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## Measuring and testing for the systemically important financial institutions

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# Measuring and testing for the systemically important financial institutions



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by C. Castro and S. Ferrari

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# Measuring and testing for the systemically important financial institutions

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## Abstract

This paper analyses  $\Delta CoVaR$  proposed by Adrian and Brunnermeier (2008) as a tool for identifying/ranking systemically important institutions and assessing interconnectedness. We develop a test of significance of  $\Delta CoVaR$  that allows determining whether or not a financial institution can be classified as being systemically important on the basis of the estimated systemic risk contribution, as well as a test of dominance aimed at testing whether or not, according to  $\Delta CoVaR$ , one financial institution is more systemically important than another. We provide two applications on a sample of 26 large European banks to show the importance of statistical testing when using  $\Delta CoVaR$ , and more generally also other market-based systemic risk measures, in this context.

*Keywords:* Systemic risk, SIFIs, interconnectedness, quantile regression, stochastic dominance test

*JEL:* C21, C58, G32

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## 1. Introduction

The 2007-2008 financial crisis has shifted the focus from the assessment of the resilience of individual financial institutions towards a more systemic or

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<sup>1</sup>The findings, recommendations, interpretations and conclusions expressed in this paper are those of the authors and not necessarily reflect the view of the National Bank of Belgium.

macroprudential approach. As illustrated by the crisis, an important aspect of systemic risk is the propagation of adverse shocks to a single institution through the rest of the system. Therefore, mitigating the risk stemming from so-called systemically important financial institutions (SIFIs) and more in general interconnectedness within the financial system have been and still are important topics on the regulatory reform agenda. In particular, capital surcharges have been imposed on global systemically important banks (G-SIBs) and many jurisdictions are in the process of developing a framework for their domestic SIFIs (D-SIFIs).

One of the challenges in macroprudential policy aimed at reducing the risk of SIFIs and interconnectedness is determining how to identify which institutions are in fact systemically important<sup>2</sup> and how to measure interconnectedness in the absence of sufficiently granular data on intrafinancial exposures. While the Basel Committee on Banking Supervision has developed an indicator-based framework for identifying G-SIBs on the basis of five dimensions of systemic importance (size, interconnectedness, substitutability, complexity and cross-border activities), data availability, especially on interconnectedness, remains an issue.

As a consequence, a strand of literature that aims at assessing systemic importance and interconnectedness on the basis of market data, such as stock returns or CDS spreads, has emerged. Within this group of measures, the so-called co-risk measures have attracted considerable attention in both academic and policy research. Intuitively, co-risk measures determine the systemic importance of a financial institution as the increase in the risk of the financial system (or other individual financial institutions in the system) when a given financial institution encounters distress. Perhaps the best known co-risk measure of systemic importance is  $\Delta CoVaR$  proposed by Adrian and Brunnermeier (2008), which refer to the increase in system-wide risk due to the distress of a financial institution (i.e., the estimated value of  $\Delta CoVaR$ ) as the “systemic risk contribution” of that financial institution.

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<sup>2</sup>In order to properly measure the systemic importance of a financial institution, the measure must concentrate on the institution’s potential impact on the system in the event of failure or distress. This may entail several identification issues, which we discuss fully in a previous survey paper (Castro and Ferrari, 2010). In particular, determining the systemic importance of a financial institution requires separating spillover or contagion effects from the effects of a systematic shock through common exposures, as well as identifying cascade or domino effects.

While  $\Delta CoVaR$  has already been extensively applied and extended both in the academic literature and by policymakers, statistical testing procedures to assess the significance of the findings and interpretations based on this co-risk measure have not yet been developed. In particular, the current applications of  $\Delta CoVaR$  do not test whether the systemic risk contribution for a given financial institution is significant, and whether the systemic risk contribution of one financial institution is significantly larger than that of another financial institution. This is of paramount importance for drawing credible conclusions that can be used for policymaking, however.

In this paper we fill this gap by deriving, within a linear quantile regression framework, two hypothesis tests and their respective test statistics. In particular, we develop a test of significance of  $\Delta CoVaR$  that allows determining whether or not a financial institution can be classified as being systemically important on the basis of the estimated systemic risk contribution, as well as a test of dominance aimed at testing whether or not, according to  $\Delta CoVaR$ , one financial institution is more systemically important than another. In addition, we provide two applications to show the importance of statistical testing when using  $\Delta CoVaR$  for identifying/ranking SIFIs and assessing interconnectedness. In particular, we provide a ranking of a set of large European banks in terms of their potential impact on the market index. We test whether the systemic risk contribution, as measured by  $\Delta CoVaR$ , for the different banks is statistically significant and whether the systemic risk contributions of different banks statistically differ from each other. Second, we apply our significance test to form a mapping of the interconnections between the 26 banks in the sample. We consider two banks as being interconnected when the impact of one institution on the other, as measured by  $\Delta CoVaR$ , is statistically significant. We find that while banks with a larger estimated  $\Delta CoVaR$  are more likely to have a statistically significant systemic risk contribution, a larger  $\Delta CoVaR$  does not necessarily imply that a bank's systemic risk contribution is significant. In addition, when categorizing institutions in terms of their systemic importance, one should not only make use of significance results, but also consider the results of pairwise dominance tests. That is, when applying a bucketing approach for ranking and regulating systemically important financial institutions, statistical tests to see whether banks in higher buckets actually have a larger systemic risk contribution than banks in lower buckets should be considered. In fact, we find that very few banks can actually be ranked according to their systemic risk contribution on the basis of  $\Delta CoVaR$ . Concerning the mapping of inter-

connections, testing for the significance of the estimated  $\Delta CoVaR$ s clearly affects the network picture one obtains. In particular, the subset of linkages that have to be analysed is substantially narrowed down. These conclusions do not only apply to  $\Delta CoVaR$ , but may generalize to other market-based systemic risk measures.

Our paper builds on and contributes to the evolving literature on market-based systemic risk measures. In addition to Adrian and Brunnermeier's (2008)  $\Delta CoVaR$  measure, well-known and often-cited examples in this literature include marginal expected shortfall (MES) suggested by Acharya et al. (2010) and Brownlees and Engle (2011) and the Shapley value approach of Tarashev et al. (2009) and Drehmann and Tarashev (2011). Alternative approaches have been suggested by for example Elsinger et al. (2006a,b), Segoviano and Goodhart (2009), White et al. (2010), Zhou (2010), Billio et al. (2011), and Huang, Zhou and Zhu (2010, 2011). Applications and extensions of  $\Delta CoVaR$  are numerous and can be found in for example Deutsche Bundesbank (2010), Brunnermeier et al. (2011), Girardi and Ergun (2011), Hautsch et al. (2011), Rodríguez-Moreno and Peña (2011), Sedunov (2011), van Oordt and Zhou (2011), and Lopez-Espinosa et al. (2012). Specific applications of (extensions of)  $\Delta CoVaR$  with the aim of assessing interconnectedness between (groups of) financial institutions include Fong et al. (2009), Chan-Lau (2009), IMF (2009), Adams et al. (2011), and Roengpitya and Rungcharoenkitkul (2011). Chan-Lau (2010) and Gauthier et al. (2011) consider (extensions of)  $\Delta CoVaR$  for determining systemic capital requirements. Jäger-Ambrozewicz (2010) and Hong (2011) theoretically analyse  $\Delta CoVaR$  in a Gaussian setting. Finally, critical assessments of market-based risk measures, including  $\Delta CoVaR$ , can be found in Danielsson et al. (2011) and Löffler and Raupach (2011). We contribute to this literature by developing significance and dominance tests for  $\Delta CoVaR$  and show the importance of statistical testing in two applications. In addition, we argue that the potential inability of  $\Delta CoVaR$  to rank financial institutions according to their systemic risk contributions, which may be due to the restrictive nature of the assumed linear relationship between the variables of interest, may limit its usefulness for supporting macroprudential policy measures towards SIFIs.

From a methodological point of view, we relate to the literature of inference in the quantile regression framework (Koenker and Machado, 1999; Koenker and Xiao, 2002; Chernozhukov and Fernandez-Val, 2005; and Chernozhukov and Hansen, 2006). In addition, we relate to the literature on tests of stochastic dominance (Linton et al., 2005). Our approach differs

from the traditional tests of stochastic dominance in two respects. First, our tests of dominance are formulated in terms of the quantile function. Second, we are interested in the conditional quantile function (or the response function) of the variable of interest, rather than an unconditional distribution or the residuals from some estimated model. More specifically, following earlier work on inference based on a quantile process, we develop a test based on the Kolmogorov-Smirnov statistic. This approach is highly attractive since the test statistic is asymptotically distribution free.

The remainder of the paper is organized as follows. Section 2 presents a review of  $\Delta CoVaR$  against the background of traditional quantile-based risk measures. Particular attention is given to the types of hypotheses regarding the systemic importance of financial institutions one may want to test in this framework. In Section 3 we relate  $\Delta CoVaR$  to the extensive literature on treatment effects. The testing procedures developed in this literature provide a basis for the significance and dominance tests in this paper. In Sections 4 and 5 we develop a series of testing procedures within the general linear testing framework for identifying and ranking SIFIs and perform a Monte Carlo experiment to determine the power of the tests, respectively. Section 6 provides two applications to show the importance of statistical testing when using  $\Delta CoVaR$  for identifying/ranking SIFIs and assessing interconnectedness. Section 7 concludes.

## 2. The Co-Risk measure $\Delta CoVaR$

The focus of risk management practice is to estimate and limit potential losses. The most commonly used risk measures are those that focus on extreme losses (i.e., the tail of the distribution): value-at-risk (VaR) and expected shortfall (ES). Both of these measures are quantile based risk measures. In particular, the VaR risk measure is equivalent to the more general concept of the quantile function, which for a random variable  $X$  with probability distribution  $F_X$ , is defined as follows:

**Definition 2.1.** For  $\tau \in (0, 1)$  the  $\tau$ -quantile function of distribution  $F_X$  is given by:

$$Q_X(\tau) := \inf \{x \in \mathbb{R} : F_X(x) \geq \tau\}.$$

Quantile functions possess some useful properties: left continuous and non-decreasing functions of  $\tau$ , equivariant to monotone transformations, among



others (see, Parzen, 1979, 2004).

