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Capturing information contagion in a stress-testing framework

Kartik Anand
(Deutsche Bundesbank)

Prasanna Gai
(University of Auckland)

Céline Gauthier
(Université du Québec en Outaouais)

Moez Souissi
(International Monetary Fund)

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Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

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Non-technical summary

Research Question

During a bank run, policymakers often cite the very real prospect of ‘information contagion’, whereby bad news about one institution precipitates a loss of confidence in the security of holdings across the banking system, as a key reason for intervention. But, notwithstanding important progress in developing top-down stress-testing models to help guide financial stability policy, quantifying the role of information contagion in systemic risk analysis has proven elusive.

Contribution

We present a model-based stress-testing framework where the solvency risks, funding liquidity risks and market risks of banks are intertwined. The key transmission mechanism is a two-way interaction between the beliefs of secondary market investors and the coordination failure between the creditors of financial institutions. Pessimism about macroeconomic fundamentals triggers creditor runs, but also increases the fire sale discount applied to illiquid assets by secondary market investors. This hampers a troubled bank’s recourse to liquidity and increases the incidence of bank runs, potentially unleashing a wave of investor pessimism that can drive otherwise solvent banks into illiquidity. We quantify this contagion channel in the context of the Bank of Canada’s model of the Canadian banking system and a stress-test scenario used by the IMF during its 2013 evaluation of the Canadian financial sector.

Results

Our results suggest that information contagion can be significant – the probability of one bank entering distress due to contagion alone is nearly 20%, and the extreme tail associated with the system-wide distribution of bank losses is materially larger. We decompose the aggregate losses to the banking system into those arising from solvency risk, funding liquidity risk and information contagion.

Nichttechnische Zusammenfassung

Fragestellung

Während eines Bank Runs berufen sich politische Entscheidungsträger oft auf die „Informationsansteckung“ als Grund für eine Intervention. Durch eine „Informationsansteckung“ führen negative Nachrichten über eine Institution zu einem generellen Vertrauensverlust in die Sicherheit von Anlagen im Bankensystem. Obwohl es in den letzten Jahren signifikante Fortschritte in der Entwicklung von Top-Down-Stresstest-Modellen gab, bleibt es weiterhin sehr schwer die Bedeutung der Informationsansteckung im Rahmen von systemischen Risikoanalysen zu quantifizieren.

Beitrag

Wir präsentieren einen modellbasierten Stresstestrahmen, der Solvenzrisiko, Finanzierungs-Liquiditätsrisiko und Marktrisiko von Banken miteinander verbindet. Der wichtigste Ansteckungsmechanismus ist eine beidseitige Interaktion zwischen den Meinungen sekundärer Marktinvestoren und dem Koordinationsversagen unter den Gläubiger der Finanzinstitution. Pessimismus bezüglich der makroökonomischen Grundlagen löst einen Ansturm der Gläubiger aus, aber vermindert auch den Verkaufswert für schwer zu liquidierende Aktivposten, den die sekundären Marktinvestoren ansetzen. Das beeinträchtigt den Zugang einer in Schwierigkeiten geratenen Bank zu Liquidität und erhöht die Wahrscheinlichkeit von Bank Runs, was wiederum den Pessimismus der Investoren weiter verstärken kann, der eine ansonsten zahlungsfähige Bank in die Zahlungsunfähigkeit treiben kann. Wir quantifizieren diesen Übertragungskanal mit einem Modell des kanadischen Bankensystems, das die Bank of Canada erstellt hat, und mit einem Stresstestszenario, das das IMF während der Bewertung des kanadischen Finanzsektors 2013 eingesetzt hat.

Ergebnisse

Unsere Resultate legen nahe, dass die Informationsansteckung signifikanten Einfluss haben kann – die Wahrscheinlichkeit einer Bank, alleine aufgrund der Ansteckung unter Stress zu geraten, beträgt fast 20 Prozent. Außerdem ist die Wahrscheinlichkeitsmasse im extremen Ende der Verteilung in einem Modell mit Ansteckung bei einer systemweiten Verteilung von Bankverlusten deutlich größer als in einem Modell ohne Ansteckung. Des Weiteren unterteilen wir die gesamten Verluste im Bankensystem in diejenigen, die jeweils auf Solvenzrisiken, Finanzierungsliquiditätsrisiken und Informationsansteckung zurückzuführen sind.

Capturing Information Contagion in a Stress-Testing Framework*

Kartik Anand
Deutsche Bundesbank

Prasanna Gai
University of Auckland

Céline Gauthier
Université du Québec en Outaouais

Moez Souissi
International Monetary Fund

Abstract

We develop an operational model of information contagion and show how it may be integrated into a mainstream, top-down, stress-testing framework to quantify systemic risk. The key transmission mechanism is a two-way interaction between the beliefs of secondary market investors and the coordination failure between the creditors of financial institutions. Pessimism about macroeconomic fundamentals triggers creditor runs, but also influences the fire sale discount applied to illiquid assets by secondary market investors. This hampers a troubled bank's recourse to liquidity and increases the incidence of bank runs, potentially unleashing a wave of investor pessimism that can drive otherwise solvent banks into illiquidity. We quantify this contagion channel in the context of the Bank of Canada's model of the Canadian banking system and a stress-test scenario used by the IMF during its 2013 evaluation of the Canadian financial sector.

Keywords: Liquidity risk, contagion, stress testing, global games.

JEL classification: G01, G21, G28, C72, E58.

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“...the Chancellor of the Exchequer...was conscious that had Northern Rock been allowed to fail, there was a substantial risk that the spectacle of depositors unable to access their funds in Northern Rock would lead depositors with other banks to lose faith in the banking system as a whole, the so-called ‘contagion effect’...”

The Run on the Rock, House of Commons Treasury Committee Report Volume 1, p55, 2008.

1 Introduction

During a bank run, policymakers often cite the very real prospect of ‘information contagion’, whereby bad news about one institution precipitates a loss of confidence in the security of holdings across the banking system, as a key reason for intervention. As the epigraph suggests, such considerations were central during the failure of Northern Rock in 2007. But, notwithstanding important progress in developing top-down stress-testing models to help guide financial stability policy, quantifying the role of information contagion in systemic risk analysis has proven elusive.

In this paper, we present a tractable approach to quantifying information contagion and integrate it into the Bank of Canada’s stress-testing model (the Macro-financial Risk Assessment Framework, or MFRAF). The model is calibrated to representative balance sheet data for the six largest Canadian banks, and we quantify the information contagion that arises from a stress-test scenario conducted by the IMF as part of its 2013 Financial Sector Assessment Program (FSAP) for Canada. Our results suggest that information contagion is significant – the probability of one major bank entering distress due to contagion alone is nearly 20% and the extreme tail associated with the system-wide distribution of bank losses is materially larger. We decompose the aggregate losses to the banking system into those arising from solvency risk, funding liquidity risk and information contagion.

The analytical foundation of our approach stems from the global games literature, which synthesizes the pure panic-based and fundamental-based views of bank runs. Work in this tradition ([Morris and Shin, 2003](#); [Rochet and Vives, 2004](#); [Goldstein and Pauzner, 2005](#)) suggests that poor fundamentals can trigger self-fulfilling beliefs about a financial crisis. A bank’s funding liquidity risk is driven by credit and market losses, funding composition and maturity profile, and concerns over the bank’s future solvency. A creditor receives a noisy signal about their bank’s losses, and decides to withdraw based on expectations over the signals of other creditors.

A bank’s market liquidity risk, by contrast, is driven by the perception of secondary market investors over the macroeconomy. This, in turn, drives the haircut that investors impose on banks’ illiquid assets. When the macroeconomy is in a ‘bad’ state, the haircut is large, while when the macroeconomy is in a ‘good’ state, the haircut is small. Importantly, these investors cannot observe the state of the macroeconomy or the losses that banks suffer. They can, however, observe whether a bank has failed or not, and impute from this the state of the macroeconomy.

Thus, the coordination failure between a bank’s creditors interacts with the views of secondary market investors. The interaction gives rise to vicious (as well as virtuous) cycles that can drive otherwise solvent banks into illiquidity. Pessimism about the macroeconomy influences the haircut applied to illiquid assets by secondary market investors,

hampering a troubled bank's recourse to liquidity. This influences the incidence of bank runs which, in turn, makes investors more pessimistic and drives down the recourse to liquidity of other banks in the system.

We demonstrate that for an arbitrary number, $N \geq 2$, of banks, vicious/virtuous cycles between investors and creditors end after, at most, N rounds. This result allows us to operationalize our model, since it ensures that the contagion dynamics will always converge and terminate after a finite number of iterations. We can, therefore, readily implement the model within a simulation based stress-test exercise. The result also ensures that we can quantify the extent of information contagion. Banks that turned illiquid during the first iteration suffered from liquidity risk. While, banks that turned illiquid during subsequent iterations did so because of contagion risk.

Our framework is intended as a demonstration of how information contagion can begin to be quantified in stress-testing work. As such, we make a number of simplifying assumptions. First, we abstract away from network externalities caused by the cascading defaults of a bank's counterparties. Second, we suppose that the loss distributions are identical for all banks and, thus, equal to the loss distribution of an 'average' bank. Third, all banks are assumed to have the same Tier 1 capital buffers, liquid assets, and common returns. Section 4 considers the implications for quantifying information contagion of relaxing these assumptions.

