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Monetary policy effects on bank risk taking

by Angela Abbate and Dominik Thaler

September 2015 No 287
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Abstract

Motivated by VAR evidence on the risk-taking channel in the US, we develop a New Keynesian model where low levels of the risk-free rate induce banks to grant credit to riskier borrowers. In the model an agency problem between depositors and equity holders incentivizes banks to take excessive risk. As the real interest rate declines these incentives become stronger and risk taking increases. We estimate the model on US data using Bayesian techniques and assess optimal monetary policy conduct in the estimated model, assuming that the interest rate is the only available instrument. Our results suggest that in a risk taking channel environment, the monetary authority should seek to stabilize the path of the real interest rate, trading off more inflation volatility in exchange for less interest rate and output volatility.

JEL Classification: E12, E44, E58.

Keywords: Bank Risk; Monetary policy; DSGE Models.

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1 Introduction

The recent financial crisis has marked the importance of monitoring the different types of risks to which the financial sector, and ultimately the real economy, are exposed. A relevant aspect is whether interest rates, and therefore monetary policy, can influence the risk-taking behavior of financial intermediaries. This transmission mechanism, known as the risk-taking channel of monetary policy,\(^1\) could have contributed to the excessive risk exposure of the financial sector in the lead-up to the 2008 crisis. In the aftermath of the financial crisis interest rates have fallen to their zero lower bound in many countries, raising concerns on whether financial market participants might be once again induced to reallocate portfolios towards riskier investments, creating the risk of yet another crisis.\(^2\)

Motivated by the empirical finding that expansionary monetary policy shocks persistently increase bank asset risk in the US, we model bank asset risk taking in a DSGE framework and assess how monetary policy should be conducted in the presence of a risk-taking channel. Building on Dell’Ariccia et al. [2014], we assume that banks can choose from a continuum of investment projects, each defined by different risk-return characteristics. Since depositors cannot observe this choice, and because bank owners are protected by limited liability, an agency problem emerges: banks are partially isolated from the downside risk of the investment and hence choose a risk level that exceeds what would be chosen if these frictions were absent. With respect to this latter benchmark case, excessive risk leads to inefficiently low levels of capital, output and consumption. Furthermore, we show that lower levels of the real risk-free rate induce banks to choose even riskier investments.

This connection between interest rates and the risk choice raises the question of whether the monetary authority should take this channel into account when setting the interest rate. Since the answer to this question is of quantitative nature, we embed our banking sector in a medium scale Smets and Wouters [2007] type DSGE model, known to fit the data well along many dimensions, and estimate it on US data. Using this model we then analyze optimal monetary policy using simple rules. We find that, if the risk-taking channel is active, monetary policy should be less responsive to inflation and output fluctuations. In this way, the monetary authority allows more inflation volatility in exchange for stabilizing the real interest rate, which in turn reduces the welfare detrimental volatility of the banks’ risk choice. The welfare gains from taking the risk-taking channel into account are significant.

Our work relates to a small but growing theoretical literature that links monetary policy to financial sector risk in a general equilibrium framework. Most of the existing works focus on funding risk, associating risk with leverage, and build on the financial accelerator framework of Bernanke et al. [1999].\(^3\) Angeloni and Faia [2013] and Angeloni et al. [2015] instead build

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\(^1\)The term was first coined by Borio and Zhu [2008].


\(^3\)For example in Gertler et al. [2012] and de Groot [2014] a monetary expansion increases banking sector leverage, which in turn amplifies the financial accelerator and strengthens the propagation of shocks to the real economy.
a model where higher leverage induced by expansionary monetary policy doesn’t just amplify other shocks but also translates into a higher fraction of inefficient bank runs.

In this paper we model bank asset risk, another aspect of financial sector risk that could have played a role in the lead-up to the 2008 financial crisis, but has not yet been explored in the monetary policy literature. This link is analyzed in a micro model by Dell’Ariccia et al. [2014], but their framework is not suitable to answer the policy questions addressed in this paper. Inefficient asset risk-taking is modelled also in Collard et al. [2012], though ultimately the financial regulator ensures that risk is always chosen optimally.4 Our paper on the contrary abstracts from regulation as an additional tool and explores how monetary policy should be conducted if the risk taking channel cannot be resolved (fully) through regulation.

The lack of theoretical papers on the asset risk taking channel is not mirrored by lack of empirical evidence. On the contrary, several studies find a causal link between monetary policy and risk taking. Most of the existing research relies on loan or bank level panel data and identification is based on the assumption that monetary policy is exogenous. Jimenez et al. [2014] use micro data of the Spanish Credit Register from 1984 to 2006 and find that lower interest rates induce banks to make relatively more loans to firms that qualify as risky ex ante (firms with a bad credit history at time of granting the loan) as well as ex post (firms that default on the granted loan). They argue that this effect is economically significant and particularly strong for thinly capitalized banks. These findings are confirmed for Bolivia using credit register data in Ioannidou et al. [2014] and for the US using confidential loan level data from the Terms of Business Lending Survey in Dell’Ariccia et al. [2013].

Evidence based on aggregate, publicly available time series data, where identification is obtained through restrictions on the dynamic responses, is less clear cut, not least because of limited data availability. Buch et al. [2013] estimate a FAVAR on a large data set of US banking-sector variables from the public summary data of the Terms of Business Lending Survey. They find that an expansionary monetary policy shock induces small domestic banks to increase the fraction of risky new loans to total new loans, but this evidence is not confirmed for the entire banking system. Afanasyeva and Guentner [2015] show, using a similar FAVAR methodology, that a monetary policy expansion decreases the net percentage of banks reporting tighter lending standards in the Fed survey of business lending. This measure, which may be considered a possible proxy for asset risk-taking, has not been found to be impacted by monetary policy in the empirical analysis5 conducted in Angeloni et al. [2015]. We complement this result in the following section, showing that further evidence of an asset risk-taking channel can be retrieved by using a more direct measure of asset risk taking.

An interesting detail on the asset risk-taking channel is found in Buch et al. [2013] and

4 Only in an extension do they briefly consider the possibility that monetary policy might have implications for optimal regulation.

5 Angeloni et al. (2015) use a recursive identification scheme on monthly US data between 1985 and 2008. They test three different measures for risk: funding risk (leverage), asset risk (percentage of loan officers reporting a tightening of loan standards in the Fed survey of business lending) and overall risk (realized volatility of bank stock prices). They find (lagged) significant positive effects only for funding and overall risk.
Ioannidou et al. [2014]. Both show that an increase in risk taking induced by low interest rates is not accompanied by an offsetting increase in the risk premium, indicating that the additional risk taking is inefficient.

Motivated by this comprehensive empirical evidence and by our own VAR analysis in section 2, we develop in section 3 a DSGE model of asset risk-taking, where banks respond to low interest rates by inefficiently taking more risk. Section 4 discusses the steady-state and dynamic implications of bank risk taking in the estimated version of the model presented in section 3. Section 5 analyses how monetary policy should be conducted if a risk-taking channel is present and section 6 concludes.

2 The asset risk-taking channel in the US

In this section we provide additional empirical evidence on the existence of the asset risk-taking channel in the US. Our starting point is a classical small scale VAR that includes inflation, output, a measure of bank risk taking and the effective federal funds rate, taken as the monetary policy instrument. Output is measured by real GDP growth, while inflation is defined as the annualised log change in the GDP deflator. Identification of the monetary policy shock is achieved through sign restrictions that do not involve the measure of bank risk, and are in this sense agnostic. We find that an unexpected decrease in the risk-free interest rate causes a persistent increase in bank risk taking, a result robust to using a recursive identification scheme. Our positive findings contradict the insignificance result on asset risk in Angeloni et al. [2015], whose specification is identical apart from the risk measure used.⁶

Measuring bank risk-taking behaviour There are many notions of asset risk. One can distinguish between ex-ante, ex-post and realized asset risk. The former is the risk perceived by the bank when making a loan or buying an asset. Banks can influence this class of risk directly, when making their investment decisions (the ex-ante risk choice). On the other hand, the ex-post risk of a bank’s balance sheet is also affected by unforeseen changes in asset riskiness, that take place after origination and are largely outside the banks’ influence. Finally, both types of risk together yield a certain stochastic return when the asset or a loan is due (materialized asset risk).

In this paper we focus on active risk taking, that is the level of ex-ante risk that intermediaries choose. Ex-ante bank risk taking is however largely unobservable. Inferring it from materialized risk (e.g. loan losses) is hardly possible with aggregate data. Inference from the spread between some measure of bank funding costs and loan rates neglects the fact that this spread not only reflects default risk but also incorporates a liquidity premium and the markup, which are likely to be affected by the same variables that influence the risk choice. Instead we use a survey based

⁶We believe that our measure of asset risk (weighted risk rating of new loans) is superior to theirs (percentage of loan officers reporting tightening of lending standards) in two ways: First, our measure provides level information while their measure indicates only the direction of change in risk. Second, our measure is based on the quantity and quality of loans granted (revealed preferences), whereas theirs is based on reported perceptions that are aggregated to an index in an arbitrary way.

3
proxy for bank risk-taking, which is provided by the US Terms of Business Lending Survey and consists in the internal risk rating assigned by banks to newly issued loans. In this survey, available only from 1997-Q1 onwards, 400 banks report the volume of loans originated in the quarter prior to filling the survey, grouped by the internal risk rating. This rating varies between 1 and 5, with 5 being the maximum level of risk. Following Dell’Ariccia et al. [2014], we construct a weighted average loan risk series, using as weights the value of loans in each risk category. We plot this measure of risk, together with the nominal risk-free interest rate, in figure 1. We can see that low levels of the monetary policy instrument tend to be associated with higher levels of bank risk (lower or decreasing levels in the blue line). This is particularly evident in the period between 2000 and 2006, when the monetary policy stance in the US was deemed to be particularly lax.

An increase in risk at times of low interest rates can be caused either by worsened macroeconomic conditions, giving banks no other choice than making riskier loans on average, or by an active choice of the banks to extend credit to riskier borrowers on average. We control for this effect, to the extent that it is reflected in inflation and output.

Figure 1: Bank risk taking and nominal interest rate: The risk measure (solid blue line, left axis) is defined such that a decrease can be associated to a higher risk-taking behaviour of the banking sector. The nominal interest rate (dashed line, right axis) is the effective federal funds rate.

Identification strategy and results We estimate a VAR on US data from 1997-Q1, the start of the survey-based proxy for risk taking, to 2008:Q3, as described in appendix A. The lag length is chosen to be 1, as indicated by the BIC information criterion.

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The average loan risk is a perfect measure for bank risk taking if we assume that the volume of loans is constant. Else, banks could also minimize their risk exposure by reducing the quantity of loans as their average quality goes down. While the correlation between risk and loan volume growth is slightly negative, it is not significant at 10%. For a more in-depth discussion of the data we refer to Buch et al. [2013].

We have decided to cut the zero-lower bound period, but it should be noted that our results still hold also when the latest available data are used.
Table 1: **Sign restriction identification scheme**: restrictions are assumed to hold on impact and on the following period. 

<table>
<thead>
<tr>
<th>Shock</th>
<th>y</th>
<th>π</th>
<th>R</th>
</tr>
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<tbody>
<tr>
<td>Monetary policy shock</td>
<td>+</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Total factor productivity shock</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
</tbody>
</table>

Figure 2: **Monetary policy shock on bank risk-taking**: Impulse responses over a 3-year horizon, identified through the sign restriction scheme in Table 1. Error bands correspond to 68% (light grey area) and to 95% (dark grey area) confidence intervals reflecting rotation uncertainty. The variable $q_{e x-a n t e}$ proxies the ex-ante safety of banks’ assets and is defined as the inverse of the average loan risk rating. See text for further details.

We identify an unexpected monetary policy shock by using a set of sign restrictions that are robust across a variety of general-equilibrium models. In particular, we assume that an expansionary monetary policy shock decreases the nominal risk-free interest rate, and increases inflation and output, both at the time of the shock and in the quarter immediately after. Note that the response of inflation ensures that this shock is identified separately from a productivity shock, as summarised in table 1.

