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**Working Paper**

## Discriminatory versus uniform-price auctions

MNB Occasional Papers, No. 111

**Provided in Cooperation with:**

Magyar Nemzeti Bank, The Central Bank of Hungary, Budapest

*Suggested Citation:* Monostori, Zoltán (2014) : Discriminatory versus uniform-price auctions, MNB Occasional Papers, No. 111, Magyar Nemzeti Bank, Budapest

This Version is available at:

<http://hdl.handle.net/10419/141995>

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Zoltán Monostori

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The views expressed here are those of the authors and do not necessarily reflect the official view of the central bank of Hungary (Magyar Nemzeti Bank).

MNB Occasional Papers 111

**Discriminatory versus uniform-price auctions**

(Diszkriminatív áras és egyenáras aukciók)

Written by Zoltán Monostori\*

Budapest, February 2014

Published by the Magyar Nemzeti Bank

Publisher in charge: Eszter Hergár

Szabadság tér 8–9., H–1850 Budapest

[www.mnb.hu](http://www.mnb.hu)

ISSN 1585-5678 (online)

\* This paper was discussed at the November 15, 2012 meeting of the Monetary Forum, it has been presented at several conferences and was published in Hungarian in the *Közgazdasági Szemle* in October 2013 (Vol. 60, pp. 1048–1074.).

I am most grateful to Edina Berlinger, my Ph. D. thesis supervisor and my colleagues Judit Antal, Csaba Balogh, Csaba Csávás, Áron Gereben, Viola Monostoriné Grolmusz and Zoltán Reppa for their suggestions and assistance. I am indebted to Gyula Magyarkuti, who taught me mathematical auction theory in a Ph. D. course. I am grateful to Alexandra Szatmári and Péter Bárczy for their valuable comments.

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# Contents

<b>Abstract</b>	5
<b>1 Introduction</b>	6
<b>2 Theoretical results</b>	8
2.1 Theorems for single-unit auctions	8
2.2 Misconceptions in connection with multi-unit auctions	9
2.3 Bid curves submitted in multi-unit auctions	10
2.4 Winner's curse	10
2.5 Risk aversion	11
2.6 Fog of war	11
2.7 Secondary market, forward market, collusion	12
2.8 Summary of theoretical results	13
<b>3 Laboratory experiments</b>	15
<b>4 Non-laboratory empirical evidence</b>	16
4.1 Empirical evidence of uniform-price auctions	16
4.2 Empirical evidence of discriminatory-price auctions	16
4.3 Empirical comparison of uniform-price and discriminatory-price auctions	18
<b>5 International practice</b>	22
<b>6 Summary and conclusion</b>	24
<b>References</b>	27
<b>Appendix</b>	32



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# Abstract

The purpose of this paper is to compare the two auction techniques (discriminatory and uniform-price auctions) most commonly used for the sale of securities. Literature tends to analyze methods from the aspect of the expected revenue from the auction. Theoretical models arrive at different rankings for expected revenue; however, they do reveal the relationship between the bids submitted and the auction technique. These results are confirmed both by 'laboratory' experiments and the empirical evidence of real-world auctions. The latter may also provide a robust answer to the question of expected revenue; the uniform-price format coming out as the more beneficial for the Treasury. Still, at present the global majority of issuers of government bonds use the discriminatory-price format and central bank instruments also tend to be sold in this format. This is because issuers may have considerations other than expected revenue.

**JEL:** C78, D44, D53, D82, H63.

**Keywords:** auction, central bank auctions, treasury auctions, discriminatory auctions, uniform-price auctions.

## Összefoglaló

A tanulmány célja az értékpapírok esetében leggyakrabban alkalmazott két aukciós eljárás (az egyenáras és a diszkriminatív áras aukció) összehasonlító elemzése. A szakirodalom elsősorban az aukció várható bevétele szempontjából elemzi a módszereket. Az elméleti modellek eltérően rangsorolják a módszerek várható bevételét, viszont jól megmutatják, hogy miként függenek a beadott ajánlatok az aukciós eljárástól. Ezeket az eredményeket „laboratóriumi” kísérletek és valós aukciók empirikus vizsgálatai is megerősítik. Utóbbiak ráadásul robusztus választ adnak a várható bevétel kérdésére is, mégpedig az egyenáras forma előnyére. Mégis, jelenleg globálisan többségben vannak a diszkriminatív áras módszert használó állampapír-kibocsátók, és a jegybanki eszközöket is többnyire ilyen módszerrel értékesítik. Ez azzal magyarázható, hogy a kibocsátó más szempontokat is figyelembe vehet a várható bevételen kívül.



# 1 Introduction

The purpose of the paper is to give a comprehensive overview of literature to discuss if the uniform-price or the discriminatory auction format is the better allocation mechanism under given conditions. This review is particularly topical as, following similar steps by other treasuries, the public debt management agency of a country in the Central-Eastern-European region, Poland, switched to the uniform-price system in January 2012. The paper concludes with a policy recommendation on whether it is expedient for the Government Debt Management Agency (ÁKK) to continue with discriminatory auctions given the current state of the government bond market and the primary dealer system. The analysis may also be useful in reconsidering the form of auction for the central bank instruments introduced during the crisis and for the design of the format for the sale of any new instruments to be launched in the future.

In most countries around the world, the issued government securities are allocated through auctions, even though subscription-based syndicated issues did survive for quite some time in England and Japan, for instance. The across-the-board popularity of auctions is attributable to the fact that they assure the scheduled, regular, safe financing of public debt at a low cost and at a close-to-market price.

The two auction methods most frequently used in this area are discriminatory and uniform-price auctions. In both cases, the issuer ranks the bids received for the homogeneous products by price, in a descending order. Then it accepts bids in that order, going from highest to lowest, until the intended volume is taken up or all the bids are accepted. (That is, the highest bids for the given volume are accepted.) If at the lowest accepted price the quantity demanded is higher than the residual quantity of issuable products, then the residual quantity is distributed among bidders according to the proportions of their submitted bids at this price. The two formats differ in that while in discriminatory-price auctions financial settlement occurs at the different prices indicated in the bids, in uniform-price auctions the winning bidders all end up paying the price indicated in the highest rejected bid.

These two auction mechanisms have the following impact on the expected revenue of the auctioneer: while participants may be assumed to submit higher bids for uniform-price auctions<sup>1</sup>, the average price of the accepted bids at discriminatory auctions may be increased by price discrimination (Chart 1). Thus a switch to the uniform-price auction method may be successful in terms of expected revenue if the area between points BCD is larger than the opportunity cost DEF, therefore the revenue from the uniform-price auction (ACFG rectangle) is greater than the revenue from the discriminatory auction (ABEG trapezoid).

Even though literature tends to examine auctions mostly from the aspect of the expected revenue of the auctioneer, we should note that the Treasury may have other considerations as well<sup>2</sup>. These may include efficiency (i.e. whether the goods end up at the participants that place the highest value on them), curtailing the possibility of collusion or other forms of manipulation, promoting competition (i.e., bringing the average auction price closer to the market price) or increasing the number of primary dealers. In the case of central bank instruments, diverting market prices may also be a priority.

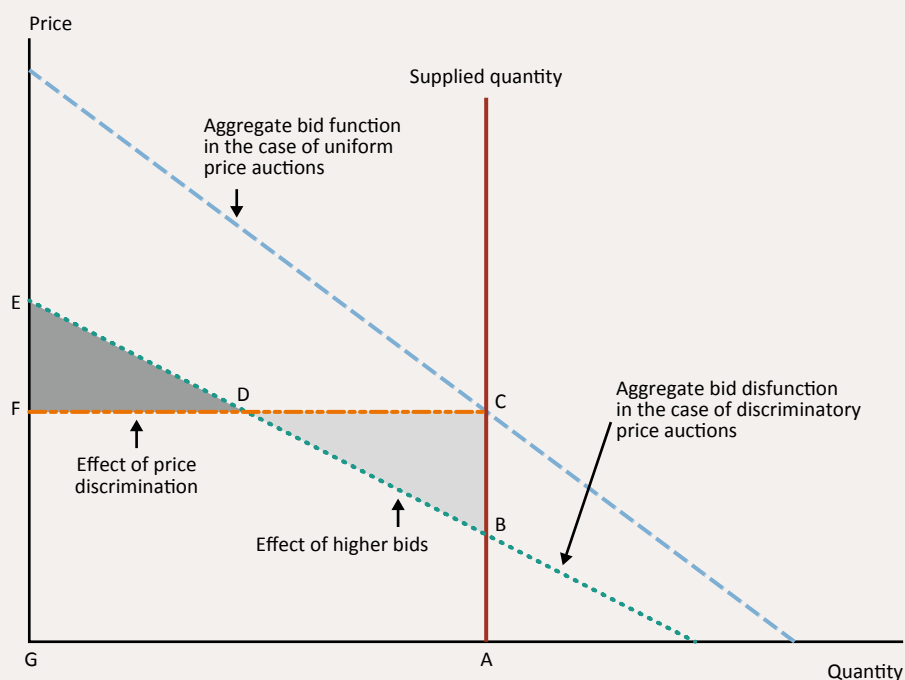
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<sup>1</sup> Almost every accepted bidder pays less than their bid.