Many of the co-risk measures that have been developed in the literature build on these quantile based risk measures. Intuitively, co-risk measures determine the systemic importance of a financial institution as the increase in the risk of the financial system when the institution in question encounters distress. Co-risk measures of systemic importance generally infer the impact of the failure or distress of a financial institution directly from market data, such as stock returns or CDS spreads, without relying on a structural credit risk model to first quantify total risk in the system. The advantage of these approaches is therefore that they require little information and make use of statistical methods with minimal assumptions, to obtain an estimate of a financial institution's potential impact on the system. Perhaps the best known co-risk measure of systemic importance is  $\Delta CoVaR$  proposed by Adrian and Brunnermeier (2008).

### 2.1. Definition

The calculation of  $\Delta CoVaR$  makes use of the risk measure VaR. In Adrian and Brunnermeier (2008),  $\Delta CoVaR$ , is a composition of the conditional and the unconditional VaR of the financial system. First, the (unconditional) VaR from the distribution of, for instance, stock returns for an index of financial institutions (the financial system)  $X^{index}$  is computed.<sup>3</sup> This represents a VaR for the financial system:

$$P(X^{index} \leq VaR_{X^{index}}(\tau)) = \tau.$$

Second, the conditional VaR (CoVaR) is computed as the VaR for the distribution of the stock returns of the index of financial institutions, conditional on the stock return of the financial institution  $i$  in question  $X^i$  being at its VaR-level (in distress):

$$P(X^{index} \leq CoVaR_{X^{index}|X^i}(\tau) \mid X^i = VaR_{X^i}(\tau_{X^i})) = \tau,$$

where  $\tau_{X^i}$  is the confidence level at which the individual institution's return  $X^i$  is evaluated; this may equal the confidence level  $\tau$  at which the system's

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<sup>3</sup>We in fact define our variables of interest as the negative of stock returns, so that the results can be interpreted in terms of losses.

return  $X^{index}$  is evaluated, but this is not necessarily the case. Without loss of generality and to simplify notation, from now on we consider the case where  $\tau = \tau_{X^i}$  and suppress it from the CoVaR notation, unless otherwise stated.

The difference between CoVaR and the unconditional VaR of the system is called  $\Delta CoVaR$ , which is the eventual measure of systemic importance:<sup>4</sup>

$$\Delta CoVaR^{index|i}(\tau) = CoVaR_{X^{index}|X^i}(\tau) - VaR_{X^{index}}(\tau). \quad (1)$$

Adrian and Brunnermeier (2008) refer to this measure as the “systemic contribution” of financial institution  $i$ . Intuitively, it measures the increase in the risk of the financial system when the institution in question encounters distress.<sup>5</sup>

## 2.2. Estimation

The estimation of the co-risk measure  $\Delta CoVaR$  can be accomplished in several ways. In their application of the measure, Adrian and Brunnermeier (2008) use a parametric approach based on quantile regression. This parametric approach, which is followed in most of the applications of  $\Delta CoVaR$ , is embedded in the extensively developed linear location-scale-model (Koenker, 2005). In this linear location-scale framework, the dependent variable, which in our application of  $\Delta CoVaR$  is the stock returns for the index of financial institutions  $X^{index}$ , follows some factor structure

$$X_t^{index} = K_t\delta + (K_t\gamma)\varepsilon_t, \quad (2)$$

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<sup>4</sup>In a revised version of the paper, Adrian and Brunnermeier define  $\Delta CoVaR$  as the difference between two conditional distributions evaluated at different points in the design space. Under this setup, the measure of systemic risk contribution is  $\Delta CoVaR^{index|i}(\tau) = CoVaR_{X^{index}|X^i=VaR_{X^i}(\tau_{X^i})}(\tau) - CoVaR_{X^{index}|X^i=VaR_{X^i}(0.5)}(\tau)$ , where the first term denotes the VaR of the system conditional on the financial institution’s return  $X^i$  being evaluated at its  $\tau_{X^i}$ -th quantile, the second term the VaR of the system conditional on the financial institution’s return  $X^i$  being evaluated at its median, and  $\tau_{X^i} > 0.5$  (e.g.,  $\tau_{X^i} = 0.99$ ).

<sup>5</sup>More generally,  $\Delta CoVaR$  can also be computed for an individual financial institution rather than the financial system. In this case,  $\Delta CoVaR^{j|i}(\tau) = CoVaR_{X^j|X^i}(\tau) - VaR_{X^j}(\tau)$  captures the impact of a financial institution  $i$  being in distress on financial institution  $j$ . For expositional reasons, we focus in the theoretical part of our paper on  $j$  being equal to the financial system. In the empirical application, we consider both cases.

where  $K_t$  is a  $k$ -dimensional vector of factors and  $t = 1 \dots T$  denotes time. The factors influencing the financial index variable in the context of  $\Delta CoVaR$  typically include the stock return  $X_t^i$  for a financial institution  $i$  of interest, a constant term and possibly a set of common variables.<sup>6</sup> The error term  $\varepsilon_t$  is assumed to be i.i.d with zero mean and unit variance, and is independent of  $K_t$  so that  $E[\varepsilon_t | K_t] = 0$ . The market variable is generated by a stochastic process within the location-scale family of distributions, implying that conditional expectation and volatility of the random variable  $X_t^{index}$  depends on the  $k$ -dimensional vector of factors,  $K_t$ . Since expression (2) represents the conditional distribution function for  $X_t^{index}$ , it can analogously be written in terms of a quantile function representation:

$$\begin{aligned} Q_{X^{index}|K}(\tau) &= K_t\delta + (K_t\gamma)Q_\varepsilon(\tau) \\ &= K_t\beta(\tau), \end{aligned} \tag{3}$$

where  $\beta(\tau) = \delta + \gamma Q_\varepsilon(\tau)$ . Note that in this model the quantile varying coefficients are identical up to a affine transformation. While  $\tau \in (0, 1)$ , we are typically interested in values of  $\tau$  close to 1, since  $\Delta CoVaR$  is a risk measure. The quantile function in (3) can be estimated via the quantile regression (see Koenker, 2005):

$$\hat{\beta}_T(\tau) = \underset{\beta(\tau)}{argmin} \sum_t \rho_\tau(X_t^{index} - K_t\beta(\tau)),$$

where  $\rho_\tau(u) = u(\tau - I(u < 0))$ .

In this quantile regression framework, the increase in system-wide risk due to the distress of financial institution  $i$ ,  $\Delta CoVaR^{index|i}(\tau)$ , can be obtained as follows. First, equation (3) is estimated with the stock return of financial institution  $i$  excluded from the explanatory variables, i.e., with only a constant term and possibly a set of common variables included in  $K_t$ . The fitted value of this regression will result in the unconditional VaR of the financial system returns  $VaR_{X^{index}}(\tau)$ . Secondly, equation (3) is estimated with the stock return  $X_t^i$  of financial institution  $i$  included (in addition to a constant term and possibly a set of common variables) in the explanatory variables  $K_t$ . The fitted value of this regression, with  $X_t^i$  evaluated a distressed level, say  $VaR_{X^i}(\tau)$ , results in the VaR of the financial system returns conditional

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<sup>6</sup>Note that this model also nest the pure location shift model when  $\gamma K_t = 1$ .

on financial institution  $i$  being in distress,  $CoVaR_{X^{index}|X^i}(\tau)$ . From the definition of  $\Delta CoVaR^{index|i}(\tau)$  in expression (1), it follows that the systemic risk contribution of financial institution  $i$  is obtained by taking the difference between the estimated values for  $CoVaR_{X^{index}|X^i}(\tau)$  and  $Var_{X^{index}}(\tau)$ .

### 2.3. Inference

Since  $\Delta CoVaR$  is a co-risk measure and therefore serves as proxy for the potential impact that the distress of a given financial institution may have on the financial system (or another financial institution), it can be considered to be a useful measure for identifying and ranking SIFIs as well as assessing interconnectedness in the financial system in general. In particular, on the basis of the  $\Delta CoVaR$  methodology, SIFIs can be identified as those institutions for which  $\Delta CoVaR^{index|i}(\tau)$  exceeds a given threshold level. In addition, financial institutions can be ranked in terms of systemic importance on the basis of a ranking of their  $\Delta CoVaR^{index|i}(\tau)$ ; institutions with a larger  $\Delta CoVaR^{index|i}(\tau)$  can be considered to be more systemically important. Such a ranking of financial institutions according to their systemic importance may be useful when policy instruments aimed at reducing the risk imposed on the system by financial institutions are levied in a differentiated way, with the instrument being more strict or binding for financial institutions that are more systemically important. Alternatively, the ranking of institutions in terms of  $\Delta CoVaR^{index|i}(\tau)$  may simply be used as a tool for determining factors that explain an institution's systemic importance. That is, estimated values of  $\Delta CoVaR$  can be regressed on a set of variables, such as banks' balance sheet characteristics, in order to determine what factors contribute to their systemic importance. Finally, when considering the impact of a given financial institution  $i$  being in distress on each other institution  $j$  in the system,  $\Delta CoVaR^{j|i}(\tau)$  may serve as a basis for mapping bilateral interconnections between the institutions in the financial system.

While this type of identifications/rankings of systemic importance and assessments of interconnectedness have been provided in several applications (and extensions) of  $\Delta CoVaR$ , the statistical significance of the results and interpretations based on  $\Delta CoVaR$  exceeding a certain threshold or  $\Delta CoVaR$  of one financial institution being larger than that of another have not been considered yet. This is of paramount importance for drawing credible conclusions that can be used for policymaking, however. We fill this gap by proposing tests for two types of hypotheses and the relevant test statistics, which we refer to as a test of significance and a test of dominance:

**Significance** As mentioned above, SIFIs can be identified as those institutions for which  $\Delta CoVaR^{index|i}(\tau)$  exceeds a given threshold level. Without loss of generality, we set this threshold level equal to zero in the development of our hypothesis test. Hence, a hypothesis test for the identification of a systemically significant institution will have the following null hypothesis:

$$H_0 : \Delta CoVaR^{index|i}(\tau) = 0, \quad (4)$$

for a given  $\tau \in (0, 1)$  or, more specifically, on a given subset of  $\mathcal{T} \subset (0, 1)$ . This implies that under the null hypothesis there is no statistical difference between the empirical conditional VaR of the financial system's returns,  $CoVaR_{X^{index}|X^i}(\tau)$ , and the unconditional VaR of the financial system's returns,  $VaR_{X^{index}}(\tau)$ . Therefore, any change in the financial institution's individual stock return does not have a significant effect on the index for financial institutions at the given quantile  $\tau$ .

