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## Firm-specific production factors in a DSGE model with Taylor price setting

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# Working paper research

n° 85 June 2006

## Firm-specific production factors in a DSGE model with Taylor price setting

Gregory de Walque Frank Smets Raf Wouters



# NATIONAL BANK OF BELGIUM

## WORKING PAPERS - RESEARCH SERIES

### FIRM-SPECIFIC PRODUCTION FACTORS IN A DSGE MODEL WITH TAYLOR PRICE SETTING

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Frank Smets (\*\*)  
Raf Wouters (\*\*\*)

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The views expressed in this paper are our own and do not necessarily reflect those of the National Bank of Belgium or the European Central Bank.

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## **Abstract**

This paper compares the Calvo model with a Taylor contracting model in the context of the Smets-Wouters (2003) Dynamic Stochastic General Equilibrium (DSGE) model. In the Taylor price setting model, we introduce firm-specific production factors and discuss how this assumption can help to reduce the estimated nominal price stickiness. Furthermore, we show that a Taylor contracting model with firm-specific capital and sticky wage and with a relatively short price contract length of four quarters is able to outperform, in terms of empirical fit, the standard Calvo model with homogeneous production factors and high nominal price stickiness. In order to obtain this result, we need very large real rigidities either in the form of a huge (constant) elasticity of substitution between goods or in the form of an elasticity of substitution that is endogenous and very sensitive to the relative price.

JEL-code : E1, E3.

Keywords: Inflation persistence, DSGE models.

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# 1 Introduction

Following the theoretical work of Yun (1996) and Woodford (2003), the New Keynesian Phillips curve (NKPC), relating inflation to expected future inflation and the marginal cost, has become a popular tool for monetary policy analysis. Typically, the elasticity of inflation with respect to changes in the marginal cost is, however, estimated to be very small (e.g. Gali and Gertler (1999), Gali, Gertler and Lopez-Salido (2001) and Sbordone (2002)). In models with constant-returns-to-scale technology, perfectly mobile production factors and a constant elasticity of substitution between goods, such low estimates imply an implausibly high degree of nominal price stickiness. For example, Smets and Wouters (2003) find that, on average, nominal prices are not re-optimised for more than two years. This is not in line with existing micro evidence that suggests that on average prices are sticky for around six months to one year.<sup>1,2</sup>

In response to these findings, a number of papers have investigated whether the introduction of additional real rigidities, such as frictions in the mobility of capital across firms, can address this apparent mismatch between the macro and micro estimates of the degree of nominal price stickiness. For example, Woodford (2005), Eichenbaum and Fisher (2004) and Altig *et al.* (2005) show how the introduction of firm-specific capital lowers the elasticity of prices with respect to the real marginal cost for a given degree of price stickiness. This paper focuses on the same issue, but differs from the previous analysis in a number of ways. In contrast to the above-mentioned papers that focus on models with Calvo (1983) style sticky prices, we introduce firm-specific factors into a general equilibrium model with overlapping price and wage contracts as in Taylor (1980). The reason for using Taylor contracts is three-fold. First, the Calvo model has the unattractive feature that at any time there are some firms that have not adjusted their price optimally for a very long time. In contrast, with Taylor contracts there is a maximum contract length. Second, while the simple Calvo model is analytically tractable, its derivation with firm-specific factors and endogenous capital accumulation is non-trivial and can not be solved in closed form. This complicates the empirical estimation of the full model. The assumption of Taylor contracts facilitates the estimation of a fully-specified linearised Dynamic Stochastic General Equilibrium (DSGE) model which embeds the pricing decisions of monopolistically competitive price and wage set-

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<sup>1</sup>See the evidence in Bils and Klenow (2004) for the US and Altissimo, Ehrmann and Smets (2005) for a summary of the Inflation Persistence Network (IPN) evidence on price stickiness in the euro area.

<sup>2</sup>However, one should be careful with using the micro-evidence to interpret the macro estimates. Because of indexation and a positive steady state inflation rate, all prices change all the time. However, only a small fraction of prices are set optimally. The alternative story for introducing a lagged inflation term in the Phillips curve based on the presence of rule-of-thumb price setters is more appealing from this perspective, as it does not imply that all prices change all the time. In that case, the comparison of the Calvo parameter with the micro evidence makes more sense. As the reduced form representations are almost identical, one could still argue that the estimated Calvo parameter is implausibly high.

ters and real rigidities such as firm-specific capital. Finally, the use of Taylor contracts in this DSGE setting also makes it easier to analyse the distribution of prices and quantities across the various sectors. This analysis is important to check whether the introduction of real rigidities leads to a realistic distribution of prices and quantities (as in Altig *et al.*, 2005). Our paper is most closely related to Coenen and Levin (2004), which also investigates the relative importance of real and nominal rigidities in a world with Taylor contracts. However, Coenen and Levin (2004) focuses on Germany and does not specify the full structural model. Finally, in contrast to most of the papers mentioned above, our paper also analyzes the implications of firm-specific labour markets.

In the rest of this paper, we proceed in several steps. First, we compare the Calvo and Taylor specification in an estimated DSGE model under the assumption that firms are price-takers in the factor markets, i.e. the labour and capital markets, and hence all firms face the same flat marginal cost curve. We find that the length of the Taylor contracts in the goods market needs to be extremely long (about five years) in order to match the data as well as the Calvo scheme. Though striking, this result is consistent with Dixon and Kara (2005a), who show how to compare the mean duration of contracts in both time-dependent price setting models. In this section, we also show that the standard way of introducing mark-up shocks in the Calvo model does not work very well with Taylor-type price setting and we propose a different way of introducing price mark-up shocks.

Next, we re-estimate the Taylor contracting models with firm-specific capital and/or firm-specific labour and analyse the impact of these assumptions on the empirical performance of the DSGE model and on the estimated contract length in the goods market. Our main findings are twofold. First, in line with the previous literature we find that introducing firm-specific capital does lead to a fall in the estimated Taylor contract length in the goods market to a more reasonable length of 4 quarters. However, the elasticity of substitution between goods of the various price-setting cohorts is estimated to be improbably high. Furthermore, the corresponding price mark-up is estimated to be smaller than the fixed cost, implying negative profits in steady state. Enforcing a steady-state zero-profit condition leads to a significant deterioration of the empirical fit. At the same time, the estimated elasticity of substitution remains very large. Moving from the traditional Dixit-Stiglitz aggregator towards Kimball's (1995) generalized aggregator helps to solve both problems. In that case, the curvature parameter is estimated to be high, which is a sign that real rigidities are at work, but both the estimated elasticity of substitution and the cost of imposing the above-mentioned zero-profit constraint are sharply reduced. These results are in line with Eichenbaum and Fisher (2004), Coenen and Levin (2004) and Altig *et al.* (2005). In this context, we also investigate the implications of the various models for the firm-specific supply and pricing decisions, which is easier to perform in a Taylor-contracting framework.



Finally, we also analyse the impact on empirical performance of introducing firm-specific labour markets. Here the results are less promising in terms of reducing the estimated degree of nominal price stickiness. The reason is that firm-specific labour markets only dampen the price impact of a change in demand for a given degree of nominal price stickiness, if the firm-specific labour markets are flexible and the firm-specific wage is responding strongly to changes in the demand for labour. Such wage flexibility is, however, incompatible with the empirical properties of aggregate wage behaviour.

The rest of the paper is structured as follows. First, we briefly review the estimated DSGE model of Smets and Wouters (2005) with a special focus on the estimated degree of price stickiness. Section 3 compares the Calvo model with the standard Taylor-contracting model. Section 4 explores the impact of introducing firm-specific production factors. The concluding remarks are in Section 5.

## 2 Calvo price-setting in a linearised DSGE model

In this Section, we briefly describe the DSGE model that we estimate using euro area data. For a discussion of the micro-foundations of the model we refer to Smets and Wouters (2005). Next, we review the main estimation results with regard to the inflation equation embedded in the DSGE model.

### 2.1 The DSGE model

The DSGE model contains many frictions that affect both nominal and real decisions of households and firms. The model is based on Smets and Wouters (2004). Households maximise a non-separable utility function with two arguments (goods and labour effort) over an infinite life horizon. Consumption appears in the utility function relative to a time-varying external habit variable. Labour is differentiated, so that there is some monopoly power over wages, which results in an explicit wage equation and allows for the introduction of sticky nominal wages à la Calvo (1983). Households rent capital services to firms and decide how much capital to accumulate taking into account capital adjustment costs.

The main focus of this paper is on the firms' price setting. A continuum of firms produce differentiated goods, decide on labour and capital inputs, and set prices. Following Calvo (1983), every period only a fraction  $(1 - \xi_p)$  of firms in the monopolistic competitive sector are allowed to re-optimize their price. This fraction is constant over time. Moreover, those firms that are not allowed to re-optimize, index their prices to the past inflation rate and the time-varying inflation target of the central bank. An additional important assumption is that all firms are price takers in the factor markets for labour and capital and thus face the same marginal cost. The marginal costs depend

on wages, the rental rate of capital and productivity.

As shown in Smets and Wouters (2004), this leads to the following linearised *inflation equation*:

$$\widehat{\pi}_t - \bar{\pi}_t = \frac{\beta}{1 + \beta\gamma_p} (E_t \widehat{\pi}_{t+1} - \bar{\pi}_t) + \frac{\gamma_p}{1 + \beta\gamma_p} (\widehat{\pi}_{t-1} - \bar{\pi}_t) \quad (1)$$

$$+ \frac{1}{1 + \beta\gamma_p} \frac{(1 - \beta\xi_p)(1 - \xi_p)}{\xi_p} \widehat{s}_t + \eta_t^p$$

$$\widehat{s}_t = \alpha \widehat{r}_t^k + (1 - \alpha) \widehat{w}_t - \varepsilon_t^a - (1 - \alpha) \gamma t \quad (2)$$

Parameters  $\alpha$  and  $\beta$  are respectively the capital share and the household's psychological discount factor. The deviation of inflation  $\widehat{\pi}_t$  from the target inflation rate  $\bar{\pi}_t$  depends on past and expected future inflation deviations and on the current marginal cost, which itself is a function of the rental rate on capital  $\widehat{r}_t^k$ , the real wage  $\widehat{w}_t$  and the productivity process, that is composed of a deterministic trend in labour efficiency  $\gamma t$  and a stochastic component  $\varepsilon_t^a$ , which is assumed to follow a first-order autoregressive process:  $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$  where  $\eta_t^a$  is an iid-Normal productivity shock. Finally,  $\eta_t^p$  is an iid-Normal price mark-up shock.

When the degree of indexation to past inflation is zero ( $\gamma_p = 0$ ), this equation reverts to the standard purely forward-looking New Keynesian Phillips curve. In that case, all prices are indexed to the inflation objective so that announcements of changes in the inflation objective will be largely neutral even in the short run while the Phillips curve will be vertical in the long run. With  $\gamma_p > 0$ , the degree of indexation to lagged inflation determines how backward looking the inflation process is, i.e. how much structural persistence there is in the inflation process. The elasticity of inflation with respect to changes in the marginal cost depends mainly on the degree of price stickiness. When all prices are flexible ( $\xi_p = 0$ ) and the price mark-up shock is zero, this equation reduces to the normal condition that in a flexible price economy the real marginal cost is constant.

