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Real-Time State-Space Method for Computing Smoothed Estimates of Future Revisions of U.S. Monthly Chained CPI

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Real-Time State-Space Method for Computing Smoothed Estimates of Future Revisions of U.S. Monthly Chained CPI

Abstract

Well known CPI of urban consumers is never revised. Recently initiated chained CPI is initially released every month (ICPI), for that month without delay within BLS and for the previous month with one month delay to the public. Final estimates of chained CPI (FCPI) are released every February for January to December of the calendar year two years before. Every month, simultaneously with the release of ICPI, we would like to have a best estimate, given current information, of FCPI for that month, which will not be released until two calendar years later. ICPI and FCPI data may be indexed in historical time by months of occurrence or in current or real time by months of observation or release. The essence of the solution method is to use data indexed in historical time to estimate models and, then, for an estimated model, to use data indexed in real time to estimate FCPI. We illustrate the method with regression and VARMA models. Using a regression model, estimated FCPI is given directly by an estimated regression line; and, using a VARMA model, estimated FCPI is computed using a Kalman smoother.

JEL-Codes: C320, C530, E170.

Keywords: Kalman smoother estimation of delayed and revised data.

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1. Introduction.

An initial estimate (ICPI) of chained CPI of urban consumers (CCPIU) is produced and released to the public every month with a month's delay, and final revised estimates of CCPIU (FCPI) are released every February for all months two calendar years earlier. Depending on the month, a final release comes 14-25 months after an initial release. ICPI differs from FCPI on average by about 22% (see the column average of the BENCH model in table 5). The difference occurs because final estimates of expenditure weights are available for a given year only two calendar years later. ICPI is an initial estimate of FCPI that uses two-year-old expenditure data. The present paper describes and illustrates a method for estimating FCPI concurrently, i.e., in the month of an ICPI release, and evaluates the accuracy of the obtained estimates in terms of root mean-squared errors (RMSE). In the method, FCPI estimates are based on estimated regression models and estimated vector autoregressive moving-average (VARMA) models.

In the regression branch of the method, FCPI is given directly by an estimated regression line. In the VARMA branch of the method, FCPI is estimated by applying the Kalman smoother to an estimated VARMA model. Because FCPI is released every February for two calendar years earlier (not in every month with a constant delay) and is observed in relatively few periods, in practice, a regression can include only current and past ICPI as explanatory variables for estimating current FCPI. By contrast, because a VARMA model and the Kalman smoother can easily handle data delays, in particular, annual February releases of FCPI, they can easily use all current and past ICPI and FCPI data when estimating current FCPI, which explains why the VARMA-Kalman-smoothing estimates of FCPI here are more accurate than the regression-line estimates.

The application here uses monthly data from January 1998 to December 2005, where the 1998-1999 data are unofficial and the 2000-2005 data are official. The results (figure 2, tables 5-6) show that the best VARMA model RVAR12 and its associated Kalman smoother estimate of FCPI imply RMSEs of 17% or 23% lower than the RMSE of 22% of ICPI as an estimate of FCPI. However, even VARMA model RVAR0, which can be produced automatically (i.e., without any judgement or intervention) produces a RMSE of 19% or 14% lower than the RMSE of 22% of ICPI. These results suggest that using the Kalman smoother is more important for accurately estimating FCPI than which particular model is used. For example, even though VARMA model RVAR0 and regression model UREG0

are theoretically equivalent, the VARMA-Kalman-smoothing estimates of FCPI yield a RMSE of 19%, whereas the regression-line estimates of FCPI yield a RMSE of 23% (see column averages of UREG0 and RVAR0 models in table 5).

We consider unrestricted and restricted regression models of FCPI regressed on current and lagged ICPI and unrestricted and restricted VARMA models of ICPI and FCPI. In unrestricted models, all coefficients of variables and disturbances lagged 1-12 months are estimated; in restricted models, insignificant coefficients are set to zero in rounds of estimation. To limit the number of estimated models, only pure VAR and pure VMA models are considered.

