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## Equilibrium selection with coupled populations in hawk-dove games: Theory and experiment in continuous time

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# DISCUSSION PAPER

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# Equilibrium Selection with Coupled Populations in Hawk-Dove Games: Theory and Experiment in Continuous Time<sup>a</sup>

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## Abstract

Standard one- and two-population models for evolutionary games are the limit cases of a uniparametric family combining intra- and intergroup interactions. Our setup interpolates between both extremes with a coupling parameter  $\kappa$ . For the example of the hawk-dove game, we analyze the replicator dynamics of the coupled model. We confirm the existence of a bifurcation in the dynamics of the system and identify three regions for equilibrium selection, one of which does not appear in common one- and two-population models. We also design a continuous-time experiment, exploring the dynamics and the equilibrium selection. The data largely confirm the theory.

Keywords: evolutionary game theory, experiment in continuous time, hawk-dove game, replicator dynamics.

JEL Classification: C62, C73, C91, C92.

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<sup>a</sup>An earlier version of this paper was entitled “Intra- and Intergroup Conflicts: Theory and Experiment in Continuous Time.”

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# 1 Introduction

Evolutionary game theory makes an important distinction as to whether players interact within a single population or between two (or more) disjunct populations (Cressman, 2003; Friedman, 1991; Weibull, 1995). When matched with opponents in a single population, players earn the expected payoff as if playing against the aggregate strategy of their own population, so only symmetric strategies can survive. With a two-population matching, each member of the group of, say, row players is matched against a rival from the group of column players. Here, polarization in behavior can occur and the populations may specialize in different strategies. The same evolutionary forces can thus imply qualitatively different results, so the distinction of single- and two-population settings is crucial.

The compartmentalization of one- and two-population models may, however, not always be appropriate. A two-population analysis requires that players exclusively receive their payoffs from interactions with the external population. Likewise, in a one-population setting, players never interact with opponents from other populations. Both these assumptions may not be warranted: why should players in a two-population game not interact at least occasionally within their own population? Why should agents in a single population setting not sometimes be also exposed to interactions with agents from other populations? It seems plausible that the interaction will often be mixed.

For non-human players, examples where the one- and the two-population cases overlap are abundant in resource conflicts. Animals will predominantly compete for resources with other members of the same species (intra-species competition). But there will also be inter-species competition (Birch, 1957)—think of different predatory mammals fighting for prey and water, or of various sessile organisms competing for light interception and soil. Inevitably, intra-specific and inter-specific competition overlap.<sup>1</sup>

An example with human players can be found in Mailath (1998). Traders bargain either within their own village or encounter visitors from a different population. The game is hawk-dove in both cases but the analysis is one-population in the first case and two-population in the second and the evolutionary selection mechanisms differ starkly. But beyond these polar cases traders may, of course, interact at the same time both with players from their own village and with visitors.<sup>2</sup>

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<sup>1</sup>Connell and Sousa (1983) and Schoener (1983) provide surveys of works on inter-specific competition. Kennedy and Strange (1986) show how the density of salmon fry in an ecological niche appears to be influenced by the presence of both older salmons (same species, different generation) and trout (different species).

<sup>2</sup>Somewhat similarly, Roll (1994) interprets stock market traders performing fundamental analysis as doves and traders gathering information only from price movements as hawks. As suggested by a referee, Roll's original model is formulated for one population but could easily be extended to consider two populations, perhaps in different countries or different types of market participants, for example, pension funds vs. hedge funds.

The notion that intragroup and intergroup interactions overlap makes sense when agents do not condition their strategy on the population from which an opponent stems. This will be the case when players cannot identify which population a rival is from, that is, when group membership is determined by indiscernible factors such as geographic location and religious or political views. Even when they can identify the groups to which other players belong, they may still not be able or willing to condition their strategy on this identification.<sup>3</sup> A firm’s strategy may involve a managerial structure or incentive scheme that cannot be switched on and off depending on whether the firm is interacting with its peers in a supply chain or with its customers or suppliers. Boundedly rational players may choose the same action for intragroup and intergroup interactions due to limited learning in complex environments. But rational agents may do the same in order to establish a global reputation, or as a result of the costly cognitive resources they employ to organize their reasoning (see section 2).

In this paper, we analyze the interaction of one- and two-population dynamics, theoretically and in experiments. We analyze a uniparametric family that combines the two models by interpolating between both extremes with a coupling parameter  $\kappa$  which measures the weight of the intergroup interaction.<sup>4</sup> One- and two-population matchings are obtained for  $\kappa = 0$  and  $\kappa = 1$ , respectively. We analyze the replicator dynamics of this system theoretically<sup>5</sup> and conduct the experiment in continuous time (Pettit et al., 2014). This is more suitable than standard discrete-time experiments for testing the predictions of evolutionary game theory, foremost because it allows for asynchronous choices and faster convergence.

Our application is a hawk-dove game. The hawk-dove game is the paradigm for the analysis of conflicts over scarce resources (Maynard Smith, 1982). Introduced by Maynard Smith and Price (1973) in the context of animal conflict, it also became highly influential for human interactions due to its fairly simple definition which nevertheless generates very rich dynamics as a population game.

Oprea et al. (2011) analyze the hawk-dove game for the sign-preserving dynamics of the one- and the two-population case. Their (continuous-time) experiment confirms the predictions in that the symmetric mixed equilibrium is more likely to be selected in the one-population treatment whereas separation is stronger in the two-population treatment.

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<sup>3</sup>Taking a different approach, Selten (1980) assumes that players can condition their strategy based on the information available to them.

<sup>4</sup>Independently, a similar approach has been developed by Friedman and Sinervo (2016, section 3.7). Their starting point is a two-population model, and they introduce coupling in the form of “own-population effects.” See also a previous analysis of evolutionary models with two groups of individuals in Cressman (1995).

<sup>5</sup>Gómez-Gardeñes et al. (2012) use a similar model to analyze simulations of games on overlapping graphs.

Our theoretical analysis confirms the existence of a bifurcation in the dynamics of the system, and the replicator dynamics predicts three regions with different stable equilibria. First, for any  $\kappa < \kappa_m^*$  the predictions for the aggregate population strategy is a non-degenerate mixed equilibrium, as in the one-population setting ( $\kappa = 0$ ). For  $\kappa > \kappa_p^*$  pure play emerges for both groups, as in the two-population analysis ( $\kappa = 1$ ). For the intermediate values  $\kappa \in (\kappa_m^*, \kappa_p^*)$ , a qualitatively novel prediction emerges where one population coordinates on a pure strategy while the second population is composed of a mixture of hawks and doves. This hybrid case does not occur with a single population or with two populations.

One way of interpreting these theoretical results is that the existing analyses of undiluted one- and two-population cases are robust with respect to perturbations. We find  $\kappa_m^* = 1/2$  and show that  $\kappa_p^*$  will vary between a half and one. In words, the prediction of the one-population case extends with much overlap to a second population. A more moderate statement can be made regarding the two-population case as the scope for pure equilibria is typically smaller than the scope for mixed equilibria. In that sense, the two-population analysis seems somewhat less robust. Nevertheless, theoretically, it appears that neither of the one- nor the two-population cases are strongly affected by perturbations.

