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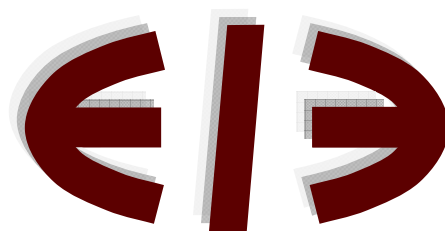
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Exploding Offers and Buy-Now Discounts*

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Abstract

A common sales tactic is for a seller to encourage a potential customer to make her purchase decision *quickly*. We consider a market with sequential consumer search in which firms often encourage first-time visitors to buy immediately, either by making an “exploding offer” (which permits no return once the consumer leaves) or by offering a “buy-now discount” (which makes the price paid for immediate purchase lower than the regular price). Prices often increase when these policies are used. If firms cannot commit to their sales policy, the outcome depends on whether consumer incur an intrinsic cost of returning to a firm: if there is no such return cost, it is often an equilibrium for firms to offer a uniform price to both first-time and returning visitors; if the return cost is positive, however, firms are forced to make exploding offers.

Keywords: Consumer search, oligopoly, price discrimination, high-pressure selling, exploding offers, buy-now discounts, costly recall.

1 Introduction

Selling techniques are rarely a focus of economic research, although they are an important aspect of the consumer experience in many markets. One important and controversial sales technique attempts to force the consumer to decide quickly whether to buy, before she has time to consider the product in depth or before she has time to explore other options available in the market. In his account of sales practices, Cialdini (2001, page 208) reports:

Customers are often told that unless they make an immediate decision to buy, they will have to purchase the item at a higher price later or they will be

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unable to purchase it at all. A prospective health-club member or automobile buyer might learn that the deal offered by the salesperson is good for that one time only; should the customer leave the premises the deal is off. One large child-portrait photography company urges parents to buy as many poses and copies as they can afford because “stocking limitations force us to burn the unsold pictures of your children within 24 hours”. A door-to-door magazine solicitor might say that salespeople are in the customer’s area for just a day; after that, they, and the customer’s chance to buy their magazine package, will be long gone. A home vacuum cleaner operation I infiltrated instructed its sales trainees to claim that, “I have so many other people to see that I have the time to visit a family only once. It’s company policy that even if you decide later that you want this machine, I can’t come back and sell it to you.”

Likewise, Bone (2006, pp. 71-73) documents how a home improvement company offers its potential customers a regular price for the agreed service, together with a discounted price—which was termed a “first call discount”—if the customer agrees to buy immediately.

There are other settings where similar tactics might plausibly be used. One can imagine a sales assistant in an electronics store offering a customer a 10% discount if the sale is made immediately (e.g., before the assistant “leaves for the day”). When searching for air-tickets online, a consumer may find a quote on one website, go on to investigate a rival seller, only to return to the original website to find the price has mysteriously risen. An academic journal may offer to publish a scholar’s paper as is, if the scholar submits it immediately before trying her luck with another outlet. A seller of life insurance may give a quote to a consumer which is valid only for 10 days, knowing that it will take the consumer more than 10 days to generate another quote (given the medical tests required).

This paper examines a seller’s incentive to encourage its potential customers to buy quickly. Two reasons why a seller might encourage immediate purchase are (i) to prevent the customer from going on to investigate other, perhaps superior, deals available in the market, and (ii) to prevent the customer from adequately evaluating the seller’s own product. For the most part we investigate the former reason, which requires the seller to operate alongside rivals. (The second reason can be examined in a simple monopoly setting, as we will do at the end of the paper.) As indicated by Bone and Cialdini, methods of encouraging quick sale include a seller telling the potential customer that she will have to pay higher price if she decides to purchase at a later date (in this paper we then say the seller offers a “buy-now discount”), or the seller may refuse to sell to a customer at all unless she buys immediately (a sales tactic for which we use the term “exploding offer”).¹

¹We use a model with rational consumers. There are many other methods to induce sales which rely on more psychological factors. These include attempts to make the prospective buyer “like” the seller (e.g., by claiming similar interests, families or social background) or attempts to make the buyer feel obligated

We employ a model of sequential consumer search with horizontal product differentiation to pursue our analysis. In markets in which consumers search through available options, it is common for a consumer to return to buy from a previously sampled seller (if the seller permits this) after investigating other sellers.² Of course, to implement a policy of exploding offers or buy-now discounts, a seller needs to be able to distinguish potential customers it meets for the first time from those who have returned after a previous visit. A sales assistant may tell from a potential customer’s questions or demeanor whether she has paid a previous visit or not, or may simply recognize her face. In online markets, a retailer using tracking software may be able to tell if a visitor using the same computer has visited the site before. Sometimes—as with job offers, tailored financial products, medical insurance, or home improvements—a consumer needs to interact with a seller to discuss specific requirements, and this process reveals the consumer’s identity. In these situations where sellers can distinguish new from returning visitors, we argue that firms often have an incentive to discriminate against returning visitors.

In our search model, there are potentially two reasons why a firm may wish to discriminate against returning visitors. First, there is a *strategic* reason, which is to deter a potential consumer from going on to investigate rival offers. If a consumer cannot return to a seller once she leaves, this increases the opportunity cost of onward search, as the consumer then has fewer options remaining relative to situation in which return is costless. Second, the observation that a consumer has come back to a seller after sampling other options reveals relevant *information* about a consumer’s tastes, and this may be a profitable basis for price discrimination. A seller may charge a higher price to those consumers who have already investigated other sellers, because their decision to return indicates they are unsatisfied with rival products.³ However, this incentive is tempered by the fact that returning consumers also do not have a strong taste for the firm’s own product, for otherwise they would have purchased immediately instead of going on to investigate alternatives.

to the seller (e.g., by means of a “free gift”). Cialdini (2001) describes these and other sales techniques in more detail, and Bone (2006) illustrates their use in the two companies he studies. Bone (page 90) describes the use of an extreme tactic: the seller “burst into tears” when the sale appeared in difficulty, claiming she would be in trouble with her boss if she didn’t make the sale.

²De los Santos (2008) presents a rare empirical study of consumer search behaviour prior to making a purchase, using data from online book purchases. De los Santos (2008, section 4) finds that three-quarters of consumers search only one retailer before making their purchase. Of the remaining consumers who search at least twice, approximately two-thirds buy from the final firm searched and one-third go back to a firm searched earlier. De los Santos also finds that the initial search is non-random, and one firm (*Amazon.com*) was sampled first by about two-thirds of all consumers making a purchase.

³This contrasts with the substantial literature about how firms can use the information of consumer *purchase* history to refine their prices. These models often predict that a firm will price low to a customer who previously purchased from a rival (or consumed the outside option in the case of monopoly), since such a customer has revealed she has only a weak preference for the firm’s product. See Fudenberg and Villas-Boas (2006) for a survey of this literature.

In section 2 of the paper, we suppose that firms can employ one of just two “buy-later” policies: consumers can freely return after leaving the firm (and buy at the same price), or exploding offers are used and first-time visitors are forced to buy immediately or never. We derive the equilibrium price when all firms use exploding offers, and show with examples that typically this price is (weakly) higher than the corresponding price with free recall. The use of exploding offers also leads to inefficient matching between products and consumers. When a firm uses an exploding offer, this makes those consumers with strong tastes for the firm’s product more likely to buy immediately, but it prevents consumers with moderate tastes from returning after they find nothing better elsewhere. We show that firms wish to use exploding offers when the consumer demand curve is concave, while when demand is convex firms choose to allow free recall. In this setting, only the strategic reason to make return costly is present, as by construction firms make no sales to returning visitors when exploding offers are used.

In section 3 we assume firms have a richer set of buy-later policies to choose from, and rather than simply banning return they can charge returning visitors a higher price; that is, they offer first-time visitors a buy-now discount. Starting from a situation in which firms treat first-time and returning consumers equally, we show under relatively mild conditions—essentially, that the demand curve be strictly logconcave—that a firm has an incentive to offer a buy-now discount. We also show that a firm wishes to set only a “moderate” buy-later price which induces some of its initial visitors to return to buy. In a duopoly example, we calculate the equilibrium prices for immediate and returning purchase, and find that the buy-now discount is largest when intrinsic search frictions are small. Because of the extra search frictions introduced by the buy-now discount, even the discounted buy-now price is higher than the non-discriminatory price. As such, this form of price discrimination lowers both consumer surplus and total welfare.

In section 4 we relax the assumption that firms can commit to their buy-later policies when consumers make their first visit. The outcome without commitment depends sensitively on whether or not consumers face an intrinsic (as opposed to artificially inflated) cost of returning to a firm. If there is no such cost, we show that it is an equilibrium for firms to offer uniform prices when the demand curve is convex, i.e., the fact that a consumer has come back to a firm after sampling other sellers gives no *ex post* incentive for the firm to surprise its returning visitors with a price rise. This implies that the informational incentive to set higher prices to returning customers is non-existent, and it is the strategic impact on a consumer’s incentive to buy immediately which is the dominant factor when a firm decides to make return costly. Thus, an inability to commit to its buy-later policy can reduce a firm’s incentive to discriminate against return visitors.

However, it is usually more realistic to suppose that consumers have at least a small intrinsic cost of returning to a previously-visited firm. Then, for reasons akin to Diamond’s (1971) famous paradox, the only credible outcome is that firms make exploding offers and

the return market collapses. With intrinsic costs of return, then, an inability to commit to its buy-later policy will amplify a firm’s incentive to discriminate against its returning visitors. In addition, if a firm has *limited* commitment power, in the sense that it can commit to an upper bound on the prices paid by returning visitors—this upper bound might simply be the displayed price of the item in the store, for instance—then an equilibrium exists which is identical to the full commitment outcome in section 3.

Finally, in section 5 we adjust the analysis so that a consumer needs time to evaluate properly the *current* firm’s offer, rather than to discover the offers made by rivals in the market. We show that if the marginal cost of supply is sufficiently small relative to the product’s expected utility, a firm has an incentive to use high-pressure sales techniques which force a consumer to reach her decision before she has time to fully evaluate the product. (This is true both in a monopoly and in a competitive search context.) The result is that consumer surplus is fully extracted, and, in the search context, consumers are matched only randomly with products.

Our paper relates to several strands of the industrial organization literature. Our underlying framework is a sequential search model with horizontally differentiated products in which consumers search both for price and product fitness, as introduced by Wolinsky (1986). Each firm has two sources of demand: consumers who buy its product on their first visit to the firm (“fresh demand”), and consumers who sample the firm, go on to sample rival products, but eventually come back to buy (“returning demand”). In Wolinsky’s model, firms cannot distinguish between these two groups and so must treat all visitors equally, while in our paper firms are able to discriminate between the two groups.

Our paper is complementary to the model of ordered search in Armstrong, Vickers, and Zhou (2009). The two papers use the same Wolinsky framework and focus on the same distinction between fresh and returning demand, but there are two major differences. First, Armstrong *et al.* suppose that firms know something about their place in the consumer search order and can set their prices accordingly, while for the most part in this paper we assume random search whereby firms do not know where they are in a consumer’s search process. Second, Armstrong *et al.*, like Wolinsky, assume that firms cannot directly distinguish between fresh and returning visitors and must treat both groups equally, while the ability to distinguish between the two groups lies at the heart of the current analysis. In Armstrong *et al.*, a firm which is more “prominent” is predicted to set a lower price than its less prominent rivals. (If a firm is far back in the search order, it knows that any consumer who reaches it must not care for the products of its rivals, and so this firm has monopoly power over its consumers.) This reflects the informational motive to set high prices to consumers who have already sampled, and rejected, rival products.

Our analysis is related to models of search with (exogenous) costly recall. Janssen and Parakhonyak (2010) extend Stahl’s (1989) model so that consumers incur an exogenous cost to return to a previous firm. This stopping rule is significantly more complicated than

when return is costless. When there are more than two firms, a consumer’s stopping rule is non-stationary and her reservation surplus level depends on her previous offers. They further show that equilibrium prices do not depend on the recall cost (unlike our model, where prices are sensitive to the endogenously generated recall costs).⁴

Firms often benefit from a reduction in consumer search intensity, since this usually softens price competition. In our model, the buy-now discount or exploding offer serves this purpose. Alternatively, Ellison and Wolitzky (2008) extend Stahl’s model so that a consumer’s incremental search cost increases with her cumulative search effort. If a firm increases its in-store search cost (say, by making its tariff harder to comprehend), this will make further search less attractive. They show that if the exogenous component of search costs falls, firms will unilaterally increase their self-determined element of search costs, with the result that equilibrium prices are unchanged. Though otherwise very different, our model and theirs study how search frictions are determined endogenously: even if intrinsic search frictions are negligible, a market may suffer from substantial search frictions—and high prices—in equilibrium. Rotemberg (2010) presents a model with non-sequential search in which sellers, by investing in sales effort, can directly affect a prospective buyer’s utility from the seller’s item or her disutility from not buying the seller’s item. (In his model, buyers can differ in their propensity to be persuaded, and sellers can differ in their degree of altruism towards their customers.)