The paper can be viewed as bridging the gap between the literature on top-down stress-testing models and the literature on information contagion. [Elsinger, Lehar, and Summer \(2006\)](#) and [Alessandri, Gai, Kapadia, Mora, and Puhf \(2009\)](#) are early contributions to the literature on systemic stress-testing. They describe frameworks used by the Austrian Central Bank and the Bank of England to quantify system-wide bank losses in ways that explicitly characterize bank balance sheets and allow for macro-credit risk, network interactions, and fire sale effects. [Aikman, Alessandri, Eklund, Gai, Kapadia, Martin, Mora, Sterne, and Willison \(2009\)](#) and [Gauthier, Souissi, and Liu \(2014\)](#) demonstrate how these frameworks might be adapted to quantify funding liquidity risk. But these analyses eschew any consideration of the information contagion channel considered here.

The literature on information contagion is sizeable, so we only mention some important recent contributions here. [Acharya and Yorulmazer \(2008\)](#) analyze the ex ante response of banks to adverse news about other banks. In their two-bank model, information contagion occurs when bank loan returns have a common systemic risk factor – the failure of one bank conveys adverse information about the common factor. This raises the borrowing costs of the surviving bank and, in order to maximize the joint probability of survival, both banks lend and take on correlated investments. [Acharya, Gale, and Yorulmazer \(2011\)](#) present a model in which the lack of arrival of good news about the value of an asset used as collateral to roll over short-term debt can be associated with a sudden collapse in the debt capacity of firms. And [Allen, Babus, and Carletti \(2012\)](#) use a network setting to tackle the link between commonality in bank asset portfolios and information contagion. With long-term debt, the asset structure of the banking system does not matter for welfare. But when debt is short-term and portfolio quality opaque, adverse signals about bank solvency can trigger foreclosures by creditors. Greater commonality in bank portfolios increases the likelihood of information contagion.

Recent work by [Li and Ma \(2013\)](#) also considers the same contagion channel that we do – namely an adverse selection problem which results in mutually reinforcing bank runs

and fire sales. But while we emphasize the sequential determination of market prices and failure conditions, Li and Ma derive a simultaneous equilibrium for the pooling price of assets and the failure conditions. Their focus on regulatory transparency and ex ante equilibrium asset prices also contrasts with our emphasis on stress-testing and systemic risk measurement.

Empirical studies of bank contagion typically test whether news of a bank failure adversely affects other banks using indicators such as stock price reactions and bank risk premia (Calomiris and Mason, 2003; Hasan and Dwyer, 1994). More recently, Iyer and Peydro (2011) use evidence from the failure of a large Indian bank to show how direct contagion can be amplified by information spillovers as banks with higher exposure to the failed bank suffered larger withdrawals by their depositors.

The paper proceeds as follows. We present the model in Section 2 and discuss the quantitative application of it in Section 3. In Section 4, we discuss the implications of relaxing our simplifying assumptions. A final section concludes.

2 Model

The banking system consists of two banks indexed by $b \in \{1, 2\}$. We consider a one-year stress test horizon punctuated by three dates: the start of the year ($t = 0$), an interim date at six months ($t = 1$), and the end of the year ($t = 2$). The interim date comprises two rounds. In Round 1, some short-term creditors of a bank have an opportunity to withdraw. In Round 2, outside investors in the secondary market for banks' risky assets revise their views on the macroeconomic fundamentals governing the economy. Additionally, short-term creditors of liquid banks can decide whether to withdraw or rollover their loans.

If funding liquidity risk does not materialize, then the illustrative balance sheet for bank b at the end of $t = 2$ resembles Table 1.

	Assets	Liabilities	
Risky assets	$Y^b - S_1^b - S_2^b$	LT^b	Long-term Debt
Liquid assets	M^b	ST^b	Short-term Debt
		$E^b - S_1^b - S_2^b$	Capital

Table 1: Balance sheet of bank b at the end of $t = 2$

In Table 1, the asset-side of the balance sheet comprises safe and liquid assets ($M^b > 0$) carried over from $t = 0$, as well as risky but illiquid assets. The net value of these risky assets depends on the investment return Y^b and on the credit losses $S_1^b \in [0, \bar{S}_1^b]$ and $S_2^b \in [0, \bar{S}_2^b]$, written down at $t = 1$ and $t = 2$ respectively. S_1^b and S_2^b are drawn i.i.d. from a probability distribution function $f^b(S)$, with cumulative distribution function $F^b(S)$.¹

¹The assumption that losses at $t = 1$ and $t = 2$ are drawn i.i.d from the same distribution simplifies exposition of the model. Our results continue to hold if different loss distributions for losses are assumed at $t = 1, 2$.

The liability-side of the balance sheet consists of long-term debt, LT^b , that matures at $t = 2$, and short-term debt maturing at $t = 1$, ST^b , that can be rolled over. Residual capital after credit loss write-downs is denoted by $E^b - S_1^b - S_2^b$, where E^b is bank b 's starting capital plus income earned over the stress-testing horizon net of dividend payments to external shareholders. We suppose that each bank has a distinct pool of short-term creditors. These creditors hold a unit of short-term debt at $t = 0$ that can be rolled over during either Round at $t = 1$. They are also indifferent between consumption at $t = 1$ and $t = 2$.

At the interim date, banks have the opportunity to sell risky assets on the secondary market to a distinct pool of deep-pocketed outside investors who consume at the final date, $t = 2$. The price per unit of asset sold depends on the ‘haircut’ on the asset value imposed by these outsiders. The haircut, in turn, depends on macroeconomic fundamentals which we represent by the binary variable $m \in \{0, 1\}$. When macroeconomic fundamentals are ‘good’, $m = 1$, and the haircut, $h_g \in (0, 1)$, is low. Accordingly, bank b receives $\psi_g \equiv 1 - h_g \in (0, 1)$ per unit of asset sales. But if macroeconomic fundamentals are ‘bad’, $m = 0$, the haircut, $h_b \in (0, 1)$ is high and bank b receives $\psi_b \equiv 1 - h_b < \psi_g$ for the asset, implying that $h_b > h_g$.

We suppose that although the state of the macroeconomy is drawn at $t = 0$, it is unobservable to all agents. Moreover, as we discuss in Section 2.1, while creditors decide to rollover after receiving noisy signals about banks’ losses, outside investors cannot observe banks’ losses, from which they could have otherwise inferred m . Instead, the outside investors entertain a common knowledge prior belief, $w_1 = \text{Prob}(m = 1)$, that fundamentals are good. The expected price that bank b obtains from selling a unit of the risky asset to an outside investor is thus

$$\bar{\psi}_1 \equiv w_1 \psi_g + (1 - w_1) \psi_b. \quad (1)$$

Bank b is insolvent at $t = 2$ if $S_1^b + S_2^b > E^b$, i.e. its capital is insufficient to cover its losses. If $S_1^b > E^b$, then it is already insolvent at $t = 1$. If $S_1^b \leq E^b$, bank b is solvent at $t = 1$, but may become illiquid if the fraction of short-term creditors who withdraw (in either first or second rounds) is too large. In what follows, a bank that is illiquid at $t = 1$ is assumed to be insolvent at $t = 2$.

The fraction of short-term creditors who withdraw from bank b during Round 1 of $t = 1$ is denoted $\ell_1^b \in [0, 1]$. At the end of Round 1, bank b is either liquid or illiquid – a state that we denote by $\eta_1^b \in \{0, 1\}$. Thus bank b is illiquid in Round 1 whenever

$$\ell_1^b > \lambda^b(S_1^b; \bar{\psi}_1) \equiv \frac{M^b + \bar{\psi}_1[Y^b - S_1^b]}{ST^b}. \quad (2)$$

The fraction $\lambda^b(S_1^b; \bar{\psi}_1)$ is the ratio of bank b 's recourse to liquidity at $t = 1$ (the sum of liquid assets plus proceeds from illiquid assets to outside investors) to its short-term liabilities. If $\ell_1^b \leq \lambda^b(S_1^b; \bar{\psi}_1)$, then bank b is liquid in Round 1 ($\eta_1^b = 0$).

Outside investors are able to revise their beliefs about the macroeconomy at the start of Round 2 of $t = 1$. Since macroeconomic fundamentals affect banks’ credit losses and, hence, their recourse to liquidity, outside investors can infer the state of the economy by observing if banks were illiquid or liquid in Round 1. Formally, outside investors determine $w_2 \equiv \text{Prob}(m = 1 | \eta_1^1, \eta_1^2)$ and so the new expected price for banks selling risky assets in

Round 2 is $\bar{\psi}_2 \equiv w_2 \psi_g + (1 - w_2) \psi_b$.

