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# **NEW KEYNESIAN PHILLIPS CURVE ESTIMATION: THE CASE OF HUNGARY /1981-2006/**

## **Abstract**

This paper investigates for the presence of a New Keynesian Phillips (NKPC) curve in Hungary in the period 1981:3-2006:2, following the methodology proposed by Gali and Gertler (1999). They claim that a potential source of inflation may be the sluggish adjustment of real marginal costs to movements in output. The empirical model we test features forward-looking firms who pre-set prices for a couple of periods ahead, using Calvo (1983) pricing rule. In addition, measures of real marginal cost are used instead of the old-fashioned output gap. The reason is that marginal costs are a better proxy for the impact of the productivity gains on inflation, which the ad hoc measure output gap misses. We also estimate a hybrid version of NKPC, where some of the firms are backward-looking, and others are forward-looking in their price-setting behavior. Real marginal costs and forward-looking behavior are statistically significant and quantitatively important in the NKPC. However, there are some econometric issues to be considered, such as the weak identification of the parameters of the structural NKPC as well as those of the hybrid NKPC.

*Keywords:* New Keynesian Phillips curve, Hungary, instrumental non-linear GMM Estimation, weak identification

*JEL Classification:* C22, C26, E24

## 1. Introduction

This paper investigates for the presence of a New Keynesian Phillips (NKPC) curve in Hungary in the period 1981:3-2006:2. The study follows the methodology proposed by Gali and Gertler (1999), who claim that a potential source of inflation may be the sluggish adjustment of real marginal costs to movements in output. The empirical model tested features forward-looking firms who pre-set prices for a couple of periods ahead, using Calvo (1983) pricing rule. In addition, measures of real marginal cost are used instead of the old-fashioned output gap. The reason is that marginal costs are a better proxy for the impact of the productivity gains on inflation, which the ad hoc measure output gap misses. A hybrid version of NKPC, where some of the firms are backward-looking, and others are forward-looking in their price-setting behavior, is also estimated.

The paper is organized as follows: section 2 describes Gali and Gertler's approach and, thus provides a brief review of the theory that gave rise to the new Phillips curve, and discusses some existing empirical results. Section 3 contains the estimates of the new Phillips curve. In section 4, the model is extended to allow for backward-looking firms and results of a so-called "hybrid Phillips curve" are presented. Section 5 concludes.

## 2. The new Phillips curve: Background theory and evidence

The setup of the model features monopolistically competitive firms who face some constraints on price adjustments. The price adjustment rule is time-dependent – every period a fraction  $1/X$  of firms set their prices for  $X$  periods ahead in the spirit of Taylor (1980). In order to keep track of the histories of all firms we use Calvo pricing (1983) rule, which simplifies the aggregation problem: in any given period each firm has a fixed probability  $1-\theta$  that it may adjust its price during that period. Therefore, the average time over which a price is fixed is given by  $(1-\theta)\sum_{k=0}^{\infty} k\theta^{k-1} = 1/(1-\theta)$ .

Another common assumption is that the monopolistic firm faces a constant price elasticity of demand curve. Then, Gali and Gertler (1999) show that the aggregate price

level  $p_t$  evolves as a convex combination of the lagged price level  $p_{t-1}$  and the optimal reset price  $p_t^*$  (the price selected by firms that are able to change the price at period  $t$ ).

So the pricing rule takes the following form:

$$p_t = \theta p_{t-1} + (1 - \theta) p_t^* \quad (1)$$

Let  $mc_t^n$  be the firm's marginal costs (as a percentage deviation from the steady-state) and  $\beta$  denotes the discount factor. Each firm chooses a price at  $t$  to maximize expected discounted profits subject to the Calvo pricing rule, so the optimal reset price is:

$$p_t^* = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ mc_{t+k}^n \} \quad (2)$$

Now let  $\pi_t = p_t - p_{t-1}$  denote the inflation rate. Combining (1) and (2), Galí and Gertler (1999) obtain the following equation for the inflation dynamics

$$\pi_t = \lambda mc_t + \beta E_t \{ \pi_{t+1} \}, \quad (3)$$

where  $\lambda = \frac{(1 - \theta)(1 - \beta\theta)}{\theta}$  depends on the frequency of price  $\theta$  adjustment and the discount factor  $\beta$ . Iterating forward for inflation they obtain

$$\pi_t = \lambda \sum_{k=0}^{\infty} \beta^k E_t \{ mc_{t+k}^n \} < \infty \quad (4)$$

So the theory says inflation is a discounted stream of expected future marginal costs. That sum is finite due to the discounting effect and the assumption that marginal costs are bounded in each period.