Our analysis of buy-now discounts is also somewhat related to the literature on auctions with a “buy now” price (see Reynolds and Wooders, 2009, for instance). Online auctions sometimes offer bidders the option to buy the item immediately at a specified price rather than enter an auction against other bidders. In these situations, a seller has one item to sell to a number of potential bidders, and so a bidder needs to pay a *high* buy-now price in order to induce the seller from going on to search for other bidders by running an auction, whereas our model involves sellers offering a *low* buy-now price so as to induce a buyer from going on to search for other sellers. Common rationales for buy-now prices in auctions are impatience or risk-aversion on the part of bidders, neither of which is needed in our search framework.

As far as we know, our paper is the first to study the use of exploding offers in consumer markets. In the alternative setting of matching markets, however, there are a number of studies in which exploding offers play a role. Exploding offers are often used in specialized labor markets, such as those for law clerks, sports players, medical staff, and student

⁴Daughety and Reinganum (1992) make the point that the extent of consumer recall may be endogenously determined by firms’ equilibrium strategies. In their model, the instrument that a firm can use to influence consumer recall is the length of time that it will hold the good for consumers at the quoted price. In contrast to our assumption that a consumer can discover a seller’s buy-later policy only after investigating that seller, Daughety and Reinganum suppose that sellers can announce their recall policies to the population of consumers before search begins.

college allocations.⁵ When exploding offers are used, these markets have a tendency to “unravel”, and employers compete to make earlier offers (Li and Rosen, 1998). The result can be significant inefficiency. Niederle and Roth (2009) run an experiment to measure the impact of a policy which bans the use of exploding offers in a laboratory matching market. They find that firms do tend to use exploding offers when they are permitted to do so, and the result is that matching occurs inefficiently early and match quality is poor, relative to the situation in which exploding offers cannot be used.

2 Exploding Offers

Our underlying model of the market is based on Wolinsky (1986). (See Anderson and Renault, 1999, for further development of Wolinsky’s model.) There are $n \geq 2$ firms in the market, each supplying a single horizontally differentiated product at a constant marginal cost which is normalized to zero. There are a large number of consumers with idiosyncratic preferences, and their measure is normalized to one. A consumer’s valuation of product i , u_i , is a random draw from some common distribution with support $[0, u_{\max}]$ and with cumulative distribution function $F(\cdot)$ and density $f(\cdot)$. We suppose that the realization of match utility is independent across consumers and products. In particular, there are no systematic quality differences across the products. Each consumer wishes to buy one item, provided an item can be found with a positive surplus. We sometimes refer to the function $1 - F(\cdot)$ as the consumer demand curve.

Consumers initially have imperfect information about the deals available in the market. They gather this information through a sequential search process, and by incurring a search cost $s \geq 0$, a consumer can visit a firm and find out (i) its price, (ii) its “buy-later” policy, and (iii) the realized match value.⁶ In this section, the only two buy-later policies available to a firm are to use an exploding offer or to allow free recall. To implement an exploding offer, firms are assumed to be able to distinguish first-time visitors from returning customers and to have the ability to commit not to serve a returning customer. (If a firm allows free recall, it sets the same price to first-time visitors and returning visitors.) After visiting one firm, a consumer can choose to buy at this firm immediately or to investigate another firm (her sampling is without replacement). If permitted, she can costlessly return to a previous firm after sampling subsequent firms.⁷ Since firms are *ex ante* symmetric, we

⁵Roth and Xing (1994, page 1001) document some examples of high-pressure job offers. For instance, in the market for judicial clerkships, some judges use exploding offers which would be withdrawn if they are not accepted in some very short time, or even during the telephone conversation itself.

⁶If the search cost is zero, we require that consumers nevertheless consider products sequentially.

⁷In most markets, even if firms allow for free recall, consumers may face some intrinsic returning costs. In most of our analysis, introducing a relatively small returning cost does not affect results qualitatively, but only complicates the analysis. We assume it away for analytical convenience. However, when we come to discuss buy-now discounts without commitment in section 4, whether an intrinsic returning cost exists

focus on symmetric situations with random search, so that a consumer is equally likely to investigate any of the remaining unsampled firms when she searches.

The timing of the game is as follows. At the first stage, firms set prices and buy-later policies simultaneously. The strategy space of each firm is then $\mathbb{R}^+ \times \{\text{free recall, exploding offer}\}$. At the second stage, consumers search sequentially and make their purchase decision after search is terminated. Consumers do not observe firms' actual choices before they start searching, but hold rational expectations of equilibrium prices and buy-later policies. Information unfolds as the search process goes on, but consumers' beliefs about the offers made by unsampled firms are unchanged, even if they observe off-equilibrium offers from some firms. Both consumers and firms are assumed to be risk neutral. We use the concept of perfect Bayesian equilibrium, and focus on symmetric pure strategy equilibria in which firms set the same price and buy-later policy based on their expectation of consumers' search behavior, and at each firm consumers hold equilibrium beliefs about unsampled firms and make their search decisions accordingly.

A piece of notation which summarizes the distribution of match utilities and the extent of search frictions is

$$V(p) \equiv \int_p^{u_{\max}} (u - p) dF(u) - s . \quad (1)$$

Thus, $V(p)$ is the expected surplus of sampling a product if a consumer expects that the price will be p , the cost of sampling the product is s , and this is the only product available. Note that $V(p)$ is decreasing but $p + V(p)$ is increasing in p . Throughout this paper we assume that the search cost s is relatively small, so that

$$V(\bar{p}) > 0 , \quad (2)$$

where $\bar{p}$ is the monopoly price which maximizes $p[1 - F(p)]$.⁸ This condition means that consumers are willing to sample a product sold even at the monopoly price. In the example where u is uniformly distributed on $[0, 1]$, which we sometimes use for illustration in the following analysis, condition (2) requires $s < \frac{1}{8}$.

2.1 The free-recall benchmark

If all firms allow free recall, the situation is as in Wolinsky (1986). For reference later, in this section we recapitulate part of his analysis. Wolinsky shows that in a symmetric equilibrium in which all firms set the same price p_0 , consumers have a stationary stopping rule whereby they buy a product immediately if they obtain a match utility u greater than a threshold a , and if no product yields that level of utility, the consumer samples all products and buys from the best of the n options provided that one option generates a

or not will make an important difference.

⁸If the demand curve $1 - F$ is logconcave, $\bar{p}$ solves the first-order condition $\bar{p} = \frac{1-F(\bar{p})}{f(\bar{p})}$ uniquely.

positive surplus. Here, the reservation utility a is determined by the formula

$$V(a) = 0 . \quad (3)$$

The expression $\int_a^{u_{\max}} (u - a) dF(u)$ in $V(a)$ is just the incremental benefit of engaging in one more search if the best current utility is a and the consumer can freely return to this best offer if the next product does not yield higher surplus. So the optimal threshold makes the consumer indifferent between searching on, which incurs the cost s , and purchasing this product with utility a . Since $V(\cdot)$ is a decreasing function, (3) has a unique solution and a decreases with s . Condition (2) is therefore equivalent to $a > \bar{p}$.

Given that other firms are charging the equilibrium price p_0 , if firm i deviates and charges $\tilde{p}$, its demand is

$$Q = \frac{1 - F(a)^n}{n(1 - F(a))} [1 - F(a - p_0 + \tilde{p})] + \int_{p_0}^a F(u)^{n-1} f(u - p_0 + \tilde{p}) du . \quad (4)$$

To understand this expression, consider the two sources of firm i 's demand. Suppose firm i is in the k_{th} position in a consumer's search order. Then to reach the firm, the consumer must have sampled, and rejected, $k - 1$ firms first, which occurs with probability $F(a)^{k-1}$ (since a consumer will buy immediately if $u_j \geq a$). If $k < n$, the consumer will buy immediately at firm i if $u_i - \tilde{p} \geq a - p_0$, which occurs with probability $1 - F(a - p_0 + \tilde{p})$. If the firm is in the final search position (i.e., $k = n$), then she will surely buy from firm i if $u_i - \tilde{p} \geq a - p_0$, since then her surplus $u_i - \tilde{p}$ is positive and higher than all other firms. Since firm i is equally likely to be in any of the search positions, the firm's demand from this source is $[1 - F(a - p_0 + \tilde{p})] \times \frac{1}{n} [1 + F(a) + F(a)^2 + \dots + F(a)^{n-1}]$, which yields the first term in (4). The second source of demand comes from the scenario in which the consumer searches all sellers and does not find any product with net surplus greater than $a - p_0$. This consumer will then buy from the firm with the greatest net surplus, if this surplus is positive. The fraction of consumers for whom this happens and then go on to buy from firm i is

$$\Pr(\max_{j \neq i} \{0, u_j - p_0\} < u_i - \tilde{p} < a - p_0) = \int_{\tilde{p}}^{a - p_0 + \tilde{p}} F(u_i - \tilde{p} + p_0)^{n-1} dF(u_i) ,$$

which equals the second term in (4) by changing variables from u_i to $u = u_i + p_0 - \tilde{p}$.

In equilibrium, firm i maximizes $\tilde{p}Q$ by choosing $\tilde{p} = p_0$, and so expression (4) implies the first-order condition for p_0 to be the equilibrium price is⁹

$$\frac{1 - F(p_0)^n}{p_0} = f(a) \frac{1 - F(a)^n}{1 - F(a)} - n \int_{p_0}^a F(u)^{n-1} f'(u) du . \quad (5)$$

⁹Anderson and Renault (1999) show that, if $1 - F$ is logconcave, the equilibrium price is increasing in the search cost s and decreasing in the number of firms (see their Proposition 1). (However, Anderson and Renault assume that all consumers buy one product, i.e., there is no outside option, and this affects the first-order condition for the equilibrium price.) It is a subtle issue in this model whether second-order conditions are satisfied in this candidate equilibrium. For discussion, see Proposition B2 in Anderson and Renault (1999). However, a sufficient condition is that the density function f be weakly increasing.

Assuming that the demand curve $1 - F$ is strictly logconcave (i.e., the distribution for the match utility has an increasing hazard rate), a finite number of firms, and condition (2), one can show that in the relevant interval $0 < p_0 < a$, expression (5) has a unique solution, and the equilibrium price lies in the range

$$\frac{1 - F(a)}{f(a)} < p_0 < \bar{p}. \quad (6)$$

As the number of firms becomes infinite, the equilibrium price converges to $p_0 = \frac{1 - F(a)}{f(a)}$. As the search cost tends to its upper bound in (2) (i.e., as a tends to $\bar{p}$), consumers stop searching whenever they find a product with positive surplus and each firm acts as a monopolist, so the equilibrium price converges to $p_0 = \bar{p}$ (which then also equals $\frac{1 - F(a)}{f(a)}$).

In the remainder of section 2, we extend this model to allow firms to use the additional instrument of exploding offers; that is to say, firms can require first-time visitors to buy their product immediately or not at all. We discuss this issue in two stages: first, we analyze equilibrium prices under an assumption that all firms use exploding offers, and second, we discuss when firms do indeed have an incentive to use this sales tactic.

2.2 Equilibrium prices with exploding offers

Suppose now that all firms force their first-time visitors to buy immediately or not at all. Suppose consumers anticipate that each firm sets the same price p . What is a consumer's optimal search strategy? As we will show, and as is intuitive, consumers become less choosy as they run out of options, and their reservation utility for purchasing decreases the more firms they have already sampled. Indeed, if they are unfortunate enough to reach the final firm they will have to accept any offer which leaves them non-negative surplus. The stopping rule is formally described in this result:

Lemma 1 *Suppose all firms use exploding offers and set the same price $p < \bar{p}$. Then a consumer who has $0 \leq m \leq n - 1$ unsampled firms remaining will buy from the current firm if and only if her current match utility satisfies $u \geq a_m$, where a_m solves the first-order difference equation*

$$a_{m+1} = a_m + V(a_m) \quad (7)$$

with $a_0 = p$ and $V(\cdot)$ defined in (1).

This stopping rule for search without recall has been derived in, for example, Lippman and McCall (1976). It can be understood as follows. If a consumer has reached the last firm (i.e., $m = 0$), she will buy if the last product has utility no less than p , so $a_0 = p$. Suppose the stopping rule holds for $m \leq n - 1$, i.e., the consumer will buy if her current match utility satisfies $u \geq a_m$ when she has m options remaining. Then $a_m - p$ is her expected surplus from participating in a no-recall search market with m products each

sold at price p . Now consider the situation when the consumer faces $m + 1$ unsampled products. If she searches on and if the next product has utility greater than a_m , then she will buy the next product, while if the next product's utility is below a_m , she will continue to search and so obtain expected surplus $a_m - p$. Hence,

$$a_{m+1} - p = \int_{a_m}^{u_{\max}} (u - p) dF(u) + (a_m - p)F(a_m) - s ,$$

which simplifies to (7) using the definition of $V(\cdot)$.

Compared to the case with free recall, the optimal stopping rule in the no-recall case exhibits several distinct features. First, whenever $V(p) > 0$ such that $a_1 > a_0 = p$, it follows from (7) that $a_{m+1} > a_m$ for all $m \geq 0$, so that a consumer is willing to accept a less suitable product as she nears the end of the search process.¹⁰ In particular, it is possible that a consumer will end up purchasing a product with lower match utility than a product she previously rejected. While with free recall the optimal stopping rule is stationary, and $a_m \equiv a$ given in formula (3). That is, consumers do not become less choosy as they near the end of the search process. Second, unlike the case with free recall, each a_m depends on price p since the starting value a_0 does so. Finally, realize from (7) that the difference $a_{m+1} - a_m$ decreases with m , and the sequence a_m converges to the free-recall reservation utility a as $m \rightarrow \infty$ (at least when $s > 0$ or when u has bounded support).