<sup>2</sup> As a very simple approximation for the effect on the expected revenue, we can state the following: the amount of the Hungarian Forint denominated government debt is approximately 13,000 billion HUFs (FX denominated debt is not allocated through auctions nowadays in Hungary: FX-bonds are allocated subscription-based at road shows, loans are naturally not auctioned). If another auction method could reduce the yields of the newly issued government debt, every basis point gained in the yearly yields could save around 0.01 percent for the state in the long term (when every previously issued paper ran out), that is *ceteris paribus* 1.3 billion HUFs yearly.

Later, in the chapter about the real-world empirical evidences, we will see that most authors have found a difference around 1-3 basis points between the revenue of the different auction methods. This might be on the one hand a significant amount for the state; on the other hand, this might be on the same order of magnitude as some distractions (like the change in liquidity premium which might also be affected by the changing market structure) or the estimation uncertainty.

**Chart 1**  
**Expected revenue from uniform-price and discriminatory-price auctions**



Source: own chart, based on (Kondrát, 1996).

Three main practical applications are generally examined in literature where a large volume of homogeneous goods are auctioned off: electricity auctions (e.g. Hudson, 2000), IPOs (e.g. Aussenegg et al., 2006; Kandel et al., 1999) and Treasury auctions. In our paper we focus on the latter, summarizing the literature on Treasury auctions.

This paper is all the more topical as in recent years several debt management agencies have switched from discriminatory-price auctions to uniform-price arrangements to sell government bonds (e.g. Poland in 2012, Korea in 2000 while Italy made the change in respect of government bonds already in 1988 in the wake of an experiment in 1985.) One might ask: should Hungary also make the change?

Furthermore, the comparison of auction methods may also be relevant because central banks tend to use the discriminatory method to auction their instruments; this is also the MNB's format of choice for all its auctioned instruments (1-week and 3-month FX swaps, 6-month variable-interest collateralized loans, FX auction). We should state right in the beginning, though, that different considerations may be relevant for the sale of central bank instruments and government bonds. As another motivation, the most recent Hungarian study of this subject<sup>3</sup> focused primarily on models based on the unit demand assumption (Kondrát, 1996), whereas researchers have demonstrated<sup>3</sup> that these findings are often not applicable to all of the multi-unit auctions.

<sup>3</sup> Important contributions to the Hungarian tradition of research into auctions include Kondrát (1996) as well as Szatmári (1996) and Eső (1997).

# 2 Theoretical results

## 2.1 THEOREMS FOR SINGLE-UNIT AUCTIONS

There is an ever more marked distinction in literature between single-unit auctions (e.g. art treasures, oil fields, mobile phone frequencies) and multiple-unit auctions (e.g., bonds, stocks, electricity) as the two scenarios may provide different incentives for the behavior of bidders. A number of papers (e.g. Binmore and Swierzbinski, 2000; Das and Sundaram, 1997) start the presentation of theoretical models with single-unit auctions. We also need to lay down some required theorems that will become important mostly for the interpretation of our results concerning multi-unit auctions.

The theorems described below apply to the simplest single-unit model: the non-repeatable auction of a single, indivisible, unique consumption (rather than investment) good where participants can be described by identical parameters, common priors, similar estimators and risk-neutral utility functions, however, their valuations of the good can be described with independent identically distributed random variables. During the auction the auctioneer has no discretion, the rules are set in advance, bidders know the rules and the identical distribution function for the valuations, then they make decisions to maximize their profits. In this scenario, for consumer goods the profit for losing bids is zero while for winning bids it is the difference between the valuation and the price paid. (Later, in the case of models assuming a secondary market, the profit for the winning bidder will be the difference between the selling price achievable on the secondary market and the price paid at the auction.<sup>4</sup> If the secondary market is introduced, the auction becomes a common-value auction.) The following five theorems are well-known in this field, and we will rely on them in later chapters. The first three theorems are illustrated in Chart 2.

- In the case of second price, sealed bid auctions the genuine, honest valuation should be submitted as the bid. (The optimal bid function<sup>5</sup> is the identity function, i.e., the 45 degree half-line.)<sup>6</sup> (Krishna, 2009, p. 13.)
- In the case of a first-price sealed-bid auction a bid below the valuation is worth submitting for each valuation. (The optimum bid function on a first price auction yields a value below the identity function for any number of participants, since bidding the true valuation would rule a positive profit out.) (Krishna, 2009, pp. 14–16.)
- If we relax the assumption of risk-neutrality: in the case of a first-price sealed-bid auction the ‘cowards are more aggressive’ (i.e., at the same valuation, the more risk-averse player submits a higher bid because this way he will win a lower value but with higher probability). This also implies that in the case of risk-aversion, the expected revenue in a first-price auction is greater than that in a second-price auction (Krishna, 2009, pp. 38–39.).
- Revenue equivalence theorem: if a few (not overly strict) additional conditions are satisfied<sup>7</sup>, the expected revenue from the auction does not depend on the auction method (that is, for instance, the expected revenue from first-price and second-price auctions is the same, but the theorem has more general application) (Krishna, 2009, p. 28). (See the proof in the appendix.)

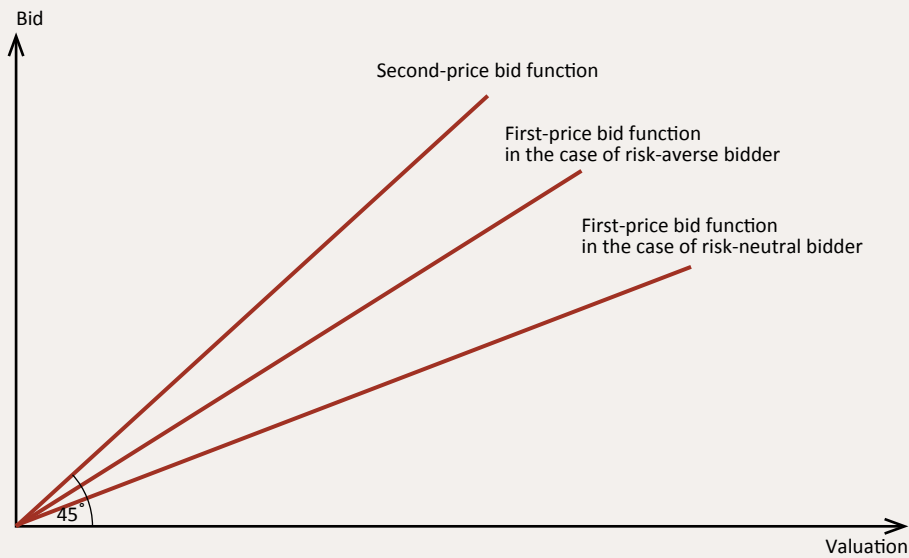
<sup>4</sup> However, this is only an assumption of the models. In reality, by a primary dealing system, primary dealers are not only motivated by the basis points between the selling price achievable on the secondary market and the price paid at the auction. They have rights and obligations as primary dealers, which might also influence their behavior.

<sup>5</sup> The bid function gives the bid submitted as a function of the valuation.

<sup>6</sup> This is because if a bid below the valuation is submitted, then, in contrast to the ‘honest’ bid, the participant gives up on cases where he could have closed the auction with a profit had he told the truth, while in the case of a bid above the valuation, a negative profit becomes possible. Everything else would be unchanged, so bidding the truth valuation is a weakly dominant strategy.

<sup>7</sup> Conditions: the theorem applies to standard auctions (that is, the highest bidder wins) and the strictly monotonous increase of the bid function is a condition in such a way that participants submit a zero bid for a zero valuation while above that level they always submit a higher bid for a higher (private) valuation. (Their valuations are independent and identically distributed.) A wit point of view: (Klemperer, 2004).

**Chart 2**  
Relative position of bid functions



Source: author's chart.

- Despite the equality of expected revenues, the different auction methods lead to different results at a number of points: e.g., the standard deviation of the expected revenue of first-price auctions is lower than that of second-price auctions (Krishna, 2009, p. 19–21).

## 2.2 MISCONCEPTIONS IN CONNECTION WITH MULTI-UNIT AUCTIONS

Many economists have tried to apply the theorems stated for single-unit auctions more generally to multi-unit auctions as well, by assuming a similarity of first-price and discriminatory auctions and of second-price and uniform-price auctions. Undoubtedly, there is some similarity but the imperfect separation of single-unit and multi-unit auctions has led to a number of misunderstandings. The most common misconception is that bidders submit their real valuation as the bid in uniform-price auctions as well. The erroneous statements below are critically quoted, inter alia, by Ausubel and Cramton (2002) and Binmore and Swierzbinski (2000).

