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Jonathan Jacob and Tobias Buer

Abstract A coalition of freight carriers is considered which has to decide how to allocate a pool of transport requests among its members. The literature is aware of a number of solution approaches which usually assume truthful behavior of the freight carriers. However, the used negotiation protocols are mostly not proven to enforce truthful behavior. This paper gives some insights into the impact of non-truthful behavior via computational experiments. We solve the collaborative problem via a genetic algorithm (GA) which is operated by an auctioneer. The GA's individuals are allocations of requests to carriers. To calculate the fitness of an individual, the carriers bid on the allocations. Bidding below a carrier's true valuation could ceteris paribus increase its profits. However, understated valuations can influence the search process negatively, in particular when a favoured allocation is dismissed wrongly. It is shown via computational experiments that for six tested instances, bidding non-truthfully is individually, but not collectively, rational and results in a kind of prisoner's dilemma.

1 Introduction

A way multiple freight carriers can establish a coalition is through collaborative transportation planning. Members of horizontal coalitions (i.e. carriers) try to increase their profits by exchanging some of their transport requests [5]. Through the exchange, they expect to find better tour plans that increase service quality and provide a higher utilization of resources. Empirical results show that horizontal collaborations are seen as beneficial, however, opportunistic behavior is perceived as a threat [5]. One of the main questions members of transport coalitions face is how to allocate requests in a way that is profitable to the coalition.

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Verdonck et al. [10] categorize request sharing techniques into either joint route planning or auction-based approaches. For joint route planning, a centralized decision maker is assumed who optimizes the decisions from the coalition's point of view. Auction-based approaches on the other hand consider that in most coalitions the carriers are autonomous, have therefore private information, and are self-interested. In an auction the requests are tendered, the carriers submit bids on the requests, and the auctioneer decides which bids win the auction. Individual carriers are responsible for their routing and valuation decisions [3, 4, 6, 11, 12] which appears to be a welcomed feature by many coalitions.

However, all of these recent studies assume truthful bidding. One reason may be that manipulations are non-trivial. The involved subproblems like the bid generation problem [4] or the winner determination problem [3] are hard to solve even without considering cheating. The Generalized Vickrey Auction, as an incentive compatible mechanism, is impracticable to apply for transport coalitions because of its high computational effort [3], its vulnerability to collusion by subsets of bidders, and its vulnerability to false-name bids [1].

In what follows, the transportation request assignment problem is introduced in Section 2 and a collaborative planning approach based on a genetic algorithm is presented in Section 3. In Section 4, the computational results on the impact of non-truthful bidding in a coalition of carriers are presented.

2 The Transportation Request Assignment Problem

Freight carriers collaborate by forming a coalition. A coalition is a set *A* of *n* selfinterested and independent agents, here denoted as carriers. The coalition considers a set *R* of freight requests for servicing. The following *pairwise disjoint* subsets of *R* are relevant: Each carrier $a \in A$ holds an initial set of requests $I_a \subset R$. These are private and not for exchange. For any request in I_a , carrier $a \in A$ is obliged to personally fulfill it or pay a penalty when it is not fulfilled. Furthermore, a broker (or one or more shippers) offers the coalition a set $P \subset R$ of requests. The coalition can either accept all requests in the pool *P* or reject all of them. If *P* is accepted, the requests have to be serviced or penalty costs incur. Altogether, *R* is defined as $R := \bigcup_{a \in A} I_a \cup P$.

The coalition's goal is to maximize the profit by jointly servicing R, taking into account that I_a are private information $(a \in A)$ and must not be revealed to other members of the coalition. The profit π_a of carrier $a \in A$ is defined in (1). It depends on a's allocated requests R_a (with $I_a \subseteq R_a \subseteq R$) and the winning bid price $b_a \in \mathbb{Z}$:

$$\pi_a(R_a, b_a) = p(R_a) + \frac{\sum_{i \in A} b_i}{n} - c(R_a) - b_a.$$
(1)

