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# RBC Models and the Hours-Wages Puzzle: Puzzle Solved!

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## **Abstract**

This paper shows that a modified real business cycle (RBC) model, one that includes home production and fiscal spending shocks, can solve one of the RBC puzzles and generates zero correlation between wages and hours. In addition, the micro-founded model presented here provides a sound theoretical model to analyze fiscal policy in a neoclassical framework and is able to capture many aspects of the data that the benchmark RBC model was missing.

**Keywords:** fiscal policy, home production, government spending shock, indivisible labor

**JEL Codes:** C63, E32, E62

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## 1. Introduction

This paper shows that a modified real business cycle (RBC) model, one that includes home production and fiscal spending shocks, can solve one of the RBC puzzles and generates zero correlation between wages and hours. In addition, the micro-founded model presented here provides a sound theoretical model to analyze fiscal policy in a neoclassical framework and is able to capture many aspects of the data that the benchmark RBC model was missing.

The puzzle, also known as the “hours-productivity puzzle in the RBC literature”, has bothered economists since the 1980s: Prescott (1986), King, Plosser and Rebelo (1988), Hansen and Wright (1992), King and Rebelo (1999) documented that the representative-agent RBC model captures private sector labor market dynamics only imperfectly. In their models, labor markets are assumed to be perfectly-competitive, which meant that the wage rate is proportional to labor productivity, measured as output produced per hour worked. In the face of technology shocks hitting the economy, wages increase, and hours supplied follow. Thus, almost by definition, hours and wages feature almost perfect correlation, while in data that correlation is very close to zero.

Therefore, there must be additional mechanism at work that breaks down the correlation between hours and productivity, which the models were missing. This might have led Kydland and Prescott (1982) to include so-called non-separability of hours in the household’s utility derived out of leisure. In other words, hours supplied in the current period were assumed to depend on the labor supply in the past with this dependence dying over time. This modeling trick solved, at least partially the puzzle, as now hours did not react that strongly to increases in productivity.

For some reason, however, this assumption was not adopted in later papers. Instead, improvements on the benchmark model focused more on realistic aspects such as the effects of fiscal policy, e.g. Christiano and Eichenbaum (1992) and Hansen and Wright (1992), among many others. Christiano and Eichenbaum (1992) added government spending as a second shock hitting the economy and showed that this extension was able to cut the hours-productivity correlation down to 0.5. The new mechanism at work was the negative wealth effect: after an unexpected increase in government spending, there were less resources available for private consumption and investment, and the household felt relatively poorer. More precisely, a shock to government spending is a shock to aggregate demand. Since leisure is assumed to be a normal good, this in turn led to an increase in hours worked. Such an effect on labor, being a “demand effect”, partially offsets the “supply effect” driven by technology shocks, and breaks down the perfect correlation between hours and wages predicted by earlier models.

In a different line of work, a similar effect on the hours-wages correlation was achieved with Rogerson’s (1988) indivisible hours extension as in Hansen (1985)

and documented in Wright and Hansen (1992). The major idea is that if the labor choice at individual level is constrained to be discrete – choosing not to work at all, or work full-time, produces a significantly different dynamics on the aggregate. Benhabib, Rogerson and Wright (1991) focus on the non-market economic activity to show that when home production is included in the model, it can also partially address the hours-productivity puzzle, by providing a second sector, which requires labor as a productive input. The presence of such a labor relocation mechanism has a significant effect on the co-movement of market hours and the wage rate (market labor productivity).<sup>1</sup>

The model economy in this paper is based on Benhabib, Rogerson and Wright (1991), who calibrate a dynamic stochastic general equilibrium model (DSGE) for an economy with household production sector. The novelty in the current paper is that restrictions on the functional forms are imposed, using results from McGrattan, Rogerson and Wright (1997) who apply maximum likelihood procedure to estimate model parameters instead of calibrating them. Their estimates cannot reject the hypothesis that household's utility function is logarithmic in consumption, households put no weight on government spending in their utility function and that the only input in the home production function is labor. The model is numerically solved by log-linearizing around the steady-state.