Regression and VAR models are estimated using linear methods such as ordinary least squares (OLS) and seemingly unrelated regression (SUR). The VMA model is estimated using the nonlinear maximum likelihood estimation (MLE) method. Whereas regression estimates of FCPI are based only on current and past ICPI and a constant regression-line "formula," VARMA-Kalman-smoothing estimates of FCPI are based on current and past ICPI and FCPI and on a monthly-varying estimation "formula," because FCPI is released only in February. In principle, regression estimates could be based on separate regression lines for each month, but there is not enough data to estimate a separate regression for each month. By contrast, a Kalman smoother automatically produces a different estimation "formula" every month, even for a time-invariant model.

Interim estimates of CCPIU (NCPI) are also released every February for the months of the previous calendar year. However, NCPI were not used here, because they are not clearly "interim" between ICPI and FCPI: in even-numbered years ICPI and NCPI are identical to within eight decimal digits and differ only in odd-numbered years. The resulting high degree of multicollinearity between ICPI and NCPI suggests that NCPI would contribute little additional information for estimating FCPI beyond that in ICPI.

The paper continues as follows. Section 2 discusses data in historical versus real-time forms, both of which the application uses. Section 3 discusses transforming the ICPI and FCPI data by logging, differencing, standardizing, and normalizing them, in order to account for trends, reduce seasonality, and simplify and improve model estimation. Section 4 describes the particular regression and VARMA models to be estimated and used for computing estimates of transformed FCPI. Section 5 reviews some related economics and statistics literature. Section 6 discusses the state-space formulation of estimated VARMA models used for computing FCPI estimates using

the Kalman smoother. Section 7 reports estimated regression and VARMA models of transformed ICPI and FCPI data and RMSEs of the FCPI estimates based on the estimated models. Section 8 concludes the paper with a summary. Section 9 contains supplemental figure 2 and table 6.

2. Data in Historical and Real-Time Forms.

This section discusses data in historical and real-time forms. The idea is that estimating a model is more easily done using the data and model in more compact historical form, but estimating FCPI with partly delayed data is more easily done using the data and model in expanded real-time form, which accounts explicitly for data delays. The historical form is usually used for storing data and estimating models. In the application, we first estimated models in historical form and, then, converted both data and models to expanded real-time form in order to compute VARMA-Kalman-smoothing estimates of FCPI.

Table 1: Data in Compact Historical Form.

| Month t of occurrence | di_t | df_t |
|-----------------------------|-----------|-----------|
| 1 | di_1 | df_1 |
| 2 | di_2 | df_2 |
| 3 | di_3 | df_3 |
| ... | ... | ... |
| 10 | di_{10} | df_{10} |
| 11 | di_{11} | df_{11} |
| 12 | di_{12} | df_{12} |
| End of year t = 1, ..., 12 | | |
| 13 | di_{13} | df_{13} |
| 14 | di_{14} | df_{14} |
| 15 | di_{15} | df_{15} |
| ... | ... | ... |
| 22 | di_{22} | df_{22} |
| 23 | di_{23} | df_{23} |
| 24 | di_{24} | df_{24} |
| End of year t = 13, ..., 24 | | |

Table 1 depicts $di_t = \ln(ICPI_t) - \ln(ICPI_{t-1})$ and $df_t = \ln(FCPI_t) - \ln(FCPI_{t-1})$ in compact historical form. In table 1, all instances of t in all columns denote the *historical month of occurrence of a datum*, regardless when it was released. Table 2 depicts di_t and df_t in expanded real-time form. As in table

1, t subscripts of di_t and df_t in table 2 denote the historical month of occurrence of a datum, but, in contrast to table 1, t in column 1 of table 2 denotes the *real-time month of observation of a datum*.

