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## Basel II and Operational Risk: Implications for risk measurement and management in the financial sector

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# NATIONAL BANK OF BELGIUM

## WORKING PAPERS - RESEARCH SERIES

### **Basel II and Operational Risk: Implications for risk measurement and management in the financial sector**

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## Editorial

On May 17-18, 2004 the National Bank of Belgium hosted a Conference on "*Efficiency and stability in an evolving financial system*". Papers presented at this conference are made available to a broader audience in the NBB Working Paper Series ([www.nbb.be](http://www.nbb.be)).

## Abstract

This paper proposes a methodology to analyze the implications of the Advanced Measurement Approach (AMA) for the assessment of operational risk put forward by the Basel II Accord. The methodology relies on an integrated procedure for the construction of the distribution of aggregate losses, using internal and external loss data. It is illustrated on a 2x2 matrix of two selected business lines and two event types, drawn from a database of 3000 losses obtained from a large European banking institution. For each cell, the method calibrates three truncated distributions functions for the body of internal data, the tail of internal data, and external data. When the dependence structure between aggregate losses and the non-linear adjustment of external data are explicitly taken into account, the regulatory capital computed with the AMA method proves to be substantially lower than with less sophisticated approaches allowed by the Basel II Accord, although the effect is not uniform for all business lines and event types. In a second phase, our models are used to estimate the effects of operational risk management actions on bank profitability, through a measure of RAROC adapted to operational risk. The results suggest that substantial savings can be achieved through active management techniques, although the estimated effect of a reduction of the number, frequency or severity of operational losses crucially depends on the calibration of the aggregate loss distributions.

JEL-code: C24, G18, G21

Keywords: Operational Risk Management, Basel II, Advanced Measurement Approach, Copulae, External Data, EVT, RAROC, Cos-benefit Analysis.

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## 1. Introduction

Since the first Basel Accord was adopted in 1978, the banking sector has been persistently complaining about the simplistic approach of risk-adjusted credit exposures based on the adoption of the Cooke ratio for the determination of economic capital. The arbitrary categorization of securities in broad risk classes was allegedly leading to overly conservative and/or inadequate capital charges. Therefore, many large institutions have developed their own proprietary model for credit and market risk exposure with the objective of convincing their corresponding regulator of the superiority of their "Internal Rating Based" approach over the Basel I standards. The need for organizing the framework under which the IRB approach is eligible to measure banks' exposures to credit risk is probably the main impetus for the revision of this system through the second Accord.

Yet, the Basel Committee on Banking Supervision (hereafter the Basel Committee) has also taken this opportunity to extend the scope of its proposals well beyond this emblematic issue. In particular, the new Accord introduces and thoroughly examines a type of risk which, although well documented in the manufacturing sector, had been somewhat overlooked by the banking industry until recently: that is, operational risk, defined by the New Accord on Capital Adequacy proposal (hereafter Basel II) as the "*risk of loss resulting from inadequate or failed internal processes, people, and systems or from external events*" (BCBS, 2003a).

This new focus of the Regulatory Authorities on operational risks has indeed had a tremendous impact on the banking sector. Unlike credit and market risks, whose awareness within the banking industry roots back very far in the past and whose importance had already been recognized by the Basel I Accord, operational risk in the financial sector is a fairly new concept and thus in need for precise modeling and measurement methodologies. Indeed, except for fraud, most banks had in the past a tendency to neglect this heterogeneous family of risks that were perceived as too diffuse and peripheral. For the same reasons, until recently, few banks had set up a systematic collection of data relative to operational losses.

Basel II leaves to banks the choice between three approaches for quantifying the regulatory capital for operational risk. First, the *Basic Indicator Approach* (BIA) defines the operational risk capital as a fraction (15%) of the gross income of the institution, thus explicitly assuming that operational risk is related to size. Gross income is the sum of the interest margin, the fee income, and the other revenues. However, internationally active banks are strongly recommended not to adopt this simple model. Second, the *Standardized Approach* (SA) slightly refines the BIA, as it calculates the operational risk capital on the basis of gross income split per business. Here, the regulator distinguishes among

different operational risk levels according to the type of activity performed. The fraction of the gross income for capital assessment varies from 12% for the least risky business lines (i.e., retail banking, asset management) to 18% for the most risky ones (i.e. trading and settlement), with an intermediate level at 15% of the gross income for other categories (corporate banking for instance). Finally, under the *Advanced Measurement Approach* (AMA), banks are free to develop their own model for assessing the regulatory capital that covers their operational risk, with a confidence interval of 99.9%. International banks are advised by the regulator to comply with the AMA, and to quickly adapt their quantitative data collection, theoretical modeling of risk exposure and statistical validation in order to be allowed to make use of a proprietary model. The choice faced by banks among several methods, although similar to the choice for credit risk modeling, is more critical in this case, as the cost-benefit trade-off of the alternative is completely unknown.

Our paper examines two key issues faced by banks in handling operational risks: the cost-benefit analysis of engaging in the AMA instead of the basic approach, and the incremental cost-benefit analysis of striving towards an efficient operational risk management system. These two levels of analysis involve a study in two stages, with a focus on the necessary trade-off between the accuracy of the modeling approach (in order to fit actual data) on the one hand, and the relative parametric simplicity of the framework (in order to conserve the possibility to perform sensitivity analyses) on the other hand.

The paper is organized as follows. The second section offers an overview of the literature. In Section 3, we discuss the modeling choices underlying the measurement of operational risk capital. Section 4 describes the database that we used in our analysis. The fifth section tests the risk measurement methodology on real data. Section 6 reviews the best practices in operational risk management and links them to the quantitative methodology. Section 7 assesses the impact of operational risk management for a bank. Finally, Section 8 presents our concluding remarks.

## **2. Literature Review**

As the concern about operational risk is rather new in the banking area, the literature on this topic, both by scientific researchers and practitioners, is currently booming, mostly on quantitative methodologies and tools than can be applied to this issue.

The Advanced Measurement Approach (AMA) proposed by the Basel II Accord encompasses all measurement techniques that lead to a precise measurement of the exposure of each business line of a financial institution to each category of operational loss events. Although AMA is in principle open to

any proprietary model, the most popular AMA methodology is by far the Loss Distribution Approach (or LDA).

The LDA approach is an application of actuarial methods that combine a frequency distribution describing the occurrence of operational losses in the organization and a severity distribution that describes the economic impact of the individual losses (see e.g. Frachot *et al.*, 2001, or Cruz, 2002, for theoretical backgrounds and Bank of America, 2003, or ITWGOR, 2003, for practitioners' points of view). Although it does not specifically consider the tail of the aggregate loss distribution, its modular structure opens the possibility to deal separately with the extreme losses, using instruments from Extreme Value Theory (EVT) to model the tail of the distribution (Embrechts *et al.*, 1997). Still, estimating high quantiles of the distribution remains a difficult problem, since the structure of operational risk data is barely consistent with standard modeling assumptions (Embrechts *et al.*, 2003). This is mostly because internally generated databases are not likely to include sufficient data to merely rely on the observation of extreme losses for the calibration of the tails of distribution.

Using external loss data to model extreme losses raises a number of methodological questions, as observed by several authors (Frachot and Roncalli, 2002; Baud *et al.*, 2002). The main issue is to identify the type of data to consider, since the processes having generated those external losses might be very different from one banking institution to another. Another question relates to the appropriate scaling of the external data in order to adjust for the size of the bank including them in its model (Shih *et al.*, 2000, or Hartung, 2003).

After modeling the loss distribution for one type of event in one business line of activities, the approach has to be extended to several business lines of activities, and several types of operational events. While, by default, Basel assumes full positive correlation between these risks, banks are nevertheless offered the possibility to estimate the correlation between risk events by appropriate techniques for dependence characterization, such as copulae. Once again, applications to operational risk are scarce; in risk management, this approach has been used so far for measurement of dependence in insurance (Klugman and Parsa, 1999), market risk (Mashal and Zeevi, 2002) or credit risk (Frey *et al.*, 2001).

In this paper, we develop an integrated LDA methodology and we apply it to real internal operational loss data from a European banking institution. To our knowledge, this is the only application in the current literature that uses a full LDA approach with real life data. Most other papers usually concentrate on technical aspects and illustrate them with simulated data. The study most closely related to ours in this respect is Fontnouvelle *et al.* (2003), which uses public operational loss databases to show that the charge for operational risk often exceeds the charge for market risk,



although the amount of regulatory capital may vary with the size and scope of a bank's activities. However, the study by Fontnouvelle *et al.* (2003) is based on an external database that is publicly available, not exhaustive and restricted to large losses.

Next to the numerous contributions on modeling, a few publications address specific issues relating to operational risk management. The Basel Committee (BCBS, 2003b) defined sound practices for the management of operational risks. Jorion (2003) summarizes some of the bank practices and recommendations previously mentioned in BIS publications. Hoffman (2002) presents the best practices in operational risk management for 20 large companies. Crouhy *et al.* (2001) and Alexander (2003) propose synthetic classifications of the different dimensions of operational risk management.

### **3. Modeling operational risk**

#### **3.1. Overview**

In the Basel II Accord, three approaches are thus proposed to compute the capital requirements for operational risk in banks. When they opt for the AMA, banks are allowed to develop in-house measurement techniques provided they fulfill qualitative and quantitative requirements. In particular, a soundness standard similar to the standard adopted for credit risk is mandatory. This standard is set to a confidence level of 99.9% for a one-year holding period. Clearly, accurate modeling of the extreme right part of the loss distribution is of crucial importance when computing an Operational Value-at-Risk (henceforth OpVaR) at such a high level of confidence.

Among eligible AMA techniques, we specifically use the Loss Distribution Approach (or LDA). This parametric technique consists in separately estimating the frequency and severity distributions of losses, then computing the aggregated loss distribution through convolution. It is usually impossible to derive analytical expressions for this kind of convolutions; hence, numerical methods such as Monte Carlo simulations are used in practice. As a consequence, a precise overall characterization of the entire severity distribution, including its body, is required.

Thus, the analyst faces the need of fitting both the body and the tail of the distribution very well to get accurate figures. A single functional form for the severity distribution lacks the necessary flexibility to correctly deal with both the body and the tail. Moreover, goodness-of-fit tests such as the Kolmogorov-Smirnov statistic will often select distributions that do a good job in fitting the body of the distribution, while under-weighting the extreme parts of the tail. A solution could be to modify

these tests by incorporating weights for the different parts of the distribution so that the extreme quantiles are adequately accounted for.

In our preliminary tests, however, we have repeatedly found that classical probability distributions are unable to model the entire range of losses in a satisfactory way (i.e., they yield a poor fit). Therefore, we propose to consider a conceptually different approach whereby the operational losses of a bank are viewed as arising from two different generating processes, so that "normal" (i.e. high frequency/low impact) losses do not stem from the same distribution as the "extreme" (i.e. low frequency/high impact) losses. As a consequence we define the severity distribution as a mixture of two distributions: the "normal" distribution and the "extreme" distribution. For simplicity, we will assume that these distributions are mutually exclusive; that is, the "normal" distribution includes all losses in a limited range denoted  $[L;H]$ ,  $L$  being the "collection" threshold used by the bank, while the "extreme" distribution generates all the losses above  $H$ .<sup>1</sup> Thus  $H$  is the "cut-off" threshold separating "normal" and "extreme" losses, as can be seen on Figure 1.

Insert Figure 1 approximately here

This idea of dealing separately with "normal" and "extreme" losses has been examined in the operational risk context by several authors (see, among others, King, 2001, and Alexander, 2003). Unfortunately, the determination of the most appropriate threshold for separating the distributions of normal and extreme losses is still heuristic, and is typically based on a graphical analysis.<sup>2</sup> With this respect, in order to achieve a fully consistent algorithmic procedure, we provide support for a different treatment of the "extreme" losses and a more rigorous way to detect the "cut-off" threshold.