In the literature, the inclusion of home production is motivated by the following three stylized facts (as presented in Greenwood, Rogerson and Wright 1995):

*Fact 1.* A typical married couple in US (PSID database) allocates 1/3 of its time for paid work and 1/4 to work in household production activities.

*Fact 2.*  $\frac{i_{mt}}{i_{nt}} = 1:15$ , dened as purchases of consumer durables and residential structures. (in our calibration it is 0).

*Fact 3.* Home production output is in the range of 20–50% of the measured market GNP.

The main idea is that in the standard RBC model labor input is mis-measured by ignoring the home production component, and erroneously lumping it together with leisure. Although it is standard in the literature to use only market hours, time surveys show work at home is an important use of total time endowment. By explicitly modeling the choice between working in the market or at home, we in-

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<sup>1</sup> The list is far from exhaustive: Albonico, Kalyvitis and Pappa (2012) include investment adjustment costs to address the hours-productivity puzzle. In a recent paper, Vasilev (2015b) introduces health shocks in a standard RBC model to show that those also bring the model correlation between hours and wages (labor productivity) closer to the estimated correlation in data. Interested readers are directed to those papers and the references therein. Bornukova (2009) goes beyond the restrictive representative-agent assumption and works with twomember households to tackle the puzzle. In the current paper we prefer to stay within the single household paradigm, though, so our results are not directly comparable.

roduce a richer dynamics in the model to describe a plausible shock propagation-mechanism. The government spending shock is interesting enough to justify this exercise in itself, since we are interested in the dynamics in Real Business Cycle (RBC) models: this a new margin of adjustment brings it closer to the data. Given the mixed evidence, modeling gives us sufficient degrees of freedom to represent richer dynamics by including sectoral effects.

We also extend and generalize the work by Hansen and Wright (1992): by putting both home production and government spending shock in the RBC model, we are able to bring down the contemporaneous correlation of productivity and market hours to zero, which is what we observe in US data. Thus, we solve one of the long-argued deficiencies of the RBC literature. Another aim of the model is to provide a useful guide for fiscal policy in the neoclassical framework, especially for countries with large agricultural sector, and/or countries with large informal sector, or when there is red tape that prevents the accumulation of market capital, as in Parente, Rogerson and Wright (1999). In the artificial economy, both the market and home production functions are subject to technology shocks, denoted by  $A_m$  and  $A_n$ , respectively. The smaller the autocorrelation between the two shocks, the lower the substitutability, the greater the effect of home production on the economy. In the calibration exercise we set it equal to zero, in order to maximize the effect of home production. Technology shocks to market output are shocks to labor demand, as they affect firm's willingness to hire workers; government shocks and shocks to household production affect household's willingness to provide labor services.

The main mechanism at work in the model is as follows: as government spending increases, people feel poorer and work more. In which sector they choose to supply hours depends on relative productivity. When  $A_m$  is relatively high (compared to  $A_n$ ), labor will flow into the market sector, resulting in a positive correlation between labor productivity and  $h_m$ . When  $A_n$  is relatively high (relative to  $A_m$ ),  $h_m$  decreases (as hours relocate to the non-market sector), and as a result, labor productivity (output per hour) raises due to the Cobb-Douglas production function. That effect generates a negative correlation between the two.<sup>2</sup> Thus with both shocks this systematic relationship between wages and hours is completely destroyed.

In this paper, there is going to be an additional twist in the model: since home production function is linear in terms of hours, working at home provides consumption directly to households. On the other hand, by working in the market sector, the effect on consumption is indirect: people generate labor and capital income, which they use to purchase market consumption. Thus market hours fall, while non-market hours increase and on the aggregate, total hours increase. Since market output is produced using Cobb-Douglas technology, when market hours fall, wages increase. The

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<sup>2</sup> In a sense, the presence of home production technology shocks has a demand effect on hours.

choice of working in two sectors destroys the perfect co-movement of wages and market hours that we observe in the benchmark RBC model. In addition, with home production, leisure appears as an inferior good, even though in structural preferences leisure is a normal good. That is, despite the negative wealth effect caused by wasteful government spending, market hours fall due to the fact that some work effort is optimally chosen to be exercised in the home production sector.

