

Vasilev, Aleksandar

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# Aggregation with a mix of indivisible and continuous labor supply decisions: the case of home production

Aleksandar Vasilev\*

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## **Abstract**

This note explores the problem of non-convex labor supply decision in an economy with both discrete and continuous labor decisions. In contrast to the setup in McGrattan, Rogerson and Wright (1997), here each household faces an indivisible labor supply choice in the market sector, while it can choose to work any number of hours in the non-market sector. We show how lotteries as in Rogerson (1988) can again be used to convexify consumption sets, and aggregation over individual preferences. With a mix of discrete and continuous labor supply decisions, disutility of non-market work becomes separable from market work, and the elasticity of the latter increases from unity to infinity.

*JEL classification:* E1, J22, J46

*Keywords:* Indivisible labor; home production; non-convexities; lotteries; discrete-continuous mix; aggregation

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\*Asst. Professor and CERGE-EI Affiliate Fellow, Department of Economics, American University in Bulgaria, 1 Georgi Izmirliev Sq., Blagoevgrad 2700, Bulgaria. Tel: 00 359 73 888 482. All remaining errors are mine. E-mail for correspondence: [avasilev@aubg.edu](mailto:avasilev@aubg.edu).

# 1 Introduction and Motivation

This note explores the problem of non-convex labor supply decision in an economy with both discrete and continuous labor decisions. In contrast to the setup presented in McGrattan, Rogerson and Wright (1997), here each household will be assumed to face an indivisible labor supply choice in the market sector, while it can choose to work any number of hours in the non-market sector. In addition to the final good production technology, there is going to be a home production technology as well. The latter also provides consumption flows. Total consumption is an aggregate of market output and services from home production. We show how lotteries as in Rogerson (1988) can again be used to convexify households' consumption sets, and aggregation over individual preferences. With a mix of discrete and continuous labor supply decisions, disutility of non-market work becomes separable from market work, but keeps its original (logarithmic) form, while the elasticity of the market hours supply increases from unity to infinity.

## 2 Model Description

### 2.1 Households' problem

The theoretical setup presented in this note is a one-period economy without physical capital. There is a unit measure of identical households, whose preferences are defined over composite consumption ( $c$ ) and leisure ( $l$ ), and utility function  $u(c, l)$  as follows:

$$u = \ln(c) + \ln(l), \quad (1)$$

where

$$c = \left[ ac_m^b + (1 - a)c_n^b \right]^{1/b}, \quad (2)$$

is, as in McGrattan, Rogerson and Wright (1997), a Constant Elasticity of Substitution (CES) aggregation of market- and non-market ("home") consumption, denoted by  $c_m$  and  $c_n$ , respectively. Parameters  $a$  and  $1 - a$ , where  $0 < a < 1$ , denote the weights attached to different consumption categories in the aggregate consumption bundle, and parameter  $b > 0$  measures the degree of substitutability between market and home production.

Each household has a unit endowment of time, which can be either supplied in the market sector, used to produce non-market output, or enjoyed as leisure, hence

$$l = 1 - h_m - h_n. \quad (3)$$

Non-market output ( $y_n$ ) is non-tradable and non-storable consumption good ( $c_n$ ), and can be produced using labor as follows:

$$y_n = c_n = h_n, \quad (4)$$

where each household can supply any amount of hours in the non-market sector. On the other hand, the labor choice in the market sector is a discrete (non-convex) one. A household can either work full-time, or not work at all, i.e.  $h_m \in \{0; \bar{h}\}$ . The hourly wage rate in the market sector is  $w$ . Finally, each household claims an equal share of the representative firm's profit, denoted by  $\pi$ .<sup>1</sup> The budget constraint that each household faces is then

$$c_m = wh_m + \pi \quad (5)$$

Household's utility maximization problem (1) s.t. (2)-(5) of choosing  $\{c_m, h_m, h_n\}$  optimally by taking  $\{w, \pi\}$  as given, can be split into two main sub-cases: let  $c_m^u$  will denote the market consumption of households that do not work, with  $c_m^u = \pi$  and  $h_m^u = 0$ . Similarly, full-time workers would enjoy  $c_m^e = w\bar{h} + \pi$  and  $h_m^e = \bar{h}$ , where superscript  $e$  denotes workers. However, household's utility maximization problem is complicated from the fact that the number of hours supplied in the market sector is a continuous decision, and those who decide to work in the home sector, would enjoy higher  $c_n$  but a lower utility of leisure due to a positive amount of hours  $h_n$  worked. Optimal non-market (home) consumption will be determined from the following first-order condition:

$$h_n : \frac{a(1-a)h_n^{b-1}}{c_m^b + (1-a)h_n^b} = \frac{1}{1 - h_m - h_n}. \quad (6)$$

As seen from the equation above, optimal non-market consumption (and hours) is a function of market consumption (and hence of market labor, wage rate and profit share).

