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Governmental platform intermediation to promote alternative fuel vehicles

by
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Abstract

Many governments promote green technological innovation within the automobile sector as a means of combating climate change. Most of these innovations are driven by alternative fuels. Buyer’s premiums and governmental investment in service infrastructure are widely used. This paper investigates the question regarding whether market intervention is adequate by considering the two-sided market character of the automobile market. This study shows that network effects, competition effects triggered by more automobile users and decreasing marginal utilities of further service stations determine the welfare-efficient extent of governmental intervention. The results of the analysis indicate that governmental promotion of service infrastructure is reasonable, although governments should be cautious about buyer’s premiums.

Keywords: network effects, two-sided markets, platform intermediation, alternative fuel vehicles, climate change, regulation

JEL: L150, L920, L980, O330, Q550, Q580

1 Introduction

To reduce greenhouse gas emissions in the transport sector, many governments promote alternatives to the currently dominant powertrain technology, namely, the gasoline-driven internal combustion engine. For example, the British and French
governments are paying buyer’s premiums for low-carbon vehicles, and both governments invest in new service infrastructure for electric vehicles. The German government still holds back with buyer’s premiums for alternative fuel vehicles, but it has decided to invest in charging for the infrastructure for electric vehicles at motorway service areas. Against this background, this work addresses the question regarding whether market intervention is adequate to promote alternative powertrain technologies. This study argues that the automobile market is a two-sided market.

This work is driven by earlier analysis of markets with network effects and two-sided markets (see Economides (1996), Shy (2011) and Rysman (2009)). Network effects arise if the utility of consuming a good increases with the number of others consuming the same good (Katz and Shapiro, 1985). This definition implies direct network effects as observed in telecommunication markets, for example. In contrast, indirect network effects occur in the case of complementarity. Originally, researchers analyzed this phenomenon for the consumer electronics market, in which buyers prefer hardware with a large variety of complementary software and in which more software is developed for hardware with a large number of users (see Gandal et al. (2000), Park (2004) or Lee (2013)). Markets for complementary products with indirect network effects can face several problems. If innovations are not compatible with the established installed base of compatible components, they face large entry barriers. The consequences may be path dependency and technological lock-in. Even excess inertia can emerge, in which a Pareto-superior technology cannot enter the market because it is not compatible with the installed base (Farrell and Saloner, 1986). Furthermore, because of indirect network effects, the market might be a multi-sided market. In this case, platform intermediation is necessary to enable different market sides to interact with one another. Recent literature (Armstrong (2004), Armstrong (2006) or Rochet and Tirole (2006), for example) states that a platform intermediary usually has market power and is able to determine strategic price setting, thereby internalizing network effects in the market and facilitating interaction.

In the network literature, it is widely recognized that the automobile market
owns network effects (see Katz and Shapiro (1985), Arthur (1989), Foray (1997) and Church et al. (2008), for example). In particular, indirect network effects arise because vehicles need fuel to be driven, and therefore, a compatible service infrastructure is needed for area-wide use. If innovations are not compatible with the installed service network, they face large entry barriers that could lead to the above-mentioned problems. Furthermore, the automobile market is two-sided. Platform intermediation occurs because automobile users and fuel retailers interact with one another via a common powertrain fuel standard. In other words, automobile users can interact with all retailers that offer the appropriate fuel for their vehicles. Therefore, allowing for interaction, the powertrain fuel standard fulfills the characteristics of a platform intermediary.

Especially in the infant market\textsuperscript{1} for alternative fuel vehicles, such as electric or hydrogen-driven cars, there are network externalities that could be internalized by a platform. Because these alternative technologies cannot be served by the installed service infrastructure, each new alternative fuel retailer generates benefits for automobile users and increases the numbers of users choosing alternative fuel vehicles. Network effects need to be considered. As car users and fuel retailers interact via a new powertrain fuel standard, two-sided platform intermediation is also relevant.

