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Accidents, Liability Obligations and Monopolized Markets for Spare Parts*

Pio Baake

Abstract

We analyze the effects of accidents and liability obligations on the incentives of car manufacturers to monopolize the markets for their spare parts. We show that monopolized markets for spare parts lead to inefficiently high prices for spare parts. Furthermore, monopolization induces the manufacturers to choose inefficiently high qualities. The key for these results is the observation that high prices for spare parts entail a negative external effect inasmuch as liability obligations imply that consumers of competing products have to pay the high prices as well.

KEYWORDS: aftermarket, monopolization, liability

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1 Introduction

The optimal extent of design protection has been extensively discussed during the past few years. This is especially true for the protection of spare parts for motor vehicles. ‘Must-match’ restrictions with respect to the exact look of visible spare parts in combination with strict design protection imply that car manufacturers and their component suppliers have almost perfect monopoly power for visible replacement parts. Concerning the economic effects of this observation, there are essentially two different views. First, monopoly power due to design protection should be evaluated in the same manner as patent protection for innovations. Furthermore, applying the Chicago school argument that there is only one monopoly rent leads to the conclusion that monopoly power on secondary markets is not detrimental for social welfare.¹ According to the second view, however, this conclusion is premature. Monopoly power on secondary markets may well lead to additional distortions and may thus increase allocative inefficiencies.

The actual policy in the European Union seems to follow the second line of reasoning. Based on the Design Directive of 1998 (Directive 98/71/EC) and the proposal for the amendment on that Directive of September 2004 (COM, 2004, 582 final), the parliament of the European Union in December 2007 backed a proposed directive which aims at liberalizing secondary markets for spare parts. The proposed directive limits design protection for visible parts to primary markets by referring to a ‘repair clause’. This clause allows competitive suppliers to produce spare parts for secondary markets, i.e., markets for repair and maintenance services. Thus, design protection is to be reduced such that market entry and competition on secondary markets is possible.

The model presented in this paper supports the approach taken by the European Union. The focus of our model is on the possibility that consumers—car drivers—cause accidents with other cars and that they are responsible for the entailed damage. The analysis of the implied economic effects shows that car manufacturers have in fact strong incentives to monopolize the markets for their spare parts as this can lead to higher profits. Social welfare, however, is lower with monopolized markets.

The key for these results is the observation that high prices for spare parts not only harm a manufacturer’s own consumers, but also entail a negative external effect for other consumers. With strictly positive probabilities of causing accidents, high prices for spare parts increase expected expenditures for all consumers. Using this correlation, each manufacturer has an incentive to choose rather high prices

¹See Posner (1976) and Bork (1978).

for spare parts but relatively low prices for cars. In contrast to the simple Chicago school argument, monopolized markets for spare parts are thus not neutral with respect to the market equilibrium. Furthermore, considering endogenous quality decisions, monopolized markets for spare parts alter the decisions of the firms such that they choose socially-inefficient high qualities. High prices for spare parts and inefficiently-high qualities imply that social welfare is unambiguously lower with monopolized markets for spare parts as compared to the case with competitive markets for spare parts.

In contrast to the majority of the literature on secondary markets (see Chen et al., 1998, for an overview), our results are based on external effects. While we assume that consumers are locked in with respect to the possible choices of spare parts, we also assume that consumers have perfect foresight and that there are no commitment problems concerning future prices. More precisely, we analyze a simple three-stage game where two car producing firms choose the qualities of their cars first. In the second stage, the firms decide on their prices for cars and spare parts. Consumers decide in the third stage which car to buy. Their decisions are based on the (given) prices and the overall expenditures they expect to incur if they buy a car from either firm. Expected expenditures comprise the price for the car bought as well as expected payments due to accidents. While each consumer can decide whether or not to repair his own car, every consumer has to pay the damages he caused to other cars. Assuming rational consumers, we suppose that consumer can perfectly anticipate the expected payments due to accidents caused. This assumption can be justified by two observations. First, the expected costs for repairing other cars can be thought as being entailed in the premiums for liability insurance car drivers are obliged to carry in most countries.² Second, at least in Germany costs for repair and maintenance services are rather well documented. For example, the largest German association of car drivers regularly publishes detailed cost indexes which comprise the cars' prices, average costs for fuel consumption as well as expected costs for repair and maintenance services.³

Our setting does not entail any aspect of price discrimination between consumers who differ with respect to their willingness to pay (see for example Chen et al., 1993, and Emch, 2003). Furthermore, with perfect foresight of consumers manufacturers cannot economize on lock-in effects or information costs (see Borenstein et al., 1995).⁴ Our assumption that all prices are chosen in the second stage rules

²Actually, it turns out that in our model the expected costs for repairing damages caused to other cars do not affect the consumers' decision which car to buy.

³See http://www1.adac.de/Auto_Motorrad/autokosten/default.asp

⁴Shapiro (1995) provides a critical discussion of monopolization incentives based on information costs.

out any commitment problem. Borenstein et al. (2000) analyze an overlapping-generation model with durable goods and service provisions. They show that even with competitive markets for durable goods, firms will charge prices above marginal costs for services if they cannot commit to future prices (see also Blair and Herndorn, 1996). Additionally, we assume that there is no imperfect information with respect to the manufacturers' qualities. This is in contrast to Schwarz and Werden (1996) who show that tying of goods and services in combination with low prices for services can be used to signal high qualities.

Our results concerning social welfare are contrary to the findings of Carlton and Waldman (2006). Their approach focuses on durable goods in conjunction with maintenance, remanufactured parts and product improvements. Carlton and Waldman show that in all these cases monopolization of the respective aftermarkets can enhance efficiency. In contrast to competitive markets, monopolization allows for pricing structures that resemble Ramsey prices and thus lead to more efficient allocations when maintenance versus replacement decisions or the purchase of either improved or upgraded products are analyzed. In the case of remanufactured parts, competition may harm social welfare because of potential cost disadvantages of competing suppliers. Compared to the model analyzed by Carlton and Waldman our model focuses on the impact of liability obligations and shows that the induced strategic incentives for the firms lead to negative welfare effects of monopolized markets for spare parts.

In the following, we first describe the model. We then characterize consumers' decisions and specify the firms' demand functions for cars and spare parts. Section 4 focuses on the benchmark case where spare parts are offered competitively. In section 5, we turn to the market outcome when spare parts are offered by the car producing firms only. Using specific functional forms, we illustrate our results in section 6, where we also consider social welfare. The final section concludes.

2 The Model

We focus on the impact that potential accidents and liability obligations have on the firms' prices for cars and spare parts. Our model is based on the following framework: There are two firms which produce cars and spare parts. Spare parts are only used to repair damages due to accidents consumers may cause. Consumers are differentiated according to the Hotelling model and can decide which car to buy. Furthermore, consumers can decide whether or not they repair their own cars when

they have caused an accident. Liability obligations, however, imply that consumers are obliged to adjust any damage they have caused to other cars.

Firms The two firms $i = 1, 2$ are located at the endpoints of the unit interval, i.e., at 0 and 1, respectively. Both firms produce cars and spare parts. For simplicity we assume that each firm produces just one spare part. Both firms decide on the quality q_i of their cars and set prices p_i and \tilde{p}_i for their cars and their spare parts, respectively. The firms have the same marginal costs functions $C(q)$ and $\tilde{C}(q)$ for producing cars and spare parts. For simplicity we assume

$$C(q) > \tilde{C}(q); C'(q), C''(q) > 0 \text{ and } \tilde{C}'(q), \tilde{C}''(q) > 0.$$

Consumers Consumers are characterized by their location $\theta \in [0, 1]$ on the unit interval. Locations are uniformly distributed, and the number of consumers is normalized to one. Consumers have a quasi-linear utility function and incur linear transportation costs. We assume that the market for cars is covered, i.e., each consumer buys exactly one car. Consumers' decisions which car to buy depend on the expected quality of the cars, expected overall expenditures, and on the consumers' locations (expected quality and expenditures will be characterized in the next section). Let the expected quality of car i be denoted by q_i^e and let m_i^e denote the expected overall expenditures associated with a purchase of car i . Then, the expected utility of a consumer located at θ if he buys car i is given by

$$Eu(q_i^e, m_i^e, \theta) = q_i^e - m_i^e - \Delta_i(\theta) \text{ with } \Delta_i(\theta) := |\theta - (i - 1)|. \quad (1)$$