Table 2: Data in Expanded Real-Time Form.

| Month t of observation | di_t | df_t | | |
|---------------------------------|-----------|------------|-----|------------|
| 1 | di_1 | na | ... | Na |
| 2 | di_2 | df_{-12} | ... | df_{-23} |
| 3 | di_3 | na | ... | Na |
| ... | ... | na | ... | Na |
| 10 | di_{10} | na | ... | Na |
| 11 | di_{11} | na | ... | Na |
| 12 | di_{12} | na | ... | Na |
| End of year $t = 1, \dots, 12$ | | | | |
| 13 | di_{13} | na | ... | na |
| 14 | di_{14} | df_0 | ... | df_{-11} |
| 15 | di_{15} | na | ... | na |
| ... | ... | na | ... | na |
| 22 | di_{22} | na | ... | na |
| 23 | di_{23} | na | ... | na |
| 24 | di_{24} | na | ... | na |
| End of year $t = 13, \dots, 24$ | | | | |

From BLS's viewpoint, di_t is released in the same month in which it occurs, with no monthly delay. Consequently, for di_t , there is no distinction between historical and real time, so that the simultaneously historical and real-time values of di_t are in the same rows in tables 1 and 2. By contrast, all 12 monthly values of df_t for a given calendar year are released in February two calendar years later. For example, December values of df_t for a given year are released 14 months later and January values of df_t for the same year are released 25 months later. Because all values of df_t for a given year are released in February two years later and because rows in table 2 are indexed by real-time months of release or observation, we need 12 columns in table 2 in order to depict the 12 monthly values of df_t released each February. Thus, actual values of df_t are placed in months or rows marked 2 and 14, which are the Februaries in table 2. For all other non-February months or rows, values of df_t are denoted "na," which means not available or missing. Consider, for example, the second February or row $t = 14$ in table 2. In principle, table 2 has 14 columns, so that the dots represent columns 4-13 which are not shown. Row $t = 14$ in table 2 contains df_0, \dots, df_{-11} , which

denote observations on df_t which occurred, respectively, in months $t = 0, \dots, -11$ or December to January two calendar years earlier.

3. Data Transformation, Trend, and Seasonality.

We now explain how and why we transformed data by logging, differencing, standardizing, and normalizing them in order to account for trends, reduce seasonality, and simplify and improve model estimation.

The $ICPI_t$ and $FCPI_t$ data were obtained from January 1998 to December 2005 (1998:1 - 2005:12) or 96 months. An initial "in-sample estimation" period of 1998:1 - 2002:12 or 60 months was used to estimate models; a subsequent "out-of-sample forecasting" period of 2003:1 - 2005:12 or 36 months was used to estimate df_t and evaluate its accuracy. Before being used for estimating models and df_t , the $ICPI_t$ and $FCPI_t$ data were transformed to normalized, standardized, month-to-month first differences of natural logarithms. The transformed data are graphed in figure 1 to get an idea of the nature and extent of autocorrelations and seasonality and, thereby, to get an idea of needed lags in the regression and VARMA models.

The given $ICPI_t$ and $FCPI_t$ data were first transformed to $di_t = \ln(ICPI_t) - \ln(ICPI_{t-1})$ and $df_t = \ln(FCPI_t) - \ln(FCPI_{t-1})$. Before being graphed or used in estimation, the differenced-logged data were standardized by subtracting in-sample means and dividing by in-sample standard deviations and were normalized by replacing outliers more than 3 standard deviations from zero with a missing-data indicator. To make the standardization and normalization realistic, the out-of-sample data were also standardized using the in-sample means and standard deviations, but were not normalized, because, when standing at the end of an in-sample period, ready to estimate df_t out of sample, we should proceed as if we did not know out-of-sample means, standard deviations, and outliers. Accordingly, the realistic practice is to estimate df_t on the assumption that in-sample and out-of-sample data are generated by the same process and to standardize in- and out-of-sample data using the same in-sample means and standard deviations. Estimating models and df_t using standardized data also results in simpler estimation because constant terms are zero.

