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Predictive Ability of Value-at-Risk Methods: Evidence from the Karachi Stock Exchange-100 Index

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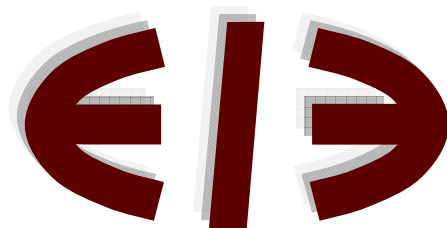
Economics and Econometrics Research Institute

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Predictive Ability of Value-at-Risk Methods: Evidence from the Karachi Stock Exchange-100 Index

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ABSTRACT

Value-at-risk (VaR) is a useful risk measure broadly used by financial institutions all over the world. VaR has been extensively used to measure systematic risk exposure in developed markets like of the US, Europe and Asia. This paper analyzes the accuracy of VaR measure for Pakistan's emerging stock market using daily data from the Karachi Stock Exchange-100 index January 1992 to June 2008.

We computed VaR by employing data on annual basis as well as for the whole 17 year period. Overall we found that VaR measures are more accurate when KSE index return volatility is estimated by GARCH (1,1) model especially at 95% confidence level. In this case the actual loss of KSE-100 index exceeds VaR in only two years 1998 and 2006. At 99% confidence level no method generally gives accurate VaR estimates. In this case 'equally weighted moving average', 'exponentially weighted moving average' and 'GARCH' based methods yield accurate VaR estimates in nearly half of the number of years. On average for the whole period 95% VaR is estimated to be about 2.5% of the value of KSE-100 index. That is on average in one out of 20 days KSE-100 index loses at least 2.5% of its value.

We also investigate the asset pricing implication of downside risk measured by VaR and expected returns for decile portfolios sorted according to VaR of each stock. We found that portfolios with higher VaR have higher average returns. Therefore VaR as a measure of downside risk is associated with higher returns.

1. INTRODUCTION

Accurate and meaningful measure of risk has always been of interest in business and finance. Starting since the mid-1990s a measure of risk known as Value-at-Risk (VaR) has emerged as the most popular risk measure for investors in financial securities, banks and investment companies and the regulating authorities that regulate this type of institutions. VaR is a single number that summarizes potential risk arising from a broad spectrum of causes e.g. investment risk, operational risk, liquidity risk and credit risk. VaR is defined as the maximum expected loss of a portfolio over a given holding period at a specified confidence level. Mathematically, let P_t be the price of a financial asset on day t. A k-day VaR on day t is defined by

$$P(P_{t-k} - P_t \leq \text{VaR}(t, k, \alpha)) = 1 - \alpha. \quad (1)$$

Figure 1 illustrates that 95% VaR is simply the 5% quantile of the return distribution. Hence unlike some risk measures which consider both upside and downside movement of asset return as risky such as beta, VaR is a true downside risk measure.

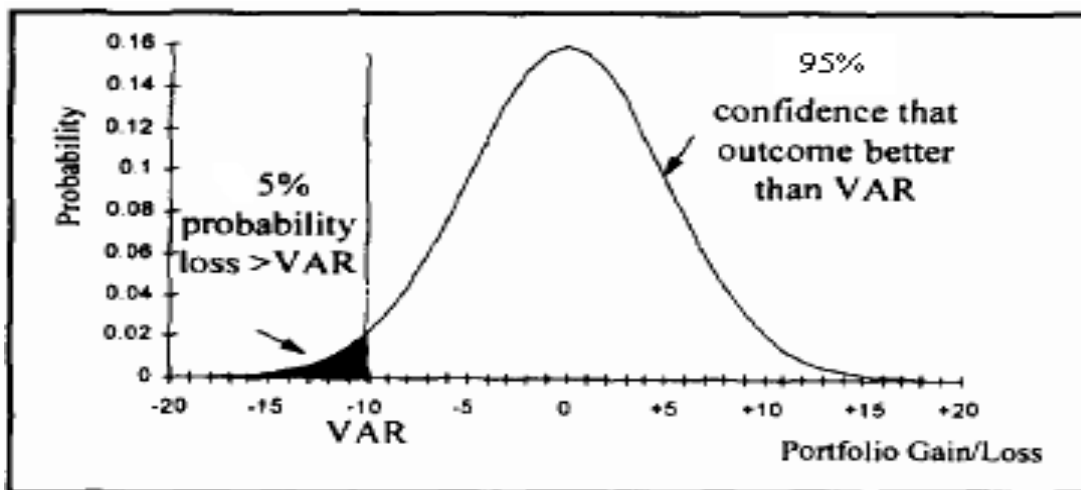


Figure 1: Illustration of 95% VaR

The concept of VaR has been adopted by regulators. For instance VaR has been a component of both the Basel I and Basel II recommendations on banking laws and regulations issued by the Basel Committee on Banking Supervision. In Pakistan also the

Security and Exchange Commission of Pakistan (SECP) has stressed the importance of VaR measures for brokerage houses to measure their risk exposure.

