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The Forward-looking Disclosures of Corporate Managers: Theory and Evidence

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The Forward-looking Disclosures of Corporate Managers: Theory and Evidence*

Abstract

We consider an infinitely repeated game in which a privately informed, long-lived manager raises funds from short-lived investors in order to finance a project. The manager can signal project quality to investors by making a (possibly costly) forward-looking disclosure about her project's potential for success. We find that if the manager's disclosures are costly, she will never release forward-looking statements that do not convey information to external investors. Furthermore, managers of firms that are transparent and face significant disclosure-related costs will refrain from forward-looking disclosures. In contrast, managers of opaque and profitable firms will follow a policy of accurate disclosures. To test our findings empirically, we devise an index that captures the quantity of forward-looking disclosures in public firms' 10-K reports, and relate it to multiple firm characteristics. For opaque firms, our index is positively correlated with a firm's profitability and financing needs. For transparent firms, there is only a weak relation between our index and firm fundamentals. Furthermore, the overall level of forward-looking disclosures declined significantly between 2001 and 2009, possibly as a result of the 2002 Sarbanes-Oxley Act.

Keywords: repeated games, asymmetric information, firms, reputation

JEL Classification: C73, D82, G30, L14

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1. Introduction

Certain statements in this report, other than purely historical information, including estimates, projections, statements relating to our business plans, objectives, and expected operating results, and the assumptions upon which those statements are based, are “forward-looking statements” within the meaning of the Private Securities Litigation Reform Act of 1995, Section 27A of the Securities Act of 1933 and Section 21E of the Securities Exchange Act of 1934. Forward-looking statements may appear throughout this report, including without limitation, the following sections: "Business," "Management’s Discussion and Analysis," and "Risk Factors." These forward-looking statements generally are identified by the words "believe," "project," "expect," "anticipate," "estimate," "intend," "strategy," "future," "opportunity," "plan," "may," "should," "will," "would," "will be," "will continue," "will likely result," and similar expressions. Forward-looking statements are based on current expectations and assumptions that are subject to risks and uncertainties which may cause actual results to differ materially from the forward-looking statements. A detailed discussion of risks and uncertainties that could cause actual results and events to differ materially from such forward-looking statements is included in the section titled "Risk Factors" (Part I, Item 1A of this Form 10-K). We undertake no obligation to update or revise publicly any forward-looking statements, whether because of new information, future events, or otherwise.

The above disclaimer from Microsoft’s 10-K report for fiscal year 2013 typifies the cautious nature of forward-looking statements made by corporate managers. It demonstrates that managers are concerned about the legal costs that could result from forward-looking disclosures, even though several regulations (like the Private Securities Litigation Reform Act of 1995) grant such disclosures a “safe harbor” status. Furthermore, it suggests that the information contained in managers’ forward-looking disclosures is less reliable than backward-looking, historical information. Hence, the value of such disclosures to external investors is unclear, and arguably depends to a high degree on the credibility of the management which releases them. In light of that ambiguity, the Securities and Exchange Commission (SEC) traditionally prohibited the inclusion of forward-looking statements in a firm’s public filings, thereby restricting managers’ disclosures to historical information. In 1973, however, the SEC repealed that prohibition, and subsequently encouraged corporate managers to include forward-looking information in their SEC filings.

Because a manager has direct control over her forward-looking disclosures, she could use them as a convenient means of reducing the information asymmetry between firm insiders and outside investors, thereby securing more favorable financing terms for her firm. Prima facie, the information content of such disclosures is ambiguous: Since her forward-looking disclosures are not verifiable ex ante, a manager might try to improve her firm’s financing terms via the release of overly optimistic statements - that is, she might attempt to mislead external investors. At the extreme, her disclosures might be totally uninformative to external investors (as in a “babbling
equilibrium”). However, anecdotal evidence suggests that a manager’s forward-looking disclo-
sures are no cheap talk; rather, they incur costs for her firm, such as legal costs or proprietary
costs. Furthermore, since her forward-looking disclosures can be verified ex post, a manager’s
repeated interaction with external investors suggests that such disclosures are subject to reput-
ation effects. In such a case, the credibility of a manager’s future disclosures is affected by the
accuracy of her current disclosures, thereby creating an intertemporal incentive that shapes her
disclosure policy. Consequently, it is risky for a manager to mislead investors by making overly
optimistic forward-looking disclosures, because doing so might harm her reputation and, as a
result, her ability to affect her firm’s financing terms in the future. Hence, corporate managers
face a trade-off between the immediate gain from an overly optimistic statement and the benefit
from retaining her reputation in the future.

Our study explores the economic mechanisms that govern a manager’s forward-looking dis-
closures in the presence of disclosure-related costs and reputation effects. We therefore build
on the framework of Mathis et al. (2009), and consider an infinitely repeated game where a
privately informed, long-lived manager must raise funds from short-lived investors in order to
finance some project. A project may be of good or bad quality, where good (bad) projects
offer a certain (uncertain) payoff and have a positive (negative) net present value. In order
to signal investors that her project is of good quality, the manager can make an optimistic
forward-looking disclosure about the project’s success potential. However, the manager’s firm
may incur costs on account of that disclosure. There are two types of managers in our model:
Honest managers, who make a public disclosure if and only if (iff) they have access to a good
project, and opportunistic managers, who follow a disclosure policy that maximizes their firm’s
discounted sum of expected payoffs. A manager’s reputation is captured by the probability
$q \in [0, 1]$ that investors assign to the event that she is honest. A strategy $x_d$ of the opportunis-
tic manager then specifies for each reputation $q$ her disclosure policy $x_d(q)$. After observing
a manager’s public disclosure, external investors assign a probability to the good project type
that depends on the manager’s reputation, denoted by $\alpha_p(q)$. A Markov perfect equilibrium
requires that for each $q \in [0, 1]$, the opportunistic manager’s disclosure policy $x_d(q)$ is optimal,
and the assigned probability $\alpha_p(q)$ is correctly specified.

We find that, when the manager’s forward-looking disclosures are associated with costs, she will
never make public disclosures that convey no information to external investors. The intuition for this result is simple: If the release of forward-looking statements is associated with costs, but offers no benefit, a manager always prefers to refrain from public disclosures. Since one can plausibly assume that the forward-looking disclosures of corporate managers are indeed associated with costs, this finding suggests that their actual disclosures do contain information for external investors. Furthermore, we find that the managers of transparent firms subject to significant disclosure-related costs never release forward-looking statements. The reason is that, for transparent firms, the information asymmetry between firm insiders and outsiders is less severe, such that the manager’s forward-looking disclosures yield the firm only a small benefit. In contrast, a policy of accurate disclosures by the opportunistic manager can be sustained for firms that are opaque or profitable. The intuition is that the managers of such firms find it valuable to retain their credibility among external investors, and that they don’t want to jeopardize this credibility by misleading current investors through overly optimistic disclosures. Under certain conditions, the release of accurate forward-looking statements is the only equilibrium policy of the manager.

In order to test our findings empirically, we construct an index that captures the quantity of forward-looking disclosures in a firm’s annual 10-K report, and link this index to a number of firm characteristics. In line with our theoretical results, we find that among opaque (i.e. unrated) firms, the index is positively correlated with a firm’s profitability and financing needs. Among transparent (i.e. rated) firms, there is a considerably weaker relation between our index and firms’ fundamentals. In addition, we find that the overall level of forward-looking disclosures saw a significant decline between 2001 and 2009. That evolution may reflect the enactment of the Sarbanes-Oxley Act in 2002, which increased the transparency of public firms to external investors.

The paper is organized as follows: Section 2 reviews the related literature, and Section 3 describes our model setup. In Section 4, we define an equilibrium concept and derive several equilibrium properties. Section 5 details the construction of our index and presents our empirical results. Section 6 concludes.
2. Literature

Our study contributes to the literature on the voluntary disclosure of private information. The defining element of this literature is an informed insider (e.g. a manager) who decides on the disclosure of her private information to uninformed outsiders (e.g. external investors). Initial studies on this topic argued that it would be optimal for the insider to make a full disclosure of her private information (Grossman and Hart (1980), Grossman (1981)). The underlying idea is that if the insider withholds some of her private information, outsiders interpret this as a negative signal, with the result that the insider would always be better off by making a full disclosure. However, the result that an insider fully discloses her private information rests on quite restrictive assumptions, inter alia the assumptions that her disclosures are costless and always truthful (Beyer et al. (2010)). Subsequent researchers loosened these assumptions, and obtained deviating results: Verrecchia (1983) and Dye (1986) show in a static framework that in the presence of disclosure-related costs, it might be optimal for an informed manager to withhold her private information instead of disclosing it to external investors. Sobel (1985) introduces a repeated cheap-talk game where the insider may misrepresent her private information, and who may be of two types: A trustworthy type, or an opportunistic type. Sobel shows that reputation concerns by the opportunistic insider incentivise her to disclose her private information truthfully in certain situations. Similar to us, Stocken (2000) employs a framework of repeated interaction between a corporate manager and an outside investor in order to examine the credibility of the manager’s unaudited disclosures of her private noisy information. He finds that under certain conditions, there always exists an equilibrium where the manager almost always discloses her private information truthfully. To our knowledge, ours is the first study which employs a repeated framework where the insider may decide to misrepresent as well as to withhold her private information.

Our work also contributes to the literature which relates a firm’s financing terms to its disclosure policy: Barry and Brown (1985) use a model with multiple securities that feature asymmetric parameter uncertainty, and show that investors’ estimation risk may affect a security’s expected return in equilibrium. Coles and Loewenstein (1988) extend this result to the case of symmetric parameter uncertainty. Both findings suggest that an increase in a firm’s information disclosure reduces its equity costs. Diamond and Verrecchia (1991) develop a model where corporate
disclosures increase the liquidity of a firm’s securities, with the result that its capital costs are lower. The findings of Easley and O’Hara (2004) suggest that more disclosures by a firm might lower its cost of capital, due to a reduced information risk. Cheynel (2013) employs a model where a firm’s disclosures are voluntary, and shows that firms that disclose their information feature lower capital costs than firms that do not disclose.

Further, our study is related to the game-theoretic literature on reputation building, established by the seminal work of Kreps and Wilson (1982) as well as Milgrom and Roberts (1982). A number of authors from this literature employed a framework of repeated cheap talk, similar to the one used by Sobel (1985): Benabou and Laroque (1992) consider an infinitely repeated cheap-talk game where the sender’s private information is noisy, and find that she can repeatedly distort her messages without losing her credibility. Morris (2001) also considers a repeated cheap-talk game, and shows that in certain situations, reputation concerns by the sender may inhibit instead of promote the transmission of information to the receiver. Similar to us, some authors explicitly focused on the role of reputation in financial markets: Diamond (1989) examines how reputation effects may alleviate problems of moral hazard in debt markets, and concludes that reputation concerns have a disciplinary effect on borrowers with a good reputation, but not on those with a bad reputation. Mathis et al. (2009) use an infinitely repeated cheap-talk game in order to examine whether reputation concerns induce credit rating agencies to provide reliable ratings. They find that an equilibrium where a rating agency provides accurate ratings is sustainable for certain model parameters, but that otherwise the agency will always be too lax in its rating standards.

Our paper also contributes to the financial literature on textual analysis, which usually extracts some qualitative information from a given text, and relates this information to financial variables. Tetlock (2007) constructs a measure of pessimism in financial media, and shows that this measure is associated with market returns and market trading volume. Tetlock, Saar-Tsechansky and Macskassy (2008) show that the fraction of negative words used by the financial press in firm-specific news stories has forecasting power for a firm’s future earnings. Loughran and McDonald (2011) argue that many words classified as negative by standard dictionaries have no negative meaning in a financial context. Consequently, they construct their own list of negative words, and find that the proportion of words from this list in a firm’s 10-K report...
is associated with the firm’s 10-K filing return. Further, Loughran and McDonald (2014) use a sample of 10-K reports filed between 1994 and 2009, and find that 10-K reports with fewer words feature lower analyst dispersion in projected firm earnings. Similar to us, a few papers from the literature on textual analysis focus on the forward-looking disclosures of corporate managers: Hussainey et al (2003) show for a sample of UK firms that profit-related forward-looking disclosures are helpful for the prediction of future earnings changes. Karapandza (2013) shows that the stocks of firms whose managers make only few promises in their 10-K reports feature positive abnormal returns, and provides evidence that this is due to a risk-premium demanded by investors. Muslu et al (2014) construct an index which captures the quantity of forward-looking disclosures in a firm’s 10-K report, and find that the managers of firms with poor information environments make more forward-looking disclosures.