We next derive the symmetric equilibrium price. Suppose $n - 1$ firms set the price p and one firm is considering its choice of price, say $\tilde{p}$. (Of course, when choosing their search strategy consumers anticipate that this firm has set the equilibrium price p .) Suppose this deviating firm happens to be in the k_{th} position of a consumer's search process, so there are $n - k$ firms remaining unsampled. Then the probability that the consumer will visit this firm is $h_1 \equiv 1$ if $k = 1$, and if $k > 1$ this probability is

$$h_k \equiv \prod_{i=1}^{k-1} F(a_{n-i}) . \quad (8)$$

She will then buy at this firm if $u - \tilde{p} > a_{n-k} - p$, which has probability $1 - F(a_{n-k} - p + \tilde{p})$, and so the firm's demand given it is in a consumer's k_{th} search position is

$$h_k [1 - F(a_{n-k} - p + \tilde{p})] . \quad (9)$$

Since the firm is in any position $1 \leq k \leq n$ with equal probability, its total demand with price $\tilde{p}$ when all other firms are expected to set price p is

$$Q = \frac{1}{n} \sum_{k=1}^n h_k [1 - F(a_{n-k} - p + \tilde{p})] ,$$

¹⁰In the alternative setting of matching markets, an applicant for a job (say) may also be reluctant to search for long because the desirable vacancies may quickly be filled.

and its profit is $\tilde{p}Q$. The firm's profit is concave in its price $\tilde{p}$ if (but not only if) each function $\tilde{p}[1 - F(a_{n-k} - p + \tilde{p})]$ is concave in $\tilde{p}$. A sufficient condition for this is that the demand curve $1 - F(\cdot)$ is concave.

Therefore, the first-order condition for p to be the equilibrium price is

$$p = \frac{\sum_{k=1}^n h_k [1 - F(a_{n-k})]}{\sum_{k=1}^n h_k f(a_{n-k})}. \quad (10)$$

Since each a_{n-k} depends on p , this equation defines p only implicitly.

As with the free-recall case, if $1 - F$ is strictly logconcave, the number of firms is finite and condition (2) holds, expression (10) has a solution in the range¹¹

$$\frac{1 - F(a)}{f(a)} < p < \bar{p}.$$

Note that assumption (2) implies that consumers are indeed willing to participate in the market. It can be shown that as the number of firms tends to infinity, this equilibrium price converges to the same lower bound $\frac{1-F(a)}{f(a)}$ as in the free-recall case. Intuitively, when the number of firms is unlimited, a consumer would never choose to return to a previously sampled firm, even if she could freely do so, and so the use of exploding offers then has no effect on the equilibrium price. It is also clear that as the search cost tends to its upper bound (i.e., as a tends to $\bar{p}$), p converges to the monopoly price $\bar{p}$.

At this level of generality, it is hard to compare market performance with and without the use of exploding offers, and the comparison between the prices in (5) and in (10) is opaque. To gain further insights consider the case of a uniform distribution for match utility. (In section 2.3, we will show that with the uniform distribution it is an equilibrium for all firms to use exploding offers.)

Uniform example: If u is uniformly distributed on $[0, 1]$, so that the demand curve is linear, then (7) implies

$$a_{m+1} = \frac{1}{2}(a_m^2 + 1) - s$$

starting with $a_0 = p$. This difference equation appears to have no analytical solution. It converges as m becomes large to $a = 1 - \sqrt{2s}$, the free-recall threshold. Except when n is small, equation (10) has no analytical solution, but it can be solved numerically. The solid curve in Figure 1a depicts how the equilibrium price p varies with the number of firms when $s = 0$. The dashed curve represents the corresponding price (5) in the free-recall

¹¹Focus on the relevant range $0 < p < a$. Then we have $a_{n-k} \in (p, a)$ for every $k \leq n-1$. (Recall $a_0 = p$, and whenever $p < a$, a_m increases with m and converges to a as $m \rightarrow \infty$.) Since $(1 - F)/f$ is a strictly decreasing function, each $\frac{1-F(a_{n-k})}{f(a_{n-k})}$ is between $\frac{1-F(a)}{f(a)}$ and $\frac{1-F(p)}{f(p)}$ (for $k = n$, it is equal to the latter), and so is the right-hand side of (10), which establishes the claim. The uniqueness of the solution can be established if the right-hand side of (10) is decreasing in p , which is true if the demand curve $1 - F(\cdot)$ is concave (given each a_{n-k} increases with p).

market. Both prices converge to zero for large n , but it seems that prices with exploding offers are approximately double those which prevail with free recall. (This figure includes the monopoly case $n = 1$, in which case the monopolist charges the price $\bar{p} = \frac{1}{2}$ and the use of exploding offers has no impact.) The difference between the two prices is greatest for an intermediate numbers of firms. In the same example, Figure 1b shows that the exploding-offer equilibrium has a higher profit level than the free-recall equilibrium except when $n = 2$.¹² Numerical calculations suggest that as the search cost gets larger, the difference between the exploding-offer and free-recall prices decreases (and if $s = \frac{1}{8}$, the difference vanishes).

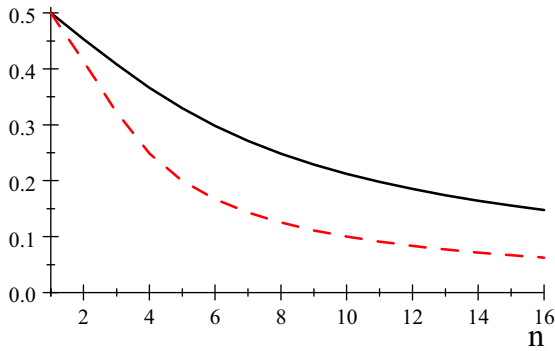


Figure 1a: Prices with exploding offers

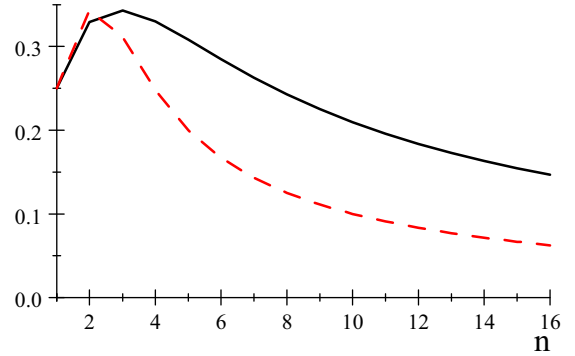


Figure 1b: Profits with exploding offers

In this uniform example, aggregate consumer surplus and total welfare (measured by the sum of consumer surplus and profit) fall when firms use exploding offers. Consumer surplus falls since the price rises compared to the free-recall situation *and* consumers are prevented from returning to a product which yields positive surplus. (Even if $p = p_0$, i.e., if using exploding offers did not change the market price, consumers would obtain lower surplus in the exploding-offer case due to the no-return restriction. The higher price $p > p_0$ only adds to their loss.) As far as total welfare is concerned, relative to the free-recall situation, the use of exploding offers not only induces suboptimal consumer search (i.e., consumers on average cease their search too early due to the “buy now or never” requirement, resulting in sub-optimal matching), but also excludes more consumers from the market, both of which harm efficiency.

Exponential example: To illustrate how the use of exploding offers need *not* increase equilibrium prices, consider a second example in which $F(u) = 1 - e^{-u/\mu}$, where μ is the expected value of match utility. The special feature of this distribution is that a monopoly

¹²The reason why industry profits increase with n for small n is that with few suppliers many consumers will not find a product which yields them positive surplus. With monopoly, for instance, half of consumers are excluded from the market, while with many firms almost all consumers will eventually find a suitable product. But with more firms profits fall with n , as the price reduction effect outweighs this market expansion effect.

firm facing this population of consumers, where each consumer has an outside option with utility $z \geq 0$, will choose the same price $p = \mu$ regardless of z .¹³ When firms use exploding offers, this immediately implies that each firm will choose $p = \mu$, regardless of the number of firms and the search cost (as long as s is relatively small such that consumers are willing to enter the market). One can also show that the same price is chosen when there is free recall, so that $p_0 = \mu$ solves expression (5) in this example for all n and a . Thus, the use of exploding offers has no impact on equilibrium prices. Nevertheless, this sales technique harms both consumers and firms, as demand and match quality are artificially restricted by the requirement that consumers cannot return to a firm. We will see in the next section that firms faced with this demand curve will not choose to use exploding offers.¹⁴

2.3 Incentives to use an exploding offer

Here we discuss when the behaviour discussed in the previous section is in fact an equilibrium. That is, if all its rivals set the price p in (10) and make exploding offers, does a firm have an incentive to deviate and allow free recall (and, possibly, set a different price as well)? Before pursuing the analysis in detail, consider this simple duopoly example with fixed prices which yields the main insight.

Suppose there are two firms, both of which set the exogenous price $p < a$. Is a firm's demand boosted or reduced if it decides to force its first-time visitors to buy immediately or not at all? First, for those consumers who first sample its rival, firm i 's decision whether or not to use an exploding offer has no impact on its demand. Therefore, the only impact on the firm's demand comes from that half of the consumer population who sample it first. If firm i allows free recall, such a consumer will buy from it immediately whenever $u_i > a$, and a consumer will return to buy from it whenever $p < u_i < a$ and $u_i > u_j$. This pattern of demand is depicted in Figure 2a below. If, instead, firm i uses an exploding offer, expression (7) implies that a consumer will buy from it if and only if $u_i > a_1 = p + V(p)$. This pattern of demand is depicted in Figure 2b.

As discussed in section 2.2, $a_1 \in (p, a)$ and so the use of an exploding offer makes a consumer more likely to buy immediately, but it eliminates all the returning demand. One can calculate that when u is uniformly distributed on $[0, 1]$, firm i 's demand in the two figures is identical, and when a firm forces immediate sale this has no net impact on its demand. More generally, the impact of using an exploding offer is to eliminate the firm's demand from "low u_i " consumers, who have match utility close to price p and might

¹³This is the "memoryless" property of the exponential distribution. With price p , the monopolist will sell to a consumer if $u - p \geq z$, and so will choose p to maximize $pe^{-(p+z)/\mu}$, a choice which does not depend on z .

¹⁴While we have been unable to make progress in comparing prices with and without exploding offers with general distributions for match utility, numerical simulations confirm that for a wide range of distributions prices are higher when exploding offers are employed. (We conjecture that this is true provided $1 - F$ is strictly logconcave.)

otherwise come back, and to boost its demand from “high u_i ” consumers, who do not wish to risk losing the existing desirable option by going on to sample the rival. If u has an increasing density (i.e., the demand curve is concave), the latter effect dominates the former, and the net impact of forcing immediate sale is to boost a firm’s demand. Similarly, if the demand curve is convex, then the former effect dominates and demand is reduced when an exploding offer is used.

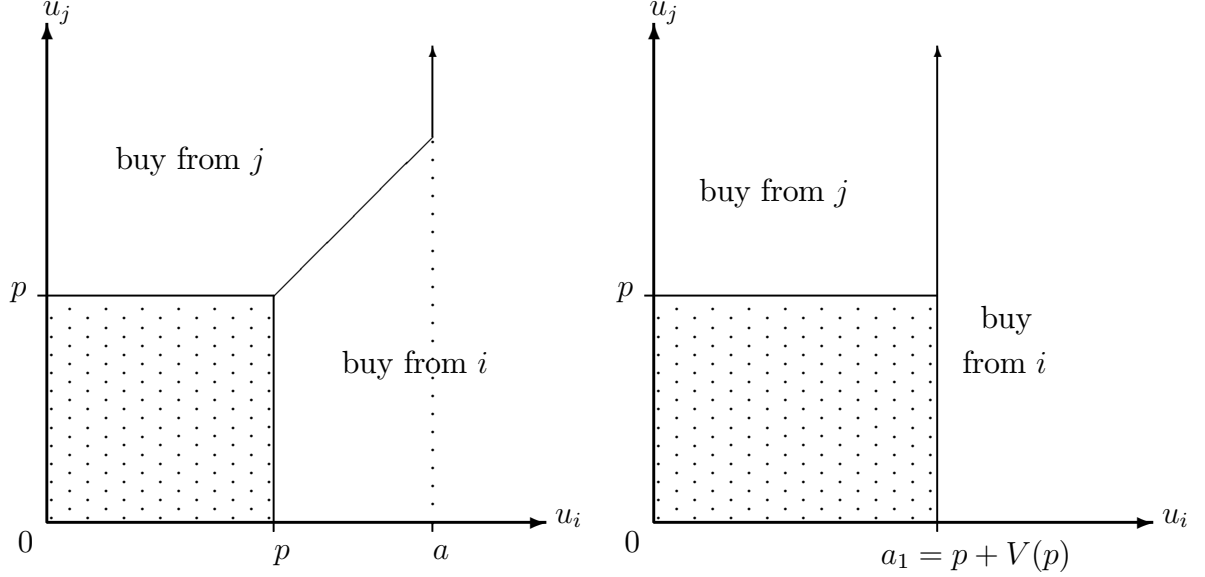


Figure 2a: Demand with free recall

Figure 2b: Demand with exploding offer

The next result proves that this insight is valid with an arbitrary finite number of firms and endogenous prices.¹⁵

Proposition 1 *Suppose the number of firms is $1 < n < \infty$.*

- (i) *If the demand curve $1 - F$ is strictly concave then every symmetric equilibrium involves firms using exploding offers;*
- (ii) *If the demand curve $1 - F$ is strictly convex then every symmetric equilibrium involves firms allowing free recall;*
- (iii) *If the demand curve $1 - F$ is linear, i.e., u is uniformly distributed, then an equilibrium with exploding offers and an equilibrium with free recall both exist.*

(All omitted proofs can be found in the appendix.)