## **3.2. Models for the distribution of losses: Internal data**

### **3.2.1. Frequency of losses**

The issue of calibrating a probability density function for the number of losses within a given time interval, i.e. the frequency of losses, is classical in risk management. For short periods of time, the choice between the homogenous Poisson distribution and the negative binomial distribution is important, as the intensity parameter is deterministic in the first case and stochastic in the second (see Embrechts *et al.*, 2003 for a discussion). However, as the prudential requirement for the computation of economic capital involves measuring the 99.9% OpVaR on a yearly period, this issue appears to be

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<sup>1</sup> The "no overlap" assumption can arguably be questioned. However, the approach described here could easily be extended to an "overlap" situation. This extension is left for further research.

<sup>2</sup> The approach advocated by Dupuis (1998) is an exception.

marginally relevant: using simulations, numerical evidence has shown us that the mere calibration of a Poisson distribution with constant parameter  $\mathbf{1}$  corresponding to the average number of observed losses during a full year provides a very good approximation of the true frequency distribution. Therefore, we choose not to focus on this particular issue in the rest of the paper.

### 3.2.2. “Normal” losses (severity distribution)

The “normal” losses are generally well represented in the collected samples and their severity can thus quite easily be modeled with a traditional Maximum Likelihood Estimation (MLE). The severity distribution can be fitted with well-known heavy-tailed distributions such as the Exponential, the Weibull, the Gamma or the Lognormal. Functional forms of these distributions are given in the Appendix 1.

As a preliminary step to measure operational risk, it is necessary to take the “collection” threshold into account when estimating the parameters of the distribution. Moreover, as can be seen in Figure 1, we also have to introduce the existence of an upper bound (the “cut-off” threshold) in the calculations. Thus, letting  $\mathbf{q}$  be the parameters vector, the “true” probability density function of the loss variable  $x$  (denoted  $f(x; \mathbf{q})$ ) is transformed as follows in order to obtain the density function  $f^*$  of the losses in  $[L; H]$  :

$$f^*(x; \mathbf{q}) = \frac{f(x; \mathbf{q}) - F(L; \mathbf{q})}{F(H; \mathbf{q}) - F(L; \mathbf{q})} \quad (1)$$

where  $F$  denotes the cumulative density function,  $L$  the collection threshold and  $H$  the “cut-off” threshold.

Thus  $f^*(x; \mathbf{q})$  is the function of interest when estimating the parameters. We use a simple Maximum Likelihood approach to estimate the distribution’s parameters. As it is more convenient to optimize the logarithmic transformation of this function, the log-likelihood function to be maximized is

$$\ell(x; \mathbf{q}) = \sum_{i=1}^N \ln \left( \frac{f_i(x_i; \mathbf{q}) - F(L; \mathbf{q})}{F(H; \mathbf{q}) - F(L; \mathbf{q})} \right) \quad (2)$$

where  $(x_1, \dots, x_N)$  is the sample of observed normal losses.

### 3.2.3. “Cut-off” threshold

To identify a threshold separating "normal" from "extreme" losses, some authors simply select an arbitrary measure such as the 90<sup>th</sup> percentile of the sample, or rely on graphical tools such as the popular Mean Excess Plot (see Embrechts *et al.*, 1997, for details).

Although the graphical approach is currently the most widely used, Dupuis (1998) describes a parametric method to perform the threshold selection. In a related research (Peters *et al.*, 2004), we propose an algorithmic alternative, which compares several thresholds and selects the best one based on an objective measure, namely a “goodness-of-fit” statistic on the upper part of the sample. However, since this issue is peripheral to the current research, we do not develop the full-fledged methodology here and directly report the main results of the algorithm.

### 3.2.4. “Extreme” losses (severity distribution)

Lack of data, resulting in small-sized samples, represents a common issue when dealing with operational losses in banks. Moreover, because of the limited collection period available nowadays (often less than 3 years), databases typically do not include very rare, but yet very severe losses. Therefore, estimating the distribution of "extreme" losses by classical maximum likelihood methods may yield distributions that are not sufficiently heavy-tailed to reflect the probability of occurrence of such exceptional losses. To resolve this issue, we rely on concepts and methods from Extreme Value Theory (EVT), and more specifically on the Peak Over Threshold (POT) approach.

This approach first requires to determine a high threshold and then to estimate the parameters of an extreme distribution using all the observations above this threshold. This procedure builds upon a classical theorem of Pickands (1975) and Balkema and de Haan (1974) which essentially states that, for a broad class of distributions, the values of the variables above a sufficiently high threshold follow the same distribution, namely the Generalized Pareto Distribution (GPD).<sup>3</sup>

In the literature, EVT is often used to estimate very high quantiles, for instance to compute Value-at-Risk figures (see McNeil, 2000, or K ellezi and Gilli, 2003). But estimating an extreme quantile of a distribution is very different from obtaining the whole PDF of the losses, which is nevertheless needed in order to compute the convolution of the severity distribution with itself (this is how we get the aggregated loss distribution). In addition, the global shape of this distribution is also important when dealing with dependence measurement techniques.

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<sup>3</sup> The complete form of the GPD is given in Appendix 1.

In our implementation, we simultaneously assess the distribution of normal losses and select the cut-off threshold. To do so, we consider  $m$  different levels for the “cut-off” threshold  $H_i$ ,  $i = 1, \dots, m$  and we estimate the parameters vector  $\mathbf{q}$  of the GPD associated with each level (i.e. based on the excesses over  $H_i$ ). Then, we compare the  $m$  selected combinations (one for each value of  $H$ ) and we select the optimal “cut-off” threshold based on a mix of goodness-of-fit statistic (Cramer-von Mises), visual inspection of the Mean Excess Plot and expert judgment.

### ***3.2.5. Mitigating risk through insurance***

Under the Basel II recommendations, banks adopting the advanced approaches are authorized to account for the risk mitigating impact of insurance in their capital charge computations, provided the implied capital reduction is less than 20%.

Concretely speaking, if an insurance policy covers the losses between the amounts  $A$  and  $B$ , all the simulated losses that fall between these two bounds and that satisfy the conditions included in the policy are fixed to 0 (or any other minimum amount specified in the contract).

In our case, such a policy does exist by our data provider so that we have accounted for it in all computations.

### **3.3. The aggregate loss distribution per business line and per event type**

Once the overall form of the severity distribution has been derived, we combine it with the frequency distribution to get the aggregated loss distribution, which is the relevant distribution when it comes to compute the required economic capital.

This aggregated distribution is obtained by  $n$ -fold convolution of the severity distribution with itself, where  $n$  is the Poisson frequency variable. We compute this convolution by Monte Carlo simulations.

### **3.4. Models for the distribution of losses: External data**

In order to fully comply with the Basel Accord, the Advanced Measurement Approach ought to specify a proper way to complete the sample of extreme losses using external loss data.

There are several ways to integrate internal and external data:

- Separate estimation of two distributions, respectively based on internal and external data, and combination of both distributions by Bayesian techniques (see for instance Chapter 7 in Alexander, 2003).
- Creation of an enlarged sample of observations containing a mix of internal and external data (Frachot *et al.*, 2002).
- Improvement of the accuracy of the tail of the severity distribution, based on the information contained in the external dataset. Relying on external data provides indeed another way of accounting for events that have never been observed at the financial institution under consideration but that could occur in the future, and is similar in spirit to Extreme Value Theory approaches.

To avoid a bias toward overestimation, the first two methods require the external dataset to have a collection threshold that is not too high when compared with the internal one. Loss data collected by pooling consortia such as ORX or the Italian initiative DIPO are thus well suited for these methods. On the other hand, data found in commercial loss databases such as OpVantage's "First" usually have a high threshold (\$1 million for First), so that they are more appropriate for the third approach. We have adopted the latter approach in our study.

Whatever the motivation for considering external data, pooling internal and external observations presents several statistical challenges. In particular, external data must be scaled appropriately to be comparable with internal data, and the threshold of collection of extreme losses is often not known precisely for external data. To date, few researchers have addressed these issues explicitly (see Baud *et al.*, 2002, Frachot and Roncalli, 2002, or Shih *et al.*, 2000).

A direct scaling method for external data consists in linearly adjusting the losses based on a given exogenous measure, such as gross income. While easy to implement, such a method is not very appealing, as the heterogeneous nature of operational risks suggests that the magnitude of each type of operational losses has no simple linear relationship with gross income.

Another methodology is thus to use a non-linear relationship between losses and gross income, similarly to Shih *et al.* (2000) or Hartung (2003). A potential drawback of these approaches is that the collection threshold of the external database is not unique, as it has to be adjusted for each event. The threshold should therefore be considered as a stochastic variable to be estimated (see Baud *et al.*, 2002, for details), unless one uses the external database for the sole purpose of completing the tail estimation of the distribution, which is the option taken in this paper. We thus follow the non-linear scaling approach of Shih *et al.* (2000) to model the tail of the severity distribution and consider the following relationship between firm size and loss magnitude:

$$Loss = R^a F(\theta) \quad (3)$$

where  $Loss$  is the loss magnitude,  $R$  is a proxy for the firm size (the gross income for instance),  $a$  is a scaling factor (when  $a = 1$ , we have the simple linear relationship) and  $\theta$  is the vector of all the risk factors not explained by  $R$ , so that  $F(\theta)$  is the multiplicative residual term not explained by any fluctuations in size.

Taking the logarithm and dividing both sides of (3) by  $\ln(R)$ , we obtain the following relationship:

$$y = a + \mathbf{b}x + \mathbf{e} \quad (4)$$

with  $y = \ln(Loss) / \ln(R)$  and  $x = 1 / \ln(R)$ .<sup>4</sup>

Once  $a$  is estimated (through a simple regression approach with Ordinary Least Squares or Weighted Least Squares), losses can easily be scaled using the formula

$$Loss_{scaled} = Loss_{raw} \left( \frac{R_{ext}}{R_{int}} \right)^a \quad (5)$$

where  $R_{ext}$  is the gross income of the external business segment (or bank) and  $R_{int}$  is the gross income of the internal business segment (or bank). If  $a = 0$ , it means that the volume of the bank's activities has no relationship with the size of the losses. If  $a = 1$ , this relationship is assumed to be linear.

### 3.5. Dealing with all business lines and event types

For each entity, operational losses (frequency and severity) have to be collected for 8 business lines and 7 event categories. This creates a matrix with 56 cells, with various characteristics on the number of observations, average and dispersion of losses.

The modeling approach outlined in Sections 3.2, 3.3 and 3.4 can be applied to each individual cell of the matrix. Then, the resulting loss distributions still have to be combined in order to derive the multivariate distribution of operational losses for the entire matrix. This modular procedure must be flexible enough to account for two difficulties:

- Some cells may be empty or quasi-empty, which creates the need for appropriate procedure inferring information from more global data. Since this issue is not central in our study, we restrict our attention to a dataset that will enable us to disregard it altogether; see Section 4.2.

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<sup>4</sup> Note that  $\beta = E[\ln F(\theta)]$  and  $\varepsilon = [\ln F(\theta) - \beta] / \ln(R)$ .

- The correlation of distributions between different cells has to be modeled in a tractable but accurate way. This requires to estimate and to model the dependency between univariate distributions and to produce their joint distribution. For this purpose, we investigate the use of copulae specifications; see Section 5.3.

## **4. Database description**

### **4.1. Internal data**

The methodology has been tested on a set of real operational loss data coming from a large banking institution in Europe, whose collection has been made in compliance with the Basel II definition of business lines and event types for the adoption of the AMA.

For the sake of data confidentiality, we have scaled the amounts of losses by a homothetic transformation, so that this does not influence the distribution and we have adjusted the time frame of data collection so as to obtain a total of 3,000 loss events. Therefore, though neither the number nor the amounts of losses could be used to assess the actual operational risk exposure of this financial institution, the internal database remains realistic.

The summary statistics displayed in Table 1 give a general overview of the amount, the nature and the distribution of the data used in this paper.

Insert Table 1 approximately here

In our study, we use external data drawn from the First database commercialized by OpVantage. Descriptive statistics for these losses are given in Appendix 2.