Profit π_a is *after* sharing the coalition's profit. The *income* of carrier *a* consists of $p(R_a)$, the sum of the paid prices for servicing requests in R_a (the price per request is given) and *a*'s share in the coalition's profit. The coalition's profit is calculated

from the sum of the winning bid prices. It is assumed to be distributed among the carriers in equal shares, i.e. $\frac{\sum_{i \in A} b_i}{n}$. The *expenses* of carrier *a* consist of its winning bid price b_a (negative prices are possible) and the costs $c(R_a)$ of its tour plan for servicing R_a . Basically, these are made up of the fixed costs per tour, the tour length costs and in particular of the penalty costs when some requests in R_a are *failed* to be serviced. We assume all requests $r \in R$ are pickup-and-delivery requests with time windows [9]. In addition, for each request $r \in R$, a price p_r and a penalty cost for non-fulfillment q_r are given. Therefore, in order to calculate $c(R_a)$, a carrier has to solve the well-known and *NP*-hard pickup-and-delivery problem with time windows (PDPTW) to service the requests in R_a for minimum cost. The extension to the traditional PDPTW is that requests bear penalty costs if they are not fulfilled.

In order to agree on an allocation of the pooled requests to the carriers, the coalition has to solve the transportation request assignment problem (TRAP), given by formulas (2) to (6). The TRAP is basically a bi-level optimization problem based on the set partitioning problem. The task is to find a partition of the set of pooled requests *P* that consists of *n* subsets. Each subset P_a is assigned to exactly one carrier.

$$\max \sum_{a \in A} \pi_a(P_a \cup I_a, b_a) \tag{2}$$

s.t.
$$\bigcup_{a \in A} P_a = P \tag{3}$$

$$P_i \cap P_j = \emptyset \quad \forall i, j \in A, i \neq j \tag{4}$$

$$\sum_{a \in A} b_a \ge 0 \tag{5}$$

$$b_a \in \mathbb{Z} \quad \forall a \in A$$
 (6)

The total profit (2) of the coalition should be maximized. All requests in the pool *P* have to be assigned to exactly one carrier, see (3) and (4). Furthermore, the sum of the carriers' bids has to be positive (5), otherwise it would be better for the coalition to reject *P*. In order to decide about the bid price b_a (6) on an allocation a carrier $a \in A$ has to calculate its marginal profits which requires solving the PDPTW.

3 A Genetic Algorithm with Bidding on Encoded Allocations

To solve the TRAP, Jacob and Buer [7] introduced a genetic algorithm (GA). Following [10], it is classified as an auction-based approach. It can be used by the mediator of the negotiation and enables collaboration of carriers while protecting private information to a large extent. The GA searches an allocation α , i.e., an assignment of all requests in *P* to carriers in *A*. To calculate the fitness of the individuals (i.e., the allocations) the carriers only revealed their ranking of the allocations; cost information remained private which is an important feature. However, the surplus profit generated by the coalition was also unknown and could not be distributed between the members of the coalition.

In order to overcome this deficit, we now propose the carriers should evaluate an allocation via a – possibly negative – monetary value, i.e., they should bid on an allocation. One distinctive feature is that only bids on complete allocations are allowed; in contrast to bidding on subsets of the auctioned request which includes as special cases bids on single requests or bids on request bundles that are tours. Although a carrier has to reveal its price for an allocation, the revealed cost structure is much less detailed than, e.g., prices of sets of requests. In addition, the sum of the bid prices for an allocation is a nice indicator for the coalition's surplus profit. The main features of the GA are as follows.

Encoding of an individual. An individual of the GA represents an allocation α of requests to carriers. It is a sequence of carriers $a \in A$ of length |P|. Each position of the sequence represents a request in *P*. For example, the individual $\alpha = (3, 1, 3, 2)$ represents an allocation of four requests where carrier a_1 gets request 2, a_2 gets request 4, and carrier a_3 receives 1 and 3.

Fitness value. Different from [7], the fitness of an individual is calculated as the sum of the bid prices. A bid $b_a(\alpha)$ of carrier $a \in A$ may be positive or negative (see below).

Crossover and mutation. A standard 2-point-crossover is applied with a probability of 90 percent. Next, mutation is applied with a 30 percent probability. If an individual is mutated, the carrier at each position is replaced by a random one with a probability of 10 percent.