Thus, we see there are plausible cases when exploding offers are used in equilibrium, as well as other plausible cases (such as the exponential distribution considered above which

¹⁵Note that if there were unlimited firms in the market ($n = \infty$), banning return or artificially raising the cost of return has no impact on a firm’s profit. As is well known, with unlimited options, consumers would not choose to return to a previously sampled option even if it was free for them to do so. As such, equilibria with exploding offers and with free recall will exist for any match utility distribution.

results in convex demand) when a firm prefers to let consumers return freely after sampling rival products. In the uniform example at least (see Figure 1a), the use of exploding offers leads to higher prices being chosen in equilibrium. In these situations, firms choose to use exploding offers and yet consumers are harmed by the practice.

Nevertheless, our analysis covers only situations with concave or convex demand (i.e., where the density for the match utility is monotonic). The reason why results are then so clear-cut is that the impact of exploding offers on a firm's demand is unambiguous, regardless of the prevailing price. With a non-monotonic density function, whether exploding offers are an equilibrium sales technique may depend on price. In particular, it may depend both on the number of firms in the market and the size of the search cost. A second factor which could come into play with non-monotonic densities is that firms may choose intermediate buy-later policies, which make return costly for their first-time visitors but not prohibitively so. For example, online sellers can ask customers to log on to their accounts or input information again; firms can ask consumers to queue again or make another appointment if they want to come back. With a monotonic density, a firm wishes either to make return impossible or free, even if it could impose intermediate returning costs.

As can be seen from the proof of Proposition 2 below, when we start from the free-recall equilibrium with price p_0 , introducing a small return cost boosts a firm's demand if

$$\int_{p_0}^a F(u)^{n-1} f'(u) du > 0 . \quad (11)$$

Whether this condition holds for non-monotonic densities depends both on the number of firms and the search cost. Consider for example a Weibull distribution with $F(u) = 1 - e^{-u^3}$ defined on $[0, \infty)$, which has a hump-shaped density with mode around 0.87. If the search cost is high enough that a is smaller than the mode, then (11) always holds. With a low search cost such that $a = 2$, say, then condition (11) always fails and free recall is the equilibrium outcome. However, if the search cost is moderate so that $a = 1$, then condition (11) holds for $n = 2$ and 3 but fails for $n \geq 4$. In this case, firms with few rivals have an incentive to make return costly, while a more competitive market will allow free recall.

In some markets consumers search in a non-random order, and a prominent seller may attract a disproportionate share of initial consumer searches. (Recall that De los Santos (2008) showed how this was so in the online book market. Likewise, a doorstep seller for, say, vacuum cleaners or home improvements, is relatively like to be the first seller encountered by the consumer.) It turns out that prominence does not affect a firm's incentive to adopt exploding offers, at least when the demand curve is concave or convex. The reason can be understood by looking at Figure 2 for the duopoly case. The decision about whether or not to use an exploding offer only affects a firm's demand from those consumers who sample it first, and this demand effect is positive (negative) if the demand curve is concave (convex), independent of the proportion of such consumers.

3 Buy-Now Discounts

An alternative framework allows a firm to charge a higher price to returning visitors instead of the drastic measure of banning return. Consider the same model as before, except that instead of choosing the extreme policies of either allowing free return or no return, each firm can choose two distinct prices: $\hat{p}$ is the price for returning customers and p is the price for first-time visitors, and the strategy space of each firm is $\mathbb{R}^+ \times \mathbb{R}^+$. (Neither price is observable to consumers before they start searching.) Whenever $\hat{p} > p$, returning to a previous firm is costly.¹⁶ Indeed, when $\hat{p}$ is sufficiently high, the firm in effect makes exploding offers. One interpretation of this discriminatory pricing is that each firm sets a regular (or “buy-later”) price $\hat{p}$ and offers first-time visitors a “buy-now” discount $\tau \equiv \hat{p} - p$. We assume for now that a firm can commit to $\hat{p}$ when it offers new visitors the buy-now price p .

3.1 Incentives to offer a buy-now discount

In this section we analyze when a firm unilaterally has an incentive to offer a buy-now discount τ , starting from the situation in which all firms offer the equilibrium uniform price p_0 in expression (5). As a preliminary result, we observe that the impact of offering a small buy-now discount on a firm’s profit is just as if the firm levies a small buy-later premium:

Lemma 2 *Starting from the situation in which all firms offer the equilibrium uniform price p_0 in (5), the impact on a firm’s profit of offering a small buy-now discount τ (so its buy-now price is $p_0 - \tau$ and its buy-later price is p_0) is equal to the impact of levying a buy-later premium τ (so its buy-now price is p_0 and its buy-later price is $p_0 + \tau$).*

Proof. Suppose all but one firm choose the uniform price p_0 in (5). If the remaining firm offers the buy-now price p and buy-later price $p + \tau$, denote this firm’s profit by $\pi(p, \tau)$. If $p \approx p_0$ and $\tau \approx 0$ we have the first-order approximation

$$\begin{aligned} \pi(p, \tau) &\approx \pi(p_0, 0) + (p - p_0)\pi_p(p_0, 0) + \tau\pi_\tau(p_0, 0) \\ &= \pi(p_0, 0) + \tau\pi_\tau(p_0, 0), \end{aligned} \tag{12}$$

where the equality follows from the assumption that p_0 is the equilibrium uniform price and subscripts denote partial derivatives. It follows that the impact on the firm’s profit is captured by the term $\tau\pi_\tau(p_0, 0)$, independent of p , which implies the result. ■

¹⁶If $\hat{p} < p$, then a consumer has an incentive to leave a firm and then return, even if she has no intention of investigating other firms. If this kind of consumer arbitrage behavior—of stepping out the door and then back in again—cannot be prevented, then setting $\hat{p} < p$ is equivalent to setting a uniform price $\hat{p}$, and so without loss of generality we assume firms are constrained to set $\hat{p} \geq p$.

Intuitively, the fact that p_0 is the equilibrium uniform price implies that a firm's profit is not affected by small changes in its uniform price, and the only impact on a firm's profit comes from its buy-now discount τ (regardless of whether this is interpreted as a discount for immediate purchase relative to the buy-later price p_0 , or as a premium for later purchase relative to the buy-now price p_0).

To illustrate the pros and cons of offering a discount most transparently, consider initially the case of duopoly. It is somewhat more straightforward to consider the incentive to set a buy-later premium, and then to invoke Lemma 2. If firm i introduces a buy-later premium, this has no impact on its demand and profit from those consumers who first sample the rival given they hold equilibrium beliefs, and so we can restrict attention to that portion of consumers who sample firm i first. A buy-later premium not only discourages consumers from searching on, as the exploding offer did in the earlier analysis, but also generates extra revenue from returning consumers.

How does τ affect a consumer's decision whether to buy immediately from firm i ? Denote by $a(\tau)$ the reservation utility which leads the consumer to buy immediately, i.e., if she finds match utility $u_i \geq a(\tau)$ at the firm she will buy without investigating the rival. Clearly if no premium is levied ($\tau = 0$) then $a(0) = a$, the free-recall reservation level in (3). By definition, if a consumer discovers utility $u_i = a(\tau)$ at firm i she is indifferent between buying immediately (thus obtaining surplus $a(\tau) - p_0$) and going on to investigate firm j , which yields expected utility

$$\underbrace{\int_{a(\tau)-\tau}^{u_{\max}} (u_j - p_0) dF(u_j)}_{\text{utility when she buys from } j} + \underbrace{F(a(\tau) - \tau)[a(\tau) - p_0 - \tau]}_{\text{utility when she returns to buy from } i} - s . \quad (13)$$

To understand expression (13), note that if the consumer finds utility u_j at the rival, she will buy from that firm if $u_j - p_0 \geq a(\tau) - p_0 - \tau$, and otherwise she will return to buy from firm i (but at the higher price $p_0 + \tau$). Equating $a(\tau) - p_0$ with expression (13) yields the following formula for $a(\tau)$ given τ :

$$V(a(\tau) - \tau) = \tau . \quad (14)$$

(Remember $V(\cdot)$ is defined in (1), and given τ this equation has a unique solution $a(\tau)$.) The pattern of demand for those consumers who first sample firm i is illustrated in Figure 3.¹⁷

¹⁷This analysis and Figure 3 presume that some consumers do return to firm i after sampling firm j , which requires that the premium τ is not too large. By examining the figure, one sees that the exact condition is $a(\tau) > p_0 + \tau$. From (14), and noting that $V(\cdot)$ is a decreasing function, this is equivalent to $\tau < V(p_0)$. This is the case for sufficiently small τ as long as $V(p_0) > 0$, which is true given (2). When the discount exceeds $V(p_0)$, the returning cost is so great that the consumer never returns to a firm once she leaves it (i.e., the firm in effect uses an exploding offer).

Note that $a(\tau)$ decreases with τ , and by differentiating (14) we obtain

$$a'(\tau) = \frac{-F(a(\tau) - \tau)}{1 - F(a(\tau) - \tau)} < 0 . \quad (15)$$

This is intuitive, as raising the cost of returning makes a consumer more likely to buy immediately (just as in the extreme case of exploding offers).

Using Figure 3, the fraction of those consumers who sample firm i first and who actually buy from the firm is

$$1 - F(a(\tau)) + \int_{p_0+\tau}^{a(\tau)} F(u - \tau) f(u) du .$$

By using (15), the derivative of firm i 's demand with respect to τ is equal to

$$\int_{p_0+\tau}^{a(\tau)} F(u - \tau) f'(u) du . \quad (16)$$

In particular, the firm's demand is boosted with a buy-later premium whenever the density is increasing, as we saw when we discussed exploding offers in section 2.3.

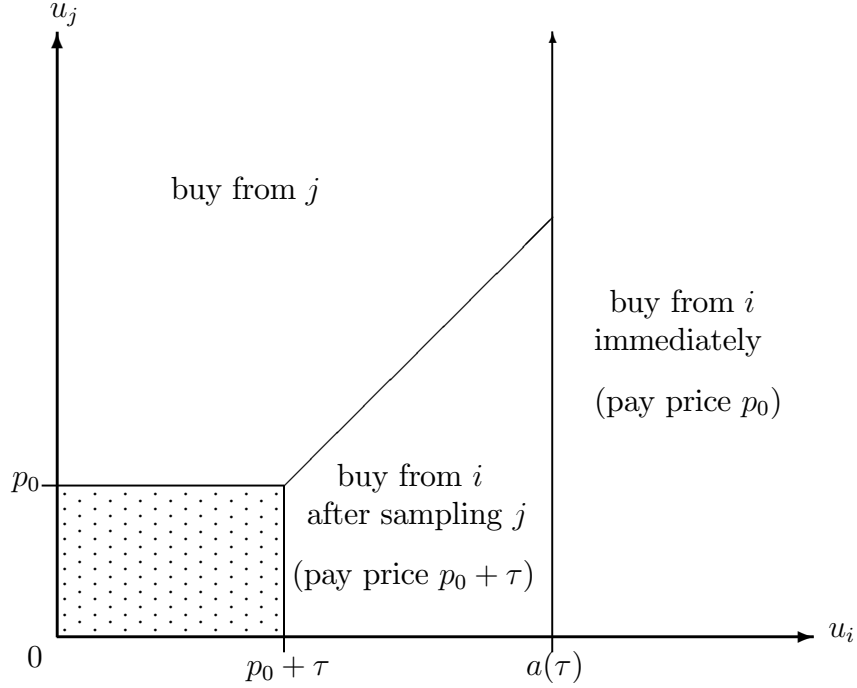


Figure 3: Pattern of demand when firm i levies buy-later premium τ

Firm i makes revenue p_0 from each of its customers, and an additional τ from each of its returning customers. It follows that the derivative of firm i 's profits with respect to τ evaluated at $\tau = 0$ is

$$\int_{p_0}^a F(u) [f(u) + p_0 f'(u)] du . \quad (17)$$

Here, $\int_{p_0}^a F f du$ is the extra revenue generated from the returning customers while $\int_{p_0}^a F f' du$ is the extra (maybe negative) demand generated by increasing the cost of return.