Accidents Each consumer who buys a car of either type can cause two different types of accidents. First, he can cause accidents involving other cars. Second, there are one-car accidents. We assume that all consumers have the same probabilities to cause accidents of the different types. For simplicity, we normalize the overall probability that a consumer causes an accident to one. The probability that a consumer causes an accident with another car is denoted by $\rho \in [0, 1]$ whereas $1 - \rho$ denotes the probability that he causes an accident where no other car is involved.⁵ Considering damages, we assume that accidents differ with respect to the severity of the implied damages. More precisely, we assume that the severity $s \in [0, 1]$ of any accident is randomly determined and that the damage D an accident of sever-

⁵Denoting the overall probability that a consumer causes an accident by γ and assuming $0 < \gamma < 1$, the probability that a consumer causes an accident of either type could be written as $\gamma\rho$ and $\gamma(1 - \rho)$, respectively. However, employing $\gamma < 1$, we would get qualitatively the same results.

ity s causes to every car involved is increasing in s as well as in the quality of the respective car.⁶

$$D(s, q_i) \text{ with } D(0, q_i) = 0; D(1, q_i) = q_i \quad (2)$$

$$\text{and } D_s(s, q_i), D_{q_i}(s, q_i) > 0 > D_{ss} \text{ and } D_{sq_i} > 0 \text{ for } s > 0. \quad (3)$$

To simplify the analysis, we assume that s is uniformly distributed on $[0, 1]$ and that all accidents are independent events. Additionally, we assume that any damage can only be fixed by using the spare part for the respective car.⁷

Given this framework, we analyze the following three-stage game, which we solve by backward induction. In the first stage of the game, both firms choose the quality of their cars. Prices for cars and spare parts are set in the second stage. In the final stage of the game, consumers decide which car to buy. Furthermore, potential accidents and the involved demand for spare parts are realized.

3 Consumers' Demand for Cars and Spare Parts

In order to characterize consumers' demand for cars and spare parts, we start by analyzing the consumers' expected costs for repairing their own cars as well as the expected quality of their cars. We then turn to the costs consumers expect to bear due to their obligations to repair all damages they cause to other cars. Combining the results from both steps and using (1), we are able to characterize the demand functions for cars and spare parts.

Assume that a consumer bought car i and has caused an accident of severity s . Then, the consumer will repair his car whenever $\tilde{p}_i \leq p_i$ and

$$q_i - \tilde{p}_i \geq q_i - D(s, q_i) \quad (4)$$

hold. While $\tilde{p}_i \leq p_i$ ensures that the consumer does not buy a new car instead of repairing his old car, (4) can be used to define a critical severity level

$$S(\tilde{p}_i, q_i) \text{ with } D(S(\tilde{p}_i, q_i), q_i) \equiv \tilde{p}_i \quad (5)$$

up to which a consumer would not repair his car. Employing S , we get the following two expressions for the the consumer's expected costs \tilde{M}_i^e for repairing his car and

⁶Subscripts denote partial derivatives. The arguments of the functions will be omitted in the following where this does not lead to any confusion.

⁷While the last assumption rules out the possibility of gradual repair services, allowing for gradual repair services or different repair costs would not change the qualitative effects which liability obligations have on the firms' incentives to increase the prices for their spare parts.

for the expected quality Q_i^e of his car

$$\tilde{M}_i^e(\tilde{p}_i, q_i) = (1 - S(\tilde{p}_i, q_i))\tilde{p}_i \quad (6)$$

$$Q_i^e(\tilde{p}_i, q_i) = q_i - \int_0^{S(\tilde{p}_i, q_i)} D(s, q_i) ds. \quad (7)$$

Note that (7) rests on the assumption that consumers are liable for all damages they cause to other cars. Any damage to a consumer's car which is caused by another consumer will be repaired and does not affect the quality the consumer expects if he buys car i .⁸

While a consumer's expected costs for repairing his own car depend on his decision which car to buy, his expected costs \tilde{M}^e for repairing the damages he may cause to all other cars are given by (again, we assume $\tilde{p}_i \leq p_i$ for $i = 1, 2$)

$$\tilde{M}^e(\tilde{p}_1, \tilde{p}_2, \alpha_1, \alpha_2) = \rho(\alpha_1 \tilde{p}_1 + \alpha_2 \tilde{p}_2). \quad (8)$$

where α_i denotes the market share of firm i in the market for cars. Since liability obligations imply that any damage caused to other cars has to be repaired, \tilde{M}^e does not depend on whether a consumer bought his car from firm 1 or firm 2.⁹ Moreover, note that $\tilde{p}_i > p_i$ would induce consumers who are liable for the repair of car i to buy a new car i instead of repairing the damaged car.

Combining (6)–(8) and using (1), the expected utility Eu_i of consumer θ if he buys car i is given by

$$Eu_i(p_i, \tilde{p}_i, q_i, \tilde{p}_j, \theta) = Q_i^e - p_i - \tilde{M}^e - \tilde{M}_i^e - \Delta_i(\theta). \quad (9)$$

Solving (9) for the consumer Θ who is indifferent between buying from firm 1 and 2, we get

$$\Theta(p_1, \tilde{p}_1, p_2, \tilde{p}_2, q_1, q_2) = \frac{1}{2}(Q_1^e - Q_2^e - \tilde{M}_1^e + \tilde{M}_2^e - p_1 + p_2). \quad (10)$$

Employing (10), the firms' demand functions for cars X_i and for spare parts \tilde{X}_i can be written as (the functions X_i and \tilde{X}_i depend on all prices and qualities)

$$X_1 = \Theta \text{ and } \tilde{X}_1 = (1 + \rho - S(\tilde{p}_1, q_1))\Theta \quad (11)$$

$$X_2 = 1 - \Theta \text{ and } \tilde{X}_2 = (1 + \rho - S(\tilde{p}_2, q_2))(1 - \Theta). \quad (12)$$

⁸Strictly speaking, this result relies on the assumption that consumers can not use side payments in order to avoid inefficient repair services.

⁹Note that this result heavily rests on the assumption that all consumers have the same probabilities of causing accidents of either type. Hence, there are no moral hazard or adverse selection problems as for example reflected in higher insurance rates for sports cars. Note further, that our simplifying assumptions allow to extend the model by taking obligatory liability insurances into account. Assuming competitive insurance markets, the insurance premium each consumer has to pay would not depend on his decision from which firm to buy.

where \tilde{X}_i follows from (6), the fact that with probability ρ every consumer causes an accident where another car is involved and that consumers are liable for the damages they caused.

4 Benchmark: Competitive Markets for Spare Parts

Considering first the case in which spare parts can be produced by competitive firms, we assume that the competitive firms have the same cost functions for producing spare parts as firms 1 and 2. Then, competitive markets for spare parts imply that the prices for spare parts $\tilde{P}_i^c(q_i)$ are given by

$$\tilde{P}_i^c(q_i) = \tilde{C}(q_i). \quad (13)$$

The profit functions Π_i^c of firms $i, j = 1, 2, i \neq j$ can thus be written as

$$\Pi_i^c(p_i, p_j, q_i, q_j) = (p_i - C(q_i))X_i. \quad (14)$$

Maximizing Π_i^c with respect to p_i , it is straightforward to show that the firms' equilibrium prices $P_i^c(q_i, q_j)$ are implicitly given by the solution of ($i = 1, 2$)

$$p_i - C(q_i) = 2X_i. \quad (15)$$

Furthermore, substituting P_i^c into the firms' profit functions, maximizing with respect to q_i , and employing the envelope theorem, the firms' optimal qualities Q_i^c are implicitly given by

$$C'(q_i) + (1 - S(\tilde{P}_i^c(q_i), q_i))\tilde{C}'(q_i) = 1 - \int_0^{S(\tilde{P}_i^c(q_i), q_i)} D_q(s, q_i) ds. \quad (16)$$

Analyzing (16) in view of (6) and (7) shows that the optimal quality of each firm is such that it maximizes the consumers' expected quality minus the firm's costs. Hence, there are no strategic interdependencies between the firms' quality decisions.¹⁰ Furthermore, liability obligations imply that the costs for repairing damages caused to other cars do not affect the firms' market shares. Therefore, the firms' quality decisions do not depend on ρ . However, taking into account the overall costs for repairing damages due to accidents where two cars are involved, we obtain that the socially-efficient qualities are the lower the higher ρ .¹¹ Thus, the equilibrium qualities Q_i^c are inefficiently high as long as ρ is strictly positive.