### **Traditional Phillips curve**

Traditional Phillips curve emphasizes the use of a proxy for real activity, namely the “output gap.” This is a measure which shows how current GDP differs from the potential one. It is obtained by taking logs from the series, seasonally adjusting the quarterly series, differencing to eliminate the unit root and applying the Hodrick-Prescott filter so that we express it as a percentage change from the steady state. Thus,  $mc_t = kx_t$ , where  $k$  is the elasticity of the marginal cost.

Plugging the expression above into the inflation equation, we obtain

$$\pi_t = \lambda k x_t + \beta E_t \{ \pi_{t+1} \} \quad (5)$$

Substituting forward, the resulting expression becomes

$$\pi_t = \lambda k \sum_{k=0}^{\infty} \beta^k E_t \{ mc_{t+k} \} < \infty \quad (6)$$

In other words, current inflation rate is the weighted discounted sum of future marginal costs.

## New Keynesian Phillips Curve

It is widely known fact that conventional measures of the output gap contain a substantial amount of measurement errors. That is primarily due to the fact that the theoretical measure of “natural level” of output is not an observable. The gap is estimated by fitting a smooth deterministic trend and subtracting it from the series. This trend-fitting itself involves measurement error. Depending on whether supply or demand shocks are predominant in the economy, that could lead to counter-intuitive signs of the coefficients.

Gali and Gertler (1999) concentrate on obtaining a measure for real marginal costs, estimated in a way that it is consistent with theory. Their theory is used as a guide for the estimation in this paper: Output is assumed to be produced by A Cobb-Douglas production function,  $Y_t = A_t K_t^{\alpha_k} N_t^{\alpha_n}$ , where  $A_t$  denotes total factor productivity,  $K_t$  capital, and  $N_t$  labor. Real marginal cost (MC) is the ratio of the real wage to the marginal product of labor (MPL). Thus,  $MC_t = (W_t / P_t)(\partial Y_t / \partial N_t) = S_t / \alpha_n$ , where  $S_t = W_t N_t / P_t Y_t$  is the labor income share. Using lowercase letters to denote percent deviation from the steady-state, the formula becomes  $mc_t = s_t$ . That measure is obtained by first taking natural logs from the series and then applying the Hodrick-Prescott filter to it. The series is stationary: Dickey-Fuller test rejects the presence of a unit root at 1% level of significance.

After plugging the expression for real MC into the inflation equation, we obtain

$$\pi_t = \lambda s_t + \beta E_t \{ \pi_{t+1} \} \quad (7)$$

Since this is a rational expectations (RE) model, the forecast of  $\pi_{t+1}$  is uncorrelated with any of the variables in the information set, i.e variables in time  $t$  or earlier. This leads to the following moment condition

$$E_t\{(\pi_t - \lambda s_t - \beta \pi_{t+1})z_t\} = 0 \quad , \quad (8)$$

where  $z_t$  is a vector composed of the variables taken from the information set, which are orthogonal to the inflation surprise. The moment condition above is used to estimate the model using the Generalized Method of Moments (GMM).

An important reason why GMM estimation is used is that non-linear least squares (NLLS) will give biased and inconsistent estimates since  $corr(\pi_t, \pi_{t+1}) \neq 0$ , and thus  $corr(\varepsilon_t, \pi_{t+1}) \neq 0$ , which violates one of the underlying assumption for using NLLS. Note that using NLLS-IV estimation with homoscedasticity assumption and no autocorellation yields exactly the GMM orthogonality condition.