From (17) and Lemma 2, the firm has an incentive to introduce a buy-now discount if the density f is increasing, i.e., whenever the demand curve is concave. But it has an incentive to introduce a discount much more generally, and the incentive is present whenever p_0 in (5) is strictly above $\frac{1-F(a)}{f(a)}$. To see this, use (5) to obtain

$$\begin{aligned} p_0 \int_{p_0}^a F(u) f'(u) du &= \frac{1}{2} \left[\frac{p_0 f(a)}{1-F(a)} (1-F(a)^2) - (1-F(p_0)^2) \right] \\ &> -\frac{1}{2} [F(a)^2 - F(p_0)^2] \\ &= -\int_{p_0}^a F(u) f(u) du, \end{aligned}$$

where the inequality follows from the assumption that $p_0 > \frac{1-F(a)}{f(a)}$. Thus, expression (17) is positive and a firm has a unilateral incentive to offer a buy-now discount.

Part (i) of the next proposition shows that this results holds for an arbitrary (finite) number of firms, while part (ii) shows that firms do not wish to set such a large buy-later price that no consumers ever return.

Proposition 2

(i) Starting from the free-recall equilibrium with uniform price p_0 in (5), a firm has a unilateral incentive to offer first-time visitors a buy-now discount if $p_0 > \frac{1-F(a)}{f(a)}$.

(ii) Starting from the exploding-offer equilibrium with price p in (10), a firm has a unilateral incentive to offer a buy-later price low enough to induce some first-time visitors to return.

As discussed in section 2, a sufficient condition to ensure $p_0 > \frac{1-F(a)}{f(a)}$ is that the demand curve is strictly logconcave and that the number of firms is finite.

Part (i) of Proposition 2 indicates that a seller typically has an incentive to offer a first-time visitor a discount on the regular price if the consumer buys immediately. The intuition for this result is as follows. As Lemma 2 shows, the impact of a small buy-now discount is the same as a small buy-later premium. A small buy-later premium has two effects: the extra revenue effect (every returning consumer now pays a premium) and the demand effect (first-time visitors become more likely to buy immediately, but potential returning consumers are less likely to come back). The second effect is similar to the demand effect caused by exploding offers, and whether it is positive or negative depends on the shape of the demand curve. However, the first revenue effect must be positive. Part (i) shows that this first effect is powerful enough for the overall effect to be positive under a mild condition on the demand curve. Part (ii) shows that a firm prefers to set a “moderate” buy-later price, rather than such a high buy-later price that none of its initial visitors returns. The intuition is that a firm can enjoy the strategic benefits of exploding offers but also generate some additional revenue if it charges returning visitors a high price instead of banning returning altogether.

3.2 Equilibrium buy-now discounts in an example

The previous result indicated that firms typically have an incentive to offer a buy-now discount. In this section we report the *equilibrium* discount and price in a duopoly example in which match utility u_i is uniformly distributed on $[0, 1]$, and compare this outcome to the situation with uniform prices.¹⁸ (The calculation is straightforward but lengthy; see our working paper Armstrong and Zhou (2010) for details, together with calculations for non-uniform distributions for match utility.)

We focus on symmetric equilibrium in which the buy-now price is p and the buy-later price is $\hat{p}$. To ensure an active market we assume $s < \frac{1}{8}$. First, we can show that the use of buy-now discounts leads to higher prices, i.e., $p_0 < p < \hat{p}$. That is, even the discounted buy-now price in the discriminatory case is higher than the uniform price, and the ability to offer discounts for immediate purchase drives up *both* prices.¹⁹ The intuition is that the buy-now discount adds to the intrinsic search frictions in the market, and this allows firms to charge a higher price. (Relative to the uniform-price case, consumers become less willing to search on, and so the firms' demand is less price elastic.) Figure 4a below depicts how the three prices vary with the search cost s , where from the bottom up the three curves represent p_0 , p and $\hat{p}$, respectively. As is expected, both the uniform price p_0 and the buy-now price p increase with the search cost. Less expected is the observation that the buy-later price $\hat{p}$ depends non-monotonically on s (and is always above the monopoly price $\bar{p} = \frac{1}{2}$ in this example).

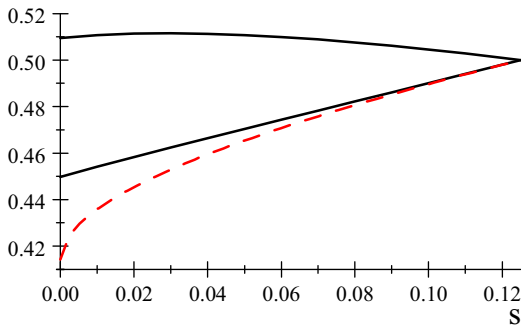


Figure 4a: Prices and search cost

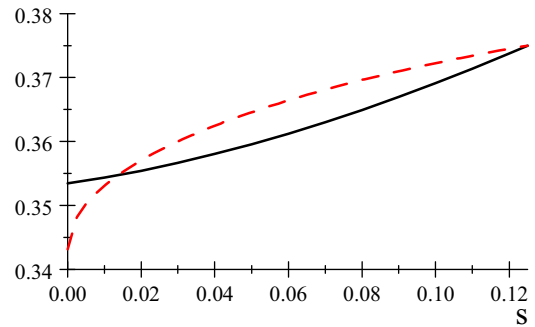


Figure 4b: Profits and search cost

Second, the equilibrium buy-now discount τ (the distance between the upper curve and the middle curve in Figure 4a) decreases with the search cost s . That is, the higher is

¹⁸It appears to be hard to characterize the equilibrium buy-now discount equilibrium for an arbitrary number of firms, as we were able to do in our discussion of exploding offers. When there are more than two firms, the consumer stopping rule with buy-now discounts is non-stationary and depends on the history of realized match utilities, and this makes the equilibrium analysis complex. (When exploding offers are used, by contrast, the stopping rule does not depend on previous offers, since the consumer has no ability to return.)

¹⁹It is not unusual that the ability to price discriminate in oligopoly leads to a fall in all prices, but cases where all prices rise are less familiar.

the intrinsic search cost, the less incentive firms have to deter consumers from searching on. In particular, when $s = 0$, we have $p \approx 0.45$ and $\hat{p} \approx 0.51$, and so $\tau \approx 0.06$. In this case, although the market has no intrinsic search frictions, firms in equilibrium impose “tariff intermediated” search frictions on consumers via the buy-now discount, which here is about 12% of the buy-later price. By contrast, in a market with $s = \frac{1}{8}$, which is the highest intrinsic search cost which induces consumers to participate, we have $p = \hat{p} = \frac{1}{2}$ and $\tau = 0$, so that there is no buy-now discount. (When $s = \frac{1}{8}$, search costs are so high that consumers will accept the first offer which yields them a non-negative surplus. In particular, there are no returning consumers even with costless recall.)

Third, since both prices rise, the buy-now discount equilibrium excludes more consumers from the market. In addition, as is expected, the use of buy-now discounts boosts fresh demand and reduces returning demand. This is illustrated for the case $s = 0$ in Table 1 (including for reference the case where exploding offers are used).

	p	$\hat{p}$	fresh	returning	excluded
no discount	0.41	0.41	41%	41%	17%
with discount	0.45	0.51	66%	11%	23%
exploding offer	0.45	n/a	73%	0%	27%

Table 1: The impact on prices and demand of buy-now discounts and exploding offers

However, whether the use of buy-now discounts leads to higher profit depends on the magnitude of the search cost. Figure 4b shows how industry profits with uniform pricing (the dashed curve) and profits with buy-now discounts (the solid curve) vary with the search cost s . We see that price discrimination leads to higher profit only if the search cost is relatively small. When the search cost is relatively high, price discrimination leads to prices which exclude too many consumers. In these cases, firms are engaged in a prisoner’s dilemma: when feasible an individual firm wishes to offer a buy-now discount, but when both do so industry profits fall. Nevertheless, as was seen in the exploding offer analysis in Figure 1b above, when there are more than two firms we anticipate that profits will rise when buy-now discounts are used, since the price-increasing effect will then outweigh the market participation effect. (When there are many firms, most consumers will eventually find a product they buy.) Finally, for similar reasons in the exploding-offer case, aggregate consumer surplus and total welfare fall when firms use buy-now discounts.

4 Buy-Later Policies Without Commitment

The preceding analysis has assumed that a firm can commit to its buy-later policy—be it an exploding offer or a buy-later price—when consumers first visit. In this section we discuss the plausibility of this assumption, and discuss whether firms will continue to discriminate against return visitors when they have less commitment power.

4.1 Exploding offers

Our analysis in section 2 relied on a firm's ability to commit to an exploding offer. If a consumer does come back to a firm after sampling a rival, the firm will have an incentive to sell to that consumer.²⁰ This credibility problem is enhanced by the fact that consumers often *will* wish to return to previous firms, since their stopping rule is such that their remaining option may have lower utility than previously rejected options. In markets where the commitment problem is hard to avoid, firms will be unable to implement exploding offers. However, the problem can sometimes be at least partially solved. First, in a dynamic environment sellers may be able to gain a reputation for sticking to exploding offers. In labour market settings, for instance, some employers may be known to keep their word. Second, in some business to business transactions, the seller may be able to offer a contract to the buyer which stipulates that the offer will not be available, or a higher price will be charged, if the buyer comes back after a specified time. Third, for those less extreme selling techniques which impose non-monetary returning costs on consumers, commitment seems easy to achieve. For example, a financial advisor can credibly ask a customer to make an appointment again if she wants to come back later.

Even if firms lack any ability to commit, firms may wish to claim to employ exploding offers if a fraction of consumers are “credulous”. When some consumers mistakenly believe a seller's claim that they must buy immediately or not at all, then Proposition 1 still applies. To see this, notice that the other rational consumers will always ignore what the sellers say about their buy-later policies and behave as in the free-recall case. So the decision about whether to use exploding offers or not depends only on the credulous consumers who behave just as the consumers analyzed in the full commitment case in section 2. While the proportion of credulous consumers does not affect the incentive to use exploding offers (at least when the demand curve is convex or concave), this proportion will affect the equilibrium price when exploding offers are used.²¹

Alternatively, in the buy-now discount model in which firms set higher prices to returning visitors rather than banning their return, we show next that if firms cannot commit to their buy-later price then exploding offers are the *only* credible equilibrium whenever consumers face a positive intrinsic cost of returning to a firm. This argument holds for arbitrary demand curves (including those which are convex).

²⁰Indeed, the quote from Cialdini in the introduction immediately goes on to say: “This, of course, is nonsense; the company and its representatives are in the business of making sales, and any customer who called for another visit would be accommodated gladly.”

²¹A conceptual issue arising in such a model with both rational and naive consumers is how they form their expectation of equilibrium prices. Our discussion here implicitly assumes that all consumers somehow hold the correct expectation about prices.

4.2 Buy-now discounts

We discuss next whether buy-now discounts are used if we relax the assumption that a firm can commit to its buy-later price when consumers first visit. The basic game structure is the same as before, except that now when a consumer discovers a firm's buy-now price, she can only form some belief about its buy-later price (the belief is of course required to be correct in equilibrium). The actual buy-later price can be learned only after she returns to the firm.

Here, unlike the rest of the paper, it makes an important difference whether or not consumers face an *intrinsic* returning cost when they come back to a previously-visited firm. Since in most situations such a returning cost does exist, we initially consider this case. (In the analysis with full commitment in sections 2 and 3, the presence of a small intrinsic return cost makes no qualitative difference, and for simplicity we assumed this cost was precisely zero.)

Positive intrinsic return cost: Suppose consumers face an intrinsic returning cost when they come back to a previously sampled firm. Proposition 3 describes the outcome when firms cannot fully commit to their buy-later price:

Proposition 3 *Suppose consumers face a positive intrinsic returning cost.*

- (i) *If firms cannot commit to their buy-later price, in equilibrium no consumers return to a previously-visited firm and the equilibrium price is as described in section 2.2.*
- (ii) *If firms can commit to an upper bound on their buy-later price, then firms in any equilibrium will choose their buy-later price to equal the upper bound, and the outcome is as if firms can fully commit to their buy-later prices as described in section 3.*

Proof. (i) Denote by $r > 0$ the intrinsic returning cost. Suppose in some equilibrium that each consumer forecasts that a firm's buy-later price is $\hat{p}(p_i)$ when its buy-now price is p_i , where $\hat{p}(\cdot)$ can take any form. Suppose that the buy-now price in this equilibrium is p^* , say, and suppose—contrary to the claim—there is some returning demand in this equilibrium. But if a consumer returns to firm i after sampling other firms, her match utility must satisfy $u_i \geq \hat{p}(p^*) + r$, since the consumer needs to pay the returning cost r . Since all its returning customers have match utility at least as great as $\hat{p}(p^*) + r$, the firm's optimal price for these customers must be at least $\hat{p}(p^*) + r$. This is because charging returning consumers $\hat{p}(p^*) + r$ will not induce any of them to leave this firm again and buy from others (since going back to any other firm also involves a returning cost r), while charging them a price below that cannot increase demand (since the deviation is not public). We thus obtain a contradiction to the assumption that $\hat{p}(p^*)$ was the correctly anticipated buy-later price. Therefore, in any equilibrium there are no returning consumers. The unique equilibrium outcome is then that firms charge first-time visitors a price as described in the exploding-offer equilibrium in section 2.2, and charge returning consumers a sufficiently high price such that consumers never come back to previously sampled firms.