3. Repeated model
3.1 Basic setup
The central agent in our model is a manager that runs a public firm for an infinite number of periods, and who acts in the interest of the firm’s long-lived, risk-neutral owners. In each period $t (t \in\{0, 1, \ldots \})$, the manager privately observes the quality $\theta \in\{\theta_b, \theta_g\}$ of an investment project which requires an initial outlay of $I > 0$. A project may be of good quality ($\theta = \theta_g$) or bad quality ($\theta = \theta_b$), and if carried out, each project type yields a verifiable, normalized payoff of either $R$ (success) or 0 (failure). For a good (bad) project, the probability of success is given by $p_g (p_b)$, where $p_g = 1$ and $0 < p_b < 1$. We assume that $R > I$ ($p_b R < I$), i.e. a good (bad) project has a positive (negative) net present value. The quality of investment projects is distributed identically and independently across periods, where the probability that a manager has access to a good project in a given period equals $\alpha \in (0, 1)$. The distribution of project types across periods as well as the project parameters are common knowledge.

In order to carry out a project, the manager has to raise funds from external investors, who are assumed to be competitive and short-lived (i.e. they live for only one period). To this end, the firm repeatedly issues a short-term security which offers its holders a fixed fraction

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1 We normalize all payoffs in our model in order to avoid discounting within a period.
2 We assume that a firm starts out without any cash, and that it distributes all profits from a given period.
of the project payoff. Consequently, the cash flow profile of external investors equals the cash flow profile of (short-term) equity investors, and is uniquely determined by their share in the project’s payoff.

Eventually, we assume that $(\alpha + (1 - \alpha)p_b)R < I$. Hence, the information asymmetry between firm insiders and outsiders is so severe that without additional information, external investors refuse to provide finance to the firm.

3.2 External investors

There are two groups of investors in our model: Conservative investors and speculative investors.

Conservative investors are financially sophisticated, i.e. they acquire more detailed information about the manager’s investment project. This information acquisition results into a privately observable signal $h \in \{h^g, h^b\}$ about project quality. We assume that (compare Chemmanur and Fulghieri (1994))

$$Pr(h = h^b | \theta = \theta^b) = 1$$

$$Pr(h = h^g | \theta = \theta^g) = \rho$$

where $0 < \rho < 1$. Bayes’ rule then implies that

$$Pr(\theta = \theta^g | h = h^g) = 1$$

$$Pr(\theta = \theta^g | h = h^b) = \frac{\alpha}{\alpha + \frac{1}{1-\rho}(1 - \alpha)} \in (0, \alpha)$$

Hence, conservative investors know that the project is good after observing $h^g$ (the “good” signal), but assign a probability smaller than $\alpha$ to the good project if they observed $h^b$ (the “bad” signal). We assume that they are willing to buy the firm’s security iff $h = h^g$, regardless of the manager’s public disclosures (see below). Thus, conservative investors are a source of informed finance. The signal $h$ cannot be manipulated by the manager, and can therefore be interpreted as the firm’s hard information. The parameter $\rho$ is common knowledge, and essentially measures the transparency of the firm.
Different from conservative investors, speculative investors are financially unsophisticated, i.e. they are incapable of acquiring more detailed information about the firm’s project. In order to signal project quality to these investors, the manager can make a (optimistic) forward-looking disclosure about her project’s success potential. Let \( d \in \{ s, n \} \) denote the manager’s action, where \( d = s \) (\( d = n \)) represents the case of a public disclosure (no public disclosure) by the manager. Speculative investors form their beliefs about project quality after observing \( d \). In the absence of a public disclosure (\( d = n \)), they are never willing to finance an available project. However, if they observed a disclosure by the manager (\( d = s \)), they assign a positive probability to the good project type, and offer financing terms to the firm which are consistent with this probability.\(^3\) Consequently, speculative investors are a source of uninformed finance in our model. Since corporate managers have direct control about their public disclosures, \( s \) can be interpreted as the firm’s soft information.

### 3.3 Choice of financing source

We assume that the manager raises either informed or uninformed finance in order to carry out an available project, i.e. we preclude the possibility that she uses some form of mixed financing. Further, we assume that the manager always tries to raise informed finance if speculative investors are unwilling to finance her project. Formally, let \( \alpha_c \in (\alpha, 1) \) represent the minimum perceived project quality for which investors are willing to buy the firm’s security, i.e. let \( \alpha_c \) satisfy \( (\alpha_c+(1-\alpha_c)p_g)R = I \).\(^4\) In addition, let \( \tilde{\alpha} \in [0, 1] \) denote the probability that speculative investors assign to the good project after observing the manager’s action \( d \). We assume the following:

**Assumption 1**: If \( \tilde{\alpha} < \alpha_c \), the manager seeks informed finance.

Since conservative investors identify and finance some of the firm’s good projects (see section 3.2), assumption 1 essentially stipulates that the manager does not forgo profitable investment opportunities. Since \( \tilde{\alpha} < \alpha_c \) whenever \( d = n \) (see section 3.5), this assumption determines the manager’s financing choice in the absence of a disclosure.

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\(^3\)This could mean that they are unwilling to buy the firm’s security, e.g. if they assign probability \( \alpha \) to the good project type

\(^4\)Our assumptions in section 3.1 ensure that \( \alpha_c \) is uniquely determined and that \( \alpha_c \in (\alpha, 1) \)
We further assume that if speculative investors are willing to finance an available project after a public disclosure, the manager sells her firm’s security to them. Hence, we make the following assumption:

**Assumption 2**: If \( d = s \) and \( \tilde{\alpha} \geq \alpha^c \), the manager seeks uninformed finance.

Consequently, we rule out the possibility that after a public disclosure, the manager tries to sell her security to conservative investors although speculative investors are willing to buy it. It can be easily shown that in equilibrium, such an event never occurs, i.e. this assumption does not affect our results.

Since we have \( \tilde{\alpha} < \alpha^c \) in case of \( d = n \), the manager’s financing choice is always uniquely determined by assumption 1 and 2. Hence, the manager’s decision problem in our model is reduced to the choice of her firm’s disclosure policy. We assume that the manager can make her disclosures contingent on \( \theta \), but not on conservative investors’ private signal \( h \) (see section 3.5).

