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## Aging, Retirement and Pay-As-You-Go Pensions

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## **ABSTRACT**

### **Aging, Retirement and Pay-As-You-Go Pensions**

In this paper we consider the effects of population aging on a pay-as-you-go financed defined contributions pension scheme. We show that when retirement decisions are endogenous, aging increases the retirement age and the steady state level of capital. The effect on pension payouts is in general ambiguous, except for the solution of full retirement, when this effect is unambiguously negative.

JEL Classification: J13, H55

Keywords: PAYG pensions, aging, retirement, overlapping generations model

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# 1 Introduction

We study the effects of population aging, both via decreasing fertility and increasing length of life, in an overlapping generations model where agents can choose their retirement age and a government provides a pay-as-you-go pension, financed with a defined contribution scheme. In particular, we would like to concentrate on the effects of aging on the steady-state level of capital, on the retirement age and on the pension payouts.

To do so, we employ a simple three-period model where all decisions are taken in the second period by perfect foresighted adult agents who maximize an expected utility function whose arguments are consumption in both periods and length of retirement. Using standard functional forms for the utility function (logarithmic) and production function (Cobb-Douglas) we derive the following analytical results: a) aging, either from falling fertility or from increasing longevity, increases the steady state level of capital; b) aging increases the retirement age; c) when the tax rate exceeds a certain threshold, there is full retirement; d) when there is full retirement, aging negatively affects pension payouts; e) otherwise, the effects of aging on pensions is ambiguous. In the last Section we provide a simulation of our model to show how this ambiguity resolves for a certain set of parameter values.

Various papers have analyzed the effects of population aging in OLG models with pension, especially in order to study the sustainability of the PAYG pension system. A possible classification of the literature distinguishes between three main effects of a falling fertility: the capital dilution effect, the intergenerational transfer effect, and the child quality effect. The first one is the classic effect that arises in all neoclassical growth models where, as a result of a fall in fertility, there is an increase in the capital labour ratio and the subsequent positive effect on output per capita (and pensions). Among the relevant papers are Michel and Pestieau (1993) and Cigno (1993). The second effect refers to the fact that in the decentralized equilibrium there might be too few workers to support the pensioners. For examples of these models see Alders and Broer (2005) or Wigger (1999). The last effect is the shift from child quantity to child quality, which increases future wages and hence tax revenues and pensions, like in Cremer et al. (2011).

More recently, the literature has also devoted attention to the issue of increasing longevity, i.e. to aging from above. For instance, Cipriani (2014), complementing the results of Fanti and Gori (2012) to include longevity, presents an OLG model with aging resulting from changes in fertility and in longevity and shows that pensions are adversely affected. However, these papers do not take retirement into account. In fact, few theoretical papers take into account the retirement decision in an overlapping generations model. Two recent works in this area are Cabo and García-Gozales (2014), that studies a parametric reform of the pension system in a dynamic game between the government and the representative consumer, and the calibrated study in continuous time by Chen and Lau (2014). Using, instead, the standard two-period OLG model, Michel and Pestieau (2013) employ a similar setting to the one we use here. However, their model does not include longevity, since the focus is not on population aging but on the relationship between retirement decisions and the size of the social security system. Other recent works include Mizuno and Yakita (2013) and Aísa et al. (2012) which focus, respectively, on fertility and longevity. However, the first paper assumes wages and interest rates as fixed, which can only be justified in a small open economy, and the second paper assumes that old agent's labour force participation is a zero-one decision. Fisher and Heijdra (2012), on the other hand, assumes a mandatory retirement age. Instead, we take a dynamic general equilibrium perspective and allow for partial participation to the labour market in the last period of life. In conclusion, with respect to all this literature, we show that when retirement is fully endogenous the effects of an aging population may not be detrimental to the PAYG pension system because agents could adjust their old-age labour supply accordingly. Incidentally, a similar result, in a computable OLG model, has recently been shown by Bagchi (2016). As a policy conclusion, we claim that governments should not restrict the elderly labour supply by imposing upper limits to the retirement age.