Equation (1) yields a direct link between the elasticity of inflation with respect to the marginal cost and the Calvo parameter. A weak reaction of inflation to the marginal cost implies a very high Calvo parameter. Shocks that affect the marginal cost will influence inflation only gradually as a consequence of the high price stickiness. However, the marginal cost is not directly observed and its definition is therefore open to discussion. It is clear from equation (1) that a smoother response of the marginal cost to shocks might result in a lower estimate of price stickiness. As variable capital utilisation mitigates the response of marginal cost to output fluctuations, it should help obtaining a low Calvo parameter. However, Smets and Wouters (2004) show that, empirically, this friction is not very important once one allows for the other frictions that smooth marginal costs such as nominal wage rigidities.<sup>3</sup>

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<sup>3</sup>In the version of the model estimated in this paper, variable capital utilisation still appears but the

The simple relation between the elasticity of inflation with respect to the marginal cost and the Calvo parameter, as appearing in equation (1), is only valid if all firms are producing at the same marginal cost. This will be the case if capital is mobile between firms at each point in time and all firms can hire labour at a given wage, determined in the aggregate labour market. In this case, the capital-labour ratio will also be equal across firms and a function of the aggregate relative price of the production factors:<sup>4</sup>

$$\frac{W_t L_{j,t}}{r_t^k K_{j,t}} = \frac{1 - \alpha}{\alpha} \forall j \in [0, 1]$$

The rest of the linearised DSGE model is summarised in the appendix. In sum, the model determines nine endogenous variables: inflation, the real wage, capital, the value of capital, investment, consumption, the short-term nominal interest rate, the rental rate on capital and hours worked. The stochastic behaviour of the system of linear rational expectations equations is driven by ten exogenous shock variables. Five shocks arise from technology and preference parameters: the total factor productivity shock, the investment-specific technology shock, the preference shock, the labour supply shock and the government spending shock. Those shocks are assumed to follow an autoregressive process of order one. Three shocks can be interpreted as “cost-push” shocks: the price mark-up shock, the wage mark-up shock and the equity premium shock. Those are assumed to follow a white-noise process. And, finally, there are two monetary policy shocks: a permanent inflation target shock and a temporary interest rate shock.

## 2.2 Findings in the baseline model

The linearised DSGE model is estimated for the euro area using seven key macro-economic time series: output, consumption, investment, employment, real wages, prices and a short-term interest rate. The data are described in section 6.1 of the appendix. The full information Bayesian estimation methodology used is extensively discussed in Smets and Wouters (2003). Table 1 reports the estimates of the main parameters governing the hybrid New Keynesian Phillips curve and compares these estimates with those obtained by Gali *et al.* (2001) which use single-equation GMM methods to estimate a 

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 adjustment cost has been given a looser prior than in Smets and Wouters (2003) (cf. appendix). This results in a much higher estimated adjustment cost. As a consequence the variable capacity utilisation plays virtually no role in the model presented in this paper. This is not necessarily a bad thing since allowing for a relatively insensitive marginal cost of changing the utilisation of capital substantially reduces the impact of introducing firm-specific capital.

<sup>4</sup>It is important to note that in the empirical exercise wages are observed (in contrast to the rental rate on capital) and as a result the response of wages to all types of shocks is restricted by the data. The smoother the reaction of wages to the different shocks the flatter the marginal cost curve and the lower will be the estimated price stickiness.

similar equation on the same euro area data set.<sup>5</sup>

Let us very briefly mention some results. First, the degree of indexation is rather limited, implying a coefficient on the lagged inflation rate of 0.15. Second, the degree of Calvo price stickiness is very large: each period 89 percent of the firms do not re-optimize their price setting. The average price contract is therefore lasting for more than 2 years. This is implausibly high, but those results are very similar to the ones reported by Gali *et al.* (2001). Our estimates generally fall in the range of estimates reported by Gali *et al.* (2001), if they assume constant returns to scale as we do in our model (Table 1).

**Table 1: Comparison of estimated Philipps-curve parameters with Gali, Gertler and Lopez-Salido (GGL, 2001)**

	SW	GGL (2001) (1)	GGL (2001) (2)
Structural parameters			
$\xi_p$	0.89 (0.01)	0.90 (0.01)	0.92 (0.03)
$\gamma_p$	0.18 (0.10)	0.02 (0.12)	0.33 (0.12)
$A$	9.1	10.0	12.5
Reduced-form parameters			
$\gamma_f$	0.84	0.87 (0.04)	0.68 (0.04)
$\gamma_b$	0.15	0.02 (0.12)	0.27 (0.07)
$\lambda$	0.013	0.018 (0.012)	0.006 (0.007)

Notes: The GGL (2001) estimates are those obtained under the assumption of constant returns to labour under two alternative specifications. Strictly speaking, the structural parameters are not directly comparable as GGL use the inclusion of rule-of-thumb price setters (rather than indexation) as a way of introducing lagged inflation.  $A$  stands for the average age of price contracts in numbers of quarters;  $\gamma_f$  is the implied reduced-form coefficient on expected future inflation;  $\gamma_b$  is the coefficient on lagged inflation and  $\lambda$  is the coefficient on the real marginal cost.

Moreover, we display in a companion paper (de Walque, Smets and Wouters (2005)) that the degree of price stickiness is one of the most costly frictions to remove in terms of

<sup>5</sup>As there are many models estimated throughout the paper and since the Monte Carlo Markov Chain sampling method used to derive the posterior distribution of the parameters is extremely demanding in computer-time for such large scale models, the MCMC sampling algorithm has only been run for some models (see Appendix 6.4). The parameters and standard errors reported in all Tables are the estimated modes and their corresponding standard error. The log data density displayed is actually the Laplace approximation. It is shown in appendix 6.4 that it is very close to the modified harmonic mean for the models for which the latter has been computed.

the empirical fit of the DSGE model. Indeed, reducing the Calvo probability of not re-optimising from the estimated 89 percent to a more reasonable 75 percent corresponding to contracts lasting since about 4 quarters reduces the log data density of the estimated model drastically (by about 75). This feature perfectly illustrates the puzzle we face. At the micro level, one observes that prices are re-optimised on average between every 6 month and one year, while at the macro level, inflation is shown to be very persistent. In the model with homogeneous production factors, inflation persistence requires large nominal price stickiness in contradiction with the micro observation.

### 3 Taylor versus Calvo-style price setting with mobile production factors

In this section, we compare the empirical performance of the Taylor price-setting model with the Calvo model discussed above, maintaining the assumption of mobile production factors. One unattractive feature of the Calvo price setting model is that some firms do not re-optimize their prices for a very long time.<sup>6</sup> As indicated by Wolman (2001), the resulting misalignments due to relative price distortions may be very large and this may have important welfare implications. The standard Taylor contracting model avoids this problem.<sup>7</sup> In this model firms set prices for a fixed number of periods and price setting is staggered over the duration of the contract, i.e. the number of firms adjusting their price is the same every period.<sup>8</sup> The explicit modelling of the different cohorts in the Taylor model also facilitates the introduction of firm-specific capital and labour as no aggregation across cohorts is required. It also has the advantage that the cohort-specific output and price levels are directly available, which is important for checking whether the dispersion of output and prices across price-setting cohorts is realistic.

In order to be able to compare the Taylor price-setting model with the Calvo model discussed above, we maintain the assumption of partial indexation to lagged inflation and the inflation objective. As discussed in Whelan (2004) and Coenen and Levin (2004), the staggered Taylor contracting model gives rise to the following linearised equations

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<sup>6</sup>See, however, Levy and Young (2003) for an exception. The 5-nickel price of a bottle of Coca-Cola has been fixed for a period of almost 80 years.

<sup>7</sup>Another alternative is the truncated Calvo model as analysed in Dotsey *et al.* (1999), Bakhshi *et al.* (2003) and Murchinson *et al.* (2004).

<sup>8</sup>See Coenen and Levin (2004) and Dixon and Kara (2005b) for a generalisation of the standard Taylor contracting model where different firms may set prices for different lengths of time.

for the newly set optimal price and the general price index :

$$\widehat{p}_t^* = \frac{1}{\sum_{i=0}^{n_p-1} \beta^i} \left[ \sum_{i=0}^{n_p-1} \beta^i (\widehat{s}_{t+i} + \widehat{p}_{t+i}) - \sum_{i=0}^{n_p-2} \left( (\gamma_p \widehat{\pi}_{t+i} + (1 - \gamma_p) \overline{\pi}_{t+i+1}) \sum_{q=i+1}^{n_p-1} \beta^q \right) \right] + d \varepsilon_t^p \quad (3)$$

$$\widehat{p}_t = \frac{1}{n_p} \sum_{i=0}^{n_p-1} \left( \widehat{p}_{t-i}^* + \sum_{q=0}^{i-1} (\gamma_p \widehat{\pi}_{t-1-q} + (1 - \gamma_p) \overline{\pi}_{t-q}) \right) + (1 - d) \varepsilon_t^p \quad (4)$$

where  $n_p$  is the duration of the contract,  $d$  is a binary parameter ( $d \in \{0, 1\}$ ) and  $\varepsilon_t^p = \rho_t^p \varepsilon_{t-1}^p + \eta_t^p$ , with  $\eta_t^p$  an i.i.d. shock. We experiment with two ways of introducing the price mark-up shocks in the Taylor contracting model. The first method ( $d = 1$ ), is fully analogous with the Calvo model. We assume a time-varying mark-up in the optimal price setting equation, which introduces a shock in the linearised price setting equation (3) as shown above. The second method ( $d = 0$ ) is somewhat more ad hoc. It consists of introducing a shock in the aggregate price equation (4).<sup>9</sup>

Similarly, we introduce Taylor contracting in the wage setting process. This leads to the following linearised equations for the newly set optimal wage and the average wage

$$\widehat{w}_t^* = \frac{1}{\sum_{i=0}^{n_w-1} \beta^i} \left[ \sum_{i=0}^{n_w-1} \beta^i \left( \sigma_l \widehat{l}_{i,t+i} + \frac{1}{1-h} (\widehat{c}_{t+i} - h \widehat{c}_{t+i-1}) - \varepsilon_{t+i}^l \right) + \sum_{i=1}^{n_w-1} \left( (\widehat{\pi}_{t+i} - \gamma_w \widehat{\pi}_{t+i-1} - (1 - \gamma_w) \overline{\pi}_{t+i}) \sum_{q=i}^{n_w-1} \beta^q \right) \right] + d \varepsilon_t^w \quad (5)$$

$$\widehat{w}_t = \frac{1}{n_w} \left[ \sum_{i=0}^{n_w-1} \widehat{w}_{i,t} + \widehat{p}_{t-i} \right] - \widehat{p}_t + (1 - d) \varepsilon_t^w \quad (6)$$