(ii) Suppose now that firms can commit to an upper bound on the price they will charge returning visitors. Suppose that firm i charges the buy-now price p_i and commits to an upper bound on its buy-later price given by $\hat{p}_i$. Then any consumer who returns to buy from firm i must expect that the firm will actually charge price $\hat{p}_i$. (Suppose to the contrary that a returning consumer anticipates that the firm will actually charge price $\tilde{p} < \hat{p}_i$. Then, following the same logic as in part (i) of this proof, firm i then has an incentive to increase its buy-later price above $\tilde{p}$ since it knows that the consumer is willing to pay at least $\tilde{p} + r$ for the product. Therefore, the only equilibrium belief can be that returning consumers anticipate that firms will set their buy-later price equal to their announced upper bound.) The firm has an incentive to raise the price above $\hat{p}_i$, as in the proof to part (i), but that is not feasible given that the firm commits to its cap. Hence, firm i will charge its returning customers exactly $\hat{p}_i$. We deduce that announcing an upper bound to the buy-later price is equivalent to committing to an actual buy-later price at the level of the cap, and so the analysis of section 3 can be applied. ■

Thus, part (i) shows that if firms cannot commit to their buy-later price and if there is an intrinsic returning cost (no matter how small), rational consumers anticipate that buy-later prices will be so high that it is never worthwhile to return to a previous firm after leaving it. In effect, firms are forced to make exploding offers, and consumers have just one chance to buy from any firm. Thus, the lack of commitment power strengthens, rather than weakens, a firm's temptation to exploit returning consumers. This result is analogous to Diamond's (1971) paradox, showing how a small search cost can cause a market to shut down. Diamond's result relies on consumers knowing their match utility in advance, and a central advantage of Wolinsky's formulation with *ex ante* unknown match utilities is that this paradox can be avoided. But even in our Wolinsky-type framework, the *returning* consumers know their match utility, and so the returning market fails for the same reason as the primary market failed in Diamond's framework.

Of course, as shown in part (ii) of Proposition 2, a firm would like to avoid this complete shut down of the return market if possible. One method, when feasible, is to commit to a buy-later price cap. For instance, in most retailing markets the price printed on the price label in the store usually has this commitment power, and a sales person has no authority to increase the price above the displayed price. (Rather the shelf price is chosen at a higher managerial level within the firm.) Similarly, as discussed in the introduction, the firm in Bone's (2006) study offered its potential customers a regular price (in the form of a written quote) if they decided to buy later. Whenever this form of partial commitment is feasible, part (ii) of the proposition shows that the equilibrium is the same as that in the full commitment case of section 3. Thus, a cap on the buy-later price—which firms can plausibly impose on themselves in many situations—can be used as a full commitment device.

Notice that this buy-later price cap can also sustain the full commitment outcome when

the intrinsic returning cost is zero, if consumers happen to have beliefs whereby they expect a firm will set its buy-later price equal to its committed cap. (With a positive return cost, this is the *only* rational belief.) However, as we discuss next, there are typically other, perhaps more plausible, outcomes in that situation.

No intrinsic return cost: Now suppose that consumers face no intrinsic returning cost. Although this situation is unrealistic, it has some theoretical interest. In this case, there is often an equilibrium in which uniform pricing (as in the benchmark model in section 2.1) is a credible strategy, so that no buy-now discount is offered. That is to say, (i) consumers do not anticipate that they will face a higher price if they return to buy from a previously sampled firm and plan their search strategy accordingly, and (ii) when a consumer does return to a firm, that firm has no *ex post* incentive to surprise the consumer with an unexpected price hike. That this is so is easy to understand in the extreme case with $s = 0$. When search costs are zero, consumers sample all firms before they purchase (given their belief that there is no buy-later surcharge), and so all buyers are returning customers. Thus, we are just in the situation of Wolinsky model with zero search costs, and the incentive to set the price to returning consumers is exactly the same as the incentive to set the uniform price p_0 in the $s = 0$ version of expression (5).

More generally we have the following result:

Proposition 4 *Suppose firms cannot commit to the buy-later price and consumers face no intrinsic returning cost. Then uniform pricing is an equilibrium outcome whenever the demand curve is weakly convex.*

Proposition 4 echoes Proposition 1, and in both situations when firms face a convex demand curve they offer the same deal to all their potential customers. The intuition is that when the buy-later premium is not anticipated by first-time visitors, it loses its strategic benefit of deterring consumers from further search. At the same time, the informational effect is also relatively weak: given a consumer comes back to a firm when she does not expect to pay a returning purchase premium and does not need to pay an intrinsic returning cost, her taste for this firm's product cannot be too strong (for otherwise she would have purchased immediately), although she dislikes other products even more. Under the conditions of Proposition 4, the informational motive to raise prices to return visitors is precisely zero.

Proposition 4 also suggests an alternative outcome which may emerge under the “price cap” regime discussed in Proposition 3. There, we suggested that when a firm commits to an upper bound on its buy-later price, there is an equilibrium in which consumers believe that sellers will set the price at this upper bound if they return. Alternatively, however, a consumer who has been offered a discount at a store on a first visit may well believe that she will be able to negotiate the same discount should she return. Formally, her belief is then that the buy-later price will be the same as the buy-now price. With these beliefs,

and when there is no intrinsic return cost, Proposition 4 shows that firms will not set their buy-later price at the level of their indicated price cap, and instead will offer the “discount” uniformly to all visitors. Therefore, when there are no intrinsic return costs and firms can commit to a buy-later price cap, there are at least two equilibria: one where consumers believe that firms will charge the same price on a return and a first visit, and one where consumers believe the discount will be available only on the first visit.

5 Prevent Learning About Firm’s Own Product

The focus of this paper has been on a seller’s strategic incentive to prevent a consumer from acquiring information about rival offerings. By making it hard to return to a firm, a consumer is reluctant to go on to investigate other deals. An alternative form of high-pressure selling is to force a potential customer to buy quickly, before she has had a chance to evaluate the *current* product adequately. If a seller forces consumers to decide quickly (or offers a discount if they buy quickly), a consumer might have to decide whether or not to purchase before she has worked out how much she actually wants the product. Without accurate information about the realized match utility, suppose that a consumer bases her purchase decision on the *expected* match utility, which is $\bar{u}$, say.

This issue can be analyzed within a monopoly framework (unlike our main model). Suppose the monopolist has marginal cost c for supplying the product. If the seller gives the consumer time to calculate her (privately observed) match utility u , the seller’s profit with price p is $(p - c)(1 - F(p))$, and the optimal price maximizes this expression. If instead the seller forces the consumer to buy immediately, knowing only her expected utility, the seller can charge $p = \bar{u}$ and obtain profit $\bar{u} - c$. Since $\bar{u} > p(1 - F(p))$ for all p , it follows that the latter strategy is more profitable whenever c is sufficiently close to zero. By contrast, if c is sufficiently large (above $\bar{u}$, for instance), then the monopolist prefers to give consumers enough time to understand the realized match utility.²²

One can also consider a search version of this problem. Consider the Wolinsky model, but suppose a consumer’s initial search is costless so that consumers are willing to participate in the market. When the marginal production cost c is small enough, it is an equilibrium for all firms to force sales before the consumer discovers her utility and to fully extract expected utility with the monopoly price $p = \bar{u}$. (Suppose all other firms do so. Then when a consumer arrives at a seller, she will never search further. So the seller acts as a monopolist and, as we have seen, its most profitable strategy is then to force a quick sale to conceal match-specific information.²³) Even with small search costs, then, all firms en-

²²For further details of the monopolist’s incentives to reveal or conceal match-specific information, see Lewis and Sappington (1994). They show that the monopolist typically will choose to reveal all information or none. Anderson and Renault (2009) discuss when a firm wishes to disclose match-specific information to consumers about a *rival*’s product.

²³If c is large enough (above $\bar{u}$, say), this high-pressure selling equilibrium cannot be sustained.

gage in this form of high-pressure selling, with undesirable results: consumers are left with no surplus; even low- u consumers buy, despite the costs of serving them, and consumers are randomly matched with sellers rather than buying the most suitable product. Thus, if firms have the ability to conceal match-specific information by means of high-pressure sales techniques they will often choose to do so, and the Diamond Paradox emerges again.

6 Conclusions

This paper has explored the incentives firms have to discourage consumer search by making it costly for consumers to return after investigating rival sellers. The use of exploding offers can be individually profitable for firms under certain conditions, such as when the demand curve is concave. A less extreme policy is to offer first-time visitors a buy-now discount, and firms have an incentive to offer such discounts under the relatively mild condition that the demand curve is strictly logconcave. Either selling technique tends to raise market prices and lower both consumer surplus and total welfare. If firms cannot commit to their buy-later price the outcome depends on whether there is an intrinsic cost of returning to a firm: if the intrinsic return cost is zero, it is often an equilibrium for firms to offer the same price to all potential customers; if the intrinsic return cost is positive, firms are forced to make exploding offers.

As demonstrated in this paper, high-pressure selling can limit a consumer's ability to make a well-informed decision, which in turn can harm market performance. Public policy has attempted to address this problem. For instance, the *Unfair Commercial Practices Directive* (adopted in 2005 across the European Union) prohibits in all circumstances "Falsely stating that a product will only be available for a very limited time, or that it will only be available on particular terms for a very limited time, in order to elicit an immediate decision and deprive consumers of sufficient opportunity or time to make an informed choice." However, the enforcement of such laws is often difficult. Often a more efficient method to tackle the issue is to restore a consumer's freedom of choice using other, indirect, means. For example, exploding offers could in essence be prohibited by mandating a "cooling off period", so that consumers have the right to return a product in some specified time after agreeing to purchase. (They could then return a product if they subsequently find a preferred option.) Many jurisdictions impose cooling off periods for some products, especially those sold in the home.

To end, we point out reasons why sales tactics which disadvantage returning visitors are not seen in *all* markets, even when their use is permitted. Proposition 4 may provide one theoretical explanation why in some markets uniform prices offered to both first-time and returning visitors, even when firms can distinguish first-time from returning visitors. Alternatively, a "behavioral" reason why firms do not surcharge their returning customers is that many consumers could be antagonized by an unexpected price rise, and decide to

buy elsewhere. But first and foremost, many retailers, especially in the traditional bricks-and-mortar sector, cannot distinguish first-time from returning visitors. Shopping in a supermarket, say, is unlikely to involve much contact with sales personnel at all, and there is no mechanism by which the firm can detect first-time from returning visitors. More generally, consumers may be able to conceal their search history (e.g., by deleting cookies on their computer). Thus, if firms discriminate against return visitors and if it is costless to pretend to be a new visitor, consumers will do this, and the market will operate as a standard search market with uniform prices.

APPENDIX

Proof of Proposition 1: Part (i): Our proof consists of two steps. First, we show that if the match utility density f is strictly increasing, then all firms using exploding offers is an equilibrium. Second, we exclude the possibility that all firms allowing free recall is also an equilibrium.

The hypothesis is that all firms choose to use exploding offers and to set the price p in (10). Suppose a deviating firm chooses price $\tilde{p}$ and allows free recall, while other firms follow the proposed equilibrium strategy. Suppose that the deviating firm is in the k_{th} position of a consumer's search process and $k < n$. (If $k = n$ then allowing free recall or not does not affect the firm's demand.) Then the probability that this consumer will visit the firm is still h_k in (8), since consumers hold equilibrium beliefs. However, her incentive to search beyond the firm is now altered. Since she can return to this firm whenever she wants, she becomes more willing to continue searching. If at the deviating firm she finds utility u such that $u - \tilde{p} \leq 0$, she will never buy from the firm (either immediately or later). So consider the situation where $u - \tilde{p} > 0$. Then if she leaves the deviating firm, she will enter a no-recall search market with $n - k$ products each being sold at price p , but now with an outside option $u - \tilde{p}$. To calculate the consumer's stopping rule in this situation, we need to calculate her expected surplus from entering such a search market.

Denote by $W_m(z)$ the expected surplus from a no-recall search market with m unsampled products with price p and outside option $z \geq 0$. It is difficult to derive an explicit expression for $W_m(z)$, and instead we use an indirect method.²⁴ Let $r_m(z)$ be the probability that the consumer will eventually consume the outside option. By standard envelope reasoning we have the following result.²⁵

²⁴By contrast, it is straightforward to derive an explicit expression for consumer surplus in the case of free recall—see expression (22) below.

²⁵A sketch of a proof goes as follows. Let Θ_m be the set of all possible stopping rules in the no-recall search market with m products and outside option z . If the consumer uses $\theta \in \Theta_m$, her expected surplus is $zR(\theta) + U(\theta)$, where $R(\theta)$ is the probability that the consumer will opt for z given the stopping rule θ , and $U(\theta)$ is the surplus from buying other products (including the expected search costs). Thus, $W_m(z) = \max_{\theta \in \Theta_m} [zR(\theta) + U(\theta)]$ and $r_m(z) = R(\theta(z))$, where $\theta(z)$ is the optimal stopping rule given z . $W_m(z)$ is convex since the objective function is linear in z , and its derivative is $r_m(z)$ almost everywhere.