The sequence of events in a given stage game is displayed in Figure 1.

```
Beginning of period
Project quality is drawn
Manager discloses or not
Manager chooses financing source
Investment decision of investors
Outcome of financed project
End of period
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**Figure 1: Sequence of events**

### 3.4 Disclosure-related costs

We allow for the possibility that the manager’s public disclosures are associated with costs for her firm (e.g. legal costs or proprietary costs). To this end, we assume that whenever the manager released a forward-looking statement before the issuance of her security, her firm’s profit from a financed project is reduced by a fraction of \( l \in [0,1] \). Hence, if the firm owners’ share in the project payoff equals \( e_M \), their corresponding profit in case of success is given by \( (1 - l)e_M R \). These disclosure-related costs arise regardless of the firm’s financing source, i.e. they arise in case of informed finance as well as in case of uninformed finance. Since there is
a chance that conservative investors finance an available good project, it might therefore be costly for the firm if its manager makes a public disclosure and raises informed finance.

3.5 Markov strategies

Our model features two types of managers: An honest type, and an opportunistic type. Honest managers are committed to make accurate disclosures, i.e. they release a forward-looking statement iff their project is of the good type. Opportunistic managers, on the other hand, choose their disclosure policy in order to maximize the discounted sum of expected payoffs to their firm. Speculative investors do not know whether a manager is honest or opportunistic, but use information from past periods in order to update their beliefs about her type (see section 3.7).

Their beliefs can be represented by the probability they assign to the event that the manager is honest, denoted by $q \in [0, 1]$. Such beliefs constitute the single state variable in our model, and can be interpreted as a manager’s reputation.

A manager’s disclosure policy must specify her behaviour for each project type, i.e. it must specify for each $\theta \in \{\theta^b, \theta^g\}$ the probability that she makes a disclosure. Let $x_b (x_g)$ denote the probability of making a disclosure in case of a bad (good) project. We make the following assumptions on the possible disclosure policies of opportunistic managers:

**Assumption 3**: There holds $x_g \in \{0, 1\}$.

**Assumption 4**: If $x_g = 0$, we have $x_b = 0$.

Assumption 3 stipulates that in case of a good project, an opportunistic manager either refrains from forward-looking disclosures, or always makes a disclosure. We make this assumption in order to rule out equilibria where the opportunistic manager’s disclosures only partially reveal good projects. Assumption 4 entails that an opportunistic manager who refrains from disclosures for good projects also refrains from disclosures for bad projects. Hence, we preclude the possibility that she makes forward-looking disclosures only for bad projects.

Following Mathis et al. (2009), we focus on stationary Markov strategies for the opportunistic manager and speculative investors. Due to assumption 3 and 4, a stationary Markov strategy

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5Since conservative investors base their investment decision solely on the firm’s hard information, they hold no beliefs about the manager’s type
for the opportunistic manager can be represented by a function \( x_d : [0, 1] \rightarrow \{0, 1\} \times [0, 1] \) which makes the manager’s disclosure policy contingent on her reputation \( q \). For given \( q \in [0, 1] \), we use the notation \( x_d(q) = (x_g(q), x_b(q)) \), where \( x_g(q) \in \{0, 1\} \) \( (x_b(q) \in [0, 1]) \) represents the probability of making a disclosure in case of a good (bad) project. Hence, if the opportunistic manager is willing to make a disclosure after observing a good project, we have \( x_g(q) = 1 \). In such a case, \( x_b(q) \) represents the probability that the manager misleads external investors, i.e. that she makes a forward-looking disclosure in case of a bad project. On the other hand, if the manager is not willing to make a disclosure for good projects (s.t. \( x_g(q) = 0 \)), assumption 2 implies that \( x_b(q) = 0 \), i.e. we have \( x_d(q) = (0, 0) \) in such a case.

We use the following definition:

**Definition 1**: We call the probability that speculative investors assign to the good project after observing \( d \in \{n, s\} \) the perceived project quality.

The investment behaviour of speculative investors directly depends on the perceived project quality. Assume that after observing \( d \), they believe that if the manager is opportunistic, her disclosure policy equals \( (x_g^p(q), x_b^p(q)) \). A stationary Markov strategy for speculative investors can then be represented by the belief function \( \alpha_p : \{n, s\} \times [0, 1] \rightarrow [0, 1] \) given by

\[
\begin{align*}
\alpha_p(n, q) &= \frac{\alpha(1-q)}{\alpha(1-q) + (1-\alpha)} \\
\alpha_p(s, q) &= \frac{\alpha}{\alpha + (1-\alpha)(1-q)x_b^p(q)}
\end{align*}
\]

If speculative investors observe the action \( d \in \{n, s\} \), \( \alpha_p(d, q) \) reflects the perceived project quality: In absentia of a disclosure \((d = n)\), they believe that if the manager is opportunistic, she either refrained from public disclosures \((x_g^p(q) = 0)\) or not \((x_g^p(q) = 1)\). In the latter case, omitted disclosures unambiguously reveal bad projects, such that \( \alpha_p(n, q) = 0 \), while in the former case, the corresponding value is derived via Bayes’ rule. It can be easily shown that \( \alpha_p(n, q) < \alpha^c \) always holds, i.e. speculative investors never finance a project in the absence of a public disclosure. On the other hand, in case of a disclosure \((d = s)\), speculative investors believe that if the manager is opportunistic, she has decided to make public disclosures \((x_g^p(q) = 1)\), and calculate the associated perceived project quality according to Bayes’ rule.
After observing the action $d$, speculative investors are willing to buy the firm’s security iff 
$\alpha_p(d, q) \geq \alpha^c$. We use the following definition:

**Definition 2**: If $\alpha_p(s, q) \geq \alpha^c$, we say that the manager’s forward-looking disclosures are 
*informative*. Otherwise, we say that her disclosures are *uninformative*.

Since $\alpha_p(n, q) < \alpha^c$, this means that speculative investors finance a project iff they observed 
an informative public disclosure by the manager.\footnote{Note that assumption 2 ensures that the manager seeks uninformed finance after an informative disclosure} In the following, we will usually write $\alpha_p(q)$ instead of $\alpha_p(s, q)$, i.e. we define $\alpha_p(q) := \alpha_p(s, q)$ for $q \in [0, 1]$.

### 3.6 Stage game payoffs

We calculate the firm’s expected payoff from a financed project via the break-even condition of 
external investors (see section A1 in the appendix). Let investors’ beliefs be exogenously given 
by (1)-(2). Depending on the quality of the manager’s project, we can distinguish two cases:
i.) $\theta = \theta^g$

In case the manager is able to raise informed finance, her firm’s corresponding expected payoff 
solely depends on her disclosure decision $x_g(q)$. If we denote this payoff by $\Pi^h_g(x_g(q))$, we have

$$\Pi^h_g(x_g(q)) = (1 - x_g(q)l)(R - I)$$

On the other hand, if the manager raises uninformed finance via the release of an informative 
forward-looking statement, her firm’s associated expected payoff, $\Pi^s_g(\alpha_p(q))$, is given by

$$\Pi^s_g(\alpha_p(q)) = (1 - l) \frac{[\alpha_p(q) + (1 - \alpha_p(q))p_b]R - I}{\alpha_p(q) + (1 - \alpha_p(q))p_b}$$

It can be easily shown that $\Pi^s_g(\alpha^c) = 0$ and $\Pi^s_g(1) = \Pi^b_g(1)$.

ii.) $\theta = \theta^b$

The manager will never raise informed finance in such a case. However, she can raise uninformed finance if she makes an informative public disclosure. The corresponding expected
payoff $\Pi_b^s(\alpha_p(q))$ is given by

$$\Pi_b^s(\alpha_p(q)) = p_b(1 - l) \left[ \frac{\alpha_p(q) + (1 - \alpha_p(q))p_b}{\alpha_p(q) + (1 - \alpha_p(q))p_b} R - I \right]$$

There holds $\Pi_b^s(\alpha^c) = 0$.

3.7 Manager reputation

Let $t \in \{0, 1, \ldots\}$ denote the current period, and let the beliefs of speculative investors in the current period be given by (1)-(2). Subsequent investors (i.e. speculative investors from any subsequent period $t + k$) use information from the current period in order to update their beliefs about the manager’s type. We assume that they are informed about the manager’s action $d \in \{n, s\}$ in the current period, i.e. we make the following formal assumption:

**Assumption 5:** Subsequent investors always observe $d$, the manager’s action in period $t$.

Further, we assume that if the manager raised uninformed (informed) finance for her current project, subsequent investors observe (do not observe) the outcome of the project. Hence, we make the following assumption:

**Assumption 6:** Subsequent investors observe the outcome of an implemented project iff it was financed by speculative investors.

Assumption 6 is a technical assumption which ensures that an opportunistic manager never reveals her type if she decides to refrain from forward-looking disclosures. Further, it implies that subsequent investors recognize uninformative public disclosures by the manager: Whenever they do not observe a project outcome after a released forward-looking statement ($d = s$), they know that the statement was uninformative for current speculative investors.

The information of subsequent investors can therefore be represented by one of the following observations: No disclosure (N), uninformative disclosure (U), project success (S) or project failure (F) after an informative disclosure. Figure 2 (Figure 3) depicts the decision tree and the associated observations which correspond to an opportunistic manager that makes uninformative (informative) disclosures.
Figure 2: Decision tree of opp. manager (uninformative disclosures)

Figure 3: Decision tree of opp. manager (informative disclosures)

Subsequent investors use their observation $O \in \{N, U, S, F\}$ in order to update their beliefs about the manager’s type. Assume that after observing $O$, subsequent investors believe that if the manager is opportunistic, her disclosure policy equals $(x_g^s(q), x_b^s(q))$. Their updated beliefs, denoted by $\psi(q|O)$, are then given by

\[
\psi(q|N) := q^N = x_g^s(q) \frac{q}{1 - (1 - q)x_b^s(q)} + (1 - x_g^s(q)) \frac{q}{1 + \frac{\alpha}{1 - \alpha}(1 - q)} \quad (3)
\]

\[
\psi(q|U) := q^U = \frac{q}{1 + (1 - q)\frac{1 - \alpha}{\alpha}x_b^s(q)} \quad (4)
\]

\[
\psi(q|S) := q^S = \frac{q}{1 + (1 - q)\frac{1 - \alpha}{\alpha}x_b^s(q)p_b} \quad (5)
\]

\[
\psi(q|F) = 0 \quad (6)
\]

In (3), subsequent investors believe that the opportunistic manager either refrained from public disclosures ($x_g^s(q) = 0$) or not ($x_g^s(q) = 1$). For both possible values of $x_g^s(q)$, the corresponding expression is derived via Bayes’ rule. Since $q^N$ is not well-defined for $(x_g^s(0), x_b^s(0)) = (1, 1)$, we follow Mathis et al. (2009) and set $q^N = 0$ in such a case. In (4) and (5), subsequent investors believe that if the manager is opportunistic, she decided to make a disclosure ($x_g^s(q) = 1$), and update their beliefs according to Bayes’ rule. (6) results from our assumption that good projects
do not fail and that honest managers never mislead external investors. Hence, an opportunistic manager loses her reputation after a failed bad project.

3.8 Value function