Milton Friedman told the Wall Street Journal: 'A [uniform-price] auction proceeds precisely as [a discriminatory auction] with one crucial exception: All successful bidders pay the same price, the cut-off price. An apparently minor change, yet it has the major consequence that no one is deterred from bidding by fear of being stuck with an excessively high price. You do not have to be a specialist. You need only know the maximum amount you are willing to pay for different quantities.' (Friedman, 1991).

In an interview with the New York Times (15 September 1991, 3:13) Merton Miller explained his view that in uniform-price auctions there is no incentive for bid shading: 'All of that is eliminated if you use the [uniform-price] auction. You just bid what you think it's worth.' (Miller, 1991).

The Joint Report on the Government Securities Market written for the US Treasury, the SEC and the Fed, lay the ground for changing the auction format for government securities, and also started from the aforementioned misconception: 'Moving to a uniform-price method permits bidding at the auction to reflect the true nature of investor preferences. [...] In the case envisioned by Friedman, uniform-price awards would make the auction demand curve identical to the secondary market demand curve.' (Department of the Treasury, 1992).

## 2.3 BID CURVES SUBMITTED IN MULTI-UNIT AUCTIONS

However, several authors (e.g. Fabra, 2003; Vickrey, 1961) have demonstrated that in the case of multi-unit auctions the uniform-price system does not guarantee bids to show real valuations. What is more: the revenue equivalence theorem is not satisfied in the case of multi-unit auctions. If the simplest single-unit model discussed in point 2.1 is modified, *ceteris paribus*, so that a multi-unit auction is held and several types of bidders participate in the auction, then the discriminatory-price auction may result in higher expected auctioneer revenue than a uniform-price format.

This is because on the one hand participants in a discriminatory auction simply submit relatively flat bid curves that have a negative slope based on their marginal profit, which results in bids close to the market price in a competitive market. On the other hand, Back and Zender (1993), LiCalzi and Pavan (2005), Maxwell (1983) and Wilson (1979) have demonstrated, *inter alia*, that in uniform-price auctions a few large actors known to be well-informed may submit steep bid curves, thereby considerably increasing the marginal cost of other participants (because they would risk their additional demand significantly raising the price), reducing competition and depressing the final price. These results appear to be robust also to the modifications of Ausubel and Cramton (2002), Biais and Faugeron-Crouzet (2002) Engelbrecht-Wiggans and Kahn (1998) and Noussair (1994).

Ausubel and Cramton describe the phenomenon of steep bid curves in the case of uniform price auctions as follows: when the model enables a multi-unit bid for the participants, after the first bid (made on the honest valuation) every additional bid raises the expected price to pay for earlier own bids with a positive probability. Therefore, the bid curves will be steeper than the honest valuations, as the marginal revenue curve of a monopolist is steeper than its demand curve: at a minimal quantity, the two curves meet at the same price, but at every additional quantity the bidden price will be below the honest valuation of the additional unit. As a result of the steep bid curve of large actors, efficiency may also be compromised since in certain cases smaller participants may purchase goods having a lower valuation relative to the large participants. On the other hand, uniform-price and discriminatory auctions cannot be ranked by efficiency. The extent of the demand reduction is effected by the market power of the biggest participants (Ausubel and Cramton, 2002).

Viswanathan and Wang (2000) argue that the auction format yielding the highest expected auctioneer revenue depends on the circumstances: if non-competitive bids are submitted for very large amounts<sup>8</sup>, the steep aggregate demand increases the expected revenue from uniform-price auctions while otherwise the discriminatory-price format yields higher revenues for the Treasury. Building on the work of Back and Zender (1993), Wang and Zender (2002) demonstrated that the uniform-price format does not dominate over the discriminatory system or vice versa, that is, either auction form may be more profitable than the other depending on the parameters.

This was a major theoretical achievement because for a long time the so-called Friedman argument prevailed, considering the uniform-price format to be dominant due to the so-called 'winner's curse'<sup>9</sup> (Friedman, 1959)

## 2.4 WINNER'S CURSE

In the context of the winner's curse the conditions of uniqueness and the consumption purpose of the auctioned goods are relaxed and we assume that participants bid to obtain investment goods at a price below the secondary market price. We also assume that there are several types of bidders in the market. Large actors (either because of their better analytical capacities or their greater role in the primary or secondary market) can predict the post-auction secondary market price more accurately than smaller ones; the latter shade their bids considerably due to the uncertainty of the expected secondary market price (Ausubel, 1997).

In this event, in a discriminatory-price auction smaller actors may fear that their valuations (and bids) may be significantly higher than the market valuation, consequently they may sustain large losses on their winning bids. In the case of uniform-

<sup>8</sup> These bids are awarded the same volume irrespective of the eventual price and bidders pay the average yield at the auction. They have priority over competitive bids that specify yields.

<sup>9</sup> Ausubel tried to split the concept into a single-unit scenario (winner's curse) and a multi-unit scenario (the phrase he suggested was 'champion's plague') (Ausubel, 1997). Though the latter phrase is also used by others, literature tends not to separate the two cases and uses the term 'winner's curse' for both.

price auctions, however, the auction price in this model will not be significantly different from the post-auction secondary market price, and thus smaller actors may feel more confident to participate in the auction. The increased volume demand may send the revenue expected from uniform-price auctions above that of discriminatory-price auctions (Bolten, 1973; Friedman, 1959; Milgrom and Weber, 1982)<sup>10</sup>.

In connection with the entry of smaller actors in the market, it is worth differentiating between markets depending on whether they have a primary dealer system. While in a number of countries, including the Hungarian government bond market, such a system is in place, and thus very small participants could not enter the market even if the uniform-price system were used, Germany, for instance, has no such system, therefore more actors could be brought into the market by a switch to the uniform-price method. Furthermore, the models examining the winner's curse also fail to take into account the possibility of non-competitive bids, in which case the bidder only states the volume and receives the securities at the average auction price. The use of non-competitive bids is also common in the Hungarian government bond market, which may also mitigate the power of the winner's curse to restrain bids.

## 2.5 RISK AVERSION

Some studies, like the article of Harris and Raviv (1981) in which the benchmark model is modified by risk aversion of the bidders and multi unit auctions are enabled, also had a profound impact on literature.

Authors in these studies often use several theorems that have only been proven for single-unit auctions. They assume that the revenue equivalence theorem applies to risk neutral participants. The introduction of risk aversion should not change the optimal strategy (of 'truthfulness': submitting a realistic valuation) in the case of uniform-price auctions while in discriminatory-price auctions, based on the 'cowards are more aggressive' principle, participants will raise their bids to increase their chances to obtain large volumes at a lower profit compared to a risk-neutral scenario. Thus the higher demand results in a higher auction price in the discriminatory-price scenario (Kondrát, 1996).

Several authors note that the introduction of risk aversion in itself is not necessarily legitimate. First, the profit achieved on auction bids is generally negligible compared to the total balance sheet of the bidding firm, thus risk plays a minor part in their decision. Second, the motivations of the person deciding about the auction bid are not necessarily the same as the motivations of the investor; therefore the principal-agent problem may raise additional questions in the context of risk aversion.

## 2.6 FOG OF WAR

According to Binmore and Swierzbinski (2000), the fog of war is the danger that other players may not act rationally and/or the game has more than one equilibrium, which may make participants cautious. The simplest theoretical example for the fog of war is in the case of private-value, single-unit auctions of consumption goods. In this scenario the second-price system may be more favorable to bidders because irrespective of any other factors, they always need to submit their private valuation as the bid. This certainty showing the right bidding strategy may also intensify auction participation; therefore second-price auctions may be advantageous for the auctioneer as well.

To give an empirical example for the fog of war, there is evidence from auctions of investment goods with uncertain value (e.g., the auction of an oil field with an unknown quantity of oil) that the ascending-price (English, open ascending second-price) auction results in a higher price than the second-price, sealed-bid auction which is considered to be strategically equivalent in theoretical models. This is because bidders at auctions of investment goods become less cautious if they see that their valuation is not far from the valuation of others. Empirical studies of government bond markets (e.g. Elsinger et al., 2012) confirm that bids are significantly affected by market uncertainty.

As uncertainty may restrain willingness to participate, the designer of an auction is well advised to make the auction predictable (e.g. by the accurate specification of rules in advance) to increase demand.

<sup>10</sup> As another important achievement, the article of Milgrom and Weber (1982) introduced into academic thinking the concept of correlated-value auctions to supplement private-value and common-value auctions.