A potential problem is that the model does not capture the correlation between the market and non-market investment. In times of high relative market productivity, agents move capital out of home and into the market. The same with labor inputs. In data,  $\text{corr}(k_{mt}, k_{nt}) = 0.3$ , though. We bypass this problem because in our model we do not have non-market capital, (McGrattan, Rogerson and Wright (1997) and so cannot reject the hypothesis that non-market capital is not significant in home production). In addition, rural production that is one form of non-market production, is usually less capital intensive than manufacturing. Thus in our model correlation is 0 by our modeling choice.<sup>3</sup>

The model has some shortcomings, however. First, it is subject to Gali's (1999) criticism, who argues that unconditional moments are not that relevant, because they can be generated close to the true moments for the wrong reasons. More important are the conditional moments: impulse responses show exactly the relative variance conditional on a certain shock. In order to study that issue, however, Gali (1999) resorts to the use of identification schemes, which are to a great extent arbitrary.

The rest of the paper is structured as follows: In section 2, the foundations of the model are laid out. The model equations are then log-linearized around the non-stochastic steadystate, and simulated moments are presented in Section 3. Section 4 presents an extension with indivisible hours in the market sector, and Section 5 concludes.

## 2. The Model

The economy consists of households, firms, and a government. Households are atomistic, infinitely many, infinitely-living with identical preferences, aggregated into a representative one. Households hold an endowment of capital stock, which they rent to the firms together with their labor services. In addition, households have access to home-production technology, which produces consumption from the hours supplied in the household production. Households pay taxes on labor and

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<sup>3</sup> Also, as argued in Bornukova (2009, 9), "the omission of capital from home production technology did not have a significant effect on the model performance, while the omission of home-production specific technology shock had important impact on the behavior of productivity."

capital income and receive government transfers. Government collects taxes on income, consumes output in a wasteful manner, and distributes lump-sum transfers to the household.

## 2.1. Household's problem

There is a representative households whose preferences are defined over composite consumption ( $c$ ) and leisure ( $l$ ), and discounted utility function as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln(c_t) + \ln(l_t) \right\}, \quad (1)$$

where  $E_0$  is the expectation operator as of period 0,

$$c_t = \left[ a c_{mt}^b + (1 - a) c_{nt}^b \right]^{1/b}, \quad (2)$$

is, as in McGrattan, Rogerson and Wright (1997), a Constant Elasticity of Substitution (CES) aggregation of market- and non-market ("home") consumption, denoted by  $c_{mt}$  and  $c_{nt}$ , respectively. Parameters  $a$  and  $1 - a$ , where  $0 < a < 1$ , denote the weights attached to different consumption categories in the aggregate consumption bundle, parameter  $b > 0$  measures the degree of substitutability between market and home production, and  $0 < \beta < 1$  is the discount factor.

The household can invest in physical capital, which follows the law of motion specied below:

$$k_{t+1} = i_t + (1 - \delta)k_t, \quad (3)$$

where  $0 < \delta < 1$  denotes the depreciation rate on capital and  $r_t$  is the return on a unit of physical capital. In addition, the representative household has a unit endowment of time, which can be either supplied in the market sector used to produce non-market output, or enjoyed as leisure, hence

$$l_t = 1 - h_{mt} - h_{nt}. \quad (4)$$

Non-market output is non-tradable and non-storable consumption good and can be produced using labor as follows:

$$y_{nt} = c_{nt} = A_{nt} h_{nt}, \quad (5)$$

where  $A_{nt}$  denotes the level of technology in the home production sector and each household can supply any amount of hours in the non-market sector. The hourly wage rate in the market sector is  $w_t$ . Finally, each household claims a share of the representative firm's profit, denoted by  $\pi_t$ . The budget constraint that each household faces is then

$$c_{mt} + i_t = (1 - \tau^l)w_t h_{mt} + (1 - \tau^k)r_t k_t + g_t^t + \pi_t, \quad (6)$$

where  $\tau^l, \tau^k$  denote the average effective tax rates applied to labor and capital income and  $g_t^t$  are government transfers.