Removing outliers moves standardized data closer to the standard-normal or $N(0,1)$ distribution. The idea is that, because estimation aims primarily to account for the systematic or nonrandom parts of a data generating process, the estimation improves if outliers are first removed. Leaving

outliers in out-of-sample data also makes evaluating estimates of df_t more realistic. For example, figure 2 and table 6 in the appendix report two large unremoved outliers in September and November 2005, due to Hurricane Katrina.

Figure 1: Transformed Data in Historical Form, Autocorrelations, and Spectra.

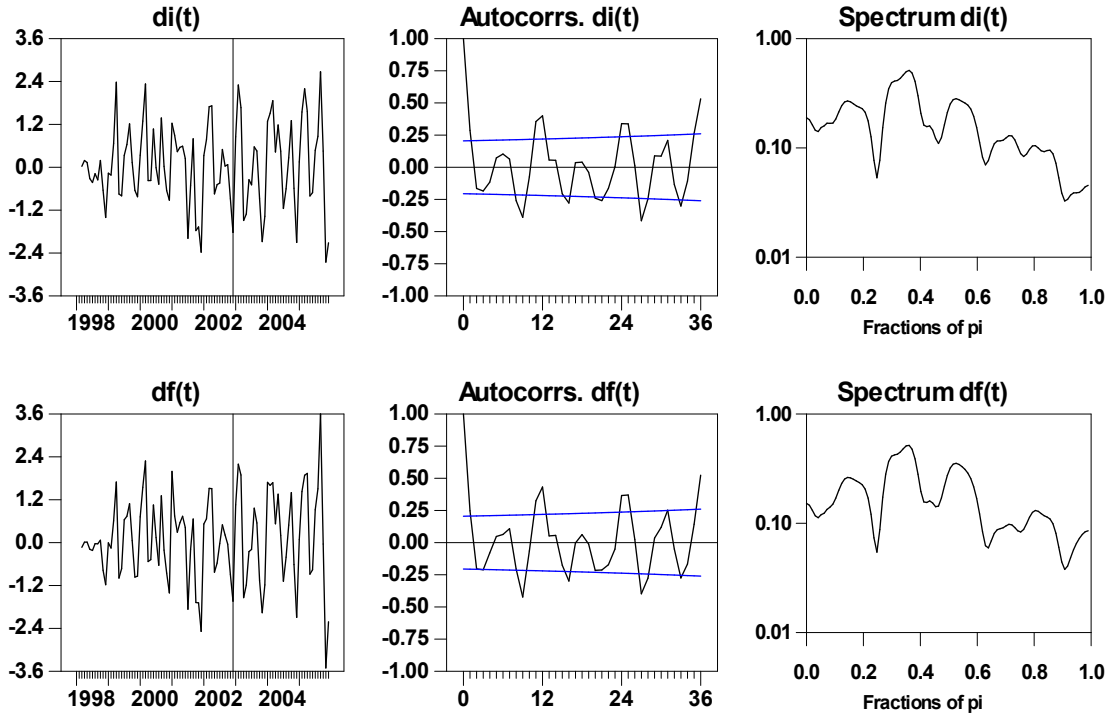


Figure 1 contains 6 graphs: the top 3 graphs depict standardized and normalized di_t , its autocorrelations, and its spectrum; the bottom 3 graphs depict standardized and normalized df_t , its autocorrelations, and its spectrum. Vertical lines in the leftmost graphs divide the sample into in- and out-of-sample periods. Unshown graphs of original data ($ICPI_t$ and $FCPI_t$) display common upward trends. Leftmost graphs of data show that di_t and df_t are approximately stationary over the sample, with no discernable trends and slightly increasing variance. Middle graphs of autocorrelations of data show significant seasonality, because autocorrelations at seasonal lags of 6, 12, 24, and 36 months are significant (outside of two-standard error confidence bounds about zero). Rightmost graphs of spectra of data, interpreted in terms of table 3, more precisely indicate the nature of the data's seasonality.