Claim 1 $W_m(z)$ is convex and $W'_m(z) = r_m(z)$ almost everywhere.

Notice that $W_m(0)$, the expected surplus from a no-recall search market with a zero outside option, is just $a_m - p$. Thus, we have

$$W_m(z) = a_m - p + \int_0^z r_m(x) dx .$$

Since $0 < r_m(z) \leq 1$, $W_m(z)$ is an increasing function with slope no greater than one. In addition, since the consumer can always consume the outside option without searching at all, we have $W_m(z) \geq z$. In particular, for a sufficiently large z (e.g., $z \geq u_{\max} - p$), the consumer will consume the outside option immediately, so $W_m(z) = z$. Hence, we can deduce that $W_m(z) = z$ and $r_m(z) = 1$ for $z \geq z_m$, where $z_m = \inf\{z : W_m(z) = z\}$ and $z_m \in (a_m - p, u_{\max} - p)$. For $z < z_m$, the consumer will search and so $r_m(z) < 1$.

When the deviating firm occupies the k_{th} position in a consumer's search order, the consumer will buy from it immediately if and only if $u - \tilde{p} \geq W_{n-k}(u - \tilde{p})$, i.e., if $u - \tilde{p} \geq z_{n-k}$, where z_{n-k} , according to its definition, satisfies

$$z_{n-k} = a_{n-k} - p + \int_0^{z_{n-k}} r_{n-k}(x) dx . \quad (18)$$

Thus, the firm's demand when it is in the k_{th} position, charges price $\tilde{p}$ and permits free return, is

$$\begin{aligned} h_k \left[1 - F(z_{n-k} + \tilde{p}) + \int_{\tilde{p}}^{z_{n-k} + \tilde{p}} r_{n-k}(u - \tilde{p}) f(u) du \right] \\ = h_k \left[1 - F(z_{n-k} + \tilde{p}) + \int_0^{z_{n-k}} r_{n-k}(u) f(u + \tilde{p}) du \right] , \end{aligned} \quad (19)$$

where the equality follows after changing variables in the integral. Compared to the demand generated with an exploding offer given in (9), it now has reduced immediate demand since $z_{n-k} > a_{n-k} - p$, but has positive returning demand comprised of the integral term.

Claim 2 Demand in (19) is smaller than that in (9) if f is strictly increasing.

Proof. We need to show

$$\int_0^{z_{n-k}} r_{n-k}(u) f(u + \tilde{p}) du < F(z_{n-k} + \tilde{p}) - F(a_{n-k} - p + \tilde{p}) . \quad (20)$$

Define

$$\phi(u) \equiv z_{n-k} + \tilde{p} - \int_u^{z_{n-k}} r_{n-k}(x) dx .$$

Note that $\phi'(u) = r_{n-k}(u)$, $\phi(z_{n-k}) = z_{n-k} + \tilde{p}$, and $\phi(0) = a_{n-k} - p + \tilde{p}$ (which follows from (18)). Then the right-hand side of (20) can be written as

$$\int_0^{z_{n-k}} r_{n-k}(u) f(\phi(u)) du .$$

Since $\phi(u) > u + \tilde{p}$ (because $r_{n-k}(x) < 1$ for $x < z_{n-k}$), expression (20) holds if f is an increasing function. ■

Therefore, for any price $\tilde{p}$, unilaterally allowing free recall causes the deviating firm's demand (and hence profit) to fall when f is increasing. (This is true regardless of the firm's position in a consumer's search order, except when it is in the final position in which case the use of exploding offers makes no difference to the firm's demand.) It follows that an equilibrium in which all firms use exploding offers exists.

The second step is to exclude the possibility of a free-recall equilibrium when f is strictly increasing. We show that, starting from the hypothetical free-recall equilibrium with price p_0 , each firm has a unilateral incentive to use an exploding offer no matter what position it is in the consumer's search process (except when it is in the final position).

As in expression (4), a firm's demand, if it is in the k_{th} position of the consumer's search process with $k < n$, is

$$F(a)^{k-1}[1 - F(a)] + \int_{p_0}^a F(u)^{n-1} f(u) du . \quad (21)$$

The first term is demand when the consumer buys the firm's product immediately, and the second term is demand when the consumer first leaves the firm but eventually comes back. Suppose now that the firm unilaterally uses an exploding offer but still charges the price p_0 . (We will show the firm's profits increase with this deviation, and hence the hypothetical equilibrium is not valid. The firm's profits would increase still further if it altered its price as well.) Define $\delta \equiv \max\{0, u_1 - p_0, \dots, u_{k-1} - p_0\}$. Then the consumer will visit the firm if and only if $\delta < a - p_0$. If she finds match utility u at the firm, she will buy (immediately) if $u - p_0$ is greater than the expected surplus from searching further.

Denote by $V_m(z)$ the expected surplus from participating in a free-recall search market with m products offered at price p_0 and an outside option $z < a - p_0$. Then²⁶

$$V_m(z) = z + \int_{z+p_0}^a [1 - F(u)^m] du . \quad (22)$$

One can check that $z \leq V_m(z) < a - p_0$.

²⁶The consumer will stop searching before she runs out of options if and only if she finds a product with match utility greater than a . (This is true regardless of z provided that $z < a - p_0$.) Her expected surplus is therefore

$$V_m(z) = [1 - F(a)^m] \cdot [E(u|u \geq a) - p_0] + \Pr(u^* < a) \cdot E[\max\{u^* - p_0, z\} | u^* < a] - sT,$$

where $u^* = \max\{u_1, \dots, u_m\}$ and $T = [1 - F(a)^m] / [1 - F(a)]$ is the expected number of searches. The first term is the surplus when the consumer ends up buying a product with match utility higher than a , and the second term is the surplus when she ends up sampling all firms. From the definition of the reservation utility a in (3), we have $s = [1 - F(a)][E(u|u \geq a) - a]$. Substituting this into $V_m(z)$ yields the formula.

The consumer will buy from firm i if and only if $u - p_0 \geq V_{n-k}(\delta)$. Here, δ is the consumer's outside option if the consumer leaves the firm and continues searching (since the firm is using an exploding offer). The c.d.f. of δ defined on $[0, u_{\max} - p_0]$ is $G(\delta) \equiv F(\delta + p_0)^{k-1}$, which has a mass point at zero. Therefore, the deviating firm's demand when it is in the k_{th} position is

$$\begin{aligned} & \Pr(\delta < a - p_0 \text{ and } u - p_0 > V_{n-k}(\delta)) \\ &= G(0)[1 - F(p_0 + V_{n-k}(0))] + \int_0^{a-p_0} [1 - F(p_0 + V_{n-k}(\delta))] \frac{dG(\delta)}{d\delta} d\delta \\ &= F(a)^{k-1}[1 - F(p_0 + V_{n-k}(a - p_0))] + \int_{p_0}^a f(p_0 + V_{n-k}(x - p_0)) V'_{n-k}(x - p_0) F(x)^{k-1} dx, \end{aligned} \quad (23)$$

where the second equality follows after integrating by parts and changing the integral variable. According to the definition of $V_m(\cdot)$ in (22), we have $V_{n-k}(a - p_0) = a - p_0$ and

$$V_{n-k}(x - p_0) = x - p_0 + \int_x^a [1 - F(u)^{n-k}] du, \quad V'_{n-k}(x - p_0) = F(x)^{n-k}.$$

Substituting these into (23) shows that the firm's demand is

$$F(a)^{k-1}[1 - F(a)] + \int_{p_0}^a F(x)^{n-1} f\left(a - \int_x^a F(u)^{n-k} du\right) dx. \quad (24)$$

Since $a - \int_x^a F(u)^{n-k} du > x$ for $x < a$, one can see that if f is strictly increasing (we actually only need f to be strictly increasing on $[p_0, a]$), demand in (24) is strictly greater than demand in (21). Therefore, the firm does have an incentive to deviate from the supposed free-recall equilibrium. This completes the proof of part (i). Parts (ii) and (iii) can be proved in a similar manner.

Proof of Proposition 2: (i) We will show that a firm has an incentive to introduce a small buy-later premium, and then invoke Lemma 2 to show that the firm also has an incentive to offer a small buy-now discount. Compared to the duopoly case analyzed in the main text, the additional analysis needed for the general n -firm case involves the extra complexity of a consumer's stopping rule. In particular, the consumer's stopping rule at a firm which offers a buy-later premium will depend on the history of offers she sees before she encounters the firm, and this feature is absent in the duopoly analysis.

Let p_0 be the price in the free-recall equilibrium defined by (5). Assumption (2) implies that $p_0 < a$. We first consider this hypothetical search problem:

A search problem: Suppose the consumer encounters firm i first, and is offered match utility u_i , the buy-now price p_0 , and a buy-later premium $\tau > 0$ (so the buy-later price at firm i is $\hat{p} = p_0 + \tau$.) Suppose she expects that all m remaining firms charge price $p_0 < a$ and allow free recall, and suppose the consumer has an outside option $\delta < a - p_0$. What is her optimal stopping rule at firm i ?

It is clear that (a) if $u_i \geq a$, the consumer will surely stop searching and buy at firm i immediately (this is even true when $\tau = 0$); and (b) if $u_i - p_0 \leq \delta$, then firm i 's offer is dominated by the outside option and the consumer will not buy from the firm (either immediately or later), and she will keep searching since $\delta < a - p_0$.

Now consider the intermediate case with $u_i - p_0 \in (\delta, a - p_0)$. If the consumer buys immediately at firm i , her payoff is $u_i - p_0$. If she leaves firm i , she will begin a free-recall search process with m firms and an outside option

$$z = \max\{\delta, u_i - \hat{p}\} < a - p_0 .$$

(Recall she will pay the higher price $\hat{p} > p_0$ if she returns to buy from firm i .) As before, the expected surplus $V_m(z)$ from entering this search market is given by (22). Given δ , z is a function of u_i and we can therefore regard $V_m(z)$ as a function of u_i : it is flat until u_i reaches $\delta + \hat{p}$ and then increases with u_i with slope less than one. (Note that we are considering the case with $u_i < a$, so the slope cannot be equal to one.) Recall from (22) that for $z < a - p_0$, $z < V_m(z) < a - p_0$.

Clearly, the consumer will buy immediately from firm i if and only if

$$u_i - p_0 \geq V_m(\max\{\delta, u_i - \hat{p}\}) . \quad (25)$$

Given the properties of $V_m(\cdot)$, the equality of (25) has a unique solution $a_m(\tau) \in (\delta + p_0, a)$. We conclude that the consumer will buy immediately from firm i if and only if $u_i \geq a_m(\tau)$.

There are then two cases, depending on the size of the premium τ :

(a) If $u_i - p_0$ crosses $V_m(z)$ at the flat portion, which occurs when $\delta + \hat{p} - p_0 > V_m(\delta)$ or $\tau > V_m(\delta) - \delta$, then

$$a_m(\tau) = p_0 + V_m(\delta) , \quad (26)$$

which does not depend on τ . In this case, the consumer will never return to firm i once she leaves because $u_i - \hat{p}$ is dominated by δ . Therefore, τ is so large that firm i has no returning demand.

(b) If $u_i - p$ crosses $V_m(z)$ at the increasing portion, which occurs when $\tau \leq V_m(\delta) - \delta$, then $a_m(\tau)$ is implicitly determined by $a_m(\tau) - p_0 = V_m(a_m(\tau) - p_0 - \tau)$, which from (22) implies $a_m(\tau)$ satisfies

$$\tau = \int_{a_m(\tau) - \tau}^a [1 - F(u)^m] du , \quad (27)$$

which does not depend on p_0 or δ . In particular, $a_m(0) = a$. Expression (27) is the generalization beyond duopoly of our earlier formula (14). In this case, the consumer will initially reject firm i 's offer if $u_i < a_m(\tau)$, but will come back to the firm after sampling the remaining m firms if $u_i - \hat{p} > \max_{1 \leq j \leq m} \{\delta, u_j - p_0\}$.²⁷ Note that the assumption $\delta < a - p_0$ implies that $V_m(\delta) - \delta > 0$, and so case (b) is relevant for all sufficiently small $\tau > 0$.

In sum, we deduce the following result:

²⁷Note that once the consumer leaves firm i , she has the outside option $z < a - p_0$ and so she will never come back before sampling all the remaining m firms.

Claim 3 *In this hypothetical search problem, the consumer will buy from firm i immediately if and only if $u_i \geq a_m(\tau)$, where $a_m(\tau)$ is defined in (26) if $\tau > V_m(\delta) - \delta$ and otherwise $a_m(\tau)$ is defined in (27).*

Finally, since $V_m(\delta) - \delta$ is decreasing in δ , the condition $\tau > V_m(\delta) - \delta$ is equivalent to $\delta \in (\delta_\tau, a - p_0)$, where δ_τ solves

$$\tau = V_m(\delta_\tau) - \delta_\tau = \int_{\delta_\tau + p_0}^a [1 - F(u)^m] du \quad (28)$$

if $\tau < V_m(0)$, and $\delta_\tau = 0$ otherwise. In particular, $V_m(\delta_0) = \delta_0 = a - p_0$.