Based on the two-sided market approach, this work examines platform-driven standard setting in the automobile market. In the first scenario, standard setting by a private monopolistic platform agent is discussed. Unlike the classic two-sided market models, such as Armstrong (2006), the marginal network utility for car buyers is decreasing with the number of fuel retailers. Additionally, competition effects in the market for fuel are considered. A second scenario addresses the case of governmental standard setting. With the recognition that private intermediation is rarely observed, the possibilities for a governmental platform agent to promote new standards in the car market are derived. This is the second innovation of the

\textsuperscript{1}For the automobile market in general, one could argue that it has already reached its maturity. Thus, the service infrastructure has reached a certain density, and no further externalities arise when another service station is established. Strategic pricing is no longer possible. Platform intermediation without strategic pricing is called an open platform by Hagiu (2006).
paper.

From a policy perspective, this analysis contributes to the research on alternative technologies in the car market. Apart from reducing manmade greenhouse gas emissions, alternative vehicles should also reduce environmental and harmful externalities as well as the dependence on fuel imports. For this reason, status quo regulations in the transport sector often address these topics (see Santos et al. (2010a) and Santos et al. (2010b)). The market entry of alternative fuel vehicles is analyzed from various perspectives. Conrad (2006, 2009) or Greaker and Heggedal (2010), for example, analyze the market entry of vehicles driven by alternative fuels, such as natural gas or hydrogen, under competition. Schwoom (2007) or Melaine (2003) estimate the minimum number and distribution of hydrogen filling stations. Other researchers, such as Achtnicht et al. (2012), study the determinants of the potential demand for alternative cars. All analyses recognize the importance of positive feedback, namely, network effects caused by a compatible service infrastructure, for the successful market entry of alternative fuel vehicles. Hence, it is known that new technologies have a competitive disadvantage over conventional technologies that rely on a dense service infrastructure, such as gasoline-driven cars. However, while Conrad (2006, 2009) recommends governmental intervention, Greaker and Heggedal (2010) posit that regulation should focus on environmental externalities only. Addressing regulatory questions, Sartzetakis and Tsigaris (2005) and Dietrich and Sieg (2014) use a dynamic approach to show that there is excess inertia in the automobile market that could be an argument for governmental intervention. Sartzetakis and Tsigaris (2005) suggest a combination of taxing dirty technologies and subsidizing green technologies. Dietrich and Sieg (2014) find that even the promotion of green but ultimately dead-end technologies could enhance welfare. Nevertheless, the question of efficient governmental intervention has not yet been completely answered. By using the two-sided market approach, this work provides further insight into this topic.
2 Simple modeling

2.1 Group setting

There are two groups of agents in the market. Both gain benefits from interacting with one another. Hence, the more members of group two are in the market, the greater the benefits for group one, and vice versa. This statement implies positive network effects. This group setting refers to the analysis of Armstrong (2006). Applied to the automobile sector, group one consists of automobile users, and group two includes fuel retailers. Positive network effects in the automobile market are assumed because the more people use automobiles, the more dense the service infrastructure is, and vice versa. For automobile users, the decision of whether to buy a vehicle depends on the number of compatible service stations. Therefore, users prefer variety and value a dense service station network. The costs per interaction are lower because searching and traveling costs are reduced. The benefit for a representative individual using a car is defined by the following utility function:

\[ u = \alpha_1 \sqrt{\phi_2(\pi)} + (\beta_1 - c_1). \]  

The utility depends on the number of service stations, \( \phi_2(\pi) \geq 0 \), and the user’s preference for a dense network, \( \alpha_1 > 0 \), determined by the costs to travel to the next service station. The number of service stations depends on the fuel retailer’s profit, \( \pi \), and its size is equal for all retailers. The more fuel stations are in the market, the more users decide to buy the technology, and therefore, the more retailers enter the market. Unlike in the approach of Armstrong (2006), diminishing marginal network benefits are assumed using the root function. This assumption considers that the positive effect of additional installed service stations, such as reduced searching costs, decreases with each new station. This standard assumption in economics is the diminishing marginal utility of consumption. The parameter \( \beta_1 \) describes the network-independent benefit of using the technology determined by the preference for mobility and technology, while \( c_1 \) are the costs of using the technology.
The case for retailers is different. Their decision to install additional service stations depends on their profits. The profit for a representative retailer equals