The household takes  $\{w_t, r_t, \pi_t, g_t^t\}_{t=0}^\infty$ , and the initial condition for capital  $k_0$  as given, and chooses  $\{c_{mt}, k_t, h_{mt}, h_{nt}\}_{t=0}^\infty$  optimally to maximize (1) s.t. (2)–(6). This produces the following first-order conditions:

$$c_{mt} : \frac{a c_{mt}^{b-1}}{[a c_{mt}^b + (1 - a) c_{nt}^b]} = \lambda_t, \quad (7)$$

$$h_{nt} : \frac{(1 - a)(A_{nt} h_{nt})^{b-1}}{[a c_{mt}^b + (1 - a) c_{nt}^b]} = \frac{1}{1 - h_{mt} - h_{nt}}, \quad (8)$$

$$h_{mt} : \frac{1}{1 - h_{mt} - h_{nt}} = \lambda_t (1 - \tau^l) w_t, \quad (9)$$

$$k_{t+1} : \lambda_t = E_t \lambda_{t+1} [1 + (1 - \tau^k) r_{t+1} - \delta], \quad (10)$$

$$TVC : \lim_{t \rightarrow \infty} \beta^t \lambda_t k_{t+1} = 0, \quad (11)$$

where  $\lambda_t$  is the Lagrangian multiplier attached to the household's budget constraint. The optimality conditions have standard interpretations in the literature: the first equates the marginal utility of market consumption to the shadow price of wealth; the second equates the cost of working an additional hour in the home sector to the benefit of the extra increase in non-market output (and thus home consumption); the third condition is the optimal labor supply in the market sector at the margin, the cost of supplying additional hour and the benefit in terms of after-tax return exactly offset each other. The fourth equation is the so-called Euler equation, which describes the optimal allocation of physical capital across any two adjacent periods. Lastly, the transversality condition (TVC) is a boundary condition that guarantees that the optimal solution is non-explosive.

## 2.2. Firms

There is a representative firm producing a homogeneous final good using labor and capital as inputs. For simplicity, its price is normalized to unity. The production function features constant returns to scale and is given by:

$$y_t = k_t^\alpha (A_{mt} h_{mt})^{1-\alpha} \quad (12)$$

The firm acts competitively by taking  $(w_t, r_t)_{t=0}^\infty$  as given, and chooses  $k_t, h_{mt}, \forall t$  to maximize profit:



$$\max_{k_t, h_{mt}} \pi = k_t^\alpha (A_{mt} h_{mt})^{1-\alpha} - w_t h_{mt} - r_t k_t. \quad (13)$$

In equilibrium, profit is zero and inputs receive their marginal returns, i.e.:

$$w_t = (1 - \alpha) \frac{y_t}{h_{mt}}. \quad (14)$$

$$r_t = \alpha \frac{y_t}{k_t}. \quad (15)$$

### 2.3. Government

There is also a government in our model, which levies taxes on labor and capital income, which are then used for government transfers and wasteful spending  $g_t^c$  and follows a balanced budget rule:

$$\tau_h w_t h_{mt} + \tau_k r_t k_t = g_t^c + g_t^t, \forall t. \quad (16)$$

Capital and labor income tax rates  $\{\tau^l, \tau^k\}$  will be fixed. Government consumption will be approximated by an AR (1) process, and government transfers will be a residually determined instrument that would guarantee that the budget is balanced in every period. Thus, government lump-sum transfers vary endogenously in response to variations in government tax revenue. Households pay a lump-sum tax if wasteful government spending needs additional financing to balance the government budget period by period.