**Dominance** In order to establish some form of ranking across the institutions according to their systemic importance, the magnitude of the estimated  $\Delta CoVaR$  could be compared for different pairs of financial institutions  $i$  and  $j$ . Since the unconditional VaR of the system,  $VaR_{X^{index}}(\tau)$ , appears in both  $\Delta CoVaR^{index|i}(\tau)$  and  $\Delta CoVaR^{index|j}(\tau)$ , this boils down to comparing  $CoVaR_{X^{index}|X^i}(\tau)$  and  $CoVaR_{X^{index}|X^j}(\tau)$ . Therefore, a hypothesis test to test whether financial institution  $i$  is statistically more systemically important than institution  $j$  will have the following null hypothesis:

$$H_0 : CoVaR_{X^{index}|X^i}(\tau) \geq CoVaR_{X^{index}|X^j}(\tau), \quad (5)$$

for a given  $\tau \in (0, 1)$  or, more specifically, on a given subset of  $\mathcal{T} \subset (0, 1)$ . As we will show in the next section, this test is equivalent to a test of stochastic dominance between two conditional distributions (or equivalently, quantile functions); we therefore refer to this hypothesis test as a test of dominance.

### 3. $\Delta CoVaR$ and quantile treatment effects

$\Delta CoVaR$  is related to a well-known concept of quantile treatment effects.  $\Delta CoVaR$  can be interpreted as a two-sample quantile treatment effect where the unconditional distribution represents the control group and the conditional distribution reflects the treatment group.

### 3.1. Two-sample treatment effects

Let  $W_1, \dots, W_T$  and  $Z_1, \dots, Z_S$  denote two random samples, and let  $G(w)$  and  $F(z)$  represent their respective unknown distribution functions. In the general model for two-sample treatment effects let  $\{W\}_{t=1}^T$  represent the data for the treatment and,  $\{Z\}_{s=1}^S$  the data for the control group. In order to determine if the treatment is unambiguously beneficial then we must test whether  $G$  is stochastically larger than  $F$ . In this two-sample case the quantile treatment effect is given by the following expression:

$$\varrho(\tau) = G^{-1}(\tau) - F^{-1}(\tau),$$

where  $G^{-1}$  and  $F^{-1}$  are the quantile functions of distributions  $G$  and  $F$ , respectively.

A natural non-parametric estimator of the treatment effect is:

$$\hat{\varrho}(\tau) = \hat{G}_T^{-1}(\tau) - \hat{F}_S^{-1}(\tau),$$

where  $\hat{G}_T$  and  $\hat{F}_S$  denote the empirical distribution functions of the treatment and control observations, based on  $T$  and  $S$  observations, respectively.

The most common types of hypothesis tests that are considered in the literature on quantile treatment effects are the following:

1. Hypothesis of no effect:  $\varrho(\tau) = 0$  for all  $\tau \in (0, 1)$ .
2. Constant effect hypothesis:  $\varrho(\tau) = \varrho$  for all  $\tau \in (0, 1)$ .
3. Dominance hypothesis:  $H_0 : \varrho(\tau) \geq 0$  for all  $\tau \in (0, 1)$  versus  $H_a : \varrho(\tau) < 0$  for some  $\tau \in (0, 1)$ .

### 3.2. $\Delta CoVaR$ as a quantile treatment effect

As presented in section 2.2, we use a linear function to represent the relationship between the random variables  $(X^{index}, K)$ , see equation (3). Assuming without loss of generality that in the remainder of the paper  $K$  only includes  $X^i$  and a constant term,  $CoVaR$  or the conditional quantile function for the response variable  $X^{index}$  given  $X^i$  can be defined as:<sup>7</sup>

$$\begin{aligned} Q_{X^{index}|X^i}(\tau) &= CoVaR_{X^{index}|X^i}(\tau) \\ &= \beta_0(\tau) + X^i \beta_1(\tau) \end{aligned} \tag{6}$$

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<sup>7</sup>Without loss of generality, in the remainder of the paper we drop the common variables  $Z_t$  from the vector of explanatory variables  $K_t$ .

Therefore using the relationships between quantile and distribution functions, the definition of  $\Delta CoVaR$  for a given level of  $\tau$  can be formulated as follows:

$$\begin{aligned}\widehat{\Delta CoVaR}^{index|i}(\tau) &= \widehat{Q}_{X^{index}|X^i}(\tau) - \widehat{Q}_{X^{index}}(\tau) \\ &= \widehat{F}_{X^{index}|X^i}^{-1}(\tau) - \widehat{F}_{X^{index}}^{-1}(\tau),\end{aligned}\quad (7)$$

where  $\widehat{F}_{X^{index}|X^i}$  and  $\widehat{F}_{X^{index}}$  denote the empirical conditional and unconditional distributions functions obtained from the stock market returns for the index of financial institutions and the individual financial institution  $i$ , respectively. From this formulation, we can easily see the equivalence between  $\Delta CoVaR$  and two-sample treatment effects. In particular,  $\widehat{F}_{X^{index}|X^i}^{-1}(\tau) = \widehat{G}_T^{-1}(\tau)$  and  $\widehat{F}_{X^{index}}^{-1}(\tau) = \widehat{F}_S^{-1}(\tau)$ .

As a consequence, we can relate our hypothesis tests, as formulated in section 2.3, to the hypothesis tests 1. and 3. considered in the literature on quantile treatment effects. In particular, the hypothesis of significance given by equation (4) relates to hypothesis test 1. (hypothesis of no effect) of the quantile treatment effects literature:

$$H_0 : \Delta CoVaR^{index|i}(\tau) = 0,$$

for a given  $\tau \in (0, 1)$  or, more specifically, on a given subset of  $\mathcal{T} \subset (0, 1)$ . The hypothesis of dominance in equation (5) is similar to hypothesis test 3. (dominance hypothesis) of the quantile effects literature:

$$H_0 : CoVaR_{X^{index}|X^i}(\tau) \geq CoVaR_{X^{index}|X^j}(\tau),$$

for a given  $\tau \in (0, 1)$  or, more specifically, on a given subset of  $\mathcal{T} \subset (0, 1)$ .

As indicated, in the case of  $\Delta CoVaR$  we are not interested in the entire domain of  $\tau \in (0, 1)$ , like in hypotheses 1.-3. in the quantile treatment effects literature, but rather in a particular quantile ( $\tau = 0.95, \tau = 0.99$ ) or on a given subset  $\mathcal{T} \subset (0, 1)$ .<sup>8</sup> Since our interest is mainly a downside risk measure this subset will generally be defined as  $\mathcal{T} := (0.90, 0.99)$ , the lower tail of the conditional distribution of the random variable of interest (losses, returns). In the next section, we will use the inference procedures developed in the quantile treatment literature for testing hypotheses 1.-3. as a basis for the

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<sup>8</sup>This is an important difference with respect to the standard statistical test for stochastic dominance.

testing procedures that we develop for the two abovementioned hypothesis tests in the context of  $\Delta CoVaR$ . In particular, the tests that we develop are based on testing the difference between a conditional and an unconditional distribution or quantile function (significance) and whether one of two conditional distributions or quantile functions stochastically dominates the other (dominance), respectively, in the domain of interest for  $\tau$ .

#### 4. Testing for the systemic importance of a financial institution

Testing procedures for the hypothesis of significance and dominance are entirely determined by the underlying statistical model and the restrictive nature of it. In a parametric approach the differences between the conditional and unconditional distribution for the system or institution's losses will be entirely determined by the location and scale parameters or linear functions of such parameters. In other words, the statistics used in the hypothesis test are linear function of the location and scale parameters.

##### 4.1. General linear testing framework

Consider a linear hypothesis of the general form:

$$H_0 : R\beta(\tau) = r(\tau), \tau \in \mathcal{T}, \quad (8)$$

where  $\mathcal{T} \subset (0, 1)$ ,  $\beta(\tau)$  is a  $p$ -dimensional vector and  $R$  denotes a  $q \times p$  matrix ( $q \leq p$ ).

From Theorem Appendix A.1 in Appendix A we can easily see that

$$\sqrt{T}(R\hat{\beta}_T(\tau) - R\beta(\tau)) \rightarrow_d (\tau(1 - \tau))^{1/2}(R\Omega(\tau)R')^{1/2}N(0, I_q). \quad (9)$$

Under the null, the Wald statistic, which is a process indexed by  $\tau$ , is:

$$w_T(\tau) = T \frac{(R\hat{\beta}(\tau) - r(\tau))'(R\hat{\Omega}(\tau)R')^{-1}(R\hat{\beta}(\tau) - r(\tau))}{(\tau(1 - \tau))}, \quad (10)$$

where  $\hat{\Omega}(\tau)$  is a consistent estimator of  $\Omega(\tau)$ .

To test the general linear hypothesis Koenker and Machado (1999) propose using a sup-Wald test, i.e., the supremum of  $w_T(\tau)$  over a given subset  $\tau \in \mathcal{T}$ .



Let  $\mathcal{B}_q(\tau)$  denote a vector of  $q$ -dimensional Brownian Bridges with distribution  $(\tau(1-\tau))^{1/2}N(0, I_q)$ .<sup>9</sup> Therefore, (9) can be expressed as

$$\sqrt{T}(R\hat{\beta}_T(\tau) - R\beta(\tau)) \rightarrow_d (R\Omega(\tau)R')^{1/2}\mathcal{B}_q(\tau). \quad (11)$$

Under suitable conditions the Wald process converges weakly to the  $q$ -dimensional Brownian Bridge process (on a given subset of  $\mathcal{T} \subset (0, 1)$ ):

$$w_T(\tau) \Rightarrow \left\| \frac{\mathcal{B}_q(\tau)}{\sqrt{\tau(1-\tau)}} \right\|^2, \tau \in \mathcal{T}. \quad (12)$$

The statistic converges in the limit to the sum of squares of  $q$  independent Bessel process. Therefore we have the following result:

$$\sup_{\tau \in \mathcal{T}} w_T(\tau) \Rightarrow \sup_{\tau \in \mathcal{T}} \left\| \frac{\mathcal{B}_q(\tau)}{\sqrt{\tau(1-\tau)}} \right\|^2, \tau \in \mathcal{T}. \quad (13)$$

The critical values for the supremum of the Bessel process of order  $q$ ,  $\sup \mathcal{B}_q^2(\tau)$ , have been tabulated by DeLong (1981) and Andrews (1993, 2003) by simulation methods, and more recently by exact methods by Estrella (2003) and Anatolyev and Kosenok (2012). For any fixed  $\tau \in (0, 1)$  we have that  $\mathcal{B}_q^2(\tau) \sim \chi_q^2$ , thus it is natural to interpret  $\mathcal{B}_q^2(\tau)$  as a natural extension of the familiar univariate  $\chi^2$  with  $q$  degrees of freedom. Furthermore, in the special case  $q = 1$ ,  $\mathcal{B}_1^2(\cdot)$  behaves asymptotically like a squared Kolmogorov-Smirnov statistic (Koenker, 2005).