Our experimental results qualitatively confirm the predictions, but there are also departures from the theory. We find that mixed behavior is observed throughout where predicted—including the pure one-population treatment and the coupled variants with  $\kappa < 1/2$ . We also see a sound separation of hawks and doves in our pure two-population ( $\kappa = 1$ ) treatment. These findings confirm and extend the experiments of Oprea et al. (2011). Among the discrepancies between the replicator dynamics prediction and the experimental results is a general bias in the mixed strategies: in the treatments where the mixed equilibrium was expected, the frequency of hawk play was lower than predicted. As for the  $\kappa \in (\kappa_p^*, 1)$  case (where a pure equilibrium is predicted), the separating effect is less pronounced than with  $\kappa = 1$ . The data further indicate that the splitting point (understood as the level of  $\kappa$  for which the populations split into two groups of “mostly hawks” and “mostly doves”) experiences notable variations across sessions.

## 2 Literature

The implementation of interactions as population games recovers the spirit of the “mass action” interpretation of the mixed Nash equilibrium (Björnerstedt and Weibull, 1996; Young, 2011). Each player in a population can simply play a pure strategy, but the frequencies of the strategies in the population may correspond to a mixed Nash

equilibrium. This relaxes the reasoning skills required for mixed play.

When the (static) game exhibits multiple equilibria, the evolutionary approach provides a powerful tool for equilibrium selection and learning (Friedman, 1996).<sup>6</sup> In a seminal contribution to evolutionary game theory, Friedman (1991) compares the theoretical conditions for static stability in evolutionary games involving one and two (or more) groups of individuals, with applications to, for example, male-female mating problems. Weibull (1995) attributes the first multi-population replicator dynamics analyses to Taylor and Jonker (1978) and Maynard Smith (1982).<sup>7</sup> Weibull (1995) considers a different version of replicator dynamics for the  $n$ -population case. Following Nowak and May (1992), a number of papers have also analyzed how the structure of a population may affect its behavior (Lieberman et al., 2005; Taylor et al., 2004).

Our starting point is that players choose the same action when playing the same game in encounters with players from different populations. This is in line with Samuelson (2001) who argues that agents facing a problem of multiple strategic interactions may balance the gains from better decision-making against the cost of using scarce cognitive resources. This can result in the application of the same choice in several of the interactions. Jehiel (2005) formalizes the notion of “analogy-based expectation equilibrium” where players best respond to beliefs that are correct, on average, over various analogous situations. Huck et al. (2011) provide experimental evidence. Grimm and Mengel (2014) analyze learning across games in experiments with the concepts of “belief bundling” and “action bundling” – both of which can imply that players simplify their decision environment by choosing the same action in different games, as a form of “best response bundling.” See also Mengel (2012) and the discussion contained therein.

### 3 Theory

We analyze the replicator dynamics with an even number of players divided into two (coupled) populations of equal size, labeled  $X$  and  $Y$ . We define the simplex  $S^X = \{s^X = (s_1^X, s_2^X) : \sum_{a=1,2} s_a^X = 1\}$  such that any point in it represents the fraction of population  $X$  employing each available strategy.<sup>8</sup>  $S^Y$  is defined analogously for population  $Y$ . The product  $\Omega = S^X \times S^Y$  is the set of strategy profiles and also the state space of the dynamical system. We focus on symmetric two-strategy games defined by  $2 \times 2$  payoff matrices, with action set  $S = \{s_1, s_2\}$ . The matrix element  $\pi_{ij}$  defines the payoff from

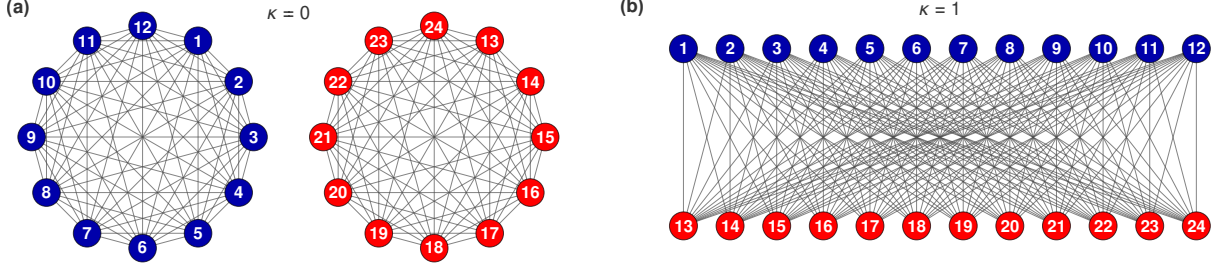
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<sup>6</sup>On learning issues, see Camerer and Ho (1999), Hopkins (2002), and Huck et al. (1999).

<sup>7</sup>Page and Nowak (2002) explain different approaches to deterministic dynamics in population games. Hofbauer and Sandholm (2002) show the connection between stochastic choice-making and deterministic dynamics. See Szabó and Fáth (2007) or Sandholm (2010) for comprehensive surveys.

<sup>8</sup>Players could also be using a mixed strategy. This would not alter the analysis.





**Figure 1. Population structures.** Network representation of (a) one-population and (b) two-population protocols with 24 players.

choosing action  $s_i$  when playing against pure strategy  $s_j$ . To simplify notation, let  $x$  and  $y$  be the share of the strategy  $s_1$  in populations  $X$  and  $Y$ , respectively. The state vector of population  $X$  is  $\omega_x = (x, 1 - x)^T$ , and  $\omega_y = (y, 1 - y)^T$  for population  $Y$ . Then, the dimensionality of the problem is reduced to two,  $x$  and  $y$ , because the state of the dynamical system is a composite in the form  $\omega = (x, 1 - x; y, 1 - y)^T \in \Omega$ . Let  $\mathcal{L}_\Pi : [0, 1] \times [0, 1] \rightarrow \mathbb{R}$  be a linear operator for any given  $2 \times 2$  payoff matrix  $\Pi$ . Its application over a bidimensional vector  $w$  is defined as  $\mathcal{L}_\Pi[w] = \langle e, \Pi w \rangle$ , where we define  $e = (1, -1)^T$  and  $\langle \cdot, \cdot \rangle$  is the inner product in the vector space.

Consider the two standard matching protocols: the one-population protocol and the two-population protocol. In the one-population case, players interact randomly with other players from their population and so, technically, every player in a population earns the payoff of her choice against the aggregate strategy of her own population. With the two-population protocol, the row players (population  $X$ ) play against the column players (population  $Y$ ). See Figure 1 for a graphical representation of these population structures. In the one-population case for, say, population  $X$ , we can write the rate of growth of the strategy  $s_1$  in the population as  $\dot{x} = x(1 - x)\mathcal{L}_{\Pi_A}[\omega_x]$ . For population  $X$  in the two-population case, we write  $\dot{x} = x(1 - x)\mathcal{L}_{\Pi_B}[\omega_y]$ . In general, we can consider different payoff matrices for the within-group and the between-groups games,  $\Pi_A$  and  $\Pi_B$ , respectively.