¹⁰This result is due to the assumption that the consumers' utility functions are quasi-linear.

¹¹This result follows immediately from using $\tilde{p}_i = \tilde{C}(q_i)$ and maximizing $Q_i^e - \tilde{M}_i^e - \tilde{M}^e$ with respect to the firms' qualities.

5 Monopolized Markets for Spare Parts

With spare parts exclusively sold by the firms 1 and 2, the firms' profit functions Π_i are given by

$$\Pi_i(p_i, \tilde{p}_i, p_j, \tilde{p}_j, q_i, q_j) = (p_i - C(q_i))X_i + (\tilde{p}_i - \tilde{C}(q_i))\tilde{X}_i. \quad (17)$$

Before we analyze the price and quality equilibria implied by (17) in more detail, note first that it is never optimal for a firm to set their prices such that $\tilde{p} > p$ holds (to simplify the exposition we drop the subscript when we refer to both firms). This result is simply due to $C(q_i) > \tilde{C}(q_i)$ and the fact that consumers always have the option to buy a new car instead of buying a spare part. Hence, we can impose $p \geq \tilde{p}$ as a restriction the firms have to obey when they decide on their prices.

Analyzing the impact of ρ on the firms' pricing and quality decisions, it turns out that $p \geq \tilde{p}$ does not bind for either firm as long as ρ is low enough. In contrast to the benchmark case, however, equilibrium prices and qualities are such that the difference between the prices for cars and spare parts is decreasing in ρ while the equilibrium qualities are increasing in ρ .

With relatively high values of ρ there can exist equilibria where the firms' quality decisions are asymmetric and where $p \geq \tilde{p}$ is only binding for the firm which chooses the higher quality. Increasing ρ further, the restriction $p \geq \tilde{p}$ becomes binding for both firms and the induced quality and pricing decisions are again symmetric.

5.1 Unconstrained Equilibria

5.1.1 Prices

We first analyze the case in which $p \geq \tilde{p}$ is not binding. Maximizing (17) with respect to p_i and \tilde{p}_i and solving the respective first order conditions, the firms' price reaction functions P_i^R and \tilde{P}_i^R for the prices for cars and spare parts are implicitly given by the solutions of the following two equations ($i, j = 1, 2$ and $i \neq j$)

$$p_i - C(q_i) = 2X_i - (1 + \rho - S(\tilde{p}_i, q_i)) (\tilde{p}_i - \tilde{C}(q_i)) \quad (18)$$

$$\tilde{p}_i - \tilde{C}(q_i) = \rho D_s(S(\tilde{p}_i, q_i), q_i). \quad (19)$$

Equations (18) and (19) reveal that \tilde{P}_i^R depends on ρ and q_i only. The optimal price P_i^R , however, depends on the prices of firm j , the qualities of both firms as well as on ρ . Thus, we have $P_i^R(p_j, \tilde{p}_j, q_i, q_j, \rho)$ and $\tilde{P}_i^R(q_i, \rho)$. Moreover, analyzing the

cross-partial derivatives of the firms' profit functions, we get

$$0 < \frac{\partial^2 \Pi_i}{\partial p_i \partial \tilde{p}_j} = \frac{1}{2}(1 - S(\tilde{P}_i^R, q_i)) < \frac{\partial^2 \Pi_i}{\partial p_i \partial p_j} = \frac{1}{2}. \quad (20)$$

While the firms' reaction functions P_i^R increase in the other firms' prices, (20) also indicates that an increase in p_j has a higher positive impact on P_i^R than \tilde{p}_j . Roughly speaking, even though prices for spare parts can be used as strategic instruments, competition on the car market continues to be driven by the prices for cars. Note also, that the last inequality in (20) implies that there exists a unique price equilibrium $P_i^*(q_i, q_j, \rho)$ and $\tilde{P}_i^*(q_i, q_j, \rho)$.

With $\rho = 0$ we get $\tilde{P}_i^* = \tilde{C}(q_i) < P_i^*$ (see (19)). If there are no external effects, i.e., if there are no accidents with other cars, the one monopoly rent argument proposed by the Chicago school applies and the equilibrium prices for the firms' spare parts are equal to the respective marginal costs. Intuitively, by choosing $\tilde{P}_i^* = \tilde{C}(q_i)$ each firm maximizes the overall surplus the firm and its consumers can get from repairing their cars. The prices for cars are then used to maximize the firms' profits.¹²

However, analyzing the impact of $\rho > 0$ on P_i^* and \tilde{P}_i^* , it turns out that liability obligations induce the firms to exploit the implied external effects by increasing the prices for spare parts and by decreasing the prices for cars.

Proposition 1 *While $\rho = 0$ leads to $\tilde{P}_i^* = \tilde{C}(q_i) < P_i^*$, an increase in ρ increases \tilde{P}_i^* but decreases P_i^* .*

Proof. See Appendix. ■

The intuition for Proposition 1 is based on the following two observations: First, a higher probability of causing accidents where other cars are involved increases the probability that a consumer driving car j causes accidents with cars i and thus has to pay the spare part for these cars. Since the induced demand for firm i 's spare parts is price inelastic, firm i has a strong incentive to increase the price for its spare parts. Second, considering the cross partial derivatives of (17) with respect to the firms' prices and using (18) and (19), we obtain

$$\frac{\partial^2 \Pi_i}{\partial p_i \partial \tilde{p}_i} = -1 + S(\tilde{p}_i, q_i) < 0. \quad (21)$$

From a firm's perspective the prices for its cars and spare parts are strategic substitutes. Since an increase in \tilde{p}_i leads to higher costs for repairing cars i , consumers

¹²The same argument applies for repair and maintenance services purely caused by abrasion.

who buy car i expect higher costs for repairing their own cars which in turn leads to a lower expected quality Q_i^e . To reach its optimal market share on the market for cars, firm i has thus to decrease p_i if it increases \tilde{p}_i .

5.1.2 Quality

Substituting P_i^* and \tilde{P}_i^* into the firms' profit functions, let Π_i^* denote the firms' reduced profit functions

$$\Pi_i^*(q_i, q_j, \rho) = (P_i^* - C(q_i))X_i + (\tilde{P}_i^* - \tilde{C}(q_i))\tilde{X}_i. \quad (22)$$

Employing the envelope theorem, and solving the first order condition with respect to the firms' quality, the firms' quality-reaction functions Q_i^R are implicitly given by

$$C'(q_i) + (1 + \rho - S(\tilde{P}_i^*, q_i))\tilde{C}'(q_i) = 1 - \int_0^{S(\tilde{P}_i^*, \rho)} D_q(s, q_i) ds + \rho D_q(S(\tilde{P}_i^*, q_i), q_i). \quad (23)$$

Inspection of (23) immediately shows that Q_i^R depends on ρ only. Thus, unrestricted pricing for cars and spare parts implies that the firms' quality decisions have no mutual strategic effects. Consequently, the equilibrium is symmetric and both firms choose the same equilibrium quality $Q_i^*(\rho) = Q_i^R(\rho)$. Additionally, considering the impact of ρ on $Q_i^*(\rho)$, we obtain

Proposition 2 *The equilibrium qualities $Q_i^*(\rho)$ are symmetric and increasing in ρ as long as $D_{ss}(s, q)$ is close to zero.*

Proof. See Appendix. ■

Compared to the case with competitive markets for spare parts, monopolized markets allow the firms to increase their prices for spare parts in order to exploit the consumers of the other firm. This observation also implies that increasing its market share by choosing higher quality is more profitable for a firm the higher ρ . Hence, the higher the probability that consumers cause accidents with other cars, the higher are the firms' incentives to increase their quality. Compared to competitive markets for spare parts, the firms' quality are thus distorted upwards.

Propositions 1 and 2 also reveal that both firms will choose $Q_i^*(\rho)$ as long as ρ is close enough to zero. Furthermore, since an increase of ρ leads to higher quality, the combined effect of ρ on the firms' prices for cars is ambiguous. On the one hand a higher ρ leads to lower car prices. On the other hand higher quality increases the firms' costs and thus tend to increase the prices for their cars. Analyzing this second effect in more detail, we obtain

Lemma 1 *Considering $q_i = Q_i^*(\rho)$, an increase in q_i strictly decreases $P_i^* - \tilde{P}_i^*$ if D_{ss} and $\tilde{C}'(Q_i^*(\rho))$ are high enough, i.e., if D_{ss} is close to zero and if*

$$1 < (2 - S(\tilde{P}_i^*, q_i)) \left[\tilde{C}'(Q_i^*(\rho)) + \rho D_{sq}(S(\tilde{P}_i^*, Q_i^*(\rho)), Q_i^*(\rho)) \right]$$

holds.