The paper is organized as follows: Section 2 presents the model, Section 3 looks at the results of a simulation and Section 4 concludes.

## 2 The model

We consider an overlapping generations model, where agents live for at most three periods. In the first period they are children and no economic decision is taken. In the second period they are adult and have to decide how much to consume and save for retirement. If they survive to the last period, they have to decide how much time to spend on the labour market, the alternative being retirement. When retired, agents receive a pension financed by the government that runs a PAYG retirement scheme. The government imposes a tax rate on wages and uses the proceedings to pay a pension to the retired households.

The expected utility function of an adult agent born in period  $t$  is the following:

$$U^t = \ln c_a^t + \beta p [\ln c_o^t + \delta \ln(1 - l^t)] \quad (1)$$

where  $c_a^t$  is consumption in young age of an agent of generation (superscript)  $t$ ,  $c_o^t$  is consumption when old,  $\beta \in (0,1)$  is a utility discount factor,  $p$  is the probability of survival to the third period of life (longevity) and  $l^t$  is labour supply in the third period of life. Therefore agents supply inelastically one unit of labour when young and enjoy  $(1 - l^t)$  units of retirement when old.

The first period budget constraint is the following:

$$c_a^t = (1 - \tau)w_t - s^t \quad (2)$$

where  $w_t$  is wage at period  $t$ ,  $s^t$  is savings of an individual belonging to generation  $t$ , and  $\tau$  is the PAYG income tax rate. The second period budget constraint is the following:

$$c_o^t = \frac{R_{t+1}}{p} s^t + (1 - \tau)w_{t+1}l^t + b_{t+1} (1 - l^t) \quad (3)$$

where  $b_{t+1}$  is pension received in period  $t+1$  by an old individual and  $\frac{R_{t+1}}{p}$  is the rate of return on savings. This rate of return assumes the presence of financial intermediaries operating under perfect competition who must take into account the risk from uncertain lifetimes. Hence  $R_{t+1}$  is the risk-free interest rate. The number of new adults at time  $t$

is  $N_t = N_{t-1}(1 + n)$  where  $n$  is the exogenous fertility rate. Since both young and old individuals can work, labour force is given by  $L_t = N_{t-1}(1 + n + pl^{t-1})$ .

On the production side, we assume a perfectly competitive output market where the production function is of the usual Cobb-Douglas form  $Y_t = AK_t^\alpha L_t^{1-\alpha}$ , with  $A > 0$  and  $0 < \alpha < 1$ . Assuming full capital depreciation after one period, the profit maximizing conditions imply that  $w_t = (1 - \alpha)Ak_t^\alpha$  and  $R_t = \alpha Ak_t^{\alpha-1}$ .

The government runs a defined contribution pay-as-you-go social security scheme with a balanced budget. Hence, labour income is taxed, both for the young and the old individuals, and the proceedings are used to pay a pension for the retired. Therefore, pensions benefits are

$$b_{t+1} = \frac{\tau w_{t+1}(1 + n + pl^t)}{p(1 - l^t)} \quad (4)$$

The household maximizes (1) subject to (2), (3) taking as given the wage, the interest rate and the pension benefit. After substituting for  $b_{t+1}$  from (4), the first order conditions with respect to savings and retirement are the following:

$$s^t = \frac{(1 - \tau) [\beta(1 + \delta)w_t R_{t+1}^e - w_{t+1}^e]}{\frac{R_{t+1}^e}{p}(1 + \beta p(1 + \delta))} \quad (5)$$

$$l^t = \frac{w_{t+1}^e [p(1 + p) - \tau((1 + n + p)(1 + p) + (1 + n)p)] - pR_{t+1}^e w_t(1 - \tau)}{pw_{t+1}^e [1 + \beta p(1 + \delta)]} \quad (6)$$