As our key novelty, we introduce a new matching protocol involving the coupling parameter  $\kappa \in [0, 1]$  which integrates the two protocols in a linear fashion. This coefficient is restricted to the unit interval, and its extremes  $\kappa = 0$  and  $\kappa = 1$  correspond to the one-population and the two-population settings, respectively. We then define the linear combination  $\dot{x} = x(1 - x)[(1 - \kappa)\mathcal{L}_{\Pi_A}[\omega_x] + \kappa\mathcal{L}_{\Pi_B}[\omega_y]]$  generalizing the study to situations with a simultaneous existence of strategic interactions at both intra- and intergroup levels. The (instantaneous) payoff function for a player belonging to population  $X$  and choosing

**Table 1. Fixed points' location.** Replicator dynamics (3) with  $v = 12$  and  $c = 18$ . See the appendix for the general solution.

Pure states	Hybrid states	Mixed states
$p_1^* = (0, 0)$	$p_5^* = (0, 2/[3(1 - \kappa)])$	$p_9^* = (2/3, 2/3)$
$p_2^* = (1, 0)$	$p_6^* = (1, (2 - 3\kappa)/[3(1 - \kappa)])$	
$p_3^* = (0, 1)$	$p_7^* = (2/[3(1 - \kappa)], 0)$	
$p_4^* = (1, 1)$	$p_8^* = ((2 - 3\kappa)/[3(1 - \kappa)], 1)$	

strategy  $s_i \in S$  is given as

$$\pi_X(s_i; x, y)(t) = (1 - \kappa)[\pi_{i1}^A x(t) + \pi_{i2}^A (1 - x(t))] + \kappa[\pi_{i1}^B y(t) + \pi_{i2}^B (1 - y(t))]. \quad (1)$$

Due to symmetry,  $\pi_Y$  is analogous and just requires the exchange of the population labels  $x$  and  $y$ .

We now simplify  $\Pi_A = \Pi_B$  and choose the hawk-dove game for our analysis of intra- and intergroup interaction. The hawk-dove game can be parametrized as

$$\Pi_A = \Pi_B = \begin{pmatrix} a + \frac{1}{2}(v - c) & a + v \\ a & a + \frac{1}{2}v \end{pmatrix}, \quad (2)$$

where the parameters  $0 < v < c$  represent the valuation of the good and the cost of the conflict, respectively, and  $a > 0$  is the players' endowment.

We obtain the dynamics of the model as a system of coupled ordinary differential equations:

$$\begin{cases} \dot{x} &= x(1 - x)\frac{1}{2}[v - c(x + \kappa(y - x))] \\ \dot{y} &= y(1 - y)\frac{1}{2}[v - c(y + \kappa(x - y))] \end{cases} \quad (3)$$

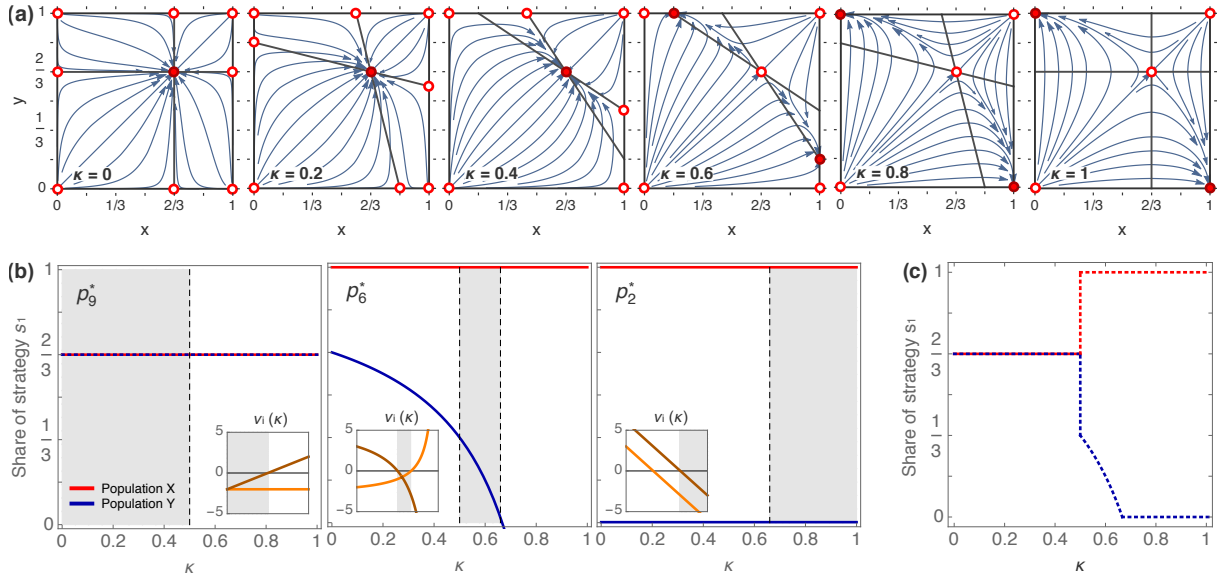
The rate of growth of each strategy in the population is determined solely as a function of: (i) the current state of the system  $(x, y)$ , (ii) the value of the good  $v$  and the cost of the conflict  $c$ , and (iii) the coupling parameter  $\kappa$ .

This parameter  $\kappa$  represents the strength of the coupling between the two populations of players, while  $(1 - \kappa)$  is the weight of the interaction within each group. Depending on the context of application of the model, this can mirror different effects. For the traders in the example in Mailath (1998),  $\kappa$  measures the fraction of players at the trade fair coming from a neighboring city. For the notion of best-response bundling in the experiments of Grimm and Mengel (2014), our model can be seen as the mean field abstraction of a treatment where  $\kappa$  tunes the frequency in which each matching protocol appears.<sup>9</sup>

The following proposition formalizes the analysis of the dynamical system (3) and

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<sup>9</sup>The definition can include asymmetric coupling with groups of different sizes or in situations where the populations weight the two conflicts in a different way, suitable in the meaning of animal competition.



**Figure 2. Replicator dynamics.** Plots with  $v = 12$  and  $c = 18$ . (a) Phase portrait of (3). White-filled points: non-stable. Red-filled points: stable. Dark lines: nullclines. (b) Share of strategy  $s_1$  (hawk) in populations depending on  $\kappa$ . Shaded areas: stability domains (eigenvalues  $\nu_i(\kappa)$  as inset and in the Appendix). (c) Resulting bifurcation diagram.

relates it to our notion of intra- and intergroup conflict. A proof can be found in the Appendix.

**Proposition 1.** *Given the replicator dynamics in (3):*

- (a) *if  $\kappa < \kappa_m^*$ , the mixed Nash equilibrium is selected,*
- (b) *if  $\kappa > \kappa_p^*$ , the pure Nash equilibria are selected,*
- (c) *in the intermediate range  $\kappa_m^* \leq \kappa \leq \kappa_p^*$ , a hybrid equilibrium is selected where one population plays a pure strategy and the other one chooses a mixture.*

The cutoff points satisfy  $\kappa_m^* = 1/2 \leq \kappa_p^*$  and  $\kappa_p^* = \max\{v/c, 1 - v/c\}$ . If  $c = 2v$  then  $\kappa_p^* = \kappa_m^* = 1/2$  and case (c) is void.

Proposition 1 contains the one-population and two-population cases from previous research (Oprea et al., 2011) as limit cases. The prediction for region (a) is as in the one-population matching ( $\kappa = 0$ ) and the one for region (b) is as in the two-population matching ( $\kappa = 1$ ). The hybrid equilibrium in region (c) is novel and exists neither as a Nash equilibrium nor as an attractor of the one-population or two-population settings. Table 1 gives the coordinates of the fixed points with parameters corresponding to the games played in the experiment (section 4).