Table 3 indicates frequencies and periodicities of harmonic monthly seasonal cycles, with angular frequencies of $\pi/6 = .5236$, ..., $\pi = 3.142$ radians and periodicities of 12, ..., 2 months. The rightmost graphs in figure 1 show that nearly identical seasonality of di_t and df_t is dominated by seasonal cycles with periodicities of 4, 6, and 12 months.

Table 3: Frequencies and Periodicities of Harmonic Monthly Seasonal Cycles.

| Cases | Frequency radians | Frequency π radians | Frequency Cycles/mon | Period mon/cycle |
|-------|----------------------|----------------------------|-------------------------|---------------------|
| 1 | .0000 | 0 | .0000 | ∞ |
| 2 | .5236 | 1/6 | .0833 | 12 |
| 3 | 1.047 | 1/3 | .1667 | 6 |
| 4 | 1.571 | 1/2 | .2500 | 4 |
| 5 | 2.094 | 2/3 | .3333 | 3 |
| 6 | 2.618 | 5/6 | .4167 | 12/5 |
| 7 | 3.142 | 1 | .5000 | 2 |

In sum, significant seasonality of di_t and df_t in figure 1 suggests that estimated regression and VARMA models should include lagged values up to about 12 months back in order to fit the stationary and seasonal di_t and df_t data adequately.

4. Estimated Regression and VARMA Models.

We now define the eight regression and VARMA models. The four regression models are denoted BENCH, UREG0, UREG12, and RREG12 and the four VARMA models are denoted RVAR0, UVAR12, RVAR12, and UVMA12. In section 7, we do not report any restricted-to-zero or estimated parameters of any models, because the parameters have no particular meanings in the context of the application.

Regression model BENCH, whose name means *benchmark regression* is defined by

$$(4.1) \quad df_t = di_t + \varepsilon_{ft},$$

with disturbance ε_{ft} and disturbance variance σ_{ff} . Although BENCH has no estimated coefficients, we think of it as regression model UREG0 with coefficient β_0 restricted to one. BENCH has only parameter σ_{ff} to be estimated.

All the regression models have the same assumptions on the disturbances, ε_{ft} , that we now make for BENCH: (i) ε_{ft} is distributed normally, identically, independently, with mean zero, and constant positive variance, $\sigma_{ff} > 0$, or $\varepsilon_{ft} \sim \text{NIID}(0, \sigma_{ff})$; and, (ii) ε_{ft} is distributed independently of current and lagged regressors, di_t, \dots, di_{t-12} .

Regression model UREG0, whose name means *unrestricted regression with 0 lags*, is defined by

$$(4.2) \quad df_t = \beta_0 di_t + \varepsilon_{ft},$$

with 2 parameters, β_0 and σ_{ff} , estimated using ordinary least squares (OLS).

Regression model UREG12, whose name means *unrestricted regression with 12 lags*, is defined by

$$(4.3) \quad df_t = \beta_0 di_t + \beta_1 di_{t-1} + \dots + \beta_{12} di_{t-12} + \varepsilon_{ft},$$

with 14 parameters, $\beta_0, \dots, \beta_{12}$, and σ_{ff} , estimated using OLS.

Regression model RREG12, whose name means *restricted regression with up to 12 lags*, is defined by

$$(4.4) \quad df_t = \beta_0 di_t + \beta_1 di_{t-1} + \beta_4 di_{t-4} + \beta_9 di_{t-9} + \varepsilon_{ft},$$

with 5 parameters, $\beta_0, \beta_1, \beta_4, \beta_9$, and σ_{ff} , estimated using OLS, as follows. In estimated UREG12, we set to zero the least significant estimated coefficient, with the highest marginal significance level or p value; we reestimated the resulting reduced regression using OLS; we set to zero the least significant resulting estimated coefficient; we continued like this until all estimated coefficients were significant at about the 10% level or $|t_{\hat{\beta}}| \geq 1.645$; and, we estimated σ_{ff} residually at the last step.