The response of bank asset risk to an expansionary monetary policy shock is shown in figure 2. An unexpected decrease in the monetary policy interest rate is followed by a moderate macroeconomic expansion: output growth increases for less than a year, while inflation displays a longer reaction of about two years. The results are also compatible with the existence of a risk-taking channel in the US: a fall in the nominal interest rate leads in fact to a decrease in the ex-ante proxy for the safety of banks’ assets, and thus to an increase in the risk rating the banks attach to the loans in their portfolio. Interestingly, the implied responses of the real interest rate and the risk measure are virtually proportional.

We assess the robustness of our results by employing a recursive identification scheme as in
Angeloni et al. [2015] with two different variable orderings. Output and inflation are always ordered before the nominal interest rate, implicitly assuming that they do not react to interest rate shocks, nor to shocks coming from the banking sector. More controversial is the ordering of the nominal interest rate and of the loan risk rating. We experiment with both orderings and show in figure 4 in appendix B that results do not depend on the ordering chosen and confirm the results obtained using sign restrictions\(^9\).

3 A Dynamic New Keynesian model with a bank risk-taking channel

In this section we build a general-equilibrium model where monetary policy can influence the risk-taking behaviour of banks, thus providing an explanation for the stylized facts observed in the data. As a starting point we use a standard New Keynesian model with imperfect competition and price stickiness in the goods market, which implies a role for monetary policy. We augment this basic framework with an intermediation sector based on Dell’Ariccia et al. [2014]: competitive banks obtain funds from depositors and equity holders, which they invest into capital projects carried out by capital producers. Every bank chooses its investment from a continuum of available capital production technologies, each defined by a given risk-return characteristic. The risk choice of the representative bank is affected by the level of the real interest rate, and can be shown to be suboptimal. This model reproduces two features found in the data: risk taking depends on the contemporaneous interest rate and is inefficient.

While the aforementioned blocks are the necessary ingredients, in order to obtain a quantitatively more realistic model we add further elements, which are typically used in the DSGE literature. In particular we allow for internal habits, investment adjustment costs and imperfect competition and wage stickiness in the labor market. Our model therefore features seven agents that are typical for DSGE models (households, unions, labor packers, capital producers, intermediate goods producers, final goods producers, central bank) and two agents that we introduce to model risk taking (banks, funds). Seven structural shocks hit the economy: these affect productivity, investment, preferences, wage and price markups shock, as well as monetary and fiscal policy. The complete set of equations that characterize the model can be found in appendix C.

3.1 Households

The representative household chooses consumption \(c_t\), working hours \(L_t\) and savings in order to maximize its discounted lifetime utility \(U(c_t, L_t)\). Saving is possible through three instruments: government bonds \(s_t\), deposit funds \(d_t\), and bank equity funds \(e_t\). The return on government

\(^9\)Our measure of risk taking does not take into account loan quantities. For example, riskier lending may go hand in hand with lower loan volumes which may leave the expected loss volume unchanged. When adding loan volumes to the VAR we find that the response of volumes is not significantly different from zero with the mean showing no clear pattern in either direction, while the other IRFs remain unchanged.
bonds purchased today is safe and equal to the nominal interest rate \( R_t \). The two funds enable the representative household to invest into the banking sector and pay an uncertain nominal return of \( R_{d,t} \) and \( R_{e,t} \). Households maximize their lifetime utility function subject to the per-period budget constraint in real terms:

\[
\max_{d_t, e_t, s_t, c_t, L_t} \sum_{t=0}^{\infty} \beta^t E_t \left[ u(c_t, L_t) \right]
\]

where \( u(c_t, L_t) = \frac{(c_t - \mu_{t-1})^{1-\sigma_c}}{1-\sigma_c} \exp \left( \varphi L_t^{1+\sigma_c} \frac{\sigma_c - 1}{1 + \sigma_c} \right) \).

subject to the per-period budget constraint in real terms:

\[
c_t + d_t + e_t + s_t + T_t = L_t w_t + d_t \frac{R_{d,t}}{\pi_t} + e_t \frac{R_{e,t}}{\pi_t} + s_t \frac{R_{t-1}}{\pi_t} + \Pi_t,
\]

where \( \pi_t \) is the inflation rate, while \( T_t \) and \( \Pi_t = \Pi_{cap} + \Pi_{equ} + \Pi_{firm} \) are respectively taxes and profits from firm ownership, expressed in real terms. We assume that in each period a preference shock \( \varepsilon_t^B \) shifts overall utility, and evolves according to a first-order autoregressive process; furthermore we allow for habits in consumption. From the first-order conditions with respect to consumption and savings we obtain the two no-arbitrage conditions:

\[
E_t \left[ \varepsilon_{t+1}^B u(c_{t+1}, L_{t+1}) \frac{R_{d,t+1}}{\pi_{t+1}} \right] = E_t \left[ \varepsilon_{t+1}^B u(c_{t+1}, L_{t+1}) \frac{R_t}{\pi_{t+1}} \right],
\]

\[
E_t \left[ \varepsilon_{t+1}^B u(c_{t+1}, L_{t+1}) \frac{R_{e,t+1}}{\pi_{t+1}} \right] = E_t \left[ \varepsilon_{t+1}^B u(c_{t+1}, L_{t+1}) \frac{R_t}{\pi_{t+1}} \right],
\]

where \( u_c(c_t, L_t) \) is the marginal period utility of consumption. The first-order condition for consumption yields the usual Euler equation:

\[
u_c(c_t, L_t) = \beta E_t \left[ \varepsilon_{t+1}^B u(c_{t+1}, L_{t+1}) R_t \right],
\]

### 3.2 Labor unions, final and intermediate good producers

The problems faced by final and intermediate good producers, as well as by labor unions, are standard. We discuss them briefly and refer to Smets and Wouters [2007] for further details. The corresponding equilibrium conditions are listed in appendix C.

Final good producers assemble different varieties of intermediate goods through a Kimball [1995] aggregator with stochastic elasticity of substitution \( \varepsilon_t^B \), taking as given both the final good price and the prices of intermediate goods. Their optimization problem yields demand functions for each intermediate good variety as a function of its relative price.

A continuum of firms produces differentiated intermediate goods using capital \( K_t \) and “packed” labor \( l_t^d \) as inputs. The production function is Cobb-Douglas and is affected by a total factor productivity shock \( A_t \), which is persistent with log-normal innovations. Firms use their monop-
olistic power to set prices, taking as given their demand schedule. As in Calvo [1983], they can reset their prices in each period with probability $\lambda^p$, otherwise they index their prices to past inflation with degree $\gamma^p$ and to steady state inflation with degree $(1 - \gamma^p)$.

The labor market resembles the product market: Packed labor is produced by labor packers, who aggregate differentiated labor services using a Kimball [1995] aggregator with stochastic elasticity of substitution $\varepsilon^w_t$.

Differentiated labor services are produced by a continuum of unions from the households labor supply. They use their monopolistic power to set wages. Wages are reset with probability $\lambda^w$, otherwise they are indexed to past inflation (with degree $\gamma^w$) and steady state inflation.

### 3.3 Equity and deposit funds

As we explain in detail below, there is a continuum of banks which intermediate the households’ savings using deposits and equity. Each bank is subject to a binary idiosyncratic shock which makes a bank fail with probability $1 - q_{t-1}$, in which case equity is wiped out completely and depositors receive partial compensation from the deposit insurance. The equity (deposit) funds’ function is to diversify away the idiosyncratic bank default risk by buying a perfectly diversified portfolio of 1 period equity (deposits) of all banks.

The deposit fund works without frictions, that is it represents the depositors’ interests perfectly as it invests into bank deposits. The deposit fund raises money from the households and invests it into $d_t$ units of deposits\footnote{We use deposits to refer to both units of deposit funds and units of bank deposits since they are equal. We do the same for equity.}. Next period the fund receives the nominal deposit rate $r_{d,t}$ from each bank that does not fail. In the case of failed banks, depositors are partially compensated by deposit insurance. Most deposit insurance schemes around the world, including the US, guarantee all deposits up to a certain maximum amount per bank account\footnote{For a comprehensive documentation see, for instance, Demirgüç-Kunt et al. [2005].}. We represent this capped insurance model by assuming that the deposit insurance guarantees deposits up to a fraction $\psi$ of total bank liabilities $l_t$, which are the sum of deposits $d_t$ and equity $e_t$. We assume that the deposit insurance cap is inflation adjusted, to avoid complicating the monetary policy trade-off by allowing an interdependence between monetary policy and deposit insurance. We furthermore assume that the deposit insurance cap is binding in equilibrium\footnote{We will show later that there is no equilibrium where the cap doesn’t bind.}, i.e. that the bank’s liabilities exceed the cap of the insurance $r_{d,t} d_t > \psi (d_t + e_t) \pi_{t+1}$. Using the definition of the equity ratio $k_t = \frac{e_t}{d_t + e_t}$, the deposit fund receives a real return of $\psi / (1 - k_t)$ per unit of deposits from each defaulting bank at $t$. Per unit of deposits the deposit fund hence pays a nominal return of:

$$R_{d,t+1} = q_t r_{d,t} + (1 - q_t) \frac{\psi}{1 - k_t} \pi_{t+1}. \quad (6)$$

Unlike the deposit fund, which is managed frictionlessly, the equity fund is subject to a simple agency problem. In particular, we assume that the fund manager faces two options. He...
can behave diligently and use the funds $e_t$, raised at $t$, to invest into $e_t$ units of bank equity. A fraction $q_t$ of banks will pay back a return of $r_{e,t+1}$ next period while the defaulting banks pay back nothing. Alternatively the fund manager can abscond with the funds, in which case he consumes a fraction $\xi$ of the funds in the subsequent period and the rest is lost. Since he is a small member in the big family of the representative household his utility from doing so is $u_c(c_{t+1}, L_{t+1})\xi e_t$. To prevent the fund manager from absconding funds, the equity providers promise to pay him a premium $p_t$ at time $t + 1$ conditional on not absconding. The associated utility for the fund manager is $u_c(c_{t+1}, L_{t+1})p_t$. The minimum premium to rule out absconding is just high enough to make him indifferent between absconding and investing, i.e. $p_t = \xi e_t$. The profit made by the equity fund manager is then rebated lump sum to the household. Once absconding is ruled out in equilibrium, the equity fund manager perfectly represents the interests of the equity providers. The nominal return on the bank equity portfolio is $q_t r_{e,t+1}$ per unit of equity, and the equity investors receive a gross nominal pay-off per unit of equity of:

$$R_{e,t+1} = q_t r_{e,t+1} - \xi p_{t+1} \cdots (7)$$

Note that, since bank equity is the residual income claimant, the return of the equity fund is affected by all types of aggregate risk that influences the surviving banks’ returns.

The two financial distortions introduced so far have important implications. The agency problem implies an equity premium, i.e. a premium of the return on equity over the risk-free rate. Deposit insurance is instead a subsidy on deposits, which further reduces the cost of deposits relative to equity. As explained below, the difference in the costs of these two funding types induce a meaningful trade-off between bank equity and bank deposits under limited liability.

### 3.4 Capital producers

We assume that the capital production process is risky in a way that nests the standard New Keynesian model. In particular, capital is produced by a continuum of capital producers indexed by $m$. At period $t$ they invest $i_t$ units of final good into a capital project of size $o_t^m$. This project is successful with probability $q_t^m$, in which case the project yields $K_{t+1}^m = (\omega_1 - \frac{a}{2} q_t^m) o_t^m$ units of capital at $t + 1$. Else, the project fails and only the liquidation value of $\theta o_t^m$ units of capital can be recovered (where $\theta < a - \frac{b}{2} q_t^m$). Each capital producer has access to a continuum of technologies with different risk-return characteristics indexed by $q^m \in [0, 1]$. Given a chosen technology $q_t^m$, the output of producer $m$ evolves as follows:

$$K_{t+1}^m = \begin{cases} (\omega_1 - \frac{a}{2} q_t^m) o_t^m & \text{ with probability } q_t^m \\ \theta o_t^m & \text{ else} \end{cases}$$

Implying that the safer the technology (higher $q^m$), the lower is output in case of success.