For meaningful measure of risk exposure, the holding period holding should be selected to approximate one's trading behavior. Liquid markets such as banks and portfolio managers typically find their portfolio change dramatically from one day to the next, and so consider a one-day holding period to be appropriate. On the other hand individual investors will typically maintain a portfolio intact for a longer period such as a month or longer. In this paper we employ one day holding period so we consider a relatively active portfolio managers' perspective.

A stock market index tracks the overall movement in the common stock prices. Many investment companies and fund managers investment in securities whose values are associated with the index value, these securities are known as index fund. This paper is concerned with the accuracy of different methods of computing VaR associated with index fund that is linked with the Karachi Stock Exchange 100 (KSE-100) index. A well functioning stock market is considered leading indicators of the economy. Accurate assessment of risk inherent in the stock index fund is therefore important at macroeconomic level as well.

2. REVIEW OF THE LITERATURE

There is now a huge and increasing literature on value-at-risk. Some selected papers are reviewed here. Darbha (2001) investigated the value-at-risk for fixed income portfolios, and compared alternative models including variance-covariance method, historical simulation method and extreme value method. He finds that extreme value method provides the most accurate VaR estimator in terms of correct failure ratio. Cheong (2006) compared the power-law value-at-risk (VaR) evaluation with quantile and non-linear time-varying volatility approaches. A simple Pareto distribution is proposed to account the heavy-tailed property in the empirical distribution of returns. The results evidenced that the predicted VaR under the Pareto distribution exhibited similar results with the symmetric heavy-tailed long-memory ARCH model. However, it is found that only the

Pareto distribution is able to provide a convenient framework for asymmetric properties in both the lower and upper tails. Inui, Kijima and Kitano (2007) shows that VaR is subject to a significant positive bias. They show that VaR has a considerable positive bias when used for a portfolio with fat-tail distribution. Lima and Neri (2007) compared four different Value-at-Risk (VaR) methodologies through Monte Carlo experiments. Their results indicate that the method based on quantile regression with ARCH effect dominates other methods that require distributional assumption. In particular, they show that the non-robust methodologies have higher probability of predicting VaR's with too many violations. McMillan and Speight (2007) investigated the value-at-risk in emerging equity markets. Comparative evidence for symmetric, asymmetric, and long-memory GARCH models is also provided. In the analysis of daily index data for eight emerging stock markets in the Asia –Pacific region, in addition to the US and the UK benchmarks, they found both asymmetric and long memory features to be important considerations in providing improved VaR estimates. Pownall, and Koedijk (1999) examined the downside risk in Asian equity markets. They observe that during periods of financial turmoil, deviations from the mean-variance framework become more severe, resulting in periods with additional downside risk to investors. Current risk management techniques failing to take this additional downside risk into account will underestimate the true value-at-risk. Lan, Hu and Jhonson (2007) employed different combinations of resampling techniques, which include the bootstrap and jackknife. Unlike previous studies that only take into consideration the uncertainty of VaR arising from the estimation of conditional volatility, they also account for the uncertainty of VaR resulted from the estimation of the conditional quantile of the filtered return series. The jackknife seems to be very useful in improving forecast precision.

Bali and Cakici (2004) is among very few papers who consider the VaR from an asset pricing perspective. They investigated the relationship between portfolios ranked according to value-at-risk and expected stock returns. They conclude that value at risk, size and liquidity can explain the cross-sectional variation in expected returns, but market beta and total volatility have almost no power to capture the cross-section of expected

returns at the stock level. Furthermore, the strong positive relationship between average returns and VaR is robust for different investment horizons and loss-probability levels.

