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How to sell renewable electricity: Interactions of the intraday and day-ahead market under uncertainty

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EWI Working Paper, No 16/04

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How to Sell Renewable Electricity - Interactions of the Intraday and Day-ahead Market Under Uncertainty

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Abstract

Uncertainty about renewable production increases the importance of sequential short-term trading. We consider a two-stage market where conventional and renewable producers compete in order to satisfy the demand of consumers. The trading in the first stage takes place under uncertainty about production levels of renewable producers, which can be associated with trading in the day-ahead market. In the second stage, which we consider as the intraday market, uncertainty about the production levels is resolved. Our model is able to capture different levels of flexibility for conventional producers as well as different levels of competition for renewable producers. We find that it is optimal for renewable producers to sell less than the expected production in the day-ahead market. In situations with high renewable production it is even profitable for renewable producers to withhold quantities in the intraday market. However, for an increasing number of renewable producers, the optimal quantity tends towards the expected production level. More competition as well as a more flexible power plant fleet lead to an increase in overall welfare, which can even be further increased by delaying the gate-closure of the day-ahead market or by improving the quality of renewable production forecasts.

Keywords: Cournot Competition, Electricity Markets, Day-ahead Market, Intraday Market, Competition Under Uncertainty

JEL classification: D81, L13, L94, Q21

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1. Introduction

The fundamental works of Cournot, Bertrand, and Stackelberg has led to a deeper understanding of competition in all kinds of markets. Although these models can provide insights into many areas, market structures may change with time and models need to be adapted accordingly. This holds especially true for electricity markets, where the whole supply structure has been changing in recent years. In the past, electricity has been predominantly generated based on conventional technologies (nuclear, coal, lignite, or gas) and hydropower. In recent years, the share of electricity generation from renewable sources (such as wind and solar) has been steadily increasing and this trend is expected to continue. In Germany, for example, the share of electricity generation from wind and solar has been increasing from 6% in 2000 to 33% in 2015 (BMW_i, 2016).

At the same time, renewable support schemes, such as the feed-in premium in Germany, have led to an integration of renewables into wholesale power markets. The number and size of aggregators who are responsible for the trading of renewable production has been steadily increasing. Wassermann et al. (2015) list 14 aggregators in the German market which trade more than 32 GW of the installed renewable capacity in 2014.¹ This amounts to 35% of the total installed renewable capacity in Germany in 2014 (BMW_i, 2016).² Renewable aggregators can lift significant scale effects by increasing their renewable portfolio. The costs for trading additional capacities are almost negligible and there may also be positive balancing effects within the portfolio. This is why the size of aggregators can be expected to grow in the future.

The increasing renewable share and the direct marketing by aggregators have implications for the market structure and competition in the electricity market. Renewable energy technologies differ in two important aspects from conventional technologies. First, renewables produce electricity at short run marginal costs of zero whereas conventional technologies have short run marginal costs greater than zero. Second, renewable electricity production depends on weather conditions that can only be predicted to a certain level. This has led to an increasing importance of sequential short-term electricity markets. Besides the day-ahead market, where electricity is traded one day before physical delivery, the intraday market has gained in importance for participants. Here, electricity can be traded down to 30 minutes before physical delivery, which becomes increasingly important for the integration of large shares of renewables into the electricity system.

In this paper, we analyze the competition between conventional and renewable producers that interact in two sequential stages by using an analytic model. The first stage is considered as the day-ahead and the

¹Wassermann et al. (2015) list only aggregators with more than 800 MW installed renewable capacity.

²The transmission system operators are still in charge of marketing the remaining share of renewable production but this amount is decreasing.

second as the intraday market. The electricity production of the renewable producer is uncertain in the first stage and is realized in the second stage. In particular, this affects renewable producers in choosing the optimal quantity to trade in both stages. Furthermore, we account for flexibility constraints of conventional power producing technologies, because not all conventional technologies are flexible enough to change their production schedules in short time intervals (e.g. 30 minutes before physical delivery). These flexibility constraints are included in our model to measure effects on profit maximizing quantities and prices. We analyze the results based on different levels of competition for the renewable producers, ranging from a monopoly to oligopolies under a flexible and less flexible power plant fleet.

Our investigation is strongly related to the branch of two-stage Cournot games as well as the literature of optimal bidding strategies for renewable producers. Concerning two-stage Cournot games, a fundamental work is given by Allaz and Vila (1993) who investigate Cournot competition of a duopoly in sequential markets. Their subject of investigation is the forward market which, however, can be transferred to our idea of a day-ahead auction before the market is finally cleared in an intraday auction. The setting differs to our model with respect to the type of players. In Allaz and Vila (1993) both players have increasing marginal costs of production and no uncertainty associated with their level of production. Similar to this, Saloner (1987) developed an extension of the classical Cournot one-shot duopoly to a model with two production stages in which the market clears only once after the second stage. In this framework Saloner showed the existence of a unique Nash-Cournot equilibrium under the possibility of a second stage response action. Nevertheless, the model does not account for different player types or uncertainty of production. Twomey and Neuhoff (2010) consider the case of renewable and conventional producers that are competing in electricity markets. They analyze the case when conventional players use market power to increase prices. With their model they are able to show that renewable producers are worse off in settings with market power. However, they do not consider the strategic behavior of renewable producers and abstract from uncertainty.

The other branch of relevant literature covers optimal bidding strategies under uncertain production of one single player. Many papers in this field analyze numerical models from a price taker perspective and focus on wind power producers. For instance, Botterud et al. (2010) numerically analyze the optimal bidding for a wind power producer in a two-stage market (day-ahead and real time market) under certain risk assumptions.³ They find that the optimal bid on the day-ahead market depends on risk behavior and the respective market prices. Furthermore, it tends towards the expected production as a deviation penalty

³Here, *real time market* means the ancillary grid services for balancing supply and demand.

between the day-ahead and the real time market is introduced. Botterud et al. (2010) focus on one specific wind power producer without considering the implications of adjusted bidding strategies on the market equilibrium. Those effects can influence the optimal bidding strategy as we will show in the investigated oligopoly cases. Further literature similar to Botterud et al. (2010) can be found in Bathurst et al. (2002), Usaola and Angarita (2007), Pinson et al. (2007), and Morales et al. (2010).

To the best of our knowledge there is no literature that focuses on competition between producers with marginal costs of zero and uncertain production levels on one side and producers with strictly increasing marginal costs of production on the other side in a generalized model framework. With this paper we intend to close this gap in the literature and facilitate the discussion about optimal market designs for short-term electricity markets.

The latter of the paper is organized as followed: In section 2 we develop the basic model framework. Section 3 analyzes the Cournot competition and the basic model is applied to the monopolistic as well as the oligopolistic case. Section 4 focuses on the impact of flexibility constraints for conventional power technologies. Section 5 sheds light onto the incentives of renewable producers to withhold capacity in the intraday market. In section 6 we show the effects on welfare, producer and consumer surplus by numerical examples. In section 7 we conclude our results and discuss possible policy implications.

2. The Model

We consider three players that interact at two stages in the wholesale market for electricity, namely consumers, conventional producers (c) and renewable producers (r). The consumers are assumed to behave completely price-inelastic in the short-run and demand a quantity D . The demand of consumers is satisfied already in the first stage, since we assume consumers as being risk-averse. On the supply side, we distinguish between conventional producers and renewable producers.

Conventional producers in the model are represented as competitive fringe. They are able to produce electricity at total costs of $C(q_c)$ where q_c is the quantity produced. These quantities are sold into the market at a uniform price of the marginal production costs. The conventional producers also act as market makers which means they always satisfy the residual demand in both stages⁴.