The bank orders the capital projects and requires the capital producer to use a certain technology, but this choice cannot be observed by any third party. Since the capital producer is
a representative firm we can drop the index \( m \). Given the technology choice \( q_t \), we can exploit the law of large numbers to derive the law of motion of capital:

\[
K_{t+1} = q_t \left( \omega_1 - \frac{\omega_2}{2} q_t \right) \alpha_t + (1 - q_t) n t \theta_t .
\] (8)

Furthermore we assume that capital, which depreciates at rate \( \delta \), becomes a project (of undefined \( q_t \)) at the end of every period. That is, existing capital may be destroyed due to unsuccessful reuse, and it can be reused under a different technology than it was originally produced\(^{13}\).

The total supply of capital projects by the capital producers is the sum of the existing capital projects \( o^\text{old}_t = (1 - \delta) K_t \), which they purchase from the owners (the banks) at price \( Q_t \), and the newly created projects \( o^\text{new}_t \), which are created by investing \( i_t \) units of the final good. We allow for investment adjustment costs, i.e. we assume that \( i_t \) units of investment yield \( 1 - S(i_t/i_{t-1}) \) units of project, where \( S = \kappa \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \). Hence \( \alpha_t = o^\text{new}_t + o^\text{old}_t \) and \( o^\text{new}_t = \left( 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right) i_t \). Capital producers maximize their expected discounted profits taking as given the price \( Q_t \) and the households stochastic discount factor\(^{14}\):

\[
\max_{i_t, o^\text{old}_t} \mathbb{E} \sum_{0}^{\infty} \beta^t u_c(c_t) e^B_t \left[ Q_t \left( 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right) i_t + Q_t o^\text{old}_t - i_t - Q_t o^\text{old}_t \right] .
\]

While the old capital projects are always reused, the marginal capital project is always a new one\(^{15}\). Hence, the price of projects \( Q_t \) is determined by new projects according to the well known Tobin’s q equation:

\[
Q_t \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) - S' \left( \frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} \right] - 1 = \beta u_c(c_{t+1}) e^B_{t+1} \left[ Q_{t+1} S' \left( \frac{i_{t+1}}{i_t} \right) \left( \frac{i_{t+1}}{i_t} \right)^2 \right] .
\] (9)

Note that our model of risky capital production boils down to the standard riskless setting of the New Keynesian model if we set \( q_t = 1 \) and \( \omega_1 - \frac{\omega_2}{2} = 1 \).

### 3.5 The Bank

The bank is the central agent of our model and is modeled similarly as in Dell’Ariccia et al. [2014]. Banks raise resources through deposits and equity and invest them into a risky project. Since depositors cannot observe the banks’ risk choice and banks are protected from the downside risk of their investment by limited liability, an agency problem arises between them when the banks choose the risk level. The less equity a bank has, the higher the incentives for risk taking.

\(^{13}\)This assumption ensures that we do not have to keep track of the distribution of different project types. Think of a project as a machine that delivers capital services and that can be run at different speeds (levels of risk). In case it is run at higher speed, the probability of an accident that destroys the machine is higher. After each period the existing machines are overhauled by the capital producers and at this point the speed setting can be changed.

\(^{14}\)Their out of steady state profits are rebated lump sum to the household.

\(^{15}\)We abstract from a non-negativity constraint on new projects.
Yet, since deposit insurance and the equity premium drive a wedge between the costs of deposits and equity, the banks’ optimal capital structure comprises both equity and deposits, balancing the agency problem associated with deposits with the higher costs of equity. We will show that the equilibrium risk chosen by the banks is excessive, and that the interest rate influences the degree of its excessiveness.

We assume that there is a continuum of banks who behave competitively and can be represented by a representative bank (we therefore omit the bank’s index in what follows). The bank is owned by the equity providers, and hence maximizes the expected discounted value of profits using the household’s stochastic discount factor. Every period the bank first raises deposits $d_t$ and equity $e_t$ from the respective funds (optimally choosing its liability structure). Then it invests these resources into $q_t$ capital projects, purchased at price $Q_t$. When investing into capital projects, the bank can choose the risk characteristic $q_t$ of the technology that the capital producer applies. This risk choice is not observable for depositors. Each bank can only invest into one project and hence faces investment risk: with probability $q_t$ the bank receives a high pay-off from the capital project; with probability $1 - q_t$ the investment fails and yields only the liquidation value $\theta$. Assuming a sufficiently low value for $\theta$, a failed project implies that the bank, which is protected by limited liability, defaults. In case of success the bank can repay its investors: depositors receive their promised return $r_{d,t}$ and equity providers get the state contingent return $r_{e,t+1}$. In case of default equity providers get nothing and depositors get the liquidation value plus the deposit insurance.

It is useful to think of the bank’s problem as a recursive two-stage problem. In the second stage, the bank chooses the optimal risk level $q_t$ given a certain capital structure and a certain cost of deposits. In the first stage, the bank chooses the optimal capital structure, anticipating the implied solution for the second stage problem. Note that not only the bank but also the bank’s financiers anticipate the second-stage risk choice and price deposits and equity accordingly, which is understood by the bank. Below we derive the solution for this recursive problem.

Before we do so, we need to establish the bank’s objective function. Per euro of funds raised (through deposits and equity) in period $t$ the bank gets $1/P_t$ units of consumption good, where $P_t$ is the current price level. These $1/P_t$ units of consumption goods are invested into a capital project of value $(1/P_t)/Q_t$ and type $q_t$, which are bought from the capital producer at price $Q_t$. If the project is successful it turns into $(\omega_1 - \frac{\omega}{2}q_t)/(Q_t P_t)$ capital goods. In the next period $t+1$, the bank rents the capital to the firm, who pays the real rental rate $r_{k,t+1}$ per unit of capital $K_{t+1}$. Furthermore the bank receives the depreciated capital, which becomes a capital

---

16 Profits in excess of the opportunity costs of equity.

17 The assumption that the bank can only invest into one project and cannot diversify the project risk might sound stark. Yet three clarifications are in place: First, our setup is isomorphic to a model where the bank invests into an optimally diversified portfolio of investments but is too small to perfectly diversify its portfolio. The binary payoff is then to be interpreted as the portfolio’s expected payoff conditional on default resp. repayment. Second, if the bank was able to perfectly diversify risk, then limited liability would become meaningless and we would have a model without financial frictions. Third, we don’t allow the bank to buy the safe asset. Yet this assumption is innocuous: since the banks demand a higher return on investment than the households due to the equity premium, banks wouldn’t purchase the safe asset even if they could.
project again, with real value of \((1 - \delta)Q_{t+1}\) per unit of capital \(K_{t+1}\). The bank’s total nominal income, per euro raised, conditional on success is therefore:

\[
(\omega_1 - \frac{\omega_2}{2}q_t) \frac{r_{k,t+1}}{Q_t} + \frac{(1 - \delta)Q_{t+1} P_{t+1}}{P_t}
\]

At the same time, the bank has to pay the per-unit funding costs \(r_{d,t}\) and \(r_{e,t+1}\). Using the equity ratio \(k_t\), the per-unit total nominal payment to be made in \(t+1\) in case of success is \(r_{e,t+1}k_t + r_{d,t}(1 - k_t)\).

The bank maximizes the expected discounted value of this excess profit, using the stochastic discount factor of the equity holders, that is the household. Given the success probability of \(q_t\) and the fact that the equity providers receive nothing in case of default, the bank’s objective function is:

\[
\max_{q_t, k_t} \beta \mathbb{E} \left[ \frac{u_c(c_{t+1}, L_{t+1}) \epsilon^B_{t+1}}{\pi_{t+1}} q_t \left( \omega_1 - \frac{\omega_2}{2}q_t \right) \frac{r_{k,t+1}}{Q_t} + \frac{(1 - \delta)Q_{t+1} P_{t+1}}{P_t} \pi_{t+1} - r_{d,t}(1 - k_t) - r_{e,t+1}k_t \right]
\]

(10)

Note that we did not multiply the per-unit profits by the quantity of investment. In doing so we anticipate the equilibrium condition that the bank, whose objective function is linear in the quantity of investment, needs to be indifferent about the quantity of investment. The quantity will be pinned down together with the return on capital by the bank’s balance sheet equation \(e_t + d_t = Q_t \alpha_t\), the market clearing and zero profit conditions.

The bank’s problem can be solved analytically, yet the expressions get fairly complex. Therefore we derive the solution of the banks problem for the case that \(\psi = \theta = 0\), that is without deposit insurance and with a liquidation value of 0. This simplifies the expressions but the intuition remains the same. Allowing \(\psi\) and \(\theta\) to be nonzero on the other hand is necessary to bring the model closer to the data.

To make notation more tractable we rewrite the bank’s objective function (10) in real variables expressed in marginal utility units:

\[
\omega_1 q_t \tilde{r}_{t,t} - \frac{\omega_2}{2} q_t^2 \tilde{r}_{t,t} - q_t r_{d,t}(1 - k_t) - q_t r_{e,t+1}k_t,
\]

(11)

For later use we rewrite the household’s no-arbitrage conditions (3) and (4) combined with the definition of the funds’ returns (6) and (7) as \(\tilde{r}_{d,t} = \frac{\tilde{R}_t}{\tilde{q}_t}\) and \(\tilde{r}_{e,t} = \frac{\tilde{R}_t + \tilde{\epsilon}_t}{\tilde{q}_t}\). Let us now solve the bank’s problem recursively.

\footnote{That is we use the following definitions: \(\tilde{r}_{t,t} = E \left[ u_c(c_{t+1}, L_{t+1}) \epsilon^B_{t+1} \left( \frac{r_{k,t+1}}{Q_t} + \frac{(1 - \delta)Q_{t+1} P_{t+1}}{P_t} \pi_{t+1} \right) \right] \); \(\tilde{r}_{d,t} = E \left[ u_c(c_{t+1}, L_{t+1}) \epsilon^B_{t+1} \frac{r_{d,t}}{\pi_{t+1}} \right] \); \(\tilde{r}_{e,t} = E \left[ u_c(c_{t+1}, L_{t+1}) \epsilon^B_{t+1} \frac{r_{e,t+1}k_t}{\pi_{t+1}} \right] \); \(\tilde{R}_t = E \left[ u_c(c_{t+1}, L_{t+1}) \epsilon^B_{t+1} \frac{\tilde{R}_t}{\pi_{t+1}} \right] \); \(\tilde{\epsilon}_t = E \left[ u_c(c_{t+1}, L_{t+1}) \epsilon^B_{t+1} \tilde{\epsilon}_t \right] \).}
3.5.1 Second-stage problem:

At the second stage, the bank has already raised $e_t + d_t$ funds and now needs to choose the risk characteristic of her investment $q_t$, such that equity holders’ utility is maximized. As already mentioned, we assume that the bank cannot write contracts conditional on $q_t$ with the depositors at stage one, since $q_t$ is not observable to them. Therefore at the second stage the bank takes the deposit rate as given. Furthermore, since the capital structure is already determined, maximizing the excess profit coincides with maximizing the profit of equity holders. The bank’s objective function is therefore:

$$
\max_{q_t} \omega_1 q_t \tilde{r}_{l,t} - \frac{\omega_2}{2} q_t^2 \tilde{r}_{l,t} - q_t \tilde{r}_{d,t}(1 - k_t) \ .
$$

(12)

Solving problem (12) with respect to $q_t$ yields the following first-order condition\(^{19}\):  

$$
q_t = \frac{\omega_1 \tilde{r}_{l,t} - \tilde{r}_{d,t} (1 - k_t)}{w_2 \tilde{r}_{l,t}} \ .
$$

(13)

3.5.2 First-stage problem:

At the point of writing the deposit contract in stage one, depositors anticipate the bank’s choices in stage two and therefore the depositors’ no arbitrage condition $\tilde{r}_{d,t} = \tilde{R}_t q_t$ must hold in equilibrium\(^{20}\). Using this equation together with the previous first-order condition (13) we can derive the optimal $q_t$ as a function of $k_t$ and $\hat{q}_{t,t}$:

$$
\hat{q}_t \equiv q_t(k_t) = \frac{1}{2 \omega_2 \tilde{r}_{l,t}} \left( \omega_1 \tilde{r}_{l,t} + \sqrt{(\omega_1 \tilde{r}_{l,t})^2 - 4 \omega_2 \tilde{r}_{l,t} \tilde{R}_t (1 - k_t)} \right) \ .
$$

(14)

We can now solve the first-stage problem of the banker. The bank chooses the capital structure $k_t$ to maximize her excess profits, anticipating the $q_t(k_t)$ that will be chosen at the second stage:

$$
\max_{k_t} \hat{q}_t \omega_1 \tilde{r}_{l,t} - \frac{\omega_2}{2} \tilde{r}_{l,t} \hat{q}_t^2 - q_t \tilde{r}_{d,t}(1 - k_t) - \tilde{r}_{e,t} k_t \ ,
$$

(15)

subject to the no-arbitrage condition for depositors ($\tilde{r}_{d,t} = \frac{\tilde{R}_t}{q_t}$) and for equity providers ($\tilde{r}_{e,t} = \frac{\tilde{R}_t + \xi_t}{q_t}$). Plugging these in and deriving, we obtain the first-order condition for $k_t$:

$$
\omega_1 \tilde{r}_{l,t} \frac{\partial \hat{q}_t}{\partial k_t} + \tilde{r}_t = \frac{\omega_2}{2} \tilde{r}_{l,t} \frac{\partial \hat{q}_t^2}{\partial k_t} = 0 \ .
$$

(16)

\(^{19}\)We focus on interior solutions and choose the larger of the two roots, which is the closest to the optimum, as we will see below.