### **4.2. Selected cells of the matrix**

Since many cells contain a small number of data, too small indeed to apply sophisticated statistical techniques, and since our purpose in this study is primarily to develop and to illustrate a methodology, we focus our analysis on a sub-matrix consisting of 2 rows and 2 columns of the original matrix: (Private banking + Asset management<sup>5</sup> / Retail banking)  $\times$  (Clients, products & business practices /

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<sup>5</sup> These two business lines, although distinct in the Basel II list, are merged for the sake of our VaR estimations. They indeed involve activities and risk exposures that are very close to each other.



Execution, delivery & process management). The distribution of loss events among these cells is given in Table 2.

Insert Table 2 approximately here

These cells involve enough data to enable us to perform a meaningful analysis on internal data, including the calibration of a proper dependence structure between business lines and event types.

## **5. Empirical results**

In this section, we first develop an internal measurement for one of the four selected cells using the methodology described in the previous sections. We treat internal data only in Section 5.1.1, then we explore the inclusion of external losses and compare the results of both methods in Section 5.1.2. Finally, once the approach is conducted for the four cells, we use copulae to introduce some dependence structure in our models. Notice that insurance policies are accounted for throughout this section.

### **5.1. Measurement for a single cell**

#### ***5.1.1. Use of internal data only***

We consider the computation of the Operational Value-at-Risk (OpVaR) for the cell “Retail Banking / Clients, Products and Business Practices”. In order to do so, we first have to split the sample into two sub-samples of “normal” vs. “extreme” losses. To identify the threshold that separates the two sub-samples, we plot the Mean Excess Function (MEF). When the graph of the MEF follows a reasonably straight line with positive gradient above a certain value, this indicates a heavy-tailed distribution (see Embrechts *et al.*, 1997, for details). As can be seen on Figure 2, a strengthening of the positive trend appears around  $u = 700$ .

As the Mean Excess Plot does not necessarily provide a reliable answer to the threshold detection problem, we complement visual inspection with a more robust algorithm (see Peters *et al.*, 2004, for details). The main results are summarized in Table 3. The minimum Cramer – von Mises (CVM) statistic is obtained for  $u = 775$ , but since a very similar result is obtained with  $u = 675$  (note the similar estimate of the tail index  $\xi$ ), we select the latter threshold so as to increase the number of extreme observations available to estimate the GPD parameters.

Insert Figure 2 approximately here

Insert Table 3 approximately here

Next, we fit a distribution on each sub-sample by a Maximum Likelihood approach adapted to the truncated sub-samples. We test three distributions for the “normal” losses (Gamma, Weibull and lognormal<sup>6</sup>) and we select the best one based on well-known goodness-of-fit indicators (Kolmogorov-Smirnov, Cramer-von Mises and Anderson-Darling). The lognormal (0.86; 2.84) provides the best fit for this specific cell. Its quantile-quantile plot (or QQ-plot) is displayed in Figure 3.

Insert Figure 3 approximately here

We then estimate the parameters of the extreme GPD distribution for the losses above the threshold  $u = 675$ . We obtain estimates of 0.735 for the shape parameter  $\alpha$  and 542 for the scale parameter  $\beta$ . More details about the results are reported in Table 4.

Insert Table 4 approximately here

Finally, we compute the OpVaR at the 99.9% confidence level with Monte Carlo simulations<sup>7</sup>. The frequency of the losses is assumed to be Poisson distributed<sup>8</sup> and the severity distribution is a mixture of a lognormal (0.86; 2.84) for the losses under 675 and a GPD (0.735; 542) for the losses above this threshold. Basel II defines the regulatory capital charge as the Unexpected Loss (defined as the difference between the OpVaR<sub>99.9</sub> and the Expected Loss) provided the bank is “*able to demonstrate to the satisfaction of its national supervisor that it has measured and accounted for its [Expected Loss] exposure*” (BCBS, 2003, al. 629). In our case, this amounts to 1.16 million – 0.10 million = 1.06 million.

### **5.1.2. Introducing external data**

We now introduce an additional component in our measurement framework, namely the external database. We scale the external data by the procedure of Shih *et al.* (2000), as described in Section 3.4. An ordinary least square technique yields an estimate of the scaling factor  $a$  (see Equations 3 to 5). For the “Retail Banking / Client, Products and Business Practices” cell, we obtain the value  $a = 0.152$ , which is in line with the findings in Shih *et al.* (2000). Such a value indicates that the relationship between losses and size is clearly non-linear. Then we scale the external data accordingly and estimate

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<sup>6</sup> These distributions are classical candidates for these kinds of applications, although other specifications could obviously be considered as well.

<sup>7</sup> We have simulated 5 sets of 10.000 years of losses and averaged the obtained quantiles.

<sup>8</sup> The alternative would be to use a negative binomial distribution and apply it similarly in our Monte Carlo experiments.

the distribution of the resulting data. For this particular case, a lognormal distribution with parameters 9.041 and 1.529 fits the data well.

Next, we compute the aggregate loss distribution based on a severity distribution that now combines three elements: a distribution for the body of the data (“high frequency/low severity” events), the GPD distribution for high losses and the external data distribution for extremely high losses as shown in Figure 4.

Insert Figure 4 approximately here

The frequency distribution, the severity distribution and the aggregated loss distribution for this cell are plotted in Figure 5.

Insert Figure 5 approximately here

To assess the impact of the introduction of external data to the high quantiles estimates of the aggregate loss distribution and the regulatory capital, Table 5 provides a comparison between results obtained with internal data only and with the inclusion of external data.

Insert Table 5 approximately here

These results suggest that replacing EVT estimates of the GPD parameters by the fitted distribution of comparable external observations considerably concentrates the tail of the aggregate loss distribution, leading to a larger value of the  $OpVaR_{95}$  and the  $OpVaR_{99}$ . However, the sign of the difference switches within the last percentile, leading to a more conservative estimate of the  $OpVaR_{99.9}$  when computed with internal data. This is due to the particular property of the GPD involved in our application of the Extreme Value Theory that results in a fatter behaviour of the very far end of the tail than if one merely fits observed values, even if they are very large, as with external data.

## **5.2. Measurement for the complete matrix**

A similar methodology has been used for the other three cells. Table 6 summarizes the corresponding results when external data is used in the modeling of the tail.

Insert Table 6 approximately here

If the operations of the bank were limited to these four cells, the results in Table 6 could provide the total required capital charge for operational risk under the assumption of perfect dependence between the cells of the matrix. Based on this default assumption of Basel II, we simply need to aggregate the OpVaR in excess of Expected Losses to get the overall capital charge. In our case, this amounts to 7.91 millions (OpVaR<sub>99,9</sub>) – 0.86 million (Expected Loss) = 7.05 millions. A more realistic approach, i.e. adequately taking dependence between risks into account, is analyzed in the next section.

Finally we have computed confidence intervals for the capital charge using bootstrapping techniques in order to test the robustness of the results. The 90%-confidence interval for the capital charge of each cell is reported in Table 7.

Insert Table 7 approximately here

Overall, our capital charge estimation for the whole four-cells bank should be within a 20% interval of our point estimate nine times out of ten. While this interval might seem broad, one should remember that we have a database of limited size. As operational losses databases increase in size, we may expect the accuracy of the estimates to improve and the confidence intervals to get narrower.

### **5.3. Introduction of a dependence structure**

#### **5.3.1. Dependence assumptions**

An important issue in operational risk modeling is the dependence assumption. Basel II assumes perfect positive dependence between risks, as it proposes to compute the total capital charge by simple addition of the capital charge for every cell of the matrix. Thus, all the severe losses are implicitly assumed to take place simultaneously. This assumption is not realistic: it is legitimate to consider that operational risks are not fully correlated in view of their heterogeneous nature.

A possible remedy is to include more appropriate dependence structures through the use of copulae. Copulae are the joint distribution functions of random vectors with standard uniform marginal distributions. They provide a way of understanding how marginal distributions of single risks are coupled together to form joint distributions of groups of risks. As a consequence, copulae could be an appealing solution to model dependence between risks. There are numerous families of copulae, each having its own specificities.

In the literature, the most usual way of studying dependence between risks is to focus on the frequency dependence rather than on the severity one (see Section 4.3 of Frachot *et al.*, 2003, for a

discussion of this topic). It seems indeed relevant to consider correlated occurrences of loss events, and this can be performed in a straightforward way. Unfortunately, this approach neglects the possible dependence (or absence thereof) of the magnitude of losses between event types and/or business lines.

We address the dependence issue in more details below but a quick look at Table 8 gives a first indication about the correlations between frequencies of risks.

Insert Table 8 approximately here

Panel A of this Table reports the Spearman's rank correlation coefficient between each of the four cells, while Panel B focuses on the two selected business lines and Panel C gives the correlations between all the business lines. In our context of strictly positive random variables following a highly skewed distribution, the use of a non-parametric indicator of dependence such as the Spearman's rank correlation coefficient is more appropriate than Pearson's product-moment coefficient (see Embrechts *et al.*, 2002).

The relatively low values of the coefficients clearly demonstrate that a perfect positive dependence assumption is probably unduly strong; this suggests that taking "real" dependence structure into account would lead to more realistic results and, probably, lower the total required capital charge.

### ***5.3.2. Assessment of the diversification effect***

Within the advanced approach proposed by Basel II, banks should consider 8 business lines and 7 event types. As a result, 56 aggregated loss distributions should be estimated and then combined to derive the overall aggregated loss distribution of the bank.

For instance, if we want to model the dependence between these 56 cells by means of a Gaussian copula, a correlation matrix should be derived. As a consequence dealing with a 56x56 matrix might lead to computational difficulties. Therefore some banks will limit themselves to the modeling of dependence between business lines. In this paper, we consider both cases, using the Gaussian copula.

When modeling dependence between business lines only (in our case, that means that we are considering a simple bivariate case), Spearman's  $r$  is 0.155, once again indicating a low dependence between the risks.

Table 9 summarizes the different values of the Operational Value-at-Risk and the capital charges reported under various dependence assumptions (see Nelsen, 1999, for a description of the computational methodology involved here).

Insert Table 9 approximately here

As can be seen in Table 9, taking the real dependence into account substantially reduces the required capital charge. In our case, this reduction is in the 30-35% range, which is similar to some results observed in the literature.<sup>9</sup> Reduction of the capital charge is thus potentially important when adequate dependence measures are introduced in the approach.

There exist many different copulae and the choice of an adequate copula is not an easy task. To assess the impact of a given copula on the OpVaR, we have conducted another study using Frank's copula (Frank, 1979). Using our data, the difference is not very significant. For instance, when only considering the business lines, the parameter estimate of the copula is 0.97, which leads to an OpVaR<sub>99.9</sub> of 5.41 millions (versus 5.38 million for the Gaussian copula) and only a 0.5 % increase of the capital charge (4.55 millions versus 4.53 millions).<sup>10</sup>

## 5.4. Conclusion

Our measurement approach includes the use of different distributions to model the body and the tail of the severity distribution, the use of external data to improve the modeling of the tail and the use of copulae to account for the real dependence structure.

By taking very large losses into account (including losses that might not yet have occurred in the bank), Extreme Value Theory (EVT) and external data open the possibility to improve models of the tail of the loss distribution. This prudential approach is compliant with Basel II requirements. Moreover, we have shown that adequately introducing the observed dependence between risks allows for a significant reduction of the capital charge (about a third).

Table 10 reports the capital charge obtained under various assumptions: the Basic Indicator Approach (BIA), the Standardized Approach (SA), and four Advanced Measurement Approaches (AMA) with different dependence assumptions (full positive dependence, dependence between business lines,

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<sup>9</sup> For instance, Frachot *et al.* (2001) reports potential reduction of 37.9% for the capital charge at the 99.9% confidence level.  
<sup>10</sup> Other copulae, such as the extreme value copulae, could lead to larger changes, but this topic is outside the scope of our study and we leave it for further research.

dependence between cells and independence). For ease of comparison, we have also performed a standardization by the BIA, SA and full-dependence AMA capital charges.

Insert Table 10 approximately here

These results show that a very conservative AMA approach (Extreme Value Theory + external data, full dependence) leads to a heavier capital charge than the Standardized Approach. However, when dependence is correctly specified, the capital charge can be reduced by more than 35% and the AMA becomes the least capital consuming approach of all.