#### 4.2. Test for significance and dominance using the quantile response function

In this subsection we derive a statistic which is the basis for the test of significance and dominance in the linear quantile regression framework. We first present the approach that allows us to perform inference on the quantile response function (properly defined in Appendix B) through the use of the general linear testing framework in quantile regressions introduced in section 4.1. Next, we derive specific testing procedures for testing the specific hypotheses in the context of  $\Delta CoVaR$ .

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<sup>9</sup>A brownian bridge is a gaussian process with mean  $E[X_t] = 0$ , and  $Cov[X_t, X_s] = \min(t, s)(1 - \max(t, s))$ .

#### 4.2.1. Inference on the quantile response function

**Theorem 4.1.** *From Theorem Appendix A.1 in Appendix A and let us define some continuous mapping  $g(\beta(\tau)) = \mathbf{X}\beta(\tau)$ , where this mapping defines the quantile response function, evaluated at some point in the design space.*

$$\sqrt{n}(\hat{Q}_{\mathbf{Y}|\mathbf{X}}(\tau) - Q_{\mathbf{Y}|\mathbf{X}}(\tau)) \rightarrow_d N(0, \tau(1 - \tau)\mathbf{X}\Omega(\tau)\mathbf{X}') \quad (14)$$

**Proof:**

Direct application of the Delta Method such that:

$$\sqrt{n}(\mathbf{X}\hat{\beta}_n(\tau) - \mathbf{X}\beta(\tau)) \rightarrow_d N(0, \tau(1 - \tau)\mathbf{X}\Omega(\tau)\mathbf{X}'). \quad (15)$$

Hence,  $\hat{Q}_{\mathbf{Y}|\mathbf{X}}(\tau)$  is weakly consistent for  $Q_{\mathbf{Y}|\mathbf{X}}(\tau)$ <sup>10</sup>.

Theorem 4.1 serves as a first step toward introducing additional inference problems, based on the quantile response function, beyond the fundamental testing problems in the quantile treatment effects literature, mentioned in Section 3.1. In particular, setting  $R = X$  in expression (9) results in equivalence between expressions (9) and (14). Statistical testing then requires that  $X$  is evaluated at some point in the design space, for example at the centroid  $R = \bar{\mathbf{X}}$  or, as in our application, a particular quantile  $R = VaR_X(\tau_X)$ .

#### 4.2.2. Test for significance and dominance

As explained in Section 3,  $\Delta CoVaR$  can be interpreted as a quantile treatment effect. Therefore, we base our test on the Kolmogorov-Smirnov (KS) type statistic. KS type test are highly attractive since they are asymptotically distribution free.<sup>11</sup> The KS test provides a natural way to measure the discrepancy between distributions (Abadie, 2002). Furthermore, variants of the two-sample KS test have been widely used for inference based on a quantile process, such as those considered in Section 3.1. Our approach differs from previous approaches, since we consider a conditional distribution, rather than an unconditional distribution, and in particular the conditional quantile response function of a linear model.

<sup>10</sup>A stronger form of consistency of the conditional quantile function requires more stringent regularity conditionals and it is explored in Basset and Koenker (1982)

<sup>11</sup>In distribution free type test we can tabulate the distribution under the null, of the statistic, without specifying the underlying distribution of the data. The distribution free property, of a statistic, is a key property of many non-parametric procedures.

Suppose we have two different (at least one column is different) design matrices  $W$  and  $Z$ . The respective empirical quantile response functions are as follows:

$$\hat{Q}_{\mathbf{Y}|\mathbf{W}}(\tau) = \mathbf{W}\hat{\beta}_T^w(\tau) \quad (16)$$

and

$$\hat{Q}_{\mathbf{Y}|\mathbf{Z}}(\tau) = \mathbf{Z}\hat{\beta}_T^z(\tau) \quad (17)$$

This setup includes the case where we either compare a continuous treatment to non-treatment of the same population  $Y$  (significance) or two different continuous treatment effects applied to the same population  $Y$  (dominance), all within the framework of a linear model that relates  $Y$  to the  $W$  and  $Z$  covariates.<sup>12</sup>

Without loss of generality, we consider equal amount of observations  $T$  throughout the design space. Therefore, we have the following parametric empirical process:

$$\begin{aligned} V_T(\tau) &= \sqrt{T}(\hat{Q}_{\mathbf{Y}|\mathbf{W}}(\tau) - \hat{Q}_{\mathbf{Y}|\mathbf{Z}}(\tau)) \\ &= \sqrt{T}(\mathbf{W}\hat{\beta}_T^w(\tau) - \mathbf{Z}\hat{\beta}_T^z(\tau)) \\ &= \sqrt{T}(\mathbf{X}\hat{\beta}(\tau)), \end{aligned} \quad (18)$$

with  $\mathbf{X} = [\mathbf{W}, -\mathbf{Z}]$  and  $\hat{\beta}(\tau) = [\hat{\beta}^w(\tau), \hat{\beta}^z(\tau)]'$ .

Given that we are in a linear location-scale framework, we can derive a statistic for the two sample tests of hypothesis embedded in the general linear hypothesis frameset:

$$v_T(\tau) = \frac{\sqrt{T}(R\hat{\Omega}(\tau)R')^{-1/2}(R\hat{\beta}(\tau) - r(\tau))}{\sqrt{\tau(1-\tau)}} \quad (19)$$

Using Theorem 4.1, we can use this statistic for testing the significance of the empirical process in expression (18). In particular, we set  $R = \tilde{\mathbf{X}}$ , with  $\tilde{\mathbf{X}} = [\tilde{\mathbf{W}}, -\tilde{\mathbf{Z}}]$  and  $\tilde{\mathbf{X}}$  implying that the quantile response functions are evaluated at a given point of the design space ( $\tilde{\mathbf{W}}$  and  $\tilde{\mathbf{Z}}$ , respectively). As mentioned above, this point can be the centroid or an (extreme) quantile of interest.

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<sup>12</sup>In addition to the continuous treatment effect, the design matrices may also contain control variables.

Depending on the specification of  $W$  and  $Z$  in  $X$  and the specification of the test as a one-sided or a two-sided test, this will result in either a test of our significance hypothesis or our test of the dominance hypothesis.

**Significance** When testing the significance hypothesis, we are interested in comparing a continuous treatment to non-treatment of the same population  $Y$ . Therefore, whereas  $W$  contains the continuous treatment,  $Z$  does not. In the context of  $\Delta CoVaR$ , we have that  $Y = X^{index}$ ,  $W = VaR_{X^i}(\tau_X)$  and  $Z = 1$ .  $\hat{\beta}^w(\tau)$  equals the quantile regression estimate of  $\beta_1$  in equation (6) and  $\hat{\beta}^z(\tau)$  denotes an estimate of the unconditional VaR of the system  $VaR_{X^{index}}(\tau)$ . Hence, when testing  $H_0 : \Delta CoVaR^{index|i}(\tau) = 0$ ,  $R$  in (19) equals  $[VaR_{X^i}(\tau), -\mathbf{1}]$ ,  $\hat{\beta}(\tau) = [\hat{\beta}_1^i(\tau), \widehat{VaR}_{X^{index}}(\tau)]'$  and  $r(\tau) = 0$ .<sup>13</sup>

The two-sided KS type statistic is

$$K_T = \sup_{\tau \in \mathcal{T}} \|v_T(\tau)\|,$$

which is indicative of the statistical difference between the two empirical quantile functions, i.e., the quantile function of the market index conditional on institution  $i$  being at its  $VaR_{X^i}(\tau_X)$  and the unconditional quantile function of the market index  $VaR_{X^{index}}(\tau)$ .<sup>14</sup>

**Dominance** When testing the dominance hypothesis, we are interested in comparing two different continuous treatment effects applied to the same population  $Y$ . Therefore,  $W$  contains one continuous treatment and  $Z$  contains another continuous treatment. In the context of  $\Delta CoVaR$ , we have that  $Y = X^{index}$ ,  $W = VaR_{X^i}(\tau_X)$  and  $Z = VaR_{X^j}(\tau_X)$ .  $\hat{\beta}^w(\tau)$  equals the

<sup>13</sup>The presence of nuisance parameters in the test statistic may jeopardize the distribution-free character of the test (the so-called Durbin problem). However, Koenker and Machado (1999) show that, in the absence of nuisance parameters in  $R$  and  $r(\tau)$ , the nuisance parameters in  $\Omega(\tau)$  can be replaced by consistent estimates without jeopardizing the distribution-free character of the test. Given that  $r(\tau) = 0$  and  $VaR_{X^i}(\tau)$  in  $R$  is estimated non-parametrically, the KS-type test remains distribution free in our framework.

<sup>14</sup>As suggested in Koenker (2005), in some situations it is desirable to restrict the interval of estimation to a closed subinterval  $[\tau_0, \tau_1]$  of  $(0, 1)$ . This can easily be accommodated by considering the renormalized statistic  $K_T = \sup_{\tau \in [\tau_0, \tau_1]} \| \tilde{v}_T(\tau) - \tilde{v}_T(\tau_0) \| / \sqrt{\tau_1 - \tau_0}$ . In our applications we consider  $\tau \in [0.90, 0.99]$ .

quantile regression estimate of  $\beta_1$  in equation (6) and  $\hat{\beta}^z(\tau)$  denotes the parameter estimate in the equivalent regression for  $X^j$  instead of  $X^i$ . Hence, when testing  $H_0 : CoVaR_{X^{index}|X^i}(\tau) \geq CoVaR_{X^{index}|X^j}(\tau)$ ,  $R$  in (19) equals  $[VaR_{X^i}(\tau_X), -VaR_{X^j}(\tau_X)]$ ,  $\hat{\beta}(\tau) = [\hat{\beta}_1^i(\tau), \hat{\beta}_1^j(\tau)]'$  and  $r(\tau) = 0$ .

