \quad (10)$$

$$= F_X(0) + f_X(0)\mathbb{E}[(v-Y)|X \approx 0] + \frac{f'_X(0)}{2}\mathbb{E}[(v-Y)^2|X \approx 0] \approx F_X(0), \quad (11)$$

which can be simplified using standard deviation σ as well as expected values μ to

$$f_X(0)(v - \mu_{Y|X \approx 0}) + \frac{f'_X(0)}{2} \left((v - \mu_{Y|X \approx 0})^2 + \sigma_{Y|X \approx 0}^2 \right) \approx 0. \quad (12)$$

Equation (12) represents a quadratic equation in $(v - \mu_{Y|X \approx 0})$ that can readily be solved for v based on the assumption of small σ_Y (so that the error of order $\mathcal{O}(\sigma_{Y|X \approx 0}^4)$ is small), such that Equation (4) follows. \square

From Equation (4), we observe that two terms including two different statistical features of the individual supplier are decisive for its contribution to supply chain reliability: its average availability μ on the one hand, and its standard deviation σ on the other, both at times of scarce capacity, i.e. for $X \approx 0$. Specifically, higher average availability at times of critical capacity directly contributes to reliability. In contrast, the effect of the standard deviation getting larger may be either positive or negative for reliability, depending on the sign of $f'_X(0)$ (i.e., the convexity of the

cdf).^{13,14} It should typically hold true that high levels of reliability are required from supply chains, such that the critical point $X = 0$ is located at the left hand side of the distribution where the probability for insufficient capacity is still low and increasing in x , which yields $f'(x) > 0$. Then, the individual supplier's contribution to reliability decreases in its standard deviation. Somewhat counterintuitive, however, the individual supplier's contribution to reliability may also *benefit* from a high standard deviation. This is the case for $f'(x) < 0$, resulting in a situation where positive deviations weight more than negative ones. This could hold true and represent an interesting feature, e.g., for supply chains that are still at an early stage of development while already facing high levels of demand, or for well-established systems that underwent a sudden and substantial increase in demand.

Two further points are worth mentioning. First, the contribution of an individual supplier to supply chain reliability may – instead of absolute numbers – be reported relative to its maximum available capacity $\bar{y} = \max Y$, i.e., as a fraction $\tilde{v} = v/\bar{y}$, with $0 \leq \tilde{v} \leq 1$. Second, Equation (4) can readily be extended to multiple (n) suppliers contributing jointly to supply chain reliability. In this case, define Y as the sum of the individual unit's generation, i.e., $Y = \sum_{i=1}^n Y_i$. Denoting with N the set of all n suppliers in the system contributing jointly to generation adequacy, we will write $v(N)$ for their joint contribution. To determine the joint contribution of a few units only, we will use the set $S \subseteq N$, and denote the corresponding contribution by $v(S)$. The joint contribution of multiple units (or, in other words, a *coalition* of units) will become important for large parts of the subsequent analysis.

3.3. Statistical properties of the contribution

In the following, three corollaries shall be discussed that describe essential properties of the individual supplier's contribution v . For the formal proofs, we build on the explicit formulation for the supplier's contribution to supply chain reliability as presented in Proposition 1. Even though derived statistically, we will also discuss first economic implications the identified properties may have.

Corollary 1. *The contribution of an individual supplier is generally subject to changing returns to scale.*

Proof. Inserting a scaled random production capacity aY in Equation (4), and using the fact that both, mean and standard deviation scale directly with the scaling factor of the random variable,

¹³Note that the density function $f(x)$ is always positive by definition.

¹⁴The effect of the standard deviation essentially stems from the difference in impact from positive and negative deviations from the average availability. If positive and negative deviations had an equal impact and would outweigh each other, the *expected* overall contribution would not change. However, the impact of the standard deviation on the reliability contribution increases with the absolute level of $f'(x) = F''(x)$, i.e., with the level of convexity of the cumulative distribution function $F(x)$. It also increases with $f(x) = F'(x)$ (i.e., the slope of the cdf) decreasing, as the difference in impact from positive and negative deviations then becomes more important.

it follows that

$$v(aS) = \mu_{aY|X \approx 0} - \frac{\sigma_{aY|X \approx 0}^2}{2} \frac{f'_X(0)}{f_X(0)} \quad (13)$$

$$= a\mu_{Y|X \approx 0} - \frac{a^2 \sigma_{Y|X \approx 0}^2}{2} \frac{f'_X(0)}{f_X(0)}. \quad (14)$$

Whereas the first term on the right hand side increases linearly with a , the second term decreases with higher order a^2 as long as $\sigma_{Y|X \approx 0} > 0$ and $f'(x) > 0$. Under these conditions, it holds that $v(aS) < av(S)$, i.e. decreasing returns to scale result when considering an increasing amount of capacity with equal availability Y . In contrast, $\sigma_{Y|X \approx 0} > 0$ and $f'(x) < 0$ yields increasing returns to scale. \square

Economically, Corollary 1 may be of particular relevance for a supply chain if it shall increasingly rely on one particular supplier. For illustration, imagine a power system that aims at replacing an increasing number of fossil power plants with wind power capacities while keeping its original reliability level.¹⁵ Corollary 1 implies that with each additional unit of wind power installed, a decreasing amount of fossil power can be safely removed from the system.

Corollary 2. *Gains of diversification may apply for the contribution of individual suppliers.*

Proof. We assess the contribution of some capacity Y_1 to supply chain reliability. In order to analyze possible gains of diversification – without loss of generality – we assume that some share $\alpha \in [0, 1]$ of capacity Y_1 can be sourced from an alternative supplier S_2 instead of S_1 . Depending on the choice of α , the joint production capacity becomes $Y_{12} = (1 - \alpha)Y_1 + \alpha Y_2$. The joint contribution of this portfolio to supply chain reliability writes as

$$v((1 - \alpha)S_1 \cup \alpha S_2) = \mu_{Y_{12}|X} - \frac{\sigma_{Y_{12}|X}^2}{2} \frac{f'_X(0)}{f_X(0)} \quad (15)$$

$$= (1 - \alpha)\mu_{Y_1|X \approx 0} + \alpha\mu_{Y_2|X \approx 0} - \left((1 - \alpha)^2 \sigma_{Y_1|X \approx 0}^2 + \alpha^2 \sigma_{Y_2|X \approx 0}^2 + 2\alpha(1 - \alpha)\sigma_{Y_1, Y_2|X \approx 0} \right) \frac{f'_X(0)}{2f_X(0)}. \quad (16)$$

To identify gains of diversification, we need to check the derivative of v with respect to α in

¹⁵Suppose a reliable power system with $f'_X(0) > 0$, and variable wind power resources with $\sigma_{Y|X \approx 0} > 0$.

the region of $\alpha = 0$, i.e.,

$$\left. \frac{\partial}{\partial \alpha} v((1-\alpha)S_1 \cup \alpha S_2) \right|_{\alpha=0} = -\mu_{Y_1|X \approx 0} + \mu_{Y_2|X \approx 0} + \left((1-\alpha)\sigma_{Y_1|X \approx 0}^2 - \alpha\sigma_{Y_2|X \approx 0}^2 - (1-2\alpha)\sigma_{Y_1, Y_2|X \approx 0} \right) \frac{f'_X(0)}{f_X(0)} \Big|_{\alpha=0} \quad (17)$$

$$= -\mu_{Y_1|X \approx 0} + \mu_{Y_2|X \approx 0} + \left(\sigma_{Y_1|X \approx 0}^2 - \sigma_{Y_1, Y_2|X \approx 0} \right) \frac{f'_X(0)}{f_X(0)}. \quad (18)$$