However, the preference of second-price auctions cannot be extended to multi-unit auctions as preference of the uniform-price format. Binmore and Swierzbinski argue that uniform-price auctions entail more uncertainty than discriminatory-price auctions (more equilibria possible, a new entrant may cause major price swings, the quantity to be sold can often be changed, participants may employ mixed strategies, and assuming private valuation there is always greater uncertainty as to the outcome of the game), which may reduce demand (Binmore and Swierzbinski, 2000).<sup>11</sup>

## 2.7 SECONDARY MARKET, FORWARD MARKET, COLLUSION

Bikhchandani and Huang (1993) assumed the existence of a secondary market in a model that contains two types of investors: the participants that are also present in the primary market have considerable analytical capacities while final investors only trading on the secondary market are price takers.

The high auction price also results in higher secondary market prices as it is possible to profit from existing own-account (long) positions; consequently, some major participants may have an interest in raising prices. This price raising strategy is much cheaper for a bidder (or a cartel) to implement in a uniform-price auction as a relatively small additional own demand may raise the price for each winning bidder. Consequently in this model the uniform-price auction yields higher expected revenue (in a single period).

In the longer term, however, it may be a more important consideration that price-increasing manipulations may undermine the efficiency of auctions<sup>12</sup> and deter potential bidders, thereby reducing demand.

A similar model was presented by Viswanathan and Wang (2000), but their version also contained the when-issued forward market<sup>13</sup>, that is, participants could buy or sell securities at predetermined prices. In the United States this market is very important as numerous institutional investors (e.g. pension funds) purchase the quantities they need in advance; consequently, many bidders start the auctions with a short position (as they have not yet purchased what they have sold). In the aforementioned model there are two forces at play: on the one hand, the expected revenue of the auctioneer is higher because the items already sold may increase risk tolerance, while on the other hand the expected revenue may be reduced by the absence of high-valuation actors from the auction as their consumer surplus is absorbed by the participants of the primary market. According to the model, the resultant of these two forces is more favorable for the issuer in the case of discriminatory-price auctions.

The fact that many actors start bidding in a short position if a when-issued forward market exists may make puffing (price-enhancing manipulation) even more attractive because if the puffer manages to obtain a significant part of the securities issued, the actors that are stuck with their short positions will be forced to buy<sup>14</sup>, driving prices even higher (short squeeze). A similar manipulation of Salomon Brothers was a major contributor to the launch of an experiment that resulted in the US Treasury introducing the right to change to quantity sold, post-auction re-issuance (where actors left in short positions may buy) and switching to uniform-price auctions. After the so-called 'Salomon squeeze' it was demonstrated empirically that short squeeze was a frequent occurrence in the US market before the reform (Sundaresan, 1994).<sup>15</sup> The uniform-price format was adopted despite the fact that price-enhancing manipulation is cheaper to achieve in the case of uniform-price auctions than in the discriminatory-price format, where free-riders would also enjoy the benefits (Bikhchandani and Huang, 1993; Nyborg and Strebulaev, 2004). The US reform should rather be interpreted as intending to mitigate the winner's curse through the introduction of uniform-price auctions, expecting smaller actors to enter the primary market as a result. Curbing manipulation was much better served by the right to change the volume sold and the introduction of re-issues.

<sup>11</sup> However we can also argue against the discriminatory auction. The differences and biases of the valuations may have an effect on the expected revenue. In the case of government bonds, bidders may have insider information, and their knowledge about each other (i.e. utility functions, risk aversion vs. profit maximizing) might be asymmetric. These could result in stronger bid shading and underbidding, which might decrease the expected revenue of the discriminatory method.

<sup>12</sup> This means that at the end of the day not all bonds are awarded to investors with the highest valuation.

<sup>13</sup> When-issued forward markets were also studied by (Aussenegg et al., 2006; Bolten, 1973; Das and Sundaram, 1997).

<sup>14</sup> These involuntary post-auction purchases are called the loser's nightmare in literature.

<sup>15</sup> However, nowadays the repurchase agreements developed the opportunity to defend against short squeezes.



It should be noted that there are also price reducing cartels. In the case of uniform-price auctions collusion requires only the cheap end of the bid curve to be in the low price range. Thus any rogue cartel member can obtain only a small additional amount by submitting a higher bid. In the case of discriminatory-price auctions, however, the price-reducing cartel submits a flat bid curve, thus a rogue cartel member may win the entire volume at a slightly higher price. As price-cutting cartels are more rare (or at least short lived) under discriminatory-price arrangements, in this model this method yields higher expected revenues for the Treasury (Daripa, 2001). We should also note that it is in the interest of the issuer to prevent the rouge cartel member from being exposed, which is an argument for limiting transparency.

However, manipulation can be very effectively combated by reserving the right to change the issued quantity (Back and Zender, 2001; Damianov and Becker, 2010; Damianov et al., 2010; Keloharju et al., 2005; Kremer and Nyborg, 2004a, 2004b); indeed, in certain cases it can be eliminated altogether (McAdams, 2007).<sup>16</sup>

## 2.8 SUMMARY OF THEORETICAL RESULTS

On the whole, the main argument against discriminatory-price auctions is that the fear of the winner's curse makes actors with limited analytical capacities shade their bids (lest they win with a too high price) or even stay away from the market, which reduces demand. On the other hand, discriminatory-price auctions offer less room to powerful market players to exercise their market power than uniform-price auctions, which results in steep bid curves. Minor arguments, possibly with weaker theoretical foundations, for discriminatory-price price auctions include the model that introduced risk aversion and the fog of war.

Even though there are several arguments for the higher expected revenue from discriminatory-price auctions, a number of studies consider the winner's curse to be the most dominant argument. Little is known about the resultant of these effects; indeed, it may vary depending on the circumstances which auction format brings the highest expected revenue to issuers.

An agent-based simulation reached a similar conclusion: in this study, bidders were able to learn and they were out to maximize long-term profits. Uniform-price and discriminatory-price auctions yield systematic differences in their outcomes and that difference is robust. In respect of the expected revenue, when the bid-to-cover ratio is low, the discriminatory-price format is more favorable for the Treasury. Where the bid-to-cover ratio is high, the uniform-price system is more advantageous. The 'cross-point' (i.e., the level of bid-to-cover ratio where the uniform-price format becomes more advantageous) is sensitive to parameters, which explains the differences in research results (both analytical and empirical) (Koesrindartoto, 2004). A study (examining the telecommunications auctions in Europe) reaches the same conclusion: an auction format that works in one market may not be appropriate for another (Klemperer, 2002). Another paper concludes

**Table 1**  
Theoretical studies comparing uniform-price and discriminatory-price auctions

Assumption different from the benchmark model	A paper focusing on the problem	Auction method with the higher revenue
Multi-unit auction, steep bid curves	(LiCalzi and Pavan, 2005)	Discriminatory
Winner's curse	(Friedman, 1959)	Uniform-price
Risk aversion	(Harris and Raviv, 1981)	Discriminatory
Fog of war	(Binmore and Swierzbinski, 2000)	Discriminatory
Secondary market, forward market, collusion	(Viswanathan and Wang, 2000)	Discriminatory

Source: author's compilation.

<sup>16</sup> There are situations, however, when the ex post modification of the quantity offered is not possible. Examples include IPO auctions (transparency is required in respect of the number of shares) or certain electricity auctions (no time to change the issued quantity). The study of LiCalzi and Pavan (2005) recommends that in such cases the issued quantity is changed according to a predetermined function. There should be a minimum price not much below the expected price, below which no bid is accepted; then as the auction price increases, so should the volume awarded increase (LiCalzi and Pavan, 2005).



that uniform-price and discriminatory auctions cannot be ranked definitively based on analytical studies; therefore it may be appropriate to approach this issue on an empirical basis (Ausubel and Cramton, 2002). This is what we will do in the following parts of this paper.

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## 3 Laboratory experiments

In laboratory experiments examining uniform-price and discriminatory-price auctions, bidders tend to be university students who receive compensation at the end of the experiment proportionate to the profit achieved in the game. The main problem with such studies is the time and funding constraints limiting experimentation.

Other experiments having quite an impact on literature included set-ups based on the unit demand assumption (Cox et al., 1984; Damianov et al., 2010), or where an open-outcry format was used (McCabe et al., 1990).

Two experiments that are relevant for securities and that focused on sealed-bid multi-unit auctions are often quoted in literature. Smith's experiment of 1967 established a new school of thought; in this, the value of homogeneous goods was uncertain. Goods were sold on average for 96 percent of the expected value at uniform-price auctions, while at discriminatory-price auctions the corresponding ratio was only 88 percent. As another lesson, the dispersion of bids was much higher at uniform-price auctions (steep bid curves) and many bids were above the expected value. However, this experiment was also severely limited by time and financial constraints as only 18 goods were auctioned and bidders could submit bids for only two units (Smith, 1967). Another frequently quoted experiment was conducted in 1996 to focus on the efficiency of collusion. The main claim of the paper is that when bidders were allowed to communicate, a price-reducing cartel was easier to maintain in the uniform-price format; that is, the discriminatory-price auction yielded a higher expected revenue for the auctioneer. However, when no communication was possible, the uniform-price format resulted in higher prices, like in the previous experiment (Goswami et al., 1996).