3. VALUE-AT-RISK METHODOLOGIES

3.1 PARAMETRIC METHODS OF VaR

The parametric methods assume a standard probability model for asset returns to simplify the calculation of VaR. In this case only the mean and the variance of portfolio returns over the holding period are required. The serial independence assumption allows the calculation of VaR over a longer horizon by multiplying the daily variance by the square root of time. The VaR estimated for time t given observations upto time $t-1$ of a single asset can thus be expressed as:

$$VaR_{t/t-1} = V_t Z_\alpha \sigma_{t/t-1} \sqrt{\Delta t} \quad (2)$$

Where V_t is the initial value of the asset, Z_α is the variate that corresponds to the confidence level α (for example, 1.645 at the 95% confidence level) $\sigma_{t/t-1}$ is the volatility of the asset returns, and Δt is the holding period.

In case of daily returns the holding period Δt is 1.i.e,

$$VaR_{t/t-1} = V_t Z_\alpha \sigma_{t/t-1} \quad (3)$$

From above equation, we know that VaR is a simple function of the return volatility. A variety of techniques can be used to estimate the volatility.

EQUALLY WEIGHTED MOVING AVERAGE METHOD (EQWMA)

The computation of Value-at-Risk by equally weighted moving average explains the probable loss for the portfolio under different circumstances. Here we describe the value at risk by EQWMA as the maximum possible loss that can enter within a certain time with a certain significance level. The basic formula for computing the value-at-risk is given in eq. (3)

Where Z_α the value of confidence level and the EQWMA figure outs the portfolio variance by employing a predetermined amount of historical data and placing equal weight on each return observation as:

$$\sigma_{t/t-1} = \sqrt{\frac{1}{k-1} \sum_{s=t-k}^{t-1} (X_s - \mu)^2} \quad (4)$$

Where σ_t is the standard deviation of variance–covariance at day t , k denotes the number of observations, X_s is the portfolio return at day s , and μ is the mean of portfolio returns.

EXPONENTIALLY WEIGHTED MOVING AVERAGE METHOD (EWMA)

The exponentially weighted moving average method places different weights to observations within the sample window. The more recent returns receive more weights than previous ones, and the weight grows exponentially. The root of variance–covariance of a portfolio is expressed as

$$\sigma_{t/t-1} = \sqrt{(1-\lambda) \sum_{s=t-k}^{t-1} \lambda^{t-s-1} (x_s - \mu)^2} \quad (5)$$

where $\lambda \in (0, 1)$ is the decay factor. In this study, we use $\lambda = 0.94$ in accordance with the RiskMetrics.

EQUALLY WEIGHTED MOVING AVERAGE WITH T-DISTRIBUTION METHOD (EQWMA_T)

The return distributions of the majority financial assets are fat-tailed, thus the supposition of a normal distribution may underestimate the VaR. To consider this inadequacy, we also assume that the returns have the Student- t distribution based on the equally weighted moving average method (EQWMA_T). Jorion (1997) suggests that the Student- t distribution with six degrees of freedom provides a better fit for most financial asset returns. We follow this suggestion in calculating the VaR. The formula for computing the value-at-risk from EQWMA_T is given by

$$VaR_{t/t-1} = V_t T_\alpha \sigma_{t/t-1} \quad (6)$$

$\sigma_{t/t-1}$ in the above equation is same as specified for EQWMA and T_α is the quantile of T -distribution.

AUTOREGRESSIVE CONDITIONAL HETROSKEDASTICITY MODEL (ARCH)

Financial market volatility is a core issue to the theory and practice of value-at-risk, asset pricing, asset allocation and risk management. It is broadly accepted among both practitioners and academics that volatility varies over time. Stock volatility is simply defined as conditional variance, or standard deviation of stock returns that is not directly observable. Since the best decision of investors relies on variance of returns that can change over time, it is important to model and forecast conditional variance. Engle (1982) developed the Autoregressive Conditional Heteroskedasticity (ARCH) model and Bollerslev (1986) generalized it to GARCH (Generalized ARCH) model.