The data used is quarterly for Hungary over the period 1981:3-2006:2. Estimation results are presented in the next section. For  $s_t$ , natural logarithm of the labor income share is used. Inflation is measured as a percentage change in the Consumer Price Index (CPI), seasonally-adjusted and differenced in order to eliminate the unit root in the series. The instrument set includes four lags of inflation, the labor income share, the output gap, the long-short interest rate spread, wage inflation and the growth in money supply (M1 aggregate).

### 3. The new Phillips curve: Estimation

We first estimate the reduced form equation, which involves only  $\lambda$  and  $\beta$ , but not the structural parameter  $\theta$ , which was the measure of price rigidity. Results are provided in Table 1 below.

**Table 1: Reduced-form estimates**

	$\beta$	$\lambda$		J-statistic
			J-statistic	9.793
LULCHP			no.obs	100
coeff.estimate	0.1247	-0.0002	df	7
st.error	0.1494	0.0007	p-value	<b>0.2006</b>
			J-statistic	8.9113
LSHLABORHP			no.obs	100
coeff.estimate	0.1135	0.0250	df	7
st.error	0.132	0.0654	p-value	<b>0.2591</b>
			J-statistic	9.1982
DLGDPSAHP			no.obs	100
coeff.estimate	0.1321	-0.0642	df	7
st.error	0.1453	0.1227	p-value	<b>0.2387</b>

Instruments used: four lags of inflation, labor income share, long-short interest rate spread, output gap, wage inflation and growth in M1. Standard err. in brackets. The automatic choice of the Newey-West estimate of the covariance matrix was used.

Neither the coefficient on the real marginal costs, nor the estimate of the discount factor  $\beta$  is statistically significant. The last result, however, is in line with Galí and Gertler's findings for US: using output gap should not generate a NKPC when quarterly data was used.

In order to recover the structural estimate of  $\theta$  non-linear instrumental GMM was also estimated. Fuhrer et al. (1995) show that in small samples GMM is sensitive to the nature of normalization of the orthogonality conditions. In this paper the ones used by Galí and Gertler (1999) are used:

$$E_t\{(\theta\pi_t - (1 - \theta)(1 - \beta\theta)s_t - \theta\beta\pi_{t+1})z_t\} = 0 \quad (9)$$



$$E_t\{(\pi_t - \theta^{-1}(1-\theta)(1-\beta\theta)s_t - \beta\pi_{t+1})z_t\} = 0 \quad (10)$$

Their claim is that the first specification minimizes non-linearities, while in the second the coefficient of inflation in the current period is restricted to be one. We do each specification for (log) labor share, (log) unit labor costs and output gap.

The results are reported in Table 2. The first two columns give the estimates of the structural parameters  $\theta$  and  $\beta$ , and the third provides the estimate for  $\lambda$ . Standard errors for  $\lambda$  were obtained using the delta method. J-statistic for overidentifying restrictions is also provided. At 5% level of significance, the model is always correctly specified.

**Table 2: Estimates of the New Keynesian Phillips Curve**

	$\theta$	$\beta$	$\lambda$		
				J-statistic	9.7527
LULCHP				no.obs	100
[1] coeff.estimate	<b>1.0002</b>	0.1241	-0.0002	df	7
st.error	0.0008	0.1493	0.0007	p-value	<b>0.203</b>
				J-statistic	9.7931
[2] coeff.estimate	<b>1.0000</b>	0.1247	<b>0</b>	no.obs	100
st.error	0.0001	0.1494	0.0000	df	7
				p-value	<b>0.2006</b>
				J-statistic	8.3573
LSHLABORHP				no.obs	100
[1]coeff.estimate	<b>0.2963</b>	<b>2.6178</b>	<b>0.1579</b>	df	7
st.errorr	0.0908	0.8581	0.0527	p-value	<b>0.3021</b>
				J-statistic	8.9167
[2] coeff.estimate	<b>0.9969</b>	0.1137	0.027	no.obs	100
st.error	0.0093	0.1321	0.0001	df	7
				p-value	<b>0.2587</b>
				J-statistic	9.7798
DLGDPSAHP				no.obs	100
[1] coeff.estimate	<b>0.4023</b>	<b>1.3682</b>	<b>0.2687</b>	df	7
st.error	0.0604	0.2757	0.0455	p-value	<b>0.2014</b>
				J-statistic	9.1927
[2]coeff.estimate	<b>1.0096</b>	0.1368	<b>-0.0083</b>	no.obs	100
st.errorr	0.0273	0.1453	0.0005	df	7
				p-value	<b>0.2391</b>