Renewable producers produce electricity at zero marginal costs. Their final production level Q is uncertain in the first stage with the probability density function $f(Q)$. The uncertainty about the production

⁴Conventional producers have a strong incentive to sell their production in a market as long as the price is above their marginal production costs. This makes it seem to be a reasonable assumption that conventional producers always satisfy the residual demand when prices are above or equal to their marginal generation costs.

level resolves over time (from stage 1 to 2; cf. Figure 1).

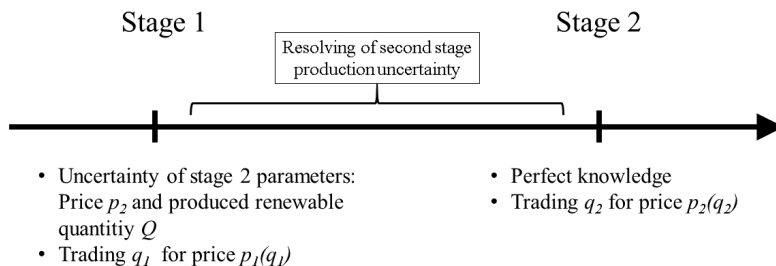


Figure 1: Basic two-stage model

Throughout our analysis we assume the probability function $f(Q)$ as symmetric. In our view this assumption is reasonable, since well behaved forecasting models should be able to produce a symmetric distribution.⁵

Conventional and renewable producers can trade electricity in the two stages ($t = 1$ and $t = 2$). For the conventional producers quantities are denoted by q_{ct} and for the renewable producer by q_{rt} . Here, we allow for q_{ct} and q_{rt} to be positive or negative. This allows producers, e.g. to sell too much production in the first stage and buy back quantities in the second stage. As already mentioned, we assume the demand of consumers (D) to get satisfied in the first stage. In the second stage, conventional and renewable producers can adjust their positions, e.g. conventional producers buy quantities from the renewable producer in order to replace their more expensive conventional production with renewable electricity. In this setting it is unclear what quantity (q_{r1}^* and q_{r2}^*) is optimal to trade in the first and second stage for the renewable producer.

The market clearing conditions at both stages can be written as

$$\text{Stage 1:} \quad D = q_{c1} + q_{r1} \quad (1)$$

$$\text{Stage 2:} \quad D = q_{c1} + q_{c2} + q_{r1} + q_{r2}. \quad (2)$$

The conventional producers produce electricity based on linear increasing marginal cost functions in both stages. In reality, marginal costs of production may change with time, which can have multiple reasons. In electricity markets this may be due to technical constraints of power plants (start-up costs, minimum load

⁵Of course the distribution would not be symmetric in cases where production is expected to be extreme in the sense of a very low (close to zero) or very high (close to the capacity limit) production. Further information on wind forecasts and uncertainty can be found in Zhang et al. (2014).

restrictions or partload-efficiency losses) or due to transaction costs of participants that do not engage in short-term trading in short intervals before production.

We account for a change in production costs by considering two different marginal cost functions $MC_1(q)$ and $MC_2(q)$ with different inclinations a_1 and a_2 . Since the number of flexible power plants is lowered the closer we get to physical delivery, a_2 has to be greater than a_1 . As explained before, the supply curve may change due to two reasons. First, technical constraints of power plants which are not able to adjust their power output in short intervals before production can lead to reduced supply. Second, there may be transaction costs for power plant operators to participate in the intraday market which is why supply is also reduced. This approach is similar to Henriot (2014).

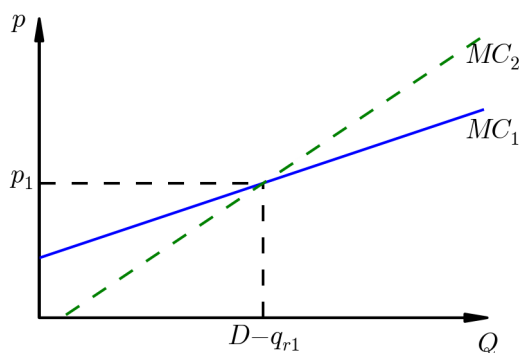


Figure 2: Marginal cost function in the first and second stage

For the analysis we have to define the properties of the marginal cost function in the second stage. Besides the increase of the slope to a_2 , the whole curve needs to cross the market clearing point from the first stage. Because if there are no adjustments in quantities the price of the first and second stage are identical. Thus the marginal cost function for the second stage can be obtained by a rotation around the market clearing point from stage 1 (cf. Figure 2). This means an increase in production comes at additional costs and a decrease in production at less savings of production costs. In combination with the market clearing conditions, this leads to the following two equations for price formation in the two stages:

$$p_1(q_{r1}) = a_1(D - q_{r1}) + b_1 \quad (3)$$

$$\mathbb{E}[p_2(q_{r1}, q_{r2})] = a_2(D - q_{r1} - q_{r2}) + b_1 + (D - q_{r1})(a_1 - a_2). \quad (4)$$

In a next step, we will derive the respective profit functions for the conventional and renewable producer.

The conventional producer's profit function is defined as

$$\Pi_c(q_{c1}, q_{c2}) = p_1 q_{c1} + p_2 q_{c2} - C_1(q_{c1}) - C_2(q_{c1} + q_{c2}) + C_2(q_{c1}). \quad (5)$$

Revenues in both stages are the products of the respective prices and quantities. Production costs depend on the power plants utilized for production. Since the marginal costs of production may change with time, the costs consist of the sum of quantities planned for production in each stage.

The profit function of the renewable producer

$$\Pi_r(q_{r1}, q_{r2}) = p_1 q_{r1} + p_2 q_{r2} \quad (6)$$

consists of the quantities traded at the respective prices in the first and second stage without associated production costs.

We are able to show how competition between renewable producers and conventional producers can be modeled by applying this framework to different settings. In this paper, we will consider three cases:

- Competition in the first stage with identical cost functions: $q_r = Q$, $a_1 = a_2 = a$
- Competition in the first stage with changing cost functions: $q_r = Q$, $a_2 > a_1$
- Competition in the first and second stage with changing cost functions: $q_r \leq Q$, $a_2 > a_1$.

3. Cournot Competition of Renewable Producers Competition in the First Stage with Identical Cost Functions

Throughout this paper we focus on a linear marginal cost function which can be regarded as the simplest case. In this section, we will first give an intuition for the results of the model based on the simple case of identical cost functions and a renewable monopolist who acts strategically in the first stage. For this part of the analysis, we assume that the renewable producer sells the complete remaining production in the second stage, meaning $q_r = Q$.⁶ In a next step, we will extend the analysis from the renewable monopoly to an oligopoly.

We can parametrize the linear marginal cost function $MC(q_c) = a q_c + b$ by the gradient $a \in \mathbb{R}_{\geq 0}$ and an offset $b \in \mathbb{R}_{\geq 0}$ with variable $q_c \in \mathbb{R}_{\geq 0}$ as the produced quantity from conventional producers. The prices

⁶Note that we assume additionally $Q \leq D$. If $Q > D$ and renewable producers have to sell their whole production in stage 2, we would force producers to bid negative prices. In such cases, we would expect that renewable producers reduce their production to avoid too low prices, e.g. below 0. This will be discussed in section 5 in which we extend the model and allow for $q_r \leq Q$.

of the linear marginal cost function can be defined as follows:

$$p_1(q_{r1}) = a(D - q_{r1}) + b \quad (7)$$

and

$$p_2(Q) = a(D - Q) + b. \quad (8)$$

3.1. Renewable Producer Monopoly

First, we look at the simple case in which all renewable production is traded by one firm. From economic literature it is well known that under the assumptions of Cournot competition, the monopolist has incentives to deviate from welfare optimal behavior in order to maximize its own profits. In our sequential market setting, this can be observed as well. By proposition 1 we show that the optimal bidding strategy for a renewable producer under a monopoly is to bid half the expected production in the first stage.