### 2.4. Stochastic Processes

The exogenous stochastic variables are total factor productivity in the market and home sector  $A_t^m, A_t^n$ , and the policy instrument government consumption  $g_t^c$  are all assumed to follow AR (1) processes in logs, in particular it follows that:

$$\ln A_{t+1}^m = (1 - \rho^m) \ln A_0^m + \rho^m \ln A_t^m + \epsilon_t^m, \quad (17)$$

where  $A_0^m = A^m > 0$  is steady-state level of the TFP process in the market sector,  $0 < \rho^m < 1$  is the first-order autoregressive persistence parameter, and  $\epsilon_t^m \text{ iid } N(0, \sigma_m^2)$  are random shocks to the TFP progress in the market sector. Hence, the innovations  $\epsilon_t^m$  represent unexpected changes in the TFP process in the market sector.

Home sector productivity is also assumed to follow AR (1) processes in logs, in particular

$$\ln A_{t+1}^n = (1 - \rho^n) \ln A_0^n + \rho^n \ln A_t + \epsilon_t^a, \tag{18}$$

where  $A_0^n = A^n > 0$  is steady-state level of the TFP process in the home sector,  $0 < \rho^n < 1$  is the first-order autoregressive persistence parameter, and  $\epsilon_t^n \sim iidN(0, \sigma_n^2)$  are random shocks to the TFP progress. Hence, the innovations  $\epsilon_t^n$  represent unexpected changes in the TFP process in the home sector.

Finally, the stochastic process for the government consumption is as follows

$$\ln g_t^c = (1 - \rho^g) \ln g_0^c + \rho^g \ln g_{t-1}^c + \epsilon_t^g \tag{19}$$

where  $g_0^c = g^c > 0$  is steady-state level of government consumption,  $0 < \rho^g < 1$  is the first-order autoregressive persistence parameter, and  $\epsilon_t^g \sim iidN(0, \sigma_g^2)$  are random shocks to government consumption. Hence, the innovations  $\epsilon_t^g$  represent unexpected changes in government consumption.

### 2.5. Decentralized Competitive Equilibrium

A Decentralized Competitive Equilibrium (DCE) is defined by allocations  $\{c_{mt}, c_{nt}, h_{mt}, h_{nt}, g_t^t\}_{t=0}^\infty$ , prices  $\{w_t, r_t\}_{t=0}^\infty$ , tax rates  $\{\tau^l, \tau^k\}$ , initial conditions for the state variables  $\{A_0^m, A_0^n, K_0\}$  and the processes for  $\{A_t^m, A_t^n, g_t^c\}_{t=0}^\infty$  s.t. (a) all households maximize utility; (b) the stand-in firm maximizes profit; (c) the government follows a balanced budget rule; (d) all markets clear.

### 2.6. Data and Model Calibration

The model will study the behavior of the US economy at quarterly frequency during the period 1947–1992. Data on real GDP, consumption, investment, government spending were obtained from US NIPA, while the time series for hours was obtained from the Bureau of Economic Analysis (BEA). We follow McGrattan, Rogerson and Wright (1997) and set the capital share  $\alpha = 0.36$  and the depreciation rate of physical capital  $\delta = 0.0235$  per quarter. This produced a discount factor of  $\beta = 0.9898$ , which is consistent with one percent quarterly return on equity. The average effective tax rates on labor and capital in the US economy over the period of investigation are  $l = 0.25$  and  $k = 0.5$ , respectively.

The persistence parameter and the standard deviation of technology process in the market sector were obtained by first obtaining the Solow residual, and then subtracting a linear trend. The detrended series are then approximated with an AR (1) process, from which we obtain the estimated persistence and volatility of the

technical progress. Due to data limitations, for the stochastic process of non-market technology, we adopt the estimates for the market technology. Lastly, parameters of government consumption were also obtained by running an AR(1) regression. Model parameters are summarized in Table 1 below. In the following section, we will simulate the model and compare theoretical to empirical second moments.