The opportunistic manager chooses her disclosure policy in order to maximize the discounted sum of her firm’s expected stage game payoffs, with the discount factor given by $\delta \in (0,1)$. A firm’s discounted sum of expected payoffs is a function of its manager’s reputation $q$, and will be denoted by $V(q)$. In the following, let $V$ be exogenously given, and let the beliefs of current as well as future investors be given by (1)-(2) and (3)-(6). Given $V$ and these beliefs, the opportunistic manager will choose her disclosure policy $x_d(q) = (x_g(q), x_b(q))$ in an optimal manner. Depending on the informativeness of her disclosures, we can distinguish two cases:

i.) $\alpha_p(q) < \alpha^c$

In such a case, it follows from assumption 1 that the manager will always seek informed finance in order to undertake an available project.

If $x_g(q) = 0$, (3) implies that the firm’s discounted sum of expected payoffs, denoted by $V_n(q)$, is given by

$$V_n(q) = \alpha \rho \Pi^h_g(0) + \delta V(q^N)$$

On the other hand, if $x_g(q) = 1$, (3)-(4) imply that the corresponding discounted sum of payoffs, $V_u(q)$, equals

$$V_u(q) = \max_{x_b \in [0,1]} \alpha \rho \Pi^h_g(1) + \delta [(\alpha + (1 - \alpha)x_b)V(q^U) + (1 - \alpha)(1 - x_b)V(q^N)]$$

ii.) $\alpha_p(q) \geq \alpha^c$

It then follows from assumption 1 that the manager seeks informed finance in case of no disclosure, while assumption 2 implies that she raises uninformed finance after a public disclosure. Hence, in case of $x_g(q) = 0$, the discounted sum of expected stage game payoffs to the firm
again equals

$$V_n(q) = \alpha p b(0) + \delta V(q^N)$$

However, if \( x_q = 1 \), it follows from (3) and (5)-(6) that the discounted sum of expected payoffs, \( V_i(q) \), is given by

$$V_i(q) = \max_{x_b \in [0, 1]} \alpha \Pi^*_g(\alpha_p(q)) + (1 - \alpha)x_b \Pi^*_p(\alpha_p(q)) + \delta[(\alpha + (1 - \alpha)x_bp_b)V(q^S) + (1 - \alpha)x_b(1 - p_b)V(0) + (1 - \alpha)(1 - x_b)V(q^N)]$$

Let \( x_b^* \in [0, 1] \) \((x_d^* \in [0, 1])\) maximize the right-hand side of \( V_n(q) \) \((V_i(q))\). In case of \( \alpha_p(q) < \alpha^c \), optimality requires that if \( V_n(q) > V_n(q) \) \((V_u(q) < V_n(q))\), there holds \( x_d = (1, x_b^*) \) \((x_d = (0, 0))\). Similarly, in case of \( \alpha_p(q) \geq \alpha^c \), we have \( x_d = (1, x_b^*) \) \((x_d = (0, 0))\) whenever \( V_i(q) > V_n(q) \) \((V_i(q) < V_n(q))\).

4. Equilibrium properties

4.1 Markov perfect equilibrium

In a (stationary) Markov perfect equilibrium, the manager’s strategy always specifies an optimal disclosure policy, current investors’ beliefs are correct, subsequent investors update their beliefs accurately, and the manager’s value function is correctly specified.

**Definition 3:** A stationary Markov perfect equilibrium (MPE) is a quadruple \((x, \alpha, \psi, V^*)\) such that for each \( q \in [0, 1] \), there holds (with \( x_d^*(q) = (x_d^*(q), x_b^*(q)) \)):

a.) Given \( \alpha_p^*(., q), \psi^*(q) \), and \( V^* \), \( x_d^*(q) \) is an optimal disclosure policy

b.) \( \alpha_p^*(., q) \) satisfies (1)-(2), where \( x_d^*(q) = x_d^*(q) \) and, if \( x_d^*(q) = 1, x_b^*(q) = x_b^*(q) \)

c.) \( \psi^*(q) \) satisfies (3)-(6), where \( x_d^*(q) = x_d^*(q) \) and, if \( x_d^*(q) = 1, x_b^*(q) = x_b^*(q) \)

d.) \( V^*(q) = \mathbb{1}_{\{\alpha_p^*(q) < \alpha^c\}} \max\{V_u(q), V_n(q)\} + \mathbb{1}_{\{\alpha_p^*(q) \geq \alpha^c\}} \max\{V_i(q), V_n(q)\} \)

where \( \mathbb{1}_{\{E\}} \) denotes the indicator function that takes value 1 iff event \( E \) is realized.

Note that in case of disclosure \((x_d^*(q) = 1)\), the randomization rule \( x_b^*(q) \) uniquely determines \( \alpha_p^*(., q) \) and \( \psi^*(q) \). However, if the manager makes no disclosures in equilibrium \((x_d^*(q) = 0)\),
\( \alpha_p^*(s, q) \) as well as \( \psi^*(q|U) \) and \( \psi^*(q|S) \) are not uniquely determined. Provided that they support \( x_d^*(q) = (0, 0) \) as part of a MPE, \( x_b^*(q) \) and \( x_s^*(q) \) can be freely chosen in such a case.

Throughout our study, we will restrict the analysis on equilibria associated with a value function \( V^* \) that is continuous on \((0, 1]\) and non-decreasing in \( q \). Let \( C_+ \) denote the set of such functions defined on \([0, 1]\).

### 4.2 Preliminaries

We will focus on equilibria where the opportunistic manager releases forward-looking statements to the public. Let \( x_d = (x_g, x_b) \) be a given strategy for the opportunistic manager. According to the one-stage deviation principle, \( x_d \) is an equilibrium strategy iff it is not profitable for the manager to deviate from \( x_d(q) \) for any \( q \in [0, 1] \). In case of \( x_d(q) = (1, x_b(q)) \), this means that two conditions must be satisfied:

(E1) The manager does not prefer the policy \((0, 0)\) to \((1, x_b(q))\)

(E2) The manager does not prefer the policy \((1, x_b')\) to \((1, x_b(q))\) for any \( x_b' \in [0, 1] \)

If the discounted sum of expected payoffs that is associated with \((1, x_b(q))\) is denoted by \( V_p(q) \), condition (E1) requires that \( V_p(q) \geq V_n(q) \). Let \( \alpha_p(s, q) \) denote the perceived project quality that corresponds to the disclosure policy \((1, x_b(q))\). Depending on the informativeness of the manager’s disclosures, condition (E2) requires that one of the following two inequalities is satisfied for any \( x_b' \in [0, 1] \):

\[
(x_b(q) - x_b')V(q^U) \geq (x_b(q) - x_b')V(q^N) \tag{UE2}
\]

\[
(x_b(q) - x_b')\left[\Pi_s^*(\alpha_p(q)) + \delta(p_bV(q^S) + (1 - p_b)V(0))\right] \geq \delta(x_b(q) - x_b')V(q^N) \tag{IE2}
\]

given that \( q^U, q^S \) and \( q^N \) are calculated on the basis of the policy \((1, x_b(q))\). Condition (UE2) ((IE2)) corresponds to the case of uninformative (informative) disclosures.

Let (E1) and \( \alpha_p(q) \geq \alpha^c \) be satisfied. According to (IE2), the disclosure policy \( x_d(q) = (1, 1) \) supports an equilibrium iff

\[
\Pi_s^*(\alpha_p(q)) + \delta(p_bV(q^S) + (1 - p_b)V(0)) \geq \delta V(q^N)
\]
On the other hand, the disclosure policy $x_b^*(q) = (1, 0)$ supports an equilibrium iff

$$\Pi_b^*(\alpha_p(q)) + \delta(p_b V(q_S^s) + (1 - p_b) V(0)) \leq \delta V(q^N)$$

The first (second) inequality simply states that if the manager makes a misleading disclosure, the resulting immediate gain and discounted expected continuation value exceeds (falls short of) the discounted continuation value that results from an omitted misleading disclosure.

### 4.3 Equilibria with uninformative disclosures

It can be shown that if forward-looking disclosures are associated with costs, a manager will never make disclosures which are uninformative for speculative investors.

**Theorem 1:** Let $\hat{x}_b(q) \in (0, 1]$, and let $(1, \hat{x}_b(q))$ be a disclosure policy for which the corresponding beliefs of current investors, $\hat{\alpha}_p(d, q)$, satisfy $\hat{\alpha}_p(s, q) < \alpha^c$. If $l > 0$, the policy $(1, \hat{x}_b(q))$ cannot be part of an equilibrium.

**Proof:** See section A2 in the appendix.

Theorem 1 has a very intuitive interpretation: If the release of a forward-looking statement is associated with costs, but offers no benefit, the firm’s manager will always prefer to make no disclosure. Consequently, if her public disclosures are costly, a manager will make them only if their effect on investors’ beliefs is sufficiently large to induce them to buy her firm’s security. Since we can plausibly assume that managers’ forward-looking disclosures are indeed associated with costs, Theorem 1 therefore suggests that the actual disclosures of corporate managers convey some information to external investors. This is an important fundamental result, since it rules out the possibility that managers’ forward-looking statements are merely uninformative babble. On the other hand, in the absence of disclosure-related costs ($l = 0$), a disclosure policy with uninformative disclosures is equivalent to a policy without disclosures, and can be sustained in equilibrium.

It can be further shown that the manager never makes public disclosures which are only marginally informative:

**Lemma 1:** Let $(1, \tilde{x}_b(q))$ represent a disclosure policy where the corresponding beliefs of current
investors, $\check{\alpha}_p(d, q)$, satisfy $\check{\alpha}_p(s, q) = \alpha^e$. Then the policy $(1, \check{x}_b(q))$ cannot be part of a Markov perfect equilibrium.

**Proof:** Assume the policy $(1, \check{x}_b(q))$ (where $\check{x}_b(q) > 0$) could be part of an equilibrium, with the corresponding value function being denoted by $\check{V} \in C_+$. Since $\check{\alpha}_p(q) = \alpha_c$, we have $\Pi_h^s(\check{\alpha}_p(q)) = \Pi_h^s(\check{\alpha}_p(q)) = 0$, and it follows from condition (IE2) that $p_b\check{V}(q^S) + (1 - p_b)\check{V}(0) \geq \check{V}(q^N)$. Since $\check{V}$ is non-decreasing in $q$, this implies that $\check{V}(0) = \check{V}(q^S) = \check{V}(q^N)$, such that $V_i(q) = \delta \check{V}(0)$. However, then follows that $V_i(q) < \alpha \rho \Pi_h^s(0) + \delta \check{V}(q^N) = V_n(q)$, which contradicts the optimality of $(1, \check{x}_b(q))$.

Lemma 1 is a consequence of our assumption that after an informative public disclosure, a manager raises external finance from speculative investors (assumption 2). The underlying intuition is that if speculative investors offer financing terms which leave no surplus to the firm, the manager is always better off by seeking informed finance, i.e. she prefers to refrain from forward-looking disclosures.

In the following, we will assume that $l > 0$. In equilibrium, the opportunistic manager will therefore either refrain from forward-looking disclosures, or her disclosures will be (strictly) informative for external investors.

### 4.4 Equilibria without disclosures

It can be shown that a manager who has been revealed to be opportunistic will never make forward-looking disclosures in equilibrium.

**Lemma 2:** In equilibrium, the manager’s strategy $x_d^*$ always satisfies $x_d^*(0) = (0, 0)$.

**Proof:** See section A3 in the appendix.

The following intuition underlies Lemma 2: Assume that a manager has been revealed to be opportunistic, such that her reputation equals $q = 0$. In such a situation, the manager’s reputation will remain at 0 throughout the rest of the supergame, regardless of her disclosure policy. Consequently, if she was able to raise external finance via a public disclosure, the manager would find it optimal to mislead speculative investors whenever possible, since this would
always yield a short-term profit, but had no effect on her future reputation. However, this would imply that, on average, speculative investors lose money if they rely on the manager’s disclosures, and cannot be consistent with an equilibrium. Therefore, in equilibrium, the manager’s forward-looking disclosures must be regarded as uninformative, and the manager decides to release no statements. De facto, this means that a manager who has been revealed to be opportunistic loses all her credibility, i.e. she is no longer able to affect her firm’s financing conditions via her forward-looking disclosures. As a consequence, the manager has to raise informed finance if she wants to carry out an available project. We obtain the following corollary from Lemma 2:

**Corollary 1**: In equilibrium, the value function $V^*$ satisfies $V^*(0) = \frac{\alpha \rho \Pi_h(0)}{1-\delta}$. 

**Proof**: Due to Lemma 2, it follows from the definition of $V_n$ that $V^*(0) = V_n(0) = \alpha \rho \Pi_h(0) + \delta V^*(0)$.

According to Lemma 2, a manager who has been revealed to be opportunistic does not make forward-looking disclosures in equilibrium. It can be further shown that under certain conditions, an opportunistic manager never releases forward-looking statements in equilibrium, regardless of her reputation $q$. To this end, we define condition (NE) by

$$\alpha \rho > (1-l)(\alpha + (1-\alpha)p_h) \quad (NE)$$

In addition, let $x^a_n$ denote the strategy of refraining from public disclosures, i.e. we have $x^a_n(q) = (0,0)$ for all $q \in [0,1]$. The following theorem holds.