Truthful bidding on an allocation. In order to calculate the fitness of an individual, each carrier bids on an allocation. To start with and in line with the vast majority of the literature [3, 4, 6, 11, 12], we assume truthful bidding. Given an allocation α , each carrier $a \in A$ calculates its bid price $b_a(\alpha)$. To this end, each $a \in A$ solves a PDPTW taking into account its initial requests I_a and its additional requests P_a for each individual in each generation. Therefore, our mechanism is computationally challenging. We use an adaptive large neighborhood search [9] to generate a set of feasible tours; then we select a proper subset of tours via solving a set covering problem. From this solution we calculate the bid price b_a that equals the marginal profit resulting from servicing P_a in addition to I_a (taking penalty costs into account). Another benefit is that in this way the marginal profit of the coalition is revealed. Note, a bid on the same allocation in a later iteration may only be increased.

Incentives for non-truthful bidding on an allocation. Our GA-based auction protocol is not proven to enforce truthful bidding. On the winning allocation, a carrier $a \in A$ increases its profit by decreasing its bid price $b_a(\alpha)$. However, the lower the sum of the bids on an allocation are, the lower are its chances to get chosen.

The question is: how strong can a non-truthful carrier understate its preferences? Non-truthful bidding is implemented via calculating the bid price according to (7). The bid price is based on the concept of marginal profits. The income of all serviced requests is $p(P_a \cup I_a)$. The expenses of the serviced requests $c(P_a \cup I_a)$ are modified by the strategy δ_a , where $\delta_a = 0$ indicates truthful bidding and $\delta_a > 0$ indicates nontruthful bidding. Without collaboration, the profit for servicing the initial requests

 I_a is denoted by $\overline{\pi}_a$. A negative bid price indicates the amount of money required to compensate the carrier for its losses due to collaborating.

$$b_a(\alpha) = p(P_a \cup I_a) - (1 + \delta_a) \cdot c(P_a \cup I_a) - \overline{\pi}_a \tag{7}$$

This non-truthful bidding scheme is implemented by a carrier consistently for all bids on all allocations throughout the complete negotiation process. The share of the coalition's profit is not considered. As the computational results in the next section show, this leads essentially to a prisoner's dilemma.

4 Results on Non-truthful Bidding and Discussion

For our tests, we created six Euclidian TRAP instances T2-1 to T2-6, each with two carriers (n = 2), twenty initial requests per carrier ($|I_a| = 20, a \in A$), and a pool of forty requests (|P| = 40). For each request $r \in R$ a price p_r was randomly chosen between 50 and 150, and a penalty cost q_r was randomly chosen between 200 and 300. Every time the GA presents an allocation α to a carrier, the carrier bids $b_a(\alpha)$ according to equation (7).

The parameter δ_a determines the bidding strategy of carrier $a \in A$. Truthful bidding is implied by $\delta_a = 0$. The greater δ_a , is the stronger a exaggerates its true costs and the lower are its bid prices. Table 1 shows the payoff matrix for a = 1, 2 and $\delta_a = 0.0, 0.35, 0.7$. The average *marginal* payoffs (Δ_1, Δ_2) for carrier 1 and carrier 2 over the instances T2-1 to T2-6 are given. Marginal payoff Δ_a is the profit of carrier $a \in A$ in the case of collaboration minus the profit without collaboration.

Table 1 Payoff matrix of averaged marginal profits (Δ_1, Δ_2)

Carrier 2

		$\delta_2 = 0.00$	$\delta_2 = 0.35$	$\delta_2 = 0.70$
	$\delta_1 = 0.00$	(1222, 1222)	(677, 1701)	(292, 1545)
Carrier 1	$\delta_1 = 0.35$	(1730,765)	(917, 1016)	(435,757)
	$\delta_1 = 0.70$	(1606, 329)	(396,219)	(357, 292)

Assume now that each carrier knows those payoff matrices from observation and sees them as a means of predicting future payoffs. Then, the different values of δ_a can be interpreted as each carrier *a*'s strategy in a game. Assuming rational behavior, carrier 1 will choose $\delta_1 = 0.35$ and carrier 2 will choose $\delta_2 = 0.35$ since this is the only Nash equilibrium [2]. But, if the carriers chose $\delta_1 = 0$ and $\delta_2 = 0$, they would both be better off. So apparently, collective rationality is not given. This holds also for the three carrier case, as is shown in the appendix.