\(^{20}\)Note that the agency problem arises from the fact that the bank does not consider this as a constraint of its maximization problem.
which (assuming an interior solution) can be solved for \( k_t \) as:

\[
\hat{k}_t \equiv k_t \left( \tilde{r}_{1,t} \right) = 1 - \frac{\xi_t (\tilde{R}_t + \xi_t) (\omega_1 \tilde{r}_{1,t})^2}{\omega_2 \tilde{R}_t \tilde{r}_{1,t} \left( \tilde{R}_t + 2 \xi_t \right)} .
\] (17)

### 3.5.3 Closing the bank model: the zero-profit condition

Since there is a continuum of identical banks, each bank behaves competitively and takes the
return on investment \( \tilde{r}_{1,t} \) as given. Perfect competition and free entry imply that banks will
enter until there are no expected excess profits to be made. In the presence of uncertainty of the
future it is natural to focus on the case that banks make no excess profit under any future state
of the world:

\[
\left( \omega_1 - \frac{\omega_2}{2} q_{t-1} \right) \left( \frac{r_{k,t} + (1 - \delta) Q_t}{Q_{t-1}} \right) - \frac{r_{d,t-1}}{\pi_t} (1 - \hat{k}_{t-1}) - \frac{r_{e,t}}{\pi_t} \hat{k}_{t-1} = 0 .
\] (18)

Using the equity and deposit supply schedules and taking expectation over this equation we get:

\[
\hat{q}_t \omega_1 \tilde{r}_{1,t} - \frac{\omega_2}{2} \tilde{r}_{1,t} \hat{q}_t^2 - q_t \tilde{r}_{d,t} (1 - \hat{k}_t) - \hat{k}_t \xi_t - \hat{k}_t \tilde{R}_t .
\] (19)

Combining (19) with the optimality conditions (14) and (17), we can derive analytical expres-
sions for the equity ratio \( k_t \), riskiness choice \( q_t \) (the last term in each row is the approximation
under certainty equivalence):

\[
k_t = \frac{\tilde{R}_t}{\tilde{R}_t + 2 \xi_t} \approx \frac{R^*_t}{R^*_t + 2 \xi} .
\] (20)

\[
q_t = \frac{\omega_1 (\xi_t + \tilde{R}_t)}{\omega_2 (2 \xi_t + \tilde{R}_t)} \approx \frac{\omega_1 (\xi + R^*_t)}{\omega_2 (2 \xi + R^*_t)} .
\] (21)

Note that, as the real risk-free rate \( R^*_t = \frac{R_t}{\omega_1 (\xi + \tilde{R}_t)} \) decreases, the riskiness of the bank \((1 - q_t)\)
imcreases\(^{21}\), while the equity ratio \( k_t \) falls as banks substitute equity with deposits.

How can this effect be explained? On one hand, a lower risk-free rate decreases the rate of
return on capital projects, reducing the benefits of safer investments, conditional on repayment.
This induces the bank to adopt a riskier investment technology. On the other hand, the lower
risk-free rate reduces the cost of funding, leaving more resources available to the bank’s owners
in case of repayment: this force contrasts the first one, making safer investments more attractive.
There is a third force: a lower risk-free interest rate means that the equity premium becomes
relatively more important. As a result the bank shifts from equity to deposits, internalizing less
the consequences of the risk decision and choosing a higher level of risk. The first and third
effects dominate, and overall a decline in the real interest rate induces banks to choose more risk.

\(^{21}\)At least up to a first order approximation, when the \( u_c (c_{t+1}, L_{t+1}) \) terms contained in the tilde variables
cancel out.
Since in a monetary model the real interest rate is a function of the nominal interest rate, which is the standard monetary policy tool, this mechanism provides an explanation for the risk-taking channel, documented in the empirical works discussed in section 2.

### 3.5.4 Excessive vs. frictionless risk taking

Limited liability and the impossibility of the banker to commit to a certain risk choice in stage 2 generate the agency problem of the bank, which leads to a suboptimal equilibrium. The importance of this friction can be assessed by comparing the solution of the bank model with incomplete contracts and limited liability to the solution of the model without any frictions. The frictionless risk choice can be derived under any of the following alternative scenarios: Either both equity premium and deposit insurance are zero (which eliminates the cost disadvantage of equity and leads to 100% equity finance), or contracts are complete and deposit insurance is zero (which eliminates the agency problem and leads to 100% deposit finance), or liability is not limited (as before), or household invests directly into a diversified portfolio of capital projects (which eliminates the financial sector all together). Since in a frictionless model \( q \) is chosen to maximize the consumption value of the expected return:

\[
\max_{q_t} \tilde{r}_{t,t}(\omega_1 - \frac{\omega_2}{2} q_t)q_t ,
\]

the optimal level of \( q_t \) trivially is \( q^o_t = \frac{\omega_1}{\omega_2} \). Comparing the frictionless risk choice \( q^o_t \) and the choice given the friction \( q^f_t \)

\[
q^f_t = q^o_t \frac{\xi_t + \tilde{R}_t}{2 \xi_t + R_t} \approx \frac{\xi + R^*}{2 \xi + R^*} ,
\]

we observe that the agency friction drives a wedge between the frictionless and the actually chosen risk level. This wedge has two important features. Firstly, it is smaller than one\(^{22}\), implying that under the agency problem the probability of repayment is too low, and hence banks choose excessive risk. Secondly, note that the wedge depends on \( R^*_t \) and that the derivative of the first order approximation of the wedge w.r.t. \( R^*_t \) is positive. This implies that the wedge increases, i.e. risk taking gets more excessive, as the real interest rate falls. Note that this feature of the model is consistent with the empirical finding of Ioannidou et al. [2014] that additional risk taking is inefficient.

But not only the bank risk choice is suboptimal. Also the capital structure is chosen sub-optimally. If banks could commit to choose the optimal level of risk, they would not need any skin in the game. Hence they would avoid costly equity and would finance themselves fully by deposits: \( k^*_t = 0 \). Instead they choose \( k^f_t = \frac{R_t}{R^*_t + 2 \xi_t} \). The wedge between the frictionless and the actual equity ratio resembles the two features of the risk wedge. First, it is bigger than one, so there is excessive use of equity funding. Second, up to a first order approximation, the equity

\(^{22}\text{This is true under certainty equivalence, i.e. up to first order approximation.}\)
wedge is increasing in $R_t$.

Both the risk and the capital structure choice have welfare implications. A marginal increase in $q_t$ means a more efficient risk choice and should hence - ceteris paribus - be welfare improving. At the same time a marginal increase in $k_t$ implies, due to the equity premium, a higher markup in the intermediation process and hence - ceteris paribus - lower welfare. Since both $q_t$ and $k_t$ are increasing functions of the real interest rate, this begs the question of whether an increase in the real rate alleviates or intensifies the misallocation due to the banking friction\textsuperscript{23}. The answer to this question depends on the full set of general equilibrium conditions. Given the estimated model, we will later numerically verify that the positive first effect dominates, i.e. an increase in $R_t$ has welfare improving consequences on the banking market\textsuperscript{24}. The existence of these opposing welfare effects further motivates our optimal policy discussion in the last section.

3.5.5 Full model with deposit insurance and liquidation value

The simplified version of the bank’s problem presented so far is useful to explain the basic mechanism. Yet deposit insurance and a non zero liquidation value are important to improve the quantitative fit of our model to the data.

The assumptions made about deposit insurance and the liquidation value imply that depositors get the maximum of the amount covered by deposit insurance and the value of the capital that remains from the failed project. That means that their return in case of default is:

$$\min \left( \frac{r_{d,t}}{\sigma_{t+1}}, \max \left( \frac{r_{k,t+1} + (1 - \delta)Q_{t+1}}{Q_t(1 - k_t)} \psi_t, \frac{1}{1 - k_t} \right) \right).$$

To make deposit insurance meaningful we assume that the cap value $\theta$ is small enough such that $\frac{r_{k,t+1} + (1 - \delta)Q_{t+1}}{Q_t(1 - k_t)} \psi_t < \frac{\psi_t}{1 - k_t}$. Furthermore there can be no equilibrium such that the cap is not binding, as proven in appendix D. Deposits therefore pay $\frac{\psi_t}{1 - k_t}$ in case of default. Combining the nominal return on the the deposit funds (6) with the households no-arbitrage condition, and defining $\tilde{\psi}_t = E \left[ u_c(c_{t+1}, L_{t+1}) \varepsilon_{t+1}^B \right] \psi_t$, we can write the deposit supply schedule as:

$$q_t \tilde{r}_{d,t} + (1 - q_t) \frac{\tilde{\psi}_t}{1 - k_t} = \tilde{R}_t.\quad (22)$$

We assume that the deposit insurance scheme, which covers the gap between the insurance cap and the liquidation value for the depositors of failing banks, is financed through a variable tax on capital. The return on loans $\tilde{r}_{l,t}$ can then be rewritten as:

\textsuperscript{23}These two opposing forces are well known from the literature on bank capital regulation, where a raise in the capital requirements hampers efficient intermediation but leads to a more stable banks.

\textsuperscript{24}The dominance of the risk-taking effect is intuitive for two reasons: First, while risk taking entails a real cost, the equity premium just entails a wedge but no direct real costs. Second, as the real interest rate moves up the equity premium becomes less important, so a more efficient allocation is intuitive.
\[
\tilde{r}_{t,1} = \frac{r_{k,t+1} + (1 - \delta)Q_{t+1} - \tau_t}{Q_t} \quad \text{where} \quad \tau_t = \frac{Q_t^{1-q_t}}{\omega_1 - \frac{\omega_2}{2} q_t} \left( \psi - \theta T_{t,t+1}^{1 - \delta} Q_{t+1} \right)
\]

We choose this tax to perfectly offset the distortion on the quantity of investment caused by the deposit insurance. In this way, deposit insurance influences only the funding decision of the bank and, through that, the risk choice. Hence, if \( q_t \) was chosen optimally (or was simply a parameter) the deposit insurance would not have any effect.