Assume that returns are generated by the ARMA (1, 1) model the ARCH (1) model can be specified as follows:

$$r_t = \beta_0 + \beta_1 r_{t-1} + u_t + \delta_1 u_{t-1} \quad u_t = v_t \sigma_t \quad (7)$$

The $\sigma_{t/t-1}$ is given by

$$\sigma_{t/t-1}^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (8)$$

To completely specify a GARCH-type model an assumption about the error distribution u_t should be made. It is more appropriate to assume that the errors have a heavy tailed distribution rather than Gaussian distribution. Here we have employed Generalized Error Distribution in GARCH estimation given by:

$$f(z_t / \nu) = \frac{\nu}{\lambda_\nu \cdot 2^{(\nu+1)/\nu} \Gamma(1/\nu)} \exp\left(-\frac{1}{2} \left| \frac{z_t}{\lambda_\nu} \right|^\nu\right); \quad \lambda_\nu = \left[\frac{2^{(-2/\nu)} \Gamma(1/\nu)}{\Gamma(3/\nu)} \right]^{1/2} \\ -\infty < z_t < \infty \quad (9)$$

Here the error term is denoted by z_t and ν is a positive shape parameter governing the thickness of the tail behavior of the distribution. For $\nu = 2$ GED reduces to the standard normal distribution, for $\nu < 2$ the distribution has tails thicker than the normal

distribution and for $\nu > 2$ the tails are thinner than the normal distribution. For $\nu \rightarrow \infty$ we get the uniform distribution.

VaR is then computed as follows

$$VaR_{t/t-1} = V_t G_\alpha \sigma_{t/t-1} \quad (10)$$

Here G_α is the 95% or 99% quantile of the GED distribution

3.2 NONPARAMETRIC METHODS OF CALCULATING VaR

Non-parametric method take a dataset of returns (historical or realistically simulated) and find the loss that is exceeded only a percent of the time corresponding to your confidence interval in the dataset. In other words, look at the actual histogram rather than the normal distribution that approximates it. And also if you do have enough tail data, the non-parametric method can give you a more accurate measure of skew and kurtosis risk and other higher moments. The normal distribution cannot fully capture the asymmetric returns. We mitigate this limitation by two non-parametric methods, the methods which come under the heading of parametric method of calculating VaR are given by Historical Simulation (HS) and Bootstrap Method (BS)

HISTORICAL SIMULATION METHOD

Historical simulation approach of value-at-risk is similar to the equally weighted moving average, as it is based on a specific quantity of past historical observations. Rather than using these observations to calculate the portfolio's standard deviation, however, historical simulation approaches use the actual percentiles of the observation period as value-at-risk measures. This method makes no assumptions on the distribution of the underlying assets, and does not need to calculate any parameters. The process of the historical method is straightforward as explained in the following two steps.

- 1- Obtain a satisfactory amount of historical returns of assets.
- 2- Sort the portfolio returns in ascending order to achieve the empirical distribution. VaR is the percentile that corresponds to the specified confidence level.

For instance, for an observation period of 500 days, the 99th percentile historical simulation VaR measure is the sixth largest losses observed in the sample of 500

outcomes. The historical simulation approach accurately reflects empirical skewness, kurtosis and any non-linear time-varying variance-covariance matrix. Jorion (1997) indicates that the historical method allows for both nonlinear and non-normal distributions, and also accounts for fat tails and avoids model risk. However, it requires a sufficient amount of history data.

The BOOTSTRAP METHOD

Bootstrapping is a re-sampling method developed by Efron (1979) that has a wide variety of applications. It can be used to simulate the sampling distribution of estimators and to derive standard errors of a complicated estimator. We assume that the log-returns of the Karachi Stock Exchange (KSE) index are independent and identically distributed. We resample $B=100$ samples each of size 500 daily returns from the empirical distribution. VaR is computed from each sample and the average VaR is employed as Bootstrap VaR estimate.

According to Jorion (1997), the bootstrap method allows for fat tails, jumps, and any departure from the normal distribution, and is able to take correlations across series into account. The main drawback of this method is that it may not approximate the actual distribution well when the sample size is small. Furthermore, any pattern of time variation is violated by random re-sampling.

4. TESTING ACCURACY OF VaR METHODS

To assess the accuracy of various VaR models we employ Kupiec (1995) Likelihood Ratio procedure of backtesting. If computed VaR underestimates the actual portfolio loss, it is denoted as an “exception.” and if say 95% VaR is accurate, the sample proportion of exceptions should not be significantly different from 5%. The following

discussion is based on Angelidis et al. (2004). Let $N = \sum_{t=1}^T I_t$ be the number of days over a T period that the portfolio loss was larger than the VaR estimate, where

$$I_{t+1} = \begin{cases} 1, & \text{if } y_{t+1} < VaR_{t+1|t} \\ 0, & \text{if } y_{t+1} \geq VaR_{t+1|t} \end{cases}$$