In Figure 2, we illustrate the equilibrium selection for the parameters used in the experiment. For values of  $\kappa < \kappa_m^* = 1/2$  the only attractor in the phase space corresponds to both populations being composed of two-thirds hawk. For  $\kappa > \kappa_p^* = 2/3$ , one

group plays purely hawk and other one plays purely dove. For the intermediate range  $\kappa \in [1/2, 2/3]$ , the replicator dynamics predict a pure-mixed configuration such that the more hawkish population plays purely hawk ( $x = 1$ ) while the more dovish group plays a completely mixed strategy.

Figure 2 (a) also shows the impact of the coupling on the phase portrait of the dynamical system. Starting with  $\kappa = 0$ , an increase in  $\kappa$  rotates the nullclines (zero-growth isoclines)  $\dot{x} = 0$  and  $\dot{y} = 0$  clockwise and counterclockwise, respectively. No qualitative change happens at the beginning, but between  $\kappa = 0.2$  and  $0.4$  these nullclines cross the upper-left and bottom-right corner of the phase space and eliminate two saddle points. This does not have a major impact on the qualitative predictions. We obtain the first bifurcation in the system for  $\kappa = 1/2$ : both nullclines coincide and their intersection transforms from attractor to saddle point. Simultaneously, the remaining saddle points located at the edges become the attractors of the system. These attractors move along the edges when  $\kappa$  continues increasing. Finally, the second bifurcation occurs when they meet the two corner points  $(1, 0)$  and  $(0, 1)$  which become the attractors of the system.

Proposition 1 generates testable hypotheses. In addition to relying on the equilibrium predictions (mixed for  $\kappa < 1/2$ , pure for  $\kappa > 2/3$ ), we will use a “separation index,”  $\Delta s(\kappa) \in [0, 1]$ . This index is defined as  $\Delta s(\kappa) = \bar{s}_1(\kappa, X) - \bar{s}_1(\kappa, Y)$ . That is,  $\Delta s(\kappa)$  is the share of the hawk strategy in the more hawkish population minus the share of the hawk strategy in the more dovish population,<sup>10</sup> for a given value of the treatment variable  $\kappa$ . Using  $\Delta s(\kappa)$  and interpreting Proposition 1 in a qualitative fashion, we obtain our main hypotheses:

$$\Delta s(0) = \Delta s(0.2) = \Delta s(0.4) < \Delta s(0.6) < \Delta s(0.8) = \Delta s(1). \quad (4)$$

## 4 Experiment

For the experiment, we choose the payoff parameters  $a = 3$ ,  $v = 12$ , and  $c = 18$ . This results in the following hawk-dove game:

$$\Pi = \begin{pmatrix} 0 & 15 \\ 3 & 9 \end{pmatrix}. \quad (5)$$

The standard two-player game has three Nash equilibria of the form  $(\sigma_X, \sigma_Y) \in \{(1, 0), (0, 1), (2/3, 2/3)\}$ , where  $\sigma_l$  denotes the probability that strategy hawk will be chosen by player  $l \in \{X, Y\}$ .

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<sup>10</sup>The label “population  $X$ ” is arbitrarily assigned to the more hawkish group in the steady state for the analysis of the experimental data in the rest of the paper.

**Table 2. Sequence of treatments in each session.**

Period	Session 1	Session 2	Session 3	Session 4	Session 5	Session 6
1	$\kappa = 0.8$	$\kappa = 0.2$	$\kappa = 1$	$\kappa = 0.8$	$\kappa = 0.4$	$\kappa = 1$
2	$\kappa = 0.2$	$\kappa = 1$	$\kappa = 0.4$	$\kappa = 0$	$\kappa = 0.8$	$\kappa = 0.6$
3	$\kappa = 0$	$\kappa = 0.6$	$\kappa = 0.6$	$\kappa = 0.4$	$\kappa = 0.2$	$\kappa = 0.2$
4	$\kappa = 0.6$	$\kappa = 0$	$\kappa = 0$	$\kappa = 0.6$	$\kappa = 1$	$\kappa = 0.4$
5	$\kappa = 0.4$	$\kappa = 0.8$	$\kappa = 0.2$	$\kappa = 1$	$\kappa = 0$	$\kappa = 0.8$
6	$\kappa = 1$	$\kappa = 0.4$	$\kappa = 0.8$	$\kappa = 0.2$	$\kappa = 0.6$	$\kappa = 0$

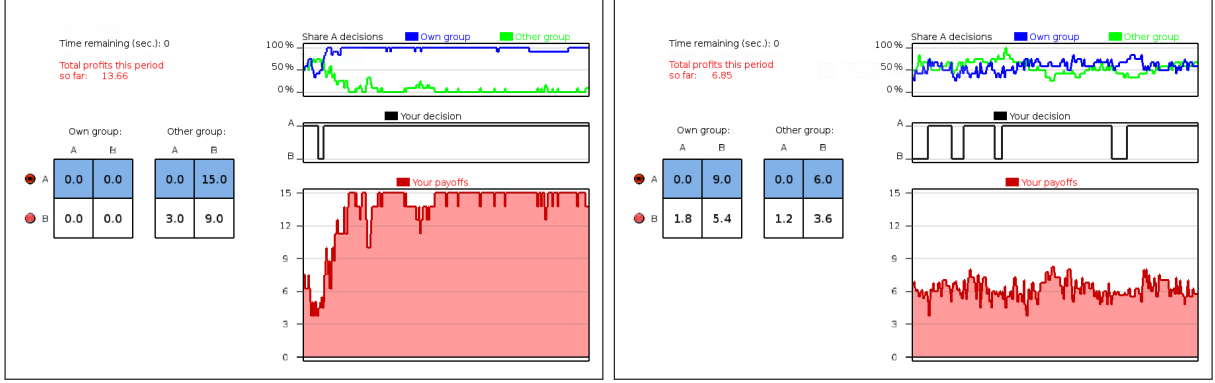
Our treatment variable is the coupling parameter  $\kappa$ . We consider  $\kappa \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$ . The cases  $\kappa \in \{0, 0.2, 0.4\}$  correspond to Proposition 1 (a), the cases  $\kappa \in \{0.8, 1\}$  to Part (b), and  $\kappa = 0.6$  corresponds to Proposition 1 (c).

We use a within-subjects design and all subjects play all six treatments consecutively. To mitigate order effects or hysteresis, we randomize the order of the treatments at the session level (see Table 2). To prevent reputation effects and to maintain the one-shot character of the experiment, we employ random matching such that the composition of the groups changes at the beginning of each treatment. Players are independently and randomly assigned their initial actions in each treatment. Furthermore, subjects are paid only for one randomly-selected treatment in order to avoid wealth effects or hedging behavior across treatments (Blanco et al., 2010). This randomization is implemented with a public dice roll at the end of each session.

Other experimental procedures were as follows. All participants received hardcopies of the instructions at the beginning of the session, and afterwards these were verbally summarized (see the instructions, available in Appendix B). Each session began with a trial part consisting of three 90-second periods in which the players had the opportunity to familiarize themselves with the software. The subjects were aware that no payoffs would result from playing these three periods and we chose payoff matrices different from the hawk-dove games that would be used in the actual treatments. The six periods in which we ran the treatments had a time length of 210 seconds each. Subjects reported a good comprehension of the task and software in an anonymous questionnaire which they filled in at the end of the sessions.