The same procedure as outlined above can be applied to obtain closed-form solutions\(^{25}\) for the risk choice and the equity ratio. The solutions can be found in appendix C. Note that the observations in subsection 3.5.4 remain valid. The deviation of the chosen risk (equity ratio) from the optimal level decreases (increases) in the real interest rate. Given our estimation, the risk effect dominates in terms of welfare implications. The intuition is similar as before. Deposit insurance makes deposits cheaper relative to equity: as a result, the bank will demand more deposits and choose a riskier investment portfolio. Deposit insurance furthermore strengthens the risk-taking channel, which is now affected not only by the importance of the equity premium relative to the real interest rate, but also by the importance the deposit insurance cap relative to the real interest rate. On the other hand, the efficient risk level is not affected by the deposit insurance.

The liquidation value is irrelevant for the banks’ and investors’ choice since it is assumed to be smaller than deposit insurance. Yet it eases the excessiveness of risk taking since it increases the optimal level of risk: \( q_t^o = \frac{\omega_1 - \theta}{\omega_2} \).

### 3.6 Monetary and fiscal policy

The central bank follows a nominal interest rate rule, targeting inflation and output deviations from the steady state:

\[
R_t - \bar{R} = (1 - \rho) (\phi_\pi \bar{\pi} + \phi_\gamma \bar{\gamma}) + \rho (R_{t-1} - \bar{R}) + \varepsilon_t^R,
\]

where \( \rho \) is a smoothing parameter, the hat symbol denotes percentage deviations from the steady state values, \( \bar{R} = \bar{\pi}_Y \) is the steady state nominal interest rate, and \( \varepsilon_t^R \) is a monetary policy shock.

In addition, the fiscal authority finances a stochastic expenditure stream \( g_y \bar{Y}_t \varepsilon_t^G \):

\[
\ln \left( \varepsilon_t^G \right) = \rho_g \ln \left( \varepsilon_{t-1}^G \right) + u_t^G + \rho_{GA} u_t^A,
\]

where we are allowing for a correlation between exogenous spending and innovations to total factor productivity\(^{26}\). For simplicity we rule out government debt \( (s_t = 0) \), implying that all

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\(^{25}\)In this case, one needs to apply the adjusted deposit supply schedule (22) and to make sensible assumptions about the relative size of parameters and about the root when solving the zero-profit equation.

\(^{26}\)This is a shortcut to take exports into account. Productivity innovations might rise exports in the data, and a way to capture it in a closed-economy model such as ours is to allow for \( \rho_{GA} \neq 0 \) as in Smets and Wouters.
expenditures are financed by lump sum taxes; i.e. \( g_y \tilde{Y}_t^G = T_t \).

4 Steady-state and dynamic implications of excessive risk taking in the estimated model

In this section we examine the steady-state and dynamic macroeconomic implications of the risk-taking channel, before we turn to discussing optimal monetary policy in section 5. Since we are interested in performing a quantitative evaluation of the risk-taking channel, the quantitative discussion is based on an estimated version of the model.

4.1 Model estimation

We have embedded our risk-taking channel in a medium-scale model which closely resembles\(^{27}\) the non-linear version of Smets and Wouters \([2007]\). This serves two purposes. First, in order to perform a sound monetary policy evaluation we need a quantitative model that is able to replicate key empirical moments of the data. Second, it helps to understand whether our channel is quantitatively important compared to other monetary and real frictions that affect the monetary policy trade-off.

A linearised\(^{28}\) version of the model is estimated with Bayesian techniques using 7 US macroeconomic time series from 1983q1 to 2007q3. These are the federal funds rate, the log of hours worked, inflation and the growth rates in the real hourly wage and in per-capita real GDP, real consumption, and real investment. For a full description of the data we refer to appendix A and to the supplementary material of Smets and Wouters \([2007]\). The observation equations, linking the observed time series to the variables in the model, as well as the prior specifications can be found in appendix C.

Table 2 summarizes the parameter values, which are broadly in line with existing empirical estimates for the US. The steady state inflation rate is estimated to be about 2.5% on an annual basis. The discount factor is estimated to be 0.996 implying an annual steady-state real interest rate of around 1.5%. Wages appear to be slower moving than prices: wages are reoptimized every year and a half, while prices are reoptimized approximately every six months. The coefficient of relative risk aversion \( \sigma_c \) is estimated to be 1.90, well above its prior mean. The posterior estimates of the Taylor rule parameters show a strong response to inflation and output (respectively 1.83 and 0.05), and a high degree of interest rate smoothing (0.85). The data are very informative on the parameters affecting the shock distribution, and no shock plays a predominant role in explaining output fluctuations, though the monetary policy shock and the two markup shocks

\(^{27}\)Of all nominal and real frictions considered in Smets and Wouters \([2007]\), we are missing the one concerning capital utilization, which is shown to be of secondary importance once wage stickiness is taken into account.

\(^{28}\)In the future we plan to estimate the model non-linearly, in order to verify whether the non linear relationship between the real interest rate and risk-taking is quantitatively important. We further plan to use banking-sector variables in order to refine the estimation of the banking sector parameters.
are found to be relatively less important. Finally, the estimated banking sector parameters are in favour of the risk-taking channel hypothesis: the posterior mean of the equity premium is around an annualised value of 6.6%, in line with the empirical estimates of Mehra and Prescott [1985], deposit insurance covers up to 79% of total bank liabilities and the recovery rate is about 50%.

4.2 Steady-state and dynamic implication of excessive risk taking

In Table 3 we compare the non-stochastic steady state of the model with banking frictions (henceforth bank model) with that of the model without banking frictions. In the latter model the capital structure is undetermined and risk is equal to the socially optimal level. For the given set of estimated values, the optimum is a corner solution: \( q^o = 1 \). In the bank model, the capital ratio is below one, implying that banks do not fully internalize the implications of their risk choice, and choose a level of risk that is not optimal from a social perspective. In the steady state of the bank model there is therefore excess risk taking (\( \bar{q}^f < \bar{q}^o \)). This implies that the capital production technology is inefficient. Consequently, the bank economy is under-capitalized in the steady state, and output, consumption and welfare are inefficiently low.

To understand the dynamic effects of the risk taking channel, we now assess how the propagation mechanism of the model differs if a risk-taking channel is present. For illustration, we discuss an expansionary monetary policy shock. As we have just seen, the economy without financial frictions and the bank economy have different steady states. This makes dynamic comparisons of the two models difficult, since both the different behaviors of \( q_t \) and \( k_t \) and the different steady states imply different dynamics. In order not to mix the two effects, we focus on comparing models with the same steady state. For this purpose we alter the model without financial frictions by treating the risk choice \( q_t \) and the equity ratio \( k_t \) as parameters, which we set to the steady state values of the bank model. This model, henceforth benchmark model, has not only the same steady state as the bank model but also corresponds to a standard New Keynesian model with a small markup in capital markets.

In figure 3 we compare the dynamic responses in the bank model (solid red lines) and in benchmark model (dashed blue lines) to an expansionary monetary policy shock, based on the posterior means reported in Table 2. A monetary policy expansion triggers a set of standard reactions, which are evident in the benchmark model. An unexpected fall in the nominal risk-free rate causes a drop in the real interest rate, since prices are sticky. Consequently, consumption is shifted forward, firms that can adjust the price do so causing an increase in inflation, while the remaining firms increase production. The risk-taking channel adds two further elements as both the risk level and the capital structure chosen by the bank respond to the real interest rate movement. On impact, the drop in the real interest rate cause banks to substitute equity for deposits, and to choose a higher risk taking (lower loan safety). The risk choice therefore moves further away from the optimal level and the expected return on aggregate investment drops. This implies that households would have to invest more to maintain the same path of
Table 2: Model estimation: prior and posterior values

<table>
<thead>
<tr>
<th>parameter</th>
<th>prior shape</th>
<th>prior mean</th>
<th>prior std</th>
<th>post. mean</th>
<th>90% HPD interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_y )</td>
<td>trend growth</td>
<td>normal</td>
<td>0.4</td>
<td>0.1</td>
<td>0.4348</td>
</tr>
<tr>
<td>( \mu_l )</td>
<td>labor normalization</td>
<td>normal</td>
<td>0</td>
<td>2</td>
<td>0.3821</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>output share</td>
<td>normal</td>
<td>0.3</td>
<td>0.05</td>
<td>0.2029</td>
</tr>
<tr>
<td>( 100^{\beta} )</td>
<td>real rate in %</td>
<td>normal</td>
<td>0.25</td>
<td>0.1</td>
<td>0.4063</td>
</tr>
<tr>
<td>( \pi )</td>
<td>inflation in %</td>
<td>gamma</td>
<td>0.62</td>
<td>0.1</td>
<td>0.6352</td>
</tr>
<tr>
<td>( \phi_p )</td>
<td>TR weight on inflation</td>
<td>normal</td>
<td>1.5</td>
<td>0.25</td>
<td>1.825</td>
</tr>
<tr>
<td>( \rho )</td>
<td>TR weight on output</td>
<td>normal</td>
<td>0.12</td>
<td>0.05</td>
<td>0.0508</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>TR persistence</td>
<td>beta</td>
<td>0.75</td>
<td>0.1</td>
<td>0.8493</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>investment adj. costs</td>
<td>normal</td>
<td>4</td>
<td>1.5</td>
<td>6.6501</td>
</tr>
<tr>
<td>( \iota )</td>
<td>habits</td>
<td>normal</td>
<td>0.7</td>
<td>0.1</td>
<td>0.7277</td>
</tr>
<tr>
<td>( \sigma_c )</td>
<td>risk aversion</td>
<td>gamma</td>
<td>1.5</td>
<td>0.375</td>
<td>1.9071</td>
</tr>
<tr>
<td>( \sigma_l )</td>
<td>disutility from labor</td>
<td>gamma</td>
<td>2</td>
<td>0.75</td>
<td>2.2369</td>
</tr>
<tr>
<td>( \theta )</td>
<td>liquidation value</td>
<td>normal</td>
<td>0.5</td>
<td>0.05</td>
<td>0.5021</td>
</tr>
<tr>
<td>( \Psi )</td>
<td>deposit insurance</td>
<td>normal</td>
<td>0.8</td>
<td>0.05</td>
<td>0.7881</td>
</tr>
<tr>
<td>( \omega_2 )</td>
<td>risk return trade-off</td>
<td>normal</td>
<td>0.41</td>
<td>0.05</td>
<td>0.4122</td>
</tr>
<tr>
<td>( \xi )</td>
<td>equity premium</td>
<td>normal</td>
<td>0.015</td>
<td>0.01</td>
<td>0.0166</td>
</tr>
<tr>
<td>( \lambda_p )</td>
<td>price calvo parameter</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5767</td>
</tr>
<tr>
<td>( \lambda_w )</td>
<td>wage calvo parameter</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
<td>0.8201</td>
</tr>
<tr>
<td>( \gamma_p )</td>
<td>price indexation</td>
<td>beta</td>
<td>0.5</td>
<td>0.15</td>
<td>0.2912</td>
</tr>
<tr>
<td>( \gamma_w )</td>
<td>wage indexation</td>
<td>beta</td>
<td>0.5</td>
<td>0.15</td>
<td>0.4881</td>
</tr>
</tbody>
</table>