If bank b turns illiquid in Round 1, it continues to remain illiquid in Round 2. If, however, bank b is liquid in Round 1, then it becomes illiquid in Round 2 if the cumulative outflow of short-term creditors who withdraw in Round 1, ℓ_1^b , and Round 2, ℓ_2^b , exceeds its ratio of liquid assets to short-term liabilities, $\lambda^b(S_1^b; \bar{\psi}_2)$. Bank b remains liquid in Round 2 provided $\ell_1^b + \ell_2^b \leq \lambda^b(S_1^b; \bar{\psi}_2)$.

Table 2 summarizes the timing of events in the model.

$t = 0$	$t = 1$ (round 1)	$t = 1$ (round 2)	$t = 2$
1. State m realized	1. Interim shock	1. Belief updated	1. Final shock
	2. <i>ST</i> debt withdrawals	2. New asset price	2. Incomes accrued
		3. <i>ST</i> debt withdrawals (from liquid banks)	3. Dividends paid

Table 2: Timeline of events

2.1 Withdrawal of short-term creditors

The decision of short-term creditors to withdraw from bank b in Round 1 is assumed to be independent of their decision to withdraw in Round 2. This assumption implies that the endogenous fraction of creditors who withdraw in Round 2, ℓ_2^b , is independent from the fraction of creditors who withdraw in Round 1, ℓ_1^b , which materially simplifies the analysis. We model the creditors' decisions in both Rounds by binary-action simultaneous-move coordination games. Table 3 summarizes the payoffs to an individual creditor during Round 1. Conditional on the creditor rolling over the claim in Round 1, their payoffs from withdrawing or rolling over in Round 2 is also summarized in Table 3.

	Solvent	Insolvent
Rollover	$1 + r^b$	0
Withdraw	1	1

Table 3: Payoffs to a short-term creditor

As Table 3 makes clear, if the creditor withdraws, then the bank returns 1 unit of funds. The creditor receives this payoff (which is immediately consumed) regardless of the outcome for the bank.² But if the creditor rolls over, the payoff depends on the outcome for bank b at $t = 2$. If bank b is solvent, then the creditor receives $1 + r^b$. In the

²This allows us to preserve the global strategic complementarity assumption needed to solve the coordination game.

event of insolvency, however, the creditor receives nothing. Insolvency can arise either because of a liquidity run at $t = 1$ or because of excessively large credit shocks.

Conditional on receiving the loss S_1^b , bank b becomes insolvent at $t = 2$ with probability

$$\mathcal{N}^b(S_1^b) = \text{Prob}(E^b - S_1^b - S_2^b < 0) = \begin{cases} 1 & \text{if } S_1^b > E^b \\ 1 - F^b(E^b - S_1^b) & \text{if } S_1^b \in [E^b - \bar{S}_2^b, E^b] \\ 0 & \text{if } S_1^b < E^b - \bar{S}_2^b \end{cases} .$$

If $S_1^b > E^b$, then the bank is always insolvent at $t = 2$ for all realizations of S_2^b . In this situation, it is a dominant strategy for all short-term creditors to withdraw in Round 1. By contrast, when $S_1^b < E^b - \bar{S}_2^b$, the bank is always solvent at $t = 2$ and the dominant strategy for creditors is to roll over during both Rounds. In the intermediate range, the probability of insolvency is non-zero and – with common knowledge over the loss S_1^b – the model exhibits the well-known multiplicity of equilibria in pure strategies highlighted by [Diamond and Dybvig \(1983\)](#). ‘Sunspots’ determine whether all creditors withdraw their claims in Round 1 during $t = 1$, or no creditor withdraws in any Round. Figure 1 depicts the tri-partite classification of the support for the credit loss S_1^b .

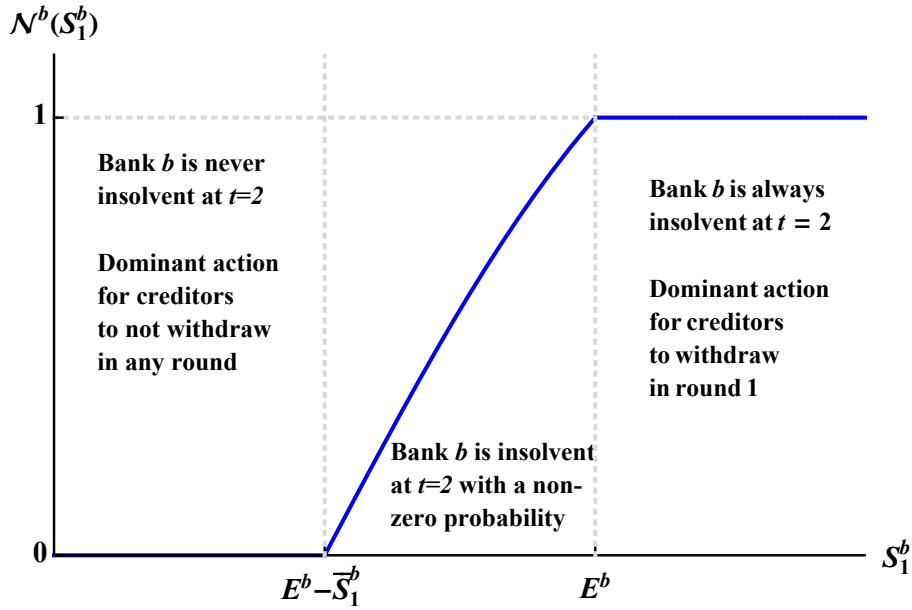


Figure 1: Tripartite classification of the shock S_1^b for bank b .

2.2 Liquidity risk and information contagion

We resolve the multiplicity of equilibria using the global games method pioneered by [Morris and Shin \(2003\)](#). We allow for the possibility of imperfect common knowledge by supposing that each short-term creditor, i , of bank b receives a noisy signal $x_{id}^b = S_1^b + \epsilon_{id}$ in each round, d , for realized credit losses. The noise terms for each creditor are independent across banks and rounds, and ϵ_{id} is uniformly distributed over the interval $[-\epsilon, \epsilon]$.³ All

³The support for the loss S_1^b must satisfy, $0 < E^b - \bar{S}_2^b - \epsilon$, and $\bar{S}_1^b > E^b + \epsilon$.

short-term creditors follow a switching strategy, i.e. there exists x_d^{b*} such that if creditor i receives a signal $x_{id}^b > x_d^{b*}$, then the creditor withdraws in Round d . But if $x_{id}^b \leq x_d^{b*}$, then creditor i does not withdraw in round d . Creditor i thus uses the signal x_{id}^b to infer the distribution of the distribution of signals received by other creditors and thus the likelihood that they withdraw. The conditions for the unique equilibrium at which liquidity risk materializes are summarized in Proposition 1 below.

Proposition 1. Critical illiquidity threshold. *In the limit of vanishing private noise, $\epsilon \rightarrow 0$, there exists a unique equilibrium in threshold strategies characterized by an illiquidity threshold, S_d^{b*} , and a signal threshold, x_d^{b*} , for each bank b in Round d . Creditor i withdraws the claim against bank b in Round d if and only if $x_{id}^b > x_d^{b*}$, and bank b is illiquid if and only if $S_1^b > S_d^{b*}$, where $x_d^{b*} \rightarrow S_d^{b*}$. The illiquidity threshold is implicitly defined by the indifference condition for the expected payoff to a creditor between rolling over and withdrawing:*

$$F^b(E^b - S_d^{b*}) \lambda^b(S_d^{b*}; \bar{\psi}_d) = \frac{1}{1 + r^b}. \quad (3)$$

The illiquidity threshold is increasing in the bank's equity, E^b , liquid assets, M^b , and returns, r^b , but is decreasing in its short-term debt level, ST^b . Finally, the threshold is also increasing in the investor's belief, w_d , and the prices, ψ_g and ψ_b .

Proof. See Appendix A. □

The comparative statics for the critical thresholds are intuitive. First, as the bank's equity, E_1^b , increases, it has more resources to withstand shocks at the interim and final dates. The bank's probability of insolvency at $t = 2$ is lower and, hence, creditors are more willing to roll over during either Round. Second, as M^b , the level of liquid assets increases, the bank has greater recourse to liquidity and is better able to satisfy creditors that choose to withdraw. Creditors, in turn, are more willing to roll over their claims. Third, as the promised return by the bank, r^b , increases, creditors benefit from waiting until $t = 2$ instead of withdrawing early during either Round at $t = 1$ and receiving only one unit of funds. Fourth, as short-term debt levels, ST^b , rise, the bank has less recourse to liquidity to meet withdrawals. This prompts creditors to withdraw earlier. Finally, increases in investor beliefs in Round d , w_d , and the secondary market prices for risky assets all improve the bank's recourse to liquidity and encourage roll overs.

Although the results of Proposition 1 hold for both Rounds at $t = 1$, it is possible for outside investors' beliefs about macroeconomic fundamentals to change between Round 1 and Round 2. In particular, if outside investors are initially pessimistic about the prospect of a good state, i.e. w_1 is low, this may precipitate liquidity risk in one bank during Round 1. On observing this outcome, investors update their beliefs and become more pessimistic, $w_2 < w_1$, reducing the second bank's recourse to liquidity in the process.