Hence, N is the observed number of exceptions in the sample. Kupiec (1995) suggests the proportion of failure test (PF test), in which a log-likelihood ratio test (LR test) is used to assess the accuracy of a VaR model. If the model is correct, the number of failures follows a binomial distribution $N \sim B(T, p)$

$$P(N / T, p) = \binom{T}{N} p^N (1-p)^{T-N} \quad (11)$$

Where N is the number of failures, T is the total number of trials, and p is the probability of a failure on any one of the independent trials. Thus, the LR test statistic is given by

$$LR = 2 \ln \left[\left(1 - \frac{N}{T}\right)^{T-N} \left(\frac{N}{T}\right)^N \right] - 2 \ln \left[(1-p)^{T-N} p^N \right] \quad (12)$$

Where N/T is observed probability of failures, and p is expected probability of failures. Under the null hypothesis that the expected exception frequency, $H_0: N/T = p$.

Asymptotically, the LR test statistic follows a Chi-Square distribution with 1 degree of freedom. For example, for the 95% VaR, if the value of Kupiec LR test is greater than 3.8414 then the null hypothesis is rejected implying that the VaR does not provide accurate loss proportion.

5. THE DATA

The sample data consists of KSE100 index daily log-returns from January 1992 to June 2008, a total 4298 daily observations. The observed returns of KSE-100 index are presented in Figure 2. The behavior of daily log returns clearly exhibit the volatility clustering phenomenon, large changes in index values tend to cluster. From the figure we

can see that the log returns are highly volatile during the time period 1998-1999 and after this year the log returns are slightly less volatile.

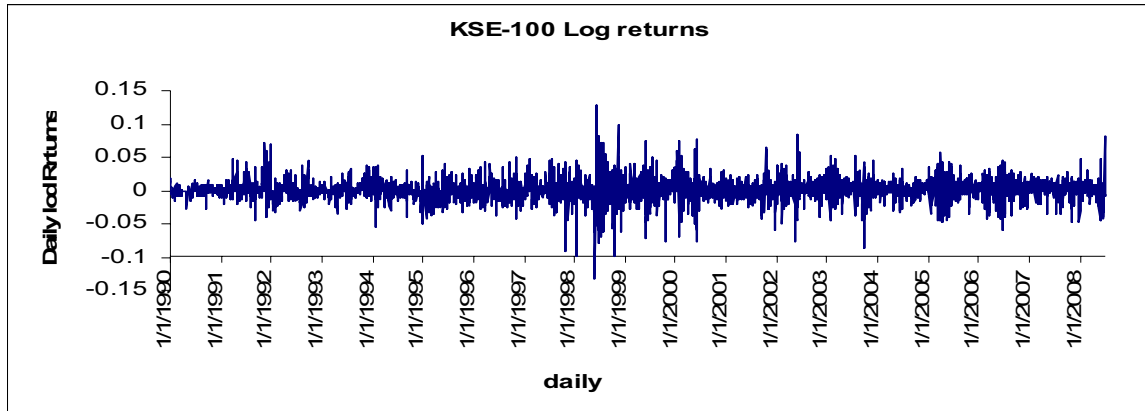


Figure 2. Time series behavior KSE-100 index returns 1991-2008

6. EMPIRICAL RESULTS

The below tables shows the summary of one-day based VaR of entire sample period of 1992 to 2008. Table 1 indicates that the estimated average 95% VaR as a percent of initial KSE-100 index portfolios is approximately 2.5% for the six methods.

Table 1: Average VaR as percent of initial portfolio (KSE-100 index) value for different methods for period 1992-2008

C.L.	95%		99%	
	Avg VaR (%)	Stdev(VaR)	Avg VaR (%)	Stdev(VaR)
1-EQWMA	2.548	0.601	3.603	0.851
2-E _{XP} WMA	2.311	1.052	3.268	1.487
3-EQWMA _T	2.458	0.580	3.975	0.938
4. GARCH	2.400	1.207	4.082	2.055
4-BS	2.547	0.721	4.361	1.518
5-HS	2.524	0.705	4.328	1.385

Table 2 presents the percentage of cases in which actual loss of KSE-100 index value exceeds the 95% VaR estimated by different methods over the sample periods. These

VaR exceptions are computed annually and for the whole period 1992-2008. The table also indicates (by * sign) the significant Kupiec LR test statistics at 5% level. Note that in some cases estimated proportion of exceptions is close to the nominal exception of 0.05, yet the differences in the number of exceptions N influence the value of Kupiec test statistic.