with

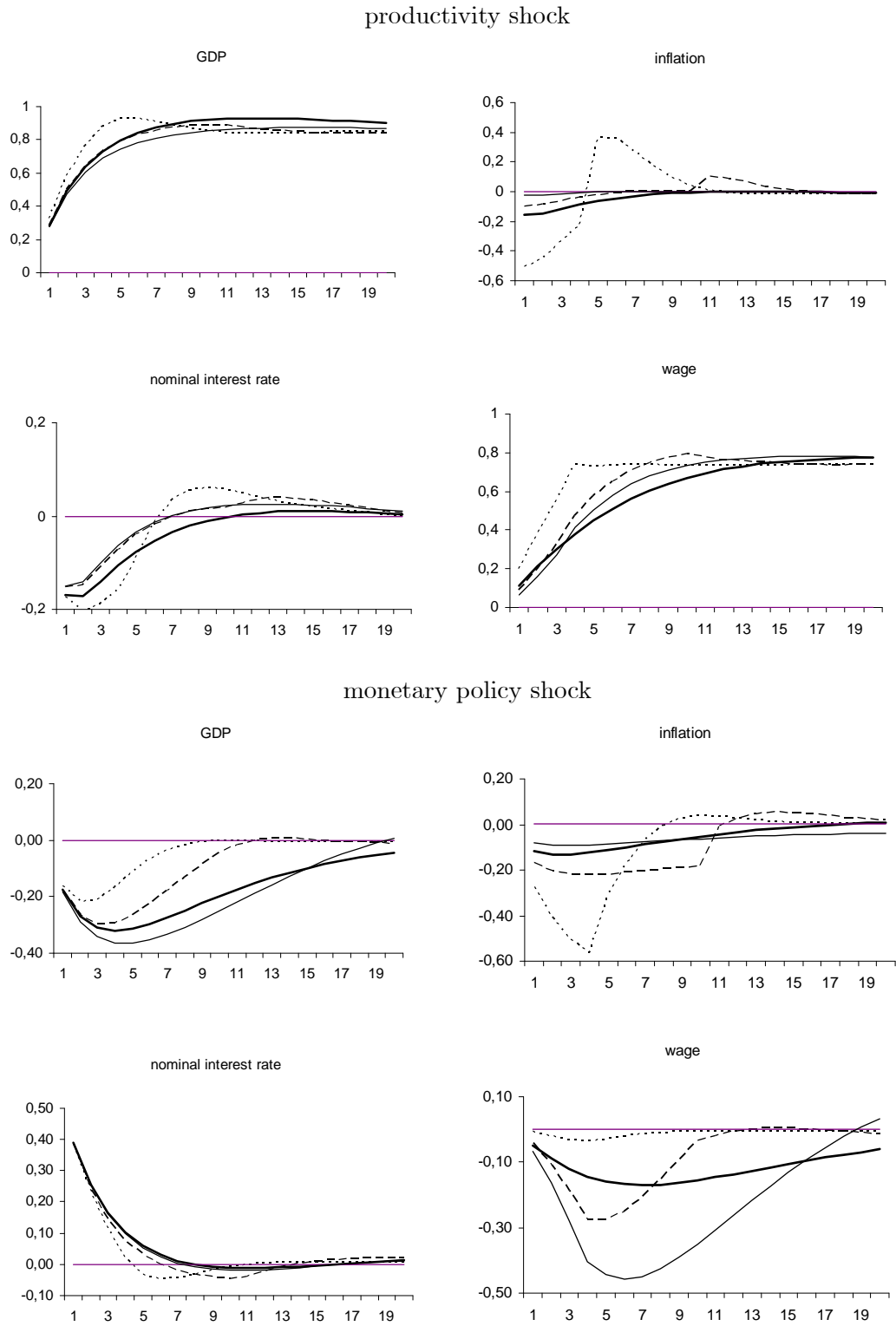
$$\widehat{w}_{i,t} = \widehat{w}_{t-i}^* + \sum_{q=0}^{i-1} (\gamma_w \widehat{\pi}_{t-1-q} + (1 - \gamma_w) \overline{\pi}_{t-q}) \quad (7)$$

$$\widehat{l}_{i,t+i} = \widehat{l}_{t+i} - \frac{1 + \lambda_w}{\lambda_w} [\widehat{w}_{i,t+i} + \widehat{p}_t - (\widehat{w}_{t+i} + \widehat{p}_{t+i})] \quad (8)$$

where  $n_w$  is the duration of the wage contract,  $\sigma_l$  represents the inverse elasticity of work effort with respect to real wage,  $\widehat{l}_t$  is the labour demand described in (A6) (cf. model appendix) and  $\widehat{l}_{i,t}$  is the demand for the labour supplied at nominal wage  $\widehat{w}_{i,t}$  by the households who re-optimised their wage  $i$  periods ago,  $h$  is the habit parameter,  $\widehat{c}_t$  is consumption,  $\varepsilon_t^l = \rho_t^l \varepsilon_{t-1}^l + \eta_t^l$  and  $\eta_t^l$  is an i.i.d. shock to the labour supply,  $\gamma_w$  is the

<sup>9</sup>This could be justified as a relative price shock to a flexible-price sector that is not explicitly modelled. Of course, such a shortcut ignores the general equilibrium implications (e.g. in terms of labour and capital reallocations).

**Figure 1: Selected impulse responses: Calvo versus Taylor contracts (baseline parameters)**



Legend: bold black line: baseline (Calvo) model; full line: 20-quarter Taylor price contract; dashed line: 10-quarter Taylor price contract; dotted line: 4-quarter Taylor price contract.

degree of indexation to the lagged wage growth rate,  $\varepsilon_t^w = \rho_t^w \varepsilon_{t-1}^w + \eta_t^w$  and  $\eta_t^w$  is an i.i.d. wage mark-up shock. Finally,  $\lambda_w$  is the wage mark-up. Note that as we did for price shocks, wage shocks have been introduced in two different ways.

Before discussing the estimation results, it is worth highlighting two issues. First, Dixon and Kara (2005a) have argued that a proper comparison of the degree of price stickiness in the Taylor and Calvo model should be based on the average age of the running contracts, rather than on the average frequency of price changes. As is well known, in a Calvo pricing model the average age of the running contracts is computed as

$$(1 - \xi_p) \sum_{i=0}^{\infty} \xi_p^i \cdot (i + 1) = \frac{1}{1 - \xi_p}$$

while the corresponding statistic for Taylor contracts is given by

$$\frac{1}{n_p} \sum_{i=1}^{n_p} i = \frac{n_p + 1}{2}$$

Thus, in order to produce the same average contract age as the one implied by a Calvo parameter  $\xi_p$ , the Taylor contract length needs to be  $\frac{1+\xi_p}{1-\xi_p}$  periods. The Calvo parameter  $\xi_p = 0.9$  estimated in section 2 above, therefore implies a long Taylor contract length of 19 quarters.

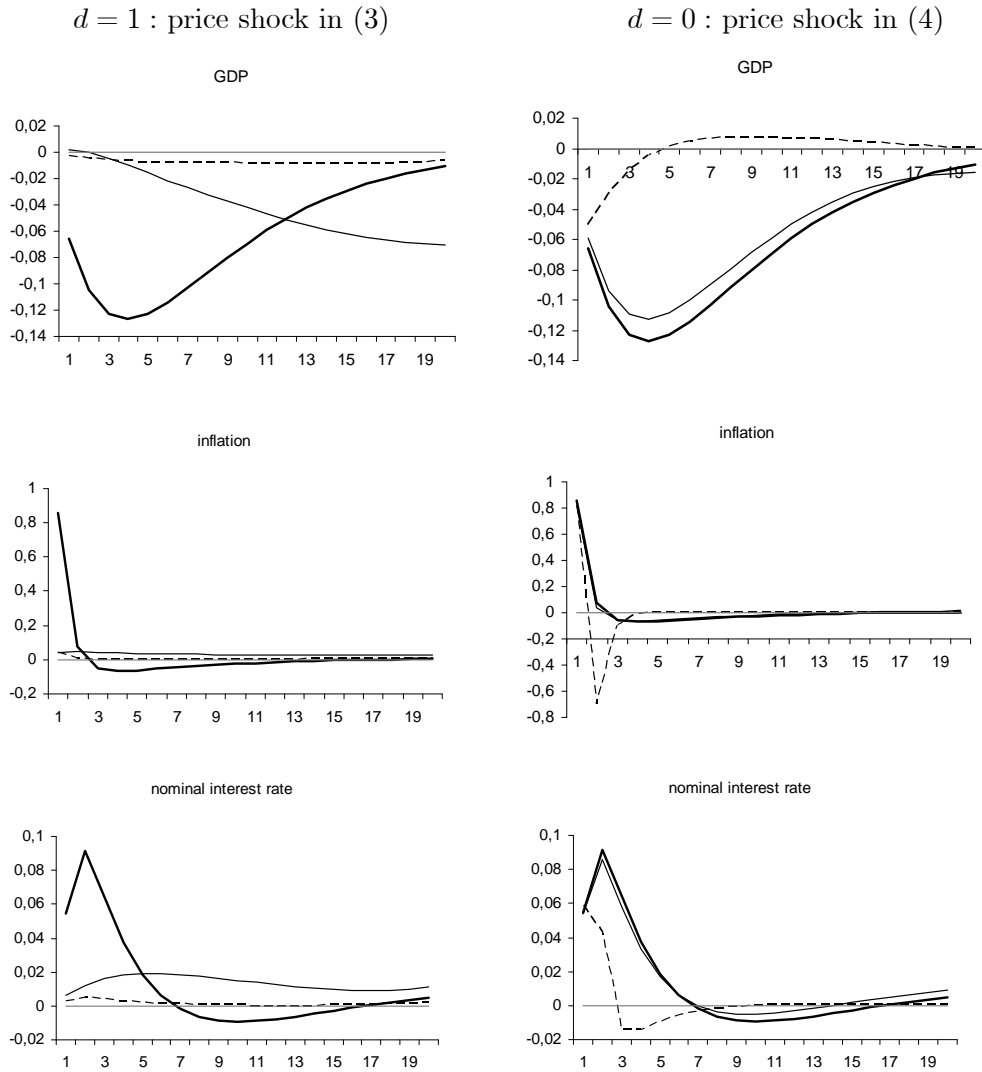
Figure 1 confirms the Dixon and Kara (2005a) analysis by comparing the impulse responses to respectively a productivity and a monetary policy shock in the baseline Calvo model and 4, 10 and 20-quarter Taylor-contracting, keeping the other parameters fixed at those estimated for the baseline Calvo model. In this figure the wage contract length  $n_w$  is fixed at four quarters. As the duration of the Taylor contract lengthens, the impulse responses appear to approach the outcome under the Calvo model. One needs a very long duration (about 20 quarters) in order to come close to the Calvo model. With shorter Taylor contracts typically the inflation response becomes larger in size, but also less persistent. Conversely, the output and real wage responses are closer to the flexible price outcome. For example, in response to a monetary policy shock the response of output is considerably smaller. Moreover, with shorter Taylor contracts the inflation response changes sign quite abruptly after the length of the contract. This feature is absent in the Calvo specification. As discussed in Whelan (2004), in reduced-form inflation equations the reversal of the inflation response after the contract length is captured by a negative coefficient on lagged inflation once current and expected future marginal costs are taken into account.

A second issue relates to the way in which the price shocks are introduced. As shown in Figure 2, the two ways of introducing price (resp. wage) shocks discussed above generate very different short run dynamics in response to such shocks. The right-hand column of Figure 2 shows that introducing a persistent shock in the GDP deflator



equation (i.e.  $d = 0$ ) allows the Taylor-contracting model to mimic most closely the response to a mark-up shock in the baseline Calvo specification.<sup>10</sup>

**Figure 2: Impulse response to a price shock in the 20-quarter Taylor model for different specifications of the price shock (baseline parameters)**



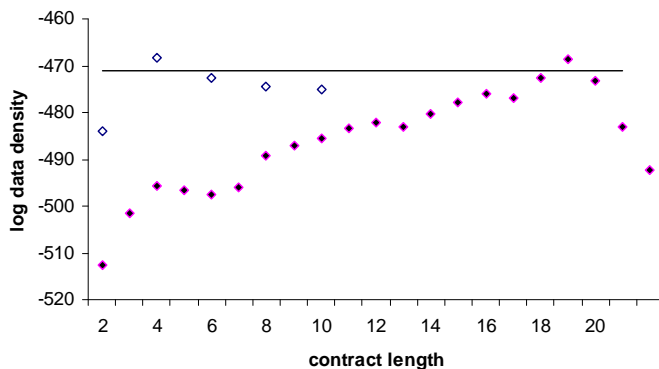
Legend: bold black line: baseline (Calvo) model; black line: 20-quarter Taylor contract with persistent price shock; dashed black line: 20-quarter Taylor contract with i.i.d. price shock.