**model parameters**

| \( \sigma_A \) | stdev TFP | uniform | 0.5 | 0.2 | 0.9167 | 0.8572 0.9684 |
| \( \sigma_B \) | stdev preference | uniform | 0.5 | 0.2 | 0.2491 | 0.0647 0.4253 |
| \( \sigma_C \) | stdev govt. spending | uniform | 0.5 | 0.2 | 0.981 | 0.9688 0.9938 |
| \( \sigma_I \) | stdev investment | uniform | 0.5 | 0.2 | 0.689 | 0.5639 0.8077 |
| \( \sigma_P \) | stdev price markup | uniform | 0.5 | 0.2 | 0.9362 | 0.883 0.9905 |
| \( \sigma_R \) | stdev monetary | uniform | 0.5 | 0.2 | 0.5933 | 0.4898 0.7103 |
| \( \sigma_W \) | stdev wage markup | uniform | 0.5 | 0.2 | 0.8823 | 0.833 0.9406 |
| \( \rho_A \) | persistence TFP | beta | 0 | 1 | 0.3788 | 0.3266 0.4268 |
| \( \rho_B \) | persistence preference | beta | 0 | 10 | 3.1301 | 1.7896 4.3806 |
| \( \rho_C \) | persistence gov. spending | beta | 0 | 10 | 2.2331 | 1.9541 2.5321 |
| \( \rho_I \) | persistence investment shock | beta | 0 | 10 | 4.9205 | 2.9521 6.633 |
| \( \rho_P \) | persistence price markup | beta | 0 | 1 | 0.098 | 0.0747 0.1228 |
| \( \rho_R \) | persistence monetary markup | beta | 0 | 1 | 0.0977 | 0.0837 0.1144 |
| \( \rho_W \) | persistence wage markup | beta | 0 | 1 | 0.4952 | 0.4322 0.5692 |
| \( \rho_{G,A} \) | correlation gov. spending & TFP | uniform | 5 | 2.8868 | 2.3345 | 1.2085 3.4284 |
| \( m_p \) | MA component of price markup | beta | 0.5 | 0.2 | 0.5861 | 0.4103 0.7905 |
| \( m_w \) | MA component of wage markup | beta | 0.5 | 0.2 | 0.9707 | 0.946 0.9948 |
Table 3: Steady state comparison: The model without banking sector frictions features an undetermined equity ratio (fixed at 0) and risk equal to the socially optimal level; i.e. $\tau^* = 1$. Parameters are fixed to the posterior mean estimates of the bank model reported in table 2.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Model with banking frictions</th>
<th>Model without banking frictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>0.954</td>
<td>1.000</td>
</tr>
<tr>
<td>$k$</td>
<td>0.181</td>
<td>0.000</td>
</tr>
<tr>
<td>$C$</td>
<td>0.632</td>
<td>0.770</td>
</tr>
<tr>
<td>$K$</td>
<td>5.690</td>
<td>15.364</td>
</tr>
<tr>
<td>$I$</td>
<td>0.142</td>
<td>0.157</td>
</tr>
<tr>
<td>$L$</td>
<td>1.000</td>
<td>0.987</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.944</td>
<td>1.130</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.006</td>
<td>1.006</td>
</tr>
<tr>
<td>$R$</td>
<td>1.010</td>
<td>1.010</td>
</tr>
<tr>
<td>Welfare</td>
<td>-1947.876</td>
<td>-1626.163</td>
</tr>
</tbody>
</table>

capital as in the benchmark case, entailing a lower consumption path. Yet this would not be optimal because of consumption smoothing and because the expected return on investment is lower. Therefore investment rises less then in the benchmark model, which makes the capital stock decline considerably. Overall, agents are worse off (in terms of welfare) in the bank model than in the benchmark economy.

Lastly, note that the responses shown in figures 3 are in accordance with the stylized facts established in section 2, in particular with the empirical finding that the response of risk is proportional to that of the interest rate.

5 Monetary policy with a risk-taking channel

In this section, we explore the implications of the risk-taking channel for optimal monetary policy. We have seen that the risk-taking channel has both static and dynamic effects. First, unless corrected for by macroprudential policy, it leads to a distorted steady state with inefficiently high levels of risk and low levels of capital and consumption. Second, variations in the real interest rate lead to variations in the risk level and in the equity ratio chosen by the bank. As the real interest rate decreases, (excessive) risk increases and the equity ratio decreases. While movements in risk and in the equity ratio have opposing impacts on the efficiency of the economy, the impulse responses have shown that the risk effect dominates.

But are these additional mechanisms implied by the risk-taking channel actually quantitatively significant for monetary policy? To answer this question we determine the optimal simple implementable monetary policy rule in the risk-taking channel model. We then compare this policy to the optimal policy in a benchmark economy without the risk-taking channel.

Our comparison of optimal policy in the benchmark and in the bank model has an interesting interpretation. Suppose that the actual economy features the risk-taking channel (the bank
model), but that the central bank believes that risk is an irrelevant constant from her point of view. The central bank would then implement optimal policy based on a wrong model (the benchmark model). Our comparison then answers the question of how important understanding the risk-taking channel is, in terms of optimal policy and welfare.

Notice that in this paper we consider a central bank that has no policy tools besides the interest rate. With a second instrument, such as capital regulation, the central bank could do better or even eliminate the friction. Exploring optimal macroprudential regulation is however beyond the scope of the present paper, and is left to future research.

In what follows, we discuss the derivation of the optimal simple implementable monetary policy rule, and then present our results.

5.1 The central bank problem

We follow Schmitt-Grohe and Uribe [2007] and characterize optimal monetary policy as the policy rule that maximizes welfare among the class of simple, implementable interest-rate feedback rules given by:

$$R_t - \bar{R} = \phi_\pi \hat{\pi}_{t+s} + \phi_y \hat{y}_{t+s} + \phi_k \hat{k}_{t+s} + \rho (R_{t-1} - \bar{R}).$$  

(24)

29For a thorough analysis of the optimal interaction between monetary and macroprudential policy in an economy with bank risk-taking see Collard et al. [2012].

30The implementability criterion requires uniqueness of the rational expectations equilibrium, while simplicity requires the interest rate to be a function of readily observable variables. For a complete discussion, see Schmitt-Grohe and Uribe [2007]. Notice that we drop their second requirement for implementability which is that an implementable rule must avoid regular zero lower bound violations.
where the hat symbol denotes percentage deviations from the steady state, and the index $s$ allows for forward- or contemporaneous-looking rules (respectively by setting $s = 1$ or $s = 0$). The policy rule specification (24) is chosen for its generality, as it encompasses both standard Taylor-type rules (setting $\phi_k = 0$), and the possibility that the central bank reacts to banking sector leverage ($\phi_k \neq 0$). Recall that the variable $k$ denotes the ratio of equity to total bank funding. A fall in the equity ratio implies that banks increase their relative debt financing, i.e. they increase leverage. When a greater fraction of assets is financed by debt, banks internalize less the consequences of their investments, and choose loans with a higher default probability. Hence, a fall in the equity ratio signals an increase in risk taking, to which the central bank may decide to respond by increasing the interest rate. We choose not to let the interest rate depend on risk taking directly, because the latter is not a readily observable variable. We furthermore impose that the parameters on output and inflation have to be non-negative and that the inertia parameter $\rho$ has to be between 0 and 1.

The welfare criterion, that defines the optimal parameter combination for rule (24), is the household’s conditional lifetime utility:

$$V = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\varepsilon_t}{\pi_t} u(c_t, L_t). \quad (25)$$

This measure is commonly used in the literature and yields the expected lifetime utility of the representative household, conditional on the economy being at the deterministic steady state.

While this measure alone allows us to compute the optimal policy, it is hard to interpret the welfare differences associated with two different rules $g$ and $b$, where $g$ is the rule that provides a higher welfare. Following Schmitt-Grohe and Uribe [2007], we define our measure for welfare comparison to be the fraction of the consumption stream that a household would need to receive as a transfer under rule $b$ to be equally well off as under rule $g$. Formally, this fraction $\Omega$ is implicitly defined by the equation:

$$V^g = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\varepsilon_t}{\pi_t} u((1 + \Omega)c^b_t, L^b_t).$$

5.2 Findings

Using the welfare criterion just described we numerically determine the coefficients of the optimal simple implementable rules in the benchmark and in the bank model using second order approximations around the non-stochastic steady state. Table 4 reports the optimal coefficients for 5 different specifications of the monetary policy rule: contemporaneous and forward-looking, without inertia and with optimal inertia, without and with reaction to current leverage. The coefficients of the optimal rules generally vary greatly between the two models. A set of results,

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31Note that reacting to leverage is impossible in the benchmark model where risk and leverage are constant.
Table 4: **Optimal simple rules**: optimal parameters for policy rules of the class $R_t - \bar{R} = \phi_\pi \pi_{t+1} + \phi_y y_{t+1} + \phi_k k_{t+1} + \rho (R_{t-1} - \bar{R})$. The hat symbol denotes percentage deviations from the steady state. Current- (forward)-looking rules let the interest rate react to current (future) deviations of variables from their steady state values. $W$ denotes the welfare level associated with the optimal rule, while $\Omega$ is the welfare cost (in % of the consumption stream) associated to implementing in the bank model the optimal policy rule of the benchmark model. For the benchmark model the restriction $\phi_k = 0$ is irrelevant, since the equity ratio is a constant in the benchmark model. Entries without digits indicate restricted parameters.

| $s$ | rule | $\rho$ | $\phi_{\pi_{t+1}}$ | $\phi_{y_{t+1}}$ | $W$ | $\rho$ | $\phi_{\pi_{t+1}}$ | $\phi_{y_{t+1}}$ | $\phi_k$ | $W$ | $\Omega$ |
|-----|------|--------|-------------------|-----------------|-----|--------|-------------------|-----------------|------|-----|--|--|
| 0   | $\phi_h, \rho = 0$ | 0      | 5.909             | 0.132           | -210.723 | 0     | 2.622             | 0.092           | 0    | -211.349 | 0.234 |
| 0   | $\phi_h = 0$      | 0.000  | 5.909             | 0.132           | -210.723 | 1.000 | 0.110             | 0.009           | 0    | -211.006 | 0.407 |
| 0   | $\rho = 0$        | 0      | 5.909             | 0.132           | -210.723 | 0     | 100.000           | 5.390           | 12.505 | -211.337 | 0.240 |
| 1   | $\phi_h, \rho = 0$| 0      | 21.182            | 0.341           | -210.703 | 0     | 3.213             | 0.118           | 0    | -211.322 | 0.923 |
| 1   | $\phi_h = 0$      | 0.000  | 21.182            | 0.341           | -210.703 | 1.000 | 0.175             | 0.014           | 0    | -210.954 | 1.108 |

which are robust across policy rule and estimation\(^{32}\) specifications, are worth noticing.

First, the optimal coefficients on inflation and output deviations are smaller in the bank model compared to the benchmark model. For any given change in inflation and output, the nominal interest rate should move less if a risk-taking channel is present. Furthermore, if the central bank can optimize over its smoothing parameter, then full interest rate smoothing is optimal in the bank model. Given that the optimal output coefficient is close to zero, the optimal rule is close to a stable real interest rate rule. In doing so, the central bank limits fluctuations in the real interest rate and hence in risk taking and slightly raises the average level of $q$ towards the efficient value, as it can be seen in table 5. At the same time inflation is significantly more volatile under the optimal rule. If a risk-taking channel is present, the central bank should accept higher inflation fluctuations in order to reduce the distortion stemming from risk taking.

What is the intuition behind this result? Monetary policy cannot affect the deterministic steady state, but it can control the real interest rate and therefore the fluctuations in excessive risk. Upward movements of the real interest rate are welfare enhancing since they lower the level of risk taking towards the efficient level, whereas downward changes of the real interest rate lead to even more excessive risk taking. But this does not mean that movements in the interest rate are irrelevant. Since the expected return on investments $q_t (\omega_1 - q^2_t q_t)$ is convex in the real interest rate, negative deviations of the real rate have a negative effect on the efficiency of the intermediation process, that is stronger than the positive effect of positive deviations of the real rate. Therefore the risk-taking channel provides a motive for keeping the real interest rate constant. This tilts the balance in the trade-off between inflation and output stabilization in the benchmark model towards less inflation stabilization.

Second, it is optimal for the central bank to respond to leverage. The reason is, that leverage

\(^{32}\)We have experimented with different estimation samples and different fixed parameters: while the optimized parameters and transfers slightly change, the qualitative results discussed in the text are very robust. Results are available upon request.
signals risk taking (there is a one to one mapping from leverage to risk). Since leverage and risk taking do not only depend on the nominal interest rate (the instrument), but also on expected inflation, this allows the central bank to steer risk taking more precisely. Once risk taking is taken care of the monetary authority can again respond aggressively to inflation deviations from the steady state. Yet the additional welfare gains of augmenting the policy rule with a response to leverage are quantitatively small.