From the derivative evaluated at $\alpha = 0$, we observe that it is positive (and hence, $v((1-\alpha)S_1 \cup \alpha S_2)$ increasing) as long as the average availability of supplier S_2 is similar to that of supplier S_1 , and their covariance is either *not* particularly strong in case of $f'_X(0) > 0$ or particularly strong otherwise. Under those conditions, the joint contribution is subject to gains of diversification. \square

The meaning of gains from diversification, as identified in Corollary 2, is quite intuitive, and essentially follows from balancing effects that may result from statistical aggregation. Hence, a diversified portfolio with the good sourced from multiple suppliers is often better able to reliably supply a certain load level. This holds true as long as the expected yield is similar and they are not highly correlated (for the typically more relevant case of $f'_X(0) > 0$).¹⁶

Corollary 3. *The contributions of individual suppliers are generally non-additive.*

Proof. Similar to Corollary 2, without loss of generality, let us consider a supply chain comprising two suppliers S_1 and S_2 with (possibly dependent) production capacities Y_1 and Y_2 , respectively. The joint contribution of S_1 and S_2 to supply chain reliability writes as

$$v(S_1 \cup S_2) = \mu_{Y_{12}|X} - \frac{\sigma_{Y_{12}|X}^2}{2} \frac{f'_X(0)}{f_X(0)} \quad (19)$$

$$= \mu_{Y_1|X \approx 0} + \mu_{Y_2|X \approx 0} - \left(\sigma_{Y_1|X \approx 0}^2 + \sigma_{Y_2|X \approx 0}^2 + 2\sigma_{Y_1, Y_2|X \approx 0} \right) \frac{f'_X(0)}{2f_X(0)}. \quad (20)$$

In contrast, if these suppliers were to be assessed independently, we can simply apply Equation

¹⁶Note that in finance, the concept of risk spreading is well-known from Markowitz' portfolio theory that shows similar characteristics when valuing properties of multi-asset portfolios (assets are here the equivalent to our suppliers). However, while the interest in a Markowitz portfolio lies in finding an efficient *tradeoff* between risk and expected returns, we are here interested in reliability contributions which are, as found in Equation (4), dependent on *both* variance *and* expectation (together with additional characteristics captured by f_X which are not occurring in Markowitz). Also note that one of the main criticisms with respect to Markowitz' portfolio theory, namely the linear dependence assumption between the joint distributions, also applies to our case here. In fact, non-linear dependencies can indeed be relevant for supply chain risks, as shown, e.g., by Wagner et al. (2009), Masih-Tehrani et al. (2011) or Elberg and Hagspiel (2015). While this would be an interesting extension of our analysis, it would go beyond the scope of this paper and is left for future research.

(4) to each of them to get

$$v(S_1) = \mu_{Y_1|X \approx 0} - \frac{\sigma_{Y_1|X \approx 0}^2}{2} \frac{f'_X(0)}{f_X(0)} \quad (21)$$

$$v(S_2) = \mu_{Y_2|X \approx 0} - \frac{\sigma_{Y_2|X \approx 0}^2}{2} \frac{f'_X(0)}{f_X(0)}. \quad (22)$$

From the above equations, we see that as long as $\sigma_{Y_1, Y_2|X \approx 0} \neq 0$, it follows that $v(S_1 \cup S_2) \neq v(S_1) + v(S_2)$. Hence, any type of (positive or negative) dependence changes the joint contribution compared to the sum of independent contributions. Specifically, the sum of the independent assessments is then inconsistent with the joint contribution. Generalizing to all n suppliers in the system, $\sum_{i=1}^n v(i) \neq v(N)$ as long as $\sigma_{Y_j, Y_k|X \approx 0} \neq 0$ for any pair j, k . \square

From Corollary 3, it follows that a comprehensive and coordinated approach is imperative to assess supply chain reliability in order to reach a consistent representation of all dependent suppliers. In other words, the *joint* contribution of all suppliers is essentially the only relevant one for supply chain reliability. However, with the goal to provide explicit reliability-related payments in order to manage and incentivize supply chain reliability, allocations need to be made to *individual* suppliers according to their *individual value*. Thus, Corollary 3 represents a particular challenge that will be tackled by a suitable payoff scheme in the next Section.

4. Payoff scheme

To approach the problem of designing a suitable payoff scheme, we apply concepts from cooperative game-theory. Specifically, we characterize the (joint) value of one or multiple suppliers for supply chain reliability as the output of a coalitional game (N, v) with transferable utility, where N is a finite set of units in the system, and v a characteristic function. $v(\cdot)$ measures the (joint) contribution of a nonempty coalition of suppliers $S \subseteq N$ as defined implicitly in Equation (3), or explicitly in Equation (4). Note that $v(S) \in \mathbb{R}^s$, i.e., for every coalition S , a corresponding contribution can be determined. A solution concept for this coalitional game is a payoff vector $\Phi \in \mathbb{R}^N$ allocating the joint value $v(N)$ to the coalition members.

The subsequent analysis consists of three steps: First, we assess the properties of the coalitional game (N, v) and implications regarding the organizational design of the supply chain. Second, we develop our solution concept, i.e., how payoffs are allocated to individual suppliers. Lastly, we investigate investment incentives resulting from our allocation rule.

4.1. Supply chain organization

Here and in the following, let us suppose it is the goal for the supply chain to reach or sustain some envisaged level of reliability R^S . In an appealing approach to organize the supply chain,

the buyer would determine his/her envisaged level of reliability, and invite tenders among the suppliers to reach it. In other words, (s)he would implement a competitive procurement of reliability contributions. For instance, in the context of a capacity procurement auction in power systems, the state, regulator, or some other independent authority could fix the envisaged reliability of power supply, and implement a platform that provides suppliers with the possibility to bid their contribution – either individually or jointly by forming coalitions.¹⁷ Hence, regulatory intervention by the authority would be limited, and the need to administratively determine individual allocations be avoided. Individual bids would be relatively easy to verify, e.g., by means of random checks and sufficiently high penalties, hence ensuring truthful individual bidding.¹⁸ However, this approach could result in unsatisfactory outcomes, as stated and proven in the following Proposition.

Proposition 2. *The equilibrium in a competitive reliability procurement is inefficient if suppliers are positively related and the buyer only verifies individual bids.*

Proof. Consider a supply chain organization that lets suppliers bid their individual contribution in a competitive procurement of reliability that features a multiunit auction. The auction is cleared by accepting all bids that are necessary to achieve the requested reliability level. To assess the quality of the auction outcome, we need to identify the suppliers' optimal bidding strategy.

To this end, without loss of generality, consider the case of two suppliers S_1 and S_2 for which the joint contribution $v(S_1 \cup S_2)$ has been derived in Equation (20) – in contrast to their individual contributions $v(S_1)$ and $v(S_2)$ (Equations (21) and (22)). As we see, a positive dependency (i.e., covariance $\sigma_{Y_1, Y_2 | X \approx 0} > 0$) implies sub-additivity of the contributions (i.e., $v(S_1 \cup S_2) < v(S_1) + v(S_2)$) and hence, disincentives to cooperate. Technically, sub-additivity implies an empty core of our game (N, v) , i.e., in every situation there is an alternative coalition able to improve the payoffs of all its members. As a consequence, the optimal bidding strategy consists of individual or coalitions of suppliers with positive dependencies bidding by themselves.

While the buyer is able to verify these individual bids (and hence, to avoid overstatements by imposing sufficiently high penalties), their sum does not correspond to the true joint contribution. In fact, the buyer accepts bids whose sum is *larger* than their actual joint contribution. Consequently, even though a competitive equilibrium exists, an insufficient amount of supply bids is contracted, and the reliability target undershot. Moreover, contracted suppliers would be overpaid for what they essentially deliver to the buyer. Hence, the auction outcome under positively related suppliers is unavoidably inefficient. \square

It is worthwhile to make a couple of remarks regarding this important proposition. First, note that the inefficiency of the procurement essentially stems from negative externalities emerging from

¹⁷For instance, the PJM capacity market has implemented such a design. It offers the possibility to make coupled offers (PJM (2015)).