<sup>10</sup>The same exercise could actually be run for a wage shock. Since it leads to similar conclusions we do not reproduce it here.

We now turn to the main estimation results. A number of results are worth highlighting. First, we confirm that, in line with the impulse responses shown in Figure 2, the specification with the persistent price shock in the GDP price equation ( $d=0$ ) does best in terms of empirical performance. For example, the log data density of the estimated model with 10-quarter Taylor contracts improves by 90 points relative to the specification with a persistent price shock in the optimal price setting equation. Similar improvements are found for other contract lengths. Moreover, the empirical performance also improves significantly by allowing for persistence in the price shocks.<sup>11</sup>

Second, as illustrated by Figure 3 which plots the log data density of the estimated Taylor model as a function of the contract length, the contract length that maximises the predictive performance of the Taylor model is 19 quarters. This again confirms the analysis of Dixon and Kara (2005a) discussed above. Table 2 compares some of the estimated parameters across various Taylor models and the Calvo model. While most of the other parameters are estimated to be very similar, it is noteworthy that the estimated degree of indexation rises quite significantly as the assumed Taylor contracts become shorter. Possibly, this reflects the need to overcome the negative dependence on past inflation in the standard Taylor contract.

**Figure 3: Log data density for Taylor contracting models with different lengths**



Legend: the black line represents the log data density in the baseline Calvo model; black diamonds are for the Taylor model with mobile production factors and a persistent price shock in the GDP price; white diamonds are for the same model but with non-mobile capital, zero profits and endogenous mark-up (see section 4.4 below).

<sup>11</sup>Similar findings have been found for various specifications of the wage shock. For that reason, we consider a persistent wage shock in the average wage equation for all the estimations performed in the rest of the paper.

**Table 2: Comparing the Calvo model with Taylor contracting models**

	Calvo	4-Q Tayl.	8-Q Tayl.	10-Q Tayl.	19-Q Tayl.
	Log data densities				
	-471.113	-495.566	-489.174	-485.483	-468.469
	Selection of estimated parameter outcomes				
$\rho_a$	0.991 (0.006)	0.980 (0.007)	0.982 (0.006)	0.962 (0.006)	0.983 (0.006)
$\sigma_a$	0.653 (0.093)	0.615 (0.068)	0.682 (0.085)	0.619 (0.076)	0.622 (0.085)
$\rho_p$	0 (-)	0.995 (0.004)	0.995 (0.004)	0.912 (0.016)	0.934 (0.018)
$\sigma_p$	0.207 (0.019)	0.406 (0.030)	0.323 (0.023)	0.277 (0.020)	0.229 (0.016)
$\rho_w$	0 (-)	0.973 (0.012)	0.966 (0.014)	0.881 (0.017)	0.955 (0.012)
$\sigma_w$	0.250 (0.021)	0.4386 (0.031)	0.453 (0.034)	0.461 (0.031)	0.454 (0.035)
$\gamma_w$	0.388 (0.197)	0.313 (0.166)	0.397 (0.205)	0.351 (0.206)	0.460 (0.188)
$\gamma_p$	0.178 (0.096)	0.859 (0.150)	0.463 (0.130)	0.436 (0.116)	0.273 (0.074)
$A$	9.1 Q	2.5 Q	4.5 Q	5.5 Q	10 Q

Note:  $\rho_a$ ,  $\rho_p$  and  $\rho_w$  are the persistency parameters associated to the productivity, the price and the wage shock respectively;  $\sigma_a$ ,  $\sigma_p$  and  $\sigma_w$  are the standard error of the productivity, the price and the wage shock respectively;  $\gamma_w$  and  $\gamma_p$  are respectively the wage and price indexation parameters;  $A$  is the average age of the price contract.

## 4 Firm-specific production factors and Taylor contracts

### 4.1 Modelling firm-specific factors

So far the model includes all kinds of adjustment costs such as those related to the accumulation of new capital, to changes in prices and wages and to changes in capacity utilisation, but shifting capital or labour from one firm to another is assumed to be costless (see Danthine and Donaldson, 2002). The latter assumption is clearly not fully realistic. In this section we instead assume that production factors are firm-specific, i.e. the cost of moving them across firms is extremely high. Although this is also an extreme

assumption, it may be more realistic. The objective is to investigate the implications of introducing this additional real rigidity on the estimated degree of nominal price stickiness and the overall empirical performance of the Taylor contracting model. As shown in Coenen and Levin (2004) for the Taylor model and Woodford (2003, 2005), Eichenbaum and Fisher (2004) and Altig *et al.* (2005) for the Calvo model, the introduction of firm-specific capital reduces the sensitivity of inflation with respect to its driving variables. Similarly, Woodford (2003, 2005) shows that firm-specific labour may also help reducing price variations and may lead to higher inflation persistence.

In the case of firm-specific factors, the key equations of the linearised model governing the decision of a firm belonging to the cohort  $j$  (with  $j \in [1, n_p]$ ) which re-optimises its price in period  $t$  are given by:

$$\hat{p}_t^*(j) = \frac{1}{\sum_{i=0}^{n_p-1} \beta^i} \left[ \sum_{i=0}^{n_p-1} \beta^i (\hat{s}_{t+i}(j) + \hat{p}_{t+i}) - \sum_{i=0}^{n_p-2} \left( (\gamma_p \hat{\pi}_{t+i} + (1 - \gamma_p) \bar{\pi}_{t+i+1}) \sum_{q=i+1}^{n_p-1} \beta^q \right) \right] \quad (3b)$$

$$\hat{p}_t = \frac{1}{n_p} \sum_{i=0}^{n_p-1} \hat{p}_t(j-i) + \varepsilon_t^p \quad (4b)$$

$$\hat{s}_{t+i}(j) = \alpha \hat{\rho}_{t+i}(j) + (1 - \alpha) \hat{w}_{t+i}(j) - \hat{\varepsilon}_{t+i}^a - (1 - \alpha) \gamma t \quad (9)$$

$$\hat{Y}_{t+i}(j) = \hat{Y}_{t+i} - \frac{1 + \lambda_p}{\lambda_p} (\hat{p}_{t+i}(j) - \hat{p}_{t+i}) \quad (10)$$

$$\hat{p}_{t+i}(j) = \hat{p}_t^*(j) + \sum_{q=0}^{i-1} (\gamma_p \hat{\pi}_{t-1-q} + (1 - \gamma_p) \bar{\pi}_{t-q}) \quad (11)$$

$$\text{with } \frac{\partial \hat{\rho}_{t+i}(j)}{\partial \hat{Y}_{t+i}(j)} > 0 \text{ and } \frac{\partial \hat{w}_{t+i}(j)}{\partial \hat{Y}_{t+i}(j)} > 0 \quad (12)$$

where  $\hat{\rho}_t(j)$  is the "shadow rental rate of capital services",<sup>12</sup> and  $\lambda_p$  is the price mark-up so that  $\frac{1+\lambda_p}{\lambda_p}$  is the elasticity of substitution between goods. The main difference with equations (3) and (4) is that the introduction of firm-specific factors implies that firms no longer share the same marginal cost. Instead, a firm's marginal cost and its optimal price will depend on the demand for its output. A higher demand for its output implies that the firm will have a higher demand for the firm-specific input factors, which in turn will lead to a rise in the firm-specific wage costs and capital rental rate. Because this demand will be affected by the pricing behavior of the firm's competitors, the optimal price will also depend on the pricing decisions of the competitors.

<sup>12</sup>Indeed, we left aside the assumption of a rental market for capital services. Each firm builds its own capital stock. The "shadow rental rate" of capital services is the rental rate of capital services such that the firm would hire the same quantity of capital services in an economy with a market for capital services as it does in the economy with firm-specific capital.

The net effect of this interaction will be to dampen the price effects of various shocks. Consider, for example, an unexpected demand expansion. Compared to the case of homogenous marginal costs across firms, the first price mover will increase its price by less because everything else equal the associated fall in the relative demand for its goods leads to a fall in its relative marginal cost. This, in turn, reduces the incentive to raise prices. This relative marginal cost effect is absent when factors are mobile across firms and, as a result, firms face the same marginal cost irrespective of their output levels. From this example it is clear that the extent to which variations in firm-specific marginal costs will reduce the amplitude of price variations will depend on the combination of two elasticities: *i*) the elasticity of substitution between the goods produced by the firm and those produced by its competitors, which will govern how sensitive relative demand for a firm's goods is to changes in its relative price (see equation (4b)); *ii*) the elasticity of the individual firms' marginal cost with respect to changes in the demand for its products (see equation (4b)). With a Cobb-Douglas production function, the latter elasticity will mainly depend on the elasticity of the supply of the factors with respect to changes in the factor prices. In brief, the combination of a steep firm-specific marginal cost curve and high demand elasticity will maximise the relative marginal cost effect and minimise the price effects, thereby reducing the need for a high estimated degree of nominal price stickiness.

Before turning to a quantitative analysis of these effects in the next sections, it is worth examining in somewhat more detail the determinants of the partial derivatives in equation (12) in each of the two factor markets (capital and labour). Consider first firm-specific capital. Given the one-period time-to-build assumption in capital accumulation, the firm-specific capital stock is given within the quarter. As a result, when the demand faced by the firm increases, production can only be adjusted by either increasing the labour/capital ratio or by increasing the rate of capital utilisation. Both actions will tend to increase the cost of capital services. It is, however, also clear that when the firm can increase the utilisation of capital at a constant marginal cost, the effect of an increase in demand on the cost of capital will be zero. In this case, the supply of capital services is infinitely elastic at a rental price that equals the marginal cost of changing capital utilisation and, as a result, the first elasticity in equation (12) will be zero. In the estimations reported below, the marginal cost of changing capital utilisation is indeed high, so that in effect there is nearly no possibility to change capital utilisation. Over time, the firm can adjust its capital stock subject to adjustment costs. This implies that the firm's marginal cost depends on its capital stock, which itself depends on previous pricing and investment decisions of the firm. As a result, also the capital stock, the value of capital and investment will be firm-specific. In the case of a Calvo model, Woodford (2005) and Christiano (2004) show how the linearised model can still be solved in terms of aggregate variables, without solving for the whole distribution of the capital stock

over the different firms. This linearisation is however complicated and remains model specific. With staggered Taylor contracts, it is straightforward to model the cohorts of firms characterised by the same price separately. The key linearised equations governing the investment decision for a firm belonging to the  $j$ th cohort are then:

$$\widehat{K}_t(j) = (1 - \tau)\widehat{K}_{t-1}(j) + \tau\widehat{I}_{t-1}(j) + \tau\varepsilon_{t-1}^I \quad (13)$$

$$\widehat{I}_t(j) = \frac{1}{1 + \beta}\widehat{I}_{t-1}(j) + \frac{\beta}{1 + \beta}E_t\widehat{I}_{t+1}(j) + \frac{1/\varphi}{1 + \beta}\widehat{Q}_t(j) + \varepsilon_t^I \quad (14)$$

$$\widehat{Q}_t(j) = -(\widehat{R}_t - \widehat{\pi}_{t+1}) + \frac{1 - \tau}{1 - \tau + \bar{\rho}}E_t\widehat{Q}_{t+1}(j) + \frac{\bar{\rho}}{1 - \tau + \bar{\rho}}E_t\widehat{\rho}_{t+1}(j) + \eta_t^Q \quad (15)$$

where  $\widehat{K}_t(j)$ ,  $\widehat{I}_t(j)$  and  $\widehat{Q}_t(j)$  are respectively the capital stock, investment and the Tobin's Q for each of the firms belonging to the  $j$ th price setting cohort. Parameter  $\tau$  is the depreciation rate of capital and  $\bar{\rho}$  is the shadow rental rate of capital discussed above, so that  $\beta = 1/(1 - \tau + \bar{\rho})$ . Parameter  $\varphi$  depends on the investment adjustment cost function.<sup>13</sup>