**Table 1. Calibration parameters**

Parameter	Value	Denition	Method
$\beta$	0.9898	Discount Factor	Calibrated
$\alpha$	0.3600	Capital share	Set
$\delta$	0.02350	Depreciation rate on physical capital	Set
$\tau^l$	0.2500	Average effective tax rate on labor income	Data avg.
$\tau^k$	0.5000	Average effective tax rate on labor income	Data avg.
$\rho^m$	0.9600	AR(1) persistence parameter, TFP market sector	Estimated
$\rho^h$	0.9600	AR(1) persistence parameter, TFP home sector	Set
$\rho^g$	0.9600	AR(1) persistence parameter, gov. cons.	Estimated
$\epsilon^m$	0.0837	st. dev, TFP market sector	Estimated
$\epsilon^h$	0.0837	st. dev, TFP home sector	Set
$\epsilon^g$	0.0210	st. dev, TFP home sector	Estimated

Source: own compilation.

### 3. Model Simulation

Results from the calibration are summarized in Table 2 on the next page and compared to a model with scal shocks only and a model with home production but without government sector as reported in Hansen and Wright (1992). Consumption relative to output varies about the same as in the data. In addition, investment varies too little compared to the data, due to the fact that in the model home production does not use capital. Hours vary about the same. In terms of getting correlations, the model performs much better, especially with the contemporaneous correlation of wages and hours. Fiscal shocks alone or home production alone bring the correlation down only to 0.49. This is in line with the findings in Christiano and Eichenbaum (1992), Hansen and Wright (1992), and McGrattan, Rogerson and Wright (1992). In general, our model capture contemporaneous correlations between market hours and wages much better than the alternatives. Note that in this class of models, the hourly market wage rate is proportional to the labor productivity, which is measured as output per market hour worked ( $y/h_m$ ). Then, when we log-linearize the model around the steady-state to study business cycle fluctuations, the variability of labor productivity is identical to that of the wage rate.

**Table 2. Cyclical Properties of US and Model-Generated Time Series**

<b>Rel. Moments</b>	<b>US Data</b>	<b>Baseline Model</b>	<b>Indiv.Mkt Hrs</b>	<b>Fiscal Shocks</b>	<b>Home Production</b>
$\sigma_c/\sigma_y$	-	0.79	0.88	-	-
$\sigma_{c_m}/\sigma_y$	0.45	0.55	0.69	0.54	0.51
$\sigma_{c_n}/\sigma_y$	-	1.84	1.56	-	-
$\sigma_i/\sigma_y$	2.78	1.5	1.76	3.08	2.73
$\sigma_{h_m}/\sigma_y$	0.78	0.87	0.53	0.55	0.75
$\sigma_h/\sigma_y$	-	0.23	0.9	-	-
$\sigma_w/\sigma_y$	0.57	0.54	0.65	0.61	0.39
<b>HH Survey</b>					
$\sigma_{h_m}/\sigma_w$	1.37	1.62	0.81	0.9	1.92
$\sigma_h/\sigma_w$	-	0.43	1.37	-	-
$corr(h_m, w)$	0.07	-0.07	0.4	0.49	0.49
$corr(h, w)$	-	0.43	0.67	-	-
$corr(h, y)$		-0.03	0.52	-	-
$corr(h_m, y)$	0.87	0.84	0.81	0.55	0.75
$corr(h_n, y)$	-0.7	0.04	-	-	
$corr(w, y)$	0.58	0.48	0.87	0.61	0.39
$corr(c, y)$	-	-0.35	0.47	-	-
$corr(c_m, y)$	0.71	0.97	0.92	0.54	0.51
$corr(c_n, y)$	-	-0.59	0.12	-	-
$corr(i, y)$	0.73	0.85	0.87	3.08	2.73

Source: own compilation.