A one-sided version of the KS type statistic is

$$K_T = \sup_{\tau \in \mathcal{T}} (v_T(\tau)),$$

which would indicate the presence of a stochastic dominance relationship between the conditional quantile functions, i.e., the quantile function of the market index conditional on institution  $i$  being at its  $VaR_{X^i}(\tau_X)$  and quantile function of the market index conditional on institution  $j$  being at its  $VaR_{X^j}(\tau_X)$ .

As mentioned above, the critical values for these KS-type tests have been tabulated by DeLong (1981) and Andrews (1993, 2003) by simulation methods, and more recently by exact methods by Estrella (2003) and Anatolyev and Kosenok (2012). In our applications, we use the exact asymptotic p-values obtained from Anatolyev and Kosenok (2012).<sup>15</sup>

## 5. Monte Carlo

In this section we report a small Monte Carlo experiment designed to evaluate the performance of the test developed in Section 4.2. We obtain the critical values for the process  $\sup \mathcal{B}_q^2(\tau)$  using the exact methods proposed in Anatolyev and Kosenok (2012). The critical values are obtained for  $q = 1$  and for the upper right quantile range  $\mathcal{T} = [0.90, 0.99]$ . To evaluate the size and power of the Kolmogorov-Smirnov type test we consider that the data are generated by the following location-scale model:

$$X_t^{index} = \alpha + X_t\beta + (X_t\gamma)\varepsilon_t, \quad (20)$$

where  $X_t$  and  $\varepsilon_t$  are both drawn as iid from  $N(0, 1)$ . Additional parameters are set as follows:  $\alpha = 0$  and the heteroscedasticity parameter  $\gamma = 0.5$ . For the estimation of the quantile regression model, required to obtain the

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<sup>15</sup>We thank, Anatolyev and Kosenok (2012) for providing the source code in GAUSS of their methodology.

quantile response function evaluated at the 99% quantile, we consider an equally spaced grid of 90 quantiles  $\mathcal{T}_n = [0.10, 0.99]$  and we obtained bootstrapped estimates of the standard errors. We consider sample sizes of  $n = 500, 1000, 5000$ . The number of iterations in all of the simulations is 1000.

**Significance** In the experiments related to the significance test, we consider the null hypothesis  $H_0 : \Delta CoVaR^{index|i}(\tau) = 0$ . When  $\beta$  is set equal to 0, the rejection rates, which are given in Table 1, provide the empirical size of the test. In other words, the null hypothesis implies that there is no difference between the conditional and unconditional distribution of the system and should not be rejected when  $\beta = 0$ . The experiment indicates that the test has some size distortions (i.e., rejects the null, whereas it should not), especially in small samples. When the parameter  $\beta$  is set to 0.5 rather than 0, the rejection rates, which are presented in Table 1, provide the empirical power of the test. Results indicate that although an increase in the sample size seems to increase power (i.e., the ability to reject the null when it should be rejected), the power does not become 1 in any circumstance. Therefore, it is important that the sample size is sufficiently large. In our empirical applications, since we use daily observations of weekly stock returns, the number of observations is close to 5000.

**Dominance** In the experiments related to the dominance test, we follow a similar procedure. The main difference with the previous experiment is that the null hypothesis now is of the form  $H_0 : CoVaR^{index|j}(\tau) = CoVaR^{index|i}(\tau)$ . Under this setup we have two parameter values for  $\beta$ , one for each institution (institution  $i$  and  $j$ ) under consideration. In addition, we consider in the testing phase  $X_t^{index}$  to be the simple average of the dependent variable generated under both values of  $\beta$ . This assumption is not far from the intended use of the test since the index of financials is a weighted average of the individual stock returns. When  $\beta^j = 0.5$  and  $\beta^i = 0.2$ , the rejection rates, which are given in Table 2, provide the empirical size of the test. The null hypothesis implies that conditional distribution of the system given that institution  $j$  is at its 99% VaR is the same as the conditional distribution of the system given that institution  $i$  is at its 99% VaR. The ex-

periment indicates that the test is oversized in particular for levels (5%, 1%); in other words rejecting at a higher rate than the nominal one. When the parameters are set to  $\beta^j = 0.9$  and  $\beta^i = 0.01$ , the rejection rates, which are presented in Table 2, provide the empirical power of the test. Results are in line with the previous results on significance, i.e., the increase in the sample size increases power, but is far from optimal. Furthermore, power increases in the sample size at a slower rate than the one observed for the significance test.

Overall the Monte Carlo experiment indicates that the test has moderate performance for the usual number of observation available for financial daily data. However, care should be taken in taking inferences further into the tails with few data points (Chernozhukov, 2000). Indeed, inference for the extremal regression quantiles needs to take into account data scarcity considerations. Chernozhukov and Umantsev (2001) mention a "rule-of-thumb" based on the effective rank ( $\frac{(1-\tau)T}{d}$ ), which takes into account the target conditional quantile function ( $\tau$ ), the number of observations ( $T$ ) and the number of regressors ( $d$ ). According to the effective rank, which measures the severity of the data scarcity problem, some asymptotic considerations should be taken into account in performing inference. Since in most cases we are interested in the 95% or 99% quantile or VaR, in our simple conditional model, it is required to have around 5000 observations in order to use regular or central asymptotic approximations as we have done. Any application of the standard inference procedures significantly below such threshold should either consider intermediate or extremal rank behavior in the data.

## 6. Empirical application

In our empirical application we apply the tests described in the previous sections to  $\Delta CoVaR$  estimated from weekly stock return data for 26 large European banks. The sample covers the period from 26 October 1993 to 13 March 2012, resulting in a dataset of 4594 daily observations of weekly returns per bank. First, we provide a ranking of the banks in terms of their potential impact on the market index. We test whether the systemic risk contribution, as measured by  $\Delta CoVaR$ , for the different banks is statistically significant and whether the systemic risk contributions of different banks statistically differ from each other. Second, we apply our significance test to form a mapping of the interconnections between the 26 banks in the sample. In particular, we consider two banks as being interconnected when the impact

of one institution on the other, as measured by  $\Delta CoVaR$ , is statistically significant.

The stock market data for the 26 large European banks are taken from Datastream. As the market index we use the STOXX Europe 600 Financials index. Since we are interested in identifying the impact of a given bank on the market (or on other banks in the sample), we control for common factors that may drive individual banks' returns and therefore also the market's return. In particular, we regress the individual bank returns on a set of common factors in a first stage, and use the residuals of these regressions for the estimation of  $\Delta CoVaR$  in the second stage. The set of common factors includes lagged values of the weekly return on the STOXX Europe 600 Basic Materials index, the weekly return on the STOXX Europe 600 Industrials index and the weekly change in the Chicago Board Options Exchange Market Volatility (VIX) index. Table 3 shows summary statistics on the individual bank returns, the market return and the first-stage control variables. Note that returns are expressed as negative returns, so that a positive  $\Delta CoVaR$  can be interpreted as an increase in extreme or tail market losses for the market (or another bank) when a given bank is in distress. The summary statistics indicate that our sample period is characterized by periods of extreme market volatility, with large swings in our variables both in the upward and the downward direction.

#### 6.1. Individual banks' impact on the market index

In this first application, we focus on the banks' systemic importance in terms of their potential impact on the market index. This impact is measured by  $\Delta CoVaR$  as in Adrian and Brunnermeier (2008):

$$\Delta CoVaR^{index|i}(\tau) = CoVaR_{X^{index}|X^i}(\tau) - VaR_{X^{index}}(\tau)$$

Table 4 provides a ranking of the 26 banks based on  $\Delta CoVaR^{index|i}(\tau)$ , with  $\tau = 0.95$  and  $\tau_{X^i} = 0.99$  ( $X^i$  evaluated at its 99% VaR). The results show that  $\Delta CoVaR^{index|i}(0.95)$  ranges between 2.40 and 6.25, and that for many institutions the values their systemic risk contributions are of quite similar order of magnitude. As argued above, statistical testing of the estimated  $\Delta CoVaR$  is important, whether one wants use the results for imposing policy measures such as capital surcharges, or simply for assessing which factors such as balance sheet indicators explain an institution's systemic importance. The banks that have a significant systemic risk contribution based



on our two-sided significance test for  $\mathcal{T} = [0.90, 0.99]$  presented in Section 4.2.2 have their value of  $\Delta CoVaR$  marked with an asterix: 12 out of 26 banks have a systemic risk contribution that is statistically significant. Generally, the banks with a larger  $\Delta CoVaR^{index|i}(0.95)$  are found to have a statistically significant systemic risk contribution. In particular, whereas 9 out of the top 13 banks have statistically significant systemic risk contribution, only 3 out of the lower 13 banks have a significant systemic risk contribution. However, these results also show that a larger  $\Delta CoVaR$  does not necessarily imply a significant systemic risk contribution.

Figure 1 graphically shows the difference between a significant systemic risk contribution and a systemic risk contribution that is not significantly different from zero. The left-hand part of the figure shows the quantile and density function of the market return conditional on ING Groep being in distress (the dotted lines), as well as the unconditional quantile and density function of the market return (the thick lines). The right-hand part similarly shows these functions for Banco Espanol de Crédito. Note that the market return's unconditional quantile and density functions (the thick lines), respectively, coincide in the left-hand and the right-hand part of the figure; only the conditional functions differ between the two examples. In the case of ING Groep, the vertical distance between tail region of the market return's conditional and the unconditional quantile function (the dotted and the thick line) is significant. For Banco Espanol de Crédito, this vertical distance between the tail region of the market return's conditional and the unconditional quantile function (the dotted and the thick line) is substantially smaller and not statistically significant.