Consider next firm-specific monopolistic competitive labour markets. In this case each firm requires a specific type of labour which can not be used in other firms. Moreover, within each firm-specific labour market, we allow for Taylor-type staggered wage setting. The following linearised equations display how a worker belonging to the  $f$ th wage setting cohort (with  $f \in [1, n_w]$ ) optimises its wage in period  $t$  for the labour it rents to the firms of the  $j$ th price setting cohort (with  $j \in [1, n_p]$ ):

$$\begin{aligned} \widehat{w}_t^*(f, j) = & \frac{1}{\sum_{i=0}^{n_w-1} \beta^i} \left[ \sum_{i=0}^{n_w-1} \beta^i \left( \sigma_l \widehat{l}_{t+i}(f, j) + \frac{1}{1-h} (\widehat{c}_{t+i} - h\widehat{c}_{t+i-1}) - \varepsilon_{t+i}^l \right) \right. \\ & \left. + \sum_{i=1}^{n_w-1} \left( (\widehat{\pi}_{t+i} - \gamma_w \widehat{\pi}_{t+i-1} - (1 - \gamma_w) \bar{\pi}_{t+i}) \sum_{q=i}^{n_w-1} \beta^q \right) \right] \end{aligned}$$

$$\widehat{w}_t(j) = \frac{1}{n_w} \left[ \sum_{i=0}^{n_w-1} \widehat{w}_t(f - i, j) + \widehat{p}_{t-i} \right] - \widehat{p}_t + \varepsilon_t^w \quad (6b)$$

$$\widehat{w}_{t+i}(f, j) = \widehat{w}_t^*(f, j) + \sum_{q=0}^{i-1} (\gamma_w \widehat{\pi}_{t-1-q} + (1 - \gamma_w) \bar{\pi}_{t-q}) \quad (7b)$$

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<sup>13</sup>As in the baseline model, there are two aggregate investment shocks:  $\varepsilon_t^I$  which is an investment technology shock and  $\eta_t^Q$  which is meant to capture stochastic variations in the external finance premium. The first one is assumed to follow an AR(1) process with an iid-Normal error term and the second is assumed to be iid-Normal distributed.

$$\widehat{l}_{t+i}(f, j) = \widehat{l}_{t+i}(j) - \frac{1 + \lambda_w}{\lambda_w} (\widehat{w}_{t+i}(f, j) + \widehat{p}_t - (\widehat{w}_{t+i}(j) + \widehat{p}_{t+i})) \quad (8b)$$

$$\widehat{l}_t(j) = -\widehat{w}_t(j) + (1 + \psi)\widehat{p}_t(j) + \widehat{K}_{t-1}(j) \quad (A6b)$$

It directly appears from these equations that there is now a labour market for each cohort of firms. Contrarily to the homogeneous labour setting, the labour demand of (cohort of) firm(s)  $j$  (equation A6b) directly affects the optimal wage chosen by the worker  $f$  (equation 5b) and consequently the cohort specific average wage (6b). When  $\gamma_w = 0$ , real wages do not depend on the lagged inflation rate.<sup>14</sup>

Due to the staggered wage setting it is not so simple to see how changes in the demand for the firm's output will affect the firm-specific wage cost (equation (12)). A number of intuitive statements can, however, be made. First, higher wage stickiness as captured by the length of the typical wage contract will tend to reduce the response of wages to demand. As a result, high wage stickiness is likely to reduce the impact of firm-specific labour markets on the estimated degree of nominal price stickiness. In contrast, with flexible wages, the relative wage effect may be quite substantial, contributing to large changes in relative marginal cost of the firm and thereby dampening the relative price effects discussed above. Second, this effect is likely to be larger the higher the demand elasticity of labour (as captured by a lower labour market mark-up parameter) and the higher the elasticity of labour supply. Concerning the latter, if labour supply is infinitely elastic, wages will again tend to be very sticky and as a result relative wage costs will not respond very much to changes in relative demand even in the case of firm-specific labour markets.

## 4.2 Alternative models

In this section we illustrate the discussion above by displaying how the output, the marginal cost and the price of the first price-setting cohort respond to a monetary policy shock. We compare the benchmark model with mobile production factors (hereafter denoted MKL) with the following three models:

- a model with homogeneous capital and firm-specific labour market (hereafter denoted NML)
- a model with firm-specific capital, homogeneous labour (hereafter denoted NMK)
- a model with firm-specific capital and labour (hereafter denoted NMKL)

Moreover, for each of those models we consider four cases corresponding to flexible and sticky wages and low (0.01) and high (0.05) mark-ups in the goods market.<sup>15</sup> Figure

<sup>14</sup>Parameter  $\psi$  is the inverse of the elasticity of the capital utilisation cost function.

<sup>15</sup>This corresponds to demand elasticities of 21 and 101 respectively. The latter is the one estimated by Altig *et al.* (2005). Furthermore, one needs rather high substitution elasticities to observe significant

4 shows the responses of the cohort that is allowed to change its price in the period of the monetary policy shock. In this Figure we assume that the length of the price and wage contracts is 4 quarters. The rest of the parameters are those estimated for the benchmark Taylor model (MKL) with the corresponding contract length. Responses are displayed for the first 10 quarters following the shock, i.e. prices are re-optimised three times by the considered cohort in the time span considered, at periods 1, 5 and 9.

Several points are worth noting. First, introducing firm-specific factors always reduces the initial impact on prices and output, while it increases the impact on the marginal cost. As discussed above, with firm-specific production factors, price-setting firms internalise that large price responses lead to large variations in marginal costs and therefore lower their initial price response. Second, the introduction of firm-specific factors increases the persistence of price changes in particular when wages are flexible. While in the case of mobile production factors with flexible wages, the initial price decrease is partially reversed after four quarters, prices continue to decrease five and nine quarters after the initial shock when factors are firm-specific. Third, in the case with mobile factors, (MKL - bold black curve), it is clear that prices and marginal cost are not affected by changes in the demand elasticity, while the firm's output is very much affected. On the contrary, for all the models with at least one non-mobile production factor, price responses decrease while marginal cost variations increase with a higher demand elasticity.

Finally, as long as wages are considered to be flexible, firm-specific labour market is the device that leads to the largest reactions in marginal cost. It is also worth to remark that the combination of firm-specific labour market and firm-specific capital brings more reaction in the marginal cost than the respective effect of each assumption separately.<sup>16</sup> However, as soon as wages become sticky, firm-specific labour markets do not generate much more variability in marginal cost. In this case, it is striking that the responses of the NMK and NMKL models gets very close to each other.

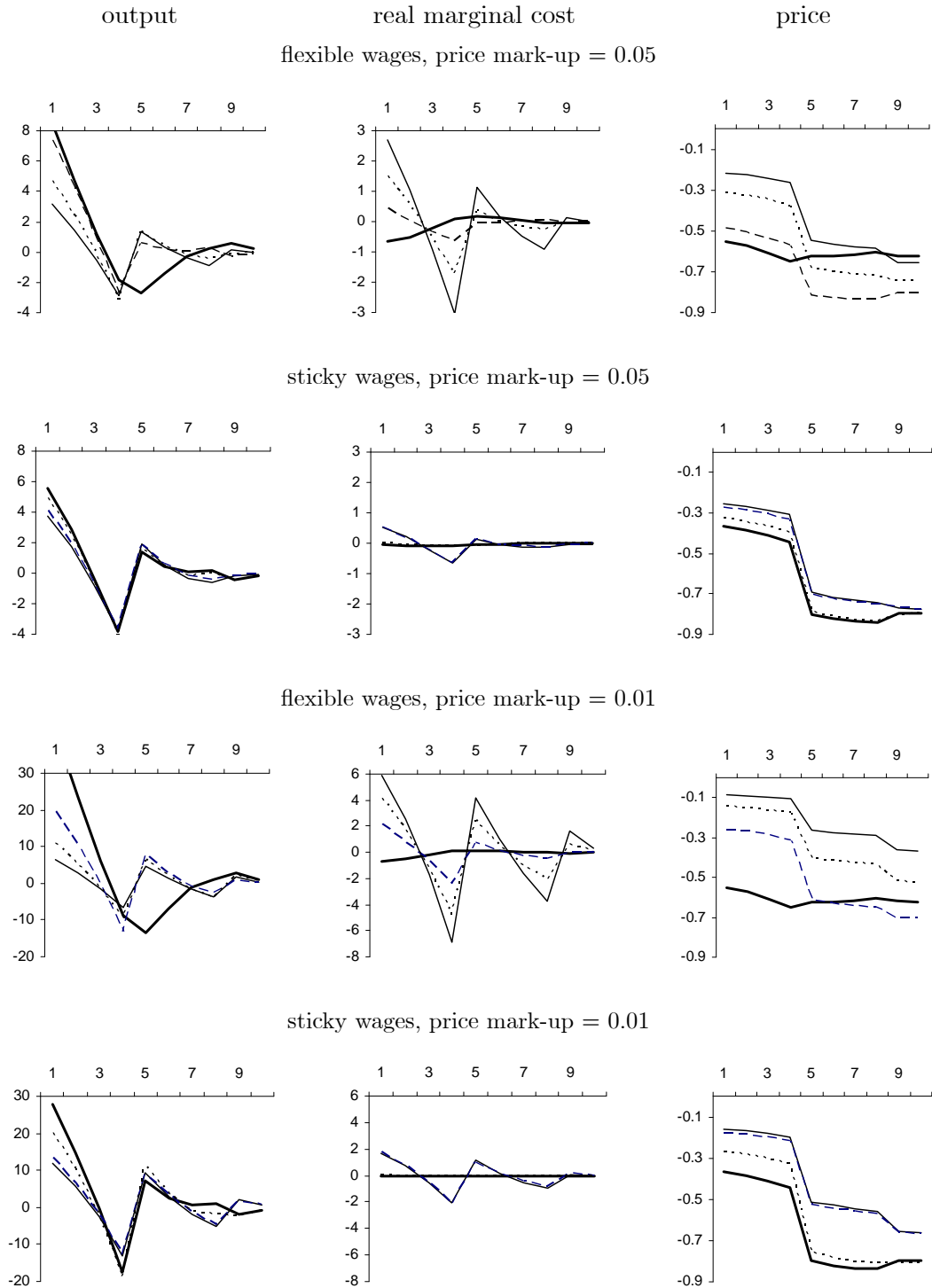
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difference between the homogeneous marginal cost model and its firm-specific production factors counterparts. So, for demand elasticities below 10, there is nearly no difference between the the MKL model and the NML, NMK and NMKL ones. This indicates again the importance of a very elastic demand curve.

<sup>16</sup>This is actually much in line with the findings of Matheron (2005) in a Calvo price-flexible wage setting with firm-specific capital and labour.