The experiment was conducted with the software ConG. This software package has been made available in open-access by Pettit et al. (2014) and allows for experiments to be played in (virtually) continuous time.<sup>11</sup> This particular setting allows the players to make their choices in a fully asynchronous manner and they experience the updating of the system in real time. This framework is particularly suitable in our case because

<sup>11</sup>We extended the basic package of ConG with a new payoff function, a customized matching scheme, and a new graphical interface adapted to the information set that needs to be displayed according to our design.



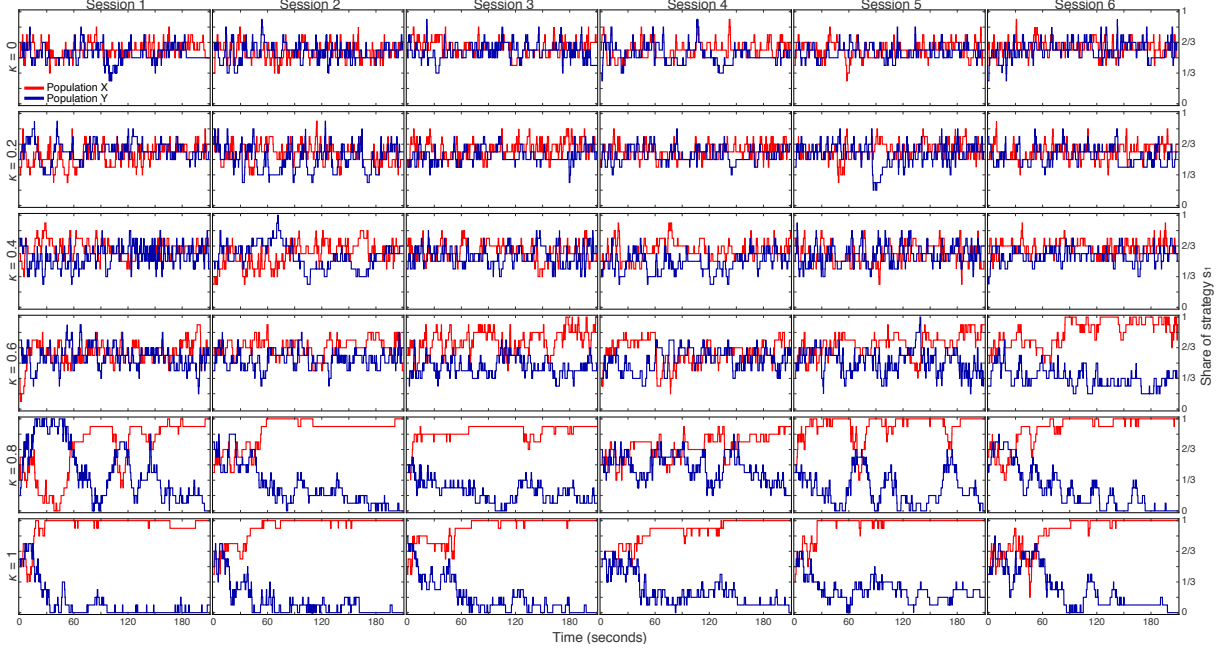
**Figure 3. Experimental display.** Screenshot (translated from German) of two terminals at the end of two treatments. Left: player in a hawkish group in  $\kappa = 1$ . Right: player in a mixed-strategy group when  $\kappa = 0.4$ .

standard evolutionary models assume asynchronous updating by the agents and make long-run predictions which may be distorted if the experiments are performed as a finite sequence of synchronous repetitions of a stage game. See Oprea et al. (2011) on this issue.

Figure 3 shows two examples of the graphical display presented to the subjects in our experiment. On the left side of the screen, players could see the payoff matrices and the selection tool. Every agent was framed as a row player who needed to choose either  $A$  (hawk) or  $B$  (dove). The selection could be made with the radio buttons or with the up and down arrow keys. Subjects could change their action at any point in time and their choices had an immediate impact on both games. The instantaneous choice was highlighted with a blue shadow in the selected row.

Players saw two payoff matrices: the left one refers to the interaction with the “own group” and the second one refers to the interaction with the “other group.” The entries of these two matrices displayed are determined as  $(1 - \kappa)\Pi$  and  $\kappa\Pi$ , respectively, with  $\Pi$  defined in (5) and  $\kappa$  being the treatment variable (not known by players). Subjects were informed that the level of their payoff flow was determined as the sum of what they were simultaneously earning in the interactions with the own and with the other group. The upper-left corner indicates the accumulated payoff during the period and the remaining time.

The right half of the screen provides the players with all relevant information on the state of play. The top chart plots the average strategy (that is, the share of subjects choosing hawk) of each population. The middle chart documents the player’s own choice, which can only alternate between  $A$  (hawk) and  $B$  (dove). The bottom chart plots a dark red solid line representing the payoff flow that the player is earning at a given point in time. The red shadow helps participants to understand that the payoff they earn is accumulated over time. The three charts share the same horizontal axis, that is, time.



**Figure 4. Experimental timelines.** Evolution over time of the aggregate strategy of both populations, by treatment  $\kappa$  and session.

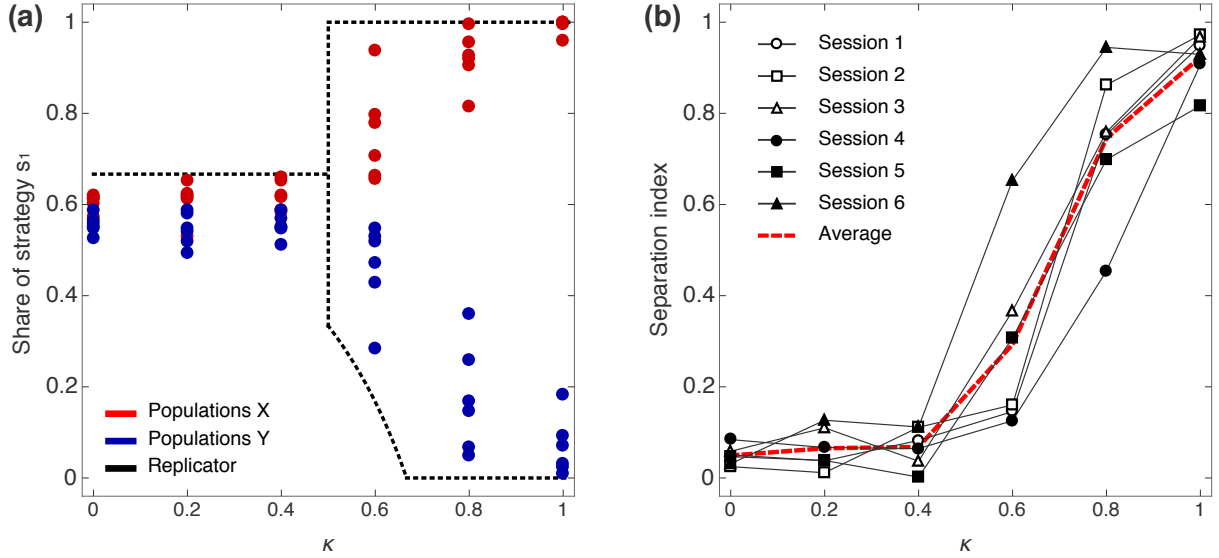
Note that all the changes of any factor are shown in the corresponding charts without any noticeable delay.