To formalize these ideas, note that Bayes' rule implies that

$$w_2 = \frac{\text{Prob}(\eta_1^1, \eta_1^2 | \psi = \psi_g) w_1}{\text{Prob}(\eta_1^1, \eta_1^2 | \psi = \psi_g) w_1 + \text{Prob}(\eta_1^1, \eta_1^2 | \psi = \psi_b) (1 - w_1)}. \quad (4)$$

Since the circumstance under which bank 1 turns illiquid is independent of those for bank 2, η_1^1 and η_1^2 are independent, and $\text{Prob}(\eta_1^1, \eta_1^2 | \psi = \psi_g) = \text{Prob}(\eta_1^1 | \psi = \psi_g) \text{Prob}(\eta_1^2 | \psi = \psi_g)$. If bank b turns illiquid in Round 1 ($\eta_1^b = 1$), then conditional on $\psi = \psi_g$, outside investors assess the likelihood of such an event to be

$$\text{Prob}(\eta_1^b = 1 | \psi = \psi_g) = \text{Prob}(S_1^b > S_d^{b*}), \quad (5)$$

where the critical threshold S_{dH}^{b*} is given by the solution to

$$G^b(E^b - S_{dH}^{b*}) \lambda^b(S_{dH}^{b*}; \psi_g) = \frac{1}{1 + r^b}. \quad (6)$$

An analogous definition holds for the critical threshold S_{dL}^{b*} when the macro-economy is in a bad state, and the haircuts are large. Propositions 2–4 present the key results of our model.

Proposition 2. Virtuous liquidity. *If both banks are liquid at the end of Round 1, then $w_2 > w_1$. Consequently, both banks remain liquid at the end of Round 2.*

Proof. See Appendix B. □

Suppose that, at the start of Round 1, outside investors hold an optimistic view of macroeconomic fundamentals, i.e. there is a high prior of a good state and w_1 is large. This, in turn, results in both banks remaining liquid at the end of Round 1. At the start of Round 2, outside investors update their belief upwards since all banks are more likely to remain liquid, i.e. $w_2 > w_1$. The banks' recourse to liquidity improves which increases the critical thresholds. Both banks continue to remain liquid and solvent.

Proposition 3. Vicious illiquidity. *Suppose bank 1 is liquid and bank 2 is illiquid at the end of Round 1. The investor become more pessimistic, $w_2 < w_1$, whenever:*

$$\frac{\text{Prob}(\eta_1^1 = 0 | \psi = \psi_g)}{\text{Prob}(\eta_1^1 = 0 | \psi = \psi_b)} < \frac{\text{Prob}(\eta_1^2 = 1 | \psi = \psi_b)}{\text{Prob}(\eta_1^2 = 1 | \psi = \psi_g)}. \quad (7)$$

If the downward revision of the belief is large enough, then bank 1 will also become illiquid at the end of Round 2.

Proof. See Appendix C. □

Outside investors entering Round 1 with pessimistic views regarding the economy (low w_1) can render bank 2 illiquid. When updating beliefs at the start of Round 2, outside investors must compute and compare four different conditional probabilities. If the inequality in (7) is satisfied, outside investors conclude that it is less likely for bank 1 to remain illiquid when assets are good quality than it is for bank 2 to turn illiquid when assets are bad quality. The failure of bank 2 has a greater weight in the investor's Bayesian updating – there is a downward revision of beliefs and a more pessimistic view of asset quality.

To study the consequences for haircuts, we suppose that (i) banks are identical, except in their reliance on short-term funding, i.e, $ST^{b'} > ST^b$; (ii) small shocks are more likely than large ones, so that $f^b = f^{b'} = f$ satisfies $f' < 0$; (iii) the probability that bank b' turns illiquid in Round 1 when macro-fundamentals are good is bounded from above, so that $1 - G(S_{1H}^{b'*}) < \bar{G}$, where \bar{G} is defined in Appendix D. Under these assumptions, we obtain the following proposition.

Proposition 4. Price and spread effects. *For a given initial belief, w_1 , and “bad-state” price, ψ_b , an increase in the “good-state”, ψ_g , increases the price spread, $\Delta\psi = \psi_g - \psi_b$. This, in turn, strengthens the pessimism condition and increases the range of parameters where the investor’s belief is revised downwards.*

On the other hand, for a given “good-state” price, ψ_g , an increase in the “bad-state” price, ψ_b , leads to a decrease in the price spread. This weakens the pessimism condition and reduces the range of parameters where the investor’s belief is revised downwards.

Proof. See Appendix D. □

As the spread between good and bad state secondary market prices increases, the probability that bank 1 remains illiquid when assets are bad quality also increases. At the same time, the probability that bank 2 turns illiquid when asset quality is bad (relative to the probability that it turns illiquid when asset quality is good) also increases. When the increase in the relative probability that bank 2 is illiquid is greater than the relative probability of bank 1 being illiquid, the net effect is to increase the range of parameters over which the pessimism condition of Proposition 4 holds. The opposite holds when the price spread decreases.

With updated beliefs, there are two possible outcomes at the end of Round 2: bank 1 remains liquid or turns illiquid. If it remains liquid there is no additional information for outside investors and their beliefs remain unchanged. But if bank 1 turns illiquid, investors turn pessimistic. In both cases, there are no further actions possible in further rounds. Generalizing to an arbitrary number, N , of banks, the following proposition summarizes the result.

Proposition 5. Convergence. *In a game involving $N \geq 2$ banks, the cycles of Bayesian updating by investors and withdrawal by creditors terminates after, at most, N rounds.*

Proof. See Appendix E. □

Proposition 5 has two practical implications for the quantification of contagion risk in banking systems. First, it ensures that the contagion dynamics will always converge and terminate after a finite number of iterations. We can, therefore, readily implement the model within a simulation based stress-test exercise. Second, the result ensures that we can quantify the extent of information contagion. Banks that turned illiquid during the first iteration suffered from liquidity risk. While, banks that turned illiquid during subsequent iterations did so because of contagion risk. The severity of the information contagion can also be measured in terms of the change in the outside investors’ beliefs, i.e. $\Delta w = w_1 - w_{d^*}$, where $d^* \leq N$ is the iteration where the contagion dynamics terminated. A large and positive Δw indicates a significant downward revision of beliefs by outside

investors, which greatly hampers secondary market liquidity. By contrast, a small and positive Δw implies limited information contagion. and, if Δw is negative, it implies that investors are optimistic following the updating of beliefs.

3 Stress testing and simulations

We now integrate the model into the Bank of Canada’s Macro-financial Risk Assessment Framework (MFRAF) and study how an extreme stress scenario for the Canadian economy might induce information contagion across the banking system. Figure 2 provides a high-level overview of MFRAF. The stress scenario has a direct impact on corporate and household defaults, resulting in credit losses for banks and an adverse impact on their capital buffers. Banks are directly connected to each other via interbank linkages, and the interbank network clears via the Eisenberg and Noe (2001) algorithm – banks repay their interbank counterparties a sum that is proportional to the original amounts due, causing counterpart credit losses. Bank balance sheets are thus brought together and integrated with the interbank network to generate illustrative distributions for system-wide losses to the banking sector.

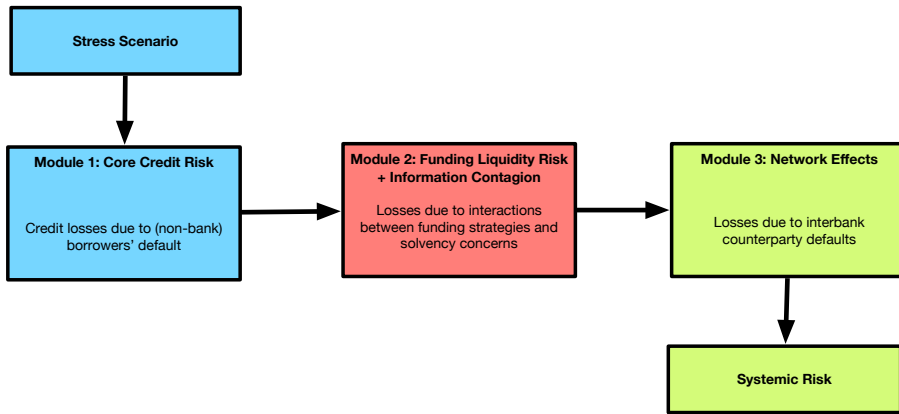


Figure 2: Schematic description of the modular structure of MFRAF

Figure 3 illustrates how funding liquidity risk and information contagion are incorporated into MFRAF. Following an episode of stress, liquidity risk can materialize endogenously as creditors decide whether or not to withdraw based on their perceptions of the future solvency of the bank (which depends on the severity of losses incurred by year-end and initial capital) relative to a supervisory threshold. The supervisory threshold is set at 7% for the ratio of tier 1 common equity to risk-weighted assets. If the threshold is breached, creditors withdraw en masse, leading to some banks failing from illiquidity. Secondary market investors update their beliefs regarding the macro-economy and adjust the price for banks’ risky assets accordingly. This, in turn, influences the withdrawal decisions of creditors for other banks across the banking system. If (pessimistic) perceptions are widespread, then contagion takes hold in the manner outlined in Section 2.1.