In a macro-prudential policy setting, one could use these results to set a rule of thumb that the institutions with a significant systemic risk contribution are the systemically relevant institutions and only impose additional policy measures on these institutions. This would be the simplest example of a bucketing approach in which stricter policy measures (e.g., capital surcharges) are imposed on banks that are more systemically important, i.e., on banks that are in a higher bucket. While in our application there are only two buckets (the banks with a significant systemic risk contribution and the banks with a systemic risk contribution that is not statistically significant), the approach can be generalized to one with more than two buckets.

The question that may arise in this context is whether the systemic risk contribution of those institutions with a statistically significant systemic risk contribution is actually larger than that of the institutions for which the

systemic risk contribution is not significantly different from zero. Or in other words, whether the systemic risk contributions of banks in a higher bucket are actually larger than that of banks in a lower bucket. One could argue that it may only be justified to impose additional regulation upon an institution if it is actually more systemically important than the others. Therefore, we apply our one-sided dominance test to all pairs of banks in the sample. Columns 4 and 8 of Table 4 list the number of banks that are dominated by the institution in question. Only ING Groep, which ranks highest in terms of  $\Delta CoVaR^{index|i}(0.95)$ , is dominating a substantial number (13) of other institutions in terms of its systemic risk contribution; 12 other banks, of which 9 have a significant systemic risk contribution, dominate 1 or 2 other banks in terms of its systemic risk contribution.

Figure 2 graphically shows the difference between a bank pair where one bank's systemic risk contribution stochastically dominates another bank's systemic risk contribution and a bank pair where this is not the case. The left-hand part of the figure shows the quantile and density function of market return conditional on ING Groep being in distress (the thick lines), as well as the quantile and density function of the market return conditional on Intesa Sanpaolo being in distress (the dotted lines). The right-hand part similarly shows these functions for ING Groep (the thick lines) and Banco Santander (the dotted lines). Note that in both cases, the market return's quantile and density functions, respectively, conditional on ING Groep being in distress (the thick lines) coincide; only the conditional functions for the second institution (the dotted lines) differ between the left-hand and the right-hand parts of the figure. The left-hand part of Figure 2 shows that the tail market losses conditional on ING Groep being in distress are substantially larger than the tail market losses conditional on Intesa Sanpaolo being in distress. Therefore, the systemic risk contribution of ING Groep stochastically dominates the one of Intesa Sanpaolo. In contrast, the right-hand part of Figure 2 shows that the vertical distance between the market return's quantile function conditional on ING Groep being in distress and the market return's quantile function conditional on Banco Santander being in distress is markedly lower; the difference between the two banks' systemic risk contributions is found not to be significant.

Table 5 provides further insight into the dominance test results for all bank pairs in the sample. Out of 325 bank pairs, there are 55 bank pairs where both banks have a significant systemic risk contribution (so both are in the higher bucket) and 105 bank pairs where both banks' systemic risk con-

tribution is not significant (both are in the lower bucket); the remaining 165 bank pairs are combinations in which one bank has a significant systemic risk contribution and the other one does not (one bank in the higher bucket and the other in the lower). In only 27 pairs out of 325, one bank's systemic risk contribution is found to stochastically dominate the one of the other bank. In 20 cases, a bank with a statistically significant systemic risk contribution (a bank in the higher bucket) is found to dominate a bank of which the systemic risk contribution is not significant (a bank in the lower bucket). In 4 cases, both the dominating bank and the dominated bank have a significant systemic risk contribution (so both are in the higher bucket), and in 3 cases, both the dominating bank and the dominated bank have a systemic risk contribution that is not significant (both in the lower bucket). There are no cases where a bank with an insignificant systemic risk contribution dominates a bank of which the systemic risk contribution is significant. Based on the latter result, i.e., on the fact that a bank from the lower bucket never dominates a bank from the higher bucket, one could argue that the rule of thumb that only those banks with a significant systemic risk contribution should be considered systemically important seems to be adequate. However, out of the 165 bank pairs where one bank with a significant systemic risk contribution and the other does not have a significant systemic risk contribution, there are only 20 for which the bank with the significant systemic risk contribution actually stochastically dominates the other bank. In the other 145 cases, the systemic risk contribution of the systemically relevant institution (according to the rule of thumb) is in fact not statistically larger than that of the bank with an insignificant systemic risk contribution. That is, in a majority of the cases, a bank from the higher bucket is found to not dominate a bank from the lower bucket. This raises serious doubts on whether additional regulation should be imposed on all these banks with a significant systemic risk contribution. Rather than using a simple rule of thumb that indicates whether a bank's  $\Delta CoVaR$  is significantly different from zero (or alternatively, exceeds some pre-specified threshold), we would suggest that the results of pairwise dominance tests are also to be taken into account when categorizing banks in terms of their systemic importance. More generally, when applying a bucketing approach for ranking and regulating systemically important financial institutions, statistical tests to see whether banks in higher buckets actually have a larger systemic risk contribution than banks in lower buckets should be considered.

To conclude, while banks with a larger estimated  $\Delta CoVaR$  are more

likely to have a statistically significant systemic risk contribution, a larger  $\Delta CoVaR$  does not necessarily imply that a bank's systemic risk contribution is significant. In addition, when categorizing institutions in terms of their systemic importance, one should not only make use of significance results, but also consider the results of pairwise dominance tests. In fact, we find that very few banks can actually be ranked according to their systemic risk contribution on the basis of  $\Delta CoVaR$ . The latter result is in line with Danielsson et al. (2011) who show - although only for four institutions - that the bootstrapped confidence intervals underlying  $\Delta CoVaR$  estimates are quite large, so it is not possible to conclude which institution is systemically riskier than the other. These results indicate that the linear relationship between the variables of interest  $(X^{index}, X^i)$  that is at the core of the  $\Delta CoVaR$  measure, may be too restrictive. In particular, the affine transformation that characterizes the construction of the conditional distribution of the variable  $X^{index}$ , is heavily stressed, by construction, at the center of the distribution rather than at the extreme. The potential inability of  $\Delta CoVaR$  to rank financial institutions according to their systemic risk contributions may limit its usefulness for supporting macroprudential policy measures towards SIFIs.

## 6.2. A mapping of the interconnections between the banks

As mentioned earlier, market data may also be used to assess how interconnected financial institutions are (in the market's view). In this second application we show how our statistical tests can be used in this context. In particular, we apply our significance test to form a mapping of the interconnections between the 26 banks in the sample. Two banks are considered as being interconnected only when the impact of one institution on the other, as measured by  $\Delta CoVaR$ , is statistically significant. We calculate the impact of bank  $i$  on bank  $j$  as:

$$\Delta CoVaR^{j|i}(\tau) = CoVaR_{X^j|X^i}(\tau) - VaR_{X^j}(\tau)$$

Table 6 presents the average impact on the other banks of the sample as measured by the average of  $\Delta CoVaR^{j|i}(\tau)$  for all  $j \neq i$  with  $\tau = 0.95$  and  $\tau_{X^i} = 0.99$ . While the ranking of institutions does not exactly match the one in Table 4, there nevertheless seems to be a large degree of consistency with the ranking in terms of impact on the market. In particular, the top 2 banks coincide and are ranked in the same order, and 12 out of the 13 top-ranked banks in Table 6 also are among the 13 top-ranked banks in Table

4. Furthermore, 10 out of the top 13 banks in Table 6 have statistically significant systemic risk contribution in terms of impact on the market (as tested in the previous subsection and marked with an asterix).

The average impact figures in Table 6 do not take into account the significance of the estimated  $\Delta CoVaRs$ , however. Table 7 provides a ranking of the banks in terms of the average impact on other banks in the sample, after taking into account the significance of the  $\Delta CoVaR$ . In particular, we set the estimated  $\Delta CoVaR$  that are found not to be significant equal to zero and recalculate the banks' average impact on the other banks in the sample. While the ranking of the banks in Table 7 is not exactly the same as the one in Table 6, taking into account the significance of the estimated  $\Delta CoVaR$  does not dramatically change the ranking of banks in terms of their average impact on the other banks in the sample.

The importance of testing for the significance of the estimated  $\Delta CoVaRs$  becomes more important when we want to draw a mapping of the interconnections between the banks in our sample. Columns 4 and 8 of Table 7 provide the number of other banks on the bank in question has a significant impact, as indicated by our significance test on  $\Delta CoVaR$ . The number of other banks on which the banks in our sample have a significant impact ranges from 1 up to 13 out of a maximum value of 25. The total number of significant linkages amounts to 150 out of 650 possible linkages. This shows the importance of significance testing in mapping interconnections on the basis of  $\Delta CoVaR$ : while there are 650 possible linkages, only 150 are of statistical relevance. Hence, the subset of linkages that have to be analysed is substantially narrowed down (and could be further reduced if statistical tests were performed on whether  $\Delta CoVaR$  exceeds a given pre-specified threshold level).

Figures 3 and 4 provide further detail on the network of interconnections. In particular, Figure 3 plots the network of significant impacts of the top 3 banks in Table 7 (ING Groep, KBC Groep and Deutsche Bank, depicted in boxes). Similarly, Figure 4 shows the network of significant impacts of the top 8 banks in Table 7 (Allied Irish Banks, Banco Espanol de Crédito, Standard Chartered, Danske Bank, Natixis, BCP-Millennium, National Bank of Greece, Landesbank Berlin-LBB Holding, depicted in boxes). Whereas with 36 out of 75 potential outgoing linkages being significant, Figure 3 shows a relatively dense network of significant impacts, the network in Figure 4 is clearly much more sparse, with only 14 out of 200 potential outgoing linkages being significant. The top 3 banks in Table 7 together also have a larger

number of different banks on which they have a significant impact (18) than the bottom 8 banks together do (9). Therefore, testing for the significance of the estimated  $\Delta CoVaRs$  clearly affects the network picture one obtains.

In summary, taking into account the significance of the estimated  $\Delta CoVaR$  does not dramatically change the ranking of banks in terms of their average impact on the other banks, at least not in our sample. However, testing for the significance of the estimated  $\Delta CoVaRs$  clearly affects the network picture one obtains. In particular, the subset of linkages that have to be analysed is substantially narrowed down.

## 7. Conclusions

After the 2007-2008 financial crisis mitigating the risk stemming from so-called systemically important financial institutions (SIFIs) and more in general interconnectedness within the financial system have been and still are important topics on the regulatory reform agenda. As data availability, especially on interconnectedness, is far from optimal to perform the crucial task of identifying/ranking SIFIs and assessing interconnecteness, market-based measures have been developed to complement balance sheet indicator-based approaches.