From here, the dynamic equation for capital per worker,  $k_{t+1} = s^t / (1 + n + pl^t)$ , can easily be derived. Assuming perfect foresight, the interior solution for the steady state is defined as:

$$k^* = \left[ \frac{\alpha \beta A p (1 - \alpha + \delta)}{\alpha(1 + n) + \alpha \beta p^2 + p[1 + \alpha \beta (1 + n)(1 + \delta)]} \right]^{\frac{1}{1-\alpha}} \quad (7)$$

$$l^* = \frac{p(1 - \alpha) - \alpha \delta (1 + n) - \tau(1 - \alpha)[(1 + n)(1 + \delta) + p]}{p(1 - \alpha + \delta)} \quad (8)$$



It is easy to show that in this steady state  $\frac{\partial k^*}{\partial n} < 0$  and  $\frac{\partial k^*}{\partial p} > 0$ , i.e. that population ageing increases the level of capital per worker via the traditional capital dilution effect, here reinforced by the effect on retirement. This last effect is the result of the fact that population aging increases the retirement age since, as can be seen from equation (8),  $\frac{\partial l^*}{\partial n} < 0$  and  $\frac{\partial l^*}{\partial p} > 0$ . The mechanisms driving these results are the following: a decrease in  $n$  or an increase in  $p$  reduces, ceteris paribus, PAYG pensions, thus inducing individuals to work more (and retire later) via the usual income effect. At the same time, an increase in  $p$  increases the expected utility from consumption in old age, thus inducing individuals to save more for retirement. In our setting this last effect prevails on the dilution effect coming from the larger number of older workers experienced by an aging economy. Note that  $\frac{\partial l^*}{\partial \tau} < 0$ : labour supply in the last period of life decreases with the tax rate and if  $\tau > \frac{p(1-\alpha)-\alpha\delta(1+n)}{(1-\alpha)[p+(1+n)(1+\delta)]}$  we have a corner solution with full retirement,  $l_c^* = 0$ , where the steady state level of capital is equal to:

$$k_c^* = \left[ \frac{\alpha\beta Ap(1-\alpha)(1-\tau)}{(1+n)(\alpha + \alpha\beta p + \tau - \alpha\tau)} \right]^{\frac{1}{1-\alpha}} \quad (9)$$

Hence, with full retirement, it is still the case, obviously, that population ageing increases the level of capital per worker in the steady state. Also, unsurprisingly,  $\frac{\partial k_c^*}{\partial \tau}$  is negative. Therefore, as the social security tax increases, labour participation in the last period of life declines and becomes zero when  $\tau$  is sufficiently high. At this point, capital per worker in the steady state starts declining with  $\tau$ .

In this framework, it is also interesting to see what happens to pension transfers when there is population aging. A number of papers in the literature have in fact concentrated on the sustainability of the pay-as-you-go pension policy. For example, Fanti and Gori (2012) show that a falling birth rate does not necessarily cause a fall of pensions in the steady state in the canonical OLG model with Cobb-Douglas production function and log utility. Cipriani (2014) shows that if longevity is introduced in their model, population aging may adversely affect the pension system in the long run. Therefore, we now move on to see what are the effects of aging on the pension payouts in the current model. To do so, let's first derive the pension payout in the steady state when agents

fully retire, because  $\tau$  is higher than the threshold defined above:

$$b_c^* = \frac{\tau A(1-\alpha)}{p} \left[ \frac{\alpha\beta Ap(1-\alpha)(1-\tau)}{\alpha + \alpha\beta p + \tau - \alpha\tau} \right]^{\frac{\alpha}{1-\alpha}} (1+n)^{\frac{1-2\alpha}{1-\alpha}} \quad (10)$$

In this case, it is easy to show that like in Cipriani (2014) aging reduces pensions in the steady state since  $\frac{\partial b_c^*}{\partial n} > 0$  and  $\frac{\partial b_c^*}{\partial p} < 0$  if  $\alpha < 1/2$ . Since a stylized fact is that the elasticity of output with respect to capital is about 1/3, these effects could be considered unambiguous.