More recent experiments have confirmed that bidders tend to submit steeper bid curves in uniform-price auctions than in the discriminatory-price format (Engelmann and Grimm, 2009; List and Lucking-Reiley, 2000). Another experiment from 2006 that also allowed for changing the quantity offered confirmed that the uniform-price auction brings higher revenues but (to some extent in contravention to theory and other experiments) it also showed that participants formed more efficient cartels in discriminatory-price auctions than in the uniform-price format (Sade et al., 2006).

The results of laboratory experiments are considerably more uncertain than real market experience as they rely on a number of simplifications: for instance, in most cases bidders were given homogeneous information and a round to simulate the secondary market is often omitted from the experiment. Consequently, actual Treasury auctions are worth analyzing; however, we should note that the steeper bid curves of uniform-price auctions were confirmed under laboratory conditions as well and results mostly showed that the uniform-price format yields higher expected revenues, which coincides with most real-life observations described in the next chapter.

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## 4 Non-laboratory empirical evidence

Comprehensive, international comparative studies of Treasury auctions are less common; there are only three frequently quoted examples in literature. Brenner et al. (2009) and Bartolini and Cottarelli (1997) collected the bond auction methods used by the various countries while Sareen (2004) described the auctions of 8 countries in detail.

The overwhelming majority of empirical studies summarizes the experiences of a single state treasury.

### 4.1 EMPIRICAL EVIDENCE OF UNIFORM-PRICE AUCTIONS

Scalia looked at Italian auctions from the 1995–1996 period. He demonstrated that the mark-up of primary market actors at the various auctions depends on the number of participants, competition and the dispersion of bids but no participant can achieve a profit significantly different from 0 in the long term. As another lesson, the introduction of re-issues systematically reduces the volatility of secondary market prices between issues. This is because participants with short positions before the auction will be able to purchase the securities on the primary market later; therefore forcing a short squeeze will not be a profitable strategy for large actors. Even though this is not the focus of the article, it does summarize how the Italian auction arrangements developed. As a result of an experiment in 1985, uniform-price auctions were introduced for longer-term bonds in 1988 while short-term T-bills continued to be sold in the discriminatory format. However, the reliability of lessons to be learned from the switch is compromised by a number of other changes: the MTS system was introduced in the same year and re-issue auctions were started in view of the increased financing requirement and to prevent short squeezes (Scalia, 1997).

Bjornnes analyzed data from Norway to demonstrate that uniform-price auctions lead to underpricing, i.e., the steep bid curves of participants may result in a low price (below the Walrasian equilibrium price) (Bjornnes, 2001).

Keloharju et al. examined the individual demand curves of bidders in uniform-price auctions held in Finland between 1992 and 1999. Despite the low number of bidders (varying between 5 and 10) the authors found that market power only has a small effect on bids in practice; they are much more influenced by volatility in the secondary market: in a volatile environment bid curves are steeper and underpricing is common. The latter finding is consistent with the results of Nyborg et al. (2002) for Swedish discriminatory-price actions. The observation that the mark-up of primary market actors did not depend on the number of auction participants is attributable to the fact that the Treasury determined the exact volume accepted only after the auction, which made manipulation more difficult. Furthermore, the Finnish Treasury does not determine the price at the point most advantageous for the current auction (maximizing short term profits) but regards issuance as a repeated game, therefore it tends to accept bids close to the estimated market price (Keloharju et al., 2005).

Kandel et al. (1999) did not focus on Treasury auctions but they are still much quoted; they examined 27 Israeli IPOs conducted in the 1993–1996 period through uniform-price auctions rather than by subscription. They found that aggregate demand was relatively flat. On the other hand, they observed an abnormal excess yield of 4.5 percent on the first trading day, which could not be explained by higher volatility, therefore the excess yield of the first day could have been the result of the underpricing caused by the uniform-price auction format (Kandel et al., 1999).

### 4.2 EMPIRICAL EVIDENCE OF DISCRIMINATORY-PRICE AUCTIONS

Looking at discriminatory US government bond auction data from the 1973–1984 period, Cammack found that participants at T-bill auctions had heterogeneous expectations concerning the post-auction secondary market prices. The average auction yields were 4 basis points higher than the corresponding secondary market prices in the period examined. The

degree of underpricing showed a positive correlation with the expectations concerning the dispersion of bids, that is, if bidders expected a wide range of Charts, underpricing tended to be significant. The auction results affected the secondary market: when bids showed a high variation, the disclosure of that fact pushed secondary market prices downwards. When the number of participants at the auction was higher than expected, prices rose on the secondary market (Cammack, 1991).

Hamao and Jegadeesh write that before 1989 the Japanese Treasury agreed on the subscription price of government bonds through negotiations with a syndicate of over 800 members consisting mostly of Japanese-owned banks and insurance companies. Under pressure from the US, between 1989 and 1990 40 percent of the 10-year bonds were sold at discriminatory-price auctions, the remaining 60 percent awarded to the syndicate at the average price in a non-competitive procedure, then in October 1990 the ratio of auctioned securities was increased to 60 percent. In their empirical study the authors analysed the 1989–1995 period, concluding that the expected profit of auction participants was not significantly different from zero. Market uncertainty and competition had no significant impact on the mark-up of participants. On the other hand, profits were higher when the relative proportion of US actors was greater within the accepted bids and the proportion of Japanese bidders was smaller. Japanese participants tended to purchase large volumes simultaneously while Americans did not exhibit a similar homogeneity (Hamao and Jegadeesh, 1998).

Gordy states that the negative-slope bid curve can be seen as a sign of risk aversion. It indicates that the bidder has an idea about future secondary market prices and the greater the expected profit per bond, the higher the amount he is willing to risk. Even though for a long time it was thought in literature that participants can be considered risk neutral as, being large corporations, they bring only a negligible part of their assets to the auction, Gordy points to the principal-agent problem in that the manager deciding on the bid may be risk averse (for merely psychological reasons or driven by his private interest<sup>17</sup>). He used data from Portugal to empirically examine why participants submit several price-quantity bids. He found that by submitting more than one bid they could approximate their average yield to the average price of the whole auction, thereby reducing the probability of the winner's curse. The empirical evidence showed that the various actors submitted a large number of bids and their submitted bids showed the greatest variance when there was high volatility on the secondary market and the expected number of well-informed bidders was also high (Gordy, 1999).

Hortacsu set up a model in which continuous demand functions are generated. A number of real characteristics have been integrated into the model (private information, reserve requirements, secondary market) while it also has some major flaws. First, the number of auction participants is determined exogenously (whereas it would be reasonable for more participants to bid when the expected profit is higher), and second, the slope of the bid functions of participants is the same, which is contrary to empirical evidence. The article is often quoted as an empirical study, which is because the paper does contain statistical data about the Turkish government bond market. However, the empirical elements were mostly aimed at testing the model. Hortacsu recommends the model for simulation studies (Hortacsu, 2002)<sup>18</sup>.

Hortacsu's model was augmented by Kang and Puller; their examination of the Korean discriminatory, then uniform-price auctions between 1999 and 2002 revealed that after filtering for the effect of the auction, the discriminatory-price format is slightly more advantageous both for the expected revenue and for efficiency (Kang and Puller, 2008). These results may not be surprising in light of the assumptions of the Hortacsu model (e.g., exogeneity of the number of participants). Korea switched to a uniform-price auction model in 2000.

Marszalec applied the models of Hortacsu (2002) and Février et al. (2004) to data from Polish discriminatory-price T-bill auctions. Both models yielded better outcomes for the discriminatory-price auction than for the uniform-price format (Marszalec, 2008). Nevertheless, the Polish central bank switched to the uniform-price auction format on 1 January 2012, but there is no in-depth analysis of the experiences with that system at the time of writing of this paper. Based on anecdotal evidence from the Polish Ministry of Finance, the introduction of uniform-price auctions was driven by two expectations. First, they wanted to reduce the fear of the winner's curse and thereby increase demand and second, they expected volatility on the secondary market to decrease (because previously winning bidders with lower prices wanted to realize profits in the

<sup>17</sup> The latter assumption contravenes the assumption used in more recent research that the portfolio manager may be a risk lover to earn his bonus.

<sup>18</sup> There is also a new and upgraded version of that article (Hortacsu and McAdams, 2010). We reviewed the older version here because that was augmented and applied by other cited authors in our article.

secondary market immediately)<sup>19</sup>. The profits of actors in the primary market decreased in excess of preliminary expectations, the average auction yield was lower than the mid prices on the secondary market (the difference reaching 6 basis points at times), and in certain periods it was below secondary market ask yields. Volatility on the secondary market (at the moments after the auctions) did not change significantly as a result of the switch to the uniform-price format. However, due to the low number of observations, we cannot rule out the hypothesis that the variance of yields has decreased.