Proposition 1. *The profit maximizing quantity for a renewable monopolist is $q_{r1}^* = \frac{\mu_q}{2}$ with μ_q the expected renewable production.*

Proof. The basic profit function of a renewable producer in our theoretical model framework is described in (6). For identical marginal cost functions, we derive the following expected profit function

$$\mathbb{E}[\Pi_r(q_{r1})] = q_{r1} (a(D - q_{r1}) + b) + \int (Q - q_{r1})f(Q) (a(D - Q) + b) dQ. \quad (9)$$

Where the first derivative results in

$$\frac{d}{dq_{r1}} \mathbb{E}[\Pi_r(q_{r1})] = a(D - q_{r1}) + b - aq_{r1} - Da \int f(Q) dQ + a \int Qf(Q) dQ - b \int f(Q) dQ. \quad (10)$$

Since $f(Q)$ is symmetric and the marginal cost function is linear, we can further simplify the expected profit function by the following substitutes:

$$\text{Expected value for } Q: \quad \int Qf(Q) dQ = \mu_q \quad (11)$$

$$\text{Distribution function has a total probability of 1:} \quad \int f(Q) dQ = 1 \quad (12)$$

This leads to the simplified necessary condition for the profit maximizing quantity q_{r1}^* as

$$\frac{d}{dq_{r1}} \mathbb{E}[\Pi_r(q_{r1})] = -Da + a\mu_q - aq_{r1} + a(D - q_{r1}) \stackrel{!}{=} 0. \quad (13)$$

Now we can solve this equation for q_{r1} which results in the profit maximizing quantity

$$q_{r1}^* = \frac{\mu_q}{2}. \quad (14)$$

In order for this being a maximum the second derivative has to be negative. This can easily be checked by calculating

$$\frac{d^2}{dq_{r1}^2} \mathbb{E}[\Pi_r(q_{r1})] = -2a. \quad (15)$$

Since a is defined as the slope of the marginal cost function and is positive by definition, $q_{r1}^* = \frac{\mu_q}{2}$ indeed describes the profit maximizing quantity for the renewable producer. \square

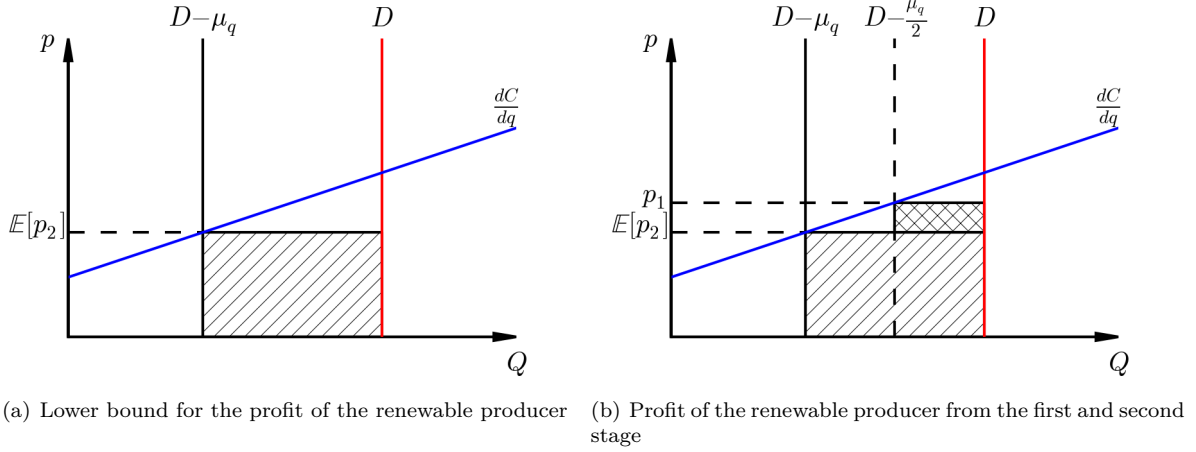


Figure 3: Optimal quantity of the renewable monopolist

The motivation of the renewable producer to bid half her expected quantity in the first stage becomes clear by analyzing Figure 3. Since we consider a linear marginal cost function, we can abstract from the uncertainty in renewable production $f(Q)$ and only consider the expected production μ_q . The profit of the renewable producer can be split into two parts. One part stems from selling the expected production into the market, as can be seen in Figure 3(a) (single hatched area). This part can be considered as a lower bound to the profit of the renewable producer and does not depend on the strategy of the renewable producer because she has to sell all production to the market in the final stage. The resulting price in the second stage is thus given by $\mathbb{E}[p_2]$. The second part of the renewable producer profit can be obtained by selling a quantity forward in the first stage at a price p_1 . In order to increase her profit, the quantity in the first stage needs to be between $D - \mu_q$ and D to obtain a higher price compared to $\mathbb{E}[p_2]$. Since the marginal cost function is linear and we have a monopolist selling forward, it is optimal to sell half her expected production because it maximizes the additional profit in Figure 3(b) (cross hatched area).

Based on this analysis, we can also draw additional conclusions for the case of a monopolist trading in multiple stages. The triangle in Figure 3(a) can be considered as the maximum profit which can be gained by selling the production of the renewable producer. When the renewable producer is able to sell this production in multiple stages, it would be optimal to sell it little by little in order to maximize her profit. This means prices in the multiple stages would be declining until the price of $\mathbb{E}[p_2]$ is reached in the final stage. Thereby, the renewable producer would be able to further increase the profit. This holds also for

conventional producers, who would be able to obtain a higher profit in the first stage, where they can sell a larger quantity at a higher price. On the other hand, consumer surplus would be lowered due to higher prices.

This leads us to the conclusion that with a renewable monopolist, different market designs can have a large impact on distributional effects between producers and consumers. Consumers lose if producers trade electricity in multiple stages. Thus, continuous trading in short-term markets lowers consumer surplus. From the view of consumers, a few separate auctions should be preferred to a continuous auction since this limits strategic behavior of a renewable monopolist.

3.2. Renewable Producer Oligopoly

In this section, we extend the monopoly case to the case of multiple symmetric renewable producers that form an oligopoly. The symmetry implies that the renewable producers have perfectly correlated generation as well as forecast errors. The remaining approach and notation are similar to previous sections. As we learned from before, the conventional producer reacts to the decision of the renewable producers and can be considered as a price taker. So we can focus on the optimal quantities of the renewable producers. We still consider a linear marginal cost function $MC(q) = aq + b$ and define the players $i = 1, \dots, N$ with their corresponding quantities in stage 1 as q_{ir1} . Furthermore, we define the sum of the quantities of all players but i as $q_{-ir1} = \sum_{j \neq i} q_{jr1}$. We find that the optimal bid of a renewable producer in the first stage is still driven by strategic behavior but tends towards the expected production level as the number of producers increases.

Proposition 2. *The optimal quantity traded in the first stage for each player is $q_{ir1}^* = \frac{1}{N+1}\mu_q$ with μ_q the total expected renewable production of all players.*

Proof. See Appendix A. □

As a direct implication from the optimal first stage bid we see that for the linear marginal cost function, the optimal strategy is still independent of the gradient or the uncertainty of production.