VARMA model RVAR0, whose name means *restricted VAR model with no lags*, is defined by

$$(4.5) \quad y_t = \varepsilon_t,$$

for bivariate data vector $y_t = (di_t, df_t)^T$ and bivariate disturbance vector $\varepsilon_t = (\varepsilon_{it}, \varepsilon_{ft})^T$, where superscript T denotes vector or matrix transposition.

All the VARMA models have the same assumptions on disturbances that we now make for model RVAR0: (i) disturbance vector ε_t is distributed normally, identically, independently, with mean zero, and constant, 2x2, symmetric, positive definite, covariance matrix, $\Sigma_\varepsilon = \begin{bmatrix} \sigma_{ii} & \sigma_{if} \\ \sigma_{if} & \sigma_{ff} \end{bmatrix}$, or $\varepsilon_t \sim \text{NIID}(0, \Sigma_\varepsilon)$; and, (ii) ε_t is distributed independently of any past variables or disturbances, $y_{t-1}, \varepsilon_{t-1}, \dots$. In RVAR0, because standardized $di_t = \varepsilon_{it}$ implies $\sigma_{ii} = 1$, only two parameters, σ_{if} and σ_{ff} , need to be estimated using MLE.

We consider theoretically equivalent UREG0 and RVAR0 as separate models in order to estimate df_t separately using regression lines and Kalman smoothing. We now illustrate this equivalence by transforming UREG0 to RVAR0. Because both β_0 of UREG0 and σ_{if} of RVAR0 account for correlation between di_t and df_t , when converting UREG0 to RVAR0, one of these parameters becomes redundant. Thus, we have $\sigma_{ii} = 1$, can set $\sigma_{if} = 0$, and obtain $y_t = \tilde{\varepsilon}_t$, where $\tilde{\varepsilon}_t \sim \text{NIID}(0, \Sigma_{\tilde{\varepsilon}})$ and $\Sigma_{\tilde{\varepsilon}} = \begin{bmatrix} 1 & \beta_0 \\ \beta_0 & \sigma_{ff} \end{bmatrix}$.

VARMA model UVAR12, whose name means *unrestricted VAR with 12 lags*, is defined by

$$(4.6) \quad y_t = \Phi_1 y_{t-1} + \dots + \Phi_{12} y_{t-12} + \varepsilon_t,$$

where the Φ_i are unrestricted 2x2 VAR coefficient matrices. The 51 parameters of UVAR12, 48 elements of the Φ_i , and 3 nonredundant elements of Σ_ε , were estimated by applying OLS to each scalar equation in (4.6).

VARMA model RVAR12, whose name means *restricted VAR with up to 12 lags*, is defined by

$$(4.7) \quad y_t = \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \Phi_4 y_{t-4} + \Phi_8 y_{t-8} + \Phi_{11} y_{t-11} + \varepsilon_t,$$

where the restrictions are limited to setting to zero certain elements of coefficient matrices Φ_i .

The 14 nonzero parameters of RVAR12 were estimated using SUR, analogous to reducing UREG12 to RREG12, as follows. In each equation of estimated UVAR12, we set to zero the least significant estimated coefficient; we estimated the resulting reduced equations using SUR; we dropped from each estimated equation the least significant estimated coefficient; we reestimated the reduced equations using SUR; we continued like this until all estimated coefficients were significant at about the 10% level; and, we estimated Σ_ε residually at the final step.