Next, we extend the model economy by introducing indivisible market hours. The conjecture is that it will affect the dynamics of the model through introducing a difference in the marginal disutility of work in the public versus the private sector. Having indivisible market hours can be interpreted as the other extreme case. In such a model, employment is the only source of fluctuation in total hours, while in the data it accounts for only two-thirds.

#### **4. Indivisible Market Hours Extension**

The baseline case is extended now to Hansen-type economy: combining indivisible labor (using Rogerson’s (1988) idea of employment lotteries) with home production. More specifically, the individual household will be allowed to supply any number of hours in the non-market sector, but will work full-time in the market sector, if it decides to work in the official sector. This discrete labor supply choice in the market sector would increase the volatility of output at the aggregate level, and decrease the correlation between hours and productivity, given the second

shock. The novelty is that household can supply market hours in a discrete fashion, while it can work any number of hours at home. The resulting discounted aggregate utility function becomes:<sup>4</sup>

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln(c_t) - \theta h_{mt} + \ln[1 - h_{nt}] \right\}, \quad (20)$$

where  $\theta$  is the (constant) disutility of working an additional hour in the market sector, and the relative weight on working in the non-market (home production) sector is unity. Note also that the number of hours supplied in the non-market sector is a function of the proportion of households supplying a full-time working week in the market sector. (For the sake of brevity we suppress that functional dependence.)

Again, the household takes  $\{w_t, r_t, \pi_t, g_t\}_{t=0}^{\infty}$ , and the initial condition for capital  $k_0$  as given, and chooses  $\{c_{mt}, k_t, h_{mt}, h_{nt}\}_{t=0}^{\infty}$  optimally to maximize (20) s.t. (2) – (6). This produces the following first-order conditions, which have qualitatively identical interpretation as in the divisible-market-hours case (note that only FOCs for market and home hours change):

$$c_{mt} : \frac{ac_{mt}^{b-1}}{[ac_{mt}^b + (1-a)c_{nt}^b]} = \lambda_t, \quad (21)$$

$$h_{nt} : \frac{(1-a)(A_{nt}h_{nt})^{b-1}}{[ac_{mt}^b + (1-a)c_{nt}^b]} = \frac{1}{1-h_{nt}}, \quad (22)$$

$$h_{mt} : \theta = \lambda_t(1-\tau^l)w_t, \quad (23)$$

$$k_{t+1} : \lambda_t = E_t \lambda_{t+1} [1 + (1-\tau^k)r_{t+1} - \delta], \quad (24)$$

$$TVC : \lim_{t \rightarrow \infty} \beta^t \lambda_t k_{t+1} = 0, \quad (25)$$

Results from the calibration exercise with indivisible market hours are shown in Table 2, again compared to a model with scal shocks only and a model with home production but without government sector, as reported in Hansen and Wright (1992). Consumption volatility is too high, investment varies more but still less than in the data, hours vary less than in the data due to all change resulting from employment. Correlations are in line correlation of hours and wages is 0.4, much better than in the original RBC model but still high. This is because of the lottery – a household wants to work more after the negative wealth effect and has to choose to supply hours in the market sector and/or home production one, but may not be chosen to work in the market. This offsets some of the negative effect on the cor-

<sup>4</sup> In Vasilev (2015a) we show how lotteries can be used to convexify consumption sets, and how aggregation of individual preferences works.

relation of hours and wages. Again, our model captures contemporaneous correlations between the variables much better than the alternatives.

## 5. Conclusions

This paper showed that a modified real business cycle (RBC) model, one that includes home production and fiscal spending shocks, can solve one of the RBC puzzles and generates zero correlation between wages and hours. In addition, the micro-founded model provided a sound theoretical model to analyze fiscal policy in a neoclassical framework and was able to capture many aspects of the data that the benchmark RBC model was missing. For future research, we plan to introduce capital in the home production and see how and if that changes the statistics of the model economy.

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