¹⁸We abstract here from additional problems such as limited liabilities, etc.

positive dependencies that are not internalized in the competitive equilibrium. Also note that if all suppliers were independent, externalities would be non-existent, and they would be strictly positive with negative relations. In the former case, suppliers would still bid individually (thus avoiding coordination effort), while in the latter case they would bid jointly into the auction (thus taking advantage of the coalition surplus). Both cases would be consistent with the true joint contribution and hence, yield an efficient equilibrium.

To overcome the inefficiency revealed in Proposition 2, two strategies could be followed: Either, the buyer could require consistency of all (accepted) bids with their true joint contribution. Suppliers would then be forced to form and bid as a coalition that is truly able to deliver the required level of reliability. This would in principle entail an efficient auction outcome. However, in order to ensure the optimal coalition really emerges, all suppliers would need to know and process the properties of all other suppliers, including their interdependencies. In practice, ensuring such a level of complex information and information processing among the suppliers would probably be hard to achieve. Moreover, related transaction costs (that would subsequently be internalized in the bids and paid by the buyer) would probably be very high. Hence, as an alternative, the buyer could centralize the process of determining the optimal coalition in the auction clearing, thus only requiring the physical properties of the suppliers along with their prices. The buyer would then determine the joint contribution of the suppliers for reliability *himself* instead of inviting tenders for them.

Either way, it is the joint contribution that essentially counts for the buyer and that suppliers should be paid for. However, in order to design at a corresponding payoff scheme, the joint contribution (or value) needs to be split and allocated to the suppliers. A suitable allocation rule that disentangles the value of individual suppliers under complex interrelations will be discussed hereafter.

4.2. Allocation rule

There is an infinite number of allocation rules that could be applied to split the joint contribution and allocate it to individual suppliers. To name a few examples, the joint contribution could simply be split into equal parts, or be allocated according to average availabilities of the suppliers. However, instead of choosing an arbitrary allocation rule, it is important to note that the specific choice may have an impact that feeds back to the supply chain performance. For instance, consider a rule that is perceived particularly unfair by some suppliers. This could, as a consequence, reduce the exerted effort or even conscious insurrection (e.g. in form of a strike) of those suppliers, and hence, trigger a suboptimal supply chain performance. Equally important are long-term investment incentives that are implicitly provided by the choice of an allocation rule. Hence, we aim at identifying an allocation rule that incorporates a number of desirable properties to suitably address the aforementioned aspects.

Proposition 3. *A normatively fair allocation of reliability-related payoffs to interdependent individual suppliers can be obtained by the Shapley value:*

$$\Phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} (v(S \cup \{i\}) - v(S)) \quad (23)$$

Proof. Taking as given the physical and strategic realities captured by the characteristic function, the Shapley value is an attempt to distribute the joint contribution of a coalition in a reasonable, or fair way. The motivation of the Shapley value is normative, and the criterion of fairness applied is egalitarianism (Mas-Colell et al. (1995)). It pays off the average marginal contribution of a supplier i to the set of predecessors, with the average taken over all orderings. It aims at incorporating the following desirable properties:

- **Static efficiency:** $\sum_i \Phi_i(v) = v(N)$, i.e., the joint value of the grand coalition is distributed and no utility is wasted. In our application, this property ensures that the sum of the individual payoffs according to the Shapley value corresponds to the joint contribution. Hence, the Shapley value ensures static efficiency of the allocation rule.
- **Symmetry:** If two suppliers i and j have an equivalent position in the game (i.e., if $v(S \cup \{i\}) = v(S \cup \{j\})$), or, in other words, if their individual contribution for reliability is the same, then their Shapley value is identical.
- **Linearity:** $\Phi_i(v_1 + v_2) = \Phi_i(v_1) + \Phi_i(v_2)$ for two characteristic functions v_1 and v_2 , and $\Phi_i(av) = a\Phi_i(v)$ for any real number a . Hence, individual payoffs are equally affected if the coalition payoff (e.g., the price in an auction for reliability) changes.
- **Dummy:** If a supplier i offers no contribution to reliability (i.e., if $v(S \cup \{i\}) = v(S)$ for all coalitions S), then the Shapley value of this supplier is zero.

All the above properties are meaningful and relevant for allocations to individual suppliers according to their reliability contribution. Shapley (1953) showed that the payoff allocation according to the Shapley value given in Equation (23) is the unique value satisfying the efficiency, symmetry, linearity and dummy axioms. \square

4.3. Investment incentives

We investigate the investment incentives induced by the above allocation rule (Equation (23)) based on the concept of biform games, studying the impact of surplus division on non-cooperative investment decisions (Brandenburger and Stuart (2007)). Investments take place in a stage prior to surplus allocation, such that the suppliers' profits are maximized. To assess the incentives of our allocation rule with respect to investment behavior, we focus on the relative weight placed on different investment options (i.e., suppliers). I.e., we analyze how investments would be *distributed*,

instead of the *absolute* level of investment.¹⁹ We compare the distribution of investments under two different premises. First, let us state the first-best benchmark. From an overall supply chain perspective, the aim is to maximize a weighted tradeoff between high reliability (i.e., high v) and high production volumes (i.e., high μ), with parameter $\beta \in [0, 1]$ reflecting the relative weight given to high generation volumes and reliability, respectively. Without loss of generality, let us suppose the case of two suppliers, and the possibility to distribute investments among this set of suppliers by adjusting the choice variable $\alpha \in [0, 1]$. Then the first-best benchmark is obtained by maximizing the following objective function:

$$\max_{\alpha} \beta[v((1 - \alpha)S_1 \cup \alpha S_2)] + (1 - \beta)[(1 - \alpha)\mu_{Y_1} + \alpha\mu_{Y_2}]. \quad (24)$$

Second, we consider the case of a price-taking independent investor who wants to extend an amount $I \in [0, \inf)$ of supply capacity, and needs to decide on the share $\gamma \in [0, 1]$ (respectively $1 - \gamma$) to be invested in supplier 1 (2). Under the assumption that reliability-related payoffs according to the Shapley value are being complemented by revenues from a production-related procurement or market, the investor's profit function becomes

$$\Pi = p_r((1 - \alpha)\Phi_1 + \alpha\Phi_2) + p_p((1 - \alpha)\mu_{Y_1} + \alpha\mu_{Y_2}) - Ic, \quad (25)$$

with p_r the price paid per reliable capacity (e.g., determined through a capacity auction, as previously discussed), p_p the price per production volume, and c the capacity cost. We assume that p_r , p_p and c are constant and independent of the supplier.

Proposition 4. *The optimal distribution of independent investments coincides with the first-best benchmark if the ratio of prices for reliability and production is properly set.*

Proof. First-best benchmark: From a social perspective, we shall derive the system-optimal diversification strategy $\alpha^* \in [0, 1]$ for two suppliers S_1 and S_2 with production capacity $(1 - \alpha)Y_1$ and αY_2 , respectively. Hence, we differentiate (24) with respect to α , which – after a few calculations – yields:

$$\alpha^* = \frac{\left[(\mu_{Y_2} - \mu_{Y_1}) \frac{1 - \beta}{\beta} + (\mu_{Y_2|X \approx 0} - \mu_{Y_1|X \approx 0}) \right] \frac{f_X(0)}{f'_X(0)} + \sigma_{Y_1|X \approx 0}^2 - \sigma_{Y_1, Y_2|X \approx 0}}{\sigma_{Y_1|X \approx 0}^2 + \sigma_{Y_2|X \approx 0}^2 - 2\sigma_{Y_1, Y_2|X \approx 0}}. \quad (26)$$

Independent investor: Now consider the case of a price-taking independent investor characterized by the profit function (25). For the case of two suppliers, the Shapley values Φ_1, Φ_2 can

¹⁹Reaching a certain *absolute* investment level would simply require the allocation of an investment volume that is sufficiently high (especially, sufficiently high to cover the costs of the investment). The linearity property of the Shapley value would ensure that increasing the investment volume would affect individual suppliers equally, and hence, that the *distribution* of investment is not altered.

easily be calculated as

$$\Phi_1 = \frac{1}{2}v(S_1) + \frac{1}{2}(v(S_1 \cup S_2) - v(S_2)) \quad (27)$$

$$\Phi_2 = \frac{1}{2}v(S_2) + \frac{1}{2}(v(S_1 \cup S_2) - v(S_1)). \quad (28)$$

Inserted in Equation (25), the optimal weight for the distribution of the investment can be derived from the first-order condition $\frac{\partial \Pi}{\partial \gamma} = 0$, which – after a few calculations – yields γ^* . We find that $\gamma^* = \alpha^*$, i.e., equality of the investor’s optimal choice and the first-best benchmark, however, only as long as the ratio of prices $(\frac{p_r}{p_p})$ equals the ratio of supply chain preferences for reliability and production $(\frac{\beta}{1-\beta})$.