$$\pi = (\alpha_2 - C)\phi_1(u) - c_2. \quad (2)$$

On the one hand, it is determined by the demand for fuel and, therefore, is positively related to the number of automobiles $\phi_1(u)$. To simplify, each user drives only one car. The more matching partners are in the market, the higher the retailer’s revenues are. The number of cars itself is determined by the user’s utility $u$, as defined in equation 1. The parameter $\alpha_2 > 0$ describes the profit of the retailer from interacting with automobile users, namely, refueling. To simplify, it is assumed that car users demand the same amount of fuel for each service station stop. It is also assumed that the frequency of interaction is equally distributed. Thus, each car user refuels his/her vehicle at each service station in equal measure. For this reason, linearity could be assumed. On the other hand, the revenue depends on the competition in the fuel market. As the number of service station owners increases with the number of automobile users, the individual revenue $\pi$ decreases. This effect refers to the work of Shriver (2015), who analyses indirect network effects in the US ethanol market. He also assumes effects of competition between ethanol retailers and therefore models a competitive market entry game. Following this argument here, the profit is downsized by $C$, the parameter representing the competition effect. The revenues are finally adjusted downward by the costs of providing the service station, $c_2$. The setting implies homogeneous preferences for all group members. Based on the setting for both groups, further calculations are made.

### 2.2 Monopolistic platform setting

To enable interaction with car users, retailers need to offer services that are compatible with the automobiles in use. Therefore, both must agree to a certain standard–here, the powertrain technology that operates using a certain fuel. User and retailers interact with one another because the capacity of vehicle tanks is limited, and they must be refilled after a certain travel distance. As a consequence,
automobile users buy and retailers sell fuel. Because the powertrain fuel standard allows interaction between buyers and sellers, it resembles a platform in the sense of two-sided markets. This is illustrated by figure 1. Here, the platform agent acts as a sponsor for a certain standard. Following the simple approach of Armstrong (2006), the platform agent is a monopolist. It follows that the platform agent has market power that allows for internalizing the positive network externalities in the market by setting an appropriate price structure. Thereby, the agent maximizes its profit. Usually, a membership fee $P_i$ is paid by each platform member to be able to interact with the other group. This implies that at least one market side (here, the car users or retailers) is willing to pay to interact with the other group via the platform. In our case, one could imagine paying the membership fee when buying a vehicle or when installing a service station. If platform could set these fees, the revenue functions of both groups could be defined as

$$u = \alpha_1 \sqrt{\phi_2(\pi)} + \beta_1 - c_1 - P_1$$

and

$$\pi = (\alpha_2 - C)\phi_1(u) - c_2 - P_2.$$  

We now discuss the platform’s maximization problem. If the standard is sponsored by a private agent, it simply maximizes its profit $\Pi$, which is the sum of membership fees multiplied by the number of cars or retailers. This is formulated
as
\[ \max_{a,\pi} \Pi = P_1 \cdot \phi_1(u) + P_2 \cdot \phi_2(\pi). \] (5)

Considering the platform’s maximizing problem and solving equations 3 and 4 for \( P_1 \) and \( P_2 \), one can determine the efficient price setting. The private platform agent maximizes its profit as shown in equation 5. The results are calculated by \( \frac{\partial \Pi}{\partial u} = 0 \) and \( \frac{\partial \Pi}{\partial \pi} = 0 \). Solving for \( P_i \) gives the profit-maximizing membership fees.

**Lemma 1** For \( P_1 = \frac{\phi_1(u)}{\phi_1(u)} - (\alpha_2 - C) \cdot \phi_2(\pi) \) and \( P_2 = \frac{\phi_2(\pi)}{\phi_2(\pi)} - \frac{\alpha_1 \cdot \phi_1(u)}{2 \sqrt{\phi_2(\pi)}} \), profit \( \Pi \) is maximized.

Proof: See the appendix.