Table 2: Percentage of Exceptions of VaR through different method

C.L.	95%					
	METHODS	EQWMA	ExpWMA	EQWMA _T	GARCH	BS
1992-2008	0.053	0.061*	0.056*	0.064*	0.054*	0.055*
1992	0.047	0.024*	0.051*	0.059	0.094*	0.098
1993	0.011*	0.038	0.011*	0.038	0.027*	0.027*
1994	0.058	0.085*	0.062*	0.069	0.077*	0.081
1995	0.085*	0.092*	0.092*	0.077	0.069*	0.077*
1996	0.065	0.061	0.073*	0.042	0.042*	0.042
1997	0.054	0.034	0.057*	0.065	0.057*	0.057
1998	0.115*	0.084*	0.123*	0.096*	0.115*	0.119*
1999	0.015*	0.027	0.019*	0.054	0.015*	0.019*
2000	0.023*	0.069	0.023*	0.05	0.023*	0.023
2001	0.031	0.073	0.034*	0.05	0.038*	0.034
2002	0.031	0.042	0.031*	0.054	0.038*	0.038
2003	0.057	0.031	0.061*	0.061	0.061*	0.061
2004	0.023*	0.061	0.023*	0.038	0.019*	0.019
2005	0.092*	0.050	0.104*	0.077	0.092*	0.096*
2006	0.092*	0.104*	0.096*	0.104*	0.077*	0.081*
2007	0.038	0.073	0.038*	0.077	0.019*	0.019
2008	0.061	0.107	0.061*	0.092	0.053*	0.046

The proportion of exceptions is the proportion of days when actual loss (i.e. difference between yesterdays and today's KSE-100 index value) exceeds VaR. For overall time period the EQWMA method provided the proportion of exception which is 0.053 that is closest to the nominal proportion of 0.05. Next comes the Bootstrap method i.e. 0.054. However it is the GARCH model which dominates since in this case in only two of the

17 years the VaR is violated. The Kupiec test is rejected by all methods in the year 1998 including GARCH. The reason might be the higher level of volatility owing to nuclear tests by Pakistan and India in 1998.

7. VaR and Expected Returns

We now investigate whether stock portfolios with higher downside risk measured by VaR earn higher expected returns. To the best of our knowledge the relationship between expected return and VaR as downside risk measure has not been investigated in emerging markets. However conducting asset pricing tests with daily data is problematic due to non-normality of daily returns and infrequent trading of stocks in an emerging market. We therefore employ monthly continuously compounded stock returns on 232 stocks from the Karachi Stock Exchange from October 1992 to June 2008. The stock prices were obtained from DataStream data base. We constructed decile portfolios by sorting stocks into 99%, 95% and 90% VaR and obtained average returns and average VaR for each decile portfolios. Following Bali and Nusret (2004) we used historical simulation method to estimate VaR. We used from 24 to 60 monthly returns (as available) to estimate the mean and the cutoff return for each confidence level. The 99%, 95% and 90% confidence level VaRs were measured by the first lowest, third lowest, and sixth lowest observation of 60 monthly returns in December of each year starting from 1995.

We tested whether the 99%, 95% and 90% VaR portfolios can produce larger and statistically significant cross-sectional variation in monthly expected returns in the emerging market under study. Starting from 1995, in December of each year, we sorted all sample of 232 KSE stocks by 99%, 95% and 90% VaR to determine the decile breakpoints for each VaR measure. Based on the breakpoints, we allocated stocks to 99%, 95% and 90% VaR deciles. Decile 1 consist of the 10 percent of stocks with the lowest VaR, and decile 10 represents the stocks with the highest VaR. We also computed the equally weighted average returns for the stocks in each decile. The portfolios are rebalanced each December in the following years. Table 3 presents the average returns of the VaR portfolios for all deciles as well as the estimated regression coefficients $\hat{\alpha}$ $\hat{\beta}$,

R^2 and the corresponding t-statistics of following cross-sectional regression of average returns of the 10 decile portfolios on the average VaR of the portfolios:

$$R_i = \alpha + \beta VaR_i + \varepsilon_i \quad i= 1, 2, ..10 \quad (13)$$

Table 3 shows that when portfolios are constructed according to 99%, 95%, and 90% VaR, average stock returns are positively correlated with VaR. In other words, stocks with the highest maximum likely loss measured by VaR have highest average returns. From the lowest 1 percent VaR decile to the highest 1 percent VaR decile, average returns on VaR portfolios increase from 0.958 percent a month to 7.829 percent a month which amounts to 82.45% annual return differential. This increase is not monotonic, for example using 99% VaR going from 8th to 9th decile portfolio result in lower average return. The overall evidence of positive risk-return relationship is nevertheless very strong. Our results are in sharp contrast to the US result of Bali and Nusret (2004) who estimated an annual return differential of 11.52% between highest and lowest VaR deciles. This is obviously an important result for investment allocation perspective. A similar strong positive relationship is also observed between average returns and the 95 percent and 90 percent VaRs.