Note that we have so far assumed a greenfield investment with no supply capacity being present. However, it is straightforward to relax this assumption, and instead allow for some arbitrarily distributed capacity already being installed. These capacities will, due to their stochastic interdependence, impact the investor’s profit (and hence, investment decisions) via the amount and shares in payoffs allocated to the investor’s capacities. However, the impact on the independent investor’s decision induced by the Shapley value is such that the additional capacity is ensured to be installed where it is supply chain optimal. The reason is that an investment in line with supply chain optimality yields highest overall contributions to reliability, which in turn is distributed among the coalition members. As the Shapley value strictly increases with the overall surplus (linearity axiom), it follows that an independent investor aligns its strategy with the supply chain optimal result. Hence, the Shapley value is able to ensure that additional investments bring the system towards the first-best optimal distribution of capacities.

Lastly, generalization to $n > 2$ suppliers is straightforward using $\text{Var}(\sum_{i=1}^n Y_i) = \sum_{i=1}^n \sum_{j=1}^n \sigma_{Y_i, Y_j} = \sum_{i=1}^n \sigma_{Y_i}^2 + 2 \sum \sum_{1 \leq i < j \leq n} \sigma_{Y_i, Y_j}$. \square

Let us briefly discuss the plausibility of the results derived in Proposition 4 and its proof. Consider the extreme case of $\beta = 0$, i.e., when full preference is given to production volumes. Then, the first-best result becomes

$$\tilde{\alpha}^* |_{\beta \rightarrow 0} = \begin{cases} 0 & : \mu_{Y_2} < \mu_{Y_1} \\ 1 & : \mu_{Y_2} > \mu_{Y_1} \\ \text{indifferent} & : \mu_{Y_2} = \mu_{Y_1}, \end{cases} \quad (29)$$

i.e., – as expected – a simple preference for the location with higher average availability if there is any, and indifference otherwise.

For the case of some preference being given to reliability, i.e., for $\beta \neq 0$, the result reveals that the optimal share depends upon the difference between the average availabilities of the two suppliers, their variances, as well as their covariance. Specifically, the following characteristics

require a stronger redistribution from supplier S_1 to S_2 : high average availability of S_2 ; low average availability of S_1 ; large variability of S_1 ; and low covariance between S_1 and S_2 .

Some further remarks concerning Proposition 4 are worthwhile: First, note that it always needs a certain price level to incentivize investments. For reliability-payoffs only, it would be individually rational to invest as long as $\frac{p_r}{c} > \frac{I}{v(\gamma S_1 \cup (1-\gamma) S_2)}$, whereas for complementary payoffs, the individual rationality constraint becomes $\frac{p_r}{c}(\Phi_1 + \Phi_2) + \frac{p_p}{c}(\mu_{Y_1} + \mu_{Y_2}) > I$. Second, we want to point the readers interest to possible extensions of our analysis of investment incentive effects: Besides the case of one investor, it would also possible to consider one or multiple players per supplier optimizing their payoffs by choosing an investment level while strategically considering the action performed by the other investors. For the case of one player per supplier, we have found an underinvestment of individual players compared to the supply chain optimal outcome, also impacting the distribution of capacities towards an inefficient balance.²⁰ As expected, this inefficiency increases with the level of interdependencies among the locations, and vanishes for independent resources. This result might indicate possible problems occurring from the exercise of market power within the Shapley value approach.

Lastly, we remark that from a supply chain organization perspective, properly setting the ratio of prices for reliability and production may indeed be challenging. An appropriate weighting of prices in the case of complementary reliability- and production-related payoffs could be ensured, e.g., through efficient prices stemming from appropriately designed markets.

5. Empirical case study: Wind power in Germany

This section presents an empirical case study for wind power embedded in the German power system. Based on real-world data, it is meant to demonstrate applicability of the concepts developed in the previous sections as well as their practical relevance, and to empirically confirm our analytical results. The case of wind power in Germany appears to be novel and pertinent. With 34.02 out of 188.68 GW production capacity being installed, wind power plays a major role in Germany's generation portfolio, while it is a declared goal to integrate the technology into general market structures. Moreover, power system reliability as well as possible regulatory interventions, such as capacity mechanisms, have been heavily discussed for several years.

5.1. Estimation procedure

Recall Equation (3) that needs to be solved for v in order to obtain the contribution of individual suppliers. We assume random variable C , i.e., the availability of the power supply fleet apart from wind power, to be independent from wind power Y and demand D , and that its distribution can be determined through convolution of the suppliers' outage probabilities (see below). In contrast,

²⁰The result shows strong similarities with the withholding situation in a Cournot-Duopoly.

the joint distribution of Y and D is estimated from simultaneous historical observations. As we only have one observation per instant in time t , we need to extend Equation (3) to multiple time periods, such that random variables Y, D may be replaced by corresponding observations y_t, d_t .²¹

$$\sum_{t=1}^T \Pr(d_t + v \leq C + y_t) \equiv \sum_{t=1}^T \Pr(d_t \leq C) \quad (30)$$

Note that summing up the probabilities over time yields the expected value during the considered number of hours T . This measure is often applied to formulate or benchmark reliability levels. For instance, a 1-day-in-10-years criterion has often been used as a benchmark or target value, both in the academic literature (e.g., see Keane et al. (2011)) and in practice (e.g., by the Midcontinent ISO or the ISO New England).²²

5.2. Data

The necessary data can be classified into three main areas: First, detailed information is needed about installed capacities and availability factors of dispatchable power suppliers apart from wind (C in Equation (30)). Second, the analysis requires high-resolution data on wind power capacities as well as their infeed profiles (y_t). Third, load levels with the same temporal resolution and regional coverage are needed to perform the calculations (d_t). Descriptions of the data along with some preparatory calculations can be found in the Appendix A. Importantly, we find clearly positive dependencies among all wind power profiles (see Appendix A.4 for a detailed analysis).

Regarding the level of detail in our analysis, we would ideally opt for a representation of each individual supplier in the system. However, for large systems with many interdependent suppliers, this would quickly involve impractical data requirements and calculation efforts. This is mainly due to the fact that each of the $2^n - 1$ possible coalitions needs to be calculated. Hence, some administrative division may provide a satisfactory level of disaggregation while still being manageable.²³ In our empirical example, we will hence aggregate wind power on the federal state level.²⁴

²¹The validity and consistency of the result obtained from this reformulation may be justified by the central limit theorem (Zachary and Dent (2011)).

