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Pure Theory of the Federal Funds Rate

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Abstract: While the target federal funds rate represents a policy instrument, the effective federal funds rate is determined in a competitive interbank market. The paper proposes a theory of its determination. This yields a specific term structure of interest rates, an account of why the 'money multiplier' approach failed, and a demonstration that interest on reserves does not change bank incentives.

Keywords: Federal funds rate; term structure of interest rates; excess reserves; money multiplier; zero lower bound.

JEL-Classification: E43, E51, E58, G01

1. Introduction

The (effective) federal funds rate is an interest rate paid on overnight loans between banks. Banks demand such loans if they wish to increase their reserve balances, and supply them in the opposite case. Thus, the federal funds rate is determined in a highly competitive market. It is often close to the *target* federal funds rate that is set by the Federal Open Market Committee (FOMC). In the fall of 2008, however, the two rates differed markedly, as will be documented later. Moreover, the federal funds rate showed large deviations from the market interest rate on long-term government bonds during the last decade. Figure 1 illustrates that during 2004–2007, the federal funds rate climbed steadily from 1 to 5.3 percent whereas the yield of 10-years government bonds did not change much and eventually even fell below the federal funds rate, an incident known as the US bond yield conundrum. In 2008, the federal funds rate plunged to almost zero, where it has remained ever since, whereas the bond rate recovered in 2009.

Figure 1: Federal funds rate (dotted line) versus bond rate (solid line). *Notes*: Weekly data, retrieved July 2016 from research.stlouisfed.org/fred2, series DFF and WGS10YR.

Why can an interest rate on overnight loans exceed the long-term interest rate? Does the federal funds rate reflect policy decisions on the target rate, or does the FOMC set the target rate in accordance with the recent effective rate, Fama (2014:181), a practice sometimes referred to as "dirty targeting"? Why is a federal funds rate at the zero lower bound (ZLB) compatible with stable prices? Answers to such questions require a theory of the effective federal funds rate.

This paper proposes such a theory. In doing so, it breaks new ground and complements the relevant empirical literature, e.g. Goda (2013), Hamilton (1996), and Rudebusch (1995). Methodically, the present approach follows the accustomed theories of the banking firm. It is also related to Poole's (1968) classical treatment of bank reserve management. None of these contributions, however, endeavored to determine the federal funds rate as an equilibrium market rate. The proposed theory is

"pure" in the sense that it abstracts from many institutional details such as banks' intraday decision-making over stochastic cash flows.

As central bank policies are quite similar internationally, the results do not pertain exclusively to the Fed and the United States but should be of general interest. Among the main findings are a term structure of interest rates, an explanation of why the 'multiplier' approach failed in the aftermath of the financial crisis, and a brief analysis of interest on reserves.

Section 2 reviews the traditional approach. Section 3 augments this model with a market for federal funds that determines the equilibrium federal funds rate. Section 4 analyzes ZLB situations, and section 5 considers interest on reserves.

2. Traditional Model

The traditional model, which can be traced back to Phillips (1920), includes a central bank and a unit continuum of identical commercial banks. It disregards currency for simplicity. The central bank buys long-term bonds, $B_t^{cb} \geq 0,$ and creates a corresponding amount of reserves, R_t. Its balance sheet reads

$$B_t^{cb} = R_t . \tag{1}$$

Each commercial bank holds exogenous reserves, R_t. In buying long-term bonds (or making loans) of the amount $B_t^b \geq 0$, the bank creates deposit money, D_t. Its balance sheet reads

$$R_t + B_t^b = D_t . \tag{2}$$

The bond interest rate, $i_t > 0,$ is given and no interest is paid on deposits. Deposits induce costs described by a short-run cost function, $J(D_t),$ satisfying $J', J'' > 0.$ Subject to a required reserve ratio $rr \in (0;1)$, which obliges banks to hold reserves of at least $rr \times D_t,$ the banks solve:

$$\begin{aligned} &\max_{B_t^b}! \quad i_t B_t^b - J(D_t) \\ &\text{s.t.} \quad rr \times D_t \leq R_t . \end{aligned} \tag{3}$$

Substituting deposits from (2) gives the Lagrange function

$$\mathcal{L} = i_t B_t^b - J(R_t + B_t^b) - \xi_t^R \left(rr(R_t + B_t^b) - R_t \right). \tag{4}$$

Differentiating with respect to the bonds as the only control variable yields

$$\xi_t^R = \frac{i_t - J'(D_t)}{rr} . \tag{5}$$

The traditional approach assumes $i_t - J(D_t) > 0$. Retaining this premise, the Lagrange multiplier is strictly positive, and Kuhn-Tucker's complementary slackness condition implies that the constraint binds at the optimum. The solutions read $B_t^b = (1 - rr)/rr\, R_t$ and $D_t = R_t / rr,$ the last being the familiar 'money multiplier' in models without currency. Banks hold only required reserves but no excess reserves.

The positive shadow price, ξ_t^R, sustains Gurley and Shaw's (1960: 271) view that reserve requirements make "commercial banks a disequilibrium sector set apart from other sectors of the economy that are guided by the pricing mechanism rather than by direct controls".