In what follows, we present results which abstract from the interbank network in order to isolate the influence of the information contagion channel. Section 4 considers the implications of relaxing this assumption.

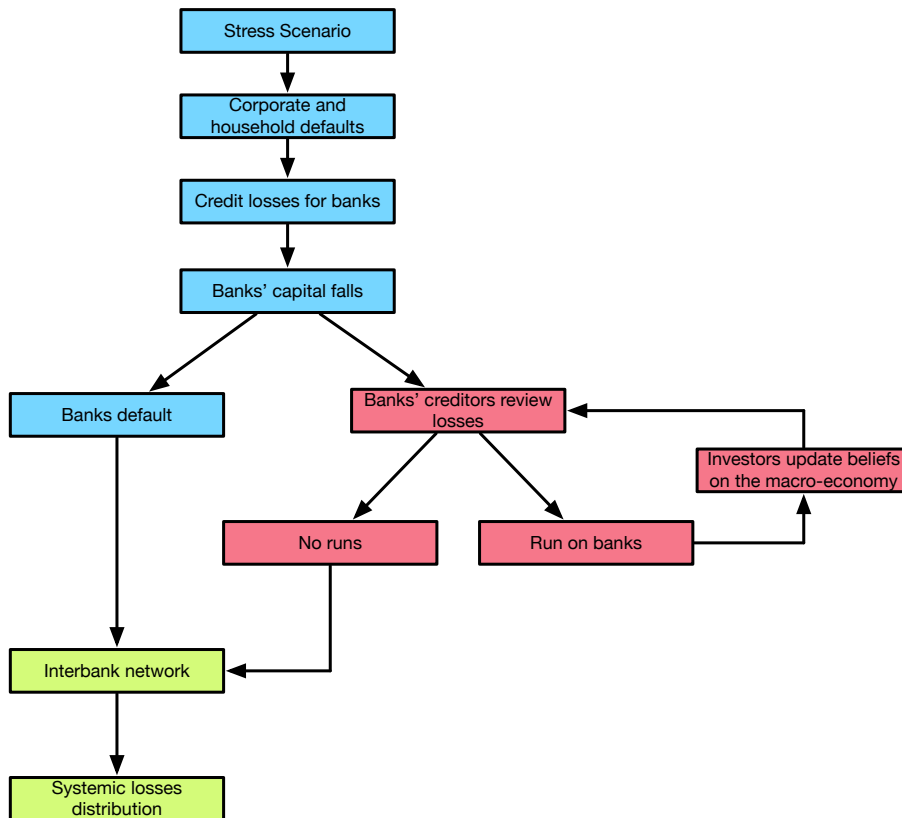


Figure 3: Flow chart for MFRAF.

3.1 Stress-test scenario

The scenario considered as part of the 2013 FSAP envisages a disorderly default in a peripheral euro-area country, which results in a banking crisis and recession in the euro area. The global economy weakens and the ensuing financial market turbulence results in an adverse shock to the Canadian economy. Specifically, the Canadian economy faces a large negative foreign demand shock, falling commodity prices, and a loss of business and consumer confidence, all of which leads to a sharp contraction in domestic demand. Tighter lending standards and house price declines ensue, generating a persistent recession more severe than those experienced since the 1980s.

Table 4 describes the scenario and places it in historical perspective. The decline in the macroeconomic variables is mapped into loan default rates on bank balance sheets, with different rates being derived for different sectors.⁴ The average default rate across sectors is 6.4%, compared with a default rate of 4.4% over the period 1988-2012.

3.2 Calibration

We run MFRAF under a ‘hybrid’ calibration of the 2013 FSAP exercise, taking some inputs from the banks own bottom-up stress-tests, and others from regulatory filings.

⁴The domestic sectors include: Accommodations, Agriculture, Construction, Manufacturing, Wholesale, Canadian governments, Financial institutions, Small business loans, Residential mortgages (uninsured), Home equity lines of credit (uninsured) and Consumer loans.

	2013 FSAP	2007–09 recession	1990s recession	1980s recession
Real GDP contraction (peak to trough, per cent)	-5.9	-4.2	-3.4	-5.1
Duration of recession (number of consecutive quarters of negative growth)	9	3	4	6
Peak increase in unemployment rate (percentage points)	5.9	2.4	4.1	5.8
House price correction (peak to trough, per cent)	-33.0	-7.6	-10.1	-4.2

Table 4: Stress-test scenario and historical comparison

Since much of the data is confidential, we only present aggregate results of the calibration publicly reported by the IMF in its 2013 FSAP exercise. Table 5 summarizes the data used along with the sources from which the calibration is based. Banks' balance sheet are calibrated using data from 2013Q1. The average common equity Tier 1 (CET1) capital ratio is 8.9%.

For the solvency risk module, banks' exposures at default (EAD), mean probabilities of default (PD) and loss given defaults (LGD) for different economic sectors were taken from the bottom-up stress-test results. The mean PDs, together with the co-variance matrix for defaults across sectors are used to generate a sequence of PDs for each bank, across the different sectors using a Student's t-distribution with 4 degrees of freedom. Averaging over the realizations, the aggregate expected loss to the banking sector is \$35 billion CAD, with a standard deviation of \$25 billion CAD.

Banks' balance sheets are calibrated using data from the first quarter of 2013. The average core equity tier 1 (CET1) capital ratio of banks is 8.9 percent. For the liquidity risk module, data on banks' liquid and illiquid assets were obtained from the Net Cumulative Cash Flow (NCCF) returns that all Canadian deposit-taking institutions must submit to the Office of the Superintendent of Financial Institutions (OSFI) on a monthly basis.⁵ Liquid assets are those that are unencumbered and eligible for central bank open market operations. These include, for example, cash and deposit accounts with the Bank of Canada, government securities (e.g., Canadian, U.S, Euro-area), bankers acceptances and National Housing Act Mortgage-Backed Securities (NHA MBS), which are government insured.

To derive secondary market haircuts, we begin with a detailed breakdown of banks'

⁵For further information see www.osfi-bsif.gc.ca/Eng/Docs/NCCF.pdf.

Module	Variables	Source
Solvency Risk	EAD, PD, LGD (by economic sectors)	Bottom-up stress test, reported by banks
	Historical covariance matrix of defaults	Bank of Canada internal model
	Operating income	Bottom-up stress test, reported by banks
Liquidity Risk	Liquid assets	Regulatory data
	Illiquid assets	Regulatory data
	Secondary market haircuts	Bank of Canada calibration, based on liaison with market participants
	Liabilities subject to a run	Regulatory data and Bank of Canada calibration based on international liquidity standards for the inclusion of funding instruments ranked by their stability
Network Effects	Interbank exposures	Regulatory data

Table 5: Data sources and calibration

assets into securities and loan portfolios. The securities include: sovereigns, mortgage-backed securities, asset-backed securities, corporate commercial paper, corporate bonds, equities, precious metals and other commodities. The loans include: residential mortgages (insured and uninsured), personal loans, credit cards, and business and government loans. For each of these items, we determine a haircut under normal, or ‘good’ economic conditions and also under stressed, or ‘bad’ conditions. Since transaction-level data was not available to back out the haircut, our estimates were based on conversations with banks and other market participants. We subsequently averaged over all loans and securities of all banks to obtain $h_g = 0.7$ and $h_b = 0.8$. Consequently, the per unit prices that banks obtain for these assets are $\psi_H = 0.3$ and $\psi_L = 0.2$, respectively.

Runnable liabilities for banks are obtained by aggregating different funding instruments and maturity profiles. We obtain data on maturing liabilities OSFI. We take into account international liquidity standards to weight the different funding instruments by their stability or likelihood to be rolled over. We then use the cumulative stock of debts coming to maturity within the first six-months of the stress-test horizon to represent the runnable liabilities in the interim date. On average, these represent 35% of total liabilities

across the six banks. Finally, The interest rate that each banks pays to its creditors was calculated as the banks’ weighted average cost of funding.

3.3 Results

For each bank, we compute the probability that its capital falls below the 7% regulatory threshold. This probability is calculated by averaging over 100x100 realizations of the interim and final date solvency shocks for each bank. We subsequently conduct an attribution analysis to determine the contributions of solvency risk, liquidity risk and information contagion to this probability. Table 6 summarizes the key findings of the stress test.

On average, the probability that a bank’s capital ratios breaches the regulatory threshold of 7% following the shock is 47%. In a crisis, uncertainty about the assets on a bank’s balance sheet increases and creditors become highly sensitive to a breach of the minimum capital requirement. A bank that breaches the threshold is deemed to be insolvent for the purposes of the exercise.⁶

Bank	Risks			Total
	Solvency	Liquidity	Contagion	
1	47.0	22.9	0.0	69.9
2	47.0	0.0	0.0	47.0
3	47.0	23.0	0.6	70.6
4	47.0	0.0	19.2	66.2
5	47.0	0.0	0.0	47.0
6	47.0	22.2	0.8	70.0

Table 6: Decomposition of risks

A bank’s balance sheet liquidity (BSL) is the ratio of its recourse to liquidity (the sum of liquid assets and sale of illiquid assets at the initial pooling price) and its liabilities that are subject to rollover risk.⁷ Assuming that secondary market investors hold prior belief $w_1 = 0.5$, the average BSL is $\bar{\lambda} = 1.08$, with three banks (banks 2, 4, 5) having a BSL ratio greater than one. For moderate shocks, and in the absence of information contagion, these banks are never illiquid. Whereas banks 1, 3, and 6 turn illiquid with positive probability, even without information contagion.