**Figure 4: The effect of a monetary policy shock on output, marginal cost and price of the first cohort in the 3 considered models**



Note: bold black curve: MKL; full curve: NMKL; dashed curve: NMK; dotted curve: NML

### 4.3 Estimation

In this section we re-estimate the model with firm-specific production factors to investigate the effects on the empirical performance of the model. Sbordone (2002) and Gali *et al.* (2001) show that considering capital as a fixed factor which cannot be moved across firms does indeed reduce the estimated degree of nominal price stickiness in US data. In particular, it reduces the implied duration of nominal contracts from an implausibly high number of more than 2 years to a duration of typically less than a year. Altig *et al.* (2005) reach the same conclusion in a richer setup where firms endogenously determine their capital stock. In this section, we extend this analysis to the case of firm-specific labour markets and test whether similar results are obtained in the context of Taylor contracts.

Table 3 reports the log data densities of the three models considered above and their flexible/sticky wages variants for various price contract lengths. A higher log data density implies a better empirical fit in terms of the model's one-step ahead prediction performance.

Table 3: Log data densities for the three models considered and their variants

	2-Q Taylor	4-Q Taylor	6-Q Taylor	8-Q Taylor
	flexible wages			
NML	-520.21	-481.86	-492.87	-490.16
NMKL	-484.92	-479.56	-481.87	-485.23
NMK	-486.50	-480.68	-482.16	-481.97
	sticky wages (4-quarter Taylor contract)			
NML	-512.50	-490.19	-484.72	-480.54
NMKL	-484.46	-466.10	-475.80	-477.23
NMK	-479.11	-464.92	-473.17	-474.30

The following findings are noteworthy. First, in almost all cases, the data prefer the sticky wage over the flexible wage version. This is not surprising as sticky wages are better able to capture the empirical persistence in wage developments. In what follows, we therefore focus on the sticky wage models. Second, with sticky wages the data prefer the model with firm-specific capital, but mobile labour. The introduction of firm-specific labour markets does not help the empirical fit of the model. The main reason for this result is that, as argued before, in order for firm-specific labour markets to help in explaining price and inflation persistence one needs a strong response of wages to changes in demand. But this is in contrast to the observed persistence in wage

developments. On the other hand, as we do not observe the rental rate of capital, no such empirical constraint is relevant for the introduction of firm-specific capital. Finally, introducing firm-specific capital does indeed reduce the contract length that fits the data best. While the log data density is maximised at a contract length of 19 quarters in the case of homogeneous production factors, it is maximised at only four quarters when capital cannot move across firms. This is clearly displayed on Figure 3 (even though for a variant model with endogenous mark-up developed in section 4.4 below). As clarified by Engin and Kara (2005a), this is equivalent in terms of price duration to a Calvo probability of not re-optimising equal to 0.6. This confirms the findings of Gali *et al.* (2001) and Altig *et al.* (2005). Moreover, it turns out that the four-quarter Taylor contracting model with firm-specific capital performs as good as the 19-quarter Taylor contract model with mobile capital and slightly better than the baseline Calvo model.

**Table 4: A selection of estimated parameters for the Taylor contracts models with firm-specific capital (NMK)**

	2-Q Taylor	4-Q Taylor	6-Q Taylor	8-Q Taylor
$\sigma_p$	0.216 (0.016)	0.225 (0.016)	0.232 (0.019)	0.230 (0.017)
$\rho_p$	0.997 (0.002)	0.979 (0.029)	0.863 (0.124)	0.802 (0.085)
$1 + \phi$	1.616 (0.093)	1.515 (0.138)	1.522 (0.111)	1.520 (0.100)
$\lambda_p$	0.0008 (0.0003)	0.004 (0.0015)	0.008 (0.003)	0.016 (0.006)
$\gamma_p$	0.067 (0.070)	0.093 (0.077)	0.149 (0.094)	0.220 (0.102)
$\gamma_w$	0.403 (0.195)	0.463 (0.210)	0.547 (0.232)	0.436 (0.231)

Note:  $\rho_p$  is the persistency parameter associated to the price shock;  $\sigma_p$  is the standard error of the price shock;  $\gamma_w$  and  $\gamma_p$  are respectively the wage and price indexation parameters;  $\phi$  is the share of the fixed cost;  $\lambda_p$  is the price mark-up.

In line with these results, in the rest of the paper we will focus on the model with firm-specific capital, homogeneous labour and sticky wages. Table 4 presents a selection of the parameters estimated for this model with various contract lengths. Note that, in comparison to the case with homogeneous production factors, we also estimate the elasticity of substitution between the goods of the various cohorts. A number of findings

are worth noting. First, allowing for firm-specific capital leads to a drop in the estimated degree of indexation to past inflation in the goods sector. In comparison with results displayed in Table 2, in this case the parameter drops back to the low level estimated for the Calvo model and does not appear to be significantly different from zero. Second, as discussed in Coenen and Levin (2004), one advantage of the Taylor price setting is that the price mark-up parameter is identified and therefore can be estimated. In contrast, with Calvo price setting, the model with firm-specific factors is observationally equivalent to its counterpart with homogeneous production factors. Table 4 shows that one needs a very high elasticity of substitution (or a low mark-up) to match the Calvo model in terms of empirical performance. It is also interesting to note that the estimated price mark-up increases with the length of the price contract, showing the substitutability between nominal and real rigidities. Finally, the persistence parameter of the price shock significantly decreases with the length of the price contract.

For the 4-quarter price contract model, the estimated parameter for the price mark-up is 0.004, which implies an extremely high elasticity of substitution of about 250. This clearly indicates that one needs large real rigidities in order to compensate for the reduction in price stickiness. However, this implies that the estimated fixed cost in production ( $1 + \phi$  stands at 1.515) very much exceeds the profit margin, implying negative profits in steady state.

In order to circumvent this problem, one may simply impose the zero profit condition in steady state. The estimation result obtained for the 4-quarter price contract model is displayed in the first column of Table 5. The empirical cost of imposing the constraint is rather high, about 15 in log data density. Furthermore, the estimated demand elasticity remains very high at about 167. Note also that the constraint leads to a much larger estimation of the standard error of the productivity shock.

#### 4.4 Endogenous price mark-up

Following Eichenbaum and Fisher (2004), and Coenen and Levin (2004), we can consider a model with an endogenous mark-up, whereby the optimal mark-up is a function of the relative price as in Kimball (1995). Replacing the Dixit-Stiglitz aggregator by the homogeneous-degree-one aggregator considered by Kimball (1995), the linearised optimal price equation (3b) becomes

$$\hat{p}_t^*(j) = \frac{1}{\sum_{i=0}^{n_p-1} \beta^i} \left[ \frac{1}{1 + \lambda_p \cdot \epsilon} \sum_{i=0}^{n_p-1} \beta^i \hat{s}_{t+i}(j) + \sum_{i=0}^{n_p-1} \beta^i \hat{p}_{t+i} - \sum_{i=0}^{n_p-2} \left( (\gamma_p \hat{\pi}_{t+i} + (1 - \gamma_p) \bar{\pi}_{t+i+1}) \sum_{q=i+1}^{n_p-1} \beta^q \right) \right] \quad (16)$$

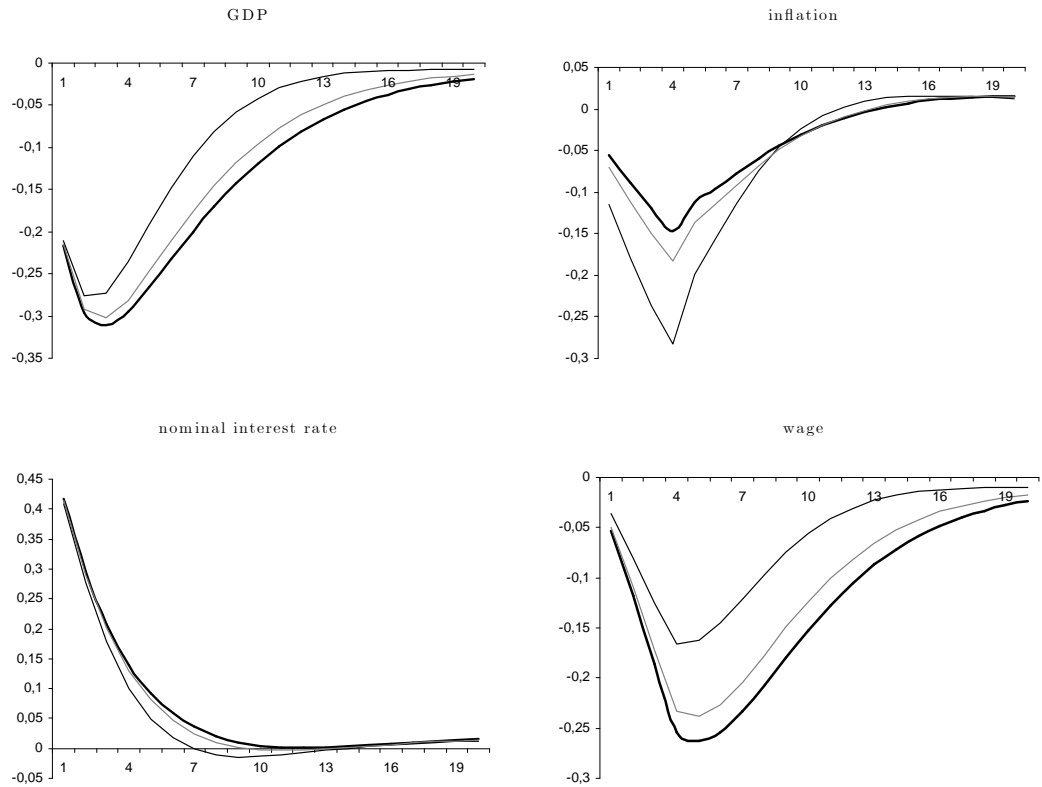
where  $\epsilon$  represents the deviation from the steady state demand elasticity following a change in the relative price, while  $\lambda_p$  is the steady state mark-up:<sup>17</sup>

$$\epsilon = \frac{\partial \left( \frac{1+\lambda_p(z)}{\lambda_p(z)} \right)}{\partial p^*} \cdot \frac{p^*}{\frac{1+\lambda_p(z)}{\lambda_p(z)}} \Bigg|_{z=1} \quad (17)$$

This elasticity plays the same role as the elasticity of substitution: the larger it is, the less the optimal price is sensitive to changes in the marginal cost. In this sense, having  $\epsilon > 0$  can help to reduce the estimate for the demand elasticity to a more realistic level.

**Figure 5: Assessing the substitutability between the steady state demand elasticity and the curvature parameter**

monetary policy shock



Legend: bold black line: estimated 4 quarter NMK model with fixed mark-up;  
black line:  $\lambda_p = 0.5$  and  $\epsilon = 20$  ; gray line:  $\lambda_p = 0.5$  and  $\epsilon = 60$ .

<sup>17</sup>Of course, the Dixit-Stiglitz aggregator corresponds to the case where  $\epsilon$  is equal to zero.

In order to illustrate this mechanism, Figure 5 displays the reactions of global output, inflation, wage and interest rate after a monetary policy shock for a model with an endogenous price mark-up. As benchmark, we use the 4-quarter price contract model with constant price mark-up estimated in Table 4 and we compare it with the model integrating both the zero-profit constraint and the endogenous price mark-up. For the latter model, we use the parameters estimated for the benchmark, except for the steady state mark-up,  $\lambda_p$ , which is fixed at 0.5, while different values are used for the curvature parameter  $\epsilon$ : 20 and 60. It is clear from Figure 5 that an endogenous price mark-up which is very sensitive to the relative price can produce the same effect on aggregate variables as a very small constant price mark-up.