Corollary 1. *The profit maximizing traded quantity in stage 1 of the above setting is identical for all players. Furthermore, q_{ir1}^* is independent of the steepness $a \in [0, \infty)$ of the marginal cost function, the offset $b \in [0, \infty)$ of the marginal cost function, and the probability distribution function $f(Q_i)$.*

According to Proposition 2 it is optimal for renewable producers to always trade less than the expected production in the first stage since this maximizes their profits. The overall quantity tends towards the overall expected quantity as the number of players increases.⁷

⁷Note that, for the moment, we assumed a linear marginal cost function which does not change between the first and the second stage.

In stage 1, this leads to an overall traded quantity of renewable production of

$$q_{r1} = \sum_{j=1}^N q_{jr1} = \frac{N}{N+1} \mu_q \quad (16)$$

with $\mu_q := \sum_{j=1}^N \mu_{jq}$. In two sequential markets, renewable producers have an incentive to trade less than the total expected renewables production in the first stage. The more players enter the market the stronger the competition and thus the traded amount in the first stage tends towards the expected production. Our results of the first stage show that, under the described setting, a renewable producer acts exactly as predicted in a standard one-shot oligopolistic Cournot game.

4. Flexibility and its Role in Short-term Markets Competition in the First Stage with Changing Cost Functions

In this section we shed light on the implications of changing cost functions in short-term markets. As mentioned before, this can happen for essentially two reasons. One reason is that not all conventional power plants are flexible enough to adjust their production capacity in stage 2 in the short run. The second reason is that there can be transaction costs for power plant operators associated with the trading in the intraday market.

The difference between the cost function of the first and second stage has implications for the optimal quantity of the renewable producers in the first stage, which we analyze here in more detail. The nomenclature corresponds to the previous sections.

Proposition 3. *The optimal quantity traded in the first stage for each renewable player is $q_{ir1}^* = \frac{1}{N+1} \mu_{iq} (N + 1 - \frac{a_1}{a_2})$, with the ratio $\frac{a_1}{a_2}$ representing the degree of flexibility of the supply side in both stages.⁸*

Proof. In a first step we will derive the optimal quantity of a player i who competes against $N - 1$ identical players⁹. According to the setup, the prices in the first and second stage can be defined as:

$$p_1(q_{ir1}, q_{-ir1}) = a_1(D - q_{ir1}) - q_{-ir1} + b_1 \quad (17)$$

$$p_2(q_{ir1}, q_{-ir1}) = a_2(D - Q_i N) + b_1 + (a_1 - a_2)(D - q_{ir1} - q_{-ir1}). \quad (18)$$

Again, we can define the expected profit function for player i , take the first derivative and integrate over f_i (which is assumed as being identical for all players). Setting the first derivative equal to zero leads us to the necessary condition for an optimal quantity:

$$-a_1 \mu_{iq} + a_2 \mu_{iq} N + a_2 \mu_{iq} - a_2 q_{-ir1} - 2a_2 q_{ir1} \stackrel{!}{=} 0 \quad (19)$$

Under the assumption that all players are identical we can set $q_{-ir1} = (N - 1)q_{ir1}$ and solve for q_{ir1} which leads to:

$$q_{ir1}^* = \frac{1}{N+1} \mu_{iq} \left(N + 1 - \frac{a_1}{a_2} \right). \quad (20)$$

⁸Small values of $\frac{a_1}{a_2}$ represent a very inflexible supply side in the second stage.

⁹The sum over all other players is still denoted by the quantity $q_{-ir1} = \sum_{j \neq i} q_{jr1}$

The second derivative of the expected profit function is negative, which proves q_{irr1}^* being a maximum for the expected profit function. \square

This means that all renewable producers together submit a quantity of

$$q_{r1}^* = \mu_q - \mu_q \frac{a_1}{a_2(N+1)} \quad (21)$$

in the first stage (with $a_2 > a_1$).

From Equation (21) we can conclude the following: (1) q_{r1}^* increases if conventional producers are less flexible ($a_2 \gg a_1$); (2) q_{r1}^* increases with an increasing number of renewable producers N . For a perfectly competitive market (with $N \rightarrow \infty$) it is optimal for each player to trade its share of the total expected quantity in the first stage.

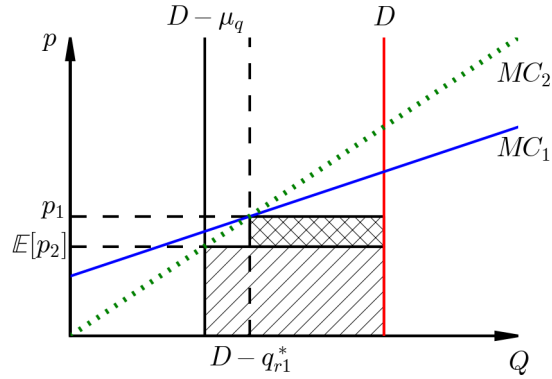


Figure 4: Profit of a renewable monopolist facing inflexible conventional producers

By looking at the example of a renewable monopolist in Figure 4, we can get a deeper understanding of the motives for a renewable producer who faces a market with inflexible conventional producers. As explained before, the marginal cost curve for the second stage rotates around the market clearing point of the first stage. The total production of the renewable producer that needs to be sold after both stages however does not change. Thus, the renewable producer has to decide what quantity to sell at a respective price in the first stage and sell the remaining quantity at a lower price in the second stage. The price is lower in the second stage due to the additional renewable quantities that are sold by the renewable producer. Basically, in Figure 4, the sum of the cross hatched area and the single hatched area needs to be maximized. The renewable producer is able to maximize both areas by a parallel shift of the marginal cost function for stage 2 (green dotted line). This means, the renewable producer has to optimize the quantity in the first stage in such a way that the profit from both stages is maximized. Summarizing, a more flexible power

plant fleet shifts the total optimal first stage bidding quantity of a renewable producer towards the expected production.

The described effects on the optimal quantity hold true for different numbers of renewable producers and different degrees of flexibility. This is shown exemplarily in Figure 5. Here, the optimal quantity converges more slowly to the expected production in the perfectly flexible case ($\frac{a_2}{a_1} = 1$) compared to a highly inflexible conventional power plant mix ($\frac{a_2}{a_1} = 4$). An increase in the number of renewable producers leads to a similar effect of a higher overall renewable quantity in the first stage.

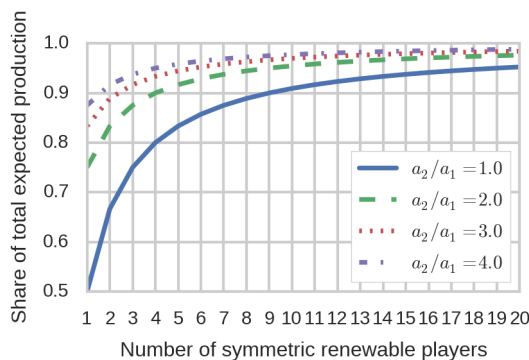


Figure 5: Optimal renewable quantity q_{r1} dependent on the number of players N and the ratio a_2/a_1

5. Incentives of Renewable Producers to Withhold Production Competition in the First and Second Stage with Different Cost Functions

In this section we extend the analysis of strategic competition in the first stage by investigating the case in which renewable producers are allowed to withhold production in the second stage. Therefore, we relax the assumption that the renewable producer needs to sell all her realized production in the second stage. This means $q_{r1} + q_{r2} \leq Q$ instead of $q_{r1} + q_{r2} = Q$. We still assume that renewable producers strictly avoid being short after stage 2, i.e. selling more production than they produce. The rationale is that high financial penalties need to be paid in case of an imbalance. All other model assumptions stay the same.