²²Alternative economic approaches would try to estimate efficient reliability levels from the value of lost load (VOLL) and costs of maintaining a certain level of supply chain reliability (Telson (1975)). A thorough discussion would clearly be beyond the scope of this paper, such that the interested reader is referred to Stoft (2002) for the necessary calculations, and attempts to quantify the VOLL, e.g., by Anderson and Taylor (1986) for Sweden or by Growitsch et al. (2014) for Germany. Noticeably, due to the fact that data requirements and estimation procedures are far from being straightforward, it is not surprising that rules of thumb and common practice, such as the 1-day-in-10-years, are often applied – both by academics as well as practitioners.

²³The same (relative) Shapley value would then apply to all units within that area.

²⁴Germany consists of 16 federal states: Baden-Württemberg (BW), Bayern (BY), Berlin (BE), Brandenburg (BB), Bremen (HB), Hamburg (HH), Hessen (HE), Mecklenburg-Vorpommern (MV), Niedersachsen (NI), Nordrhein-Westfalen (NW), Rheinland-Pfalz (RP), Saarland (SL), Sachsen (SN), Sachsen-Anhalt (ST), Schleswig-Holstein (SH), and Thüringen (TH).

5.3. Results

5.3.1. Reliability of Germany's dispatchable power fleet

To determine the cumulative distribution function F_C describing the availability distribution of Germany's power supply fleet apart from wind power, we assume that each supplier is either fully available or not,²⁵ and that individual failure probabilities are independent. Based on these assumptions, F_C can be derived via convolution of the individual suppliers' failure probabilities. We implemented the algorithm developed in Hasche et al. (2011) which proved to be fast enough to calculate the cdf for our supply chain (consisting of nearly 900 dispatchable power supply units) in less than a minute on a standard laptop. The resulting (complementary) cumulative distribution function is shown in Figure 1, together with a histogram of load levels. Load levels are for the most part far left of the critical range of around 90 GW where the complementary cdf begins to drop sharply.

From Equation (30) and our data, we obtain a basically perfect reliability of supply of $R^S = 1 - 1.15e^{-12}$ hours/year.²⁶ Hence, installed capacities appear to be largely sufficient to reliably cover today's load profile.

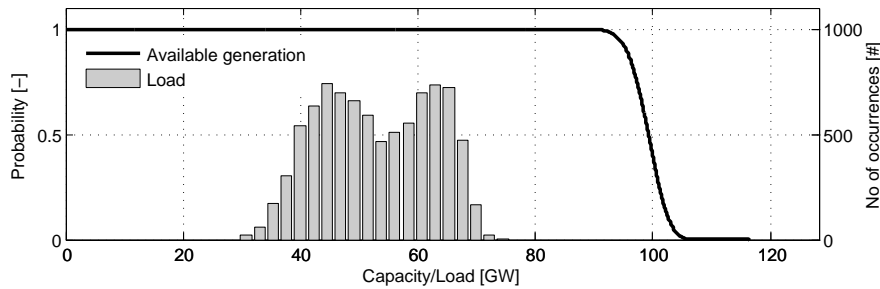


Figure 1: Complementary cdf of Germany's power plants without wind power together with a histogram of 2013 load levels

Having previously mentioned the often applied 1-day-in-10-years target, we find that a 30% increase in current (2013) load levels could be sustained in order to reach that threshold, again indicating large amounts of over-capacity in the German system.

5.3.2. The contribution of wind power to reliability of supply

We find the joint contribution of Germany's aggregated wind power to reliability of supply $v(N)$ to be 3376 MW, corresponding to $\tilde{v}(N) = 9.9\%$ relative to the installed capacity of 34.02 GW.

²⁵I.e., no partial failures are taken into account.

²⁶Note that R^S is a risk measure, not implying that a certain number of load shedding events effectively occurs. Hence, the figures presented here and in the following should not be confused with realized statistical numbers, such as the Average System Interruption Duration Index (ASIDI).

To provide empirical evidence for the decreasing returns to scale found in Corollary 1, we scale today's wind power capacity by factors of 0 to 3, while assuming unchanged characteristics of load and dispatchable power (Figure 2, left hand side). As expected, the contribution decreases monotonically along a convex function. To empirically confirm gains of diversification (Corollary 2), we first calculate the contribution of wind capacities being installed in two states (S_1 and S_2) separately, ranging from 100 to 900 MW. We take Sachsen (SN) and Thüringen (TH) as an example.²⁷ We then calculate the joint contribution of a diversified portfolio of the same aggregated capacity being installed in both the states (capacity split half-half, i.e., $v(\frac{1}{2}S_1 \cup \frac{1}{2}S_2)$). Whereas the joint contribution of the diversified portfolio lies in between the separate states for small capacities, it clearly yields higher contributions for increasing penetration levels. Especially, the rate at which the diversified contribution drops is significantly smaller.

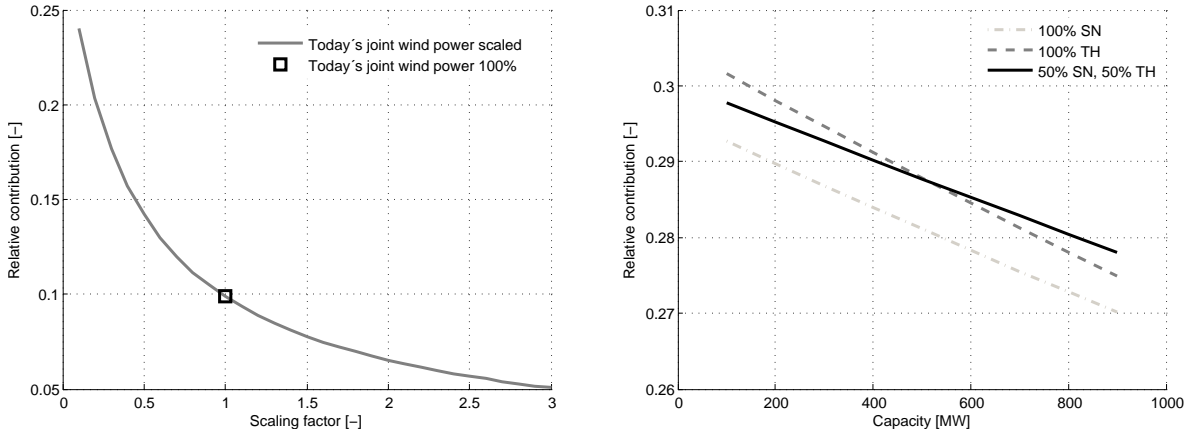


Figure 2: Decreasing returns to scale (left); Gains of diversification (right)

The following Table 1 presents today's installed wind power capacities per state, along with the individual absolute ($v(S_i)$) and relative contributions ($\tilde{v}(S_i)$). Individual relative contributions vary significantly, ranging from 13.9 to 31.3%. Summing up the individual contributions would clearly yield a false joint contribution of 19.6%. Compared to the previously determined consistent joint contribution of 9.9%, this is a huge overestimation that would be induced by neglecting the positive interdependencies between the resource availability at different locations. This empirical finding underlines the importance of the non-additivity property, as stated in Corollary 3.

5.3.3. Supply chain organization

For our case study, empirical evidence shows that our game is largely subadditive (and non-convex), and that the core is empty. The following Figure 3 is meant to illustrate the subadditive nature of our problem from two perspectives. The left hand side shows the coalition's joint contribution vs. its size (in terms of coalition members) for all 65535 possible coalitions,²⁸ clearly

²⁷These two states have a similar average availability of wind power and a correlation coefficient of 0.59.

²⁸For a game with 16 players, the number of possible coalitions is $2^{16} - 1 = 65535$.