**Theorem 2**: If condition (NE) is satisfied, the unique equilibrium strategy is given by $x^a_n$.

**Proof**: See section A4 in the appendix.

Theorem 2 suggests that the managers of public firms which are highly transparent (large $\rho$) and feature significant disclosure-related costs (large $l$) always decide to refrain from forward-looking disclosures. Note that the relevance of Theorem 2 results from our assumption that managers’ disclosures are associated with costs ($l > 0$): In absentia of such costs ($l = 0$), condition (NE) would never be satisfied, and an equilibrium without disclosures would be equivalent.
to an equilibrium with uninformative disclosures.

4.5 Equilibria with informative disclosures

It can be shown that for certain model parameters, an opportunistic manager with a positive reputation $q$ will accurately disclose project quality to external investors. The following theorem provides a necessary and sufficient condition for the sustainability of the disclosure policy $x_d(q) = (1, 0)$ in equilibrium.

**Theorem 3:** Let $q > 0$. The disclosure policy $x_d(q) = (1, 0)$ can be part of a MPE iff

$$\frac{p_b}{1 - p_b} \leq \frac{\alpha}{1 - \delta} \left(1 - \frac{\rho}{1 - l}\right)$$

(AE)

*Proof:* See section A5 in the appendix.

In order to satisfy inequality (AE), a necessary condition is that $\rho < 1 - l$. Hence, a policy of accurate disclosures can be sustained more easily for the managers of firms that are opaque (small $\rho$) or feature low disclosure-related costs (small $l$). The economic intuition is that for opaque firms, the hard information of conservative investors often does not reveal an available good project. Consequently, the loss in reputation that might result from a misleading forward-looking disclosure (s.t. $q = 0$) is very costly for the managers of such firms, implying that it is easier to discipline them in an equilibrium with accurate disclosures. On the other hand, higher disclosure-related costs reduce the profitability of good projects compared to bad projects, with the result that a policy of accurate disclosures is harder to sustain in equilibrium.

If the aforementioned necessary condition holds, a higher discount factor $\delta$ makes it more likely that inequality (AE) is satisfied. This is intuitive, since a manager who cares more about her firm’s future payoffs puts a larger weight on the ability to attract external funds via her future disclosures than on the immediate short-term gain that results from a misleading statement. This finding is also consistent with the Folk Theorem for Infinitely Repeated Games in Fudenberg and Maskin (1986). Further, given that $\rho < 1 - l$, Theorem 3 suggests that the managers of more profitable firms (higher $\alpha$) are more likely to disclose project quality accurately. The underlying intuition is that for profitable firms, the hard information of conservative investors
frequently does not reveal good projects (namely with probability \( \alpha(1 - \rho) \)). Consequently, the managers of such firms find it more valuable to retain the ability to raise external funds via their forward-looking disclosures.

We also see that a lower probability of success for the bad project \((p_b)\) makes it more likely that the forward-looking disclosures of an opportunistic manager are accurate. There are two reasons for this: Firstly, a smaller success probability reduces the firm’s expected payoff from a bad project, i.e. it reduces the short-term benefit from a misleading disclosure. Secondly, it implies that a financed bad project is more likely to fail, with the result that the manager’s type is revealed to the public. Hence, the smaller \(p_b\), the less likely it is that the manager “gets away with lying”. Instead, she will more often loose her reputation and, as a result, her credibility.

It can be further shown that if inequality (AE) holds strictly, the accurate disclosure of project quality is the only possible disclosure policy in equilibrium. To this end, let \(x^a_d\) represent the strategy where an opportunistic manager releases accurate forward-looking statements whenever her reputation \(q\) is positive, and otherwise refrains from disclosures. Hence, let \(x^a_d\) be given by

\[
x^a_d(q) = \begin{cases} (0, 0) & \text{if } q = 0 \\ (1, 0) & \text{if } q > 0 \end{cases}
\]

Then the following theorem holds.

**Theorem 4:** If inequality (AE) holds strictly, the unique equilibrium strategy is given by \(x^a_d\).

**Proof:** See section A6 in the appendix.

Hence, if inequality (AE) holds strictly, Theorem 4 suggests that whenever the manager’s reputation \(q\) is positive, her forward-looking disclosures are accurate. Note, however, that this result strongly rests on our restriction that the equilibrium value function \(V^*\) is non-decreasing and continuous on \((0,1]\).
5. Empirical analysis

In order to test our above findings empirically, we construct an index which captures the quantity of forward-looking disclosures contained in a firm’s annual 10-K report, and link the value of this index to contemporaneous firm characteristics that, according to our model, are related to a manager’s disclosure policy. Since we employ a panel dataset for our analysis, our regression model always features fixed effects that control for unobserved heterogeneity across different calendar years. Further, we include either industry-level or firm-level fixed effects in our regressions.

5.1 Empirical predictions

In our model, corporate managers make forward-looking disclosures in order to reduce information asymmetries between themselves and outside investors, resulting into better financing terms for their firms. We therefore expect that corporate managers make more disclosures whenever their firms have higher external financing needs. Further, according to Theorem 3, only the managers of opaque firms ($\rho < 1 - l$) potentially release accurate forward-looking statements to the public. Given that her firm is opaque, Theorem 4 suggests that a higher profitability (larger $\alpha$) makes it more likely that a manager’s disclosures are accurate. In addition, our model predicts that in the presence of disclosure-related costs, a manager refrains from public disclosures whenever she has lost her credibility (Lemma 2). Since only a policy of accurate public disclosures sustains a manager’s credibility in the long run, we therefore expect that among the group of opaque firms, the quantity of forward-looking disclosures is positively correlated with a firm’s profitability and financing needs.

On the other hand, Theorem 3 implies that the managers of transparent firms ($\rho \geq 1 - l$) never comply with a policy of accurate disclosures, i.e. they either make no disclosures, or some of their disclosures are misleading. In the latter case, the manager will lose her credibility at some point in time, and stop making public disclosures. Further, Theorem 2 suggests that for firms with a high level of transparency, corporate managers might find it optimal to refrain from the release of forward-looking statements. Overall, we expect that for transparent firms, the relation between the quantity of forward-looking disclosures and firm characteristics is less pronounced than for opaque firms.
In order to classify a public firm as opaque or transparent for external investors, we will use two measures: The existence of a rating and firm size. It is intuitively clear that rated firms are more transparent for external investors than unrated firms. Further, due to their stricter disclosure requirements and higher public attention, it appears plausible to assume that external investors consider larger firms to be more transparent. As a well-established measure for a firm’s (re)financing needs in the near future (see Almeida et al. (2011)), we use its long-term debt due in the coming year. Consequently, we test the following hypotheses:

**Hypothesis 1**: Among the group of unrated firms, the quantity of forward-looking disclosures is positively correlated with a firm’s profitability and debt due in the coming year, and negatively correlated with its size.

**Hypothesis 2**: The correlation pattern between managers’ forward-looking disclosures and firm fundamentals is more pronounced for unrated firms than for rated firms.

### 5.2 Index construction

Our goal is to construct an index which captures the quantity of forward-looking disclosures in a given 10-K report. To this end, we apply the so-called “Bag-of-Words” scheme, i.e. we count all expressions in the report that are contained in a prespecified set of words, and use the determined number for the construction of our index. We use the Harvard General Inquirer in order to obtain word lists that contain expressions which indicate the presence of a forward-looking statement. We use word lists coming from an external dictionary because we think that such external lists result into a more objective classification scheme than an endogenously chosen set of expressions. For the construction of our index, we use the word lists that constitute the “Goal” and “Try” category of the Harvard General Inquirer System. These two categories appeared most appropriate to capture managers’ forward-looking statements, and include expressions like “aim”, “intend”, “target” or “aspire”. We employ computer-intensive techniques in order to count the number of words that fall into either of these two categories. For a given 10-K report, we then calculate our index \( i \) in the following way:

\[
i = \frac{Nr(Goal \ or \ Try)}{Nr(Total)} \times 100
\]
where \( Nr(Goal \ or \ Try) \) denotes the number of words that fall into the “Goal” or “Try” category, and \( Nr(Total) \) represents the total number of words contained in the report. Hence, our index (which we call “FL-index”) captures the percentage of words in a given 10-K report that fall into either of the two selected categories.

We download all 10-K reports filed between 1993 and 2009 from the Securities and Exchange Commission (SEC) website using their ftp interface (ftp.sec.gov), and compute the value of the FL-index for each of them.

5.3 Firm characteristics

We obtain data on firm fundamentals from the Compustat North America Fundamentals Annual database. We require a firm to provide valid fiscal year end data on its book value of assets (\( AT \)), short-term debt (\( DS \)), long-term debt (\( DL \)), market value (\( MV \)), and book value of equity (\( BV \)). If a firm’s market value is missing in Compustat, we approximate it as the number of common shares outstanding times the closing price of the firm’s stock at fiscal year end. In addition, we require a firm to provide valid data on its earnings before interest and taxes (\( EBIT \)), net income, and long-term debt due in the next year. We deflate all variables by the US consumer price index in order to obtain real values, and keep only those observations made under the most recent fiscal year convention.\(^7\) Further, we use the Compustat North America Ratings database in order to construct a dummy which captures whether a firm had a S&P Domestic Long-Term Issuer Credit Rating in its fiscal year end month.

As is standard in the literature, we define a firm’s size as the (real) book value of its assets, its book leverage as \( (DS + DL)/AT \), and its market-to-book ratio as \( (AT - BV + MV)/AT \). In order to allow for a better comparison, we also divide a firm’s long-term debt due in the next year by \( AT \). Following Denis and Mihov (2003), we define a firm’s profitability as the average of \( EBIT/AT \) over the last three fiscal years (including the current fiscal year). Further, we construct a dummy variable which equals one iff a firm’s net income for the current fiscal year was negative, i.e. iff the firm makes losses. We include this dummy as well as a firm’s market-to-book ratio and book leverage as control variables within our regressions.

---

\(^7\)Since we have 10-K reports only until 2009, this means that we keep only those observations made under the fiscal year convention prevalent in 2009 or earlier.
5.4 Sample construction