The two specifications yield some heterogeneity in the results: the estimate of  $\theta$  is either unity (all the firms adjust), 0.3 in the case of log-labor income share, and 0.4 in the regression with the output gap. The estimates for  $\lambda$  and  $\beta$  are in the majority of the cases not statistically different from zero.

Generally, estimates are very sensitive to the GMM normalization procedure, and sometimes to the initial values chosen. The problem was that the program gives highly negative and statistically significant  $\beta$ , which is in conflict with the economic logic. The reason is that the reduced form model is identified, while the structural one is not. The latter has multiple solutions, and that is formally shown in the appendix. Therefore doing Continuous Updating (CU) will not solve the problem. Using Maximum Likelihood Estimation (MLE) is also of no help since the identification issue is not solved. Mavroeidis (2007) points out that Wald and LR test are not robust to failure of the identification assumption. That is a serious issue to be considered for all Neo-Keynesian economists who have NKPC equation in their models. In very recent working paper, Boug, Capellen and Swensen (2007) show that the estimate surface is flat, which is a sign of weak identification. Hendry (2004) also advises that NKPC specification be used with caution.

In the other camp, Martins and Gabriel (2005) try to save the model by using Generalized Empirical likelihood. Stock and Wright (2000) develop confidence set estimation to fix weak identification. They admit, however, encountering problems with fixing Wald statistics. It is worth noting that Gali and Gertler (1999) do not discuss this econometric problem. They only mention several other reasons that may cause the estimate of  $\theta$ , to have an upward bias. The first one is statistical: our measures of the real marginal cost are just proxies, and thus contain measurement error. Thus, the parameter  $\lambda$  is biased towards zero and appear insignificant, while in reality MC are an important factor for determining inflation. The second reason lies in the theory, which serves as a basis for the model. It assumes a constant mark-up of prices over MC. If mark-up is allowed to vary over the business cycle, however, then price setting becomes less sensitive to MC,

and this explains why  $\lambda$  is not statistically significant as well. In a recent paper, Gali, Gertler et al. (2005) still claim their results are robust, again failing to mention the identification issue.

In the next section, an alternative, called “hybrid” NKPC, is considered. It is a more sophisticated model of inflation dynamics. Unfortunately, much of the criticism in the paragraphs above is relevant for the hybrid version, as the problem of weak identification is even bigger in that specification.

#### 4. Hybrid Phillips curve

Inflation in data features a significant amount of inertia. Thus, in this section we extend the basic Calvo model, and allow for inertia in inflation. Now the environment includes two groups of firms – not only forward-looking, but also backward-looking ones. The latter use a rule of thumb when setting prices or is behaving in an adaptive way. In this case we can see what share of firms is not optimizing, and therefore not acting rationally.

We the share of the backward-looking firms is denoted by  $\omega$ . The aggregate price level now evolves according to the following formula

$$p_t = \theta p_{t-1} + (1 - \theta) \bar{p}_t^* \quad (11)$$

where  $\bar{p}_t^*$  is an index of the prices that were reset in period  $t$ . Let  $p_t^f$  denote the price set by a forward-looking firm at  $t$  and  $p_t^b$  the price set by a backward-looking firm. Then the index of the newly-set prices may be expressed as

$$\bar{p}_t^* = (1 - \omega) p_t^f + \omega p_t^b \quad (12)$$

Accordingly,  $p_t^f$  may be expressed as

$$p_t^f = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ mc_{t+k}^n \} \quad (13)$$