Nyborg et al. examined data from Swedish discriminatory-price auctions between 1990 and 1994 and found that the volatility of yields significantly contributed to bid shading.<sup>20</sup> This phenomenon is attributable to the fear of the winner's curse (Nyborg et al., 2002).

Elsinger and Zulehner examined the changes in the bids of various actors at discriminatory-price Austrian government bond auctions between 1991 and 2006. Bidders adapted to market circumstances (such as uncertainty, the number of bidders, the volume offered) in diverse ways: for instance, by the degree of bid shading or by changing the quantity demanded or the variance (steepness) of bids. Among market circumstances, market uncertainty (volatility) has the greatest effect on the following: degree of bid shading, individual variation of bidders, profits as well as the concentration of winning bids by participant; the demanded quantity is the only factor that does not depend on volatility. According to the authors, these findings confirm the common-valuation condition of theoretical models; however, the fact that the concentration of winning bids increases in a volatile environment suggests that the assumption of the symmetry of bidders is not realistic (Elsinger et al., 2007). In a more recent study the same authors looked at the effects of Austria's EU accession on the government bond market. Before 1995 only domestic entities participated in issuances while after the EU accession intensifying competition significantly depressed yields and reduced the bidders' profit (Elsinger et al., 2012).

Rocholl analysed discriminatory-price German auctions from the 1998–2002 period. Similarly to the results of the Finnish paper (Kelojarju et al., 2005), he also found that the issuer, instead of maximizing its revenues at a single auction, sets the auction price close to the market price. This can be interpreted as the Treasury building up its reputation because it treats the issue as a repeated game. He also looks at the ratio of competitive to non-competitive bids. In Germany 30 percent of all bids are normally non-competitive while 50 percent of the accepted bids fall into this category. According to Rocholl, as volatility increases, more non-competitive bids are submitted while the price and quantity of competitive bids decrease and their dispersion increases. As an important finding, even though the auction is discriminatory, the profit of bidders is not significantly different from zero while literature tends to assume positive profits (Rocholl, 2005).

Discriminatory-price repo auctions in the Euro area were examined by two teams. Bindseil et al. (2002) found that, unlike the empirical evidence of government bond auctions, the winner's curse and private information are not the driving forces for the outcome of repo auctions while secondary market yields and interest expectations have a major influence. They also concluded that large bidders achieved better average prices than their smaller counterparts. Bruno et al. (2005) found that large actors participate in auctions more regularly and submit less steep bid curves. The volatility of yields has a significant effect on bids: a volatile environment reduces the probability of bid submission and makes bids less dependent on interbank yields. Linzert et al. (2006) examined the repo auctions of the Bundesbank and, similarly to the previous studies, they also found no evidence for the winner's curse but, as opposed to the experience of the ECB, they did not notice any significant effect of interest expectations on bidder behaviour.

### 4.3 EMPIRICAL COMPARISON OF UNIFORM-PRICE AND DISCRIMINATORY-PRICE AUCTIONS

The past decades have facilitated numerous empirical observations to compare discriminatory and uniform-price auctions. Experiments fall into two categories. The auction format of identical goods was changed at a certain point in time in Zambia, Germany and Mexico; a similar comparison was also made in the United States, and the experiment of the IMF with the sale of gold also fall in this category. There were two experiments in the United States and one in Norway to auction different

<sup>19</sup> This expectation contradicts the findings of several studies as literature tends to link higher volatility to uniform-price auctions.

<sup>20</sup> As Krishna puts it: bid shading (or as other authors would say: demand reduction) means that at a given price bids are submitted for smaller quantities than they would be based on the honest valuation (Krishna, 2009).

products in a close-to-identical time interval with different methods. However, experiments in both categories have been plagued by the identification problem, that is, the change caused by the auction method is difficult to tell apart from the effects of other circumstances. In order to avoid this problem, an empirical study was conducted in Switzerland, which we will describe after the aforementioned experiments.

Between 1976 and 1980 the IMF sold one fifth of its gold stock, with the comparison of auction formats among the objectives. This experiment demonstrated a significant advantage of the uniform-price format as the margin was approximately 6 basis points lower relative to the previous day's prices on the international gold market than in the case of discriminatory-price auctions (Feldman and Reinhart, 1995).

According to Tenorio's article, between October 1985 and January 1987 Zambia sold dollars to importers at auctions on a weekly basis on a total of 68 occasions, using a uniform-price format the first 42 times and discriminatory-price auctions from the 43rd auction onwards. In the course of the experiment the volume sold increased and the exchange rate of the kwachas declined. The participants (particularly bidders submitting high bids) responded to the change of auction format with a delay. According to Tenorio, if we filter out other factors, uniform-price auctions yielded higher revenue due to the higher participation relative to the volume offered. It should be noted that the elimination of other factors relied on a large number of assumptions; furthermore, the Treasury used a minimum price at which bids were received regularly, which could have also had a significant effect (Tenorio, 1993).

Nautz (1995) analysed the repo transactions of the Bundesbank where the central bank borrowed securities from credit institutions. The repo transactions were sold in uniform-price auctions up to 1988, then the discriminatory-price format was used. While uniform-price auctions brought steep bid curves and higher-than-market prices, which the authors attributed to the format (contrary to most of the literature), discriminatory auctions were afflicted by the winner's curse, therefore price levels were lower in later auctions. The Bundesbank attached great importance to whether the auction yields were close to the market yields and if not, how markets perceived that phenomenon. Empirical evidence showed that market yields responded significantly to auction results when the assumed effect of the auction method fell short of the customary: that is, when the price was lower than usual in uniform-price auctions or higher than usual in discriminatory auctions.

Umlauf examined the auctions of 30-day Mexican T-bills between 1986 and 1991. The discriminatory auction model was replaced by the uniform-price method in July 1990, facilitating the comparison of the auction methods as 181 discriminatory and 26 uniform-price issues occurred in the period examined. At the time of discriminatory-price auctions the 6 largest of the participants (out of the 25 bidders participating on average) submitting competitive bids won 72 per cent of the issued volume. This is because Mexican law did not impose sanctions on collusion at that time. The members of the presumed cartel sold in the secondary market the overwhelming majority of the T-bills purchased in the afternoon of the auction. Looking at the profits thus generated Umlauf found that the average cartel profit of 2.06 basis points measured at discriminatory auctions fell to an average of 0.44 basis points after the introduction of uniform-price auctions. Critics find it rather surprising that the cartel was able to achieve relatively low extra profits even before the rules of the game were changed; others explain this by the fact that the Treasury had considerable discretion in respect of the volume of issue, which made manipulation more difficult. Importantly, Umlauf also found that at times of volatility the margin was greater, which is explained by the fact that bidders with more limited analytical capacities vary their bids more for fear of the winner's curse. In the period examined, Mexico faced major macroeconomic challenges; consequently, the Treasury eventually decided for discriminatory-price auctions with a view to reducing the volatility of yields (Umlauf, 1993).

In the literature reviewed the only government bond market experiment that showed higher expected revenue from discriminatory auctions was conducted in the United States between 1973 and 1976. First, 6 uniform-price auctions were held, followed by 10 discriminatory auctions after August 1974. However, the low number of observations in itself detracts from the reliability of the conclusions, and the experiment did not allow for observing the learning process of bidders. The parameters altered in the course of the auctions included forward market trading (allowed or not), and interest rates were also modified. Simon demonstrated that even though there was little difference in the profit of bidders (0.34 percentage point), other factors must also be taken into account: with discriminatory auctions a 7-8 basis point saving can be achieved relative to the uniform-price format, which may cause a revenue drop of as much as 0.75 per cent for the Treasury (Simon, 1994). However, this is the study that had the most uncertainty in its methodology, which is indicated by the fact that other

authors concluded from the same experiment that the greater demand at uniform-price auctions would be reflected in higher revenues for the Treasury in the long term (Tsao and Vignola, 1977) cited by (Mester, 1995).

Berg et al. concluded from data from Norway, Israel and Switzerland that the aggregate bid curve can be described with the same S-shaped function irrespective of the auction format. Then they looked at data from Norwegian auctions between 1991 and 1996, when discriminatory-price auctions for short-term T-bills and uniform-price auctions for long-term government bonds ran in parallel. They assume that the distributions of the bids for short and long term securities can be described with similar parameters. Applying the empirical function they found that the uniform-price auctions of Norwegian bonds saved 24 basis points of taxpayer money on average and the same savings could have been achieved by the uniform-price auction format of short-term securities as well (Berg et al., 2000). The latter result is hard to believe as the yields of short-term securities follow the expected curve of the central bank base rate all over the world (with the exception of high-risk countries).