In this paper we analysed one such popular market-based measure,  $\Delta CoVaR$  proposed by Adrian and Brunnermeier (2008), and developed a test of significance of  $\Delta CoVaR$  that allows determining whether or not a financial institution can be classified as being systemically important on the basis of the estimated systemic risk contribution, as well as a test of dominance aimed at testing whether or not, according to  $\Delta CoVaR$ , one financial institution is more systemically important than another. In addition, we provided two applications on a sample of 26 large European banks to show the importance of statistical testing when using  $\Delta CoVaR$ , and more generally also other market-based systemic risk measures, for identifying/ranking SIFIs and assessing interconnectedness. One of our main messages is that when categorizing institutions in terms of their systemic importance, one should not only make use of significance results, but also consider the results of pairwise dominance tests. That is, when applying a bucketing approach for ranking and regulating systemically important financial institutions, statistical tests to see whether banks in higher buckets actually have a larger systemic risk contribution than banks in lower buckets should be considered. In fact, we find that very few banks can actually be ranked according to their systemic

risk contribution on the basis of  $\Delta CoVaR$ . We argue that this potential inability of  $\Delta CoVaR$  to rank financial institutions according to their systemic risk contributions, may be due to the restrictive nature of the assumed linear relationship between the variables of interest and may limit its usefulness for supporting macroprudential policy measures towards SIFIs.

Therefore, while the testing procedures developed in this paper entail a first step in the right direction, further work is required in order to adjust the asymptotics for some of the extremal regression quantiles that are used in such quantile-based measures (see Chernozhukov, 2000; Chernozhukov and Umansev, 2001). A medium term goal of this research agenda is to develop proper stochastic dominance test at the extremum for a general class of conditional and unconditional quantile functions. Such type of test are of interest for a much needed inferential-based analysis that will hopefully allow to statistically compare loss distributions in risk management.

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## Tables and figures

Table 1: Size and Power of the test:  $H_0 : \Delta CoVaR^{index|i}(\tau) = 0$

n	$\beta = 0$			$\beta = 0.5$		
	10%	5%	1%	10%	5%	1%
500	0.09	0.07	0.05	0.68	0.64	0.50
1000	0.07	0.06	0.03	0.76	0.68	0.59
5000	0.06	0.04	0.02	0.92	0.90	0.89

Notes: n denotes the sample size used in the Monte Carlo experiment. Each cell reports the proportion of rejections reported under  $\beta = 0$ , the size (under  $\beta = 0.5$ , the power) at the designated level of significance.

Table 2: Size and Power of the test:  $H_0 : CoVaR^{index|j}(\tau) = CoVaR^{index|i}(\tau)$

	$\beta^j = 0.5, \beta^i = 0.2$			$\beta^j = 0.9, \beta^i = 0.01$		
n	10%	5%	1%	10%	5%	1%
500	0.13	0.12	0.11	0.69	0.62	0.52
1000	0.09	0.08	0.08	0.75	0.69	0.57
5000	0.01	0.01	0.01	0.86	0.84	0.79

Notes: n denotes the sample size used in the Monte Carlo experiment. Each cell reports the proportion of rejections reported under  $\beta^j = 0.5$  and  $\beta^i = 0.2$ , the size (under  $\beta^j = 0.9$  and  $\beta^i = 0.01$ , the power) at the designated level of significance.

Table 3: Summary statistics

variable	obs	mean	min	max
bank return	119444	-0.16	-191.23	81.27
STOXX Europe 600 Financials return	4594	-0.08	-28.41	26.07
STOXX Europe 600 Basic Materials return	4594	-0.22	-32.38	21.60
STOXX Europe 600 Industrials return	4594	-0.14	-25.24	19.25
VIX index change	4594	0.01	-26.38	27.09

Notes: The summary statistics on bank returns are based on pooled data for all banks. The number of observations per bank is 4594. Returns and changes of the variables are weekly.

Table 4: Ranking of banks in terms of their impact on the market

	bank	$\Delta CoVaR$	dom	bank	$\Delta CoVaR$	dom
1	ING Groep	6.25*	13	14	Standard Chartered	4.21
2	Banco Santander	5.83*	1	15	Banco Popular Espanol	4.14
3	Credit Suisse Group	5.64*	2	16	Danske Bank	4.06
4	Société Générale	5.54	1	17	Bank of Ireland	3.89
5	HSBC Holding	5.51*	1	18	Svenska Handelsbanken	3.84
6	Deutsche Bank	5.46*	1	19	Royal Bank of Scotland Group	3.79*
7	BBVA	5.35*	1	20	National Bank of Greece	3.63*
8	BNP Paribas	5.24*	1	21	Barclays	3.53*
9	Unicredit	4.99	1	22	Natixis	3.46
10	UBS	4.97*	2	23	BCP-Millennium	3.23
11	KBC Groep	4.85*	0	24	Landesbank Berlin-LBB Holding	2.79
12	Intesa Sanpaolo	4.75	0	25	Allied Irish Banks	2.55
13	Commerzbank	4.61	1	26	Banco Espanol de Crédito	2.40

Notes:  $\Delta CoVaR$  is the impact of the bank in question on the market index, as measured by  $\Delta CoVaR^{index|i}(\tau)$  with  $\tau = 0.95$  and  $\tau_{X^i} = 0.99$ . The values of  $\Delta CoVaR$  of the banks for which the systemic risk contribution is statistically significant for  $\mathcal{T} = [0.90, 0.99]$  are marked with an asterix. The columns with header dom indicate the number of other banks in the sample whose systemic risk contribution is stochastically dominated by the one of the bank in question for  $\mathcal{T} = [0.90, 0.99]$ .

Table 5: Dominance test results

variable	bank pairs with dominance	total bank pairs
total	27	325
significant dominates significant	4	55
significant dominates insignificant	20	165
insignificant dominates significant	0	
insignificant dominates insignificant	3	105

Notes: The reference to (in)significant in the first column refers to banks for which the systemic risk contribution in Table 4 is statistically (not) significant for  $\mathcal{T} = [0.90, 0.99]$ . Out of 325 bank pairs, there are 55 bank pairs where both banks have a significant systemic risk contribution and 105 bank pairs where both banks' systemic risk contribution is not significant; the remaining 165 bank pairs are combinations in which one bank has a significant systemic risk contribution and the other one does not.



Table 6: Ranking of banks in terms of their impact on the other banks in the sample

	bank	average $\Delta CoVaR$		bank	average $\Delta CoVaR$
1	ING Groep	6.50*	14	Danske Bank	4.34
2	Banco Santander	5.79*	15	Intesa Sanpaolo	4.26
3	Deutsche Bank	5.57*	16	Bank of Ireland	4.19
4	BBVA	5.45*	17	Natixis	4.16
5	Société Générale	5.40	18	Svenska Handelsbanken	3.99
6	KBC Groep	5.34*	19	Royal Bank of Scotland Group	3.86*
7	Credit Suisse Group	5.25*	20	Standard Chartered	3.80
8	UBS	5.25*	21	Barclays	3.71*
9	Commerzbank	5.22	22	BCP-Millennium	3.68
10	BNP Paribas	5.11*	23	National Bank of Greece	3.64*
11	HSBC Holding	5.07*	24	Allied Irish Banks	2.76
12	Unicredit	5.01	25	Banco Espanol de Crédito	2.73
13	Banco Popular Espanol	4.38	26	Landesbank Berlin-LBB Holding	2.67

Notes: Average  $\Delta CoVaR$  denotes the average impact of the bank in question on the other banks in the sample, as measured by the average of  $\Delta CoVaR^{j|i}(\tau)$  for all  $j \neq i$  with  $\tau = 0.95$  and  $\tau_{X^i} = 0.99$ . The values of  $\Delta CoVaR$  of the banks for which the systemic risk contribution in Table 4 is statistically significant for  $\mathcal{T} = [0.90, 0.99]$  are marked with an asterix.

Table 7: Ranking of banks in terms of their impact on the other banks in the sample: significance-adjusted

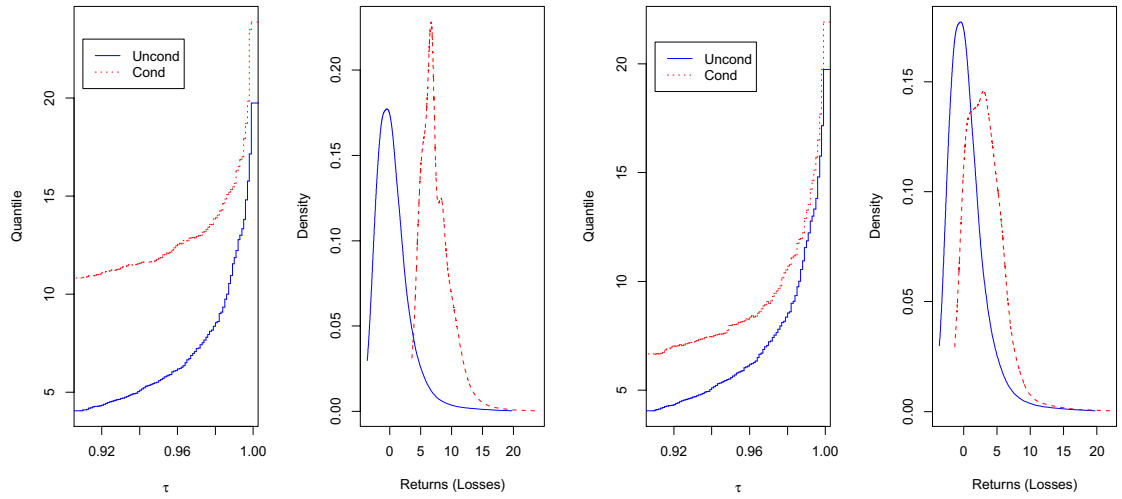
	bank	average $\Delta CoVaR$	sign. impact	bank	average $\Delta CoVaR$	sign. impact
1	ING Groep	3.22	12	14 Bank of Ireland	0.91	4
2	KBC Groep	2.74	13	15 Intesa Sanpaolo	0.89	5
3	Deutsche Bank	2.60	11	16 Credit Suisse Group	0.83	4
4	Banco Santander	2.28	10	17 Banco Popular Espanol	0.68	4
5	BBVA	2.03	10	18 Svenska Handelsbanken	0.68	4
6	Commerzbank	1.92	10	19 Allied Irish Banks	0.45	1
7	Barclays	1.76	12	20 Banco Espanol de Crédito	0.41	3
8	BNP Paribas	1.74	8	21 Standard Chartered	0.36	2
9	Société Générale	1.54	7	22 Danske Bank	0.34	3
10	HSBC Holding	1.22	5	23 Natixis	0.26	2
11	UBS	1.22	5	24 BCP-Millennium	0.15	1
12	Royal Bank Scotland Group	1.14	7	25 National Bank of Greece	0.13	1
13	Unicredit	0.92	5	26 Landesbank Berlin-LBB Holding	0.09	1

Notes: Average  $\Delta CoVaR$  denotes the average impact of the bank in question on the other banks in the sample, as measured by the average of  $\Delta CoVaR^{ji}(\tau)$  for all  $j \neq i$  with  $\tau = 0.95$  and  $\tau_{X^i} = 0.99$ , and the insignificant estimates of  $\Delta CoVaR^{ji}(\tau)$  for  $\mathcal{T} = [0.90, 0.99]$  set equal to zero. The column sign. impact presents the number of other banks in the sample on which the bank in question has a significant impact.