⁶Clearly, the standard criteria of negative net worth entails lower solvency and liquidity risk.

⁷The BSL measure is different from the LCR and should not be viewed as a proxy for the LCR. The denominator for the BSL represents liabilities subject to withdrawal at the interim date in the model and is calculated as all liabilities with a maturity date falling within six months of the start of the exercise. The LCR, on the other hand, considers cash outflows over a one-month horizon only. The two measures also differ in their assumptions on the proportion of liabilities that are subject to withdrawal, and on the haircuts to illiquid assets.

When we turn on the information contagion channel, secondary market investors update their belief about the macro-economy following the failure of banks 1, 3, and 6, and turn pessimistic. This, in turn, reduces bank 4’s recourse to liquidity and its BSL ratio falls below one. The probability that bank 4 is illiquid increases by 19.2%.

The system-wide loss distribution is determined as follows. Insolvent banks losses are equal to their credit shocks plus a bankruptcy cost, equal to 2.25% of risk-weighted assets. Banks that fail because of illiquidity suffer an additional bankruptcy cost, equal to the spread between the initial secondary market price (where $w_1 = 0.5$) and the price offered by investors after the contagion dynamics play out, multiplied by banks’ illiquid assets. Thus, the losses are proportional to the extent of information contagion.

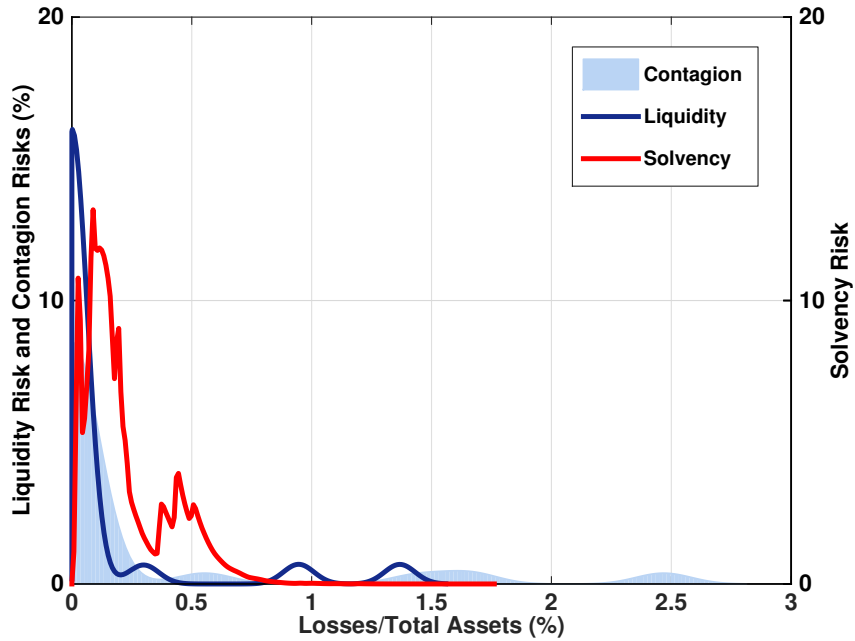


Figure 4: Aggregate loss distribution

Figure 4 shows how the three types of risk impact on the aggregate loss distribution (expressed as a percentage of total banking system assets). When only the direct impact of the stress scenario is considered (the red line called ‘solvency risk’), maximum system-wide losses do not exceed 2% of total assets. Average losses amount to less than 0.5% of total assets. Funding liquidity risk fattens the tail, however (the blue line, called ‘liquidity’). The tail of the distribution is fattened even further with the inclusion of information contagion risk (light blue shaded area) – now losses approach 3% of system assets. Thus, the failure to account for contagion risk can significantly underestimate the extent of systemic risk. Banks considered liquid in isolation can very easily be rendered illiquid due to a failure to internalize the system-wide effects of fire sales by pessimistic agents.

Table 7 presents a second set of simulations in which the stock of runnable liabilities for banks 2 and 5 are increased such that their BSL ratios are equivalent to bank 4. Banks 2 and 5 continue to withstand liquidity risk in Round 1. But following revisions to priors, the pooling price falls and their BSL ratios fall below one. This increases the probability of both banks succumbing to illiquidity due to contagion effects.

	Risks			
Bank	Solvency	Liquidity	Contagion	Total
1	47.0	22.9	0.0	69.9
2	47.0	0.0	22.6	69.6
3	47.0	23.0	0.6	70.6
4	47.0	0.0	19.2	66.2
5	47.0	0.0	19.7	46.7
6	47.0	22.2	0.8	70.0

Table 7: Decomposition of risks with lower BSLs for banks 2 and 5

We also investigate how changes in the prior belief, w_1 , and the price spread $\psi_g - \psi_l$ influence liquidity risk and contagion. The top panel of Figure 5 plots the total probability of liquidity risk as a function of initial beliefs. As secondary market investors become optimistic that macro-economy is in a good state, the probability of liquidity risk falls. The three different curves in the top panel reflect changes to the haircut, h_b , in the bad state. For the 10% (20%) curve, the haircut is increased by 10% (20%) relative to the baseline case (the 0% curve). Consequently, the price that investors offer in the bad state, ψ_b , is reduced by 10% (20%) relative to the baseline. A decrease in the low quality price increases the price spread and liquidity risk is heightened as a result. The middle panel of Figure 5 depicts the liquidity risk after the first round of contagion dynamics. As the prior belief increases, the first-round liquidity risk decreases more rapidly than the total risk. But, as demonstrated in the bottom panel of Figure 5, the decline in first-round liquidity risk is supplanted by an increase in contagion risk. When the price spread is large, the first-round liquidity risk is higher for larger values of the belief. As the belief continues to increase, the liquidity risk is replaced by contagion risk and total liquidity risk is unchanged. Figure 5 thus corroborates the theoretical result in Proposition 4.

4 Relaxing the simplifying assumptions

The stress-test results in the previous section were for a banking system wherein all banks have the same assets and credit losses, but differ in terms of their liability structures. Moreover, we ignored knock-on credit contagion between banks via the interbank market. In what follows, we relax these assumptions and investigate their implications for the stress-testing results.

4.1 Interbank credit contagion

We introduce the interbank contagion effects module described in Figures 2 and 3. As Gauthier et al. (2014) describe, this module is based on the Eisenberg and Noe (2001)

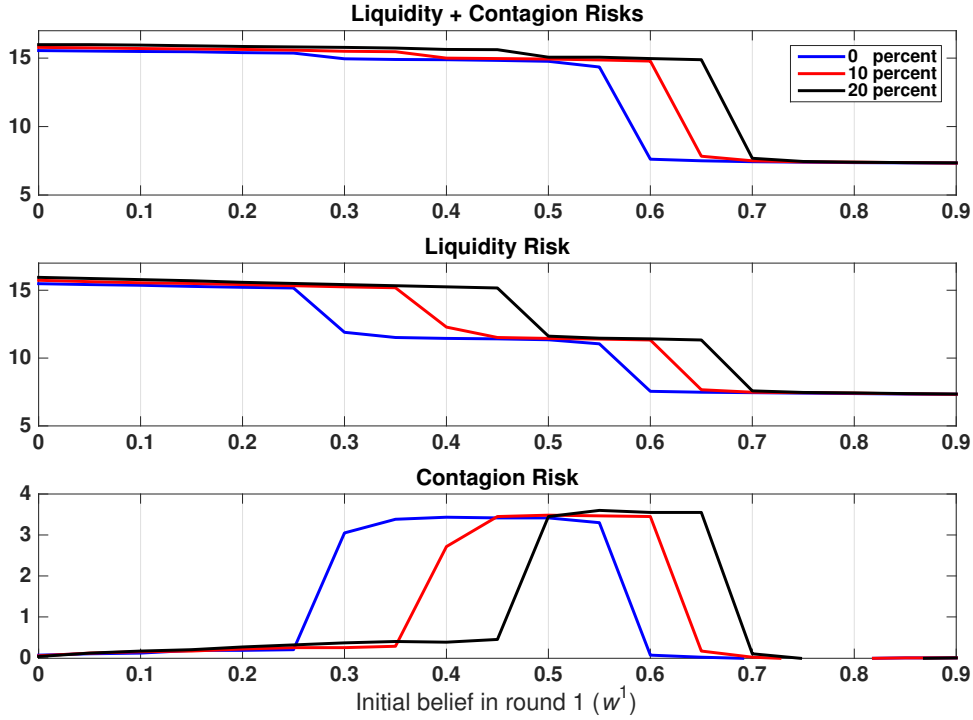


Figure 5: Decomposition of liquidity risk and contagion

clearing algorithm. Following realizations of $t = 1$ solvency shocks, runs by short term creditors and information contagion, and solvency shocks at $t = 2$, a subset of banks turn insolvent. Some of these banks may also have obligations to other solvent and insolvent banks, which cannot be serviced in full. The partial default of insolvent banks on their claims to solvent banks may, in turn, force the solvent banks into insolvency, as they cannot service their own interbank obligations in full. The Eisenberg and Noe (2001) algorithm formalizes this dynamic and calculates an endogenous ‘clearing-vector’ of payments that banks make to each other, and, thus, estimates the interbank losses. In addition, we assume that banks that turn insolvent due to network effects suffer a bankruptcy cost equal to 2.25% of risk-weighted assets.