**Table 5: Estimated models with constrained and/or endogenous demand elasticity (some selected parameters)**

	$\phi = \lambda_p$ and $\epsilon = 0$	$\phi = \lambda_p$ and $\epsilon \neq 0$
log data density	-479.671	-468.344
$\sigma_p$	0.208 (0.015)	0.178 (0.013)
$\rho_p$	0.829 (0.086)	0.539 (0.056)
$\sigma_a$	1.099 (0.153)	0.650 (0.088)
$\rho_a$	0.960 (0.011)	0.981 (0.007)
$\lambda_p = \phi$	0.006 (0.001)	0.489 (0.128)
$\frac{\epsilon}{1+\epsilon}$	0 -	0.986 (0.004)

Note:  $\rho_a$  and  $\rho_p$  are the persistency parameter associated to the productivity and the price shock respectively;  $\sigma_a$  and  $\sigma_p$  are the standard error of the productivity and the price shock respectively;  $\gamma_w$  and  $\gamma_p$  are respectively the wage and price indexation parameters;  $\phi$  is the share of the fixed cost;  $\lambda_p$  and  $\lambda_w$  are respectively the price and the wage mark-up;  $\epsilon$  is the curvature parameter.

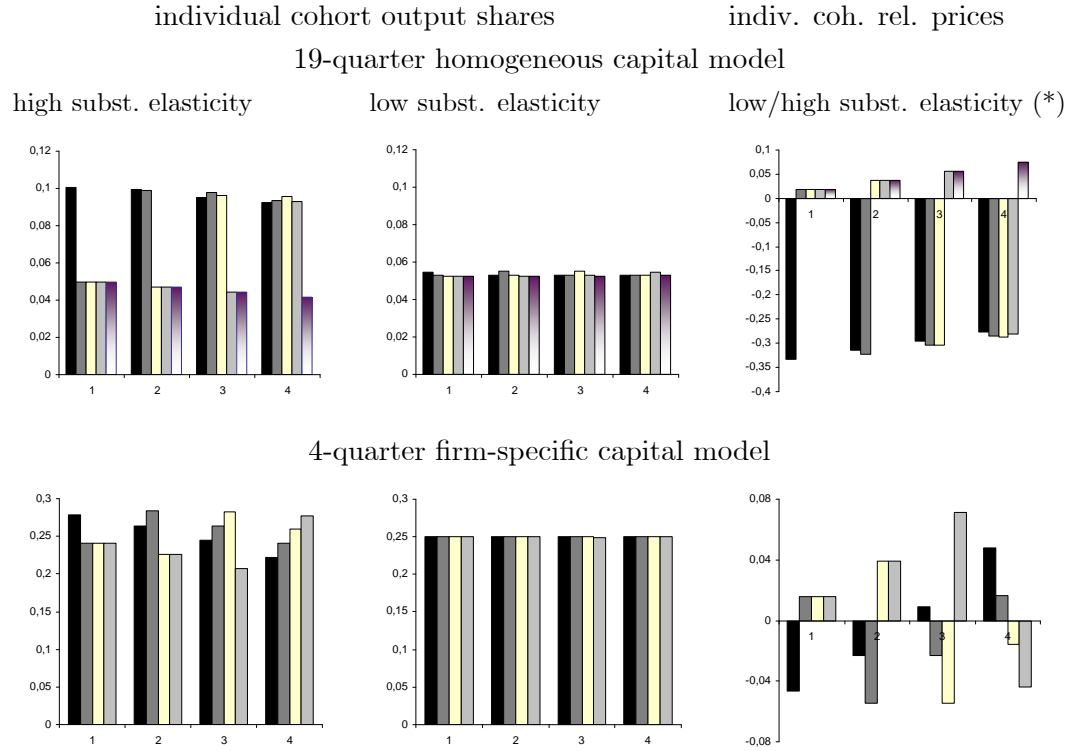
The next step is to re-estimate the NMK model with 4-quarter price and wage Taylor contracts but adding the modifications discussed above, i.e. imposing the price mark-up to equate the share of the fixed cost ( $\phi = \lambda_p$ ) and allowing  $\epsilon$  to be different from zero. The results are displayed in column 2 of Table 5. When the share of the fixed cost is forced to equate the mark-up, shifting from a final good production function with a constant price mark-up to one with a price mark-up declining in the relative price, the estimated steady state price mark-up becomes much larger, implying a demand elasticity of about 3. This helps to reduce the cost of the constraint and the log data density is improved by 11. The very high estimated curvature parameter  $\epsilon$  (about 70) reveals the need for real rigidities.

#### 4.5 Comparing models

Based on the log data density of the estimated models, we are not able to discriminate between the model with homogeneous capital and very long price contracts and the model with firm-specific capital, endogenous mark-up and short price contracts. We are then somewhat in the same position as Altig *et al.* (2005) who have to compare two models that are observationally equivalent from a macroeconomic point of view. These authors reject the model with homogeneous capital for two reasons. First, it implies a price stickiness not in line with micro evidence and second, it generates too high volatility in cohort-specific output shares. In this section we compare the various models in terms of their implied behaviour of cohort-specific output shares and relative prices. The latter allows us to confront the models also with the micro evidence on firms price setting which finds that price changes are typically large.

Figure 6 compares the evolution of the output share (as a percentage deviation from the steady-state) of the first four cohorts of firms during the first four periods following a monetary policy shock and the corresponding relative price changes. We run this comparison for four models: *(i)* the 4-quarter Taylor contract model with non-mobile capital and a high elasticity of substitution (Table 4 column 2), *(ii)* its variant with constrained elasticity of substitution and endogenous mark-up (Table 5 column 2), *(iii)* the 19-quarter Taylor contract model with mobile capital and an elasticity of substitution equal to 250 and *(iv)* the same model with a substitution elasticity of 3.

**Figure 6: output shares and relative prices for the first 4 periods after a monetary policy shock in homogeneous and firm-specific capital models**



Legend: column from left to right are for cohort 1 to cohort 4. Column 5 is for the fifteen cohorts that have not yet had the opportunity to re-optimize their price.

(\*) see footnote 18

First, focusing on the evolution of the relative prices in these models, we observe that relative prices vary much more across cohorts in the homogeneous factor model than in the model with firm-specific capital.<sup>18</sup> There are two reasons for such a higher volatility: the fact that the marginal cost is independent of firm-specific output and the length of the price contract which implies that only a small fraction of firms can actually change its price. The corollary of this high relative price variability is a much larger variability in the market shares of firms in the model with homogeneous capital and a high substitution elasticity. In that case, the first cohort to reset optimally its price nearly doubles its share in production. Even though this result is less extreme than the one presented in Altig *et al.* (2005),<sup>19</sup> such a high variability in output shares following a

<sup>18</sup>Note that the relative prices are displayed only for the model with firm-specific capital and endogenous mark-up and for the model with mobile capital. Indeed, in the case of mobile capital, the relative prices are not influenced by the substitution elasticity. For the two models with firm-specific capital, the numbers for relative prices are extremely close and showing them twice would prove redundant.

<sup>19</sup>In their model, with their estimated parameters, at the fourth period after the monetary policy shock, 57% of the firms produce 180% of the global output, leaving the remaining firms with a negative



monetary policy shock is empirically implausible. However, reducing the huge elasticity of substitution to the level consistent with a zero profit condition, we observe that the variability of the market share becomes quite small in both models, which weakens the argument made by Altig *et al.* (2005) in favour of the model with firm-specific capital. Furthermore, it is also clear from Figure 6 that the model with firm-specific capital fails to reproduce the large price changes observed at the micro level.

To conclude this section, the introduction of firm-specific capital helps to reconcile the macro models with the micro evidence concerning the frequency of the price changes. However, the mechanism for this achievement is entirely based on a very strong reaction of the marginal cost to output changes, which implies very small relative price variations. Such small relative price changes are incompatible with the micro evidence which typically finds that the average size of price changes is quite large.

## 5 Conclusion

In this paper we have introduced firm-specific production factors in a model with price and wage Taylor contracts. For this type of exercise, Taylor contracts present a twofold advantage over Calvo type contracts: (i) firm-specific production factors can be introduced and handled explicitly and (ii) the individual firm variables can be analysed explicitly. This allows a comparison of the implications of the various assumptions concerning the firm-specificity of production factors not only for aggregate variables, but also for cross-firm variability.

Our main results are threefold. First, in line with existing literature we show that introducing firm-specific capital reduces the estimated duration of price contracts from an implausible 19 quarters to an empirically more plausible 4 quarters. Firm-specific production factors makes the marginal costs of individual firms steep and very reactive to output changes. Since individual firms output depends of their relative prices, firms will hesitate to make large price adjustments. Second, introducing firm-specific labour markets does not help in improving the empirical performance of the model. The main reason is that observed wages are sticky and therefore large variations in firm-specific wages, which help in generating steep marginal costs, are empirically implausible. Overall, it thus appears that rigidities in the reallocation of capital across firms rather than rigidities in the labour market are a more plausible real friction for reducing the estimated degree of nominal price stickiness. Third, in order to obtain this outcome one needs a very high demand elasticity, implying implausibly large variations in the demand faced by the firms throughout the length of the contract. Imposing the zero-profit

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output.

condition drastically reduces the estimated demand elasticity and leads to a corresponding reduction in the volatility of output across firms. However, in this case, the need for important real rigidities becomes evident through a high estimated curvature of the demand curve.

To compare the respective merits of the models with mobile production factors (flat marginal cost) and firm-specific production factors (increasing marginal cost), it is important to remember what are the main findings emerging from micro data on firms pricing behaviour: price changes are at the same time frequent and large (cf. Bils and Klenow, 2004, Angeloni *et al.* (2006)). The model with flat marginal costs does lead to large price changes, but requires a high degree of nominal stickiness to reproduce inflation persistence. The introduction of firm-specific marginal cost does lead to less nominal stickiness, but implies small relative price variations across firms. It thus seems that, so far, neither models can simultaneously satisfy both stylized facts. Altig *et al.* (2005) favour the model with firm-specific marginal cost on the basis of the argument that it produces less extreme variations in output shares after an exogenous shock. We have, however, shown that this outcome relies heavily on the price contract length and on the very large demand elasticity. Introducing additional curvature in the demand function as in Kimball (1995) significantly reduces the variability of output shares in the model with flat marginal costs. Overall, we therefore conclude that other elements such as the presence of firm-specific shocks will have to be introduced to match all the important micro stylised facts. Further research on the relationship between prices, output and marginal costs at the firm level would be very useful in this respect.

Finally, note that in this paper and in contrast to Coenen and Levin (2004), we did not allow for heterogeneity in the contract length. Such heterogeneity is another important stylised fact of the micro data. Moreover, such heterogeneity could help explain the tension between the finding of macro persistence and micro flexibility to the extent that the presence of sectors with long price durations can have a disproportionately large effect on the aggregate inflation behaviour (cf. Dixon and Kara, 2005b). Further research along these lines would be worthwhile.

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## 6 Appendix

### 6.1 Data appendix

All data are taken from the AWM database from the ECB (see Fagan *et al.*, 2005). Investment includes both private and public investment expenditures. The sample contains data from 1970Q2 to 2002Q2 and the first 15 quarters are used to initialize the Kalman filter. Real variables are deflated with their own deflator. Inflation is calculated as the first difference of the log GDP deflator. In the absence of data on hours worked, we use total employment data for the euro area. As explained in Smets and Wouters (2003), we therefore use for the euro area model an auxiliary observation equation linking labour services in the model and observed employment based on a Calvo mechanism for the hiring decision of firms. The series are updated for the most recent period using growth rates for the corresponding series published in the Monthly Bulletin of the

ECB. Consumption, investment, GDP, wages and hours/employment are expressed in 100 times the log. The interest rate and inflation rate are expressed on a quarterly basis corresponding with their appearance in the model (in the graphs the series are translated on an annual basis).