Other elements are noteworthy: first, if the capital charge obtained with the SA seems quite low as compared to the default AMA, this is partly due to the nature of our dataset. Indeed, the SA derives the capital charge, for each business lines, by simply applying a given factor (called “beta”) to the business line’s gross income. This factor varies from 12 to 18%. The business lines considered in this study (Retail Banking and Asset Management) both have the lowest beta factors in the Basel II framework (12%). Thus the total capital charge for the SA is particularly attractive in our case. Moreover, while the operational losses of the four cells (used to fit the distributions in the AMA) represent more than 70% of the total database, the corresponding gross income (used in the other two approaches) only amounts to 35% of the total gross income of the bank. Here again, it is thus not very surprising to see a relatively low capital charge for the SA when compared to the AMA.

## **6. Managing operational risk**

At the present time, the assessment of operational risk still remains a delicate endeavor, due in part to the intrinsic difficulty of the exercise, to its exploratory stage of development, to the scarcity of data, and to the new regulatory definitions of operational risk events and of business lines of activity. Furthermore, unlike credit risk or market risk, operational risk is endogenous to the institution. It is linked to the nature and the complexity of the activities, to the processes and the systems in place, and to the quality of the management and of the information flows, to name but a few factors. For this reason, superficially similar financial institutions might end up with very different operational losses.

The Basel Committee on Banking Supervision is well aware of these difficulties and adopts a pragmatic approach to operational risk supervision, leaving banks free to assess their operational risk profile themselves provided that they display sufficient sound practices of operational risk supervision and management.

But even before the consequences of the enforcement of the Accord are considered, this new banking regulation has had a tremendous impact on the organization and on the intensity of the operational risk management in banking institutions.

Of course, operational risk management is not really new in the banking sector. Long before regulators addressed these issues, internal and external fraud were monitored and prosecuted by the internal audit department. Information Technology departments and IT controllers were already aiming at preventing breaches of security on the information system, guarantying data integrity, and protecting web-sites from hacking attempts. To insure the going-concern of the activities in case of major system breakdowns or physical damages, Business Continuity Plans had been set up and tested in most large financial institutions.

The great merit of the Basel reform is to have put a common name on a myriad of existing practices. Shedding light on these heterogeneous practices, the Basel requirements for operational risk management and supervision have provided a powerful incentive to improve the organization and to expand the scope of this activity.

The specificity of operational losses and their link to the unique features of each institution makes both the modeling of these losses more difficult, and the active risk management techniques absolutely necessary. These risk management actions may in turn influence the value of the input parameters of risk assessment models, as we will show in the following section.

Operational risks include events as various as fraud, business disruption, processing errors, or business malpractice. Despite its heterogeneous nature, some key techniques of operational risk management emerge both from the banking sector and from the literature. These techniques can be classified in various ways, according to their goals, their nature, or their stage of actions, but the general principles underlying risk management approaches remain the same.

Risk management involves four stages: risk identification, measurement, monitoring, and management. Risk identification is typically performed via risk and controls self-assessment at the department level, by analyzing internal audit report, and checking lists of key risks indicators. But the internal incident reporting database is also a powerful tool to assess potential or existing operational risks in an institution. Splitting the analysis among frequency and severity of events helps identifying potential large losses, and possible breaches in control.

Rare events implying large loss amounts are the first candidates in the identification of uncapped risks. When the incident database includes abnormal amounts of losses, specific investigations are required



in order to precisely identify the circumstances that have led to such losses. Likewise, recurrent losses of small amounts identified in the incident database require, at least once, further investigation. They might either be the consequence of an effective cap of losses in an activity that is highly exposed to operational risks, or a structural flaw in the process, possibly leading to systematic, or frequent losses, with very large amounts at stake.

After the risk identification stage, risk measurement constitutes the quantitative aspect of risk management. It has been extensively dealt with in other sections of this paper.

Risk monitoring implies a dynamic analysis of the evolution of the losses. This is best performed by Key Risk Indicators (KRI) analysis and operational dashboards, specifically designed per activity. Dashboards of operational events are useful tools to involve the management of a department in the operational risks issues. Efficient dashboards are synthetic, issued on a regular basis in a standardized format, listing top events with their main cause, and relating the amounts of losses as a percentage of the gross margin.

Operational risk management involves a multitude of techniques and approaches that essentially serve two purposes: average loss reduction and catastrophic loss avoidance. Some of these techniques help the average loss reduction, some the event avoidance, some both.

The next section simulates the impact of specific risk management actions on loss distribution, and on average financial performance.

## **7. Assessing the impact of OR management**

### **7.1. Mapping of risk management actions on loss distributions**

One of the main motivations underlying the new Basel Accord is to encourage banks to adopt effective management procedures against operational risks since any reduction in risk exposure should lead, in principle, to a reduction of the associated capital charge.

The somewhat recent history of data in our sample does not allow us to empirically test the impact of ORM on the losses of a financial institution. However, we can best guess the possible effects of some risk management actions: Table 11 reviews some active management actions and their possible impact on the parameters of the loss distributions, either in frequency or in severity.

Insert Table 11 approximately here

On the basis of the first four actions listed in Table 11, we have performed a sensitivity analysis on both the expected losses and the economic capital required. This kind of analysis serves the purpose of indicating where managers should target their efforts in order to undertake the most efficient actions, be it in terms of their impact on regulatory capital, or in terms of loss reduction.

**7.2. Impact of ORM on the RAROC**

The methodology developed here produces the necessary tools to estimate the quantitative impact of these approaches on the RAROC and, in turn, on the tariffs applicable to financial products.

RAROC stands for *Risk Adjusted Return on Capital*. This performance measure – initially developed by consultant experts in the banking sector in the early nineties – expresses the adjusted return of an investment for its risk, related to the economic capital consumed when undertaking this investment. RAROC calculations may be equally well applied to a single transaction (a loan authorization, for instance), a client (e.g. the total business generated with a given client), a segment of clientele (retail, SME’s...), or even a business unit.

The general formula for RAROC writes:

$$RAROC = \frac{Revenues - EL}{Economic Capital} \tag{4}$$

The adjustment for risk in RAROC takes place both at the numerator and the denominator of the ratio. The nominal return of the investment considered is first adjusted by reducing its amount by the expected losses (EL) that are assessed for a transaction of this type. The expected losses can be defined as the average losses previously observed for similar operations.

The denominator – the economic capital – also reflects the risk taken with a transaction, since it is the capital internally calculated as the amount of own funds necessary to cover the losses with the confidence interval required for this activity.

Until recently, the RAROC performance measure has been mostly used in the credit activities of banks. The underlying idea is to make sure that the revenues generated by a loan or by a client are sufficient to cover the remuneration of the regulatory capital that it consumes.

With the Basel Accord now defining regulatory capital for operational risks as well, banks should apply an analogous RAROC approach with operational risk. In the spirit of this foreseeable evolution of the sector, we are left with the task of adapting the general RAROC formula to the specific case of operational risk.

In order to obtain a proper RAROC measurement adapted to operational risk, we must introduce :

- the Expected Losses due to operational events;
- the Economic Capital necessary to cover the unexpected operational losses;
- the Revenues generated by taking operational risks.

The first two inputs are readily obtained with our methodology, as the fitted (multivariate) distribution of operational losses provides both the expected aggregate loss and the quantile for the regulatory 99.9% OpVaR used to determine regulatory capital. Note that the SA and the BIA do not entail the computation of the expected losses. Nevertheless, we consider that a natural way to tackle this measurement is to fit a distribution to the entire set of observed losses and to take the expected value of this distribution for each cell. For our dataset, this yields fitted lognormal distributions with parameters (0.964 ; 2.604) for “Asset Management/Private Banking” and (0.262 ; 2.510) for “Retail Banking”. Of course, this rather arbitrary method is to be considered for the sole purpose of illustration.

The estimation of the revenues associated with operational risks represents a more complex challenge. In RAROC, revenues equal the gross pre-tax margin generated by an activity, plus fees and other revenues; all costs besides the risk costs that are represented by the EL are neglected. But unlike credit risk whose counterpart in revenues can be identified, we reach here the fundamental question of the existence of operational revenues in counterpart of operational risks. Strictly speaking, operational revenues are null. We plead for a less restrictive view, though, since even pure market or credit activities, and *a fortiori* those that generate other types of revenues like the fee business (asset management, private banking, custody, payments and transaction) involve relatively large components of business risk as well as operational risk that call for compensation through an adequate tariff policy. These risks are no longer mapped on particular securities or portfolios like credit or market risks, but at a broader level on the aggregate profit center that generates them.

We do acknowledge the existence of business risk in activities such as asset management, retail banking or private banking. However, Basel II does not prescribe regulatory capital to cover business risk. Since we cannot distinguish operational risk from business risk in these activities, we will make some assumptions regarding the proportion of total revenues that are generated as a counterpart for operational risk.

In the spirit of this discussion, the “operational” RAROC of business line  $i$  writes:

$$RAROCO(i) = \frac{Gross\ income_{op}(i) - EL_{op}(i)}{Economic\ Capital_{op}(i)} \quad (5)$$

Based on the quantitative analysis in the previous sections, we have all data needed to calculate the Operational RAROC (RAROCO): Revenues, Expected Losses, and Economic Capital. We start by assuming that the revenues due to operational risk represent a fixed proportion of the revenues generated by a business line. We set this proportion to an arbitrary 5% for the sake of the illustration, although we do not view this percentage as unrealistic. The results are reported in the first three columns of Panels A and B in Table 12:

Insert Table 12 approximately here

In Panel B of Table 12, we obtain that the RAROCO for the business line “Asset Management & Private Banking” is equal to 27.7%, 34.62% and 22.58% with the Basic Indicator Approach, the Standardized Approach and the Advanced Measurement Approach, respectively. For the “Retail Banking” Business line, we get values of 29.46% for BIA, 36.83% for SA and 23.12% for AMA. Overall, the RAROC is maximized when one uses the AMA with explicit account for dependence between cells, with a value of 36.26%

Once again, the Standardized Approach seems to yield the most favorable rates of return for this particular sample. Yet, these results are extremely contingent on the choice of the expected loss for the BIA and the SA, whose impact on RAROC is very important. Of much more interest is the great improvement that can be achieved by the dependence-corrected RAROC, showing an increase of more than 50% in risk-adjusted profitability and closely matching the very generous estimates for the BIA and SA.

### 7.3. Sensitivity analysis

Table 11 presented several possible risk management actions that can and could be implemented in financial institutions entering the AMA. We detail here the impact of the first four lines: the “Lessons Learned”, assumed to limit the risk of future large losses by removing the  $k$  largest losses for the cells under consideration; the use of “Dashboards”, meant to reduce the expected frequency of events by  $x\%$ ; the “audit tracking”, aimed at reducing the expected frequency by  $x\%$  and the magnitude of the

severity by  $y\%$  for the “Internal Fraud” and “Execution, Process and Delivery Management” event types; and finally the “Business Line Reorganization”, whose purpose is to reduce frequency and severity of the losses in business line  $i$  by  $x\%$  and  $y\%$  respectively. Other actions provided in Table 11 are for illustrative purposes only, as they focus on business lines and/or event types that are not covered in our 4-cells study.

To run our analysis we assume the following scenario: the Bank has adopted the AMA approach to compute the capital charge for operational risk and it has the stated objective of a target *Return On Equity* (ROE) of 12%. Target profitability levels are indeed frequently used in practice, and this one corresponds to a very common value. This corresponds to a RAROC hurdle rate at 18% after accounting for the tax rate, since RAROC is a performance measure before tax. Moreover, the Board of the Bank intends to reduce the Expected Loss (EL) by 15% for strategic purposes. The Risk Manager is thus asked to look at different possible actions and to run a cost-benefit analysis of each of these actions.

We adopt a two-stage approach to solve this problem: first, we assess the performance (i.e. the value of the parameters  $k$ ,  $x$  and  $y$  above) required for each action in order to reach a 15% reduction of the EL. Then, we couple these performance requirements with the return constraint (i.e. keeping a RAROCO equal to 18%) to measure the maximum acceptable cost for each action. If the cost to reach the performance requirements for a given action is higher than this maximum acceptable cost, the action is rejected. Otherwise, it can enter into consideration.