The motivation for the relaxation of the second stage restriction to sell the whole production is threefold. First, we note that, in general, it is technically possible to reduce production for renewable producers. This happens for photovoltaic in critical grid situation if the voltage level extends a critical value (automatic shut down around 50.2 Hertz) or for wind turbines during storms. Second, a reduced production could be economically profitable in specific situation. Especially if prices are negative or, like in the investigated case, if market prices could be increased profitably by withholding production. Third, market manipulation by a

withhold of renewable production is not easy to prove by the regulator. It is hard to detect whether a wind turbine does not produce due to maintenance, local wind conditions or strategic production withholding.

5.1. Model Adjustment

We extend the model with cost functions by replacing the constraint $q_{r1} + q_{r2} = Q$ with $q_{r1} + q_{r2} \leq Q$. Based on this model we obtain the following results.

Proposition 4. *If renewable producers are allowed to withhold production, they only withhold production after the second stage if the expected production of all producers is high compared to the demand D , i.e. if $\mu_q > \frac{a_2 N(N+1)}{a_2(N+1)^2 - a_1 N} D + \frac{a_2(N+1)}{a_1(a_2(N+1)^2 - a_1 N)} b$. This means the expected renewable production needs to be at least $\frac{D}{2}$. Otherwise, renewable producers sell the total realized production into the market (same result as of proposition 3).*

Proof. We use the same model as in section 4 (and corresponding proposition 3). The only difference is the relaxed constraint $q_{r1} + q_{r2} = Q$ by $q_{r1} + q_{r2} \leq Q$. This allows the renewable producer to withhold production and to increase prices in the second stage. Since we adjusted an equality constraint by an inequality constraint, we face now a convex optimization problem with inequalities and can use the Karush-Kuhn-Tucker (KKT) conditions to solve it. The full proof can be found in the appendix. \square

The main finding is that renewable producers have an incentive to withhold production after the second stage only if the (expected) production exceeds a threshold value which is at least $\frac{D}{2}$ (but dependent on a_1, a_2, b and N).¹⁰ The exact threshold value is

$$Q_{threshold} := \frac{a_2(N+1)}{a_2(N+1)^2 - a_1 N} D + \frac{a_2(N+1)}{a_1(a_2(N+1)^2 - a_1 N)} b. \quad (22)$$

As long as the (expected) production is below this threshold, the renewable producers will sell their total realized production in the second stage. Nevertheless, the production is split between first and second stage to increase profits. By analyzing this threshold we find the following

- $Q_{threshold}$ is increasing in N : The more producers exist, the higher the threshold. Therefore, more competition between renewable producers limits the incentive for renewable producers to withhold quantities in the second stage.
- $Q_{threshold}$ is decreasing in a_2 (with a_1 fixed): The more inflexible the power plant fleet, the lower is the threshold. Therefore, renewable producers start to withhold production at a lower level of expected renewable production.
- $Q_{threshold}$ converges to $\frac{N}{N+1} \left(D + \frac{b}{a_1} \right)$ for $a_2 \rightarrow \infty$ but is strictly above $\frac{D}{2}$.

¹⁰In stage 1, the expected production is the relevant quantity while in stage 2 the realized production is the relevant quantity. If both, expected and realized production, deviate from each other, it is possible that the renewable producers pursue a different strategy in each stage.

To sum up, renewable producers only have an incentive to withhold quantities in situations with very high renewable generation compared to the demand. Additional renewable producers as well as more flexible conventional producers increase the threshold ($Q_{threshold}$) to withhold production quantities.

6. Prices, Welfare, Producer Surplus and Consumer Surplus

Trading in the day-ahead and intraday market has implications for overall welfare, producer surplus and consumer surplus. So far, we focused on the quantities of the renewable producers that maximize their respective profits. They determine the quantities that are traded by the conventional producers and thereby the prices in both stages. In order to disentangle the effects on overall welfare, producer and consumer surplus, we will first analyze the effects on prices in the two stages.

Since we found in section 5 that renewable players only withhold production at very high production levels compared to demand D , we focus on the case in which renewable producers sell all their production after stage 2 (the case $q_{r1} + q_{r2} = Q$).¹¹

6.1. Prices and the Role of Arbitrageurs

By plugging in the optimal quantity from Equation (20) into the price equations for the case with flexibility constraints (Equation (3) and (4)) we obtain the following prices:

$$p_1 = Da_1 + b_1 - \frac{a_1}{a_2} \frac{\mu_q}{N+1} (a_2(N+1) - a_1) \quad (23)$$

$$\mathbb{E}[p_2] = Da_1 + b_1 - \frac{a_1}{a_2} \frac{\mu_q}{N+1} (a_2(N+2) - a_1). \quad (24)$$

From these two equations we can already see that the price in the first stage is higher than in the second stage. This becomes obvious by taking the difference between the two prices:

$$p_1 - \mathbb{E}[p_2] = \frac{a_1 \mu_q}{N+1}. \quad (25)$$

We can observe the following implications: First, the price difference between stage 1 and 2 is independent of the change in the slope of the marginal cost function (a_2). The renewable producers choose their quantity dependent on the slope (a_2). This has an effect on the absolute prices in the two stages but the price delta stays constant. Second, with a higher overall expected production from renewables (μ_q) also the price difference increases. The quantity that is withheld from trading in the first stage increases with the expected production and, thereby, the price difference increases. Third, the price difference decreases with

¹¹For a realistic number of renewable players $N > 5$ and an arbitrary ratio of a_2 to a_1 , the threshold $Q_{threshold}$ is at least $0.85D$.

an increasing number of renewable producers (N). In a perfectly competitive market (with $N \rightarrow \infty$), prices in both stages are equal. As we can observe in Figure 5, the quantity in the first stage tends towards the overall expected quantity and hereby prices in both stages converge.

Based on the price difference in both stages one could suspect arbitrageurs to be entering the market. By obtaining a short position in the day-ahead market and adjusting their position in the intraday market, they would be able to make a profit. The optimal strategy of an arbitrageur is therefore identical with the strategy of the renewable players. The only difference is that arbitrageurs do not necessarily own production assets. Each additional arbitrageur that would enter the market can nevertheless be regarded as an additional renewable player. This would in turn decrease the price difference between the day-ahead and intraday market (cf. Figure 6).