The first term of both price equations in lemma 1 is related to the sensitivity of participating in the platform; it is interpreted as monopolistic rent based on Rochet and Tirole (2006). Those authors show that according to Lerner’s index, it depends on the price elasticity. Because of its market power, the monopolistic platform agent is able to set this markup. The second term of \( P_1 \) is determined by the network externalities that car users exert on retailers and the effect of competition among service station owners. As a consequence, for \( \alpha_2 > C > 0 \), the network effect is larger than the competition effect, and car buyers pay membership fees that are lower than the monopolistic rent. For \( 0 < \alpha_2 < C \), the membership fee is larger than the rent. The second term of \( P_2 \) considers the network effect that retailers exert on automobile users. Because of diminishing marginal utilities of further service stations, the term is multiplied by \( \frac{1}{2 \sqrt{\phi_2(\pi)}} \). Assuming positive network effects, sellers pay membership fees that are lower than the monopolistic rent. However, the relationship is non-linear because of diminishing marginal benefits from new service stations. In sum, a monopolistic platform agent sets a price structure that considers the network effects in the market. Because of its market power, it can set a markup that depends on the platform participation sensitivity of both groups. Additional effects, such as competition in the fuel market and the diminishing marginal gains from further service stations for automobile users, determine the final price structure.
In reality, there is no platform intermediation in the market for alternative fuel vehicles. There might be at least one reasonable explanation for the absence of intermediation. Platform agents promoting a new powertrain fuel standard would compete with the old conventional powertrain fuel standard. Hence, there would be a standard or, rather, platform competition in the market. As also shown by Armstrong (2006), platform competition limits platform agents’ opportunities for rent seeking. If rent seeking is no longer possible, the membership fees of both groups, $P_1$ and $P_2$ in lemma 1, might be negative, and the revenue of the platform agent would become negative. Because we are assuming $\alpha_i > 0$, this would be true for $\alpha_2 > C$. In other words, subsidies are necessary on both sides of the market to attract buyers and sellers to enable interaction between automobile users and fuel retailers. As a consequence, competition between the old powertrain fuel standard with an installed base of complementary infrastructure and a new standard without such an installed base can result in excess inertia. Some papers (Sartzetakis and Tsigaris, 2005; Dietrich and Sieg, 2014) argue that excess inertia can arise when mitigating green technologies in the automobile market and that governmental intervention could be necessary when welfare economics are considered. For this reason, the next section explores governmental platform setting in the market for alternative fuel vehicles.

2.3 Governmental platform setting

It is also possible that the government provides a new powertrain fuel standard. Especially with regard to alternative powertrain technologies, it might be plausible for a governmental agent to promote or even provide a certain standard aiming for (faster) market development. The governmental sponsor could thus account for buyer and seller surpluses, $v_1(u)$ and $v_2(\pi)$. Hence, the governmental agent could maximize welfare. Instead of prices, the platform agent pays subsidies to enable interaction, as shown in figure 2. The membership fees $P_i$ in lemma 1 are replaced with the membership premiums $T_i$, which are received when consumers decide to buy a vehicle or when an entrepreneur invests in service stations. The
maximization problem of the governmental platform agent is formulated as follows:

$$\max_{u, \pi} W_T = T_1 \cdot \phi_1(u) + T_2 \cdot \phi_2(\pi) + v_1(u) + v_2(\pi).$$  \hspace{1cm} (6)$$

In the case of the infant market for alternative powertrains, the government could sponsor a standard for battery-driven or hydrogen-driven vehicles, for example. Assuming a welfare-maximizing governmental platform agent facilitates the efficient implementation of alternative standards.

By assumption, the platform provides the standard without any additional costs. Let the aggregate surplus of automobile users be $v_1(u)$, which satisfies the envelope condition $v'_1(u) \equiv \phi_1(u)$, and let $v_2(\pi)$ be the aggregate surplus of fuel retailers that satisfies the envelop condition $v'_2(\pi) \equiv \phi_2(\pi)$. For the relevant calculations, see the appendix. The governmental platform maximizes welfare by solving $\frac{\partial W_T}{\partial u} = 0$ and $\frac{\partial W_T}{\partial \pi} = 0$ for $T_i$.

**Lemma 2** For $T_1 = (\alpha_2 - C) \cdot \phi_2(\pi)$ and $T_2 = \frac{\alpha_1 \cdot \phi_1(u)}{2\sqrt{\phi_2(\pi)}}$, welfare $W_T$ is maximized.