Proof. See Appendix. ■

Combining Propositions 1 and 2 as well as Lemma 1, we get

Corollary 1 *With endogenous quality, an increase in ρ strictly decreases $P_i^* - \tilde{P}_i^*$ if the conditions stated in Lemma 1 are satisfied.*

Under the conditions stated in Lemma 1, a higher probability of causing accidents not only leads to higher quality, but it also lowers the relation between the firms' equilibrium prices for cars and spare parts. Hence, ρ being high enough may well lead to the case in which the restriction $p \geq \tilde{p}$ is binding.

5.2 Constrained Equilibria

Analyzing the firms' pricing and quality decisions if $p \geq \tilde{p}$ is binding, we start by characterizing the firms' pricing decisions. To capture potentially asymmetric equilibria, we distinguish the cases where $p \geq \tilde{p}$ is binding for only one firm and where $p \geq \tilde{p}$ is binding for both firms. It turns out that the conditions stated in Lemma 1 also imply that the firms' quality reaction functions entail an upward jump and that asymmetric equilibria may exist.

5.2.1 Prices

Consider first the asymmetric case where $p \geq \tilde{p}$ is binding for only one firm. Assuming $p_i = \tilde{p}_i$ and $p_j > \tilde{p}_j$ with $i, j = 1, 2; i \neq j$, the firms' profit functions Π_i^A and Π_j^A can then be written as

$$\Pi_i^A(p_i, p_j, \tilde{p}_j, q_i, q_j, \rho) = (p_i - C(q_i))X_i + (p_i - \tilde{C}(q_i))\tilde{X}_i \quad (24)$$

$$\Pi_j^A(p_j, \tilde{p}_j, p_i, q_j, q_i, \rho) = (p_j - C(q_j))X_j + (\tilde{p}_j - \tilde{C}(q_j))\tilde{X}_j. \quad (25)$$

Since $p_j > \tilde{p}_j$ does not bind by assumption, firm j 's price reactions functions P_j^{RA} and \tilde{P}_j^{RA} are again implicitly given by (18) and (19), i.e., we have

$$P_j^{RA} = P_j^R \text{ and } \tilde{P}_j^{RA} = \tilde{P}_j^R. \quad (26)$$

Solving the first order condition for firm i 's price reaction function $P_i^{RA}(p_j, \tilde{p}_j, q_i, q_j, \rho)$, we obtain that P_i^{RA} satisfies (using $S_i := S(p_i, q_i)$)

$$p_i = \Psi \left[C(q_i) + 2X_i - (1 + \rho - S_i) (p_i - \tilde{C}(q_i)) \right] \quad (27)$$

$$+ (1 - \Psi_i) \left[\tilde{C}(q_i) + \rho D_s(S_i, q_i) \right]$$

with: $\Psi_i := \frac{(2 - S_i)D_s(S_i, q_i)}{(2 + \rho - S_i)(2 - S_i)D_s(S_i, q_i) + 2X_i} < 1$.

Hence, P_i^{RA} is a convex combination of P_i^R and \tilde{P}_i^R where the relative weight of P_i^R is lower given a higher ρ . Intuitively, the higher ρ , the more firm i gains from a high price for its spare parts and from distorting the price for its cars upwards in order to ensure that $p_i = \tilde{p}_i$ holds. For later reference let P_i^{A*} and $P_j^{A*}, \tilde{P}_j^{A*}$ denote the solutions of (26) and (27) and let $\Pi_i^{A*}(q_i, q_j, \rho)$ and $\Pi_j^{A*}(q_i, q_j, \rho)$ denote firm i 's and firm j 's reduced profit functions.

Turning to the case with $p = \tilde{p}$ for both firms and maximizing the firms' profit functions ($i, j = 1, 2, i \neq j$)

$$\Pi_i^S(p_i, p_j, q_i, q_j, \rho) = (p_i - C(q_i))X_i + (p_i - \tilde{C}(q_i))\tilde{X}_i$$

with respect to the firms' prices, it is easy to show that both firms' price reaction functions P_i^{RS} are determined by (27). We thus have

$$P_i^{RS} = P_i^{RA} \text{ for } i = 1, 2 \quad (28)$$

as long as $p = \tilde{p}$ is binding for both firms. For later reference, let P_i^{S*} denote the solutions of the system of equations given by (28), and let $\Pi_i^{S*}(q_i, q_j, \rho)$ and $\Pi_j^{S*}(q_i, q_j, \rho)$ denote the firms' reduced profit functions.

5.2.2 Quality

We again start with the asymmetric case. Assuming $p_i = \tilde{p}_i$ and $p_j > \tilde{p}_j$, analyzing the quality decisions implied by $\Pi_i^{A*}(q_i, q_j, \rho)$ and $\Pi_j^{A*}(q_i, q_j, \rho)$ and comparing the respective first order conditions for the firms' optimal quality, we immediately get that firm j 's optimal quality decision $Q_j^{RA}(\rho)$ is the same as in the unconstrained case, i.e.,

$$Q_j^{RA}(\rho) = Q_j^R(\rho). \quad (29)$$

While the quality decision of firm j does not change, a complete characterization of the optimal quality choice of firm i turns out to be rather involved. We thus focus on firm i 's behavior when the restriction $p \geq \tilde{p}$ starts to bind. Defining $Q_i^{RA}(\rho, q_j) := \arg \max \Pi_i^{A*}(q_i, q_j, \rho)$, we obtain

Lemma 2 *Assume that the conditions stated in Lemma 1 hold and that there exists a $\rho^A < 1$ such that $P_i^*(Q_i^R(\rho^A), q_j, \rho^A) = \tilde{P}_i^*(Q_i^R(\rho^A), q_j, \rho^A)$. Then, we have*

$$Q_i^{RA}(\rho^A, q_j) > Q_i^R(\rho^A) \text{ and } \frac{\partial P_j^{A*}}{\partial q_i} > 0 \text{ for } q_i = Q_i^R(\rho^A) \text{ and } \rho = \rho^A.$$

Proof. See Appendix. ■

Whereas the first inequality in Lemma 2 shows that firm i 's optimal quality is higher when the restriction $p = \tilde{p}$ starts to bind, the second inequality reveals that a higher q_i softens the restriction $p \geq \tilde{p}$ for firm j . The intuition for these two results relies on the fact that an increase in q_i leads to a higher increase in firm i 's car price if $p_i = \tilde{p}_i$ is binding (see Corollary 1). Employing (20) then shows that the equilibrium price P_j^{A*} reacts more strongly to an increase in q_i as compared to the reaction of P_j^* . Therefore, an increase in q_i leads to more profitable price changes for firm i if $p_i = \tilde{p}_i$ is binding. Additionally, $\partial P_j^{A*} / \partial q_i > 0$ indicates that asymmetric equilibria may well exist. More precisely, while firm i has an incentive to increase its quality above $Q_i^R(\rho)$ for all ρ close enough to ρ^A , firm j may well stick to $Q_j^R(\rho)$ as the constraint $p_j \geq \tilde{p}_j$ is softened.¹³

Essentially the same line of reasoning applies for the symmetric case where $p \geq \tilde{p}$ is binding for both firms. Proceeding as above and defining $Q_j^{RS}(\rho, q_i) := \arg \max \Pi_j^{S*}(q_j, q_i, \rho)$ we get

Lemma 3 *Assume that the conditions stated in Lemma 1 hold and that there exists a $\rho^S < 1$ such that $P_j^{A*}(Q_j^{RA}(\rho^S), q_i, \rho^S) = \tilde{P}_j^{A*}(Q_j^{RA}(\rho^S), q_i, \rho^S)$. Then, we have*

$$Q_j^{RS}(\rho^S, q_i) > Q_j^{RA}(\rho^S).$$

Proof. See Appendix. ■

Again, imposing $p_j = \tilde{p}_j$, an increase in q_j leads to a higher increase in firm j 's car price as compared to the case where $p_j = \tilde{p}_j$ does not bind. Since the firms' prices for cars are strategic complements, the equilibrium price changes induced by an increase in q_j are more profitable for firm j which also implies that firm j 's incentives to choose a high quality are stronger when $p_j = \tilde{p}_j$ binds.¹⁴

However, considering the quality decision of firm i , we obtain

¹³Since we have $Q_i^{RA}(\rho^A, q_j) > Q_i^R(\rho^A)$, continuity of the respective profit functions of firm i implies that firm i has an incentive to choose $Q_i^{RA}(\rho, q_j)$ for all ρ close enough to ρ^A .