To understand how different the equilibria associated to the two optimal rules are, and therefore how important it is for the central bank to take the risk-taking channel into account, we compute the cost of applying in the bank model the rule that is optimal for the benchmark model. These costs, expressed in % of the lifetime consumption stream, are reported in the last column of table 4. Though the costs vary a lot across policy specifications, they are always non negligible. For the best performing policy (last row of table 4), the costs of applying the benchmark policy in the bank model is around 1.11% of the lifetime consumption stream in every quarter. Hence, internalising the feedback effect that the nominal interest rate has on bank risk taking pays off in terms of welfare.

6 Conclusions

The recent financial crisis has highlighted the importance of monitoring the level of risk to which the financial sector is exposed. In this paper we focus on one aspect of financial sector risk, ex-ante bank asset risk, and on how the latter can be influenced by monetary policy.

First, we provide new empirical evidence of the impact of monetary policy on bank risk taking. We document that unexpected monetary policy shocks, identified through sign restrictions in a classical VAR framework, increase a measure for ex-ante bank risk taking in the US. This conclusion, robust to using a recursive identification scheme, is compatible with the monetary policy transmission mechanism in the theoretical model that we build to explain the effects of monetary policy on risk taking.
For this purpose, we extend the work of Dell’Ariccia et al. [2014] and build a general-equilibrium model where low levels of the risk-free interest rate induce banks to extend credit to riskier borrowers. At the core of this mechanism is an agency problem between depositors and equity providers: the latter choose the level of risk but are protected by limited liability. This friction leads to a steady state with excessive risk taking, and inefficiently low levels of capital, output and consumption. Furthermore, risk taking alters the dynamic response of the economy to shocks. In particular, an expansionary monetary policy shock has unintended consequences: because banks choose a riskier investment strategy, the growth of capital, output and consumption will be lower than in the model without the risk-taking channel.

In order to assess the importance of the risk-taking channel and to study optimal monetary policy, we estimate the model on US data. We study optimal monetary policy in this environment, using optimal simple rules. Our results suggest that, if a risk-taking channel is present and the interest rate is the only instrument available to the monetary authority, the optimal rule should stabilize the path of the real interest rate more than without the risk-taking channel. This implies that the central bank should tolerate higher inflation volatility in order to reduce welfare detrimental fluctuations in risk taking. The welfare gains of taking the channel into account are found to be non negligible, yet a direct response to leverage is not of primary importance. Nevertheless, these results do not rule out that an alternative instrument could perform better at maximizing consumer welfare, an issue that deserves to be investigated in future work.
References


Appendix A: Data description

Table 6: Data description: All variables are expressed in per-capita terms (divided by $N$). Hours are measured as $H_1 \cdot H_2 / N$ where $H_1$ is converted into an index. The nominal wage $W_1$ is deflated by the GDP deflator. All indexes are adjusted such that $2009 = 100$. Civilian The estimation sample spans from 1983Q1 to 2007Q3.

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>SERIES</th>
<th>MNEMONIC</th>
<th>UNIT</th>
<th>SOURCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>REAL GROSS DOMESTIC PRODUCT</td>
<td>GDPC96</td>
<td>IN, USD</td>
<td>FRED / BEA</td>
</tr>
<tr>
<td>$P$</td>
<td>GDP DEFLATOR</td>
<td>GDPDEF</td>
<td>INDEX</td>
<td>FRED / BEA</td>
</tr>
<tr>
<td>$R$</td>
<td>EFFECTIVE FEDERAL FUNDS RATE</td>
<td>FEDFUNDS</td>
<td>%</td>
<td>FRED / BOARD OF GOVERNORS</td>
</tr>
<tr>
<td>$C$</td>
<td>PERSONAL CONSUMPTION EXPENDITURE</td>
<td>PCEC</td>
<td>IN, USD</td>
<td>FRED / BEA</td>
</tr>
<tr>
<td>$I$</td>
<td>FIXED PRIVATE INVESTMENT</td>
<td>FPI</td>
<td>IN, USD</td>
<td>FRED / BEA</td>
</tr>
<tr>
<td>$H_1$</td>
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<td>CES160V</td>
<td>THOUSANDS</td>
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<tr>
<td>$W_1$</td>
<td>NONFARM BUSINESS (…) HOURLY COMPENSATION</td>
<td>INDEX</td>
<td>PRS85006103</td>
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</tr>
<tr>
<td>$N$</td>
<td>CIVILIAN POPULATION</td>
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<td>LNS10000000</td>
<td>BLS</td>
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<tr>
<td>$q$</td>
<td>AVERAGE WEIGHTED LOAN RISK</td>
<td>%</td>
<td>BOARD OF GOVERNORS</td>
<td></td>
</tr>
</tbody>
</table>
Appendix B: Empirical motivation - recursive identification scheme

Figure 4: An expansionary monetary policy shock - Recursive identification scheme. Ex-ante \( q \) is measured as the inverse of the average loan risk rating such that an decrease pertains to an increase in bank ex-ante asset risk. The error bands shown correspond to a 68\% confidence interval obtained by bootstrap.
Appendix C: The full model - list of equations and estimation details

List of equations: We report here the equations that enter the non-linear model. Following Smets and Wouters [2007] we assume (1) internal habits in consumption, (2) a utility function that is non-separable in consumption and labor, \( U(c_t, L_t) = \frac{(c_t - \bar{c}_t)^{1-\sigma_c}}{1-\sigma_c} \exp \frac{L_t^{1+\sigma_l}}{1+\sigma_l} \), and (3) that different varieties of intermediate goods and of labor are assembled through a Kimball [1995] aggregator, rather than a Dixit-Stiglitz one. This latter assumption is introduced in order to obtain estimates of price and wage rigidity that are closer to micro estimates.

\[
c_t + d_t + e_t + g_y e_t^G = \left( \frac{L_t}{w_t} + \Pi_t^{\text{firm}} + \Pi_t^{\text{cap}} + c_{t-1} \xi + d_{t-1} \frac{q_t r_{d,t} + (1 - q_t) \frac{\psi}{1-k_t} \pi_{t+1}}{\pi_t} + e_{t-1} \frac{q_t r_{c,t+1} - \xi \pi_{t+1}}{\pi_t} \right) = 0
\]

(26)

\[
\Pi_t^{\text{cap}} = S'(a_t^{\text{new}})a_t^{\text{new}}
\]

(27)

\[
\Pi_t^{\text{firm}} = y_t - r_{k,t} K_t - w_t L_t
\]

(28)

\[
E_t \left[ \ell_{t+1} B_{t+1} u_c(c_{t+1}, L_{t+1}) \frac{q_t r_{d,t} + (1 - q_t) \frac{\psi}{1-k_t} \pi_{t+1}}{\pi_{t+1}} \right] = E_t \left[ \ell_{t+1} B_{t+1} u_c(c_{t+1}, L_{t+1}) \frac{\alpha}{\pi_{t+1}} \right]
\]

(29)

\[
E_t \left[ \ell_{t+1} B_{t+1} u_c(c_{t+1}, L_{t+1}) \frac{q_t r_{c,t+1} - \xi \pi_{t+1}}{\pi_{t+1}} \right] = E_t \left[ \ell_{t+1} B_{t+1} u_c(c_{t+1}, L_{t+1}) \frac{R_t}{\pi_{t+1}} \right]
\]

(30)

\[
u_c(e_t, L_t) = \beta E_t \left[ \ell_{t+1} B_{t+1} u_c(c_{t+1}, L_{t+1}) R_t \right],
\]

(31)

\[
u_c = (c_t - \bar{c}_t)^{-\sigma_c} \bar{L} \exp \frac{L_t^{1+\sigma_l}}{1+\sigma_l}
\]

(32)

\[
\frac{L_t^\alpha}{K_t^{1-\alpha}} = \frac{r_{k,t}}{w_t}
\]

(33)

\[
m_c t = \frac{1}{A_t} \alpha^{-\sigma_c} r_{k,t}^{\alpha} w_t^{1-\alpha} (1 - \alpha)^{\alpha-1}
\]

(34)

\[
\pi_t^* = \Phi_p Z_{t,1}^p + \frac{k_p \Phi_p}{\epsilon^p (1 + k_p)} \pi_{t+1+k_p}^e Z_{t,3}^p Z_{t,2}^p
\]

(35)

\[
Z_{t,1}^p = u_c m_c Y L k_m t^e (1+k_p_t) + \beta \lambda_p H_{p,t+1}^e Z_{t+1,1}^p
\]

(36)
\[ Z_{t,2}^p = \varepsilon_{t+1}^B u_c Y_t L \text{lim}_t^{\epsilon(1+k_p)} + \beta \lambda_p H_{p,t+1}^{\epsilon(1+k_p)-1} Z_{t+1,2}^p \] (37)

\[ Z_{t,3}^p = \varepsilon_{t+1}^B u_c Y_t + \beta \lambda_p H_{p,t+1}^{-1} Z_{t+1,3}^p \] (38)

\[ \frac{1}{1+k_p} L \text{lim}_t + \frac{k_p}{1+k_p} D p l = 1 \] (39)

\[ D p K_t = \frac{1}{1+k_p} D p_t \cdot D p L \text{lim}_t^{\epsilon(1+k_p)} + \frac{k_p}{1+k_p} \] (40)

\[ D p_t = (1 - \lambda_p) \pi_t^{1-\epsilon(1+k_p)} + \lambda_p D p_{t-1} H_{p,t}^{\epsilon(1+k_p)} \] (41)

\[ D p L \text{lim}_t = (1 - \lambda_p) \pi_t^{1-\epsilon(1+k_p)} + \lambda_p D p L \text{lim}_{t-1} H_{p,t}^{\epsilon(1+k_p)-1} \] (42)

\[ D p l_t = (1 - \lambda_p) \pi_t^* + \lambda_p D p_{t-1} H_{p,t}^{-1} \] (43)

\[ L \text{lim}_t^{1-\epsilon(1+k_p)} = D p L \text{lim}_t \] (44)

\[ \pi_t^* = \Phi_w Z_{t,2}^w + \frac{k_w \Phi_w}{\epsilon^w (1+k_w)} W_t^{1+\epsilon^w(1+k_w)} Z_{t,3}^w \] (45)

\[ Z_{t,1}^w = \varepsilon_t^W u_c \epsilon_t L L_{1+\epsilon} \text{lim}_t^{1-\epsilon} W_t^{\epsilon(1+k_w)} (C_t - \epsilon C_{t-1})^{1-\sigma} \exp L_t^{1-\epsilon} \text{lim}_t^{1+\sigma} L \text{lim}_t \epsilon \text{lim}_t + \beta \lambda_w \exp (\epsilon^w (1+k_w)) \log H_{w,t+1}^1) Z_{t+1,2}^w \] (46)

\[ Z_{t,2}^w = \varepsilon_t^W u_c \epsilon_t L L_{1+\epsilon} \text{lim}_t^{1+\epsilon^w(1+k_w)} \text{lim}_t^{\epsilon^w(1+k_w)} + \beta \lambda_w \exp ((\epsilon^w (1+k_w) - 1) \log H_{w,t+1}) Z_{t+1,2}^w \] (47)

\[ Z_{t,3}^w = \varepsilon_t^W u_c \epsilon_t L L_{1+\epsilon} \text{lim}_t^{1-\epsilon} H_{w,t+1}^1 Z_{t+1,3}^w \] (48)

\[ \frac{1}{1+k_w} L \text{lim}_w + \frac{k_w}{1+k_w} D w l = 1 \] (49)

\[ D p K_t = \frac{1}{1+k_w} \frac{D w_t \cdot D w L \text{lim}_t^{\epsilon(1+k_w)}}{1-\epsilon^w (1+k_w)} + \frac{k_w}{1+k_w} \] (50)
\[
Dw_t = (1 - \lambda_w) W^\epsilon w(1 + k_w) W_t^{-1} w(1 + k_w) + \lambda_p Dw_{t-1} \left( \frac{W_t}{W_{t-1}} \right)^{\epsilon w(1 + k_w)} H_{w,t}^{1-\epsilon w(1 + k_w)}
\]