We ran six experimental sessions at the DICE Lab for experimental economics in Düsseldorf in April and May 2015. Each session comprised 24 subjects (two populations of 12 subjects each) with 144 subjects in total. Generally, participants were recruited from the local subject pool which contains students of various fields at the Heinrich Heine University of Düsseldorf, using ORSEE (Greiner, 2015).

## 5 Results

Figure 4 presents the data from all sessions and from all treatments. The vertical axis represents the share of players who chose hawk in the two populations of each session, at each instant. The horizontal axis represents time in seconds. This figure shows the evolution of the average strategy of the populations over time.

Consider first  $\kappa \in \{0, 0.2, 0.4\}$  where both populations are expected to converge to the mixed equilibrium. Observed group behavior is in line with the predictions of two-thirds hawk, as can be seen in the first three rows of the plot. Also, the average strategies oscillate around that value in all 18 charts. When we consider the last 60 seconds of play as the steady state of the system, we find that the separation index  $\Delta s(\kappa)$  is between 0 and 0.13 in all six sessions of these three treatments.



**Figure 5. Aggregate results.** (a) Share of hawk choices in steady state by  $\kappa$  in all sessions, compared to the bifurcation diagram in Figure 2. (b)  $\Delta s(\kappa)$  for each session.

For  $\kappa = 0.6$ , theory suggests the hybrid case where one population should choose purely hawk while in the other population one-sixth hawkish play should emerge. Figure 4 shows this kind of outcome for sessions 3, 5, and 6 where we observe  $\Delta s = 0.37$ , 0.31, and 0.65, respectively. In the other three sessions, the steady state is more in line with the mixed equilibrium and we observe separation indices in the vicinity of 0.15, resembling the data in the treatments with a lower  $\kappa$ . The level of  $\kappa$  for which the populations separate appears to vary from session to session.

With  $\kappa \in \{0.8, 1\}$ , the average strategies of the two populations are predicted to converge to pure play in the steady state. In Figure 4, the last two rows of the graph show much polarization between the two populations at the end of the treatments. The average separation in the two-population treatment ( $\kappa = 1$ ) is 0.93 whereas we obtain  $\Delta s(0.8)$  between 0.70 and 0.95, except in session 4. In that session, we observe a period of experimentation with mixing behavior, with a slow and delayed departure toward a pure equilibrium, such that  $\Delta s(0.8) = 0.45$ .

Figure 5 summarizes how the populations in all sessions behave in the steady state (last 60 seconds of play). Panel (a) compares the aggregate results of all sessions to the bifurcation predicted by the replicator dynamics. We arbitrarily assign the population label  $X$  to the more hawkish population in the steady state. We observe three main effects. First, for  $\kappa \in \{0, 0.2, 0.4\}$ , mixing behavior occurs, but with a general downward bias in the share of hawk choices in all populations. Second, the scatter plot for  $\kappa = 0.6$  and 0.8 shows considerable dispersion, and even though the existence of the bifurcation is apparent, the intensity of polarizing behavior is clearly weaker than predicted by the replicator dynamics. Third, most of the deviation from perfect separation in the last two



treatments ( $\kappa \in \{0.8, 0.1\}$ ) is driven by the dovish populations. When  $\kappa = 1$ , the share of hawk in the steady state of the hawkish populations is larger than  $x = 0.99$  in five cases and  $x = 0.96$  in the sixth (Session 1). By contrast, the share of strategy hawk in the dovish populations is non-zero throughout and reaches values of almost 20% in several sessions. For  $\kappa = 0.8$  both the hawkish and the dovish populations deviate from perfect separation, but a similar argument still applies.

The finding that pure-strategy behavior is more pronounced for the hawk populations can be explained as follows. The separation into hawk and dove populations induces substantial payoff inequalities. (See also the discussion in Oprea et al., 2011, section 4.) In the more dovish population, some individuals foresee that their group is doomed to earn the lowest payoff and thus deviate systematically from their best response. There are two pure equilibria, and such deviating behavior of doves can be seen as an attempt to break the coordination and push play toward the more profitable equilibrium. This kind of behavior is rather apparent for  $\kappa = 0.8$  (see Figure 4). In some sessions, the identity of the more hawkish population changes several times.

Panel (b) of Figure 5 plots the separation indices for each treatment, classified by session. Despite the subtleties described above, the statistical analysis qualitatively confirms the hypotheses (4) of the replicator dynamics. Wilcoxon signed-rank tests yield the following results, where two-sided  $p$  values above the (in)equality signs indicate whether or not the according null hypothesis is rejected:

$$\Delta s(0) \overset{p>0.999}{=} \Delta s(0.2) \overset{p>0.999}{=} \Delta s(0.4) \overset{p=0.031}{<} \Delta s(0.6) \overset{p=0.031}{<} \Delta s(0.8) \overset{p=0.062}{<} \Delta s(1). \quad (6)$$

There are no significant differences between the consecutive separation indexes with  $\kappa \in \{0, 0.2, 0.4\}$ , consistent with our prediction. We also confirm (4) in that  $\Delta s(0.4) < \Delta s(0.6)$  and  $\Delta s(0.6) < \Delta s(0.8)$  are statistically significant. Finally, and in a deviation from the prediction, the separation index for  $\kappa = 1$  is weakly significantly larger than for  $\kappa = 0.8$ .

## 6 Conclusion

Equilibrium selection and learning in populations are prime targets for evolutionary game theory. We analyze what happens when intra- and inter-group interactions overlap. We predict a dynamical bifurcation from symmetric mixed to asymmetric pure equilibria in the hawk-dove game which depends on the frequency of interaction in the own vs. another (second) population. The transition occurs at an intermediate range of the coupling parameter  $\kappa$ . In the transition range, one population coordinates on a pure strategy while the second population is composed of a mixture of hawks and doves.

We also analyze to what extent human behavior matches the bifurcation in continuous-

time experiments, extending a previous study by Oprea et al. (2011). Our experimental results largely confirm the hypotheses. One implication of the results is that the predictions for one-population and two-population settings are robust with respect to the presence of overlapping inter- and intragroup interactions. Nevertheless, the transition regime experiences notable variation across experimental sessions.

Our paper demonstrates the usefulness of continuous-time experiments in the analysis of intra- and intergroup interactions. Observations of actual bifurcations, together with other recent developments in the field such as the analysis of limit-cycles in rock-paper-scissors games (Cason et al., 2014), show a much improved degree of resolution with which experiments can study evolutionary forces.