**Proof:** See the appendix.

Lemma 2 shows that efficient standard sponsoring depends on the network effects that one market side provides to the other. For automobile users, the competition effect of additional vehicles must also be considered. As discussed in section 2.1, it is argued that the market entry of additional vehicles strengthens the competition between fuel retailers simply because a larger market for alternative fuel attracts more retailers. In contrast to the positive network effect, stronger
competition results in lower profits for the individual retailer. For this reason, the competition effect must also be considered. For retailers, diminishing marginal benefits must be considered. Hence, in the case of a dense service infrastructure, an additional retailer does not significantly influence the utility of a single car user, and this is true for an additional retailer in a small service network. Compared with the price setting of the monopolistic platform agent, the governmental platform agent does not seek monopolistic rents. This is noted in proposition 1.

**Proposition 1** \(|T_i| = |P_i - \frac{\phi_i}{\phi_i'}(\cdot)|\) holds, and the governmental platform agent is not rent seeking.

Proof: See the appendix.

We now more closely examine the premiums received by automobile users for purchasing a car. As already discussed in section 2.2, there are two opposing effects determining its amount. If the positive network effect is larger than the competition effect, then governmental platform intermediation means charging automobile users for buying an alternative fuel car. In contrast, if the negative competition effect is larger than the network effect, then buyers must be taxed. The two effects may be equal. In this case, a governmental platform intermediary does not take any action. Proposition 2 indicates this non-distinct relation.

**Proposition 2** For \(\alpha_2 = C\), \(T_1 = 0\) holds, and no intervention for buying a car is welfare efficient. For \(\alpha_2 > C\), \(T_1 < 0\), and buyers are subsidized. For \(\alpha_2 < C\), \(T_1 > 0\), and buyers are taxed instead.

For fuel retailers, the case is distinct. Given the above-mentioned assumptions, charging retailers for installing a new service station that offers alternative fuel is always reasonable, as shown by proposition 3.

**Proposition 3** \(T_2 < 0\) holds, and subsidizing service station owners for installing a new service station network is welfare efficient.

Intuitively, the results make sense. As mentioned in section 1, the literature on network effects was motivated by earlier works on complementary goods. From this perspective, vehicles and fuel infrastructure must be interpreted as complements.
Without fuel, a vehicle cannot run. Studies addressing the potential demand for alternative fuel vehicles (Achtnicht et al., 2012) strengthen the importance of complementary infrastructure for the purchase decision. The work of Shriver (2015) examining the US ethanol market estimates that 5.6% of car purchases can be explained by network effects. Therefore, investment in a service infrastructure seems to be an adequate way to support the purchase of these vehicles indirectly. In fact, 5.3% of the market entry decision can be explained by network effects. This finding indicates that network effects are also significant for fuel retailers. In Germany, for example, the conventional fuel market is highly competitive (Prof. Dr. Schneck Rating (2006) and statista (2015)). The situation may be similar for alternative fuel, and this would emphasize the consideration of competition effects. Additionally, the demand for buyer’s premiums for alternative fuel vehicles is driven by their high purchase prices (Achtnicht et al., 2012). This is a signal that these new technologies are not yet competitive with conventional cars. However, from a welfare economic perspective, this is not a reason for governmental intervention. Therefore, further investigation should be performed before implementing a buyer’s premium to address network effects.

3 Conclusion

Motivated by the public discourse in Germany and other countries regarding governmental promotion of alternative powertrains for automobiles, this work emphasizes the issue of governmental platform behavior. If a government decides to intervene in the car market as a promoter of a new powertrain standard that needs new service infrastructure, the market must be interpreted as two-sided. Therefore, the governmental agent must act as a platform intermediary. In contrast to a private platform intermediary, a governmental agent could pursue societal interests because it would not seek monopolistic rents. A governmental platform sponsor could set membership fees efficiently according to network and competition effects. If marginal revenues were diminishing, that issue must also be considered. It follows that public installation of new service infrastructure is reasonable to in-
directly promote alternative powertrain technologies. Furthermore, governmental agents should be cautious with regard to paying buyer’s premiums for alternatively driven vehicles because new vehicles not only induce positive network effects but also strengthen competition in the market for services. This effect relativizes the argument of subsidizing the purchase of alternative vehicles in the early stage of market development. In considering the implementation of membership premiums, a governmental platform has the opportunity to charge or disburse agents. Buyers can be charged when registering their vehicle, if necessary. Otherwise, car purchase taxes could be adopted. Retailers could apply for public grants when they plan to invest in new service infrastructure, or the government can invest in public service stations.