To reach a 15% reduction of the EL<sup>11</sup>, the various actions must fulfill some performance requirements:

- The “Lessons Learned” action allows an overall 15% reduction of the EL if the parameter  $k$  is 2. Note that in this case, the EL reductions for the four cells are 37%, 15%, 18% and 11% for “Asset Management / Clients...”, “Asset Management / Execution,...”, “Retail Banking / Clients...” and “Retail Banking / Execution...”, respectively.
- The “Dashboards” action can obviously provide the wanted reduction by decreasing the EL of each “cell” by 15%, which corresponds to a reduction of loss frequency of 15%, 14%, 15% and 14% for “Asset Management / Clients...”, “Asset Management / Execution,...”, “Retail Banking / Clients...” and “Retail Banking / Execution...”, respectively. This indicates a quasi-linear relationship between frequency and Expected Loss.
- The “Audit Tracking” action only impacts the “Internal Fraud” and “Execution, Process and Delivery Management” event types. As we do not cover “Internal Fraud” in our examples, only two cells will be impacted by this action: “Asset Management / Execution...” and “Retail Banking

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<sup>11</sup> In our case, the EL must be reduced from about 855.000 units to about 725.000.

/ Execution...”. The estimated severity reduction is quite low with this type of action, so that we assume a 4% severity reduction for these two cells. The needed frequency reductions are then 15% and 13%, respectively.

- The “Business Line Reorganization” only impacts the related business line, of course. In our case, we assume a reorganization of the Retail Banking business line. To illustrate an action having a larger impact on severity than the “Audit Tracking” one, we assume a 10% reduction of severity for the “Retail Banking” business line. This leads to an additional requirement of a 12% reduction of the loss frequency for this activity.

Table 13 summarizes the impact of these actions on the important input of the bank’s profitability.

Insert Table 13 approximately here

Notice that different actions, while leading to the same reduction of the expected losses, have different impacts on the unexpected loss and thus on the regulatory capital. Comparison of the “Lessons Learned” and “Dashboards” effects reveals that the effort to reduce frequency of losses appears to be more effective than the cutting of the more extreme ones. In general, our analysis renders more favorable outcomes for actions whose targets are more directed to the business line “Retail Banking” (cells (2,1) and (2,2)). Due to the calibration of the loss distribution function performed in Table 6 for this Business Line, the effect of a reduction in expected loss on the unexpected loss – and thus on the OpVaR – is greater than for the “Asset Management” Business Line. This phenomenon is not readily observable from the values reported in Table 6, and is apparently not related to the balance of efforts between frequency and severity of losses either. Although one could argue that the high weight of cell (2,2) drives the impact of managerial actions on regulatory capital, this conjecture is invalidated by the relatively lower impact of the “Audit Tracking” scenario, directly targeting this cell, with respect to the “Dashboards” scenario that splits its effect throughout the matrix. Rather, the impact of managerial actions seems to be due to the particular calibration of the distribution functions for the aggregate losses, as the non-systematic behavior of the relationship between expected and unexpected losses suggests.

Table 14 provides an overview of the major results linked with a successful implementation of these actions.

Insert Table 14 approximately here

In all cases, by reducing the EL and the Economic Capital, operational risk management measures improve the RAROC performance of the Business Lines to a significant extent. Panel B of Table 14 shows that RAROC increases from 23.2% to around 27% after completion of the management actions.

If the performance requirements described above are met, the loss reduction objective subject to the profitability constraint is achieved. But such actions are not for free and a cost-benefit analysis is needed to ensure the costs associated with putting the action into place are less than the benefits it provides. To do so, we can apply a backward reasoning and assess the percentage of revenues needed to cover the operational expected losses, and the operational economic capital, in order to maintain a RAROC at 18%. The lower this percentage, the better it is for the business line.

This more normative method allows us to draw a direct relationship between the cost supported by a financial institution to bring some corrective measures to its operational risk exposure and the impact of this particular action on its risk exposure. Specifically, we obtain values of 4.37%, 3.89%, 3.80%, 3.86% and 3.78% for the default AMA, “Lessons Learned”, “Dashboard”, “Audit Tracking” and “Business Line Reorganization” actions, respectively. The maximum acceptable cost is simply the difference between the revenues needed to cover operational risk before and after the action is undertaken.

Table 14 provides the maximum acceptable cost associated with each action, both in terms of currency units (Panel C) and percentage of total income (Panel D). For instance, if the costs of launching a “Dashboards” action, allowing for a 15% reduction of the EL, amounts to less than 230,000 currency units, then the required profitability level is maintained and the action can be accepted.

Of course, the assessment of the business implications of these figures for the desirability of such actions for an individual bank is beyond the scope of this paper. At the very least, our approach may enable financial institutions to readily and consistently assess the impact of their risk mitigating decisions on their tariffs and, ultimately, on their profitability.

## **8. Concluding remarks**

In this paper, we have attempted to provide some elements to consistently address two major issues triggered by the emergence of operational risk coverage in the scope of the Basel II Accord. The red wire of our approach was the constant care about the trade-off between parsimony in the parameterization of the distribution of operational losses, and the accuracy of the resulting fit.

As for the first research question, namely the cost-benefit analysis of adopting the AMA for covering operational losses instead of a less sophisticated method, two major conclusions can be drawn.

First, the dependence structure of operational losses per business lines and/or event types and the behavior of extremely large losses reported by other institutions in an external database are both likely to significantly affect the cost-saving properties of the AMA choice. Since this approach aims at capturing rare events, it tends to be overly conservative when the basic assumption of additive capital charges (perfect correlation) is adopted. The reduction in risk exposure is significant when dependence is taken into account in a reasonable way. More surprisingly, a proper handling of external data allows a refinement of the analysis of the tail of the aggregate loss distribution, which may lead to reduction of operational capital charges thanks to the richer sample of high severity data.

Second, the differential capital charge between the Standardized Approach and the AMA, and thus the opportunity cost of adopting a heavy operational risk management system, significantly hinges on the discretionary weight assigned to the business lines. For the selected business lines, the Basel Committee has set the lowest weighting coefficients (12%) among the partition of activities of financial institutions. On the basis of our results, these low values result in quite limited gains in regulatory capital charges by the AMA approach. This leaves a very important question open: is the choice of these beta coefficients by the Basel Committee likely to favor the adoption of the SA by financial institutions whose activities lean towards the business lines with lower weighting coefficients? If this is the case, one should observe in the near future a banking behavior towards “regulatory arbitrage”, where financial institutions would be eager to select the most favorable approach on the basis of the more or less important advantage brought by beta coefficients that are most relevant to their activities. Alternatively, and probably more prudently, one may argue that the beta coefficients of the Standardized Approach represent an average proportion of gross income that should be allocated as regulatory capital by financial institutions. As we have mentioned earlier, unlike credit or market risk, operational risk is primarily a matter that is internal and specific to each bank. Therefore, the choice of the SA may be favorable to some banks whose actual risk is greater than average, and unfavorable to others. A study focusing on a single institution is by no means able to answer the crucial question of the fairness of the beta coefficients, but this paper at least provides useful information for the bank under study about the implications of the alternative

Our answer to the second question, namely the cost-benefit of adopting a full-fledged operational risk management system, has slightly less normative content than methodological substance. We have shown that the concept of RAROC can be adopted if adapted in conjunction with our modular estimation technique in order to yield a mapping between the results of a particular action and its associated cost. With controlled scenarios, we have documented that managerial actions are likely to bring significant improvements on the risk-adjusted profitability of the institution.



Interestingly, the arbitrage between different managerial actions is not necessarily tied to the focus on the number, frequency or severity of the losses, but rather to the very distributional behavior of the aggregate loss for each business line and event type. This finding suggests that the analysis of risk management effects is all but trivial. A proper examination of managerial efforts to reduce operational risks intimately builds on a thorough understanding of statistical characteristics of operational losses.

We believe that this experiment contributes to understanding the profit side of operational risk management, and should usefully be matched with a more industrial view on the cost-side of these types of actions, which is beyond the scope of our study of course.

Both aspects of this research may bear numerous extensions, provided more extensive and robust databases are made available. We emphasize that this paper has to be primarily taken for its methodological aspects, even though the use of real data may have contributed to the persuasiveness of our results. Only when banks have eventually collected operational data – on loss events but also on corrective devices – on a systematic basis, the full potential of this very promising area of research will find a practical achievement.

## **References**

- Alexander, C., (2003): *Operational Risk: Regulation, Analysis and Management*, FT Prentice Hall, London.
- Balkema, A. A. and de Haan, L. (1974): "Residual life time at great age", *Annals of Probability*, 2, 792-804.
- Bank of America, (2003): "Implementing a comprehensive LDA", Proceedings "Leading Edge Issues in Operational Risk Measurement" Conference, May 2003, New York Federal Reserve
- Baud, N., Frachot, A., and Roncalli, T. (2002): "Internal data, external data and consortium data for operational risk measurement: How to pool data properly", Working Paper, Groupe de Recherche Opérationnelle, Crédit Lyonnais.
- Basel Committee on Banking Supervision (2003a): "The New Basel Capital Accord," Consultative Document.
- Basel Committee on Banking Supervision (2003b): "Sound practices for the management and supervision of operational risk".
- Crouhy, M., Galai, D. and Mark, R. (2001): *Risk Management*, McGraw Hill, New York.
- Cruz, M. G. (2002): *Modeling, Measuring and Hedging Operational Risk*, Wiley Finance, New York.
- Dupuis, D. J. (1998): "Exceedances over high thresholds: A guide to thresholds selection", Working Paper, Department of Engineering Mathematics, Dalhousie University, Halifax.
- Embrechts, P., Furrer, H. and R. Kaufmann (2003): "Quantifying regulatory capital for operational risk", Working Paper, RiskLab, ETH Zürich.
- Embrechts, P., Klüppelberg, C., and Mikosch, T. (1997): *Modelling Extremal Events for Insurance and Finance*, Springer Verlag, Berlin.
- Embrechts, P., McNeil, A., and Strautmann, D. (2002): *Correlation and Dependence in Risk Management: Properties and Pitfalls*, In "Risk management: value at risk and beyond", M.A.H. Dempster (Ed.), Cambridge University Press, Cambridge.
- Fontnouvelle, de, P., Jordan, J. and E. Rosengren (2003): "Using loss data to quantify operational risk", Working Paper, Federal Reserve Bank of Boston.
- Frachot, A., P. Georges, and T. Roncalli (2001): "Loss distribution approach for operational risk", Working Paper, Groupe de Recherche Opérationnelle, Crédit Lyonnais.
- Frachot, A., O. Moudoulaud and T. Roncalli, (2003): "Loss distribution approach in practice", Working Paper, Groupe de Recherche Opérationnelle, Crédit Lyonnais.

- Frachot, A., and Roncalli, T. (2002): “Mixing internal and external data for managing operational risk”, Working Paper, Groupe de Recherche Opérationnelle, Crédit Lyonnais.
- Frank, M. J. (1979): “On the simultaneous associativity of  $F(x,y)$  and  $x+y-F(x,y)$ ”, *Aequationes Mathematicae*, **19**, 194-226.
- Frey, R., McNeil, A.J. and M.A. Nyfeler (2001): “Copulas and credit models”, *RISK*, October 2001.
- Hartung, T. (2003): “Considerations to the quantification of operational risks”, Working Paper, University of Munich.
- Hoffman, D. G., (2002): *Managing Operational Risk: 20 Firmwide Best Practices Strategies*, John Wiley & Sons Ed., New York.
- Industry Technical Working Group on Operational Risk, (2003): “An LDA-Based Advanced Measurement Approach for the Measurement of Operational Risk: Ideas, Issues and Emerging Practices”, Proceedings "Leading Edge Issues in Operational Risk Measurement" Conference, May 2003, New York Federal Reserve.
- Jorion, P. (2003): *Financial Risk Manager Handbook*, 2nd edition, Wiley Finance.
- King, J. L. (2001): *Operational Risk, Measurement and Modelling*, Wiley Finance, New York.
- Këllezli, E., and M. Gilli (2003): “An Application of Extreme Value Theory for Measuring Risk”, Working Paper, University of Geneva.
- Klugman, S.A., and Parsa, R., (1999): “Fitting bivariate loss distributions with copulas”, *Insurance: Mathematics and Economics*, **24**, 139–148.
- Mashal, R. and Zeevi, A. (2002): “Beyond correlation: Extreme co-movements between financial assets”. Working Paper, Columbia Business School.
- McNeil, A. J. (2000): “Extreme Value Theory for Risk Managers”, in: P. Embrechts (Ed.), *Extremes and Integrated Risk Management*, Risk Books, London.
- Nelsen, R. B. (1999): *An Introduction to Copulas*, Springer, New York.
- Peters, J.-P., Crama, Y. and Hübner, G. (2004): "An algorithmic approach for the identification of extreme operational losses threshold", Working Paper, Université de Liège.
- Pickands, J. (1975): “Statistical inference using extreme order statistics”, *The Annals of Statistics*, **3**, 119-131.
- Shih, J., Samad-Khan, A.H. and Medapa, P. (2000): “Is the size of an operational risk related to firm size?”, *Operational Risk*, January 2000.