(51)

\[
DwLkim_t = (1 - \lambda_w) W_t^{1-\epsilon w(1 + k_w)} W_t^{1} w(1 + k_w) -1 + \lambda_w DwLkim_{t-1} \left( \frac{W_t}{W_{t-1}} H_{w,t} \right)^{\epsilon w(1 + k_w) -1}
\]

(52)

\[
Dwl_t = (1 - \lambda_w) \frac{W_t^*}{W_t} + \lambda_w DwL_t \left( \frac{W_{t-1}}{W_t} \right) H_{w,t-1}
\]

(53)

\[
Lkimw_t^{1-\epsilon w(1 + k_w)} = DwLkim_t
\]

(54)

where \( \epsilon^p = \eta^p / (\eta^p - 1) \), \( \epsilon^w = \eta^w / (\eta^w - 1) \), \( k_p = -10/\epsilon^p \), \( k_w = -10/\epsilon^w \), \( \Phi^p = \epsilon^p (1 + k_p) / (\epsilon^p (1 + k_p) - 1) \)

and \( \Phi^w = \epsilon^w (1 + k_w) / (\epsilon^w (1 + k_w) - 1) \), and it is assumed that the Kimball aggregator’s curve is 10 for both goods and labor inputs.

\[
H_{p,t} = \pi_t \frac{\pi_t^{1-\gamma_p}}{\pi_t^{\gamma_p}}
\]

(55)

\[
H_{w,t} = \pi_t \frac{\pi_t^{1-\gamma_w}}{\pi_t^{\gamma_w}}
\]

(56)
\[
oindent \alpha_{new}^t = I_t \left(1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1\right)^2\right) \tag{63} \]

\[
\epsilon_t + d_t = Q_t \alpha_t \tag{64} \]

\[
\epsilon_t = [\epsilon_t + d_t] k_t \tag{65} \]

\[
Q_t \left(1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1\right)^2 - \kappa \frac{i_t}{i_{t-1}} \left(\frac{i_t}{i_{t-1}} - 1\right)\right) \varepsilon_t + \beta Q_{t+1} \frac{u_c(c_{t+1}, L_{t+1})}{u_c(c_t, L_t)} \kappa \left(\frac{i_{t+1}}{i_t} - 1\right) \left(\frac{i_{t+1}}{i_t}\right)^2 = 1 \tag{66} \]

\[
q_t = 1 - \frac{\tilde{R}}{\tilde{\psi}_t} + \frac{\sqrt{\omega_2 (\tilde{R}_t - \tilde{\psi}_t) \left(2\omega_1 \tilde{\psi}_t (\tilde{R}_t + \bar{\xi}_t) + \omega_2 (\tilde{R}_t - \tilde{\psi}_t)\right)}}{\omega_2 \psi_t} \tag{67} \]

\[
k_t = \frac{\tilde{R}_t - \tilde{\psi}_t}{\tilde{R}_t + 2\bar{\xi}_t} \tag{68} \]

\[
\left(\omega_1 - \frac{\omega_2}{2} q_{t-1}\right) \frac{r_{e,t} + (1 - \delta) Q_t}{Q_{t-1}} = \frac{r_{d,t}}{\pi_{t+1}} (1 - k_t) - \frac{r_{c,t+1}}{\pi_{t+1}} k_t + \tau_t = 0 \tag{69} \]

\[
\bar{\xi}_t = \xi E_t [u_c(c_{t+1}, L_{t+1})] \tag{70} \]

\[
\tilde{R}_t = E_t \left[u_c(c_{t+1}, L_{t+1}) \frac{R_{t+1}}{\pi_{t+1}}\right] \tag{71} \]

\[
\bar{\tau}_t = E_t [u_c(c_{t+1}, L_{t+1}) \tau_t] \tag{72} \]

\[
\tilde{\psi}_t = E_t [u_c(c_{t+1}, L_{t+1}) \psi_t] \tag{73} \]

\[
R_t - \bar{R} = (1 - \rho) \left(\phi_x \bar{\pi}_t + \phi_y \bar{y}_t\right) + \rho \left(R_{t-1} - \bar{R}\right) + \sigma_R \varepsilon_t^R \tag{74} \]

\[
\log (\varepsilon_t^B) = \rho_P \log (\varepsilon_{t-1}^B) + \sigma^B u_t^B \tag{75} \]

\[
\log (\varepsilon_t^I) = \rho_I \log (\varepsilon_{t-1}^I) + \sigma^I u_t^I \tag{76} \]

\[
\log (\varepsilon_t^P) = (1 - \rho_P) \log (\varepsilon_{t-1}^P) + \rho_P \log (\varepsilon_{t-1}^P) + \sigma^P (u_t^P + m_p u_{t-1}^P) \tag{77} \]
\[ \log (\varepsilon_t^W) = (1 - \rho_W) \log (\varepsilon_{t-1}^W) + \rho_W \log (\varepsilon_{t-1}^W) + \sigma^W (\omega_t^W + m_W u_{t-1}^W) \]  
(78)

\[ \log (A_t) = \rho_A \log (A_{t-1}) + \sigma^A u_t^A \]  
(79)

\[ \log (\varepsilon_t^R) = \rho_R \log (\varepsilon_{t-1}^R) + \sigma^R u_t^R \]  
(80)

\[ \log (\varepsilon_t^G) = \rho_G \log (\varepsilon_{t-1}^G) + \sigma^G u_t^G + \rho_G \sigma^A u_t^A \]  
(81)

**Observational equations and prior specifications:**

The observation equations, linking the observed time series (left hand-side) to the variables in the non-linear model (right hand-side) are the following:

\[ 100 \Delta \log \left( \frac{Y_t}{Y_{t-1}} \right) = 100 \Delta \log \left( \frac{y_t}{y_{t-1}} \right) + 100 \mu_Y \]

\[ 100 \Delta \log \left( \frac{C_t}{C_{t-1}} \right) = 100 \Delta \log \left( \frac{c_t}{c_{t-1}} \right) + 100 \mu_Y \]

\[ 100 \Delta \log \left( \frac{I_t}{I_{t-1}} \right) = 100 \Delta \log \left( \frac{i_t}{i_{t-1}} \right) + 100 \mu_Y \]

\[ 100 \Delta \log \left( \frac{W_t}{W_{t-1}} \right) = 100 \Delta \log \left( \frac{w_t}{w_{t-1}} \right) + 100 \mu_Y \]

\[ 100 \Delta \log \left( \frac{P_t}{P_{t-1}} \right) = 100 \pi \]

\[ 100 \log \left( \frac{H_t}{H} \right) = 100 \log \left( \frac{L_t}{L} \right) + 100 \mu_L \]

\[ \left( \frac{R_t}{4} \right) = 100 \tilde{R} \]

where \( H \) are hours worked in 2009. Since there is no growth in the model, we estimate the mean growth rates in the data \( \mu_Y \) and \( \mu_L \). The means of the inflation rate and of the nominal interest rate in the data contain instead information used for the estimation of the steady-state inflation rate and of the discount factor, which are assumed to follow a gamma prior distribution respectively centered around 2.5% (annualised) and 0.99.

We fix parameters that are not indentified to values commonly used in the literature. In particular, we choose a depreciation rate \( \delta \) of 0.025, a steady-state wage markup \( \varepsilon^W \) of 1.05, a steady-state spending to GDP ratio \( g_y \) of 18%, a weight of labor in the utility function \( \tilde{L} \) such
that steady-state hours are equal to 1, and curvatures of the Kimball aggregator for goods and labor varieties of 10.

For all structural shocks, we employ a non-informative uniform distribution, with lower bound 0 and upper bound of 1 or 10 depending on the scale of the shock. The persistences of the shock processes are assumed to have a beta prior distribution centered at 0.5, and with standard deviation of 0.2. Following Smets and Wouters [2007], we further assume that the two markup shows have a moving average component.

The priors of the Taylor rule parameters are centered around very common values: the smoothing parameter has a Beta distribution with a mean of 0.75, while the responses to inflation and output are assumed to follow a Normal distribution with a mean of 1.5 and of 0.5/4 = 0.125.

The parameters affecting price and wage stickiness have a beta distribution centered at 0.5 with standard deviation of 0.1. Our prior is that prices and wages are reoptimized on average every 6 months, and that the degree of indexation to past inflation is only up to 50%. The steady-state price markup is assumed to be centered around 1.25, slightly above the steady-state wage markup.

We employ very common priors for all the parameters of the utility function. Habits are centered around 0.7, the intertemporal elasticity of substitution $c_l$ has a prior mean of 1.5, while the elasticity of labor supply $l_t$ has a prior mean of 2. The capital share in production has a prior mean of 0.3 while the investment adjustment costs parameter has a loose prior around 4.

The non-standard set of parameters are those pertaining to the banking sector. We center the prior for the equity premium $\xi$ to 1.5%, matching the average real excess returns of stocks over the risk-free interest rate. The recovery rate $\theta$ is set to 0.5. The means of the priors for the parameters $\omega_2$ (loan profitability) and $\psi$ (deposit insurance) are jointly set to match a steady-state annual default rate of 6%, an equity ratio of 10%. $\omega_2$ is defined as a function of the other bank parameters such that one unit of consumption good is expected to produce one unit of capital good in steady state. The first two targets are roughly in line with empirical values, the latter assumption makes sure the model is comparable with standard models of capital accumulation.

Appendix D: Proof

The proof is by contradiction: Assume that there exists an equilibrium with no excess profits where the bank would issue so little deposits that the promised repayment $r_{d,t}(1 - k_t)$ would be lower than the cap on deposit insurance $\psi / (1 - k_t)$. In this case the deposit rate $r_{d,t}$ would be equal to the risk free rate $R_t$.

\footnote{A quarterly equity premium of 1.5% is compatible with the 6% historical annual equity premium found by Mehra and Prescott [1985]. Similar values for the equity premium are furthermore obtained by computing the excess return on bank equity over the risk-free interest rate.}
The first step maximization problem of the bank would then be
\[
\max_{\hat{q}_t} q_t \omega_1 \tilde{r}_{1,t} - \frac{\omega_2}{2} \tilde{r}_{1,t} q_t - q_t \hat{R}_t (1 - k_t)
\]
and its solution
\[
\hat{q}_t \equiv q_t(k_t) = \min \max \left( \frac{\omega_1 \tilde{r}_{1,t} - \hat{R}_t (1 - k_t)}{\omega_2 \tilde{r}_{1,t}} , 0 \right , 1)
\]

The second step maximization problem would be
\[
\max_{k_t} V(k) = \hat{q}_t \omega_1 \tilde{r}_{1,t} - \frac{\omega_2}{2} \tilde{r}_{1,t} q_t^2 - \hat{q}_t (1 - k) - (\xi + \hat{R}_t) k
\]

If \(q\) is at a corner solution the objective function of the bank is decreasing in \(k\), hence \(k = 0\) is optimal. Let's assume for now an interior solution for \(\hat{q}_t\). Using the envelope theorem for the first part of this expression it is easy to see that the first and second derivatives of the objective function are
\[
\hat{q}_t \hat{R}_t - (\xi + \hat{R}_t)
\]
and
\[
\hat{R}_t \frac{\partial \hat{q}_t}{\partial k_t} = \frac{\hat{R}_t^2}{b \tilde{r}_{1,t}} > 0
\]

The function is therefore convex with a single minimum. There can therefore be only a corner solution for \(k_t\) either at 0 or 1. But which corner? Assume \(k_t = 1\) is optimal. This is true iff \(V(1) > V(0)\) and \(q_t(1)\) is interior. The latter holds iff \(\omega_1 < \omega_2\). Comparing \(V(1)\) and \(V(0)\) it is easy to show that the former can’t hold under this condition. Optimality with full insurance therefore requires that the bank uses only deposits. This implies that any insurance cap smaller than 100% would be exceeded by the deposit liabilities in case of default. Depositors are therefore never fully insured.


265. “Assessing the role of ageing, feminising and better-educated workforces on TFP growth”, by A. Ariu and V. Vanderberghe, Research series, October 2014.


286. “Monetary policy effects on bank risk taking”, by A. Abbate and D. Thaler, Research series, September 2015.