If, however, there is no full retirement, pensions are given by the following:

$$b^* = \frac{A\tau(1-\alpha)^2(1+n+p+\delta+n\delta)(1-\tau)}{\delta(\pi + \alpha + \alpha n) + (1-\alpha)(1+n+p+\delta+n\delta)\tau} k^{*\alpha} \quad (11)$$

In this case, the effects of aging on pensions are in general ambiguous. On the one hand, like before, aging reduces pension payouts because it increases the number of pensioners for each worker. On the other hand, population aging now increases the retirement age as shown before. The overall effect is thus ambiguous. For this reason, in the next section using a set of parameter values we will work out a numerical example of the model.

### 3 Simulations

In the previous section we have analytically derived the following results:

1. Population aging, either from falling fertility or from increasing longevity, increases the steady state level of capital.
2. Population aging increases the retirement age.
3. If the tax rate is above a certain level, there is full retirement, i.e. zero labour supply when old.
4. With full retirement, aging has an adverse effect on pension payouts.
5. In general, when retirement is not full, the effects of aging on pensions is ambiguous.

In this section we employ a numerical example to illustrate the effects of aging on pensions when retirement is not a corner solution. To this end, we assume the following values for the various parameters:

$$A = 10 \quad \alpha = 1/3 \quad \beta = 0.95$$

$$\tau = 0.15 \quad \delta = 0.1 \quad n = 0.01 \quad \text{or} \quad p = 1$$

The parameter  $\alpha$  is fixed given the well-known empirical regularity that the capital share in added value is about one third. Similarly, the fertility rate and the social security tax rate resembles those of the OECD average and the other two parameters,  $A$  and  $\delta$ , are calibrated in such a way to get a realistic length of the working life (about 40 years), of the real interest rate (about 4%) and of the net replacement rate (61%) in the steady state with low fertility and high life expectancy. Given these values, the following figures show the effects of longevity on the retirement age and pensions in the steady state, assuming that  $n = 1\%$ .

Figure 1: Labour supply of old agents and longevity

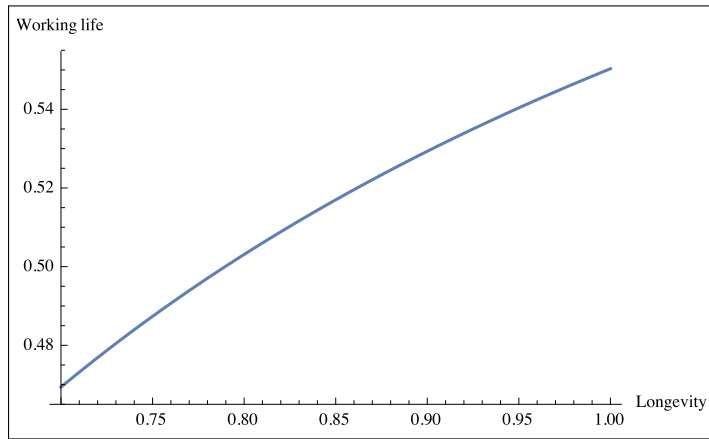


Figure 2: Pensions and longevity

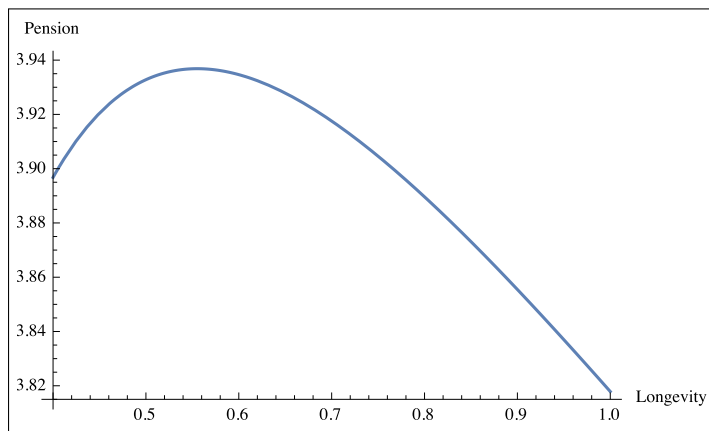


Fig. 1 confirms the positive relationship between longevity and the retirement age. However, as can be seen from Fig. 2, the relationship between pensions and longevity may be non monotonic: initially increasing with longevity and then declining after a certain level. Similarly, the following figures show the effect of fertility on retirement age and pension in the steady state, assuming that  $p = 1$ .