Figure 1

Splitting of the losses

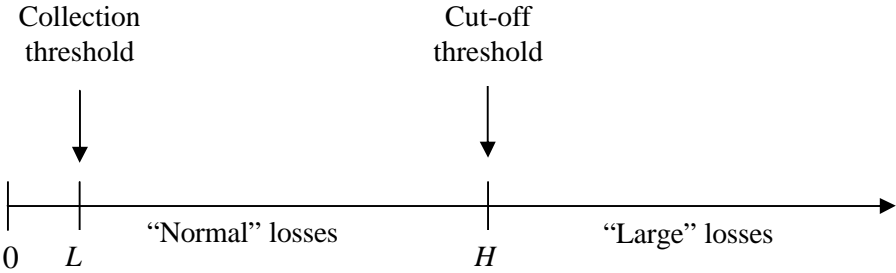
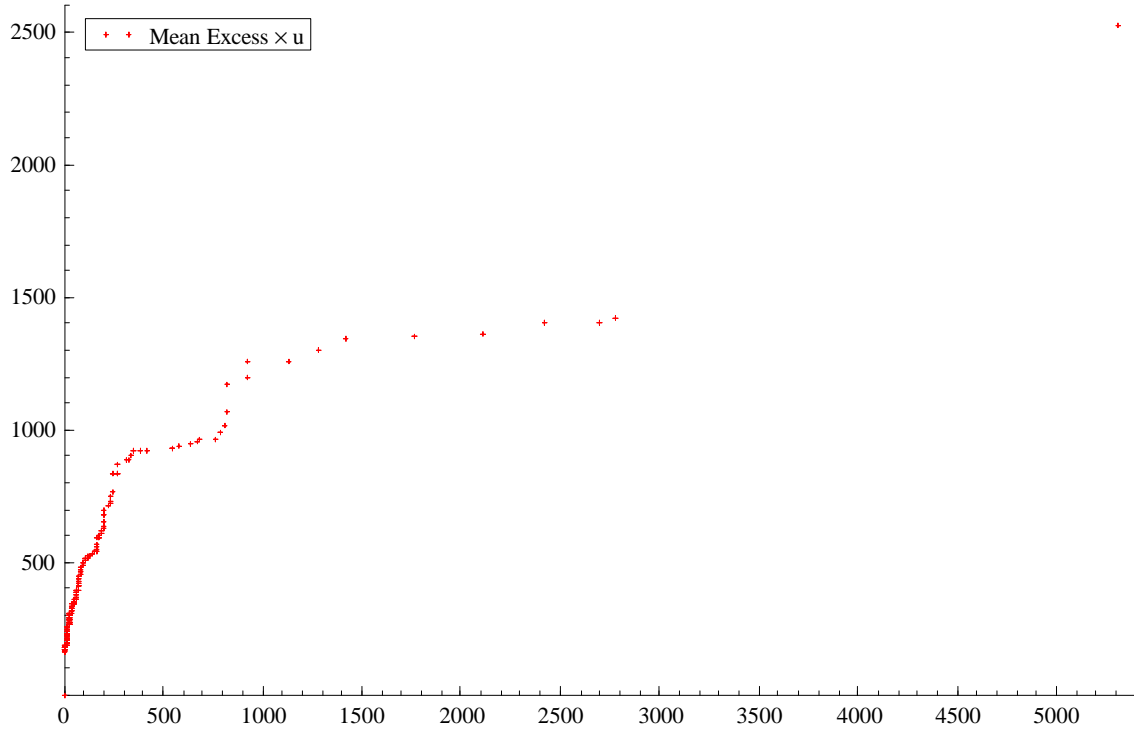


Figure 2

Mean Excess Plot (MEP) for the cell “Retail Banking / Clients, Products and Business Practices” (Cell (2,1))



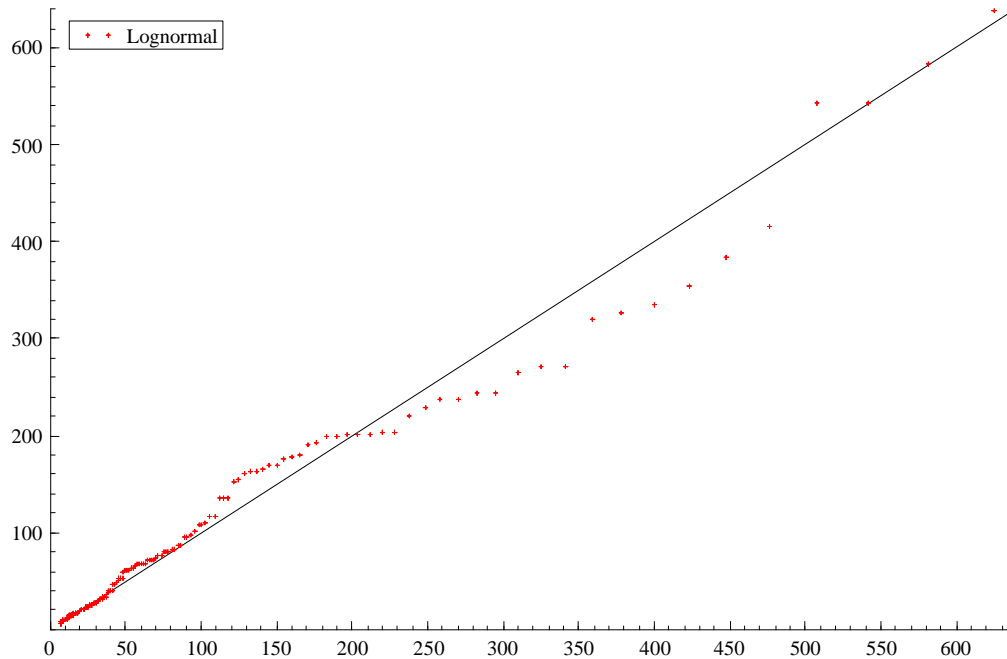
This graph represents the function  $\{(X_{k,n}, e_n(X_{k,n})) : k = 1, \dots, n\}$  where  $e_n$  is the empirical mean excess function whose mathematical expression is

$$e_n(u) = \frac{1}{n_u} \sum_{i=1}^{n_u} (X_i - u), \quad u \geq 0.$$

where  $u$  is the threshold and the  $X_i$ 's are the  $n_u$  observations such that  $X_i > u$ . The MEP should be approximately a straight line with slope  $\mathbf{x} / (1-\mathbf{x})$ . So the goal is to detect a significant shift in slope at some high point. When the empirical plot seems to follow a reasonably straight line with positive gradient above a certain value, this indicates a heavy-tailed distribution.

**Figure 3**

Quantile-Quantile plot (QQ-plot) for the “normal” losses (i.e. < 675) with a Lognormal (0.86, 2.84) distribution for the cell “Retail Banking / Clients, Products and Business Practices”.

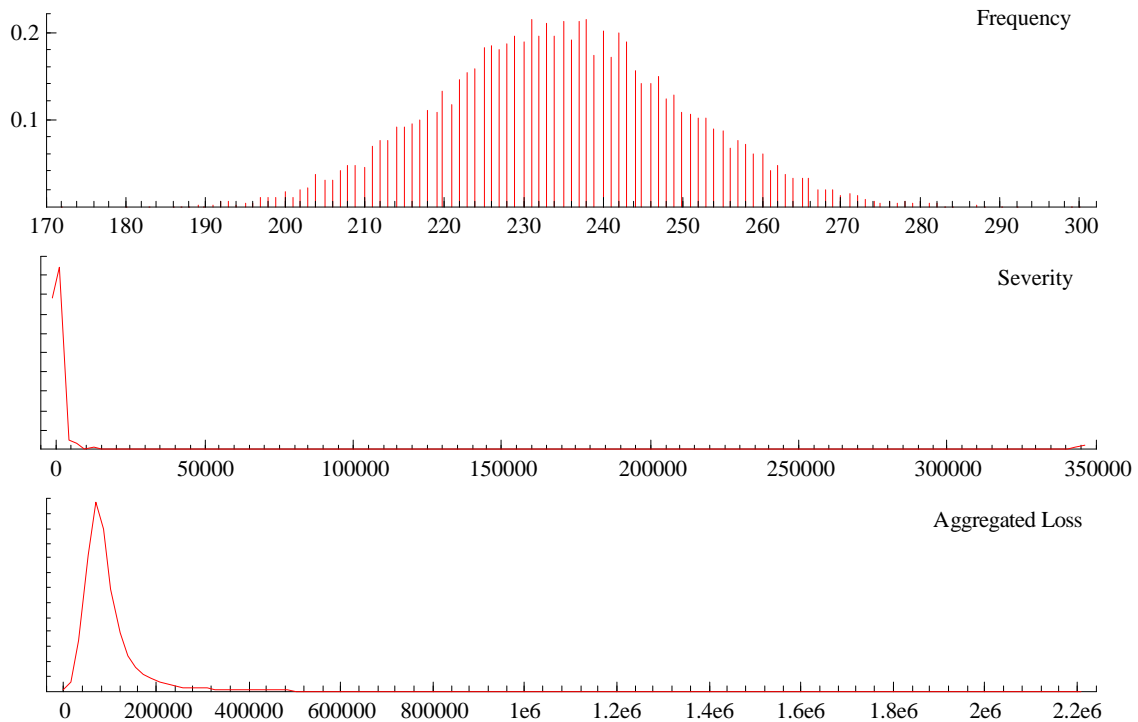


This graph plots the sample quantiles of X versus theoretical quantiles of the tested distribution, which is a Lognormal (0.86, 2.84) in this example. If the fit is good, the points should form a straight line.

The QQ-Plot can thus be expressed as  $\left\{ \left( X_{k,n}, F^{\leftarrow} \left( \frac{n-k+1}{n+1} \right) \right) : k = 1, \dots, n \right\}$ .

Figure 4

Frequency, Severity and Aggregated Loss distributions for the cell “Retail Banking / Clients, Products and Business Practices”



These graphs plots the three distributions involved in the computation of the regulatory capital charge. The upper graph is the frequency distribution, fitted here with a  $\text{Poisson}(235)$  distribution. The middle graph is the severity distribution, composed of a mix of three distributions: a  $\text{lognormal}(0.86, 2.84)$  for the losses under 675, a  $\text{GPD}(0.735, 542)$  for the losses between 675 and the external data threshold and a  $\text{Lognormal}(9.04, 1.53)$  for the losses above this threshold. The lower graph is the  $n$ -fold convolution of the severity distribution with itself ( $n$  being a random variable that follows the selected frequency distribution), obtained with 10,000 Monte Carlo simulations.