Gali and Gertler derive a rule based on the recent pricing behavior of the competitors as follows

$$p_t^b = \bar{p}_{t-1}^* + \pi_{t-1} \quad (14)$$

Then they obtain the hybrid Phillips curve by combining (13) and (14),

$$\pi_t = \lambda mc_t + \gamma_f E_t \{ \pi_{t+1} \} + \gamma_b \pi_{t-1}, \quad (15)$$

where  $\lambda = (1 - \omega)(1 - \theta)(1 - \beta\theta)\phi^{-1}$ ,  $\gamma_f = \beta\theta\phi^{-1}$ ,  $\gamma_b = \omega\phi^{-1}$ , and  $\phi = \theta + \omega[1 - \theta(1 - \beta)]$ . Note that when  $\omega = 0$ , this means that all the firms are forward-looking, and we are back to the NKPC. While the reduced form in this case is identified, the hybrid NKPC adds another dimension of non-linearity and makes the identification problem even more severe.

#### 4.1 Estimation and specification

In this section we provide the estimates of the empirical hybrid NKPC and evaluate its overall performance. Log labor share is again used as a measure of MC. To check for robustness, the regression is run with unit labor costs and output gap as well. In this case the model takes the following form

$$\pi_t = \lambda s_t + \gamma_f E_t \{\pi_{t+1}\} + \gamma_b \pi_{t-1} + \varepsilon_t \quad (16)$$

Results are provided in Table 3 below. Gamma coefficients are not significant, while lambda estimates are. However, their sign is negative, which makes no economic sense. Still, the J-test does not reject the null of correct specification.

**Table 3: Hybrid NKPC reduced-form estimates**

	$\gamma_f$	$\gamma_b$	$\lambda$	J-stat	J-statistic
LULCHP				8.672	
coeff.estimate	0.0575	0.0946	-0.0023	no.obs	100
st.error	0.1733	0.1073	0.0007	df	6
				p-value	<b>0.1929</b>
				J-stat	7.7079
LSHLABORHP	0.1182	0.1278	-0.1543	no.obs	100
coeff.estimate	0.1693	0.1192	0.0919	df	6
st.error				p-value	<b>0.2603</b>
				J-stat	8.5487
DLGDPSAHP				no.obs	100
coeff.estimate	0.1828	0.064	0.2551	df	6
st.error	0.1522	0.1231	0.2433	p-value	<b>0.2387</b>

The paper proceeds with the structural estimation procedure using again non-linear instrumental GMM estimator. In this case the same two alternatives are used as in the

previous section: the first specification minimizes non-linearities, while in the second one we restrict the coefficient of inflation in the current period to one.

$$E_t\{(\phi\pi_t - (1-\omega)(1-\theta)(1-\beta\theta)s_t - \theta\beta\pi_{t+1})z_t\} = 0 \quad (17)$$

$$E_t\{(\pi_t - (1-\omega)(1-\theta)(1-\beta\theta)\phi^{-1}s_t - \theta\beta\phi^{-1}\pi_{t+1})z_t\} = 0 \quad (18)$$

Results are provided in Table 4 below.