We merge the dataset on our index and the dataset on firm characteristics. In the merging procedure, we require that a 10-K report was filed not later than 90 days after the fiscal year end that corresponds to the matched observation on firm fundamentals. This is in line with the requirement of the SEC that a firm has to submit the 10-K report for a given fiscal year no later than 90 days after the end of this fiscal year. We apply the following data screens to the combined dataset: We drop all financial institutions from our sample (SIC codes between 6000 and 6999), which is standard in the literature on firm-level studies. Following Loughran and McDonald (2014), we also require a firm to have a stock price of at least 3$ at fiscal year-end, and to feature at least 2000 words in its 10-K report. Further, we drop all observations with a non-positive value for a firm’s book value of assets or book value of equity.

Our final sample comprises 23833 observations which span a time period from 1993 to 2009. In order to reduce the effect of outliers, we winsorize all financial variables (book value of assets, book leverage, MtB ratio, profitability, debt due in the next year) and the FL-index at the 1%- and the 99%-level.

5.5 Descriptive statistics

Table 1a and 1b display the descriptive statistics for the subsample of rated firms and the subsample of unrated firms.

<table>
<thead>
<tr>
<th>Table 1a: Rated firms</th>
<th>Observations</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>FL-index</td>
<td>7649</td>
<td>0.1649</td>
<td>0.1295</td>
<td>0.0271</td>
<td>0.7197</td>
</tr>
<tr>
<td>Total assets(in Mio. $)</td>
<td>7649</td>
<td>5369.96</td>
<td>7768.75</td>
<td>57.23</td>
<td>33215.64</td>
</tr>
<tr>
<td>Book leverage</td>
<td>7649</td>
<td>0.3118</td>
<td>0.1525</td>
<td>0</td>
<td>0.6704</td>
</tr>
<tr>
<td>MtB ratio</td>
<td>7649</td>
<td>1.7140</td>
<td>0.9483</td>
<td>0.7101</td>
<td>9.2236</td>
</tr>
<tr>
<td>Firm profitability</td>
<td>7649</td>
<td>0.0925</td>
<td>0.0627</td>
<td>-0.5877</td>
<td>0.3120</td>
</tr>
<tr>
<td>Debt due in coming year</td>
<td>7649</td>
<td>0.0164</td>
<td>0.0263</td>
<td>0</td>
<td>0.1529</td>
</tr>
<tr>
<td>Loss dummy</td>
<td>7649</td>
<td>0.1622</td>
<td>0.3687</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Unsurprisingly, there are significant differences between rated and unrated firms: Rated firms tend to be larger, more indebted, and more profitable than unrated firms. Further, the unrated firms in our sample on average have a higher market-to-book ratio, and are more often in a loss situation than rated firms. One can also see that the average value of our index is larger for the subsample of unrated firms.

Further, we display the evolution over time of the average FL-index level for our two subsamples in Figure 1.

### Table 1b: Unrated firms

<table>
<thead>
<tr>
<th>Variable</th>
<th>Observations</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>FL-index</td>
<td>16184</td>
<td>0.1938</td>
<td>0.1457</td>
<td>0.0271</td>
<td>0.7197</td>
</tr>
<tr>
<td>Total assets(in Mio. $)</td>
<td>16184</td>
<td>434.38</td>
<td>1449.38</td>
<td>10.97</td>
<td>33215.64</td>
</tr>
<tr>
<td>Book leverage</td>
<td>16184</td>
<td>0.1429</td>
<td>0.1607</td>
<td>0</td>
<td>0.6704</td>
</tr>
<tr>
<td>MtB ratio</td>
<td>16184</td>
<td>2.2064</td>
<td>1.6411</td>
<td>0.7101</td>
<td>9.2236</td>
</tr>
<tr>
<td>Firm profitability</td>
<td>16184</td>
<td>0.0484</td>
<td>0.1566</td>
<td>-0.5877</td>
<td>0.3120</td>
</tr>
<tr>
<td>Debt due in coming year</td>
<td>16184</td>
<td>0.0135</td>
<td>0.0258</td>
<td>0</td>
<td>0.1529</td>
</tr>
<tr>
<td>Loss dummy</td>
<td>16184</td>
<td>0.2520</td>
<td>0.4342</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

![Figure 4: Evolution of mean FL-index value](image-url)
In each calendar year covered by our sample, the average index level was larger for the group of unrated firms than for the group of rated firms. Further, one can see that for both groups, the average level of our index was fairly stable over the period from 1994 to 2000, but featured a strong decline from 2001 until 2009. To be precise, the average FL-index level for the group of unrated (rated) firms shrank from around 0.26% (0.22%) in 2000 to approximately 0.10% (0.10%) in 2008. One possible explanation for this strong decline might be the enactment of the Sarbanes-Oxley Act (SOX) in July 2002: As a result of the stricter requirements on financial disclosures that came into effect via the SOX, public firms became more transparent for external investors (Arping and Sautner (2013)). Our findings in Theorem 2 and Theorem 3 then suggest that the managers of public firms might have reacted to this by decreasing their forward-looking disclosures. Due to the pronounced year-to-year changes of the FL-index during our sample period, we include year fixed effects in each of our regressions.

### 5.6 Regression results

We estimate the following model:

\[
\iota_{it} = \beta_0 + \beta'_1 x_{it} + \zeta_i + \mu_t + \epsilon_{it}
\]  

(7)

where \( \iota_{it} \) represents the FL-index, and \( x_{it} \) the vector of firm characteristics of firm \( i \) for calendar year \( t \). \( \mu_t \) represents year fixed effects, where we use the year 2000 as reference year. \( \zeta_i \) represents fixed effects at the 2-digit SIC level, the 3-digit SIC level, or at the firm level. We estimate model (7) separately for the subsample of rated firms and the subsample of unrated firms. In all our estimation procedures, we employ robust standard errors.

The estimation results for the subsample of unrated firms are displayed in Table 2a. We display the estimated year fixed effects only for the period from 2001 to 2009, since for all the remaining years, the estimated coefficients were almost never statistically significant. We do not display any estimated industry or firm-level fixed effect.
Table 2a: Unrated firms

<table>
<thead>
<tr>
<th></th>
<th>Model I</th>
<th>Model II</th>
<th>Model III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total assets</td>
<td>$-0.1349^{***}$</td>
<td>$-0.1323^{***}$</td>
<td>$-0.0576$</td>
</tr>
<tr>
<td>Book leverage</td>
<td>$-0.0316^{**}$</td>
<td>$-0.0357^{***}$</td>
<td>$-0.0183$</td>
</tr>
<tr>
<td>MtB ratio</td>
<td>$-0.0001$</td>
<td>$0.0000$</td>
<td>$0.0005$</td>
</tr>
<tr>
<td>Firm profitability</td>
<td>$0.0530^{***}$</td>
<td>$0.0402^{***}$</td>
<td>$0.0343^{*}$</td>
</tr>
<tr>
<td>Debt due in coming year</td>
<td>$0.1257^{**}$</td>
<td>$0.0936^{**}$</td>
<td>$-0.0285$</td>
</tr>
<tr>
<td>Loss dummy</td>
<td>$-0.0023$</td>
<td>$-0.0011$</td>
<td>$0.0047$</td>
</tr>
<tr>
<td>Year FE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>$-0.0303^{***}$</td>
<td>$-0.0318^{***}$</td>
<td>$-0.0279^{***}$</td>
</tr>
<tr>
<td>2002</td>
<td>$-0.0648^{***}$</td>
<td>$-0.0664^{***}$</td>
<td>$-0.0629^{***}$</td>
</tr>
<tr>
<td>2003</td>
<td>$-0.0821^{***}$</td>
<td>$-0.0840^{***}$</td>
<td>$-0.0771^{***}$</td>
</tr>
<tr>
<td>2004</td>
<td>$-0.1077^{***}$</td>
<td>$-0.1082^{***}$</td>
<td>$-0.1026^{***}$</td>
</tr>
<tr>
<td>2005</td>
<td>$-0.1237^{***}$</td>
<td>$-0.1243^{***}$</td>
<td>$-0.1201^{***}$</td>
</tr>
<tr>
<td>2006</td>
<td>$-0.1372^{***}$</td>
<td>$-0.1378^{***}$</td>
<td>$-0.1330^{***}$</td>
</tr>
<tr>
<td>2007</td>
<td>$-0.1425^{***}$</td>
<td>$-0.1429^{***}$</td>
<td>$-0.1382^{***}$</td>
</tr>
<tr>
<td>2008</td>
<td>$-0.1560^{***}$</td>
<td>$-0.1555^{***}$</td>
<td>$-0.1440^{***}$</td>
</tr>
<tr>
<td>2009</td>
<td>$-0.1521^{***}$</td>
<td>$-0.1530^{***}$</td>
<td>$-0.1496^{***}$</td>
</tr>
<tr>
<td>2-digit SIC FE</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>3-digit SIC FE</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Firm-level FE</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>16184</td>
<td>16184</td>
<td>16184</td>
</tr>
</tbody>
</table>

***, ** and * indicate statistical significance at the 1%, 5% and 10% level.

We find robust evidence that specific characteristics of unrated firms are related to the forward-looking disclosures of their managers: The point estimates for firm size are negative across all three models, and statistically significant at the 1% level for two of them. Hence, it appears that the managers of large unrated firms make fewer forward-looking disclosures than the managers of small unrated firms. In addition, we find a statistically significant, positive effect of firm profitability on our index across all three models. Thus, there is strong evidence that the managers of more profitable unrated firms release more forward-looking statements to the public. Further, the estimated coefficient for debt due in the next year is positive and statistically significant for two models, i.e. it appears that the managers of unrated firms increase their forward-looking disclosures prior to periods of increased financing needs. All these findings are consistent with our predictions in Hypothesis 1. We also find some evidence that the managers of more indebted unrated firms make fewer disclosures. Besides that, we find strong evidence for a general decline in the quantity of forward-looking disclosures during the period from 2001-2009: Relative to the year 2000 (the reference year),
the estimated time fixed effect is statistically significant for each year between 2001 and 2009. The point estimates suggest that the decline began in 2001 and lasted until 2009. The estimation results for the subsample of rated firms are displayed in Table 2b. For similar reasons like before, we display the estimated year fixed effects only for the period from 2001 to 2009.

### Table 2b: Rated firms

<table>
<thead>
<tr>
<th></th>
<th>Model I</th>
<th>Model II</th>
<th>Model III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total assets</td>
<td>−0.0255***</td>
<td>−0.0202</td>
<td>−0.0061</td>
</tr>
<tr>
<td>Book leverage</td>
<td>0.0190</td>
<td>0.0208</td>
<td>−0.0378*</td>
</tr>
<tr>
<td>MtB ratio</td>
<td>0.0000</td>
<td>0.0021</td>
<td>0.0047*</td>
</tr>
<tr>
<td>Firm profitability</td>
<td>0.0762**</td>
<td>0.0633*</td>
<td>−0.0186</td>
</tr>
<tr>
<td>Debt due in coming year</td>
<td>−0.0312</td>
<td>−0.0258</td>
<td>−0.0765</td>
</tr>
<tr>
<td>Loss dummy</td>
<td>−0.0041</td>
<td>−0.0025</td>
<td>−0.0040</td>
</tr>
<tr>
<td>Year FE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>−0.0228**</td>
<td>−0.0235***</td>
<td>−0.0175**</td>
</tr>
<tr>
<td>2002</td>
<td>−0.0427***</td>
<td>−0.0441***</td>
<td>−0.0409***</td>
</tr>
<tr>
<td>2003</td>
<td>−0.0599***</td>
<td>−0.0604***</td>
<td>−0.0600***</td>
</tr>
<tr>
<td>2004</td>
<td>−0.0801***</td>
<td>−0.0811***</td>
<td>−0.0809***</td>
</tr>
<tr>
<td>2005</td>
<td>−0.0977***</td>
<td>−0.0983***</td>
<td>−0.0973***</td>
</tr>
<tr>
<td>2006</td>
<td>−0.1043***</td>
<td>−0.1063***</td>
<td>−0.1013***</td>
</tr>
<tr>
<td>2007</td>
<td>−0.1093***</td>
<td>−0.1115***</td>
<td>−0.1057***</td>
</tr>
<tr>
<td>2008</td>
<td>−0.1189***</td>
<td>−0.1206***</td>
<td>−0.1096***</td>
</tr>
<tr>
<td>2009</td>
<td>−0.1256***</td>
<td>−0.1328***</td>
<td>−0.1223***</td>
</tr>
</tbody>
</table>

|                | Yes           | No            | No            |
| 2-digit SIC FE |               |               |               |
| 3-digit SIC FE | No            | Yes           | No            |
| Firm-level FE  | No            | No            | Yes           |
| Observations   | 7649          | 7649          | 7649          |

***, ** and * indicate statistical significance at the 1%, 5% and 10% level

For the subsample of rated firms, we find only weak evidence that a firm’s characteristics are related to its manager’s forward-looking disclosures: Merely in the first model, a firm’s size has a statistical significant, negative effect on our index. In absolute terms, all point estimates are considerably smaller than the corresponding point estimates for the subsample of unrated firms. We find somewhat stronger evidence that the managers of profitable rated firms make more forward-looking disclosures than the managers of unprofitable rated firms. However, we do not find a statistically significant effect of a firm’s debt due in the next year on its manager’s disclosure level. Hence, different from the managers of unrated firms, there is no evidence that the managers of rated firms increase their forward-looking disclosures prior to periods of in-