Figure 1: Graphical presentation of systemic risk contribution: significance

(a) ING Groep

(b) Banco Espanol de Credito



Notes: Uncond refers to the unconditional quantile/density function of the market index. Cond (dotted line) refers to the quantile/density function of the market index conditional on ING Groep and Banco Espanol de Crédito being in distress, respectively.

Figure 2: Graphical presentation of systemic risk contribution: dominance

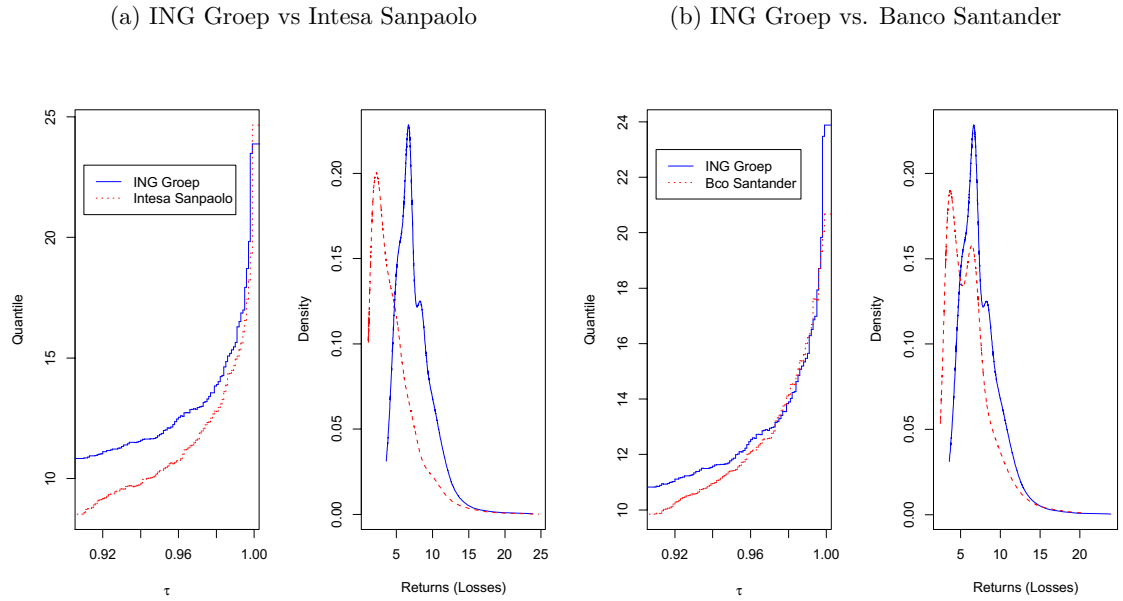
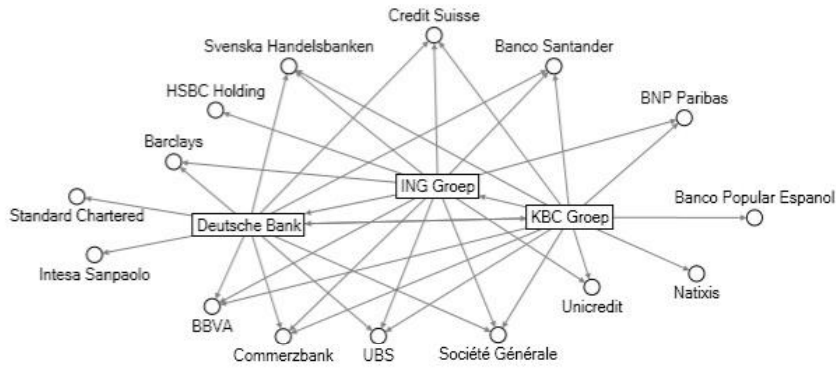
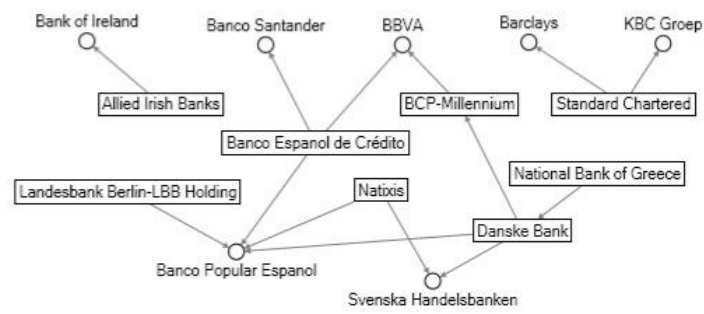


Figure 3: Network of significant impact of top 3 banks



Notes: The bank labels for which the outgoing significant impacts are plotted are depicted in boxes.

Figure 4: Network of significant impact of bottom 8 banks



Notes: The bank labels for which the outgoing significant impacts are plotted are depicted in boxes.

## Appendix A. Inference for quantile regression

For a general form of the linear quantile regression model, the independent random variables  $\{Y\}_{t=1}^T$  and  $\{X\}_{t=1}^T$  will have conditional distribution functions  $F_1, \dots, F_T$ , respectively. The conditional distribution functions will be denoted as follows:

$$Q_{\mathbf{Y}|\mathbf{X}}(\tau) = F_{Y_t|X_t}^{-1}(\tau) \equiv \xi_t(\tau) \quad (\text{A.1})$$

We state Theorem 4.1 of Koenker (2005), in order to derive the distribution of the estimator  $\hat{\beta}_T(\tau)$  obtained in section 2.2. Before restating the theorem we need a series of regularity conditions:

- **Condition 1:** The distribution functions  $F_t$  are absolutely continuous, with continuous densities  $f_t(\xi)$  uniformly bounded away from 0 and  $\infty$  at the points  $\xi_t(\tau)$ .
- **Condition 2:** There exist positive definite matrices  $Q$  and  $D(\tau)$  such that:

1.  $\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T x_t x_t' = \Omega$ .
2.  $\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T f_t(\xi_t(\tau)) x_t x_t' = D(\tau)$ .
3.  $\max_{i=1, \dots, T} \frac{\|x_t\|}{\sqrt{T}} \rightarrow 0$ .

**Theorem Appendix A.1.** *Under conditions 1 and 2,  $\hat{\beta}_T(\tau)$  is consistent and asymptotically normally distributed, such that,*

$$\sqrt{T}(\hat{\beta}_T(\tau) - \beta(\tau)) \rightarrow_d N(0, \tau(1-\tau)\Omega(\tau)) \quad (\text{A.2})$$

where  $\Omega(\tau) = D^{-1}(\tau)\Omega D^{-1}(\tau)$ . In the i.i.d. error model:

$$\sqrt{T}(\hat{\beta}_T(\tau) - \beta(\tau)) \sim N(0, \omega\Omega^{-1}) \quad (\text{A.3})$$

where  $\omega = \frac{\tau(1-\tau)}{f^2(\xi_t(\tau))}$ .

## Appendix B. Quantile response functions

Let  $\mathbf{Y} = (Y_1, \dots, Y_T)$  denote a vector of independent random variables and a design matrix  $\mathbf{X}$  of size  $T \times p$ . Denote  $\hat{\beta}(\tau)$  as the quantile regression process, such that:

$$\hat{\beta}_T(\tau) = \underset{\beta \in \mathbb{R}^p}{\operatorname{argmin}} \sum_{t=1}^T \rho_\tau(y_t - \mathbf{x}_t \beta) \quad (\text{B.1})$$

where  $\rho_\tau(u) = u(\tau - I(u < 0))$  and  $\tau \in (0, 1)$  (Koenker, 2005).

The conditional quantile function for the response variable  $\mathbf{Y}$  given  $\mathbf{X}$  can be defined as

$$Q_{\mathbf{Y}|\mathbf{X}}(\tau) = \mathbf{X}\beta_T(\tau) \quad (\text{B.2})$$

Some important equivariance properties, with respect to scale, location and reparametrization of the design matrix, for the conditional quantile function can be found in Theorem 2.3 of Basset and Koenker (1982).

The empirical counterpart of the conditional quantile function or the expected value of such response function is defined as:

$$\hat{Q}_{\mathbf{Y}|\mathbf{X}}(\tau) = \mathbf{X}\hat{\beta}_T(\tau) \quad (\text{B.3})$$

Note that like a quantile treatment effect, the empirical conditional quantile function will not necessarily satisfy the fundamental monotonicity requirement of a quantile function (i.e. that the function is non decreasing in  $\tau$ ). The estimated conditional quantile function is subject to possible quantile crossings. As pointed out in Theorem 2.5 of Koenker (2005) these crossings are generally confined to the outlying regions of the design space. Therefore in the centroid of the design space  $\bar{\mathbf{X}}$  the estimated conditional quantile function

$$\hat{Q}_{\mathbf{Y}|\bar{\mathbf{X}}}(\tau) = \bar{\mathbf{X}}\hat{\beta}_T(\tau) \quad (\text{B.4})$$

is more likely to remain monotone in  $\tau$ . Hence also the expectation of the response function evaluated at the centroid of the design space is monotone with respect to  $\tau$ .

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