Figure 3: Labour supply of old agents and fertility

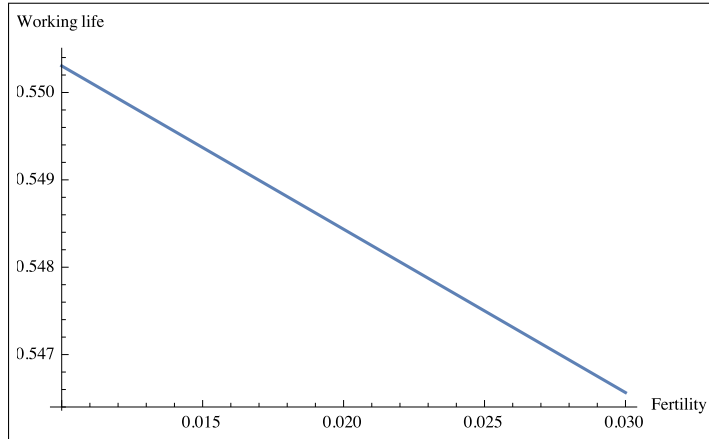


Figure 4: Pensions and fertility

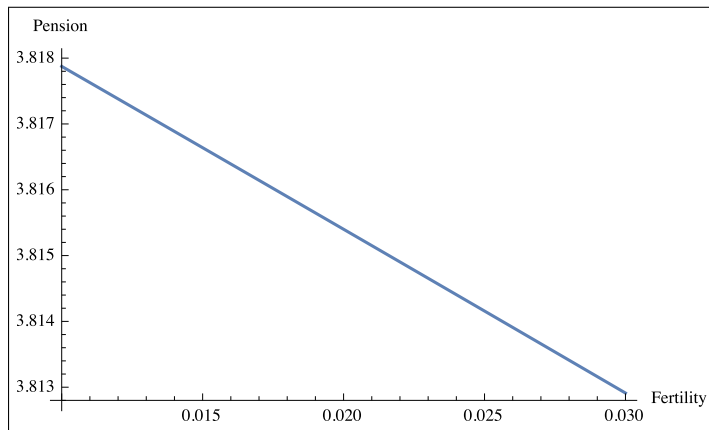


Fig. 3 confirms the negative relationship between fertility and the retirement age: the latter increases as fertility falls, like in the benchmark corner solution. Fig. 4 shows that, with our set of parameters, the relationship between fertility and pension is negative because the capital deepening effect prevails on the direct effect arising from a change in the number of workers who support the pensioners.

## 4 Conclusions

In a dynamic general equilibrium OLG model we studied endogenous retirement decisions when there is a social security system of a PAYG type in an aging economy. Increasing longevity and decreasing fertility has been a widespread experience in many countries in the last decades. Hence, it appears important to study the effects of aging on individual choices of retirement and savings. One of our main results is that optimal retirement increase with longevity. From here, a policy conclusion could be immediately drawn: governments should not restrict the elderly labour supply by imposing upper limits to the retirement age. We also find that the effects of aging on pensions may not be negative if the elderly are free to choose their retirement age, while they are always negative in the case of full retirement. These results are novel and different from those obtained in models where retirement is not endogenous. In fact, for the standard overlapping generation model with pay-as-you-go pensions and full retirement, Cipriani (2014) shows that ageing has always a negative effect on pension benefits.

The model could be extended in various ways. The most obvious extension would be a setting with endogenous fertility. This would allow to see how fertility changes, jointly with savings and retirement, as longevity increases. Then, the model could be generalized without assuming specific functional forms for the utility or the production function. However, relaxing these assumptions would forbid the derivation of analytical solutions. Finally, one may wish to study different types of pension schemes and compare the effects of population aging under the various schemes.

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