**Table 4: Estimates of the new hybrid Phillips curve**

	$\omega$	$\theta$	$\beta$	$\gamma_b$	$\gamma_f$	$\lambda$		
							J-statistic	8.2554
LULCHP							no.obs	100
[1] coeff.estimate	<b>0.358</b>	<b>1.001</b>	-0.3517	<b>0.4093</b>	<b>-0.4025</b>	<b>-0.0008</b>	df	6
st.error	0.1319	0.0008	0.1279	0.1277	0.1319	0.0000	p-value	<b>0.22</b>
							J-statistic	7.5946
							no.obs	100
[2]coeff.estimate	0.7018	<b>1.0054</b>	0.9465	0.4204	0.57	-0.0005	df	6
st.errorr	0.6067	0.031	0.8394	0.1202	0.0399	0.0017	p-value	<b>0.2693</b>
							J-statistic	8.1486
LSHLABORHP							no.obs	100
[1]coeff.estimate	<b>0.3792</b>	<b>0.9763</b>	<b>-0.3439</b>	<b>0.4420</b>	<b>-0.3913</b>	0.0199	df	6
st.errorr	0.1273	0.0805	0.1031	0.1034	0.1292	0.0794	p-value	<b>0.2274</b>
							J-statistic	8.6256
							no.obs	100
[2]coeff.estimate	0.0611	<b>1.014</b>	0.0575	0.0601	0.0573	-0.0129	df	6
st.errorr	0.1339	0.0737	0.1071	0.1142	0.1068	0.0735	p-value	<b>0.1958</b>
							J-statistic	8.5918
DLGDPSAHP							no.obs	100
[1] coeff.estimate	0.1593	<b>0.6384</b>	-0.112	<b>0.2327</b>	-0.1044	<b>0.5920</b>	df	6
st.error	0.097	0.1034	0.1544	0.153	0.0975	0.1044	p-value	<b>0.1979</b>
							J-statistic	8.4586
							no.obs	100
[2]coeff.estimate	0.0428	<b>0.9846</b>	0.0822	0.0433	0.0819	0.01494	df	6
st.errorr	0.1264	0.1485	0.1302	0.1292	0.0808	0.0961	p-value	<b>0.2064</b>

Notes: This table reports GMM estimates of the structural parameters of the NKPC.

Data period is 1981:3-2006:2. Instruments used: four lags of inflation, labor income share long-short interest rate spread, output gap, and wage inflation .Standard errors in brackets.

The automatic choice of the Newey-West estimate of the covariance matrix was used

The estimate of  $\theta$  is almost everywhere 1, except for the case where the output gap is used, where it is 0.64. All other coefficients are not significant, with the exception for the regression with unit labor cost. That equation, however, gives puzzling results because the share of forward-looking firms is negative, which makes no economic sense. Still, the J-test confirms that the model is correctly specified.

The effect of the output gap was also found to be zero by Roberts (1997, 1999) when quarterly data are used, while Fuhrer (1997) obtains a significant effect of the output gap in a model with a lot of restrictions. One explanation, aside from the identification issue, is that compared to the US, Hungary is a small open economy, so firms take international prices as given. In a regime of free trade those firms have to adjust quickly and act in a very competitive environment, as compared to the US firms which may be acting indeed as monopolistic producers and can afford to run a band of inaction. Indeed, the degree of backwardness is not statistically different from 0, and markup is seriously squeezed (theoretically equals the transportation costs of the foreign import companies).

## **5. Conclusion**

This paper investigated for the presence of a New Keynesian Phillips (NKPC) curve in Hungary in the period 1981:3-2006:2. The study followed the methodology proposed by Gali and Gertler (1999), who claim that a potential source of inflation may be the sluggish adjustment of real marginal costs to movements in output. The empirical model tested featured forward-looking firms who pre-set prices for a couple of periods ahead, using Calvo(1983) pricing rule. In addition, measures of real marginal cost were used instead of the old-fashioned output gap. The reason was that marginal costs are a better proxy for the impact of the productivity gains on inflation, which the ad hoc measure output gap misses. A hybrid version of NKPC, where some of the firms are backward-looking, and others are forward-looking in their price-setting behavior, was also estimated. However, there are some econometric issues to be considered, such as the weak identification of the parameters of the structural NKPC as well as those of the hybrid NKPC.



## APPENDIX 1:

### New Keynesian Phillips Curve identification

We want to show whether  $E(g_t(\delta)) = 0$  only at  $\delta = \delta_0$ , where  $\delta = \begin{pmatrix} \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \\ \beta \end{pmatrix}$

We need to consider two sub-cases:

#### 1. The reduced-form case

$$g_t(\delta) = z_{t\bullet}(\pi_t - \lambda s_t - \beta E_t \pi_{t+1} + \lambda_0 s_t + \beta_0 E_t \pi_{t+1} - \lambda_0 s_t - \beta_0 E_t \pi_{t+1}) = z_{t\bullet} \varepsilon_t + \\ - z_{t\bullet}(\lambda - \lambda_0)s_t - z_{t\bullet}(\beta - \beta_0)E_t \pi_{t+1};$$

Therefore,  $E(g(\delta)) = 0$  iff  $\lambda = \lambda_0$  and  $\beta = \beta_0$ .