State	Installed Capacity [MW]	Absolute Contribution [MW]	Relative Contribution
BW	624	156	24.9%
BY	1066	158	14.8%
BE	2	1	31.3%
BB	5233	1013	19.4%
HB	114	34	30.0%
HH	55	16	28.1%
HE	961	145	15.1%
MV	2301	588	25.6%
NI	7676	1392	18.1%
NW	3473	483	13.9%
RP	2366	395	16.7%
SL	223	44	19.9%
SN	1055	281	26.6%
ST	4093	955	23.3%
SH	3683	723	19.6%
TH	1097	295	26.9%

Table 1: Installed capacity, absolute and relative contributions calculated for each state individually

indicating an inverse relation. Similarly, the right hand side shows the error that would be induced by neglecting the (positive) interdependencies by plotting each possible coalition’s sum of individual contributions vs. its consistent joint contribution.²⁹ Whereas superadditivity would require all points to be above the perfect additivity line, we find that virtually all points lie well below, with the error substantially increasing for larger individual contributions (essentially due to decreasing returns to scale, i.e., in line with Corollary 1).

The results of both figures are driven by the positive interdependencies among the 16 states that are illustrated in Figure A.4. They demonstrate the empirical relevance of Proposition 2, i.e., the inefficiency of a competitive equilibrium in a reliability procurement where only individual bids are verified.

5.3.4. Payoff allocation according to the Shapley value

Turning to the core result, Figure 4 presents the relative Shapley value $\tilde{\Phi}_i(v(N))$, i.e. the payoffs relative to installed capacities allocated to each of the 16 states (green squares). To put these values into perspective, they are presented in combination with the suppliers’ average availability (μ_i) as well as with their separately calculated individual contribution ($\tilde{v}(S_i)$). We find that the individual contribution as well as the Shapley value tends – as expected – to decrease with average availability. However, the Shapley values also show pronounced deviations from the joint contribution (9.9%), as well as from the ordering of the individual contributions and average availabilities. For instance, Niedersachsen (NI) and Sachsen-Anhalt (ST) both have high average availabilities, but also large installed capacities as well as pronounced positive dependencies with

²⁹The most exterior point, i.e. the grand coalition of all states, has already been discussed in the previous paragraph.

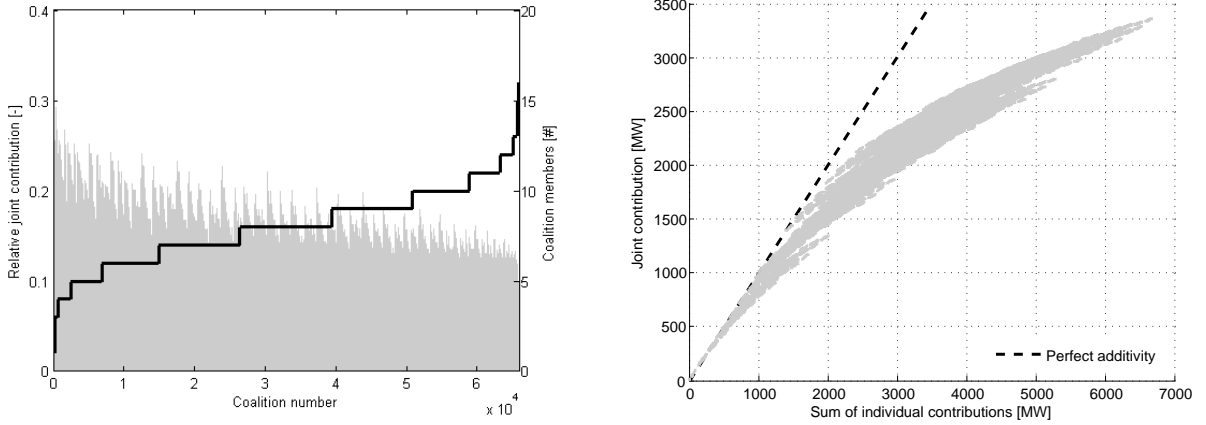


Figure 3: Relative joint contribution vs. size of coalition (left); Sum of individual contributions vs. joint contribution (right)

other states – and hence comparatively low Shapley values. Overall, the Shapley values range from 15.1% for Baden-Württemberg (BW) and Sachsen (SN) to 5.7% for Bavaria (BY).

Relating the Shapley value to the possibility of emerging coalitions in a competitive environment (Proposition 2), we find that 50337 out of 65535 possible coalitions would be able to block this allocation, due to the subadditivity found above. Hence, it seems indeed reasonable to transfer the process of determining the joint contribution of the coalition and allocating individual payoffs to an independent central authority.

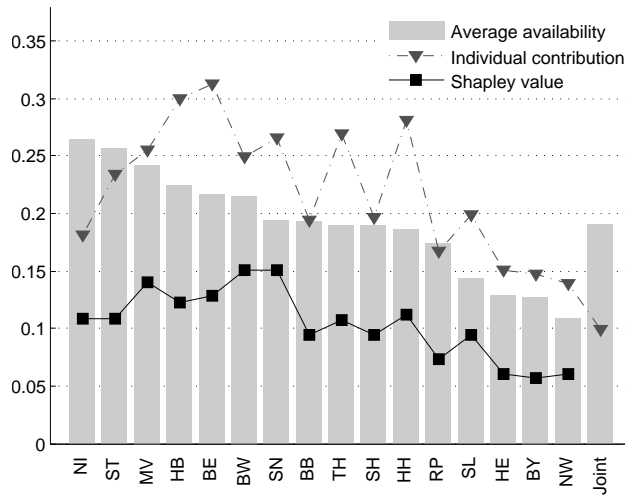


Figure 4: Average availabilities, individual contributions and Shapley values (all relative)

In practice, it will be most plausible to pay power supply units according to a weighted production- and reliability-related preference (i.e., if existing markets for energy are or were to be complemented by a capacity mechanism). As discussed in Section 4.3 and Proposition 4, it would then be important to find a balance between high production volumes (i.e., high average

availability) and high Shapley values. Figure 5 presents this tradeoff, showing that states perform strikingly different on both dimensions. Separating the field into quadrants, we find that Baden-Württemberg (BW) and Sachsen (SN) perform particularly well for the case of more preference given to reliability, Niedersachsen (NI) and Sachsen-Anhalt (ST) for the case of more preference on production volumes, and Mecklenburg-Vorpommern (MV) for an equal balance. Nordrhein-Westfalen (NW), Bavaria (BY) and Hessen (HE) perform poorly with respect to both properties. Moreover, it should be noticed that even though a general positive trend can be found, the tradeoff is widely scattered around a straight line. In practice, this property would offer the possibility to effectively incentivize investments that contribute much better to reliability than purely production-based decisions by means of appropriate payoffs reflecting (weighted) preferences for production volumes *and* reliability.

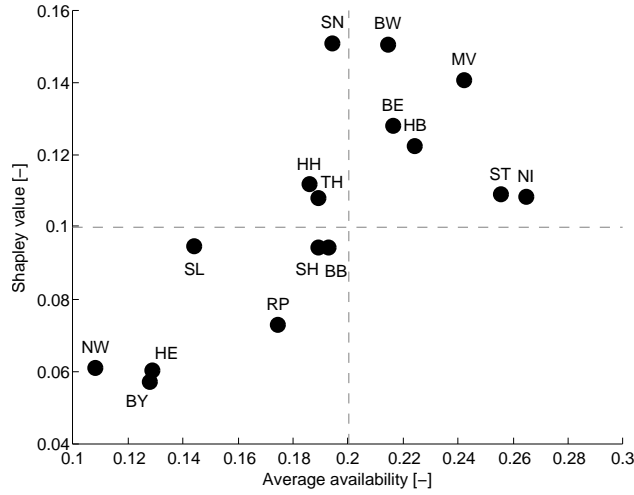


Figure 5: Average availability vs. Shapley value

6. Conclusions

Supply chain reliability is a timely and relevant subject that has been studied intensively in the academic literature. However, the specific role of individual, possibly stochastic and interdependent suppliers for supply chain reliability has so far been disregarded. To fill this gap, we have thoroughly investigated the issue based on statistical and economic analyses. Especially, we have derived an analytical solution for the contribution of individual suppliers to reliability as well as a corresponding payoff scheme that accounts for the statistical properties of the problem. Practical applicability and relevance has been demonstrated with a numerical example based on wind energy in the German power system, thereby confirming our analytical findings.