3. The Market for Federal Funds

A market for reserves is now introduced. At the going federal funds rate, $i_t^F \geq 0$, each bank can borrow an amount of $F_t^d > 0$ in the interbank market to increase its reserves to $R_t + F_t^d$. Alternatively, it can lend an amount of $F_t^d < 0$, thus reducing its reserves. Thus, bank solve

$$\begin{aligned} \max_{B_t^b, F_t^d}! \quad & i_t B_t^b - i_t^F F_t^d - J(D_t) \\ \text{s.t.} \quad & rr \times D_t \leq R_t + F_t^d \ . \end{aligned} \tag{6}$$

Borrowing funds extends a bank's balance sheet to $(R_t + F_t^d) + B_t^b = D_t + F_t^d$. Lending, by contrast, constitutes an asset swap, an exchange of reserves for claims against fellow banks. Since neither of the two actions affects the identity $R_t + B_t^b = D_t$, the latter is still used in forming the Lagrange function

$$\mathcal{L} = i_t B_t^b - i_t^F F_t^d - J(R_t + B_t^b) - \xi_t^R \left(rr(R_t + B_t^b) - R_t - F_t^d \right) \tag{7}$$

Differentiating with respect to the bonds recovers equation (5). Differentiating with respect to the federal funds gives the additional optimality condition

$$i_t^F = \xi_t^R \ . \tag{8}$$

This characterizes the effective federal funds rate as a Lagrange multiplier. Opening a market for federal funds converts the shadow price of reserves into an explicit market price whose value is determined by the equilibrium condition $F_t^d = 0$. As opposed to the traditional model, each bank is now satisfied with its reserve balances. The main result, a specific *term structure of interest rates*, follows from combining equations (5) and (8):

$$i_t^F = \frac{i_t - J'(D_t)}{rr} \ . \tag{9}$$

The interest rate i_t drives the federal funds rate because the bond market is large, compared to the federal funds market, and unlikely to be directly affected by conventional open market operations; see Thornton (2014: 206).

Three interesting conclusions emerge. First, $\partial i_t^F / \partial i_t > 0$; the federal funds rate responds positively to the bond interest rate. Second, $\partial i_t^F / \partial R_t < 0$ because higher reserves boost deposits and increase marginal costs; the federal funds rate responds negatively to reserves. Third, there is nothing unusual about an inverse term structure. Consider $rr = 1/3$ and $J'(D_t) = 0$, which implies $i_t^F = 3 i_t$. At a bond rate of 5%, banks would offer up to 15% in the federal funds market. Why? Because an additional reserve unit enables a bank to expand credit by three units that earn

3×5% in total. From this perspective, the US bond yield conundrum indicates that monetary policy kept reserves in short supply. As the demand for reserves stems from a unique statutory requirement, the federal funds rate can markedly deviate from the bond interest rate, which is largely determined by intertemporal exchange motives.

4. Pushing a String

Figure 2 shows the interplay of the federal funds rate, an endogenous variable, and the target federal funds rate, a policy instrument. As Ihrig et al. (2015) describe, the Fed normally influences the federal funds rate through modest open market operations that affect bank reserves. This changed drastically in September 2008 when the Fed expanded its balance sheet through large-scale asset purchases. Consequently, the federal funds rate fell below the target rate until 13 December 2008, when the target rate was replaced by a corridor of 0-0.25 percent.

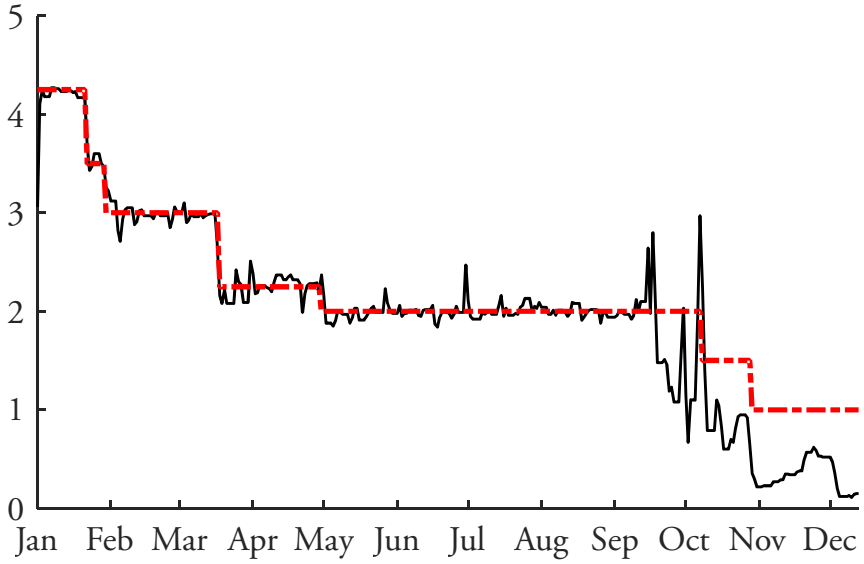

Figure 2: Federal funds rate (solid line) versus target rate (dash-dot line). *Notes*: Daily data, retrieved July 2016 from research.stlouisfed.org/fred2, series DFF and DFEDTAR.