creased financing needs. Overall, it appears that for rated firms, the firm fundamentals have a weaker effect on managers’ forward-looking disclosures than for unrated firms. This finding is consistent with Hypothesis 2.

As for the group of unrated firms, we find strong evidence for a general decline in the level of forward-looking disclosures between 2001 and 2009. The similar evolution of our index for both groups suggests that a change in the regulatory environment was the cause for the decline over this period. As already mentioned above, a possible explanation for this decline might be the enactment of the Sarbanes-Oxley Act in July 2002. The continuous decline until 2009 would be consistent with the stepwise implementation of the SOX, as well as with a stepwise adaptation of firms to the requirements of this act.

6. Conclusions

We consider an infinitely repeated game of incomplete information where an informed, long-lived manager has to raise funds from short-lived external investors for the financing of a project. A project may be of good or bad quality, where good (bad) projects offer a certain (uncertain) payoff to investors. The manager raises funds either from imperfectly informed conservative investors or uninformed speculative investors. In order to signal project quality to speculative investors, the manager can make a forward-looking disclosure about the success potential of her project. However, such a disclosure might be costly, in the sense that it might reduce the firm’s payoff from a financed project.

We find that in situations where a manager’s forward-looking disclosures entail costs, she will never make disclosures that convey no information to external investors. The intuition is simple: If the release of forward-looking statements is associated with costs, but offers no benefit, a manager always prefers to make no disclosures. Further, we find that the managers of firms that are highly transparent and subject to significant disclosure-related costs will always refrain from forward-looking disclosures. On the other hand, we show that a disclosure policy where managers always release accurate forward-looking statements can be sustained more easily for firms that are more opaque or more profitable. The underlying intuition is that the managers of profitable or opaque firms find it more valuable to retain their credibility among future investors, and that they don’t want to jeopardize this credibility by misleading current investors.
via overly optimistic disclosures. It can be shown that under certain conditions, a policy of accurate disclosures is the only possible disclosure policy for the manager.

In order to test our results empirically, we construct an index which captures the quantity of forward-looking disclosures in a firm’s annual 10-K report, and regress this index on firm characteristics that are predicted to affect the manager’s disclosure policy. Consistent with our theoretical results, we find that for opaque (i.e. unrated) firms, our index is positively correlated with a firm’s profitability and financing needs. For transparent (i.e. rated) firms, we find only a weak relation between our index and a firm’s fundamentals. We also find that the overall level of forward-looking disclosures significantly declined between 2001 and 2009. A possible explanation for this development might be the enactment of the Sarbanes-Oxley Act in 2002, which presumably increased the transparency of public firms for external investors.
References


Appendix

A1.)

Assume that external investors assign probability $\tilde{\alpha} \geq \alpha^c$ to the good project. If we denote the firm’s share in the project return by $e_M$, the break-even condition of investors requires that

$$(1 - e_M)(\tilde{\alpha} + (1 - \tilde{\alpha})p_b)R = I$$

$\iff e_M = e_M(\tilde{\alpha}) = \frac{(\tilde{\alpha} + (1 - \tilde{\alpha})p_b)R - I}{(\tilde{\alpha} + (1 - \tilde{\alpha})p_b)R}$

Depending on the project quality $\theta$, we can distinguish two cases:

1.) $\theta = \theta^g$

The manager’s expected payoff $\Pi_g$ is a function of $\tilde{\alpha}$ and her disclosure decision $x_g(q)$:

$$\Pi_g(\tilde{\alpha}, x_g(q)) = (1 - x_g(q)l)e_M(\tilde{\alpha})R$$

Conservative investors finance a project iff they observe a good hard signal ($h = h^g$). In case they do, they know that the project is of good quality, i.e. we have $\tilde{\alpha} = 1$. We then define $\Pi^h_g(x_g(q)) := \Pi_g(1, x_g(q))$.

On the other hand, speculative investors buy the firm’s security only after observing an informative public disclosure, such that $x_g(q) = 1$ and $\tilde{\alpha} = \alpha_p(q)$. In such a case, we define $\Pi^s_g(\alpha_p(q)) := \Pi_g(\alpha_p(q), 1)$.

2.) $\theta = \theta^b$

The manager’s expected payoff, denoted by $\Pi_b$, again depends on $\tilde{\alpha}$ and $x_g(q)$:

$$\Pi_b(\tilde{\alpha}, x_g(q)) = p_b(1 - x_g(q)l)e_M(\tilde{\alpha})R$$

Like before, speculative investors finance an available project only if $x_g(q) = 1$ and $\tilde{\alpha} = \alpha_p(q)$. We then define $\Pi^s_b(\alpha_p(q)) := \Pi_b(\alpha_p(q), 1)$.

A2.)

The following lemma will be helpful for the proof of Theorem 1:
Lemma A1: In equilibrium, there holds $V_n(q) \geq \frac{\alpha\rho\Pi^h_q(0)}{1-\delta}$ for any $q \in [0,1]$.

Proof of Lemma A1: Let $V \in C_+$ denote the value function in a given equilibrium. Then holds $V(0) \geq V_n(0)$, such that the definition of $V_n(0)$ yields

$$V_n(0) \geq \alpha\rho\Pi^h_q(0) + \delta V_n(0)$$

$$\Leftrightarrow V_n(0) \geq \frac{\alpha\rho\Pi^h_q(0)}{1-\delta}$$

Since $V(q^N) \geq V(0) \geq V_n(0)$, it then follows from the definition of $V_n(q)$ that

$$V_n(q) \geq \alpha\rho\Pi^h_q(0) + \delta \frac{\alpha\rho\Pi^h_q(0)}{1-\delta}$$

$$\Leftrightarrow V_n(q) \geq \frac{\alpha\rho\Pi^h_q(0)}{1-\delta}$$

Proof of Theorem 1: Assume there exists an equilibrium strategy $\hat{x}_d$ where $\hat{x}_d(q) = (1, \hat{x}_b(q))$ and $\hat{\alpha}_p(s, q) < \alpha^c$. Let $\hat{V} \in C_+$ denote the value function which corresponds to this strategy. Condition (UE2) then requires that $\hat{V}(q^U) \geq \hat{V}(q^N)$. Since $\hat{V}$ is non-decreasing in $q$, it follows from $q^U \leq q \leq q^N$ that $\hat{V}(q^U) = \hat{V}(q^N) = \hat{V}(q)$, such that $V_u(q) = \frac{\alpha\rho\Pi^h_q(1)}{1-\delta}$. If $l > 0$, we have $\Pi^h_q(1) < \Pi^h_q(0)$. From Lemma A1, we then get $V_u(q) = \frac{\alpha\rho\Pi^h_q(1)}{1-\delta} < \frac{\alpha\rho\Pi^h_q(0)}{1-\delta} \leq V_n(q)$, a contradiction to the optimality of $(1, \hat{x}_b(q))$.

A3.

Proof of Lemma 2: Assume that the manager’s equilibrium strategy satisfies $x^*_b(0) = (1, x_b)$ (with $x_b \in [0,1]$), and let the corresponding perceived project quality after a public disclosure be given by $\alpha^*_p(0)$. Since $l > 0$, Theorem 1 and Lemma 1 imply that $\alpha^*_p(0) > \alpha^c$, such that $\Pi^h_b(\alpha^*_p(0)) > 0$. Further, according to (3) and (5), we have $q^N = q^S = 0$ if $q = 0$. Condition (IE2) then requires that $(x_b - x'_b)\Pi_b^h(\alpha^*_p(0)) \geq 0$ for all $x'_b \in [0,1]$, implying that $x_b = 1$. However, if $x^*_b(0) = (1, 1)$, it follows from (2) that $\alpha^*_p(0) = \alpha < \alpha^c$, a contradiction.

On the other hand, an equilibrium where $x^*_b(0) = (0, 0)$ is always sustainable: Let $V^*$ denote the value function which corresponds to such an equilibrium. Further, let $x^*_b(0)$ be sufficiently large (e.g. $x^*_b(0) = 1$) such that the corresponding out of equilibrium beliefs of current speculative
investors satisfy \( \alpha_p(0) < \alpha^c \). Due to (3) and (4), there holds \( q^N = q^U = 0 \) whenever \( q = 0 \), such that \( V_u(0) = \alpha \rho \Pi_g^h(0) + \delta V^*(0) \). Since \( l > 0 \), it follows that \( V_u(0) = \alpha \rho \Pi_g^h(0) + \delta V^*(0) > V_u(0) \), i.e. deviating from \((0, 0)\) is not profitable. Hence, the manager’s equilibrium strategy always satisfies \( x_d^*(0) = (0, 0) \).

A4.)

Proof of Theorem 2:

Existence: From (3), it follows that \( q^N = 1 \) whenever \( q = 1 \). Consequently, if the manager’s equilibrium strategy is given by \( x_d^q \), there holds \( V(1) = V_n(1) = \alpha \rho \Pi_g^h(0) + \delta V(1) \), i.e. we have \( V(1) = \frac{\alpha \rho \Pi_g^h(0)}{1-\delta} \). Due to Corollary 1, the unique value function \( V \in C_+ \) which corresponds to the strategy \( x_d^q \) therefore satisfies \( V(q) = \frac{\alpha \rho \Pi_g^h(0)}{1-\delta} \) for all \( q \in [0, 1] \). Since \( l > 0 \), this implies that \( V_n(q) > V_u(q) \) for all \( q \in [0, 1] \). Furthermore, it follows that if \( q = 1 \), the optimal deviation from the policy \((0, 0)\) is given by \((1, 1)\), with an associated discounted payoff of \( V_i(1) = \alpha \Pi_g^s(1) + (1 - \alpha) \Pi_b^s(1) + \delta \frac{\alpha \rho \Pi_g^h(0)}{1-\delta} \). Straightforward algebraic manipulations show that \( V_n(1) > V_i(1) \) iff condition (NE) holds. It can be easily shown that \( V_i(1) \geq V_i(q) \) for any \( q \in [0, 1] \). Hence, if condition (NE) is satisfied, it follows that \( V_n(q) > V_i(q) \) for any \( q \in [0, 1] \), i.e. the manager has no incentive to deviate from the equilibrium policy \((0, 0)\).

Uniqueness: Let the manager’s equilibrium strategy and value function be given by \( x_d^q \) and \( V^* \in C_+ \). It suffices to show that for any \( q \in [0, 1] \), \( x_d^q(q) \neq (0, 0) \) is not possible in equilibrium. We showed in Lemma 2 that \( x_d^q(0) = (0, 0) \) always holds in equilibrium. Hence, let \( q > 0 \), and assume that the manager’s equilibrium strategy satisfies \( x_d^q(q) = (1, x_b) \), with \( x_b \in [0, 1] \). Since \( l > 0 \), Theorem 1 and Lemma 1 imply that the corresponding perceived project quality for \( d = s \), \( \alpha_p^*(s, q) \), satisfies \( \alpha_p^*(s, q) > \alpha^c \). We can distinguish two cases:

1.) \( x_b > 0 \)

Condition (IE2) then requires that

\[
\Pi_b^s(\alpha_p(q)) + \delta (p_b V^*(q^S) + (1-p_b) V^*(0)) \geq \delta V^*(q^N)
\]

Using this inequality and the non-decreasing behaviour of \( V^* \), \( \Pi_g^s \) as well as \( \Pi_b^s \) in the definition
of $V_i(q)$ yields

$$V^*(q) = V_i(q) \leq \alpha \Pi_g^x(1) + (1 - \alpha) \Pi_b^x(1) + \delta[(\alpha + (1 - \alpha)p_b)V^*(q) + (1 - \alpha)(1 - p_b)V^*(0)]$$

Then follows

$$V^*(q) - V^*(0) \leq \alpha \Pi_g^x(1) + (1 - \alpha) \Pi_b^x(1) - (1 - \delta)V^*(0) + \delta(\alpha + (1 - \alpha)p_b)(V^*(q) - V^*(0))$$

$$\Leftrightarrow V^*(q) - V^*(0) \leq \frac{1}{1 - \delta(\alpha + (1 - \alpha)p_b)} \left[ \alpha \Pi_g^x(1) + (1 - \alpha) \Pi_b^x(1) - (1 - \delta)V^*(0) \right]$$

According to Corollary 1, we have $(1 - \delta)V^*(0) = \alpha \rho \Pi_g^b(0)$. Then holds

$$\alpha \Pi_g^x(1) + (1 - \alpha) \Pi_b^x(1) - (1 - \delta)V^*(0)$$

$$= (R - I)((1 - l)(\alpha + (1 - \alpha)p_b) - \alpha \rho) < 0$$

This implies that $V^*(q) < V^*(0)$, which contradicts the fact that $V^*$ is non-decreasing.

2.) $x_b = 0$

From (3) and (5), it follows that $q^N = q^S = q$ in equilibrium. According to the definition of $V_i$, we then have $V_i(q) = \alpha \Pi_g^x(1) + \delta V^*(q)$. Further, we have $V_b(q) = \alpha \rho \Pi_g^b(0) + \delta V^*(q)$. Condition (E1) then requires that $\Pi_g^x(1) \geq \rho \Pi_g^b(0)$, which holds iff $(1 - l) \geq \rho$. However, this is a contradiction to (NE), since $(1 - l)(\alpha + (1 - \alpha)p_b) > (1 - l)\alpha$.

A5.)

*Proof of Theorem 3:*

1.) $\Rightarrow$: Assume there exists an equilibrium strategy $x_d$ with $x_d(q) = (1, 0)$ for some $q > 0$. Then follows from (2) that $\alpha_p(q) = 1$, as well as from (3) and (5) that $q^N = q^S = q$. If we denote the equilibrium value function by $V^a$, condition (IE2) therefore implies that

$$\Pi_g^x(1) + \delta(p_b V^a(q) + (1 - p_b) V^a(0)) \leq \delta V^a(q) \quad (A5.1)$$