¹⁴Again, $Q_j^{RS}(\rho^S, q_i) > Q_j^{RA}(\rho^S)$ implies that firm j will choose $Q_j^{RS}(\rho, q_i)$ for all ρ close enough to ρ^S .

Lemma 4 Assume that there exists a $\rho^S < 1$ such that $P_j^{A*}(Q_j^{RA}(\rho^S), q_i, \rho^S) = \tilde{P}_j^{A*}(Q_j^{RA}(\rho^S), q_i, \rho^S)$. Then, we have

$$Q_i^{RS}(Q_j^{RA}(\rho^S), \rho^S) < Q_i^{RA}(Q_j^{RA}(\rho^S), \rho^S)$$

as long as

$$\frac{\partial P_i^{RA}}{\partial q_i} > \frac{1}{2 - S(P_i^{A*}, q_i)} \left(1 - \int_0^{S(P_i^{A*}, q_i)} D_q(s, q) ds \right)$$

holds for $q_j = Q_j^{RA}(\rho)$ and $\rho = \rho^S$.

Proof. See Appendix. ■

In contrast to Lemma 2 and 3, Lemma 4 shows that firm i 's optimal quality may be lower if $p = \tilde{p}$ starts to bind for firm j . Intuitively, since the unconstrained price \tilde{P}_j^{RA} does not depend on p_i and since P_j^{RS} is a convex combination of P_j^{RA} and \tilde{P}_j^{RA} , an increase in p_i leads to a lower increase of firm j 's car price when $p_j = \tilde{p}_j$ binds. Hence, as long as $\partial P_i^{RA} / \partial q_i$ and thus $\partial P_j^{A*} / \partial q_i$ are high enough, a switch from the asymmetric to the symmetric equilibrium reduces firm i 's optimal quality.

Summarizing the results of Lemma 2, 3 and 4, and focusing on the firms' equilibrium qualities, we get

Proposition 3 Under the conditions stated in Lemma 1, there may exist asymmetric equilibria such that the restriction $p = \tilde{p}$ is only binding for the firm that provides the higher quality. Switching from an asymmetric to a symmetric equilibrium where $p = \tilde{p}$ is binding for both firms can induce quality choices that lie in between the choices in the asymmetric equilibrium.

6 Numerical Example

In order to illustrate the above findings we now turn to a specific numerical example. We assume the following functional forms for the firms' cost functions and the damages caused by accidents

$$C(q) = \frac{1}{4}q^2, \quad \tilde{C}(q) = \frac{1}{8}q^2 \quad \text{and} \quad D(s, q) = sq. \quad (30)$$

Solving for the equilibrium qualities in the benchmark case with competitive markets for spare parts, we obtain

$$q_1^c = q_2^c \approx 1.4. \quad (31)$$

The equilibrium qualities with monopolized markets for spare parts are given by the solution of¹⁵

$$Q_1^*(\rho) = Q_2^*(\rho) = 16 - \frac{8}{\sqrt{3}}\sqrt{10 - \rho^2} \quad (32)$$

as long as the restriction $p = \tilde{p}$ does not bind for either firm. Evaluating the firms' equilibrium prices shows that the difference $P_i^*(Q_1^*(\rho), Q_2^*(\rho), \rho) - \tilde{P}_i^*(Q_1^*(\rho), Q_2^*(\rho), \rho)$ is strictly decreasing in ρ . Moreover, we obtain

$$P_i^*(Q_1^*(\rho), Q_2^*(\rho), \rho) = \tilde{P}_i^*(Q_1^*(\rho), Q_2^*(\rho), \rho) \text{ for } \rho \approx 0.464,$$

which indicates that asymmetric equilibria can exist. More specifically, considering the firms' optimal quality in the asymmetric case, we assume without loss of generality that $p = \tilde{p}$ binds for firm 1 only. Calculating firm 1's optimal quality in this case and comparing the respective profit with firm 1's profit in the unconstrained case, we get that firm 1 is indifferent between choosing $Q_1^{RA}(Q_j^*(\rho), \rho)$ and $Q_1^*(\rho)$ for

$$\rho \approx 0.46. \quad (33)$$

Analyzing firm 2's optimal quality decision, it turns out that—given $Q_1^{RA}(Q_j^*(\rho), \rho)$ —firm 2 will stick to $Q_2^*(\rho)$ as long as ρ is close enough to 0.46. More precisely, we get

$$P_2^{A*}(Q_2^{A*}(\rho), Q_1^{A*}(\rho), \rho) \geq \tilde{P}_2^{A*}(Q_2^{A*}(\rho), Q_1^{A*}(\rho), \rho) \text{ for } \rho \leq 0.465$$

and that firm 2's best response to $Q_1^{RA}(Q_j^*(\rho), \rho)$ is $Q_2^*(\rho)$ for $\rho \in [0.46, 0.462]$. Increasing ρ above 0.462 induces firm 2 to increase its quality such that $p = \tilde{p}$ becomes binding for both firms. For $\rho > 0.462$ the firms' equilibrium qualities are thus given by the equilibrium qualities $Q_i^{S*}(\rho)$ in the symmetric equilibrium, i.e., by the solution of¹⁶

$$q = Q_i^{RS}(q, \rho). \quad (34)$$

Figure 1 depicts the equilibrium qualities and confirms the results stated in Propositions 2 and 3. Whereas the equilibrium qualities increase in ρ as long as ρ is low enough, the asymmetric equilibrium is characterized by $Q_1^{RA}(Q_2^*(\rho), \rho) > Q_2^*(\rho)$. Increasing ρ above 0.462 induces the firms to switch in the symmetric equilibrium which is characterized by $Q_2^*(\rho) < Q_i^{S*}(\rho) < Q_1^{RA}(Q_2^*(\rho), \rho)$ as long as ρ is close enough to 0.462. Note further that $Q_i^{S*}(\rho)$ is again strictly increasing in ρ .

Considering the firms' profits, Figure 2 reveals that both firms are better off under monopolized markets for spare parts (Π_i^* , Π_i^C as well as Π_i^{A*} and Π_i^{S*} with

¹⁵A more detailed formal analysis of the example is provided in the Appendix.

¹⁶Using $Q_i^{S*}(\rho)$ and calculating the profit of firm j when it chooses its quality such that $p = \tilde{p}$ does not bind for both firms, we get that with $\rho \geq 0.465$ deviations from $Q_j^{S*}(\rho)$ are not worthwhile.

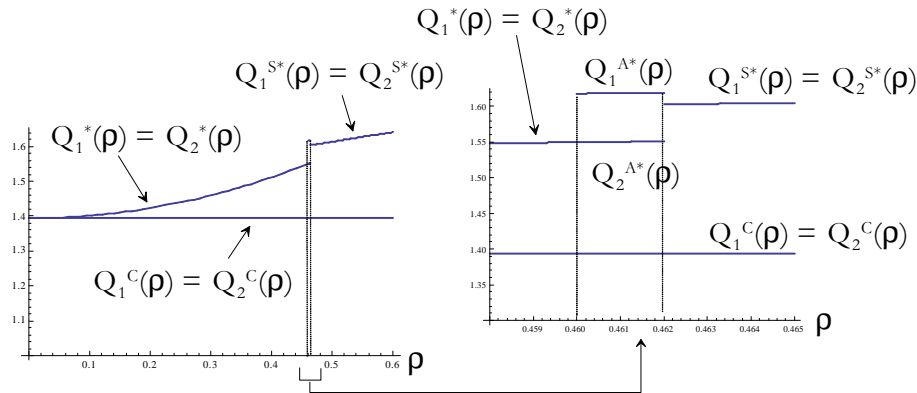


Figure 1: Equilibrium qualities

$i = 1, 2$ denote the firms' profits evaluated at the equilibrium qualities). Although this relation is strict only if the constraint $p = \tilde{p}$ is binding for at least one firm, i.e., only if ρ is high enough, our results nevertheless indicate that the firms may well agree to act against any liberalization of the markets for their spare parts.¹⁷ Note further that in the asymmetric equilibrium firm 1's profit is lower than the profit of firm 2. While each firm may have an incentive to increase its own quality, each firm also benefits from the other firm's constrained pricing behavior.