For our stress-test simulations we use a snap-shot of the Canadian domestic interbank network from 2013Q1. The data is drawn from the ‘Interbank and Major Exposure Return’. This return collects information on exposures between the six Canadian Domestic Systemically Important Banks (D-SIBS), as well as their total exposures to other financial institutions, both domestic and foreign (for a certain number of geographical areas), and their major exposures to key domestic and foreign counterparties. The return is filed on a monthly basis and contains granular information on interbank exposures across different instruments, including: secured lending (repos), money market placements (including deposits and bankers acceptances), unsecured lending (including contingent lines of credit), marketable debt and equity securities, and Over-the-Counter derivatives exposures. For the stress-test, we aggregated exposures across all instruments. The average exposure is \$13 billion CAD.

Figure 6 illustrates the aggregate loss distribution with network effects. We clearly see a fattening of the tail of the loss distribution. To further investigate the effects, Figure

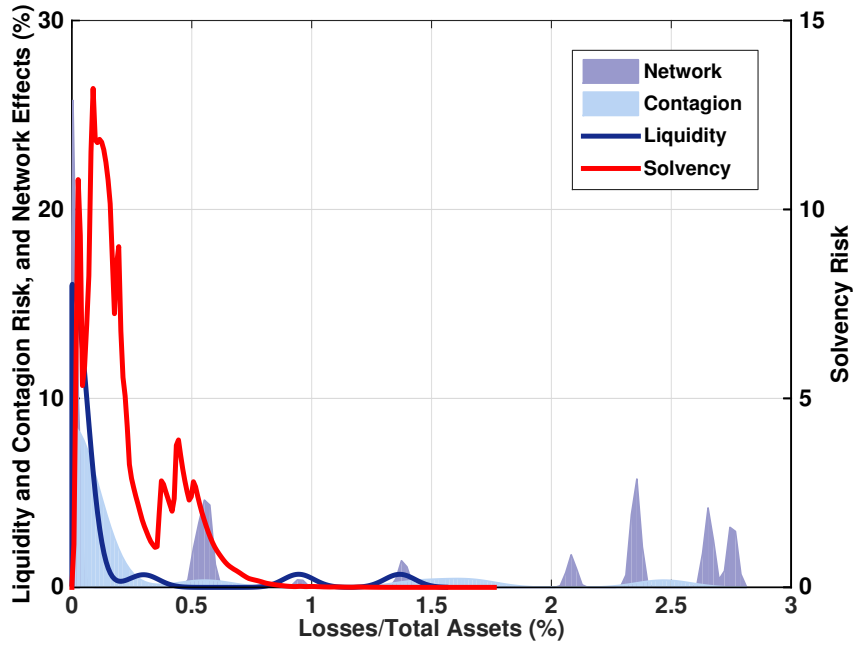


Figure 6: Aggregate loss distribution with network effects

7 plots the cumulative loss distributions with and without network effects. As expected, the loss distribution with network effects dominates the loss distributions without network effects in a first-order stochastic dominance sense. Moreover, the area between the cumulative loss distribution with network effects and the cumulative distribution without network effects measures the contribution of interbank network contagion to the aggregate loss distribution.

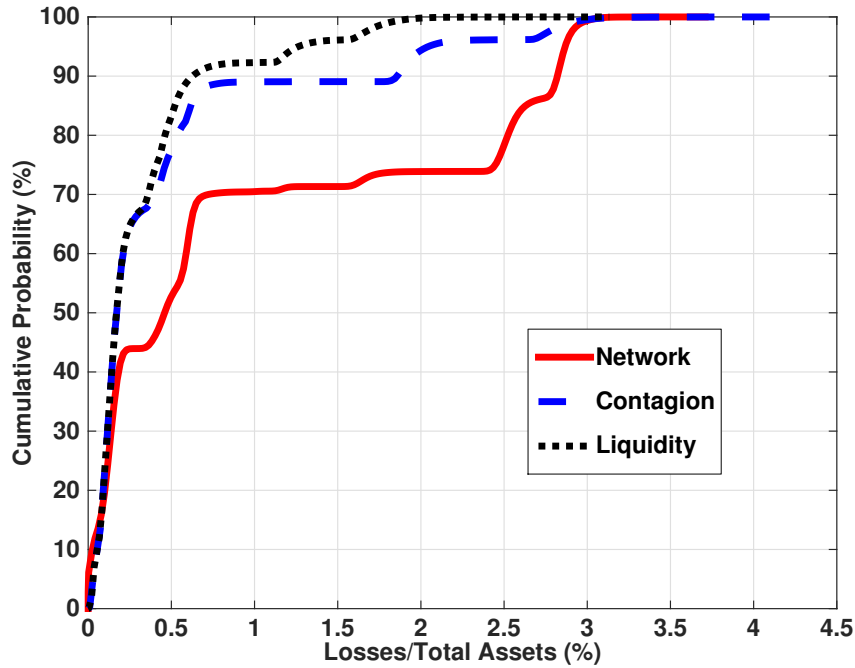


Figure 7: Cumulative loss distributions

4.2 Heterogenous balance-sheets

We finally consider the case where the different banks have different asset portfolios and loss distributions. To this end, we more fully utilize the granular balance sheet data and bottom-up stress-test results provided by the banks. Figure 8 reports the aggregate cumulative loss distribution. We note the following. The cumulative probability of obtaining a loss-to-total-assets ratio of 0.5% with network effects is, roughly, 65%. Compared with Figure 7, where we assumed a homogenous asset portfolio for banks, the cumulative probability was only 50%. The influences of information contagion and liquidity risk remain largely unchanged. The analysis, thus, suggests that, while network effects are less pronounced with heterogenous balance sheets, the results for liquidity risk and information contagion remain robust.

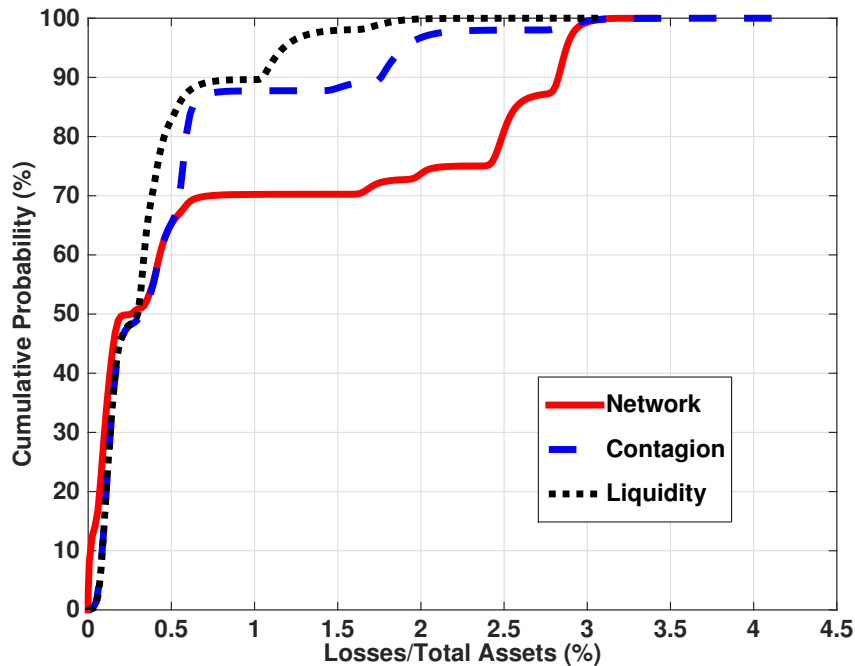


Figure 8: Cumulative loss distributions with heterogenous balance sheets

5 Conclusion

Our paper has proposed a tractable method for capturing information contagion in a top-down stress-testing framework. The key transmission mechanism is a two-way interaction between the adverse selection of secondary market investors and the coordination failure between the creditors of financial institutions. Bank runs emerge endogenously and the initial beliefs of investors crucially determines the trajectory of systemic stability. We show, in particular, that pessimism amongst investors can reduce banks' recourse to liquidity, with the inability to fire sell assets readily making them more susceptible to runs. This, in turn, generates a wave of investor pessimism.

We illustrate how such dynamics play out in a realistic stress-testing exercise, using the IMF's 2013 FSAP scenario calibrated to Canadian banking data. Information contagion

is shown to be potentially significant in this context, notwithstanding the simplifying assumptions used.

Systemic financial crises are invariably associated with large-scale information contagion. A proper understanding and quantification of the channels through which such contagion spreads is crucial for effective systemic-risk management. Our model represents a small first step towards an operational framework for dealing with this most challenging issue.