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<sup>1</sup>This would guarantee positive consumption even for unemployed people.

## 2.2 Firms

There is a representative firm producing a homogeneous final consumption good using labor as the only input. For simplicity, its price is normalized to unity. The production function features decreasing returns to scale and is given by

$$Y = f(H), f'(H) > 0, f''(H) < 0, f'(0) = \infty, f'(\bar{h}) = 0. \quad (7)$$

The last constraint means that the firm faces a capacity constraint: If all households work full-time in the market sector, the marginal product of an hour worked is zero. Together with the decreasing returns to scale that would produce positive economic profit (and thus guarantee a positive level of market consumption) in equilibrium.

The firm acts competitively by taking the wage rate  $\{w\}$  as given, and chooses hours  $\{H\}$  employed to maximize profit:

$$\max_H \Pi = f(H) - wH \quad \text{s.t.} \quad 0 \leq H \leq \bar{h}. \quad (8)$$

## 2.3 Decentralized Competitive Equilibrium

A Decentralized Competitive Equilibrium (DCE) is defined by allocations  $\{c_m^u, c_m^e, h_m, h_n, H\}$ , wage rate  $\{w\}$ , and aggregate profit  $\Pi(= \pi)$  s.t. (i) all households maximize utility; (ii) the stand-in firm maximizes profit; (iii) all markets clear.

## 2.4 Characterizing the DCE

It will be shown that in the DCE, if it exists, only some of the households will be employed, but everyone enjoys the same utility level. Denote this mass of employed by  $\lambda$ , and the unemployed by  $1 - \lambda$ . Thus, in equilibrium,  $H = \lambda\bar{h}$ . From the firm's problem, the wage rate is

$$w = f'(\lambda\bar{h}), \quad (9)$$

and economic profit equals

$$\pi = \Pi = F(\lambda\bar{h}) - F'(\lambda\bar{h})\lambda\bar{h} > 0, \quad (10)$$

which follows from the assumption that the production function features decreasing returns to scale. Next, to show that the DCE actually exists, it is sufficient to show the existence of a fixed point  $\lambda \in (0, 1)$  by analyzing a non-linear equation using the fact that in equilibrium utility is the same for all households. Note that from (6), (9) and (10),  $h_n = h_n(\lambda)$ , i.e., non-market hours are a function of the proportion of households employed in the market sector. Furthermore, using the Implicit Function Theorem on (6), we can show  $\frac{dh_n}{d\lambda} > 0$ , which follows from the complementarity between the two types of consumption in the household's utility function.

It is trivial to show that everyone working ( $\lambda = 1$ ) is not an equilibrium, since then  $w = f'(\bar{h}) = 0$ . From the ex ante symmetry assumption for households, Eq. (1) and (2), market consumption would be the same for both workers and non-workers, while non-workers would enjoy higher utility out of leisure (holding  $h_m$  fixed), hence there is no benefit of working. Similarly, nobody working ( $\lambda = 0$ ) is not an equilibrium outcome either, since the firm would offer a very high wage for the first unit of labor, and by taking a full-time job a worker could increase his/her utility.

Thus, if there is a DCE, then it must be that not all households would receive the same consumption bundle. If  $\lambda$  is an equilibrium, then total utility for households that work in the market sector should equal to the utility of households that do not supply any hours in the market sector:

$$\begin{aligned} & \frac{1}{b} \ln \left[ a[f'(\lambda \bar{h}) \bar{h} + f(\lambda \bar{h}) - f'(\lambda \bar{h}) \lambda \bar{h}]^b + (1-a)[h_n(\lambda)]^b \right] + \ln \left[ 1 - \bar{h} - h_n(\lambda) \right] \\ &= \frac{1}{b} \ln \left[ a[f(\lambda \bar{h}) - f'(\lambda \bar{h}) \lambda \bar{h}]^b + (1-a)[h_n(\lambda)]^b \right] + \ln \left[ 1 - h_n(\lambda) \right]. \end{aligned} \quad (11)$$