Figure 5

Integration of external data to model the tail of the severity distribution

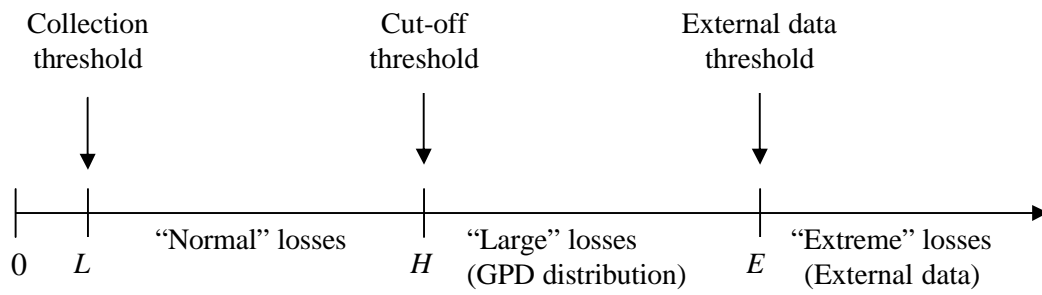




Table 1

Summary statistics for the operational loss database

**Panel A: Operational losses per business lines**

Business Lines	Amounts of Losses	Max Loss Amount	Average Loss Amount	Number of Losses
Corporate Finance	874	428	175	5
Trading & Sales	62,638	10,197	1,027	61
Retail Banking	368,686	51,051	181	2,033
Private Banking	382,047	347,115	3,537	108
Commercial Banking	9,631	3,032	419	23
Payment and Settlement	70,653	19,050	1,070	66
Agency Services	11,194	228	95	118
Asset Management	166,832	91,878	285	585
Retail Brokerage	102	102	102	1
<i>Total</i>	<i>1,072,554</i>	<i>347,115</i>	<i>358</i>	<i>3,000</i>

**Panel B: Operational losses per event types**

Event Type	Amounts of Losses	Max Loss Amount	Average Loss Amount	Number of Losses
Internal fraud	55,470	51,051	6,163	9
External fraud	131,754	15,706	308	428
Employment practices & workplace safety	5,142	2,487	1,714	3
Damage to physical assets	0	0	-	0
Clients, Product & Business Practices	411,601	347,115	1,169	352
Business disruption & system failures	17,797	2,055	157	113
Execution, Delivery & Process Management	450,790	91,878	215	2,095
<i>Total</i>	<i>1,072,554</i>	<i>347,115</i>	<i>358</i>	<i>3,000</i>

This table reports summary statistics of our sample of operational losses. The monetary unit has been rescaled with a constant amount. The time scale has been set to the necessary length to collect 3000 losses. All amounts reported in the table are in hundreds of units.

Table 2

Summary statistics for the selected sub-sample

Panel A: Number of events for the selected sub-matrix

Business line	Event type		<i>Total</i>
	Clients, Product & Business Practices	Execution, Delivery & Process Management	
Private Banking + Asset Management	75	613	688
Retail Banking	235	1,395	1,630
<i>Total</i>	310	2,008	2,318

Panel B: Descriptive statistics for the selected cells

	Business Line &Event Type			
	Asset Mgmt + Private Banking / Clients, ... (Cell 1,1)	Asset Mgmt + Private Banking / Execution ... (Cell 1,2)	Retail Banking / Clients, ... (Cell 2,1)	Retail Banking / Execution, ... (Cell 2,2)
No. Obs.	75	613	235	1,395
Median	51	28	29	20
Mean	4,797	303	178	100
Std. Dev.	40,063	3,812	505	471
Total loss	359,781	186,009	41,747	139,479

This table reports summary statistics of our selected sub-sample of operational losses. The cell coordinates correspond to the ones reported in Table 2.a.

Table 3

Selection of the cut-off threshold for Cell(2,1)

Threshold	$\sigma$	$\xi$	CVM	Percentage of extreme losses
300	1720.7	0.337	1.123	11.06%
325	1677.7	0.346	1.003	10.64%
350	2439.7	0.274	1.444	9.79%
375	2650.8	0.261	1.485	9.36%
400	2945.6	0.244	1.553	8.94%
425	3357.9	0.224	1.650	8.51%
450	2520.3	0.274	1.211	8.51%
475	1874.4	0.335	0.825	8.51%
500	1365.4	0.409	0.500	8.51%
525	956.3	0.505	0.244	8.51%
550	1541.9	0.390	0.476	7.66%
575	1079.3	0.485	0.240	7.66%
600	1225.2	0.457	0.269	7.23%
625	814.5	0.579	0.110	7.23%
650	915.3	0.553	0.118	6.81%
<b>675</b>	<b>542.6</b>	<b>0.735</b>	<b>0.060</b>	<b>6.81%</b>
700	1508.4	0.425	0.217	5.96%
725	929.3	0.571	0.082	5.96%
750	475.6	0.823	0.096	5.96%
775	621.0	0.737	0.059	5.53%
800	1214.8	0.513	0.099	5.10%

This table reports relevant fitted parameters for different candidate thresholds. Estimates  $\sigma$  and  $\xi$  correspond to the parameters estimates of the GPD distribution. The Cramer-von Mises' goodness-of-fit statistic (CVM) is reported for each threshold. The last column represents the percentage of extreme losses for a given threshold. The line corresponding to the selected threshold is reported in bold.

Table 4

Calibration of the fitted distributions for the cell Retail Banking / Clients, Products and Business Practices (Cell (2,1))

Distribution	"Normal Losses"			"Extreme Losses"
	Gamma	Weibull	Lognormal	GPD
Parameter 1	0.00007	1.0848	0.8614	0.7351
Parameter 2	0.00397	0.2247	2.8399	542.47
Log-likelihood	-1090.8	-1089.1	-1087.5	
KS	0.0811	0.0475	0.0485	-
CVM	0.3642	0.0626	0.0592	-
AD	1.7727	0.3822	0.3766	-

This table reports calibrated parameters (using MLE) and goodness-of-fit statistics for the body ("Normal Losses") and tails ("Large Losses") of the severity distribution of the cell Retail Banking / Clients, Products and Business Practices. The threshold separating both parts of the distribution is 675.

Table 5

Comparison of results obtained with only internal data versus external data beyond the cut-off threshold for the cell “Retail Banking / Clients, Products and Business Practices”

	Data used beyond cut-off threshold	
	Internal Measurement	External Data
Real Total Loss	41,747	41,747
Median	77,639	78,692
Mean	94,947	100,095
OpVaR90	139,937	159,452
OpVaR95	183,015	219,599
OpVaR99	373,095	447,089
OpVaR99.5	528,393	579,121
OpVaR99.9	1,466,094	1,157,484
OpVaR99.95	1,859,723	1,384,734

This table presents estimates of the OpVaR for the selected cell. The fitted distributions are a lognormal (0.86, 2.84) for the “normal” losses (from the collection threshold,  $L$ , to the cut-off threshold,  $U$ , in Figure 5), a GPD (542, 0.735) for the “large” losses (from  $U$  to the external data threshold,  $E$  in Figure 5) and a lognormal (9.04, 1.53) for the “extreme” losses (above  $E$  in Figure 5).

**Table 6**

Calibration of the fitted distributions using EVT and external data to model the tail

	Business Line &Event Type			
	Asset Mgmt / Clients...	Asset Mgmt / Execution...	Retail Banking/ Clients...	Retail Banking / Execution...
“Normal Losses”	Pareto	Log-Normal	Log-Normal	Log-Normal
Parameter 1	6.79	2.33	0.86	1.62
Parameter 2	0.06	1.78	2.84	0.17
Threshold	375	350	675	225
Percentage of losses above the threshold	10.7 %	6.5 %	6.8 %	6.6 %
GPD 1 ( <b>x</b> )	1.938	0.852	0.735	0.991
GPD 2 ( <b>s</b> )	183	677	542	205
External data - Parameter 1 ( <b>m</b> )	10.02	9.11	9.04	10.02
External data - Parameter 2 ( <b>s</b> )	1.45	1.54	1.53	1.67
Real Total Loss	359,781	186,009	41,747	139,479
Median	35,158	178,140	78,692	334,888
Mean	48,873	220,789	100,095	485,652
OpVaR_99	319,665	862,272	447,089	2,476,987
OpVaR_99.5	499,438	1,117,328	579,121	2,979,825
OpVaR_99.9	1,098,948	1,844,714	1,157,484	3,807,644
<b>Regulatory Capital</b>	<b>1,050,075</b>	<b>1,623,925</b>	<b>1,057,389</b>	<b>3,321,992</b>

This table reports distributional properties (“Normal Losses”, “Large Losses” and “Extreme Losses”) of the severity of losses in the cells identified in Table 2. Three distributions are compared based on goodness-of-fit tests and the best one is used to model the “normal” losses. “Large” losses are modeled by the Generalized Pareto distribution while external data are fitted with a lognormal distribution. The mean loss is the proxy for the Expected Loss.

Table 7

90%-confidence intervals for the capital charge

	Lower bound	Estimate	Upper bound
Cell(1,1)	750,579	1,050,075	1,349,571
Cell(1,2)	1,216,364	1,623,925	2,031,486
Cell(2,1)	696,580	1,057,389	1,418,198
Cell(2,2)	3,032,751	3,321,992	3,611,233
<b>Total – ind.</b>	<b>5,696,274</b>	<b>7,053,381</b>	<b>8,410,488</b>
<b>Total – dep.</b>	<b>6,368,072</b>	<b>7,053,381</b>	<b>7,738,690</b>
Cell(1,1)	71%	100%	129%
Cell(1,2)	75%	100%	125%
Cell(2,1)	66%	100%	134%
Cell(2,2)	91%	100%	109%
<b>Total – ind.</b>	<b>81%</b>	<b>100%</b>	<b>119%</b>
<b>Total – dep.</b>	<b>90%</b>	<b>100%</b>	<b>110%</b>

This table reports intervals obtained through a bootstrapping approach using 500 iterations and applying the Central Limit Theorem with the quantiles estimates. It is assumed that  $CC_i \sim N(\mu_i, \sigma_i)$ , with  $i = 1, \dots, 4$  and where  $CC_i$  is the Capital Charge of cell  $i$ ,  $\mu_i$  is the mean of the capital charge estimates for cell  $i$  and  $\sigma_i$  is the standard deviation of the capital charge estimates for cell  $i$ . If the individual capital charges estimates are assumed to be independent, then  $TCCI \sim N(\mu_T, \sigma_T)$  where  $TCCI$  is the total capital charge,  $\mathbf{m}_T = \sum_{i=1}^4 \mathbf{m}_i$  and  $\mathbf{s}_T = \sqrt{\sum_{i=1}^4 \mathbf{s}_i^2}$ . If the individual capital charges estimates are assumed to be fully positive dependent, then  $TCCD \sim N(\mu_T, \sigma_T)$  where  $TCCD$  is the total capital charge,  $\mathbf{m}_T = \sum_{i=1}^4 \mathbf{m}_i$  and  $\mathbf{s}_T = \sum_{i=1}^4 \mathbf{s}_i$ .

Table 8

Spearman's rank correlations between frequencies of risks

Panel A. Spearman's rank correlation matrix for the selected cells

	Cell(1,1)	Cell(1,2)	Cell(2,1)	Cell(2,2)
Cell(1,1)	1.000			
Cell(1,2)	-0.253	1.000		
Cell(2,1)	0.565	0.000	1.000	
Cell(2,2)	-0.408	0.260	-0.248	1.000

Panel B. Spearman's rank correlation matrix for the selected business lines

	Asset Management / Private Banking	Retail Banking
Asset Management / Private Banking	1.000	
Retail Banking	0.155	1.000

Panel C. Spearman's rank correlation matrix for all the business lines

	Trading & Sales	Retail Banking	Commercial Banking	Payment & Settlement	Agency Services	Asset Mgt / Private Bkg
Trading & Sales	1.000					
Retail Banking	0.162	1.000				
Commercial Banking	0.000	0.084	1.000			
Payment & Settlement	0.299	0.253	0.197	1.000		
Agency Services	0.293	0.156	0.565	0.423	1.000	
Asset Management / Private Banking	0.063	0.155	0.269	0.187	0.291	1.000

This table reports the rank correlation coefficient (Spearman's rho) between observations of operational loss events in the selected cells. Two business lines (Corporate Finance and Retail Brokerage) have not been considered due to the very low number of observations (5 and 1, respectively).



Table 9

Comparison of results obtained with different dependence assumptions.