With the concepts developed, the reliability of supply chains could in the future be designed and managed more efficiently. For instance, our payoff scheme could be applied to write payoff contracts

in monopsonistic markets or public procurements from various suppliers, or to allocate individual payoffs in a bonus system. To name a practical example, the approach could improve the design of capacity mechanisms in power systems, where the state or regulator procures reliability from power supply units. In fact, our analysis reveals that existing designs to incorporate variable renewable energies into capacity-mechanisms contain substantial shortcomings.³⁰ In contrast, the properties of the approach developed in this paper would ensure a level playing field for all suppliers, and clearly outperform existing, currently applied alternatives.

Our analysis could be extended in several directions: It would be worthwhile to consider a strict optimization applied within our framework with costs attached to individual suppliers, e.g., to determine an optimal subset among a broader set of interdependent suppliers. The investment incentives provided by our Shapley value approach should be studied in more detail, including variations in the number and role of market participants (e.g., market power) as well as a more sophisticated representations of investment decisions. The effect of an endogenous demand-side response on reliability would also be interesting. Our approach could be applied to other supply chains incorporating stochastic suppliers and demand, especially those with positive dependencies. For instance, it would be interesting and relevant to assess the value of an additional call center for reliable customer services, or to design suitable payoffs for individual drivers in a large network providing transport services.

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³⁰For instance, the selective Spanish capacity mechanism excludes intermittent resources by allocating zero reliable capacity, hence inducing a situation of static inefficiency as soon as there is any positive contribution – which is likely to be the case as presented in our empirical case study. Moreover, the Spanish design fails to incentivize investments into suppliers that are beneficial for reliability, hence also inducing an additional long-term inefficiency. As a serious design flaw in the PJM capacity auction, they attempt to prequalify suppliers individually – which contradicts the non-additivity we have found in Corollary 3. Moreover, while PJM allows suppliers using intermittent resources (i.e., wind and solar power) to cooperate and bid jointly, there will be disincentives to do so as implied in Proposition 2. These shortcomings will necessarily entail an inefficient outcome. Drawbacks in the UK and French capacity mechanisms can be found in the average payoff allocated to each unit in the same technology-class. While the approach ensures static efficiency by complying with the consistent joint contribution, it lacks the provision of locational investment incentives. In addition, the approach may face opposition from underrated suppliers whose individual contribution to reliability lies well above the average level.

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Appendix A. Data and preparatory calculations

Appendix A.1. Installed capacities and availability factors

For information about currently installed power supply units, we use the List of Power Plants prepared and provided by the Federal Network Agency (Bundesnetzagentur (2014)). It lists all existing units in Germany with a net nominal electricity capacity of at least 10 MW.³¹ Moreover, supply facilities of less than 10 MW are also included on an aggregated basis grouped by energy source. Extracted net nominal capacities by fuel type are depicted in Table A.1. As for wind power, 34.02 GW were installed by mid 2014, distributed among the 16 federal states as shown in Figure A.1.

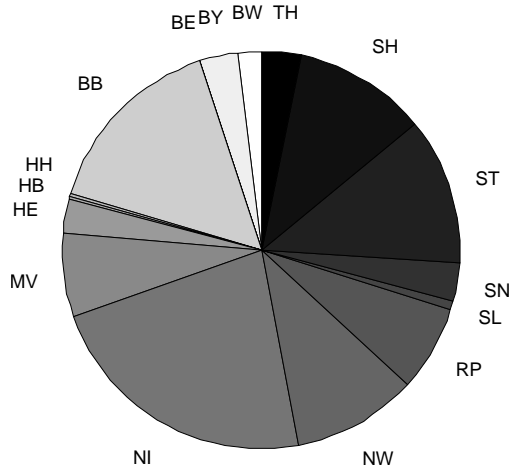


Figure A.1: Distribution of installed wind power capacities (34.02GW total)

Regarding the suppliers' availabilities, we assume fuel-type specific factors according to historical observations, taken from VGB and Eurelectric (2012), and complemented with dena (2010) for Hydro, Geothermal and Biomass, as reported in Table A.1. For PV, we assume an availability factor of 0%.³²

Appendix A.2. Wind speed data and wind to power conversion

As we want to focus on the supply side uncertainty of wind power in our empirical example, coverage and resolution of the wind data are crucial for obtaining reliable results. Consequently, we use hourly wind speed data of 32 years (1982-2013) to cover a broad range of possible wind patterns in Germany, provided by the national climate monitoring of the German Weather Service (DWD

³¹In addition, it also comprises capacities directly feeding into the German grid from Luxembourg, Switzerland and Austria (which we assume to contribute to Germany's generation adequacy)

³²This is a conservative estimate, however, consistent with the observation that highest load hours occur in the late evening during winter time.

Fuel type	Availability	Capacity [GW]
Biomass	88.0%	6.38
Coal	83.9%	27.73
Gas	88.3%	25.42
Geothermal	90.0%	0.03
Hydro (pump) storage	90.0%	10.63
Hydro run-of-river	40.0%	3.92
Lignite	85.3%	20.95
Nuclear	83.3%	12.07
Oil	89.2%	4.14
Others (Waste, Landfill gas, etc.)	90.0%	5.32
PV	0.0%	37.45
Wind	<i>to be calculated</i>	34.02

Table A.1: Availability factors and installed capacities per fuel type

(2014)).³³ We select one representative location per federal state according to the agglomeration of wind turbines within each state.³⁴

State	DWD-ID	Observatory name
BW	4887	Stötten
BY	5705	Würzburg
BE	3987	Potsdam
BB	164	Angermünde
HB	691	Bremen
HH	1975	Hamburg-Fuhlsbüttel
HE	1420	Frankfurt
MV	4271	Rostock
NI	891	Cuxhaven
NW	2483	Kahler Asten
RP	2385	Idar-Oberstein
SL	4336	Saarbrücken
SN	1048	Dresden
ST	1957	Halle-Kröllwitz
SH	4466	Schleswig
TH	1270	Erfurt-Weimar

Table A.2: Selected DWD observatories

The conversion of wind speed to electrical power output is described by a turbine-specific power curves. As a representative power curve, we use the Nordex S77 turbine with a hub height of 77 meters and a rated power of 1.5kW (Nordex (2007)). Wind speed to power conversion is

³³In case of missing data, empty entries are replaced by interpolations based on the previous and next available value if the empty space is not exceeding 12 hours. If the gap is longer, entries are replaced by data of the same station and same hours of the previous year. As measurements are taken a couple of meters above ground only, wind speeds are scaled to the wind turbines' hub height assuming a power law: $v_{h_1} = v_{h_0}(h_1/h_0)^a$, where h_0 is the measurement height, h_1 the height of interest and a the shear exponent. According to Firtin et al. (2011), a is assumed to be 0.14.

³⁴The list of selected observatories is presented in the following Table A.2.

implemented via lookup-tables with linear interpolation.

To validate this relatively simple model for generating wind power profiles, we compare statistical profiles and volumes with our synthetic model-generated data. Figure A.2 compares statistical yearly production per federal state (available from Agentur für erneuerbare Energien (2014) for the years 2001-2012, except for 2007) against our modeling results (based on historically installed wind capacities taken from Bundesverband WindEnergie (2014) and wind speeds from the corresponding years). As can be seen, our model overestimates production for the first years, whereas satisfactory conformity is reached for more recent periods. This is probably due to improvements in turbine technologies over the years (remember that we have applied the power curve of a relatively modern turbine).