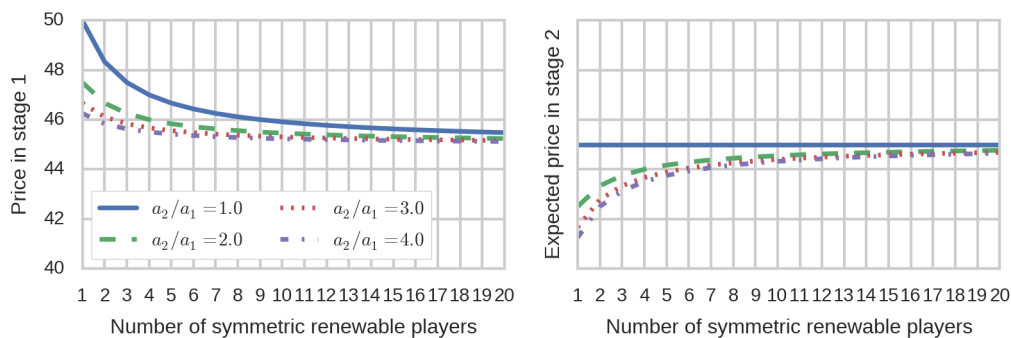


Figure 6: Prices in the two stages for an example with $D = 70$, $\mu_q = 20$, $\sigma_q = 5$, $b_1 = 20$ and $a_1 = 0.5$

Still, electricity markets have some unique features that may prevent arbitrageurs from engaging in short-term electricity markets. First, the assets that are traded are not only financial but physical obligations to produce and deliver electricity. Therefore, some short-term market platforms restrict the participation to firms with physical production assets. This prevents for example banks from entering these markets. Second, there may be information asymmetries between renewable producers and arbitrageurs that may be hard to overcome. For example renewable producers can be assumed as having better knowledge about the expected production level of their assets. For the following discussions we will thus not focus on the case of additional arbitrageurs entering the market. Nevertheless, the implications of arbitrageurs entering the market can be observed implicitly by considering an increase in the number of renewable players (N).

In order to gain a deeper understanding of the effects from changing cost functions and increased competition on prices, we plot this relationship in Figure 6 for an exemplary case. The direction of the effects

will stay the same for arbitrary a_1, a_2 with $a_2 \geq a_1$ and arbitrary D, Q and b with $(D - Q)a_1 + b \geq 0$ ($Q \sim \mathcal{N}(\mu_q, \sigma_q)$).

In Figure 6, we chose the values such that one can easily find similarities to the German electricity market. A demand D of 70 GW can be observed during peak times, where also an expected renewable production μ_q of 20 GW is quite common. Furthermore the parameters of the marginal cost function were chosen such that they represent common price levels.¹²

We can see that the prices in stage 1 and 2 converge to the same value with an increasing number of players. This benchmark is set by the perfectly flexible case ($\frac{a_2}{a_1} = 1$), where the price in the second stage stays constant. In the next sections we will analyze the effects on producer surplus, consumer surplus and overall welfare.

6.2. Producer Surplus

The producer surplus is defined as the sum of the renewable producer surplus and the conventional producer surplus. For the case with a changing marginal cost function, the conventional producer surplus can be defined as

$$\mathbb{E}[\Pi_c(q_{r1})] = p_1(D - q_{r1}) + p_2 \int (q_{r1} - Q)f(Q)dQ - C_1(q_{r1}) - \int C_2(q_{r1})f(Q)dQ. \quad (26)$$

It is the difference between the income from sold quantities in stage 1 and 2 and the associated costs with the production of electricity.

The first stage costs C_1 in our model depend on the quantities offered by the renewable producers q_{r1} . We can thus obtain the costs in the first stage by integrating over the marginal cost function MC_1

$$C_1(q_{r1}) = \frac{1}{2}a_1(D - q_{r1})^2 + b_1(D - q_{r1}). \quad (27)$$

The formulation is more complex for the costs that are associated with the second stage of production. First, it depends on the quantity that is traded in the first stage by the renewable producer q_{r1} . Second, it depends on the realization of the final renewable production Q . In the first stage, the conventional producers plan to produce a certain quantity $D - q_{r1}$. In the second stage, this quantity has to be adjusted to meet the total residual demand of $D - Q$. This means if the renewable production turns out to be higher than the traded quantity in the first stage, the conventional producers need to reduce their planned production and can buy back quantities at a lower price. Meanwhile the slope of the cost function has changed from a_1

¹²Of course a linear marginal cost function is a crude assumption in this case, but it allows us to show the overall effects.

to a_2 . This leads us to the following expected cost function for the second stage:

$$\mathbb{E}[C_2(q_{r1})] = \int \int_{q_{r1}}^{D-Q} (a_2 q_{c2} + (a_1 - a_2)(D - q_{r1}) + b_1) dq_{c2} f(Q) dQ \quad (28)$$

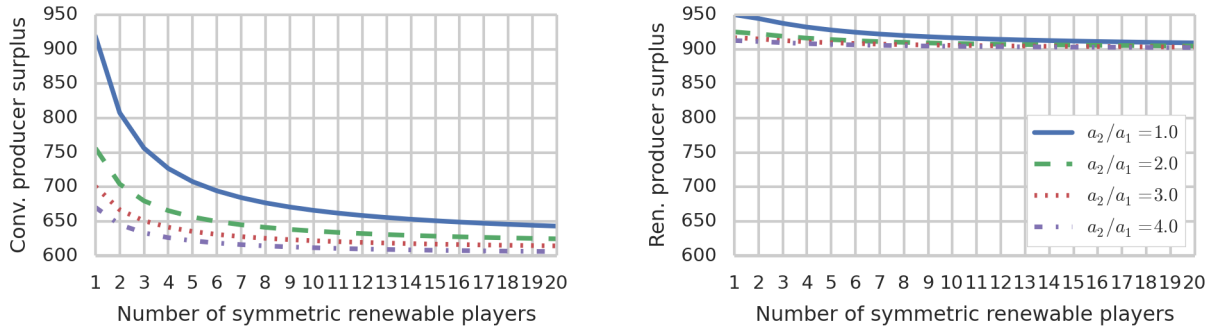
$$= (Da_1 - a_1 q_{r1} + b_1)(\mu_q - q_{r1}) + a_2 q_{r1}(\mu_q - \frac{q_{r1}}{2}) - \frac{a_2 n^2}{2} (\mu_{iq}^2 + \sigma_{iq}^2). \quad (29)$$

What is especially noticeable in this equation, is that for the first time in our analysis also the standard deviation (σ_{iq}) of the expected renewable production plays a role. The reason for this lies in the non-linear cost function of the conventional producers. Here, deviations from the expected value are not multiplied by a linear curve and weighted equally but weighted by the non-linear function. This is why the standard deviation plays an important role. By inserting Equation (27) and (29) in (26), we obtain the total conventional producer surplus.

In the same way, we can also derive the producer surplus for the renewable producers.

$$\mathbb{E}[\Pi_r(q_{r1})] = p_1 q_{r1} + \int p_2(Q - q_{r1}) f(Q) dQ \quad (30)$$

By plugging in the results from Equation (20) it is possible to quantify the renewable and conventional producer surplus. We plot this for an exemplary cases in Figure 7.



(a) Expected conventional producer surplus

(b) Expected renewable producer surplus

Figure 7: Expected producer surplus for an example with $D = 70$, $\mu_q = 20$, $\sigma_q = 5$, $b_1 = 20$ and $a_1 = 0.5$.

As we could already see from Figure 6, prices in the first stage decrease with an increase in competition or a less flexible supply curve. At the same time prices in the second stage increase. This results in both, a dampening and an increasing effect on producer surplus. From Figure 7 we can observe that the decreasing effect of the first stage outweighs the increasing effect in the second stage. Overall, we see that the producer surplus decreases with the number of renewable producers N and with a less flexible power

plant mix. Especially the decrease in conventional producer surplus is noticeable. For renewable producers the decrease in surplus is not as prominent, since they are able to reduce the effects by adjusting their optimal quantity q_{r1} . For example the overall quantity traded by renewable producers (q_{r1}^*) is increased when more renewable producers compete in the first stage. Also a less flexible power plant mix leads to a higher optimal quantity for renewable producers in the first stage (cf. Figure 5).