	Full positive dependence	Dependence between business lines	Dependence between cells	Independence
Real Total Loss	727,014	727,014	727,014	727,014
Median	626,878	666,647	698,404	699,953
Mean	855,409	846,516	853,259	849,140
OpVaR_95.00	2,139,472	1,947,965	1,811,360	1,781,489
OpVaR_99.00	4,106,013	3,320,426	3,032,034	2,868,935
OpVaR_99.90	7,908,790	5,379,562	4,487,290	4,192,533
OpVaR_99.95	9,184,315	6,098,291	4,863,845	4,390,021
<b>Capital Charge</b>	<b>7,053,381</b>	<b>4,533,046</b>	<b>3,634,030</b>	<b>3,343,393</b>

This table reports capital charge estimates under different dependence assumptions. The full positive dependence is Basel II's default assumption. Insurance policies are taken into account.

Table 10

Comparison of total capital charges

	Capital Charge		Base 100	
Basic Indicator Approach	7,470,036	100	106	125
Standardized Approach	5,976,029	80	100	85
AMA – Full dependence	7,053,381	94	118	100
AMA – BL dependence	4,533,046	61	76	64
AMA – Cell dependence	3,634,030	49	61	52
AMA – Independence	3,343,393	45	56	47

This table reports capital charge estimates obtained when adopting the AMA approach (with different dependence assumptions) versus other Basel II approaches. In the third, fourth and fifth columns, the figures have been normalized by the estimates obtained with the BIA, SA and full-dependence AMA figures, respectively, in order to ease comparison.

Table 11

Overview of selected managerial actions and their consequences

<b>Risk Management Action</b>	<b>Impact on the distribution</b>
<u>Lessons learned</u> Analysis of largest losses in Business Line (BL) “i”	Cut off the $x$ top losses, all Business Lines
<u>Dashboard</u> Systematic reduction of events in BL “i”, event types “j,k,l”	Minus $x\%$ in the number of events in Business Line “i”, for the event types “j,k,l”.
<u>Audit tracking</u> Application of audit recommendations in BL “i”	Minus $x\%$ in the number of events in BL “i”, minus $y\%$ in the severity of losses for event types: <ul style="list-style-type: none"> <li>• internal fraud</li> <li>• processing errors</li> </ul>
<u>Business line reorganization</u> New product review process for all BL	Minus $x\%$ in frequency and minus $y\%$ in severity for event types “Clients, products and business practices”,
<i><u>Rapid reaction to OR event</u></i>	<i>Minus <math>x\%</math> in severity, all BL and all event types</i>
<i><u>Business Continuity Plan</u></i>	<i>Minus <math>x\%</math> in severity for event types of business disruption and system failure (if non existent in the original distribution)</i>
<i><u>External Insurance Policies</u></i>	<i>Truncation of the distribution at the level of the amounts insured</i>

This table reports an overview of possible managerial actions taken in order to reduce the exposure to operational risk. The first four actions are examined in the scenario analysis. The actions in italics are reported for illustrative purposes.

Table 12

Base-case effects of operational losses on RAROC

Panel A: Value estimates

	Gross Income	Actual Loss	EL BIA and SA	EL AMA
BL1 – Asset Management/Private Banking	17,463,358	545,790	147.619	269,434
BL2 – Retail Banking	32,336,881	181,226	187.773	604,381
<b>TOTAL</b>	<b>49,800,239</b>	<b>727,016</b>	<b>335.392</b>	<b>873,815</b>

Panel B: Operational RAROC

RAROCO	BIA	SA	Default AMA	Default AMA - Copula
BL1 – Asset Management/Private Banking	27.70%	34.62%	22.58%	-
BL2 – Retail Banking	29.46%	36.83%	23.12%	-
<b>TOTAL</b>	<b>28.84%</b>	<b>36.05%</b>	<b>22.91%</b>	<b>36.26%</b>

Panel C: Compensation levels (in currency units) for operational risk.

Minimum Income	BIA	SA	Default AMA	Default AMA - Copula
BL1 – Asset Management/Private Banking	619,130	524,828	750,754	-
BL2 – Retail Banking	1,060,869	886,250	1,392,670	-
<b>TOTAL</b>	<b>1,679,998</b>	<b>1,411,077</b>	<b>2,143,424</b>	<b>1,662,464</b>

Panel D: Compensation levels (in percentage of total income) for operational risk

Income in % of total revenues	BIA	SA	Default AMA	Default AMA - Copula
BL1 – Asset Management/Private Banking	3.55%	3.01%	4.30%	-
BL2 – Retail Banking	3.28%	2.74%	4.31%	-
<b>TOTAL</b>	<b>3.37%</b>	<b>2.83%</b>	<b>4.30%</b>	<b>3.34%</b>

This table presents the effects of our estimates of operational risks on the bank's risk-adjusted profitability, measured by the Operational RAROC (RAROCO). For Panel B, the operational income is set to 5% of the total revenue. For Panels C and D, target RAROCO is set to 18%. For BIA and SA, the expected loss is taken as the expected value taken for fitted distributions for observed data: Lognormal (0.964 ; 2.604) for BL1 and Lognormal (0.262 ; 2.510) for BL2.

**Table 13**  
Impact of scenarios on the loss distributions

Induced changes	Lessons Learned				Dashboards				Audit Tracking				BL Reorganization			
	(1,1)	(1,2)	(2,1)	(2,2)	(1,1)	(1,2)	(2,1)	(2,2)	(1,1)	(1,2)	(2,1)	(2,2)	(1,1)	(1,2)	(2,1)	(2,2)
number	-2	-2	-2	-2	-	-	-	-	-	-	-	-	-	-	-	-
frequency	-	-	-	-	-15%	-14%	-15%	-14%	-	-15%	-	-13%	-	-	-12%	-12%
severity	-	-	-	-	-	-	-	-	-	-4%	-	-4%	-	-	-10%	-10%
Expected Loss	-37.4%	-15.2%	-18.3%	-11.2%	-14.9%	-14.5%	-15.3%	-15.2%	-	-18.8%	-	-18.0%	-	-	-20.2%	-22.3%
Unexpected Loss	-9.3%	-11.2%	-10.4%	-3.3%	-9.7%	-4.9%	-12.0%	-9.8%	-	-9.0%	-	-10.0%	-	-	-22.0%	-13.0%
<b>Reg. Capital (by cell)</b>	<b>-8%</b>	<b>-10.7%</b>	<b>-9.6%</b>	<b>-2.1%</b>	<b>-9.5%</b>	<b>-3.6%</b>	<b>-11.7%</b>	<b>-9.1%</b>	-	<b>-7.0%</b>	-	<b>-9.0%</b>	-	-	<b>-22.0%</b>	<b>-12.0%</b>
<b>Reg. Capital (by BL)</b>	<b>-9.6%</b>		<b>-3.9%</b>		<b>-5.9%</b>		<b>-9.7%</b>		<b>-4%</b>		<b>-7%</b>		<b>-4%</b>		<b>-15%</b>	
<b>Reg. Capital (total)</b>		<b>-6.1%</b>				<b>-8.2%</b>					<b>-5.8%</b>				<b>-9.1%</b>	

This table reports the impact of the four different scenarios on the ingredients of the RAROC. Changes in the number, frequency and severity of losses have been calibrated so as to reach a target -15% of total expected operational loss

Table 14

Scenario analysis on Operational RAROC of managerial actions

Panel A: Capital charge associated with managerial actions

	Default AMA	Lessons Learned	Dashboards	Audit Tracking	BL Reorganization
BL1 – Asset Management/Private Banking	2,674,000	2,416,723	2,517,013	2,556,748	2,674,000
BL2 – Retail Banking	4,379,381	4,207,900	3,954,635	4,085,998	3,739,444
<b>TOTAL</b>	<b>7,053,381</b>	<b>6,624,623</b>	<b>6,471,648</b>	<b>6,642,746</b>	<b>6,413,444</b>

Panel B: Operational RAROC

RAROCO	Default AMA	Lessons Learned	Dashboards	Audit Tracking	BL Reorganization
BL1 – Asset Management/Private Banking	22.57%	27.11%	25.54%	25.23%	22.57%
BL2 – Retail Banking	23.54%	25.94%	28.32%	27.37%	31.02%
<b>TOTAL</b>	<b>23.17%</b>	<b>26.36%</b>	<b>27.24%</b>	<b>26.55%</b>	<b>27.49%</b>

Panel C: Maximum acceptable cost (in currency units) by action

Minimum Income	Default AMA	Lessons Learned	Dashboards	Audit Tracking	BL Reorganization
BL1 – Asset Management/Private Banking	-	98,003	67,550	62,663	-
BL2 – Retail Banking	-	91,112	165,387	140,041	243,922
<b>TOTAL</b>	<b>-</b>	<b>189,114</b>	<b>232,937</b>	<b>202,704</b>	<b>243,922</b>

Panel D: Maximum acceptable cost (in percentage of total income) by action

Income in % of total revenues	Default AMA	Lessons Learned	Dashboards	Audit Tracking	BL Reorganization
BL1 – Asset Management/Private Banking	-	0.56%	0,39%	0.36%	-
BL2 – Retail Banking	-	0.28%	0,51%	0.43%	0.75%
<b>TOTAL</b>	<b>-</b>	<b>0.38%</b>	<b>0,47%</b>	<b>0.41%</b>	<b>0.49%</b>

This table presents the effects of managerial actions leading to a 15% decrease of the Expected Loss on several indicators. In Panel B, Operational Income = 5% Total Income. In Panels C and D, Target-RAROCO is 18%.

## Appendix 1 – Functional form of the statistical distributions

### a. “Normal” losses

Here are the functional forms of the distributions used in this study:

	Probability Distribution Function
Poisson( $\lambda$ )	$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$
Weibull( $a, b$ )	$f(x) = ab x^{b-1} \exp(-ax^b)$
Pareto( $q, a$ )	$f(x) = aq^a x^{-(a+1)}$
Lognormal( $m, s$ )	$f(x) = \frac{1}{x\sqrt{2p}} \exp\left[\frac{-(\log x)^2}{2}\right]$

### b. “Large” losses

The POT approach of the Extreme Value Theory is based on the Generalized Pareto Distribution. The GPD with threshold  $u$  has two parameters:  $s$  (the scale parameter) and  $x$  (the tail index). Its cumulative density function (CDF) can be expressed as

$$F(y) = 1 - \left(1 + \frac{x}{s} y\right)^{-1/x}$$

where the “threshold excess”  $y$  is simply  $x - u$ . The GPD can thus be thought of as the conditional distribution of  $X$  given  $X > u$ .

### c. Copulae

The bivariate version of Frank’s copula can be described as

$$C(u, v) = -\frac{1}{a} \ln \left( 1 + \frac{(\exp(-au) - 1)(\exp(-av) - 1)}{(\exp(-a) - 1)} \right)$$

while the Gaussian copula is expressed as

$$C(u, v) = \Phi_r \left( \Phi^{-1}(u), \Phi^{-1}(v) \right)$$

where  $\Phi_p$  is the bivariate normal distribution with correlation  $\rho$  and  $\Phi$  is the standard normal distribution.

## Appendix 2 – Descriptive statistics of the external database

Here is a summary of the descriptive statistics for the external data (in millions USD)

	Cell (1,1)	Cell (1,2)	Cell (2,1)	Cell (2,2)	<b>Total</b>
No. Observations	38	3	161	23	<b>224</b>
Mean (raw losses)	39.2	242,297.2	52.8	45.2	<b>49.6</b>
Mean (scaled losses)	27.7	2,150.6	50.5	46.9	<b>45.8</b>
Std.Dev. (raw losses)	109.6	419,600.4	179.7	161.3	<b>166.7</b>
Std.Dev. (scaled losses)	65.6	3,540.9	188.2	168.6	<b>166.9</b>
Median (raw losses)	7.5	80.0	7.0	5.6	<b>6.7</b>
Median (scaled losses)	6.8	211.9	6.4	5.2	<b>6.4</b>

This table reports summary statistics of the filtered external losses database. Filters have been applied to keep only losses coming from banks of similar countries (Europe, USA, Japan, Australia) and whose gross income was available (for scaling purposes).



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