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## A Proof of Proposition 1

**Location of fixed points.** We obtain the zero-growth curves directly from (3). Because of the linearity of the fitness functions, all the nullclines are simple lines in the plane:

$$\begin{aligned} \dot{x} = 0 &\rightarrow x = 0 & \dot{y} = 0 &\rightarrow y = 0 \\ \dot{x} = 0 &\rightarrow x = 1 & \dot{y} = 0 &\rightarrow y = 1 \\ \dot{x} = 0 &\rightarrow x = \frac{v - \kappa c y}{(1 - \kappa)c} & \dot{y} = 0 &\rightarrow y = \frac{v - \kappa c x}{(1 - \kappa)c}. \end{aligned} \tag{7}$$

Fixed points are located at the different intersections of a horizontal and a vertical nullcline. Obviously, the corners of the state space are fixed points and represent possible equilibria in which both populations play pure strategies,

$$p_1^* = (0, 0), p_2^* = (1, 0), p_3^* = (0, 1), p_4^* = (1, 1). \tag{8}$$

There are four other possible points where one population plays a pure strategy while the other is mixed,

$$\begin{aligned} p_5^* &= (0, v/[(1-\kappa)c]), & p_6^* &= (1, [v-\kappa c]/[(1-\kappa)c]), \\ p_7^* &= (v/[(1-\kappa)c], 0), & p_8^* &= ([v-\kappa c]/[(1-\kappa)c], 1). \end{aligned} \quad (9)$$

Finally, we also obtain a possible configuration in which both populations mix strategies in a symmetric manner,

$$p_9^* = (v/c, v/c). \quad (10)$$

Point  $p_9^*$  is always inside the unit square because  $0 < v < c$ . Nevertheless, the fixed points  $p_5^*$  and  $p_7^*$  only exist for  $\kappa \in [0, 1 - v/c]$  while  $p_6^*$  and  $p_8^*$  only exist for  $\kappa \in [0, v/c]$ .

**Linear stability analysis.** The Jacobian  $J$  with matrix elements  $J_{mn} = \partial \dot{m}/\partial n$  is defined by

$$\begin{aligned} 2 \times J_{xx}(x, y) &= v - 2vx + c[3x^2(1-\kappa) - \kappa y - 2x(1-\kappa-\kappa y)] \\ 2 \times J_{xy}(x, y) &= \kappa cx(1-x), \end{aligned} \quad (11)$$

with  $J_{yx}$  and  $J_{yy}$  given by symmetry ( $x \leftrightarrow y$ ). Stable points are those fixed points for which both eigenvalues of  $J$  (evaluated at the point's coordinates) are negative (Hofbauer and Sigmund, 2003). The eigenvalues are the two roots of the characteristic polynomial  $\det[\nu \mathbb{I}_2 - J]$ .

For  $p_1^*$  and  $p_4^*$ , we obtain  $\nu_1 = \nu_2 = v/2 > 0$ , and  $\nu_1 = \nu_2 = (c-v)/2 > 0$ , respectively. These two symmetric equilibria in pure strategies are never attractors of the system. For the asymmetric equilibria in pure strategies ( $p_2^*, p_3^*$ ), the eigenvalues are  $\nu_1 = (c-v-\kappa c)/2$ , and  $\nu_2 = (v-\kappa c)/2$ . If  $c < 2v$ , then  $\nu_2 > \nu_1$  and the asymmetric pure equilibria are stable for  $\kappa \in [v/c, 1]$ . If  $c > 2v$ , then  $\nu_1 > \nu_2$  and they are stable for  $\kappa \in [1 - v/c, 1]$ .

Considering  $p_5^*$  and  $p_7^*$ , the eigenvalues are  $\nu_1 = v - v/[2(1-\kappa)]$ , and  $\nu_2 = [v^2 - (1-\kappa)cv]/[2c(1-\kappa)]$ . Note that  $\nu_1$  is negative for  $\kappa \in (1/2, 1)$ , and  $\nu_2$  is negative for  $\kappa < 1 - v/c$ . These two points are stable for  $\kappa \in [1/2, 1 - v/c]$ , when  $c > 2v$ . For the points  $p_6^*$  and  $p_8^*$ , the eigenvalues are  $\nu_1 = (c-v)(2\kappa-1)/[2(\kappa-1)]$ , and  $\nu_2 = (c-v)(\kappa c - v)/[2c(1-\kappa)]$ .  $\nu_1$  is negative for  $\kappa \in (1/2, 1)$  and  $\nu_2$  is so for  $\kappa < v/c$ . Thus, these points are stable for the range  $\kappa \in [1/2, v/c]$ , provided  $c < 2v$ .

Finally, the symmetric equilibrium in mixed strategies  $p_9^*$  yields eigenvalues  $\nu_1 = v(v-c)/2c$ , and  $\nu_2 = v(c-v)(2\kappa-1)/2c$ . The first one is constant and always negative since  $c > v$  in the hawk-dove game. The second is negative for  $\kappa < 1/2$ , so  $p_9^*$  is stable when  $\kappa \in [0, 1/2]$ .

Thus, we have characterized the attractors of the dynamical system (3) to be selected

for each region of the coupling parameter  $\kappa$ . These results are summarized in Proposition 1 and illustrated in Figure 2.

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# Experimental instructions

(intended for publication as Supplementary Material)

## Welcome to this experiment on economic decision making!

Please read these instructions carefully. The experiment is conducted anonymously. This means you will not get to know which of the other participants are interacting with you or which participant is acting in which role. Please note that you must not talk to other participants once the experiment has started. Also note that the use of mobile phones or similar devices is prohibited for the duration of the experiment. If you have any questions after reading these instructions, please raise your hand and we will come to your cubicle to answer your questions personally.

There are several peculiarities about this experiment:

- The experiment consists of six parts, but only one randomly selected part will be paid.
- You will play in multiple groups at the same time.
- The experiment is conducted in continuous time.

## The different parts of the experiment

As mentioned above, there are six parts. At the end of the experiment, a random draw of the computer will determine which part is going to be paid. Please try to play each part as if it was the only one.

## You will play in multiple groups at the same time

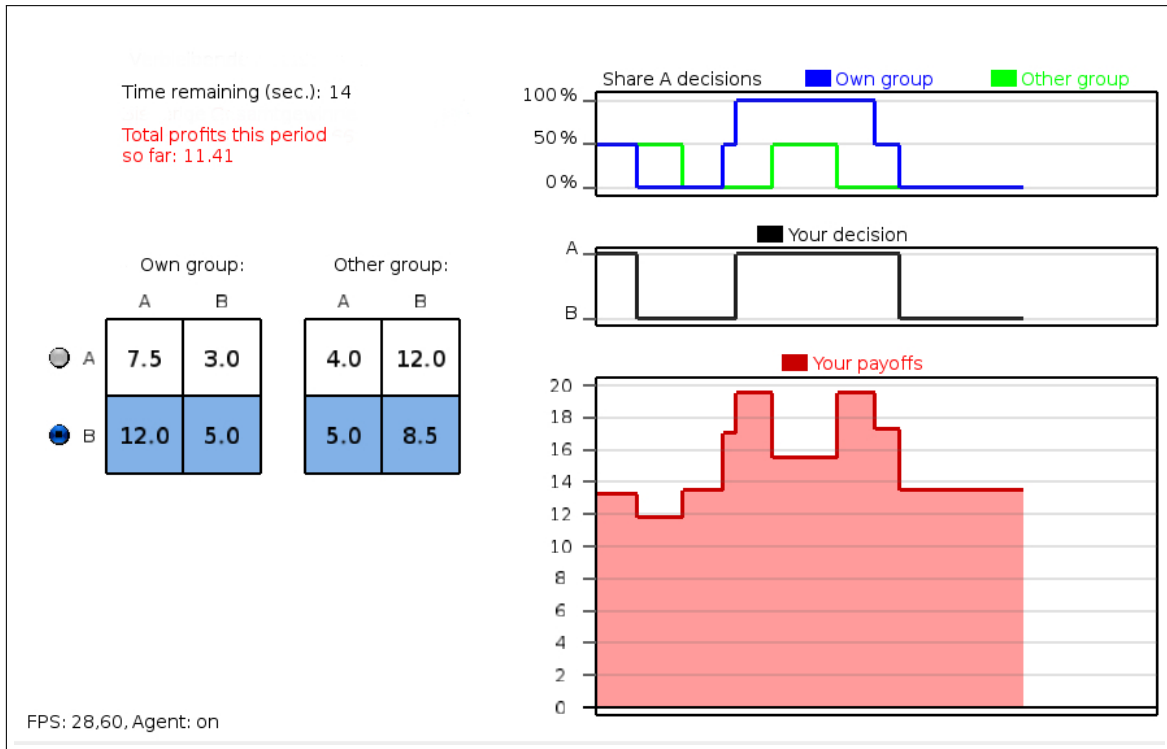
At the beginning of each part, the participants will be sorted into two groups. This sorting is random and will take place anew at the beginning of each part. Thus, the composition of the groups will change in each part. That means that participants who were in your group in one part may be in the other group in the next part, and vice versa.