## 6.2 Model appendix

This appendix describes the other linearised equations of the Smets-Wouters model (2003-2004).

Indexation of nominal wages results in the following *real wage equation*:

$$\begin{aligned}
\widehat{w}_t = & \frac{\beta}{1+\beta} E_t \widehat{w}_{t+1} + \frac{1}{1+\beta} \widehat{w}_{t-1} + \frac{\beta}{1+\beta} (E_t \widehat{\pi}_{t+1} - \bar{\pi}_t) \\
& - \frac{1+\beta\gamma_w}{1+\beta} (\widehat{\pi}_t - \bar{\pi}_t) + \frac{\gamma_w}{1+\beta} (\widehat{\pi}_{t-1} - \bar{\pi}_t) \\
& - \frac{1}{1+\beta} \frac{(1-\beta\xi_w)(1-\xi_w)}{\left(1+\frac{(1+\lambda_w)\sigma_l}{\lambda_w}\right)\xi_w} \left[ \widehat{w}_t - \sigma_l \widehat{l}_t - \frac{1}{1-h} (\widehat{c}_t - h\widehat{c}_{t-1}) + \varepsilon_t^l \right] \\
& + \eta_t^w
\end{aligned} \tag{A1}$$

The real wage  $\widehat{w}_t$  is a function of expected and past real wages and the expected, current and past inflation rate where the relative weight depends on the degree of indexation  $\gamma_w$  to lagged inflation of the non-optimised wages. When  $\gamma_w = 0$ , real wages do not depend on the lagged inflation rate. There is a negative effect of the deviation of the actual real wage from the wage that would prevail in a flexible labour market. The size of this effect will be greater, the smaller the degree of wage stickiness ( $\xi_w$ ), the lower the demand elasticity for labour (higher mark-up  $\lambda_w$ ) and the lower the inverse elasticity of labour supply ( $\sigma_l$ ) or the flatter the labour supply curve.  $\varepsilon_t^l$  is a preference shock representing a shock to the labour supply and is assumed to follow a first-order autoregressive process with an iid-Normal error term:  $\varepsilon_t^l = \rho_l \varepsilon_{t-1}^l + \eta_t^l$ . In contrast,  $\eta_t^w$  is assumed to be an iid-Normal wage mark-up shock.

The dynamics of *aggregate consumption* is given by:

$$\begin{aligned}
\widehat{c}_t = & \frac{h}{1+h} \widehat{c}_{t-1} + \frac{1}{1+h} E_t \widehat{c}_{t+1} + \frac{\sigma_c - 1}{\sigma_c(1+\lambda_w)(1+h)} (\widehat{l}_t - E_t \widehat{l}_{t+1}) \\
& - \frac{1-h}{(1+h)\sigma_c} (\widehat{R}_t - E_t \widehat{\pi}_{t+1} + \varepsilon_t^b)
\end{aligned} \tag{A2}$$

Consumption  $\widehat{c}_t$  depends on the ex-ante real interest rate ( $\widehat{R}_t - E_t \widehat{\pi}_{t+1}$ ) and, with external habit formation, on a weighted average of past and expected future consumption. When  $h = 0$ , only the traditional forward-looking term is maintained. In addition, due to the non-separability of the utility function, consumption will also depend on expected employment growth ( $E_t \widehat{l}_{t+1} - \widehat{l}_t$ ). When the elasticity of intertemporal substitution

(for constant labour) is smaller than one ( $\sigma_c > 1$ ), consumption and labour supply are complements. Finally  $\varepsilon_t^b$ , represents a preference shock affecting the discount rate that determines the intertemporal substitution decisions of households. This shock is assumed to follow a first-order autoregressive process with an iid-Normal error term:  $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$ .

The *investment equation* is given by:

$$\widehat{I}_t = \frac{1}{1+\beta} \widehat{I}_{t-1} + \frac{\beta}{1+\beta} E_t \widehat{I}_{t+1} + \frac{1/\varphi}{1+\beta} \widehat{Q}_t + \varepsilon_t^I \quad (\text{A3})$$

where  $\varphi = \overline{S}''$  depends on the adjustment cost function ( $S$ ) and  $\beta$  is the discount factor applied by the households. As discussed in CEE (2001), modelling the capital adjustment costs as a function of the change in investment rather than its level introduces additional dynamics in the investment equation, which is useful in capturing the hump-shaped response of investment to various shocks including monetary policy shocks. A positive shock to the investment-specific technology,  $\varepsilon_t^I$ , increases investment in the same way as an increase in the value of the existing capital stock  $\widehat{Q}_t$ . This investment shock is also assumed to follow a first-order autoregressive process with an iid-Normal error term:  $\varepsilon_t^I = \rho_I \varepsilon_{t-1}^I + \eta_t^I$ .

The corresponding *Q equation* is given by:

$$\widehat{Q}_t = -(\widehat{R}_t - \widehat{\pi}_{t+1}) + \frac{1-\tau}{1-\tau+\bar{r}^k} E_t \widehat{Q}_{t+1} + \frac{\bar{r}^k}{1-\tau+\bar{r}^k} E_t \widehat{r}_{t+1}^k + \eta_t^Q \quad (\text{A4})$$

where  $\tau$  stands for the depreciation rate and  $\bar{r}^k$  for the rental rate of capital so that  $\beta = 1/(1-\tau+\bar{r}^k)$ . The current value of the capital stock depends negatively on the ex-ante real interest rate, and positively on its expected future value and the expected rental rate. The introduction of a shock to the required rate of return on equity investment,  $\eta_t^Q$ , is meant as a shortcut to capture changes in the cost of capital that may be due to stochastic variations in the external finance premium. We assume that this equity premium shock follows an iid-Normal process. In a fully-fledged model, the production of capital goods and the associated investment process could be modelled in a separate sector. In such a case, imperfect information between the capital producing borrowers and the financial intermediaries could give rise to a stochastic external finance premium. Here, we implicitly assume that the deviation between the two returns can be captured by a stochastic shock, whereas the steady-state distortion due to such informational frictions is zero.

The *capital accumulation equation* becomes a function not only of the flow of investment but also of the relative efficiency of these investment expenditures as captured by the investment-specific technology shock:

$$\widehat{K}_t = (1-\tau)\widehat{K}_{t-1} + \tau\widehat{I}_{t-1} + \tau\varepsilon_{t-1}^I \quad (\text{A5})$$

The equalisation of marginal cost implies that, for a given installed capital stock, *labour demand* depends negatively on the real wage (with a unit elasticity) and positively on the rental rate of capital:

$$\widehat{l}_t = -\widehat{w}_t + (1 + \psi)\widehat{r}_t^k + \widehat{K}_{t-1} \quad (\text{A6})$$

where  $\psi = \frac{\psi'(1)}{\psi''(1)}$  is the inverse of the elasticity of the capital utilisation cost function.

The *goods market equilibrium condition* can be written as:

$$\widehat{Y}_t = (1 - \tau k_y - g_y)\widehat{c}_t + \tau k_y \widehat{I}_t + g_y \varepsilon_t^g + k_y \frac{1 - \beta(1 - \tau)}{\beta} \psi \widehat{r}_t^k \quad (\text{A7a})$$

$$= \phi \left[ \alpha(\widehat{K}_{t-1} + \psi \widehat{r}_t^k) + (1 - \alpha)(\widehat{l}_t + \gamma t) \right] - (\phi - 1)\gamma t \quad (\text{A7b})$$

where  $k_y$  is the steady state capital-output ratio,  $g_y$  the steady-state government spending-output ratio and  $\phi$  is one plus the share of the fixed cost in production. We assume that the government spending shock follows a first-order autoregressive process with an iid-Normal error term:  $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g$ .

Finally, the model is closed by adding the following empirical *monetary policy reaction function*:

$$\begin{aligned} \widehat{R}_t = & \bar{\pi}_t + \rho(\widehat{R}_{t-1} - \bar{\pi}_{t-1}) + (1 - \rho) \left[ r_\pi(\widehat{\pi}_{t-1} - \bar{\pi}_{t-1}) + r_Y(\widehat{Y}_{t-1} - \widehat{Y}_{t-1}^p) \right] + \\ & r_{\Delta\pi} [(\widehat{\pi}_t - \bar{\pi}_t) - (\widehat{\pi}_{t-1} - \bar{\pi}_{t-1})] + r_{\Delta Y} \left[ (\widehat{Y}_t - \widehat{Y}_t^p) - (\widehat{Y}_{t-1} - \widehat{Y}_{t-1}^p) \right] + \eta_t^R \end{aligned} \quad (\text{A8})$$

The monetary authorities follow a generalised Taylor rule by gradually responding to deviations of lagged inflation from an inflation objective and the lagged output gap defined as the difference between actual and potential output. Consistently with the DSGE model, potential output is defined as the level of output that would prevail under flexible price and wages in the absence of the three “cost-push” shocks. The parameter  $\rho$  captures the degree of interest rate smoothing. In addition, there is also a short-run feedback from the current changes in inflation and the output gap. Finally, we assume that there are two monetary policy shocks: one is a temporary iid-Normal interest rate shock ( $\eta_t^R$ ) also denoted a monetary policy shock; the other is a permanent shock to the inflation objective ( $\bar{\pi}_t$ ) which is assumed to follow a non-stationary process ( $\bar{\pi}_t = \bar{\pi}_{t-1} + \eta_t^\pi$ ). The dynamic specification of the reaction function is such that changes in the inflation objective are immediately and without cost reflected in actual inflation and the interest rate if there is no exogenous persistence in the inflation process.

### 6.3 Description of the priors

Some parameters are fixed. They are principally parameters related to the steady-state values of the state variables. The discount factor  $\beta$  is calibrated at 0.99, corresponding with an annual steady-state real interest rate of 4%. The depreciation rate  $\tau$  is set at



0.025, so that the annual capital depreciation is equal to 10 percent. The steady state share of capital income is fixed at  $\alpha = 0.24$ . The share of steady-state consumption in total output is assumed equal to 0.65 and the share of steady-state investment to 0.17.

The priors on the other parameters are displayed in tables of the next appendix. The first column is the description of the parameter, the second the prior distribution and the two next columns give respectively the prior mean and standard error. Most of the priors are the same as in Smets and Wouters (2003). However, an important difference is to note for the capacity utilisation adjustment cost parameter ( $\psi$ ). Instead of estimating  $\frac{1}{\psi}$  with a prior [Normal 0.2 0.075], we now estimate  $cz = \frac{\psi}{1+\psi}$  with a prior [beta 0.5 0.25], which actually corresponds to a much looser prior since it allows for values of the elasticity of the capital utilisation cost function between 0.1 and 10. Some new parameters appear: the price and wage mark-ups, which are given a rather loose prior of [beta 0.25 0.15], and the curvature parameter which is estimated via  $eps = \frac{\epsilon}{1+\epsilon}$  with a prior of [beta 0.85 0.1]. The latter allows for values of parameter  $\epsilon$  between 1.5 and 100.

For the rest, as in Smets and Wouters (2003), the persistency parameters are given a Normal prior distribution with a mean of 0.85 and a standard error of 0.10. The variance of the shocks are assumed to follow an inverted Gamma distribution with two degrees of freedom.

#### 6.4 Parameter estimates for the main models

The Metropolis-Hastings algorithm has been run with 250 000 draws. Convergence is assessed with the help of Cumsum graphs and using the Brooks and Gelman (1998) uni-and multivariate tests performed by the Dynare software.





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