We now prove Proposition 2. Starting from the free-recall equilibrium with price p_0 , suppose firm i unilaterally introduces a returning purchase premium $\tau > 0$ but keeps the buy-now price unchanged at p_0 . Suppose firm i happens to be in the k_{th} position of the consumer's search process. If $k = n$, then τ has no impact on firm i 's profit. In the following, we show that for any $k < n$, introducing a small premium $\tau > 0$ is profitable for the firm.

As in the proof of Proposition 1, let $\delta \equiv \max\{0, u_1 - p_0, \dots, u_{k-1} - p_0\}$ be the best offer from the previous $k - 1$ firms. A consumer will visit firm i if $\delta < a - p_0$. If the consumer arrives at firm i and discovers match utility u_i and the buy-later premium τ (but still holds the equilibrium belief about the remaining $n - k$ firms' policies), she faces the search problem we have just analyzed with $m = n - k$, and her stopping rule will depend on her best previous offer δ . Let us focus on a relatively small τ such that $\tau < V_{n-k}(0)$ and define δ_τ as in (28) with $m = n - k$. Then if $\delta \in (\delta_\tau, a - p_0)$, the reservation utility according to (26) is $a_{n-k}(\tau) = p_0 + V_{n-k}(\delta)$. In this case, the consumer will buy immediately if $u_i \geq a_{n-k}(\tau)$, and otherwise she will keep searching and never come back. Alternatively, if $\delta \leq \delta_\tau$ the reservation utility $a_{n-k}(\tau)$ is as given in (27) with $m = n - k$. In this case, even if the consumer leaves firm i first (i.e., if $u_i < a_{n-k}(\tau)$), she will eventually come back after sampling all remaining firms if $u_i - p_0 - \tau$ is greater than their offered surplus *and* the outside option δ which represents the best offer among the previous $k - 1$ firms. Explicitly, firm i 's returning demand in this case is

$$\begin{aligned} & \Pr(\max_{j > k} \{\delta, u_j - p_0\} < u_i - p_0 - \tau < a_{n-k}(\tau) - p_0 - \tau) \\ &= \int_{p_0 + \tau}^{a_{n-k}(\tau)} F(u_i - \tau)^{n-1} dF(u_i) = \int_{p_0}^{a_{n-k}(\tau) - \tau} F(u) f(u + \tau) du . \end{aligned}$$

(Note δ is also a random variable with c.d.f. $G(\delta) = F(\delta + p_0)^{k-1}$, and the second step follows after changing the integral variable.) Therefore, firm i 's profit if it is in the k_{th} search position and charges the buy-later premium τ is

$$p_0 \int_{\delta_\tau}^{a-p_0} [1 - F(p_0 + V_{n-k}(\delta))] dG(\delta) + p_0 G(\delta_\tau) [1 - F(a_{n-k}(\tau))]$$

$$+ (p_0 + \tau) \int_{p_0}^{a_{n-k}(\tau) - \tau} F(u)^{n-1} f(u + \tau) du . \quad (29)$$

Note from (27) that

$$(1 - a'_{n-k}(0))(1 - F(a)^{n-k}) = 1 . \quad (30)$$

By using the observations $V_{n-k}(\delta_0) = \delta_0 = a - p_0$ and (30), the derivative with respect to τ of firm i 's profit in (29) when it is in the k_{th} position (with $k < n$), evaluated at $\tau = 0$, is

$$\int_{p_0}^a F(u)^{n-1} [f(u) + p_0 f'(u)] du , \quad (31)$$

which generalizes the duopoly expression (17). Here, $\int_{p_0}^a F^{n-1} f du$ is the extra revenue generated from the returning customers, while $\int_{p_0}^a F^{n-1} f' du$ is the extra demand generated by increasing the cost of return. That (31) is positive when $p_0 > \frac{1-F(a)}{f(a)}$ follows the argument given in the main text for duopoly. Since (31) is positive (and the same) for all $k < n$, the proof of part (i) is complete.

(ii) We show that in the buy-now-discount model (where firms can commit to the buy-later price), there is no equilibrium in which firms set such high buy-later prices that no consumers return to previously visited firms. Suppose by contrast that there is an equilibrium without returning demand. Let $\bar{\tau}$ be the minimum buy-later premium needed for such an equilibrium. Then $\bar{\tau}$ satisfies

$$a_{n-1} = p + \bar{\tau} , \quad (32)$$

where p is the no-recall equilibrium price defined in (10). (Recall $\{a_m\}$ is the sequence of reservation utilities in the no-recall case.) To see this, notice that a_m is increasing in m and so if a consumer does not want to go back to the first sampled firm (at which she was most picky), she also does not want to go back to any other firm. That is, (32) implies

$$a_m < p + \bar{\tau} \quad (33)$$

for any $m \leq n - 2$.

Starting from the hypothetical equilibrium in which each firm sets a buy-later premium $\bar{\tau}$,²⁸ suppose firm i deviates and sets a buy-later premium $\bar{\tau} - \varepsilon$ where $\varepsilon > 0$ is sufficiently small (but keeps its buy-now price p unchanged). First of all, realize that this small deviation will not affect the search behavior of consumers who sample any other firm first because of (33). Therefore, we focus on those consumers who sample firm i first.

Given a buy-later premium smaller than $\bar{\tau}$, the consumer will become more likely to search on at firm i . Let $\tilde{a}_{n-1} > a_{n-1}$ be the new reservation utility. For a small deviation, $\tilde{a}_{n-1}$ must satisfy

$$\tilde{a}_{n-1} - p = W_{n-1}(\tilde{a}_{n-1} - p - \bar{\tau} + \varepsilon) . \quad (34)$$

²⁸The same argument applies if firms charge buy-later premia greater than $\bar{\tau}$.

The right-hand side is the expected surplus from participating a “no-recall” market with $n - 1$ firms and a positive outside option $\tilde{a}_{n-1} - p - \bar{\tau} + \varepsilon$ which is available if the consumer comes back to firm i .²⁹ (Recall the notation $W_m(\cdot)$ introduced in the proof of Proposition 1.) Note that $\tilde{a}_{n-1} \rightarrow a_{n-1}$ as $\varepsilon \rightarrow 0$. Let $\tilde{a}_{n-1} \approx a_{n-1} + \theta\varepsilon$ be the first-order approximation of $\tilde{a}_{n-1}$, where θ is to be determined. Then (32) and (34) imply

$$\begin{aligned} a_{n-1} - p + \theta\varepsilon &\approx W_{n-1}((1 + \theta)\varepsilon) \\ &\approx W_{n-1}(0) + (1 + \theta)\varepsilon W'_{n-1}(0) \\ &= a_{n-1} - p + (1 + \theta)\varepsilon r_{n-1}(0) . \end{aligned}$$

The equality used Claim 1 in the proof of Proposition 1, and $r_{n-1}(0)$ is the probability that the consumer will purchase nothing when firms make exploding offers. Thus, θ satisfies

$$\theta = (1 + \theta)r_{n-1}(0) . \quad (35)$$

For a small ε , firm i 's fresh demand from those who visit firm i first will be reduced by

$$f(a_{n-1})\theta\varepsilon . \quad (36)$$

On the other hand, the reduction of the buy-later premium will generate new returning demand. Those consumers who find $u \in [p + \bar{\tau} - \varepsilon, \tilde{a}_{n-1}]$ at firm i will search on first and eventually come back with a probability approximately equal to $r_{n-1}(0)$. Since the length of the above interval is (approximately) $(1 + \theta)\varepsilon$, the returning demand is

$$f(a_{n-1})(1 + \theta)r_{n-1}(0)\varepsilon . \quad (37)$$

From (35), one can see that (36), the decrease of the fresh demand is actually equal to (37), the increase of the returning demand. But each returning consumer pays more than each first-time visitor ($p + \bar{\tau} - \varepsilon > p$). Hence, the deviation is profitable. In other words, in the buy-now-discount model, there must exist returning consumers in any equilibrium.

Proof of Proposition 4: We show that when firms cannot commit to the buy-later price and consumers face no intrinsic returning cost, the following configuration is an equilibrium when the demand curve is convex (i.e., the density f is weakly decreasing): (i) each firm charges p_0 to any visitor, where p_0 is defined in (5); (ii) each consumer holds the belief that a firm's buy-later price will be the same as its buy-now price (even if she observes an off-equilibrium buy-now price), and conducts sequential search accordingly.

Suppose firm i sets a slightly different buy-now price p and surprises the returning

²⁹The consumer's reservation utility at the subsequent firms may also change, but (33) still holds for a sufficiently small ε . That is why we can regard the subsequent market as a no-recall market.

consumers with a small premium $\tau \geq 0$, while all other firms charge the uniform price p_0 .³⁰ Consumers believe that if they come back to firm i , they will only pay p instead of $p + \tau$. Let $\Pi(p, \tau)$ be firm i 's deviation profit when it offers this alternative tariff. When p is close to p_0 and τ is close to zero, as with expression (12) above we have

$$\Pi(p, \tau) \approx \Pi(p_0, 0) + \tau \Pi_\tau(p_0, 0) .$$

Thus, this deviation is unprofitable if $\Pi_\tau(p_0, 0) < 0$. This implies that we need only consider the deviation with an unchanged buy-now price p_0 and a small buy-later premium τ . Notice that this deviation will only affect firm i 's returning demand given that consumers hold equilibrium beliefs.

We claim that firm i 's deviation returning demand is

$$\frac{n-1}{n} \Pr \left(\max_{j \neq i} \{p_0 + \tau, u_j + \tau\} < u_i < a \right) = \frac{n-1}{n} \int_{p_0+\tau}^a F(u-\tau)^{n-1} f(u) du .$$

To see this, note that if a consumer samples firm i last (which occurs with a probability $1/n$), there is no returning demand from her and the buy-later premium τ has no impact. Suppose now that the consumer has firm i in the k_{th} position in her search order where $k \leq n-1$, which occurs again with probability $1/n$. In order for firm i to have any return demand, a consumer must first reach the firm, which she does if she rejects all the previous $k-1$ offers (i.e., when previous firms provide match utility below a). Given that she believes that firm i 's price will not change if she returns, she will leave firm i first if $u_i < a$ and then come back if all other firms provide match utility lower than u_i . Surprised by firm i 's deviation price $p_0 + \tau$ after she comes back, the consumer will buy at firm i only if $u_i - p_0 - \tau > \max\{0, u_j - p_0\}$ since she can leave the market or go back to other firms. The likelihood of this whole event is the probability term on the left-hand side above (which does not depend on k if $k < n$). Summing all such probabilities over $k < n$ yields the claim.

Therefore, firm i 's profit from returning consumers when it increases the buy-later price by τ is $(p_0 + \tau)$ multiplied by this returning demand. One can check that $\Pi_\tau(p_0, 0)$ has the sign of

$$\begin{aligned} \Pi_\tau(p_0, 0) &\stackrel{\text{sign}}{=} \int_{p_0}^a F(u)^{n-1} f(u) du - p_0 \left\{ f(p_0) F(p_0)^{n-1} + (n-1) \int_{p_0}^a F(u)^{n-2} f(u)^2 du \right\} \\ &= \frac{1}{n} (F(a)^n - F(p_0)^n) - p_0 \left\{ f(a) F(a)^{n-1} - \int_{p_0}^a F(u)^{n-1} f'(u) du \right\} . \end{aligned}$$

³⁰We consider only local deviations. Given that firm i has no profitable local deviation, it also has no profitable global deviation if its profit function is quasiconcave in p and τ , for instance. Although in our search model it is hard to derive more primitive conditions, we can show that it is true at least for a uniform distribution of match utilities. Note that setting $\tau < 0$ will only reduce each returning consumer's payment but not increase the returning demand since consumers observe this deviation only after they come back to the firm and all of them value the product at $u_i \geq p$. Thus, the firm will never choose to surprise a returning visitor with a price reduction.

(The equality follows after integration by parts.) The first term is just firm i 's returning demand in equilibrium, so it reflects the marginal benefit from each returning consumer paying the premium τ . The second term is the loss due to the contraction of returning demand caused by the unexpected premium τ .

Using the expression for the equilibrium uniform price (5), it follows that $\Pi_\tau(p_0, 0) < 0$ if and only if

$$\frac{f(a)}{1 - F(a)} - \frac{nf(a)F(a)^{n-1}}{1 - F(a)^n} < \frac{1}{p_0}.$$

If f is weakly decreasing, (5) implies

$$f(a) \frac{1 - F(a)^n}{1 - F(a)} < \frac{1}{p_0}.$$

So it suffices to show that

$$\begin{aligned} & \frac{f(a)}{1 - F(a)} - \frac{nf(a)F(a)^{n-1}}{1 - F(a)^n} \leq f(a) \frac{1 - F(a)^n}{1 - F(a)} \\ \Leftrightarrow & \frac{f(a)F(a)^n}{1 - F(a)} \leq \frac{nf(a)F(a)^{n-1}}{1 - F(a)^n} \\ \Leftrightarrow & F(a) \frac{1 - F(a)^n}{1 - F(a)} = F(a)[1 + F(a) + \cdots + F(a)^{n-1}] \leq n, \end{aligned}$$

which must be true. This completes the proof.

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