6.3. Consumer Surplus

In our model, consumers are represented as being completely inelastic in their demand behavior. This means that in theory consumer surplus is infinite. We are thus not able to quantify the absolute value of the consumer surplus. Nevertheless, changes in consumer surplus can be translated into changes in consumer expenses (CE). Because the demand is already satisfied in the first stage and it does not change over time, we can quantify the consumer expenditures as $p_1 D$. The consumer expenses multiplied by -1 give us the respective change in consumer surplus. By plugging in the price formulation for the first stage from Equation (3), we get

$$CE = -D^2 a_1 - D b_1 + D \frac{a_1}{a_2} \frac{\mu_q}{N+1} (a_2(N+1) - a_1). \quad (31)$$

We can now compare the consumer expenses for the different combinations of N and a_2/a_1 . As explained before, we are not able to quantify the consumer surplus in absolute terms and need to define a reference point for comparison. The consumer surplus is the lowest for the case of a renewable monopolist and perfectly flexible conventional producers. This is why we choose this setup as a reference point for our comparison.

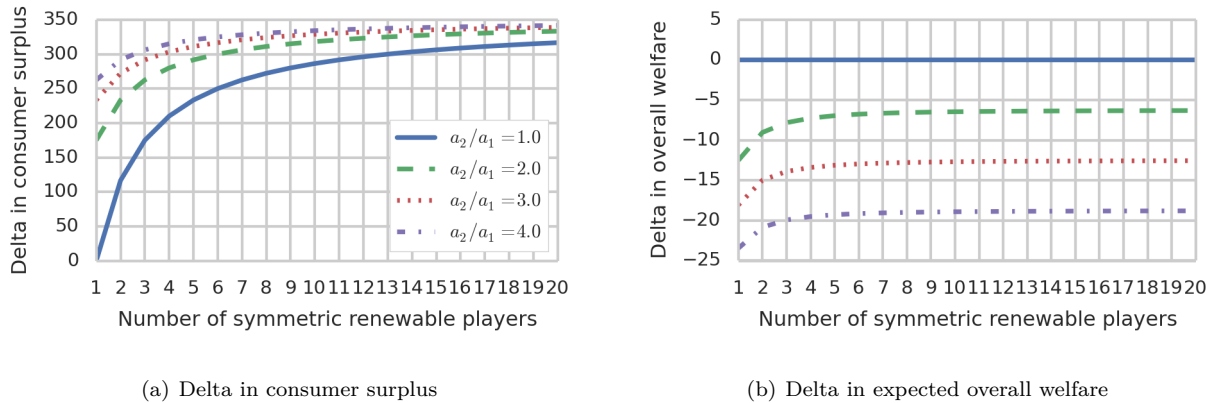


Figure 8: Delta in consumer surplus and expected overall welfare for an example with $D = 70$, $\mu_q = 20$, $\sigma_q = 5$, $b_1 = 20$ and $a_1 = 0.5$

As one could already expect from decrease in prices with an increasing number of players in Figure 6, the consumer surplus increases with the number of players. What may be counter intuitive is that consumers can

profit from a less flexible power plant mix. The lower flexibility of conventional producers leads renewable producers to adjust their quantity, which has a price dampening effect for the first stage. Consumers can therefore profit from the lower prices in the first stage as it is shown exemplarily in figure 8(a).

6.4. Welfare

Combining the effects on producer and consumer surplus leads to changes in overall welfare. As we can only analyze differences in consumer surplus this also holds for the case of overall welfare. Again, we define the perfectly flexible case with a monopolistic renewable producer as a reference point for the analysis (cf. section 6.3). The difference in overall expected welfare to the monopolistic case can be defined as

$$\Delta\mathbb{E}[\mathbb{W}(q_{r1})] = -\Delta\mathbb{E}[CE(q_{r1})] + \Delta\mathbb{E}[\Pi_p(q_{r1})]. \quad (32)$$

In Figure 8(b) we can observe these effects on overall welfare. The overall welfare stays constant for the case of a perfectly flexible power plant mix. In this case, the demand is always satisfied at the same costs which does not lead to a change in overall welfare. Negative effects on overall welfare occur only if the total production costs for electricity increase, i.e. if conventional power producers are less flexible. Especially if the power plant mix is highly inflexible, as in the case with $\frac{a_2}{a_1} = 4$, it will lead to a substantial decrease in overall welfare. Generally we can observe two effects. First, the effect on welfare has a smaller magnitude than the isolated effects on producer surplus or consumer surplus. The increase in consumer surplus and decrease in producer surplus counteract each other and lead only to a slightly reduced effect on overall welfare. Second, the welfare is generally decreased in a setting with less flexible power plants.

In a last step, we analyze the effects of uncertainty on overall welfare. So far, we assumed the production of the renewable producer in the final stage to be forecasted with a standard deviation of $\sigma_q = 5$ in the numerical examples. Now, we assume that if forecasts are improved or trading time is delayed, the standard deviation decreases, as to Foley et al. (2012). A decrease in standard deviation could also be accomplished by delaying trading of the first stage (e.g. by trading in the evening of the day before physical delivery instead of at noon). We quantify the welfare effects by comparing them to the case with no uncertainty ($\sigma_q = 0$) and a perfectly competitive market ($q_{r1} = \mu_q$). From Figure 9 we can observe that a larger standard deviation results in welfare losses. From this we can conclude that it is desirable to increase the quality of forecasts or to change the timing of trading in order to increase overall welfare.

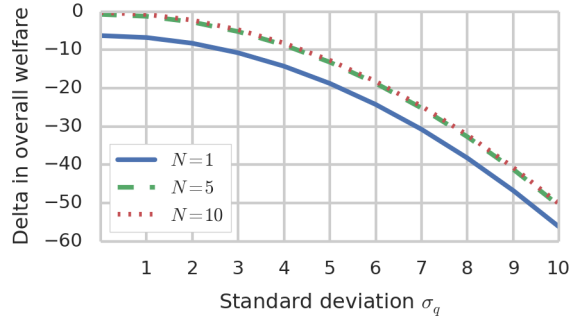


Figure 9: Delta in expected overall welfare for varying standard deviation of the forecast σ_q ($D = 70$, $\mu_q = 20$, $b_1 = 20$, $a_1 = 0.5$ and $a_2 = 1$)

7. Concluding Remarks

We derive the optimal quantities for renewable producers that are strategically selling their production in a two-stage game with uncertainty about production in stage 1 and knowledge about the realization of their production in stage 2. It is profit maximizing for renewable producers to bid less than their expected total quantity in the first stage, which we consider as the day-ahead market. Renewable producers are able to increase their profits by selling only part of their expected production in the first stage and thus raising the price in the first stage. The optimal quantity in the first stage tends towards the overall expected quantity with an increasing number of renewable producers. Conventional producers are considered as a competitive fringe that satisfies the residual demand in both markets. If conventional power producers are less flexible in their operation, renewable producers have a larger incentive to increase the traded quantity in the first stage. In general, prices in the first stage (day-ahead) are higher compared to the second stage (intraday), but with an increasing number of renewable producers or with arbitrageurs entering the market this difference decreases. In situations with very high production levels, that are at least able to serve half of the demand, renewable producers have an incentive to withhold production in the second stage. This effect is decreased by an increasing number of players but increases in a setting with low flexibility of conventional producers.

A reduced forecast uncertainty leads to an increase in overall welfare. This leads us to two conclusions. First, overall welfare can be increased by delaying the trade in the day-ahead market closer to the time of physical delivery. For example by shifting the auction from noon to the evening. Second, an increase in forecast quality has a positive effect on overall welfare.

Based on the results it becomes obvious that in a future electricity system with high shares of renewables,

regulators need to pay attention to the possible abuse of market power by large renewable producers. In situations with low liquidity and the absence of arbitrageurs this could lead to significant distributional effects and even welfare losses.