Consistent with the view that banks hold only a trifling amount of excess reserves, which is due to stochastic cash flows, US banks held excess reserves of $2 bn. in early September 2008. By 1 October 2008, this figure had risen to $135 bn. The sharp increase accords with figure 2 and the implication of formula (9) that boosting reserves will depress the federal funds rate.

However, the unprecedented amount of excess reserves did not entail a comparable expansion of deposits, nor did it produce inflation. This section offers an explanation: Banks must keep their leverage in a range acceptable for markets and regulators. In a setting without explicit equity, a *leverage requirement* restricts a bank's total liabilities by some given limit, L. For banks borrowing in the federal funds market, total liabilities equal $D_t + F_t^d$; for lending banks they equal D_t because lending

constitutes an asset swap. With limited leverage, each bank solves (6) under the additional constraint $D_t + \max\{F_t^d; 0\} \leq L$. The associated Lagrange function reads

$$\mathcal{L} = i_t B_t^b - i_t^F F_t^d - J(R_t + B_t^b) - \xi_t^R \left(rr(R_t + B_t^b) - R_t - F_t^d\right) - \xi_t^L \left(R_t + B_t^b + \max\{F_t^d, 0\} - L\right). \tag{10}$$

Any increase in R_t slackens the first constraint and tightens the second. With a small amount of reserves, the reserve requirement binds and the leverage requirement has slack. As ξ_t^L, the shadow price of the leverage requirement, vanishes in this case, the solutions are still given by (5) and (8). If the central bank conducts large-scale asset purchases, however, the reserve requirement gets slack at the equilibrium, $F_t^d = 0$, and only the leverage requirement binds. Considering $\xi_t^R = 0$ and differentiating with respect to bond demand yields

$$\xi_t^L = i_t - J'(R_t + B_t^b) > 0\,. \tag{11}$$

To characterize equilibrium in the federal funds market, one needs a direct argument since the leverage requirement is non-differentiable at the origin: For $i_t^F > 0$, every $F_t^d > 0$ is suboptimal because it reduces profit, leaves the reserve requirement unaffected ($\xi_t^R = 0$), and tightens the leverage requirement ($\xi_t^L > 0$). Some $F_t^d < 0$ are optimal because they increase profit and leave both requirements unaffected. Therefore, $i_t^F = 0$. Only with the federal funds rate at the ZLB is $F_t^d = 0$ weakly optimal for every bank, and this condition must hold in an equilibrium.

Quantitative easing drives banks into a regime where their inclination to expand credit and deposits is not restricted by the level of reserves but by their leverage. As long as the leverage requirement binds, further increases in bank reserves have no effect on credit and money and leave the effective federal funds rate at the ZLB.

5. Interest on Reserves

Effective from 17 December 2015, the Fed increased the target corridor for the federal funds rate from 0-0.25 to 0.25-0.50 percent. The effective federal funds rate, which fluctuates, rose by about 0.25 percent, contradicting the claim that its unique equilibrium value were zero. However, the Fed also increased the interest on required and excess reserve balances from 0.25 to 0.50. To take account of this instrument, an additional term $i_t^R(R_t + F_t^d)$ is added to the profit function in (10), where the new symbol i_t^R indicates the interest rate on reserve balances. Differentiating yields the following characterization of the effective federal funds rate:

$$i_t^F = \xi_t^R + i_t^R. \tag{12}$$

Thus, an increase in the interest rate on reserves induces a commensurate increase in the effective federal funds rate. This holds irrespective of which constraint binds. Since any change in interest on reserves is fully absorbed by a response of the effective federal funds rate, interest on reserves do not affect bank incentives.

References

Fama, E. F. (2013) Does the Fed Control Interest Rates? *Review of Asset Pricing Studies* 3: 180-199.

Goda, T., P. Lysandrou and C. Stewart (2013) The Contribution of US Bond Demand to the US Bond Yield Conundrum of 2004–2007: An Empirical Investigation. *Journal of International Financial Markets, Institutions & Money* 27: 113-136.

Gurley, J. G. and E. S. Shaw (1960) Money in a Theory of Finance, Washington: Brookings.

Ihrig, J. E., E. E. Meade and G. C. Weinbach (2015) Rewriting Monetary Policy 101: What's the Fed's Preferred Post-Crisis Approach to Raising Interest Rates? *Journal of Economic Perspectives* 29: 177-198.

Hamilton, J. D. (1996) The Daily Market for Federal Funds. *Journal of Political Economy* 104: 25-56.

Phillips, C. A. (1920) *Bank Credit*. New York: Macmillan.

Poole, W. (1968) Commercial Bank Reserve Management in a Stochastic Model: Implications for Monetary Policy. *Journal of Finance* 23: 769-791.

Rudebusch, G. D. (1995) Federal Reserve Interest Rate Targeting, Rational Expectations, and the Term Structure. *Journal of Monetary Economics* 35: 245-274.

Thornton, D. L. (2014) Monetary Policy: Why Money Matters (and Interest Rates Don't). *Journal of Macroeconomics* 40: 202-213.