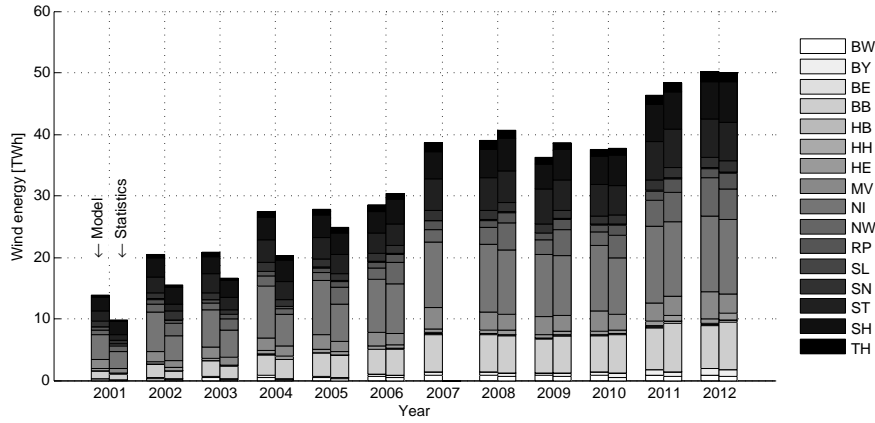


Figure A.2: Yearly wind power production per federal state (model: left, statistics: right)

Another step of validation has been carried out for the hourly profiles, based on a comparison of the distributions of the model-based data with historical production profiles (available for the years 2010-2013) by means of QQ-plots (Figure A.3). Distributions are found to be very similar for those years, however, with slightly decreasing conformity for upper quantiles. Conducting a simple regression analysis yields an R^2 of 0.80, 0.81, 0.82 and 0.81 for the years 2010-2013. For completeness, summary statistics of the obtained profiles (based on 2014 wind capacities) are also provided in Table A.3.

Appendix A.3. Load profiles

Germany's load levels are reported on an hourly basis by ENTSO-E (2014), representing the hourly average active power absorbed by all installations connected to the transmission or distribution network. Instead of using multiple years of load data, we restrict our attention to the most recent year 2013 in order to focus on the supply side uncertainty. We hence refer to reliability under current load levels and profiles. Summary statistics of the load profile are comprised in Table A.3.

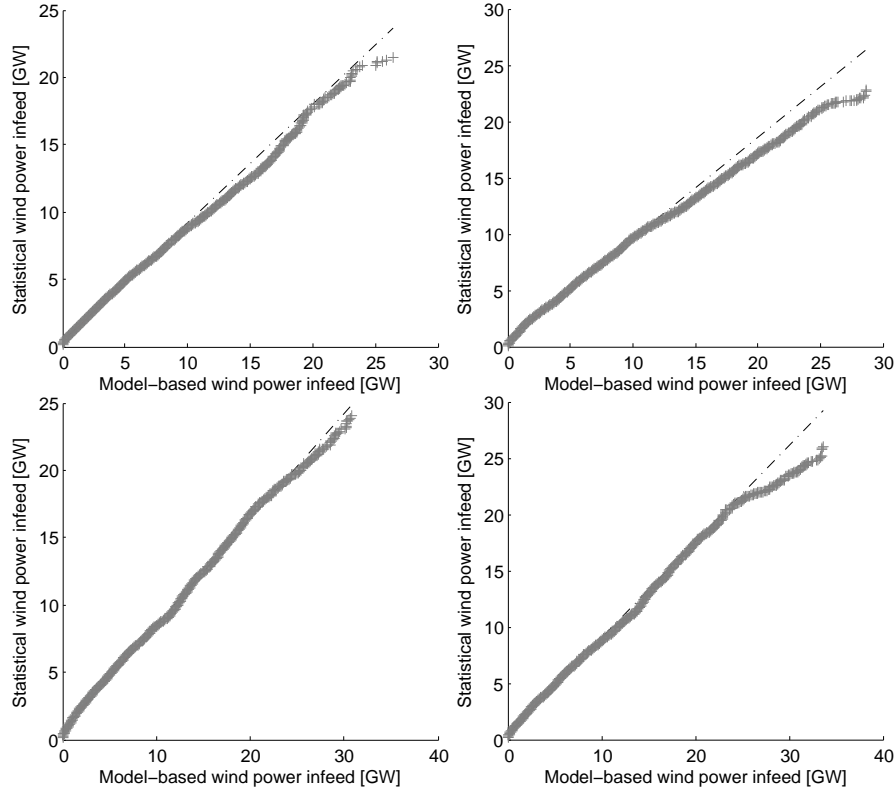


Figure A.3: QQ-plots of model-based and historical production profiles, for the years 2010-2013

Profile	Min	Max	Mean	Std
Load Germany (2013)	29.55	75.623	52.8916	9.6327
Wind Power BW (1982-2013)	0	0.6241	0.1340	0.1828
Wind Power BY (1982-2013)	0	1.0666	0.1363	0.2513
Wind Power BE (1982-2013)	0	0.002	0.0004	0.0005
Wind Power BB (1982-2013)	0	5.2331	1.01	1.4062
Wind Power HB (1982-2013)	0	0.1142	0.0256	0.0321
Wind Power HH (1982-2013)	0	0.0551	0.0102	0.0138
Wind Power HE (1982-2013)	0	0.961	0.1239	0.2147
Wind Power MV (1982-2013)	0	2.3014	0.5572	0.731
Wind Power NI (1982-2013)	0	7.6757	2.0334	2.3978
Wind Power NW (1982-2013)	0	3.4732	0.3751	0.7232
Wind Power RP (1982-2013)	0	2.3661	0.4126	0.6694
Wind Power SL (1982-2013)	0	0.2233	0.0322	0.051
Wind Power SN (1982-2013)	0	1.0545	0.2048	0.2865
Wind Power ST (1982-2013)	0	4.0925	1.0465	1.3257
Wind Power SH (1982-2013)	0	3.6828	0.6963	0.932
Wind Power TH (1982-2013)	0	1.0965	0.2073	0.3079

Table A.3: Summary statistics of load and wind power profiles in [GW]

Appendix A.4. Correlation analysis

As has been noticed several times during the theoretical analysis in Sections 3 and 4, covariance among the power production of the different locations crucially impacts the properties and results of the problem. In order to get an impression of the dependencies characterizing wind power and load in Germany, Figure A.4 shows the matrix of linear correlation coefficients ρ .

As can be seen, ρ among the wind power profiles is in a range of $[0.10, 0.77]$, with a mean of 0.48 (excluding diagonal values). Correlations between wind power and load are in a range of $[0.06, 0.17]$, with a mean of 0.13. Hence, all dependencies are clearly positive.³⁵

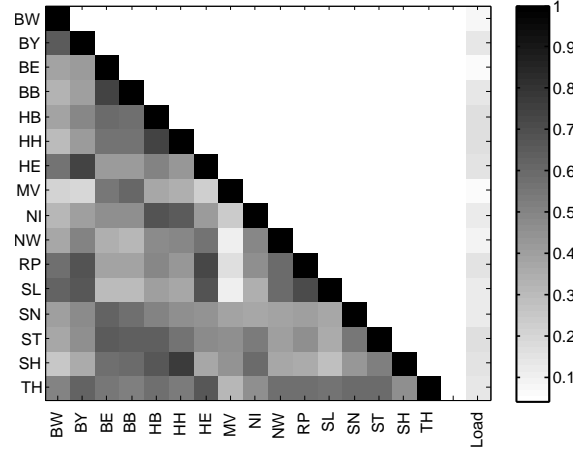


Figure A.4: Correlation matrix of wind power profiles

³⁵As reliability of supply is particularly relevant during tight capacities, we recalculate the same numbers for the data corresponding to the 5% highest load hours, resulting in a range of $[0.15, 0.79]$ and a mean of 0.51 for correlations among wind power, and $[-0.02, 0.03]$ and a mean value of 0.00 for correlations between wind power and load. Hence, wind power dependencies are even slightly more pronounced during high-load-hours, whereas wind power and load are independent.