A possible instrument to induce truthful bidding is to introduce a deposit that each carrier has to pay in order to become a part of the coalition. If P gets success-

fully allocated, each carrier gets its deposit back. If, however, no feasible solution of the TRAP can be found, the deposits get returned unevenly: The higher a carrier's average bids are, the higher will be the amount it receives. How to choose the amount of the deposit and the exact mechanism to return the deposits in case no feasible solution is found may be the object of future research.

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6

Appendix

Apart from the tests on the two carrier case, we also ran tests on six instances T3-1 to T3-6 with three carriers. As for the two player case, we assumed $\delta_a \in \{0.0, 0.35, 0.7\}$ for all carriers $a \in A$ and that the carriers are able to observe average payoff matrices and use them to predict future payoffs. For finding equilibria on our results for the three carrier case, we used the software Gambit [8]. We ran two different types of tests.

Firstly, we assumed that one of the three carriers will choose a certain value of δ_a in advance, which the other carriers correctly anticipate. We tested this case for all $a \in A$ being the one choosing a fixed value of δ_a . The payoff matrices and equilibrium strategies for this case are shown in Tables 2 to 13.

Secondly, we assumed that no carrier chooses its strategy in advance. The equilibrium strategies for this case are shown in Table 14.

As Table 15 shows, there are some equilibrium strategies where single carriers are better off than in the collaborative case. However, in terms of social welfare, i.e., the sum of all carriers' profits, collaboration is optimal. It should be noted that some of the found equilibria ($\{1,9,15,19\},\{4,12,14,18\},\{11,22\}$) consist of essentially the same strategies; however, since they were reached under different assumptions, we list them separately.

Since without predetermined strategies for any carrier there exists an equilibrium (see Table 15, eq. 19) where one carrier improves its payoff in comparison to the collaborative case, the three carrier case tested is not a pure prisoner's dilemma. Still, in four out of five equilibria all carriers are worse off than in the collaborative case. Also, assuming that each equilibrium gets reached with the same probability, on average, all carriers are worse off in the non-collaborative case. So the general conclusion holds and developing an auction protocol for solving the TRAP which enforces truthful bidding should be beneficial to all carriers.

		Carrier 2			
		$\delta_2=0.00$	$\delta_2 = 0.35$	$\delta_2 = 0.70$	
	$\delta_1 = 0.00$	(1288, 1288, 1288)	(1047, 1980, 1047)	(668, 2555, 668)	
Carrier 1	$\delta_1 = 0.35$	(1924, 987, 987)	(1464, 1545, 1047)	(1236, 2043, 406)	
	$\delta_1 = 0.70$	(2208,679,679)	(2027, 1263, 428)	(1062, 1083, 142)	

Table 2 Payoff matrix of averaged marginal profits $(\Delta_1, \Delta_2, \Delta_3)$ with $\delta_3 = 0.00$

Table 3 Payoff matrix of averaged marginal profits $(\Delta_1, \Delta_2, \Delta_3)$ with $\delta_3 = 0.35$

			Carrier 2	
		$\delta_2 = 0.00$	$\delta_2 = 0.35$	$\delta_2 = 0.70$
	$\delta_1 = 0.00$	(871, 871, 1848)	(532, 1547, 1585)	(293, 1835, 1139)
Carrier 1	$\delta_1 = 0.35$	(1345, 531, 1424)	(1178, 1010, 1078)	(540, 1061, 575)
	$\delta_1 = 0.70$	(1835, 309, 1159)	(367,229,219)	(329, 317, 195)

Table 4 Payoff matrix of averaged marginal profits $(\Delta_1, \Delta_2, \Delta_3)$ with $\delta_3 = 0.70$

			Carrier 2	
		$\delta_2 = 0.00$	$\delta_2 = 0.35$	$\delta_2 = 0.70$
	$\delta_1 = 0.00$	(494, 494, 2547)	(273, 928, 1491)	(107, 679, 714)
Carrier 1	$\delta_1 = 0.35$	(939, 344, 1574)	(571,586,992)	(156, 289, 361)
	$\delta_1 = 0.70$	(666,61,691)	(325, 177, 326)	(0, 0, 0)