The reduced form model is identified.

#### 2. The structural parameter case

Here,  $E(g_t(\delta)) = 0$  iff  $\beta = \beta_0$  and  $\frac{(1-\theta)(1-\beta\theta)}{\theta} = \frac{(1-\theta_0)(1-\beta_0\theta_0)}{\theta_0}$

By assumption  $\theta_0 > 0$  (some of the firms always adjust). Therefore,

$$\theta_0(1 - \beta\theta - \theta + \beta\theta^2) = \theta_0(1 - \beta_0\theta_0 - \theta_0 + \beta_0\theta_0^2) \text{ or}$$

$$\theta_0 - \beta\theta\theta_0 - \theta\theta_0 - \beta\theta^2\theta_0 = \theta_0 - \beta_0\theta_0\theta - \theta_0\theta - \beta_0\theta_0^2\theta$$

Cancelling equal terms on both sides

$$\theta_0 + \beta\theta^2\theta_0 = \theta_0 + \beta_0\theta_0^2\theta$$

Imposing  $\beta = \beta_0$ , we obtain  $\theta_0 - \beta_0\theta^2\theta_0 = \theta_0 - \beta_0\theta_0^2\theta$

Thus,  $(\theta - \theta_0)(1 + \beta_0\theta\theta_0) = 0$ , which holds when  $\theta = \theta_0 \cup \theta = -\frac{1}{\beta_0\theta_0}$

The second possibility creates a problem in the sense that the structural model is not identified – the t-statistics are not normally distributed.

## Hybrid New Keynesian Phillips Curve Identification

We need to consider two sub-cases:

### 1. The reduced-form case

$$g_t(\delta) = z_{t\bullet}(\pi_t - \lambda s_t - \gamma_f E_t \pi_{t+1} - \gamma_b \pi_{t-1} + \lambda_0 s_t + \gamma E_t \pi_{t+1} + \gamma_{b0} \pi_{t-1} - \lambda_0 s_t - \gamma_f E_t \pi_{t+1} - \gamma_{b0} \pi_{t-1}) = z_{t\bullet} \varepsilon_t - z_{t\bullet}(\lambda - \lambda_0) s_t - z_{t\bullet}(\gamma_f - \gamma_{f0}) E_t \pi_{t+1} - z_{t\bullet}(\gamma_b - \gamma_{b0}) \pi_{t-1};$$

Again,  $E(g_t(\delta)) = 0$  iff  $\lambda = \lambda_0$ ,  $\gamma_f = \gamma_{f0}$  and  $\gamma_b = \gamma_{b0}$ .

The reduced form is identified.

### 2. The structural parameter case

Here,  $E(g_t(\delta)) = 0$  iff

$$\begin{pmatrix} \lambda \\ \gamma_f \\ \gamma_b \end{pmatrix} = \begin{pmatrix} \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\theta + \omega[1-\theta(1-\beta)]} \\ \frac{\beta\theta}{\theta + \omega[1-\theta(1-\beta)]} \\ \frac{\omega}{\theta + \omega[1-\theta(1-\beta)]} \end{pmatrix} = \begin{pmatrix} \frac{(1-\omega_0)(1-\theta_0)(1-\beta_0\theta_0)}{\theta_0 + \omega_0[1-\theta_0(1-\beta_0)]} \\ \frac{\beta_0\theta_0}{\theta_0 + \omega_0[1-\theta_0(1-\beta_0)]} \\ \frac{\omega_0}{\theta_0 + \omega_0[1-\theta_0(1-\beta_0)]} \end{pmatrix} = \begin{pmatrix} \lambda_0 \\ \gamma_{f0} \\ \gamma_{b0} \end{pmatrix}$$

Note that the derivations for NKPC correspond to a specification with  $\omega = 0$ , and it was not identified. Now we allow for additional layer of non-linearity, therefore this model is not identified either and we can prove this using Monte Carlo simulations.

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