From the definition of $V_i$, we get $V^a(q) = \frac{\alpha 1}{1-\delta}$. Using this and Corollary 1 in (A5.1) gives

$$\Pi^s_b(1) + \delta \left( p_b \frac{\alpha \Pi^h_b(1)}{1-\delta} + (1 - p_b) \frac{\alpha \rho \Pi^h_g(0)}{1-\delta} \right) \leq \delta \frac{\alpha \Pi^h_g(1)}{1-\delta}$$

Simple algebraic manipulations then yield inequality (AE).

2.) “⇒”: Assume that (AE) holds, and let $q > 0$. We need to show that $x_d(q) = (1, 0)$ supports an equilibrium. Let $V^a$ denote a value function which corresponds to $x_d(q)$, i.e. let $V^a(q) = \frac{\alpha 1}{1-\delta}$. Further, let the beliefs of current and subsequent investors be correctly specified, such that $\alpha q_p(q) = 1$ and $q^N = q^S = q$. Condition (E1) then requires that $V_i(q) \geq V_n(q)$, which holds iff $(1 - l) \geq \rho$. Since the left-hand side of (AE) is positive, this is satisfied. Moreover, analogous manipulations like in 1.) show that if (AE) holds, we have

$$\Pi^s_b(1) + \delta (p_b V^a(q) + (1 - p_b) V^a(0)) \leq \delta V^a(q)$$

which is the variant of (IE2) that corresponds to the disclosure policy $x_d(q) = (1, 0)$. Hence, condition (E2) is satisfied as well, i.e. $x_d(q) = (1, 0)$ supports an equilibrium.

A6.)

We will use three additional lemmas (Lemma A2-A4) for our proof of Theorem 4. In each lemma, let $x_d^* = (x^*_g, x^*_b)$ denote an equilibrium strategy, with $V^* \in C_+$ and $\alpha^*_p$ representing the associated value function and beliefs of current investors.

Lemma A2: Assume that $x_d^*(1) = (1, 0)$ and that (AE) holds strictly, and let $\tilde{q} \in (0, 1)$. If $x_d^*(\tilde{q}) = (1, 0)$, we have $x_d^*(q) = (1, 0)$ for all $q \in (\tilde{q}, 1)$.

Proof of Lemma A2: From $x_d^*(1) = (1, 0)$ follows that $V^*(1) = \frac{\alpha 1}{1-\delta}$. Using this and Corollary 1, it can then be shown that (AE) holds strictly iff $\Pi^s_b(1) + \delta (p_b V^*(1) + (1 - p_b) V^*(0)) < \delta V^*(1)$. Further, we have $\frac{\alpha^*_p \Pi^h_g(0)}{1-\delta} < \frac{\alpha 1}{1-\delta}$ whenever (AE) is satisfied.

Let $x_d^*(\tilde{q}) = (1, 0)$ for a given $\tilde{q} \in (0, 1)$, and assume that $x_d^*(\hat{q}) = (0, 0)$ for some $\hat{q} \in (\tilde{q}, 1)$. Then holds $\hat{q}^N \leq \hat{q}$, and it follows that $V^*(\hat{q}) = V_n(\hat{q}) \leq \frac{\alpha^*_p \Pi^h_g(0)}{1-\delta}$. Due to Corollary 1, this implies that $V^*(\hat{q}) = \frac{\alpha 1}{1-\delta}$. However, then follows from condition (AE) that $V^*(\hat{q}) < \frac{\alpha 1}{1-\delta} = V^*(\tilde{q})$, which contradicts the restriction that $V^* \in C_+$. Hence, we have $x_d^*(q) = 1$ for all $q \in (\tilde{q}, 1)$. 


Further, if $x^*_d(\tilde{q}) \in (0,1]$ for some $\tilde{q} \in (\tilde{q},1)$, we have $\tilde{q}^N > \tilde{q} > \tilde{q}$. Condition (IE2) then implies that $\Pi_\delta^\alpha(\alpha_p^*(\tilde{q})) + \delta(p_b V^* (\tilde{q}^s) + (1 - p_b) V^* (0)) \geq \delta V^* (\tilde{q}) = \delta V^* (1)$. Because of $V^* (\tilde{q}^S) \leq V^* (1)$ and $\Pi_\delta^\alpha(\alpha_p^*(\tilde{q})) \leq \Pi_\delta^\alpha(1)$, this contradicts (AE). It follows that $x^*_d(q) = (1,0)$ for all $q \in (\tilde{q},1)$.

**Lemma A3:** Let $q^i \in (0,1)$, and assume we have $x^*_d(1) = (1,0)$ and $x^*_d(q) \in \{(0,0), (1,0)\}$ for $q \in (q^i, 1)$. If (AE) holds strictly, it follows that $x^*_d(q) = (1,0)$ for all $q \in (q^i, 1)$.

**Proof of Lemma A3:** Let (AE) hold strictly. Then holds $\Pi_\delta^\alpha(1) + \delta(p_b V^* (1) + (1 - p_b) V^* (0)) < \delta V^* (1)$ and $\frac{\alpha \Pi_\delta^\alpha(0)}{1 - \delta} < \frac{\alpha \Pi_\delta^\alpha(1)}{1 - \delta}$. Further, assume we have $x^*_d(\tilde{q}) = (0,0)$ for some $\tilde{q} \in (q^i, 1)$. If $S := \{q \in (q^i, 1)|x^*_d(q) = (0,0)\}$, it then follows that $S \neq \emptyset$ and that $s := \sup(S) \geq \tilde{q}$. Without loss of generality, we can assume that $s < 1$ and that $s \notin S$. For $q \in [s, 1)$, we then have $x^*_d(q) = (1,0)$ and $V^*(q) = \frac{\alpha \Pi_\delta^\alpha(1)}{1 - \delta}$. Further, Lemma A2 implies that $x^*_d(q) = (0,0)$ for all $q \in (q^i, s)$, such that $V^*(q) = \frac{\alpha \Pi_\delta^\alpha(0)}{1 - \delta} < V^*(s)$ for $q \in (q^i, s)$. However, this is a contradiction to the continuity of $V^*$ at $q = s$. Hence, we have $x^*_d(q) \neq (0,0)$ for any $q \in (q^i, 1)$, implying that $x^*_d(q) = (1,0)$ for all $q \in (q^i, 1)$.

**Lemma A4:** Assume we have $x^*_d(1) = (1,0)$ and $x^*_d(q) \neq (1,1)$ for all $q \in (0,1)$. Further, let $Q := \{q \in (0,1)|x^*_d(q) = (1,0)\} \neq \emptyset$, and let inequality (AE) hold strictly. If $\underline{q} := \inf(Q) > 0$, there holds $x^*_d(\underline{q}) = (1,0)$.

**Proof of Lemma A4:** Like before, we have $\Pi_\delta^\alpha(1) + \delta(p_b V^* (1) + (1 - p_b) V^* (0)) < \delta V^* (1)$ and $\frac{\alpha \Pi_\delta^\alpha(0)}{1 - \delta} < \frac{\alpha \Pi_\delta^\alpha(1)}{1 - \delta}$. Let $q > 0$. There holds $x^*_d(q) = (1,0)$ for any $q > q$: If $x^*_d(\tilde{q}) \neq (1,0)$ for some $\tilde{q} > q$, Lemma A2 would imply that $x^*_d(q) \neq (1,0)$ for all $q < \tilde{q}$, i.e. $\tilde{q} > \inf(Q)$ would be a lower bound of $Q$, a contradiction. We therefore have $V^*(q) = \frac{\alpha \Pi_\delta^\alpha(1)}{1 - \delta}$ for $q > \underline{q}$.

Assume we had $x^*_d(\underline{q}) = (0,0)$, such that $V^*(\underline{q}) = \frac{\alpha \Pi_\delta^\alpha(0)}{1 - \delta}$. This would imply that for $q > \underline{q}$, we had $V^*(q) = \frac{\alpha \Pi_\delta^\alpha(1)}{1 - \delta} > V^*(\underline{q})$, which contradicts the continuity of $V^*$ at $q = \underline{q}$. It follows that $x^*_d(q) = \{1, x^*_d(q)\}$. We show that $x^*_d(q) \in (0,1)$: Assume we had $x^*_d(q) \in (0,1)$. Then holds $\underline{q}^N > q$, such that $V^*(\underline{q}^N) = \frac{\alpha \Pi_\delta^\alpha(1)}{1 - \delta} = V^* (1)$, and condition (IE2) requires that $\Pi_\delta^\alpha(\alpha_p^*(\underline{q})) + \delta(p_b V^*(\underline{q}^S) + (1 - p_b) V^* (0)) = \delta V^* (1)$. However, since $\Pi_\delta^\alpha(\alpha_p^*(\underline{q})) \leq \Pi_\delta^\alpha(1)$ and $V^*(\underline{q}^S) \leq V^* (1)$, this is a contradiction to inequality (AE). We therefore have $x^*_d(q) \notin (0,1)$.

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8 We showed in the proof of Lemma A2 that $V^*(q) = \frac{\alpha \Pi_\delta^\alpha(0)}{1 - \delta}$ whenever $x^*_d(q) = (0,0)$
Since, by assumption, \( x^*_d(q) \neq (1, 1) \), it follows that \( x^*_d(q) = (1, 0) \).

**Proof of Theorem 4:** Using the definition of \( \Pi^h_g(1) \) and \( \Pi^s_b(1) \), it can be shown that inequality (AE) holds strictly iff \( \Pi^s_b(1) < \alpha(1 - p_b)\delta \left( \Pi^h_g(1) - \rho \Pi^h_g(0) \right) \). Note that this requires that \( \Pi^h_g(1) > \rho \Pi^h_g(0) \).

**Existence:** If inequality (AE) holds strictly, it follows from Lemma 2 and Theorem 3 that \( x^*_d \) is an equilibrium strategy. Obviously, the corresponding equilibrium value function is continuous on \((0, 1]\) and non-decreasing in \( q \).

**Uniqueness:** Let \( x^*_d = (x^*_g, x^*_b) \) represent a generic equilibrium strategy, with \( V^* \in C_+ \) and \( \alpha^*_p \) denoting the associated value function and beliefs of current investors. Due to Theorem 3, \( x^*_d(q) = (1, 0) \) is always possible for \( q > 0 \). We distinguish between two cases:

1.) \( q = 1 \)

In such a case holds \( q^S = q^N = 1 \). Assume that \( x^*_d(1) = (0, 0) \), such that \( V^*(1) = \frac{\alpha \Pi^s_b(0)}{1 - \delta} \).

Since \( \alpha^*_p(1) = 1 \), deviating to the policy \((1, 0)\) is associated with a discounted payoff of \( V_i(1) = \alpha \Pi^h_g(1) + \delta V^*(1) \). Optimality of \( x^*_d(1) = (0, 0) \) then requires that \( V^*(1) \geq V_i(1) \), which holds iff \( \rho \Pi^h_g(0) \geq \Pi^h_g(1) \), and therefore contradicts condition (AE). It follows that \( x^*_d(1) \neq (0, 0) \), i.e. we have \( x^*_g(1) = 1 \) and \( x^*_b(1) \in [0, 1] \).

We rule out that \( x^*_b(1) > 0 \): If \( x^*_b(1) = 1 \), it follows from the definition of \( V^*(1) \) and from Corollary 1 that

\[
V^*(1) - V^*(0) = \frac{1}{1 - \delta(\alpha + (1 - \alpha)p_b)} \left( \alpha \Pi^h_g(1) + (1 - \alpha) \Pi^s_b(1) - \alpha \rho \Pi^s_b(0) \right) \tag{A6.1}
\]

Further, condition (IE2) requires that \( \Pi^s_b(1) \geq \delta(1 - p_b)(V^*(1) - V^*(0)) \). Combining this with (A6.1), it follows after some straightforward manipulations that \( \Pi^s_b(1) \geq \alpha(1 - p_b)\delta \left( \Pi^h_g(1) - \rho \Pi^h_g(0) \right) \), which is a contradiction to our precondition.

Further, if \( x^*_b(1) \in (0, 1) \), condition (IE2) requires that \( \Pi^s_b(1) + \delta(p_b V^*(1) + (1 - p_b) V^*(0)) = \delta V^*(1) \). One can use this in the definition of \( V^*(1) \) in order to show that \( V^*(1) = \frac{\alpha \Pi^s_b(1)}{1 - \delta} \). Combined with Corollary 1, it then follows from (IE2) that \( \Pi^s_b(1) = \alpha(1 - p_b)\delta \left( \Pi^h_g(1) - \rho \Pi^h_g(0) \right) \), which also contradicts our precondition.

Consequently, there holds \( x^*_d(1) = (1, 0) \), such that \( V^*(1) = \frac{\alpha \Pi^s_b(0)}{1 - \delta} \). Using this and Corollary 1,
it then follows from our precondition that

\[ \Pi_b^*(1) + \delta(p_b V^*(1) + (1 - p_b) V^*(0)) < \delta V^*(1) \]  

(A6.2)

2.) \( 0 < q < 1 \)

We can directly rule out that \( x_d^*(q) = (1, 1) \): If \( x_d^*(q) = (1, 1) \), there holds \( q^N = 1 \), such that, according to condition (IE2), we have \( \Pi_b^*(\alpha_p^*(q)) + \delta(p_b V^*(q^S) + (1 - p_b) V^*(0)) \geq \delta V^*(1) \).

However, since \( \Pi_b^*(\alpha_p^*(q)) \leq \Pi_b^*(1) \) and \( V^*(q^S) \leq V^*(1) \), this is a contradiction to (A6.2). Hence, we have \( x_d^*(q) \neq (1, 1) \) for all \( q \in (0, 1) \).

Further, since \( V^* \) is continuous at \( q = 1 \), (A6.2) implies that there is a \( q^c \in (0, 1) \) such that

\[ \Pi_b^*(\alpha_p^*(q)) + \delta(p_b V^*(q) + (1 - p_b) V^*(0)) < \delta V^*(q) \text{ for all } q \in (q^c, 1). \]

Because of \( V^*(q^S) \leq V^*(q) \leq V^*(q^N) \), it then follows from condition (IE2) that \( x_d^*(q) \notin \{1 \times (0, 1) \text{ for } q \in (q^c, 1) \}. \)

Hence, we have \( x_d^*(q) \in \{(0, 0), (1, 0)\} \) for all \( q \in (q^c, 1) \), such that, according to Lemma A3, there holds \( x_d^*(q) = (1, 0) \) for \( q \in (q^c, 1) \). Consequently, we have \( Q := \{q \in (0, 1) | x_d^*(q) = (1, 0) \} \neq \emptyset \) and \( q := \inf(Q) \leq q^c \).

We show that \( q = 0 \): Assume we had \( q > 0 \). Lemma A2 and Lemma A4 would then imply that \( x_d^*(q) = (1, 0), \alpha_p^*(q) = 1 \) and \( V^*(q) = V^*(1) \) for all \( q \in [q, 1) \). Due to (A6.2), we then had \( \Pi_b^*(1) + \delta(p_b V^*(q) + (1 - p_b) V^*(0)) < \delta V^*(q) \). Since \( V^* \) is continuous at \( q = q \), it would follow that \( \exists \tilde{q} < q \) such that \( \Pi_b^*(\alpha_p^*(q)) + \delta(p_b V^*(q) + (1 - p_b) V^*(0)) < \delta V^*(q) \) for \( q \in (\tilde{q}, 1) \).

A similar reasoning like before would then imply that \( x_d^*(q) \in \{(0, 0), (1, 0)\} \) for all \( q \in (\tilde{q}, 1) \), such that, due to Lemma A3, we had \( x_d^*(q) = (1, 0) \) for \( q \in (\tilde{q}, 1) \), which contradicts the fact that \( q = \inf(Q) \). We therefore have \( q = 0 \).

From \( q = 0 \) follows that \( x_d^*(q) = (1, 0) \) for all \( q \in (0, 1) \): Assume that \( x_d^*(\hat{q}) \neq (1, 0) \) for some \( \hat{q} \in (0, 1) \). Due to Lemma A2, this would imply that \( x_d^*(q) \neq (1, 0) \) for all \( q \in (0, \hat{q}) \).

Consequently, \( \hat{q} > q \) is a lower bound of \( Q \), which is a contradiction.

Due to 1.) and 2.), there holds \( x_d^*(q) = (1, 0) \) for \( q > 0 \). Since, by Lemma 2, we have \( x_d^*(0) = (0, N) \), it follows that \( x_d^* = x_d^* \).

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\(^9\)Define the function \( f(q) = \delta(1 - p_b)(V^*(q) - V^*(0)) - \Pi_b^*(1) \), and use that \( f \) is continuous at \( q = 1 \) and that \( f(1) > 0 \) as well as \( \Pi_b^*(1) \geq \Pi_b^*(\alpha_p^*(q)) \)