Table 5 Equilibrium strategies for carriers 1 and 2 with fixed δ_3

		$p(\delta_1)$			$p(\delta_2)$		
		0.00	0.35	0.70	0.00	0.35	0.70
	eq. 1	0.0%	0.0%	100.0%	0.0%	100.0%	0.0%
$\delta_3 = 0.00$	eq. 2	0.0%	100.0%	0.0%	0.0%	0.0%	100.0%
	eq. 3	0.0%	26.5%	73.5%	0.0%	23.6%	76.4%
$\delta_3 = 0.35$	eq. 4	0.0%	100.0%	0.0%	0.0%	0.0%	100.0%
$\delta_{3} = 0.70$	eq. 5	0.0%	100.0%	0.0%	0.0%	100.0%	0.0%

Table 6 Payoff matrix of averaged marginal profits $(\Delta_1, \Delta_2, \Delta_3)$ with $\delta_2 = 0.00$

		Carrier 3	
	$\delta_3=0.00$	$\delta_3 = 0.35$	$\delta_3 = 0.70$
.00	(1288, 1288, 1288)	(871,871,1848)	(494, 494, 254

	$\delta_1 = 0.00$	(1288, 1288, 1288)	(871, 871, 1848)	(494, 494, 2547)
Carrier 1	$\delta_1 = 0.35$	(1924, 987, 987)	(1345, 531, 1424)	(939, 344, 1574)
	$\delta_1 = 0.70$	(2208,679,679)	(1835, 309, 1159)	(666,61,691)

Table 7 Payoff matrix of averaged marginal profits $(\Delta_1, \Delta_2, \Delta_3)$ with $\delta_2 = 0.35$

		Carrier 3			
		$\delta_3=0.00$	$\delta_3 = 0.35$	$\delta_3 = 0.70$	
	$\delta_1 = 0.00$	(1047, 1980, 1047)	(532, 1547, 1585)	(173, 928, 1491)	
Carrier 1	$\delta_1 = 0.35$	(1464, 1545, 663)	(1178, 1010, 1078)	(571,586,992)	
	$\delta_1 = 0.70$	(2027, 1263, 428)	(367,229,219)	(325, 177, 326)	

Table 8 Payoff matrix of averaged marginal profits $(\varDelta_1,\varDelta_2,\varDelta_3)$ with $\delta_2=0.70$

			Carrier 3	
		$\delta_3=0.00$	$\delta_3 = 0.35$	$\delta_3=0.70$
	$\delta_1 = 0.00$	(668, 2555, 668)	(293, 1835, 1139)	(107, 679, 714)
Carrier 1	$\delta_1 = 0.35$	(1236, 2043, 406)	(540, 1061, 575)	(156, 289, 361)
	$\delta_1 = 0.70$	$\left(1062,1083,142\right)$	(329, 317, 195)	(0, 0, 0)

Table 9 Equilibrium strategies for carriers 1 and 3 with fixed δ_2

			$p(\delta_1)$			$p(\delta_3)$		
		0.00	0.35	0.70	0.00	0.35	0.70	
	eq. 6	0.0%	0.0%	100.0%	0.0%	100.0%	0.0%	
$\delta_2 = 0.00$	eq. 7	0.0%	100.0%	0.0%	0.0%	0.0%	100.0%	
	eq. 8	0.0%	75.7%	24.3%	0.0%	35.8%	64.2%	
	eq. 9	0.0%	0.0%	100.0%	100.0%	0.0%	0.0%	
$\delta_2 = 0.35$	eq. 10	0.0%	100.0%	0.0%	0.0%	100.0%	0.0%	
	eq. 11	0.0%	23.7%	76.3%	30.4%	0.0%	69.6%	
$\delta_2 = 0.70$	eq. 12	0.0%	100.0%	0.0%	0.0%	100.0%	0.0%	

Table 10	Payoff matrix	of averaged	marginal	profits ($(\Delta_1, \Delta_2, \Delta_3)$) with $\delta_1 = 0.00$
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		Carrier 3					
		$\delta_3=0.00$	$\delta_3 = 0.35$	$\delta_3=0.70$			
Carrier 2	$\delta_2 = 0.00$	(1288, 1288, 1288)	(871,871,1848)	(494, 494, 2547)			
	$\delta_2 = 0.35$	(1047, 1980, 1047)	(532, 1547, 1585)	(273, 928, 1491)			
	$\delta_2 = 0.70$	(668, 2555, 668)	(293, 1835, 1139)	(107, 679, 714)			