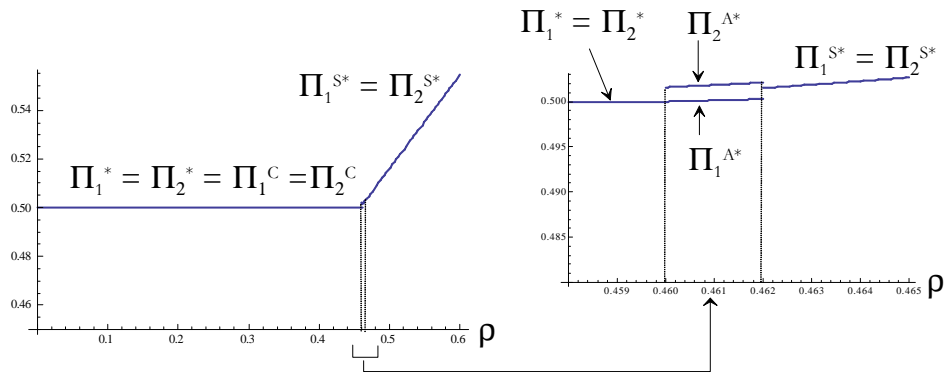


Figure 2: Firms' equilibrium profits

¹⁷The result $\Pi_i^* = \Pi_i^C$ relies on the specific characteristics of the Hotelling model. With covered markets and symmetric firms, the firms' equilibrium profits do not depend on their costs or their prices for spare parts (see also (18)).

Finally, turning to social welfare defined as the sum of the firms' profits and consumers' surplus, Figure 3 shows that social welfare is unambiguously lower with monopolized than with competitive markets for spare parts ($W^C(\rho)$ denotes social welfare with competitive markets for spare parts, while $W(\rho)$ denotes social welfare with monopolized markets).

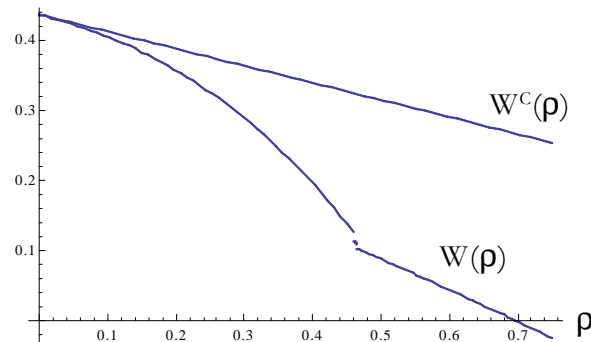


Figure 3: Social welfare with monopolized and competitive markets for spare parts

This negative result is due to the distortions high prices for spare parts induce with respect to the consumers' decisions to repair their own cars. Moreover, compared to the case with competitive markets, monopolized markets for spare parts induce the firms to choose even more distorted quality which again lowers social welfare.

7 Conclusion

The results presented in the last section indicate that monopolization of markets for spare parts can be detrimental for social welfare. Positive probabilities of causing accidents together with liability obligations imply that high prices for spare parts not only harm the firms' own consumers but also the consumers of other firms. The relation between the prices for the firms' cars and their spare parts is not neutral with respect to the firms' market shares. By choosing a relatively high price for spare parts but a relatively low price for cars each firm economizes on the external effects implied by liability obligations and positive probabilities of causing accidents with other cars. Additionally, compared to competitive markets for spare parts, the firms' qualities are inefficiently high which again reduces social welfare.

Although these results are based on a rather simple model, the underlying reasoning should continue to hold under more general assumptions. Most obviously, considering the realistic case where a number of different spare parts have to be used in order to repair potential damages, the car producing firms have to be modelled as multi-product firms offering a set of different spare parts with interdependent demands. While this leads to a more complex analysis, it does not alter the conclusions with respect to the external effects implied by accidents and liability obligations. Similarly, in order to endogenize the consumers' probabilities of causing accidents, one has to extend the model by allowing for consumers' heterogeneity towards different driving behaviors as well as different car attributes like acceleration and maximum speed. Incorporating these aspects points to a comprehensive model where potential moral hazard and adverse selection problems in conjunction with strategic price setting behavior of insurance companies can be analyzed. Despite the potential merits of such a model, some of the basic relations analyzed in our model should be preserved. As long as insurance rates are based on expected damages, insurance rates are positively related to the (average) prices of spare parts. Thus, the external effects and the implied incentives of the firms to distort the relative prices for cars and spare parts continue to exist even though customized insurance rates may be based on personnel accident statistics and may differ according to the cars consumers use.

Appendix

Proof of Proposition 1 Defining $S_i := S(\tilde{P}_i^*, q_i)$ and using (18) and (19), simple comparative statics with respect to ρ leads to ($i, j = 1, 2$ and $i \neq j$)

$$\frac{\partial}{\partial \rho} P_i^* = -\frac{(1 - S_i)D_s(S_i, q_i)^2}{D_s(S_i, q_i) - \rho D_{ss}(S_i, q_i)} - \frac{\rho}{3} (2D_s(S_i, q_i) + D_s(S_j, q_j)) < 0 \quad (35)$$

$$\frac{\partial}{\partial \rho} \tilde{P}_i^* = \frac{D_s(S_i, q_i)^2}{D_s(S_i, q_i) - \rho D_{ss}(S_i, q_i)} > 0. \quad (36)$$

Proof of Proposition 2 Differentiating $\partial \Pi_i^* / \partial q_i$ with respect to ρ and using $D_{ss}(s, q) \leq 0$, we obtain (using $S_i := S(\tilde{P}_i^*, q_i)$)

$$\begin{aligned} \text{sign} \frac{\partial^2 \Pi_i^*}{\partial \rho \partial q_i} &= \text{sign} [D_{sq}(S_i, q_i)D_s(S_i, q_i) \\ &\quad + (\tilde{C}'(q_i) - D_q(S_i, q_i))D_{ss}(S_i, q_i)]. \end{aligned} \quad (37)$$

Hence, $D_{ss}(S_i, q_i) = 0$ implies $\partial Q_i^*(\rho)/\rho > 0$.

Proof of Lemma 1 Using (18) and (19), simple comparative statics with respect to q_i lead to (again, we use $S_i := S(\tilde{P}_i^*, q_i)$)

$$\begin{aligned} \frac{\partial P_i^*}{\partial q_i} - \frac{\partial \tilde{P}_i^*}{\partial q_i} &= \frac{1}{3(D_s(S_i, q_i) - \rho D_{ss}(\tilde{P}_i^*, q_i))} [2C'(q_i)(D_s(S_i, q_i) - \rho D_{ss}(S_i, q_i)) \\ &+ \tilde{C}'(q_i)(D_s(S_i, q_i)(2\rho + S_i - 4) - 2\rho D_{ss}(S_i, q_i)(1 + \rho - S_i)) \\ &- D_s(S_i, q_i)(2\rho D_q(S_i, q_i) + 3\rho D_{sq}(S_i, q_i)(2 - S_i) + \int_0^{S_i} D_q(s, q_i)ds - 1) \\ &+ \rho D_{ss}(S_i, q_i)(D_q(S_i, q_i)(2\rho - 3S_i + 6) + \int_0^{S_i} D_q(s, q_i)ds - 1)]. \end{aligned} \quad (38)$$

Differentiating (38) with respect to $C'(q_i)$, we get

$$\frac{\partial}{\partial C'(q_i)} \left[\frac{\partial P_i^*}{\partial q_i} - \frac{\partial \tilde{P}_i^*}{\partial q_i} \right] > 0. \quad (39)$$

Furthermore, re-arranging (23) leads to

$$\int_0^{S(\tilde{P}_i^*, \rho)} D_q(s, q_i)ds = 1 - C'(q_i) - (1 + \rho - S_i)\tilde{C}'(q_i) + \rho D_q(S_i, q_i) \quad (40)$$

for $q_i = Q_i^*(\rho)$. Using $D_q > 0$, we must thus have

$$C'(Q_i^*) \leq 1 - \tilde{C}'(Q_i^*)(1 + \rho - S_i) + \rho D_q(S_i, Q_i^*). \quad (41)$$