In the second experiment conducted in the United States (1992-1998) two- and five-year bonds were sold at uniform-price auctions and other bonds continued to be offered under the discriminatory format. Nyborg and Sundaresan (1996) examined the profits achieved at auctions in 1992 and 1993 at both uniform-price (two- and five-year) and discriminatory-price auctions. In contrast, Archibald et al. (1995) compared the profits on two- and five-year bonds at the discriminatory auctions before the start of the experiment and the uniform-price auctions under the experiment. The main conclusion of the experiment is that uniform-price auctions entailed higher demand and stronger dispersion of bids. These observations are attributable to two reasons: large actors submitted steeper bid curves, and the mitigation of the winner's curse encouraged bidders with smaller analytical capacities to participate. None of the experiments showed major differences in the profit of bidders: the comparison of different asset classes in the same period brought mixed results, with a tendency to prefer uniform-price auctions, while research of the same bonds in different periods clearly showed a slight cost advantage of uniform-price auctions. As a result of the experiment, the Treasury switched to the uniform-price auction format for all of its instruments in 1998.

In order to avoid identification problems, Heller et al. transformed the aggregate demand curves of the Swiss uniform-price bond auctions to discriminatory demand curves, though they used strong assumptions for this purpose. They assumed that the uniform-price bids observed reflect actual demand, that is, no bid shading occurs, which contradicts a number of research results. They also assumed that valuations are independent and each participant is negligibly small. Briefly, under this procedure the bids received in uniform-price auctions were used to estimate the probable distribution of the cut-off price for the uniform-price format, then for the discriminatory format. Then, the estimated distribution of the cut-off price of discriminatory auctions was used to estimate hypothetical discriminatory-price bids, which in turn allowed for projecting the Treasury's revenue. Significantly higher expected revenues were found to apply to the uniform-price scenario (more than half percent funding advantage, which means a smaller spread in case of long-term bonds, naturally) (Heller and Lengwiler, 2001).

As a result of an experiment conducted in 1985, Italy has auctioned its long-term government bonds under a uniform-price format since 1988; Korea (2000) and Poland (2012) have also switched to uniform-price auctions. In contrast, Mongolia has replaced the uniform-price auction method by the discriminatory format, while the United Kingdom, for instance, uses uniform-price auctions only for its variable interest rate bonds (Kaminska, 2010). These scenarios may also allow for interesting research but we have not found any in-depth analysis of them in the literature surveyed.

Among others, Mester collects the empirical conclusions relating to the effect of auction format on expected Treasury revenues in a summary table (Mester, 1995). A more comprehensive set of titles is presented in Table 2.

We can observe that in most empirical experiments spreads were smaller in uniform-price auctions. This seems to suggest that uniform-price auctions yield higher revenues for the Treasury<sup>21</sup>. It should be emphasised that all the thoroughly planned and controlled experiments (IMF, Norwegian and the later US experiment) lean towards the uniform-price format while the methodology of the (earlier US) experiment that recommended the discriminatory-price format is highly uncertain. Spreads

<sup>21</sup> However, in theory the auction format may also have an impact on the final investor (secondary market or forward) prices used for the comparison.

**Table 2**  
**Studies comparing uniform-price and discriminatory-price auctions**

Academic article	Market	Between	Higher revenue	
			Uniform-price	Discriminatory
(Feldman and Reinhart, 1995)	IMF; gold	1976–1980	x	
(Tenorio, 1993)	Zambia; USD	1985–1987	x	
(Umlauf, 1993)	Mexico; T-Bill	1986–1991	x	
(Simon, 1994)	USA; T-Bill	1973–1976		x
(Tsao and Vignola, 1977)	USA; T-Bill	1973–1976	x	
(Berg et al., 2000)	Norway; T-Bill and T-Bond	1991–1996	x	
(Nyborg and Sundaresan, 1996)	USA; T-Bill and T-Bond	1992–1993	Difficult to decide	
(Archibald et al., 1995)	USA; T-Bond	1991–1995	x	
(Heller and Lengwiler, 2001)	Switzerland; T-Bond	1993–2000	x	

*Source: author's table based on (Mester, 1995).*

of a few basis points may represent significant differences in Treasury revenues as maturities increase. It should be noted, however, that uniform-price auctions were suspended in Zambia to avoid the excessive devaluation of the kwachas and in Mexico due to the dominance of foreign investors submitting aggressive bids<sup>22</sup> and the volatility of yields (Kondrát, 1996).

However, experiments described in the literature covered have been plagued by the identification problem; in other words, the change caused by the auction method is difficult to tell apart from the effects of other circumstances. This problem could be resolved only with the use of strong assumptions. It would be a real scientific breakthrough, though, to set up an experiment in which the same product would be sold simultaneously in both uniform-price and discriminatory auctions. Even though fewer conclusions could be drawn than in the previously proposed arrangement (due to the repetition of auctions), it would be instructive to see an experiment where primary market actors have to submit bids for both auction formats, then the real format would be decided by drawing lots. We should note, however, that the experiment may increase the 'fog of war', the strategy space may become even more complicated and the number of possible equilibria may increase to extreme heights.

<sup>22</sup> The lesson that switching to the uniform-price format increased the weight of foreign actors may also be relevant for the Hungarian government bond market. The relationship between the role of foreign investors and the auction format is not discussed in any other study surveyed, though. This finding should be treated with reservation, though, as the auction format favourable for foreign (potentially large) actors may depend on the specific rules of the issuance, such as the right to change the volume.



# 5 International practice

Brenner et al. (2009) examined the countries using uniform-price government bond auctions and discriminatory-price auctions. They demonstrated in statistical terms that countries with market-oriented economies<sup>23</sup> and those that practice common law tend to use a uniform-price method, whereas those that are less market oriented, perhaps employ more severe restrictions on participation in auctions and practice civil law tend to use discriminatory-price auctions for the sale of government bonds. Countries employing uniform-price auctions are generally less corrupt and have a more simple business environment.

Furthermore, the authors found that in countries with concentrated banking systems (the value of the assets of the three largest banks as a share of all banking assets is higher than a certain threshold) had a tendency to use discriminatory-price auctions. The authors explain this with the argument that as large primary market actors (mostly large banks) could obtain higher profits at discriminatory auctions, they try to influence auction rule makers in that direction (Brenner et al., 2009).

**Table 3**  
Countries using the various auction methods for sovereign debt

Discriminatory	Uniform price	Both
Bangladesh	Argentina	Brazil
Belgium	Australia	Canada
Cambodia	Columbia	Ghana
Cyprus	Korea	Italy
Ecuador	Norway	Mexico
France	Singapore	New-Zealand
Germany	Switzerland	Sierra-Leon
Greece	Trinidad and Tobago	Slovenia
Hungary	USA	United Kingdom
Israel		
Jamaica		
Latvia		
Lithuania		
Macedonia		
Malta		
Mauritius		
Mongolia		
Panama		
Portugal		
Solomon Islands		
Sweden		
Turkey		
Venezuela		

Sources: National central banks; Benner et al. (2009); Morgan Stanley (2012).

<sup>23</sup> Stock market capitalization as a percentage of GDP was used as a proxy for this.

The claim that discriminatory-price auctions favor large banks needs to be investigated further. It is indeed advantageous for large actors that bidders with smaller analytical capacities may shade their bids in discriminatory auctions for fear of the winner's curse or they may stay away from the auction altogether. However, collusion and other forms of manipulation (characteristic of large actors) tend to be present in the uniform-price format. Little is known about the resultant of these two effects. If we accept the part of the reasoning of the authors that discriminatory-price auctions favor large actors, we may explain this by the fact that in most countries uniform-price auctions are (also) organised with terms that make manipulation very expensive (e.g., through the ability to change the volume sold). Consequently, the dominant considerations may indeed be the winner's curse and the resulting bid shading, and we can assume that in reality, large banks benefit from discriminatory-price auctions.

## 6 Summary and conclusion

Uniform-price and discriminatory-price auctions cannot be ranked based on theoretical models by the expected revenue of the seller, and the format that is more advantageous in a given situation probably depends on the circumstances (bid-to-cover ratio, number of participants, market uncertainty, etc.).

The amount of evidence from 'laboratory' experiments is limited but it does coincide with real life experience. Empirical evidence shows that there are countries employing discriminatory-price auctions where the profit of bidders is not significantly different from 0 basis point (Germany, Japan), but in most countries uniform-price auction would probably bring higher expected revenue for the issuer than discriminatory-price auctions do.

The data collected shows that globally there are more government bond issuers that employ the discriminatory-price format. The exclusive use of the uniform-price format is rare among countries similar to Hungary. Central bank instruments are also sold mostly under the discriminatory arrangement.

In addition to the aforesaid, the following conclusion drawn from the literature reviewed may be particularly relevant.