One of the two groups will consist of you and 11 other participants. This group is referred to as “your group” or “own group.” The second group consists of 12 other participants and is referred to as the “other group.”

You will always interact with both groups at the same time. The details on this interaction are discussed in a later section of these instructions. For now, just note that you (and all other participants) always need to choose between the options “A” and “B” and that your choice affects the interaction with both groups.

## The experiment is conducted in continuous time

Each part lasts 210 seconds. At the beginning of a part, the computer will randomly select one of the options “A” or “B” for you to start with. After that, you can change your decision at any point in time and as often as you want. Below, you can see a screenshot of the experimental software. The left part of the screen informs you of the game that is played in the current part. This is also where you make your decision.



## How do I make or change my decision?

The radio buttons on the far left of the screen determine your current choice. To change your decision, you can use the mouse to click on the corresponding radio button or you may also use the up and down keys on the keyboard to change your decision. The up key will select option “A” and the down key will select option “B.” As previously mentioned, you can change your decision at any point in time during the entire part.

## How can I see what the others are doing or what I did at an earlier point in time?

The decisions of participants are recorded and displayed in the upper-right area of the screen. The black line documents your own decisions over time. It depicts whether you chose “A” or “B” at a given point in time. The blue and the green line in the upper chart document the decisions of the participants in your own group and in the other group, respectively. Either line displays the share of participants who choose “A” in the corresponding group.



### How do I see how much money I earn?

In this experiment, you earn a steady income that accumulates over time. The current income is documented in the red chart in the lower-right area of the screen. The higher the red line, the higher the income you earn. Note that your income is a flow of payoffs. If you realize an income of 10 for the entire duration of the part, you would earn a total payoff of 10 in that part. Analogously, an income of 15 means that you would receive 15 Euros if your income was constantly 15 and the corresponding part was randomly selected as the one chosen to be paid.

Your total profits are represented by the red area under the red line. Moreover, the red text in the upper-left area of the screen will always show the total payoffs you have realized so far.

### How is my income calculated?

As mentioned above, you are always playing in two groups at the same time. The interaction is described by two payoff matrices: the left matrix applies to the interaction with your own group, the right matrix describes the interaction with the other group. The matrices inform you of the income you can generate while playing with either group.

Consider the following example (from the screenshot):

	Your group		Other group	
	A	B	A	B
A	7.5	3.0	4.0	12.0
B	12.0	5.0	5.0	8.5

Assume you and all other participants (in both groups) choose “B.” In this case, your income from your own group will be 5 (indicated in the lower-right corner of the left matrix) and your income from the other group would be 8.5. Your total income would thus be 13.5.

Another example, assume that one half of the participants in your group choose “A” while the other half of your group chooses “B.” Moreover, assume that 70% of the participants in the other group choose “A” and that the remaining 80% choose “B.”

- If you are among those who choose “A”:  
Income from your group:  $0.5 \times 7.5 + 0.5 \times 3 = 5.25$ .  
Income from other group:  $0.7 \times 4 + 0.3 \times 12 = 6.4$ .  
Total income:  $5.25 + 6.4 = 11.65$ .
- If you are among those who choose “B”:  
Income from your group:  $0.5 \times 12 + 0.5 \times 5 = 8.5$ .

Income from with other group:  $0.7 \times 5 + 0.3 \times 8.5 = 6.05$ .

Total income:  $8.5 + 6.05 = 14.05$ .

In general, the income streams are calculated as follows. The matrices below are general,  $OwnPayXY$  and  $OthPayXY$  are just placeholders for the entries of the matrix with your group and with the other group, respectively. The terms  $OwnShareA$ ,  $OwnShareB$ ,  $OthShareA$  and  $OthShareB$  represent the share of participants who choose “A” or “B” in either group, respectively.

		Your group		Other group	
		A	B	A	B
A		$OwnPayAA$	$OwnPayAB$	$OthPayAA$	$OthPayAB$
B		$OwnPayBA$	$OwnPayBB$	$OthPayBA$	$OthPayBB$

Your decision determines the relevant row of the payoff matrices. If you choose “A” only the first row is used, and if you choose “B” only the second row is used.

Your income from your group would be:

- If you choose “A”:  
 $OwnPayAA \times OwnShareA + OwnPayAB \times OwnShareB$
- If you choose “B”:  
 $OwnPayBA \times OwnShareA + OwnPayBB \times OwnShareB$

The same logic also applies to the interaction with the other group. Here, your income would be:

- If you choose “A”:  
 $OthPayAA \times OthShareA + OthPayAB \times OthShareB$
- If you choose “B”:  
 $OthPayBA \times OthShareA + OthPayBB \times OthShareB$

The only difference between the interactions with your own group and with the other group is that your own decision will have an impact on  $OwnShareA$  and  $OwnShareB$  but not on  $OthShareA$  and  $OthShareB$ .

Your total income is the sum of your income from your group and the income from the other group. Keep in mind that the red line will always show your current total income and that you may change your choice at any point in time. Moreover, the red text in the

upper-left area of the screen will keep you informed of the payoffs you have accumulated so far.

### **Summary**

- There are six parts. Only one randomly selected part will be paid in the end. Each part lasts 210 seconds.
- There are always two groups. The composition of these groups is random and will change in every part.
- You will play in two groups at a time: one with your own group and one with the other group.
- Both groups generate a flow of income and your actual payoff will accumulate over time.
- You can change your decision at any point in time. Use the radio buttons on the left side of the screen or the Up/Down keys on the keyboard to do so.

The experiment will start with three short (90 seconds) trial parts that will not affect your payoff. These are simply to familiarize you with the payoff structure and the software for this experiment.

If you have any further questions, please raise your hand and we will come to your cubicle to answer the questions personally.

## PREVIOUS DISCUSSION PAPERS

- 222 Benndorf, Volker, Martinez-Martinez, Ismael and Normann, Hans-Theo, Equilibrium Selection with Coupled Populations in Hawk-Dove Games: Theory and Experiment in Continuous Time, June 2016.  
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