Table 3: Average Monthly Return of Portfolios sorted by 99%, 95%, and 90% VaR, August 1992-June 2008

Decile	99% VaR	Return %	95% VaR	Return %	90% VaR	Return %
Low VAR	2.85	0.96	0.32	1.03	0.74	1.66
2	17.72	2.39	10.85	3.42	5.88	0.82
3	21.67	4.07	13.95	1.92	9.13	4.08
4	25.16	3.73	16.03	2.88	11.08	3.65
5	28.57	3.38	17.87	4.74	12.59	4.03
6	31.89	5.23	20.36	5.57	14.10	4.32
7	35.44	5.14	22.48	5.43	16.09	4.46
8	40.45	5.89	25.34	5.47	18.20	6.62
9	47.64	4.68	28.62	6.19	20.64	6.39
High VAR	60.68	7.83	34.76	6.48	24.51	7.09
Coefficients	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
	0.93	0.11	0.94	0.18	0.75	0.27
t-Statistics	2.45*	8.04*	1.95**	9.79*	1.07	6.76*
R-Square	0.86		0.83		0.86	

Note: T-statistics are based on heteroskedasticity consistent standard error.

* and ** represent significance at 5% and 10% level of significance respectively.

The results show that the greater a portfolio's potential fall in value, the higher is the expected return. The portfolios of higher-VaR stocks appear to produce higher returns than the portfolios of lower-VaR portfolios. To measure the degree of positive correlation between average stock returns and VaR, we regressed the average returns from the decile portfolios on the average level of 99%, 95%, and 90% VaR. The results indicate that the coefficients on VaR are highly significant, with R^2 values ranging from 83 percent to 86 percent.

8. CONCLUSION

Measuring the exposure to market risk of portfolio investments has always been an important issue for investors, financial institutions and regulators. Various risk exposure methods have been employed. Value-at-risk has recently become very popular measure of risk exposure since it describes systematic risk in term of a monetary number which are easier to understand by practitioners. This can be comparable to other risk measure such as beta risk which is standardized covariance of asset return and the market returns which is not easier to understand.

There are few studies that have compared the accuracy of VaR models in measuring accuracy of risk exposure to securities in emerging markets. Since the economic, financial, political and regulatory environment of emerging market is different form the developed markets, such a contribution is important.

In this study we have employed several parametric and non-parametric methods to estimate value-at-risk of funds represented by Karachi Stock Exchange-100 index portfolio. The methods include four parametric methods 'equally weighted moving average', 'exponentially weighted moving average using normal and t-distribution of return distribution and the GARCH model. We also employed two non-parametric

methods namely 'Historical Simulation' and the 'Bootstrap' method where the quantiles of simulated return are employed in VaR calculation.

In each case VaR is computed using 95% and 99% confidence level. The accuracy of each method is tested using Kupiec LR test. We analyzed the accuracy of one day VaR methods using 17 years of daily data from 1992 through 2008 on KSE-100 index returns. The results indicate that in general at 95% confidence level 'exponentially weighted moving average method appears to yield accurate VaR estimates in 12 out of 17 years period on annual basis. 'Historical Simulation' method appears to be a close competitor in which Kupiec test is not rejected in 11 out of 17 years. Overall we found that VaR measures are more accurate when KSE index return volatility is estimated by GARCH (1,1) model especially at 95% confidence level. In this case the actual loss exceeds VaR in only two years 1998 and 2006 i.e. on an annual basis in 15 of the 17 years Kupiec test is not rejected. On average for the whole period daily VaR is estimated to be about 2.5% of the value of KSE-100 index.

Finally using monthly data of 232 stocks from the KSE we found that higher VaR stock is associated with higher average return. On average the higher VaR portfolio generates 84% higher return than the lowest VaR portfolio. This result has obvious implication for investment allocation and financial analysis.

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