Finally, employing $D_{ss} = 0$, substituting (40) into (38) and using (41) as well as $D_{ss} = 0$, we obtain

$$1 < (2 - S_i) \left[\tilde{C}'(Q_i^*) + \rho D_{sq}(S_i, Q_i^*) \right] \Rightarrow \left[\frac{\partial P_i^*}{\partial q_i} - \frac{\partial \tilde{P}_i^*}{\partial q_i} \right] \Big|_{q_i=Q_i^*} < 0. \quad (42)$$

Proof of Lemma 2 Employing (26) and (27), comparative statics with respect to q_i shows that $\partial P_j^{A*}/\partial q_i$ evaluated at $q_i = Q_i^R(\rho)$ and $\rho = \rho^A$ can be written as

$$\frac{\partial P_j^{A*}}{\partial q_i} \Big|_{q_i=Q_i^R(\rho), \rho=\rho^A} = \Phi \left[\frac{\partial P_i^*}{\partial q_i} - \frac{\partial \tilde{P}_i^*}{\partial q_i} \right] \Big|_{q_i=Q_i^R(\rho), \rho=\rho^A} \quad (43)$$

where Φ is given by (using $S_i := S(\tilde{P}_i^*, Q_i^R(\rho))$)

$$\Phi := -\frac{2(D_s(S_i, Q_i^R(\rho)) - \rho D_{ss}(S_i, Q_i^R(\rho)))X_i}{3(2 - S_i)^2 D_s(S_i, Q_i^R(\rho))^2 + 4(D_s(S_i, Q_i^R(\rho)) - \rho D_{ss}(S_i, Q_i^R(\rho)))X_i} < 0. \quad (44)$$

Furthermore, evaluating the derivative of Π_i^{A*} with respect to q_i and using the envelope theorem, we get

$$\left[\frac{\partial \Pi_i^{A*}}{\partial q_i} + \frac{\partial \Pi_i^{A*}}{\partial p_j} \frac{\partial P_j^{A*}}{\partial q_i} \right]_{q_i=Q_i^R(\rho), \rho=\rho^A} \geq 0 \Leftrightarrow \left[\frac{\partial P_j^{A*}}{\partial q_i} \right]_{q_i=Q_i^R(\rho), \rho=\rho^A} \geq 0 \quad (45)$$

$$\Leftrightarrow \left[\frac{\partial P_i^*}{\partial q_i} - \frac{\partial \tilde{P}_i^*}{\partial q_i} \right]_{q_i=Q_i^R(\rho), \rho=\rho^A} \leq 0. \quad (46)$$

Proof of Lemma 3 Employing (26) and (27), comparative statics with respect to q_j shows that $\partial P_i^{A*} / \partial q_j$ evaluated at $q_j = Q_j^{RA}(\rho)$ and $\rho = \rho^S$ can be written as

$$\frac{\partial P_i^{A*}}{\partial q_j} \Big|_{q_j=Q_j^{RA}(\rho), \rho=\rho^S} = \tilde{\Phi} \left[\frac{\partial P_j^*}{\partial q_j} - \frac{\partial \tilde{P}_j^*}{\partial q_j} \right]_{q_j=Q_j^{RA}(\rho), \rho=\rho^S} \quad (47)$$

where $\tilde{\Phi}$ is given by (again, all functions are evaluated at $q_j = Q_j^{RA}(\rho)$ and $\rho = \rho^S$ and we use $S_j := S(\tilde{P}_j^{A*}, Q_j^{RA}(\rho))$)

$$\tilde{\Phi} = -\frac{1}{\Omega} (2 - S_j) \Pi_i^{A*} X_j \frac{D_s(S_j, Q_j^{RA}(\rho)) - \rho D_{ss}(S_j, Q_j^{RA}(\rho))}{4D_s(S_j, Q_j^{RA}(\rho))X_i^2} \quad (48)$$

$$\text{with : } \Omega = \frac{\partial^2 \Pi_i^S}{\partial p_i \partial p_i} \frac{\partial^2 \Pi_j^S}{\partial p_j \partial p_j} - \frac{\partial^2 \Pi_i^S}{\partial p_i \partial p_j} \frac{\partial^2 \Pi_j^S}{\partial p_j \partial p_i} > 0. \quad (49)$$

Differentiating firm j 's reduced profit functions we obtain

$$\left[\frac{\partial \Pi_j^{S*}}{\partial q_j} + \frac{\partial \Pi_j^{S*}}{\partial p_i} \frac{\partial P_i^{S*}}{\partial q_j} \right]_{q_j=Q_j^{RA}(\rho), \rho=\rho^S} \geq 0 \Leftrightarrow \left[\frac{\partial P_j^{A*}}{\partial q_j} - \frac{\partial \tilde{P}_j^{A*}}{\partial q_j} \right]_{q_j=Q_j^{RA}(\rho), \rho=\rho^S} \leq 0. \quad (50)$$

Finally, using $D_{ss} = 0$ we again have

$$1 < (2 - S_j) \left[\tilde{C}'(Q_j^{RA}(\rho)) + \rho D_{sq}(S_j, Q_j^{RA}(\rho)) \right] \quad (51)$$

$$\Rightarrow \left[\frac{\partial P_j^{A*}}{\partial q_j} - \frac{\partial \tilde{P}_j^{A*}}{\partial q_j} \right]_{q_j=Q_j^{RA}(\rho), \rho=\rho^S} < 0$$

which together with (47)–(50) leads to the result.

Proof of Lemma 4 The result is based on a comparison between

$$\left[\frac{\partial \Pi_i^{A*}}{\partial q_i} + \frac{\partial \Pi_i^{A*}}{\partial p_j} \frac{\partial P_j^{A*}}{\partial q_i} \right]_{q_j=Q_j^{RA}(\rho), \rho=\rho^S} \quad \text{and} \quad \left[\frac{\partial \Pi_i^{S*}}{\partial q_i} + \frac{\partial \Pi_i^{S*}}{\partial p_j} \frac{\partial P_j^{S*}}{\partial q_i} \right]_{q_j=Q_j^{RA}(\rho), \rho=\rho^S}. \quad (52)$$

Using $q_j = Q_j^{RA}(\rho)$ and $\rho = \rho^S$, we obtain (again, in the following all functions are evaluated at $q_j = Q_j^{RA}(\rho), \rho = \rho^S$)

$$\frac{\partial \Pi_i^{A*}}{\partial q_i} = \frac{\partial \Pi_i^{S*}}{\partial q_j} \quad \text{as well as} \quad \frac{\partial \Pi_i^{S*}}{\partial p_j} = \frac{\partial \Pi_i^{A*}}{\partial p_j} + \frac{\partial \Pi_i^{A*}}{\partial \tilde{p}_j}. \quad (53)$$

Hence, we also have

$$\frac{\partial \Pi_i^{A*}}{\partial q_i} + \frac{\partial \Pi_i^{A*}}{\partial p_j} \frac{\partial P_j^{A*}}{\partial q_i} \geq \frac{\partial \Pi_i^{S*}}{\partial q_i} + \frac{\partial \Pi_i^{S*}}{\partial p_j} \frac{\partial P_j^{S*}}{\partial q_i} \Leftrightarrow \quad (54)$$

$$\frac{\partial \Pi_i^{A*}}{\partial p_j} \frac{\partial P_j^{A*}}{\partial q_i} \geq \left[\frac{\partial \Pi_i^{A*}}{\partial p_j} + \frac{\partial \Pi_i^{A*}}{\partial \tilde{p}_j} \right] \frac{\partial P_j^{S*}}{\partial q_i}. \quad (55)$$

Furthermore, solving

$$\frac{\partial P_j^{A*}}{\partial q_i} = \frac{\partial P_j^{RA}}{\partial q_i} + \frac{\partial P_j^{RA}}{\partial p_i} \frac{\partial P_i^{A*}}{\partial q_i} \quad (56)$$

and the corresponding equations for $\partial \tilde{P}_j^{A*} / \partial q_i$, $\partial P_i^{S*} / \partial q_i$ and $\partial P_j^{S*} / \partial q_i$ we obtain

$$\frac{\partial P_j^{A*}}{\partial q_i} = \frac{\frac{\partial P_i^{RA}}{\partial q_i} \frac{\partial P_j^{RA}}{\partial p_i} + \frac{\partial P_j^{RA}}{\partial q_i}}{1 - \frac{\partial P_j^{RA}}{\partial p_i} \frac{\partial P_i^{RA}}{\partial p_j}} \quad (57)$$

$$\frac{\partial P_j^{S*}}{\partial q_i} = \frac{\frac{\partial P_i^{RS}}{\partial q_i} \frac{\partial P_j^{RS}}{\partial p_i} + \frac{\partial P_j^{RS}}{\partial q_i}}{1 - \frac{\partial P_j^{RS}}{\partial p_i} \frac{\partial P_i^{RS}}{\partial p_j}} \quad (58)$$

where (57) follows from the fact that $\partial \tilde{P}_j^{RA} / \partial p_i = \partial \tilde{P}_j^{RA} / \partial p_j = \partial \tilde{P}_j^{RA} / \partial q_i = 0$.