- Re-issues after the auction may reduce the success of manipulation strategies based on short squeeze, thus they may reduce the volatility on the secondary market (Department of the Treasury, 1992; Scalia, 1997).
- The right to change the volume offered decreases the probability of manipulation (Back and Zender, 2001; Damianov and Becker, 2010; Damianov et al., 2010; Keloharju et al., 2005; Kremer and Nyborg, 2004a, 2004b; McAdams, 2007).
- A distinction can be drawn between pre-trade and post-trade transparency (see Balogh and Kóczán, 2008). Pre-trade transparency (e.g., the clarity of rules) may be important in avoiding the fog of war (Binmore and Swierzbinski, 2000). On the other hand, a high degree of post-trade transparency may be conducive to the survival of cartels as it may expose rogue cartel members (Kondrát, 1996).
- If instead of pursuing short-term profits, the Treasury sets the cut-off price close to the market price, it may promote its reputation, which can be more profitable in the long run (Keloharju et al., 2005; Rocholl, 2005).
- An experiment with two types of auctions being held simultaneously for a given instrument could be a scientific breakthrough.

Even though studies of auction formats tend to focus on the effect on expected revenue, the issuer may have a number of other motives and the considerations to be used to optimize the choice are far from clear. **If the objective of the auction of a good is...**

- **... to maximize revenue:** empirical evidence indicates that the *uniform-price* format may be more advantageous in most markets in the long term (see the papers overseen in Table 2).
- **... to issue near the market price:** actually, this is the original question answered in Table 2. These papers are discussing the expected revenue through the differences between primary and secondary market yields. Empirical evidence shows that *uniform-price* auctions may bring lower average margins in the longer term, thus the outcome may be closer to the market price on average (see the papers overseen in Table 2). However, this is not necessarily true for individual auctions due to higher volatility on the primary market.

- **... to assure the continuity of financing and reduce volatility on the primary market:** there is empirical evidence that bids submitted in uniform-price auctions have a higher dispersion (Ausubel and Cramton, 2002; Back and Zender, 1993; Biais and Faugeron-Crouzet, 2002; Engelbrecht-Wiggans and Kahn, 1998; LiCalzi and Pavan, 2005; Maxwell, 1983; Noussair, 1994; Wilson, 1979), thus prices may show greater variation and the *discriminatory-price* model may be more advantageous.
- **... to find out about honest valuations:** in contrast to earlier misconceptions (Friedman, 1991; Miller, 1991; Department of the Treasury, 1992), the *discriminatory-price* model is likely to be more favorable (because in the uniform-price format bidders may submit steeper bid curves than their valuation would justify [Ausubel and Cramton, 2002; Back and Zender, 1993; Biais and Faugeron-Crouzet, 2002; Engelbrecht-Wiggans and Kahn, 1998; LiCalzi and Pavan, 2005; Maxwell, 1983; Noussair, 1994; Wilson, 1979]).
- **... to increase the number of bidders and strengthen the role of the primary market:** the *uniform-price* auction is preferred, because of the less significant winner's curse (Ausubel, 1997; Bolten, 1973; Friedman, 1959; Milgrom and Weber, 1982), as also shown by empirical evidence (Archibald et al., 1995).
- **... to increase efficiency<sup>24</sup>:** no ranking is possible on an analytical basis but experiments and the empirical studies based on the Hortacsu model favor the *discriminatory-price* format (Hortacsu, 2002).
- **... to prevent collusion:** based on both theory (Bikhchandani and Huang, 1993; Daripa, 2001; Nyborg and Strebulaev, 2004) and laboratory experience (Goswami et al., 1996; Sade et al., 2006), *discriminatory-price* auctions are more favorable, while there is analytical and empirical evidence that the exact rules of the auction (e.g., right to change the volume sold) have the greatest effect on the possibility of collusion (Back and Zender, 2001; Damianov and Becker, 2010; Damianov et al., 2010; Keloharju et al., 2005; Kremer and Nyborg, 2004a, 2004b; McAdams, 2007).
- **... to orientate the market price (in the case of central bank instruments):** difficult to answer because the reviewed literature has not examined this issue. As there is no empirical study available, we should take a theoretical approach. Market price is probably best oriented if there are more participants or high-volume bids in the auction, in which case the auction price may have a greater impact on the secondary market as well. The problem of bidders may be interpreted as a private-value auction (i.e., not as a common-value auction frequently studied in literature), that is, bidders do not ask the simple question of how to win instruments below the post-auction market price. Instead, banks would like to obtain cheaper funds even if they have to pay the price of reputation risk ('stigma effect' that can be evaluated bank by bank because participation in FX swap and central bank credit auctions may be seen as a sign of difficulties in raising funds in the money market, which may raise the risk premium of the bank concerned: stigma effect). This problem may turn the entire auction into a private-value auction due to the private valuation of the reputation loss. Assuming risk aversion, a private-value auction may result in bids being submitted at discriminatory-price auctions even at a smaller 'expected financial gain – reputation loss' difference because the price a bank paid for the funds if their bid is accepted will be known. In contrast, in the case of uniform-price auctions the high degree of 'fog of war' means that bidders may be uncertain about the bid price and less clear about the expected financial gain. The above reasoning relies on a large number of assumptions. Consequently, more studies and model experiments would be needed on the subject of the orientation of market prices. In the case of central bank instruments currently sold at auctions (one-week and three-month FX-swaps, six-month variable interest rate collateralized loans, FX-auctions) the orientation of the market price may be important, and several factors may need to be taken into account.

Overall, for the time being there seems to be no strong argument for the adoption of the uniform-price format in the case of central bank auctions, in contravention to the international practice of central banks, thus discriminatory-price auctions may continue to be appropriate allocation mechanisms.

In the case of the auction of government bonds, maximizing the expected revenue of the issuer may also be important. In this respect the overwhelming majority of empirical evidence shows that uniform-price auctions have an advantage of a few basis points or at least the average profit of bidders was lower in uniform-price auctions in most of the cases reviewed. It should be noted, however, that discriminatory auctions may be more advantageous in an uncertain market environment or

<sup>24</sup> An auction is efficient if the goods are awarded to those bidders whose honest valuation is the highest.

where the bid-to-cover ratio is low. Consequently, amidst the present uncertainties, switching to the uniform-price format could be hazardous. Changing the auction format (or conducting an experiment into such a change) would be relevant if volatility remained persistently low with consistently high bid-to-cover ratios. However, the adoption of the uniform-price format may be worth considering under better market conditions in the hope of cheaper funding. In order to suppress the possibility of manipulation, in the uniform-price system it is particularly important for the issuer to be able to change the volume sold, and the success of a switch depends to a large extent on the probability of the market entry of smaller participants, thus also on the characteristics of the primary dealer system. The publication of an in-depth study on the change implemented by the Polish treasury in 2012 could shed light on important considerations for the Hungarian auction system.

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# Appendix

## The Revenue Equivalence Theory (from: Krishna, 2009, p. 28)

**“Proposition.** Suppose that values are independently and identically distributed and all bidders are risk neutral. Then any symmetric and increasing equilibrium of any standard auction, such that the expected payment of a bidder with value zero is zero, yields the same expected revenue to the seller.

**Proof.** Consider a standard auction form,  $A$ , and fix a symmetric equilibrium  $\beta$  of  $A$ . Let  $m^A(x)$  be the equilibrium expected payment in auction  $A$  by a bidder with value  $x$ . Suppose that  $\theta$  is such that  $m^A(0) = 0$ .

Consider a particular bidder – say, 1 – and suppose other bidders are following the equilibrium strategy  $\beta$ . It is useful to abstract away from the details of the auction and consider the expected payoff of bidder 1 with value  $x$  and when he bids  $\beta(z)$  instead of the equilibrium bid  $\beta(x)$ . Bidder 1 wins when his bid  $\beta(z)$  exceeds the highest competing bid  $\beta(Y_1)$ , or equivalently, when  $z > Y_1$ . His expected payoff is:

$$\pi^A(z, x) = G(z)x - m^A(z)$$

where as before  $G(z) \equiv F(z)^{N-1}$  is the distribution of  $Y_1$ . The key point is that  $m^A(z)$  depends on the other players' strategy  $\beta$  and  $z$  but is independent of the true value,  $x$ . Maximization results in the first-order condition

$$\frac{\partial}{\partial z} \pi^A(z, x) = g(z)x - \frac{d}{dz} m^A(z) = 0$$

At an equilibrium it is optimal to report  $z = x$ , so we obtain that for all  $y$

$$\frac{d}{dy} m^A(y) = g(y)y$$

Thus,

$$m^A(x) = m^A(0) + \int_0^x yg(y)dy = \int_0^x yg(y)dy = G(x) \times E(Y_1 | Y_1 < x)$$

since, by assumption,  $m^A(0) = 0$ . Since the right-hand side does not depend on the particular auction form  $A$ , this completes the proof.”

**MNB Occasional Papers 111**

Discriminatory versus uniform-price auctions

February 2014

Print: D-Plus

H-1037 Budapest, Csillaghegyi út 19-21.