This equation is monotone in  $\lambda$ , as the utility function is a sum of monotone functions, and the  $h_n(\lambda)$  term is quantitatively small. Thus we can explore the behavior of the function above as we let  $\lambda$  vary in the  $(0, 1)$  interval. As  $\lambda \rightarrow 0$

$$\begin{aligned} & \frac{1}{b} \ln \left[ a[f'(0) \bar{h} + f(0) - f'(0) 0]^b + (1-a)[h_n(0)]^b \right] + \ln \left[ 1 - \bar{h} - h_n(0) \right] \\ & > \frac{1}{b} \ln \left[ a[f(0) - f'(0) 0]^b + (1-a)[h_n(0)]^b \right] + \ln \left[ 1 - h_n(0) \right], \end{aligned} \quad (12)$$

since  $f'(0) = \infty$ ,  $f(0) = 0$ , and  $f'(0)0 = 0$  ( $\lambda$  does faster to zero compared to the speed at which  $f'(\lambda\bar{h})$  term diverges to infinity.) Thus for any  $\bar{h} > 0$  it is obvious that

$$\begin{aligned} & \frac{1}{b} \ln \left[ a[f'(0)\bar{h}]^b + (1-a)[h_n(0)]^b \right] + \ln \left[ 1 - \bar{h} - h_n(0) \right] \\ & > \frac{1}{b} \ln \left[ (1-a)[h_n(0)]^b \right] + \ln \left[ 1 - h_n(0) \right]. \end{aligned} \quad (13)$$

Similarly, as  $\lambda \rightarrow 1$

$$\begin{aligned} & \frac{1}{b} \ln \left[ a[f'(\bar{h})\bar{h} + f(\bar{h}) - f'(\bar{h})\bar{h}]^b + (1-a)[h_n(1)]^b \right] + \ln \left[ 1 - \bar{h} - h_n(1) \right] \\ & < \frac{1}{b} \ln \left[ a[f(\bar{h}) - f'(\bar{h})\bar{h}]^b + (1-a)[h_n(1)]^b \right] + \ln \left[ 1 - h_n(1) \right], \end{aligned} \quad (14)$$

since by assumption  $f'(\bar{h}) = 0$  and thus

$$\begin{aligned} & \frac{1}{b} \ln \left[ a[f(\bar{h})]^b + (1-a)[h_n(1)]^b \right] + \ln \left[ 1 - \bar{h} - h_n(1) \right] \\ & < \frac{1}{b} \ln \left[ a[f(\bar{h})]^b + (1-a)[h_n(1)]^b \right] + \ln \left[ 1 - h_n(1) \right]. \end{aligned} \quad (15)$$

The inequality above holds since for any  $\bar{h} > 0$ ,

$$\ln \left[ 1 - \bar{h} - h_n(1) \right] < \ln \left[ 1 - h_n(1) \right]. \quad (16)$$

Therefore,  $\exists \lambda \in (0, 1)$  that is consistent with equilibrium. The existence follows from continuity of the utility function. In addition,  $\lambda$  is unique which follows from the monotonicity of the underlying functions. More precisely, there are a lot of equilibria: in each equilibrium the names of the people working are different, but in every equilibrium the same fraction of people  $\lambda$  that works is the same. Let  $c_m^e$ ,  $c_m^u$ , and  $\lambda$  characterize the allocations in the equilibrium, where  $c_m^e$  is the market consumption of the individuals who work,  $c_m^u$  is the market consumption of individuals who do not, with  $c_m^e > c_m^u$ .

Given the indivisibility of the labor supply in the market sector, the equilibrium allocation obtained above is not Pareto optimal, as demonstrated in Rogerson (1988). More specifically, a Social Planner (SP) could make everyone better off by using employment lottery and choosing the fraction  $\lambda$  of individuals to work and give everyone consumption  $\lambda c_m^e + (1 - \lambda) c_m^u$ . In order to show this, we need to check that such an allocation is feasible, and that it provides

a higher level of total utility. Showing feasibility is trivial as total market labor input and total market consumption are identical to the corresponding individual equilibrium values.