In our whole analysis, we assumed the generation of all renewable producers to be perfectly correlated, as well as their forecast errors. This is not the case in reality and could be further investigated. Additionally, it would be possible to quantify welfare implications of improved forecast quality and alternative market designs at concrete examples.

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Appendix

AppendixA. Proof of Proposition 2

Proof. Because all players are symmetric we can denote the total traded renewable production of all players in stage 1 by $q_{r1} = q_{ir1} + q_{-ir1}$ (where q_{-ir1} aggregates all players but not player i), the realized production in stage 2 by $Q = NQ_i$, and the expected quantity by $\mu_q = N\mu_{iq}$. With these definitions, Equation (7) and (8) still hold for the oligopoly case.

The profit function of renewable producer i can be derived by plugging in those values into

$$\Pi_{ir}(q_{ir1}) = p_1(q_{r1})q_{ir1} + p_2(D - Q)q_{ir2} \quad (\text{A.1})$$

so that the profit function results in

$$\Pi_{ir}(q_{ir1}) = (a(D - q_{ir1} - q_{-ir1}) + b)q_{ir1} + (a(D - NQ_i) + b)(Q_{ir} - q_{ir1}). \quad (\text{A.2})$$

Remember that $q_{ir2} = Q_i - q_{ir1}$ and that we assume Q_i to be uncertain. In order to derive the expected profit function we have to integrate for Q_i over the distribution $f(Q_i)$, where $f(Q_i)$ is the probability density function for Q_i . After taking the first derivative, setting it equal to zero and replacing the expected values (analogous to Equations (11) and (12)), we get the necessary conditions

$$\frac{d}{dq_{ir1}} \mathbb{E}[\Pi_{ir}(q_{ir1})] = a(\mu_q - q_{ir1} - q_{-ir1}) \stackrel{!}{=} 0 \quad (\text{A.3})$$

and the corresponding solution is

$$q_{ir1}^* = \frac{1}{2}N\mu_{jq} - \frac{1}{2}q_{-ir1} \quad (\text{A.4})$$

for $i = 1, \dots, N$.

In an equilibrium of identical players we have identical solutions which results in $q_{-ir1} = (N - 1)q_{ir1}$. With this, we derive

$$q_{ir1}^* = \frac{1}{2}N\mu_{iq} - \frac{1}{2}(N - 1)q_{ir1}^* \quad (\text{A.5})$$

$$\Leftrightarrow q_{ir1}^* = \frac{1}{N + 1}\mu_q. \quad (\text{A.6})$$

Because the second derivative of Equation (A.2) is negative, we found the profit maximizing quantity q_{ir1}^* \square

AppendixB. Proof of Proposition 4

Proof. As before, we assume N identical (symmetric) renewable producers. Let us define our inequality constraint for producer i by

$$g(q_{ir1}, q_{ir2}) := q_{ir1} + q_{ir2} - Q_i \leq 0 \quad (\text{B.1})$$

Then the Lagrange function is

$$\begin{aligned} L(q_{ir1}, q_{ir2}, \lambda) := & q_{ir1} (a_1 (D - q_{ir1} - q_{jr1} (N - 1)) + b) + \\ & q_{ir2} (a_2 (D - q_{ir1} - q_{ir2} - q_{jr1} (N - 1) - q_{jr2} (N - 1))) + \\ & q_{ir2} (b + (a_1 - a_2) (D - q_{ir1} - q_{jr1} (N - 1))) \int f_i(Q_i) dQ_i + \\ & \lambda (Q_i - q_{ir1} - q_{ir2}), \end{aligned} \quad (\text{B.2})$$

which is the corresponding profit function of the first and second stage minus the function g . The conditions of the KKT which need to be fulfilled are

$$\text{Stationarity: } \frac{\partial L}{\partial q_{irk}} = 0, \quad k = \{1, 2\} \quad (\text{B.3})$$

$$\text{Primal feasibility: } q_{ir1} + q_{ir2} \leq Q_i \quad (\text{B.4})$$

$$\text{Dual feasibility: } \lambda \geq 0 \quad (\text{B.5})$$

$$\text{Complementary slackness: } \lambda(q_{ir1} + q_{ir2} - Q_i) = 0. \quad (\text{B.6})$$

We need to consider two cases: $\lambda = 0$ or $q_{ir1} + q_{ir2} = Q_i$ (binding capacity constraint).

To case 1 ($\lambda = 0$):

From (B.3) we derive two equations which we can solve for q_{ir1} and q_{ir2} . Since we focus on symmetric probability distribution functions f_i for the renewable production, we can substitute $\int f_i(Q_i) dQ_i = 1$. Furthermore, due to symmetric renewable producers, we can plug in $q_{ir1} = q_{jr1}$ and $q_{ir2} = q_{jr2}$ for all renewable producers i and j . Therefore, the equilibrium solution aggregated for all identical renewable producers are

$$q_{r1}^* = \frac{a_2 N (N+1) - a_1 N}{a_2 (N+1)^2 - a_1 N} D + \frac{1}{a_1} \frac{a_2 N (N+1) - a_1 N}{a_2 (N+1)^2 - a_1 N} b \quad (\text{B.7})$$

$$q_{r2}^* = \frac{a_1 N}{a_2 (N+1)^2 - a_1 N} D + \frac{N}{a_2 (N+1)^2 - a_1 N} b. \quad (\text{B.8})$$

Note that the individual quantities are $q_{irk} = q_{rk}/N$ for $k = \{1, 2\}$.

Now, we can plug the optimal quantities into the equation of the investigated case, i.e. into $q_{r1} + q_{r1} < Q$. This gives us the threshold value above which the renewable producers start to withhold production to increase prices. The threshold is

$$Q_{threshold} := \frac{a_2 N (N+1)}{a_2 (N+1)^2 - a_1 N} D + \frac{a_2 (N+1)}{a_1 (a_2 (N+1)^2 - a_1 N)} b. \quad (\text{B.9})$$

If the overall expected renewable production μ_q exceeds this threshold, the renewable producers withhold production. Otherwise, the sold quantities are constraint and we are in case 2.

Note that the expected production has to reach a high level relative to the demand such that renewable producers withhold production. μ_q has to be at least $\frac{D}{2}$ (for the monopoly situation with a infinite inflexible power plant fleet) but increases with increasing number of players or more flexible power plant fleet (for a duopoly it is at least $\frac{2D}{3}$).

To case 2 ($q_{ir1} + q_{ir2} = Q_i$): This is the same case as shown in proposition 3. Therefore the optimal quantities for each individual renewable producer is

$$q_{ir1}^* = \frac{1}{N+1} \left(N+1 - \frac{a_1}{a_2} \right) \mu_{iq} \quad (\text{B.10})$$

$$q_{ir2}^* = \frac{1}{N+1} \frac{a_1}{a_2} \mu_{iq}. \quad (\text{B.11})$$

and for all renewable producers together are

$$q_{r1}^* = \mu_q - \frac{a_1}{a_2(N+1)} \mu_q \quad (\text{B.12})$$

$$q_{r2}^* = \frac{a_1}{a_2(N+1)} \mu_q \quad (\text{B.13})$$

if $\mu_q \leq \frac{a_2(N+1)}{a_2(N+1)^2 - a_1 N} D + \frac{a_2(N+1)}{a_1(a_2(N+1)^2 - a_1 N)} b$. Remember that $N\mu_{qi} = \mu_q$. This closes the proof. \square