A Proof of Proposition 1

We establish the existence of a unique Bayesian equilibrium for each bank, b , in threshold strategies. [Morris and Shin \(2003\)](#) show that only threshold strategies survive the iterated deletion of strictly dominated strategies.

Each creditor, i of bank b uses a threshold strategy, x_{id}^b is round d , whereby debt is rolled over if and only if the private signal suggests that the credit shock is small, $x_{id}^b < x_d^{b*}$. Hence, for a given realization of the shock S_1^b , the proportion of creditors who do not roll over debt is

$$\ell_d^b(S_1^b, x_d^{b*}) = \text{Prob}(x_{id}^b > x_d^{b*} | S_1^b) = \text{Prob}(\epsilon_{id} > x_d^{b*} - S_1^b) = 1 - \frac{x_d^{b*} - S_1^b - \epsilon}{2\epsilon}. \quad (8)$$

The critical mass condition states that bank b is illiquid when the credit shock reaches a threshold S_d^{b*} , such that

$$\ell_d^b(S_d^{b*}, x_d^{b*}) = \frac{M^b + \bar{\psi}_d [Y^b - S_d^{b*}]}{ST^b}. \quad (9)$$

A creditor who receives the signal x_d^{b*} is indifferent between rolling over its debt and not. Thus, the indifferent condition is

$$G^b(E^b - S_d^{b*}) \lambda^b(S_d^{b*}; \bar{\psi}_d) = \frac{1}{1 + r^b}. \quad (10)$$

Equation (9) defines the critical signal, while Equation (10) defines the critical threshold for bank b . In the limit $\epsilon \rightarrow 0$, it is easy to verify that $x_d^{b*} \rightarrow S_d^{b*}$.

B Proof of Proposition 2

The belief at the end of round 1 is

$$\begin{aligned} w_2 &= \left(\text{Prob}(\eta_i = 0 | \psi = \psi_g) \text{Prob}(\eta_j = 0 | \psi = \psi_g) w^1 \right) \\ &\quad / \left(\text{Prob}(\eta_i = 0 | \psi = \psi_g) \text{Prob}(\eta_j = 0 | \psi = \psi_g) w^1 \right. \\ &\quad \left. + \text{Prob}(\eta_i = 0 | \psi = \psi_b) \text{Prob}(\eta_j = 0 | \psi = \psi_b) (1 - w^1) \right). \end{aligned} \quad (11)$$

To show that $w_2 > w_1$, we must have that

$$\begin{aligned} &\text{Prob}(\eta_i = 0 | \psi = \psi_g) \text{Prob}(\eta_j = 0 | \psi = \psi_g) \\ &\geq \text{Prob}(\eta_i = 0 | \psi = \psi_g) \text{Prob}(\eta_j = 0 | \psi = \psi_g) w^1 \\ &+ \text{Prob}(\eta_i = 0 | \psi = \psi_b) \text{Prob}(\eta_j = 0 | \psi = \psi_b) (1 - w^1), \end{aligned} \quad (12)$$

which, on rearranging, yields

$$\begin{aligned} &\text{Prob}(\eta_i = 0 | \psi = \psi_g) \text{Prob}(\eta_j = 0 | \psi = \psi_g) \\ &> \text{Prob}(\eta_i = 0 | \psi = \psi_b) \text{Prob}(\eta_j = 0 | \psi = \psi_b), \end{aligned} \quad (13)$$

which is always true.

C Proof of Proposition 3

The belief at the end of round 1 is

$$\begin{aligned}
w_2 &= \left(\text{Prob}(\eta_i = 0 | \psi = \psi_g) \text{Prob}(\eta_j = 1 | \psi = \psi_g) w^1 \right) \\
&\quad / \left(\text{Prob}(\eta_i = 0 | \psi = \psi_g) \text{Prob}(\eta_j = 1 | \psi = \psi_g) w^1 \right. \\
&\quad \left. + \text{Prob}(\eta_i = 0 | \psi = \psi_b) \text{Prob}(\eta_j = 1 | \psi = \psi_b) (1 - w^1) \right). \tag{14}
\end{aligned}$$

Consequently, for $w_2 < w_1$, we must have that

$$\text{Prob}(\eta_i = 0 | \psi = \psi_g) \text{Prob}(\eta_j = 1 | \psi = \psi_g) \tag{15}$$

$$< \text{Prob}(\eta_i = 0 | \psi = \psi_b) \text{Prob}(\eta_j = 1 | \psi = \psi_b), \tag{16}$$

which is identical to the condition in Equation (7).

D Proof of Proposition 4

Let us define

$$\rho_0 = \frac{G(S_{1H}^{b*})}{G(S_{1L}^{b*})}, \tag{17}$$

which is the left-hand side of the pessimism condition, and

$$\rho_1 = \frac{1 - G(S_{1L}^{b'*})}{1 - G(S_{1H}^{b'*})}, \tag{18}$$

which is the right-hand side. The derivative of ρ_0 with respect to ψ_g is

$$\frac{d\rho_0}{d\psi_g} = \frac{g(S_{1H}^{b*})}{G(S_{1L}^{b*})} \frac{dS_{1H}^{b*}}{d\psi_g} > 0, \tag{19}$$

while the derivative of ρ_1 with respect to ψ_g is

$$\frac{d\rho_1}{d\psi_g} = \frac{1 - G(S_{1L}^{b'*})}{\left(1 - G(S_{1H}^{b'*})\right)^2} g(S_{1H}^{b'*}) \frac{dS_{1H}^{b'*}}{d\psi_g} > 0. \tag{20}$$

From the implicit function theorem, it follows that $\frac{dS_{1H}^{b'*}}{d\psi_g} = \frac{dS_{1H}^{b*}}{d\psi_g}$. Moreover, since $g' < 0$, it follows that $g(S_{1H}^{b'*}) > g(S_{1H}^{b*})$. Finally, we have that $\frac{d\rho_1}{d\psi_g} > \frac{d\rho_0}{d\psi_g}$ as long as

$$\frac{1 - G(S_{1L}^{b'*})}{\left(1 - G(S_{1H}^{b'*})\right)^2} > \frac{1}{G(S_{1L}^{b*})}. \tag{21}$$

Turning to the effects of a change in ψ_b :

$$\frac{d\rho_0}{d\psi_b} = -\frac{G(S_{1H}^{b*})}{G(S_{1L}^{b*})^2} g(S_{1L}^{b*}) \frac{dS_{1L}^{b*}}{d\psi_b} < 0, \quad (22)$$

and

$$\frac{d\rho_1}{d\psi_b} = \frac{-1}{1 - G(S_{1H}^{b'^*})} g(S_{1L}^{b'^*}) \frac{dS_{1L}^{b'^*}}{d\psi_b} < 0. \quad (23)$$

As in the previous case, we have that $g(S_{1L}^{b'^*}) > g(S_{1L}^{b*})$, and that $\frac{dS_{1L}^{b'^*}}{d\psi_b} = \frac{dS_{1L}^{b*}}{d\psi_b}$. We thus obtain $\frac{d\rho_1}{d\psi_b} < \frac{d\rho_0}{d\psi_b}$ whenever

$$\frac{1}{1 - G(S_{1H}^{b'^*})} > \frac{G(S_{1H}^{b*})}{G(S_{1L}^{b*})^2}. \quad (24)$$

Combining Equations (21) and (24), we obtain the sufficient condition for our result that

$$1 - G(S_{1H}^{b'^*}) < \bar{G} \equiv \min \left\{ \frac{G(S_{1L}^{b*})^2}{G(S_{1H}^{b*})}, \sqrt{G(S_{1L}^{b*}) (1 - G(S_{1L}^{b*}))} \right\}. \quad (25)$$

E Proof of Proposition 5

Base case: In the case $N = 2$ at the end of Round 1, either both banks have turned illiquid, only one bank has turned illiquid or both remain liquid. In the first case, investors update their beliefs and become pessimistic, but there are no further actions to take. In the third case, Proposition 2 implies that investors become optimistic when they update their beliefs, and no banks suffer from illiquidity. Finally, if only one bank defaults, then investors may become more pessimistic when they update their beliefs. In the worst case, this will lead to the second bank turning illiquid in Round 2, after which there are no further actions, and the game terminates.

Induction Hypothesis: In the case of $N > 2$ banks, the game terminates after, at most, N Rounds.

Inductive Step: In the case of $N + 1$ banks, suppose that at the end of N Rounds, there are $N + 1 - k$ banks liquid and k banks illiquid, where $k \leq N + 1$. If $k = N + 1$, then all banks are illiquid, and the game ends. If $k = N$, then for the lone liquid bank, in round $N + 1$, investors update their beliefs and post a new pooling price. The creditors of the bank subsequently decide whether or not to withdraw. If they do not withdraw, then the bank remains liquid, and there is no further information to be gained for the investors, and the game terminates. If, however, they all withdraw, then the bank turns illiquid. While investors update their beliefs, there are no further actions to take and, hence, the game also terminates. For $k < N$, it follows that in round $N + 1 - k$ there were no new banks turning illiquid, and, hence, beliefs did not update, implying that the game terminated.

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