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Appendix

Proof of lemma 1

To prove lemma 1, we must calculate the first derivatives of equation 5. Using equations 3 and 4 gives

$$\frac{\partial \Pi}{\partial u} = (\alpha_1 \sqrt{\phi_2(\pi)} + \beta_1 - c_1 - u + (\alpha_2 - C) \phi_2(\pi)) \phi'_1(u) - \phi_1(u)$$  \hspace{1cm} (7)

$$\frac{\partial \Pi}{\partial \pi} = \left( (\alpha_2 - C) \cdot \phi_1(u) - C - \pi \right) \cdot \phi'_2(\pi) + \frac{\alpha_1 \phi_1(u) \phi'_2(\pi)}{2 \sqrt{\phi_2(\pi)}} - \phi_2(\pi).$$  \hspace{1cm} (8)

We can then prove lemma 1. For

$$P_1 = \phi_1(u) \cdot \left( \frac{\alpha_1 \phi_1(u) \phi'_2(\pi)}{2 \sqrt{\phi_2(\pi)}} \right) \cdot (\alpha_2 - C) \cdot \phi_2(\pi),$$

$$P_1 + (\alpha_2 - C) \phi_2(\pi)) \phi'_1(u) - \phi_1(u) = 0$$  \hspace{1cm} (9)

$$\frac{\partial \Pi}{\partial u} = 0,$$  \hspace{1cm} (10)

and for

$$P_2 = \frac{\phi_2(\pi)}{\phi'_2(\pi)} - \frac{\alpha_1 \phi_1(u)}{2 \sqrt{\phi_2(\pi)}},$$

$$P_2 \cdot \phi'_2(\pi) + \frac{\alpha_1 \phi_1(u) \phi'_2(\pi)}{2 \sqrt{\phi_2(\pi)}} - \phi_2(\pi) = 0$$  \hspace{1cm} (11)

$$\frac{\partial \Pi}{\partial \pi} = 0,$$  \hspace{1cm} (12)

and the profit $\Pi$ is maximized. $\square$

Envelop conditions

Using equation 3, we define the consumer surplus per automobile user as

$$CS = \int_0^{\phi_2^*} u(\phi_2) \, d\phi_2 = \phi_2^* (\beta_1 - c_1) + \int_0^{\phi_2^*} \alpha_2 \phi_2 \, d\phi_2.$$  \hspace{1cm} (13)

The aggregate consumer surplus sums to

$$v_1(u) = \sum_0^{\phi_1} CS = \phi_1 \cdot CS = \phi_1(u) \cdot \int_0^{\phi_2^*} u(\phi_2) \, d\phi_2$$

$$= \phi_1(u) \left( \phi_2^* (\beta_1 - c_1 - P_1) + \int_0^{\phi_2^*} \alpha_2 \phi_2 \, d\phi_2 \right).$$  \hspace{1cm} (14)
The first derivative with respect to $\phi_2$ gives the optimal value function
\[
\frac{\partial v_1}{\partial \phi_2} = \phi_1(u)(\beta_1 - c_1 - P_1 + \alpha_1 \phi_2) = \phi_1(u) \cdot u(\phi_2). \tag{16}
\]
Solving the optimal value function for the first derivative with respect to $u$ gives
\[
\frac{\partial}{\partial u} = \phi_1'(u)u(\phi_2) + \phi_1(u) \cdot 1. \tag{17}
\]
Using the envelop theorem, we obtain the envelop condition
\[
v_1'(u) \equiv \phi_1(u). \tag{18}
\]
The derivation of $v_2'(\pi) \equiv \phi_2$ is equivalent. □