Furthermore, simple comparative statics reveals $\partial P_i^{RA} / \partial q_i = \partial P_i^{RS} / \partial q_i$ as well as

$$\frac{\partial P_j^{RA}}{\partial q_i} = \mu_i \frac{\partial P_i^{RA}}{\partial p_i} \quad \text{and} \quad \frac{\partial P_j^{RS}}{\partial q_i} = \mu_i \frac{\partial P_j^{RS}}{\partial p_i} \quad (59)$$

$$\text{with : } \mu_i = \frac{1 - \int_0^{S_i} D_q(s, q_i) ds}{2 - S_i} \quad (60)$$

and

$$\frac{\partial P_i^{RS}}{\partial p_j} = v_i \frac{\partial P_i^{RA}}{\partial p_i}; \quad \frac{\partial \Pi_i^{A*}}{\partial p_j} + \frac{\partial \Pi_i^{A*}}{\partial \tilde{p}_j} = v_i \frac{\partial \Pi_i^{A*}}{\partial p_j} \quad (61)$$

$$\text{with : } v_i = 2 - S_j. \quad (62)$$

Finally, using $D_{ss} = 0$, we also get

$$\frac{\partial P_j^{RS}}{\partial p_i} = \frac{(2 - S_j)D_s(S_j, Q_j^{RA}(\rho))^2}{(2 - S_j)^2 D_s(S_j, Q_j^{RA}(\rho))^2 + [D_s(S_j, Q_j^{RA}(\rho)) - \rho D_{ss}(S_j, Q_j^{RA}(\rho))] X_j} \frac{\partial P_j^{RA}}{\partial p_i} \quad (63)$$

Substituting (57)–(63) into (55) and simplifying shows that $D_{ss} = 0$ implies

$$\frac{\partial P_i^{RA}}{\partial q_i} > \frac{1}{2 - S_i} \left(1 - \int_0^{S_i} D_q(s, q_i) ds \right) \Rightarrow \left[\frac{\partial \Pi_i^{A*}}{\partial p_j} \frac{\partial P_j^{A*}}{\partial q_i} \right]_{q_j=Q_j^{RA}(\rho), \rho=\rho^S} > \left[\frac{\partial \Pi_i^{A*}}{\partial p_j} + \frac{\partial \Pi_i^{A*}}{\partial \tilde{p}_j} \right] \frac{\partial P_j^{S*}}{\partial q_i} \Big|_{q_j=Q_j^{RA}(\rho), \rho=\rho^S}.$$

Analysis of the Example Starting with rather low values of ρ and considering the unconstrained equilibria, the firms' price reaction functions $P_i^R(p_j, \tilde{p}_j, q_i, q_j, \rho)$ and $\tilde{P}_i^R(q_i, \rho)$ can be written as (using $\tilde{c} = 1/8$ and $c = 1/4$)

$$P_i^{RA}(p_i, q_i, q_j, \rho) = \frac{1}{4q_j} \left(\begin{array}{l} \tilde{p}_j(2q_j - \tilde{p}_j) + \\ q_j(2 + 2p_j + q_i(2 + \rho^2 + q_i(2c - \tilde{c}(2 - \tilde{c}q_i))) \\ - 4\rho(1 - \tilde{c}q_i)) - 2q_j \end{array} \right) \quad (64)$$

$$\tilde{P}_i^R(q_i, \rho) = \tilde{c}q_i^2 + \rho q_i \quad (65)$$

Solving (64) and (65), the equilibrium prices $P_i^*(q_i, q_j, \rho)$ are given by

$$P_i^*(q_i, q_j, \rho) = \frac{1}{6} \left(\begin{array}{l} 6 + (2 + \rho(\rho - 6))q_i + (4c - 2\tilde{c} + 6\rho\tilde{c})q_i^2 + \tilde{c}^2 q_i^3 \\ - q_j(2 + \rho^2 - 2(c + \tilde{c})q_j + \tilde{c}^2 q_j^2) \end{array} \right) \quad (66)$$

Substituting (65) and (66) into the firms' profit functions and solving the first order conditions for the firms' optimal quality, we get the equilibrium qualities given in (32).

Turning to the asymmetric case and assuming that $p = \tilde{p}$ binds only for firm 1, firm 2's price reaction functions are again given by (64) and (65). Firm 1's equilibrium price $P_1^{A*}(q_1, q_2)$ can be calculated by solving

$$\begin{aligned} 0 &= 2(p_1 - 2q_1)(p_1^2 + q_1^3(c + (1 + \rho)\tilde{c}) - p_1 q_1(2 + \rho + \tilde{c}q_1))q_2 \quad (67) \\ &+ \frac{1}{2}(2p_1 - q_1(2 + \rho + \tilde{c}q_1))q_2 \cdot \\ &(p_1^2 - 4p_1 q_1 + q_1(6 + 2q_1 - q_2(2 + \rho^2 - 2(c + \tilde{c})q_2 + \tilde{c}^2 q_2^2))) \end{aligned}$$

for p_1 and using firm 1's second order condition in order to ensure that the solution is in fact a maximizer. Differentiating (67) with respect to q_2 and using the implicit function theorem, we can also calculate $\partial P_1^{S^*}(q_1, q_2) / \partial q_2$. The equilibrium qualities $Q_1^{A^*}(\rho)$ and $Q_2^{A^*}(\rho)$ are then obtained by solving the firms' first order conditions for their optimal qualities numerically.

Finally, considering the symmetric case in which $p = \tilde{p}$ is binding for both firms, the firms' price reaction functions $P_i^{RS}(p_j, q_i, q_j)$ satisfy

$$\begin{aligned} 0 = & 2(p_i - 2q_i)(8p_i^2 + (3 + \rho)q_i^3 - p_i q_i(16 + 8\rho + q_i))q_j \\ & + (16p_i - q_i(16 + 8\rho + q_i)) \cdot \\ & (p_i^2 q_j + q_i(4p_j q_j - p_j^3 + 2q_j(1 - 2p_i + q_i - q_j))) \end{aligned} \quad (68)$$

Solving this system of equation numerically and checking the firms' second order conditions, we get the equilibrium prices $P_i^{S^*}(q_i, q_j)$. Furthermore, differentiating (68) with respect to q_j and using the implicit function theorem, we can calculate $\partial P_i^{S^*}(q_i, q_j) / \partial q_j$ which allows us to solve the firms' first order conditions for their optimal qualities, i.e.,

$$\begin{aligned} \frac{\partial \Pi_i^{S^*}}{\partial q_i} = & \frac{1}{32q_i^3 q_j} (16P_i^{S^*4} q_j - 8(6 + \rho)P_i^{S^*3} q_i q_j - \\ & P_i^{S^*2} q_i(8P_j^{S^*2} - 32P_j^{S^*} q_j + ((7 + \rho)q_i^2 - 16(1 - q_j))q_j) + \\ & P_i^{S^*} q_i^3 (4P_j^{S^*} q_j - P_j^{S^*2} + 2(17 + 14q_i + 4\rho(2 + q_i) - q_j)q_j) + \\ & 2(3 + \rho P)q_i^4 (P_j^{S^*2} - 4P_j^{S^*} q_j - q_j(2 + 3q_i - 2q_j)) \\ & + \frac{(8P_i^{S^*2} + (3 + \rho)q_i^3 - P_i^{S^*} q_i(16 + 8\rho + q_i))(P_j^{S^*} - 2q_j)}{16q_i q_j} \frac{\partial P_i^{S^*}}{\partial q_j} \end{aligned} \quad (69)$$

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