Table 11 Payoff matrix of averaged marginal profits $(\varDelta_1, \varDelta_2, \varDelta_3)$ with $\delta_l = 0.35$

		Carrier 3					
		$\delta_3 = 0.00$	$\delta_3 = 0.35$	$\delta_3 = 0.70$			
	$\delta_2 = 0.00$	(1924, 987, 987)	(1345, 531, 1424)	(939, 344, 1574)			
Carrier 2	$\delta_2 = 0.35$	(1464, 1545, 663)	(1178, 1010, 1078)	(571, 586, 992)			
	$\delta_2 = 0.70$	(1236, 2043, 406)	(540, 1061, 575)	(156, 289, 361)			

Table 12 Payoff matrix of averaged marginal profits $(\varDelta_1,\varDelta_2,\varDelta_3)$ with $\delta_l=0.70$

		Carrier 3					
		$\delta_3=0.00$	$\delta_3 = 0.35$	$\delta_3 = 0.70$			
Carrier 2	$\delta_2 = 0.00$	(2208,679,679)	(1835, 309, 1159)	(666,61,691)			
	$\delta_2 = 0.35$	(2027, 1263, 428)	(367,229,219)	(325, 177, 326)			
	$\delta_2 = 0.70$	(1062, 1083, 142)	(329, 317, 195)	(0, 0, 0)			

Table 13 Equilibrium strategies for carriers 2 and 3 with fixed δ_1

		$p(\delta_2)$			$p(\delta_3)$		
		0.00	0.35	0.70	0.00	0.35	0.70
$\delta_1 = 0.00$	eq. 13	0.0%	0.0%	100.0%	0.0%	100.0%	0.0%
$\delta_1 = 0.35$	eq. 14	0.0%	0.0%	100.0%	0.0%	100.0%	0.0%
	eq. 15	0.0%	100.0%	0.0%	100.0%	0.0%	0.0%
$\delta_1 = 0.70$	eq. 16	0.0%	0.0%	100.0%	0.0%	100.0%	0.0%
	eq. 17	0.0%	20.2%	79.8%	0.0%	32.8%	67.2%

Table 14 Equilibrium strategies for carriers 1, 2, and 3

	$p(\boldsymbol{\delta}_1)$			$p(\delta_2)$			$p(\delta_3)$		
	0.00	0.35	0.70	0.00	0.35	0.70	0.00	0.35	0.70
eq. 18	0.0%	100.0%	0.0%	0.0%	0.0%	100.0%	0.0%	100.0%	0.0%
eq. 19	0.0%	0.0%	100.0%	0.0%	100.0%	0.0%	100.0%	0.0%	0.0%
eq. 20	0.0%	21.8%	78.2%	0.0%	51.9%	48.1%	71.5%	28.5%	100.0%
eq. 21	0.0%	30.1%	69.9%	0.0%	80.4%	19.6%	55.5%	28.1%	16.3%
eq. 22	0.0%	23.7%	76.3%	0.0%	100.0%	0.0%	30.4%	0.0%	69.6%

 Table 15 Expected payoffs in equilibrium strategies

		$E(\Delta_1)$	$E(\Delta_2)$	$E(\Delta_3)$	$\sum_{a\in A}(E(\Delta_a))$
	eq. 1	2027	1263	428	3718
	eq. 2	1236	2043	406	3685
fixed δ_3	eq. 3	1290	1338	278	2906
	eq. 4	540	1061	575	2176
	eq. 5	571	586	992	2149
	eq. 6	1835	309	1159	3303
	eq. 7	939	344	1547	2830
	eq. 8	1084	348	1360	2792
fixed δ_2	eq. 9	2027	1263	428	3718
	eq. 10	1178	1010	1078	3266
	eq. 11	843	595	484	1922
	eq. 12	540	1061	575	2176
	eq. 13	293	1835	1139	3267
	eq. 14	540	1061	575	2176
fixed δ_1	eq. 15	2027	1263	428	3718
	eq. 16	329	317	195	841
	eq. 17	639	569	200	1408
	eq. 18	540	1061	575	2176
no fixed δ_a	eq. 19	2027	1263	428	3718
	eq. 20	1216	1061	345	2622
	eq. 21	1164	928	444	2536
	eq. 22	843	594	484	1921
collaborative case		1288	1288	1288	3864