Proof of lemma 2

To prove lemma 2, we must calculate the first derivatives of equation 6. Using the modified equations 3 and 4 ($P_i = T_i$) and the envelop conditions gives
\[
\frac{\partial W}{\partial u} = \left(\frac{\alpha_1}{2\sqrt{\phi_2(\pi)}} + \beta_1 - c_1 - u + (\alpha_2 - C)\phi_2(\pi)\right)\phi_1'(u) - \phi_1(u) + v_1'(u) = 0 \tag{19}
\]
\[
\frac{\partial W}{\partial \pi} = \left(\frac{(\alpha_2 - C) \cdot \phi_1(u) - C - \pi}{\phi_2(\pi)}\right) \cdot \phi_2'(\pi) + \frac{\alpha_1 \phi_1(u)\phi_2'(\pi)}{2\sqrt{\phi_2(\pi)}} - \phi_2(\pi) + v_2'(\pi) = 0. \tag{20}
\]
We can then prove lemma 2. For $T_1 = -(\alpha_2 - C) \cdot \phi_2(\pi)$,
\[
(T_1 + (\alpha_2 - C)\phi_2(\pi))\phi_1'(u) = 0 \tag{21}
\]
\[
\frac{\partial W}{\partial u} = 0, \tag{22}
\]
and for $T_2 = -\frac{\alpha_1 \phi_1(u)}{2\sqrt{\phi_2(\pi)}}$,
\[
T_2 \cdot \phi_2'(\pi) + \frac{\alpha_1 \phi_1(u)\phi_2'(\pi)}{2\sqrt{\phi_2(\pi)}} = 0 \tag{23}
\]
\[
\frac{\partial W}{\partial \pi} = 0, \tag{24}
\]
and welfare $W$ is maximized. □
Proof of proposition 1

Because

\[ |\frac{\phi_1(u)}{\phi'_1(u)}| \geq 0, \]  
(25)

\[ |-(\alpha_2 - C) \cdot \phi_2(\pi)| = |-(\alpha_2 - C) \cdot \phi_2(\pi) + \frac{\phi_1(u)}{\phi'_1(u)} - \frac{\phi_1(u)}{\phi'_1(u)}| \]  
(26)

\[ |T_1| = |P_1 - \frac{\phi_1(u)}{\phi'_1(u)}| \]  
(27)

and because

\[ |\frac{\phi_2(\pi)}{\phi'_2(\pi)}| \geq 0, \]  
(28)

\[ |-\frac{\alpha_1 \cdot \phi_1(u)}{2\sqrt{\phi_2(\pi)}}| = |-\frac{\alpha_1 \cdot \phi_1(u)}{2\sqrt{\phi_2(\pi)}} + \frac{\phi_2(\pi)}{\phi'_2(\pi)} - \frac{\phi_2(\pi)}{\phi'_2(\pi)}| \]  
(29)

\[ |T_2| = |P_2 - \frac{\phi_2(\pi)}{\phi'_2(\pi)}|, \]  
(30)

\[ |T_1| = |P_i - \frac{\phi_i(u)}{\phi'_i(u)}| \text{ holds. } \Box \]

Proof of proposition 2

Let (as in lemma 2) \( T_1 = -(\alpha_2 - C) \cdot \phi_2(\pi) \).

Then, for \( \alpha_2 = C \geq 0 \),

\[ \alpha_2 - C = 0 \]  
(31)

\[ -(\alpha_2 - C) \cdot \phi_2(\pi) = 0 \]  
(32)

\[ T_1 = 0 \]  
(33)

The cases of \( \alpha_2 < C \) and \( \alpha_2 > C \) are calculated to be equivalent. \( \Box \)
Proof of proposition 3

Let (as in lemma 2) \( T_2 = -\frac{\alpha_1 \cdot \phi_1(u)}{2\sqrt{\phi_2(\pi)}} \)

Then, for \( \alpha_1 \geq 0 \),

\[
-\frac{\alpha_1 \cdot \phi_1(u)}{2\sqrt{\phi_2(\pi)}} \leq 0 \tag{34}
\]

\[
T_2 \leq 0 \tag{35}
\]

\( \square \)
TU Braunschweig, Economics Department, Working Paper Series

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