Next, we will show that the new allocation, which is independent of household's employment status, makes households better off since it generates higher utility on average. This is indeed the case, as

$$\begin{aligned}
& \lambda \left\{ \frac{1}{b} \ln \left[ a[\lambda c_m^e + (1 - \lambda)c_m^u]^b + (1 - a)(h_n(\lambda))^b \right] + \ln[1 - \bar{h} - h_n(\lambda)] \right\} \\
& + (1 - \lambda) \left\{ \frac{1}{b} \ln \left[ a[\lambda c_m^e + (1 - \lambda)c_m^u]^b + (1 - a)(h_n(\lambda))^b \right] + \ln[1 - h_n(\lambda)] \right\} \\
= & \frac{1}{b} \ln \left[ a[\lambda c_m^e + (1 - \lambda)c_m^u]^b + (1 - a)(h_n(\lambda))^b \right] + \lambda \ln[1 - \bar{h} - h_n(\lambda)] + (1 - \lambda) \ln[1 - h_n(\lambda)] \\
> & \lambda \frac{1}{b} \left\{ \ln \left[ a[c_m^e]^b + (1 - a)[h_n(\lambda)]^b \right] + (1 - \lambda) \frac{1}{b} \ln \left[ a[c_m^u]^b + (1 - a)[h_n(\lambda)]^b \right] \right. \\
& \left. + \lambda \ln[1 - \bar{h} - h_n(\lambda)] + (1 - \lambda) \ln[1 - h_n(\lambda)] \right),
\end{aligned}$$

where the strict inequality follows from the convexity of the CES aggregation and the concavity of the logarithmic function. Thus, the SP is indeed giving in expected utility terms an allocation that is an improvement over the initial equilibrium allocation. Next, imposing the restriction on consumption  $c_m = c_n = f(\lambda \bar{h})$ , SP's problem can be rewritten as

$$\max_{\lambda \in (0,1)} \frac{1}{b} \ln \left[ a f[\lambda \bar{h}]^b + (1 - a)[h_n(\lambda)]^b \right] + \lambda \ln[1 - \bar{h} - h_n(\lambda)] + (1 - \lambda) \ln[1 - h_n(\lambda)] \quad (17)$$

Observe that per-capita hours worked in the market sector are  $h_m = \lambda \bar{h}$ , or

$$\lambda = \frac{h_m}{\bar{h}} = \frac{1 - l - h_n(\lambda)}{\bar{h}}, \quad (18)$$

where  $l$  is leisure. Rearrange and group terms together to obtain the modified objective function from (17)

$$\ln \left[ a f[\lambda \bar{h}]^b + (1 - a)[h_n(\lambda)]^b \right]^{\frac{1}{b}} + \lambda \left[ \ln[1 - \bar{h} - h_n(\lambda)] - \ln[1 - h_n(\lambda)] \right] + \ln[1 - h_n(\lambda)] \quad (19)$$

Thus aggregate utility equals

$$\ln(c) + \frac{1 - l - h_n(\lambda)}{\bar{h}} [\ln(1 - \bar{h} - h_n(\lambda)) - \ln(1 - h_n(\lambda))] + \ln[1 - h_n(\lambda)]. \quad (20)$$

For small  $\bar{h}$

$$\frac{[\ln(1 - \bar{h} - h_n(\lambda)) - \ln(1 - h_n(\lambda))]}{\bar{h}} \approx \frac{1}{1 - h_n(\lambda)}. \quad (21)$$

We can simplify further by substituting (21) into (20) to obtain

$$\ln(c) + 1 - \frac{1}{1 - h_n(\lambda)}l + \ln[1 - h_n(\lambda)], \quad (22)$$

or

$$\ln(c) + \frac{1 - h_n(\lambda) + h_m - h_n(\lambda) - 1}{1 - h_n(\lambda)} + \ln[1 - h_n(\lambda)]. \quad (23)$$

Finally

$$\ln(c) - Ah_m + \ln(1 - h_n(\lambda)), \quad (24)$$

where  $A = \frac{1}{1 - h_n(\lambda)} > 0$ . The resulting aggregate utility function is of an interesting and novel form: On the aggregate, when each household faces a mix of divisible and indivisible labor choices, the representative agent obtained from the aggregation features different preferences of work: disutility of work in the market sector is now linear, while the disutility of home labor  $h_n$  is logarithmic as in the individual utility function. The split of the two types of labor is a novel result that is driven by the fact that market labor was indivisible, while non-market hours were divisible. To the best of our knowledge, such a case has not been discussed in the literature. In addition, the discrete-continuous mix of labor supply decisions could be quantitatively important for the transmission of business cycle shocks and welfare effects of different government policies. However, such investigations are beyond the scope of this note.

As a possible venue for future research, we plan to extend the model to a dynamic and stochastic setting, and feed the derived aggregate utility function above in a sophisticated Real-Business-Cycle model with physical capital, distortionary taxation, government spending, and home production to investigate the effect of those preferences for the transmission of technology and fiscal shocks.

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