VARMA model UVMA12, whose name means *unrestricted VMA with 12 lags*, is defined by

$$(4.8) \quad y_t = \Theta_1 \varepsilon_{t-1} + \dots + \Theta_{12} \varepsilon_{t-12} + \varepsilon_t,$$

where the Θ_i are unrestricted 2x2 VMA coefficient matrices. The 51 parameters of UVMA12, 48 elements of the Θ_i and 3 nonredundant elements of Σ_ε , were estimated simultaneously by applying MLE to (4.8). As usual, the estimated UVMA12 model was restricted to be invertible. We tried to estimate a subsequently reduced VMA model with insignificant coefficients set to zero, analogous to reducing UVAR12 to RVAR12, but this failed because the estimation algorithm stepped into parameter regions of inadequately fitting models and failed to converge.

5. Structural News-Noise and Unobserved-Components Models.

So far, we have considered nonstructural models, i.e., not motivated by economic or statistical theories, and now consider news-noise and unobserved-components models as examples of economic and statistical structural models. News-noise models have received much attention in economic discussions of data revisions and unobserved-components models are used often in statistics for modelling time series.

First, if

$$(5.1) \quad df_t = di_t + \xi_t,$$

and di_t and ξ_t are generated by separate, orthogonal, scalar, ARMA processes, then, ξ_t is considered the "news" in di_t as an estimate of df_t and $y_t = (di_t, df_t)^T$ is generated by a restricted, structural, bivariate, ARMA process. Alternatively, if di_t and df_t switch roles,

$$(5.2) \quad di_t = df_t + \xi_t$$

replaces (5.1), and df_t and ξ_t are generated by separate, orthogonal, scalar, ARMA processes, then, ξ_t is considered the "noise" in di_t as an estimate of df_t and y_t is generated by a different, restricted, structural, bivariate, ARMA process. Examples of news-noise models are in Mankiw and Shapiro (1986), Sargent (1989), Kishor and Koenig (2009), and Jacobs and van Norden (2011).

Second, if,

$$(5.3) \quad y_t = e \cdot dp_t + \xi_t,$$

where $y_t = (di_t, df_t)^T$, $e = (1, 1)^T$, and dp_t and ξ_t denote unobserved "true" d-form CCPIU and unobserved noise, generated by orthogonal, univariate and bivariate, ARMA processes, then, (5.3) is an unobserved-components model for y_t . Examples of unobserved-component models are in Howrey (1978, 1984), Hillmer and Trabelsi (1987), Trabelsi and Hillmer (1989), Shumway and Katsoff (1991), Patterson (1994), and Chen and Zadzorny (2002).

After assuming that right-side terms in equations (5.1)-(5.3) are generated by VARMA processes, we can restate the equations as VARMA models of y_t , subject to nonlinear structural restrictions on their parameters in terms of the parameters of the VARMA processes of the right-side terms. For example, if di_t and ξ_t in equation (5.1) are generated by the orthogonal, univariate, ARMA processes $\alpha(L)di_t = \beta(L)\varepsilon_{1t}$ and $\delta(L)\xi_t = \gamma(L)\varepsilon_{2t}$, where L is the lag operator, the ε_t 's are orthogonal white noises, and $\alpha(L)$, $\beta(L)$, $\delta(L)$, and $\gamma(L)$ are polynomials of finite degree in nonnegative powers of L , then, y_t

is generated by $A(L)y_t = B(L)\varepsilon_t$, where $A(L) = \begin{bmatrix} \alpha(L)\delta(L) & 0 \\ 0 & \alpha(L) \end{bmatrix}$, $B(L) = \begin{bmatrix} \delta(L)\beta(L) & \alpha(L)\gamma(L) \\ \beta(L) & 0 \end{bmatrix}$, $\varepsilon_t = (\varepsilon_{1t}^T, \varepsilon_{2t}^T)^T$, and similarly for (5.2) and (5.3).

Although, in principle, a structural model could produce more accurate estimates of df_t , the narrow range of $RMSE_f$ in table 5 suggests that this is

where $\mathbf{x}_t = (y_t^T, \dots, y_{t-25}^T, \varepsilon_t^T, \dots, \varepsilon_{t-11}^T)^T$ is a 76×1 state vector, F is a 76×76 transition matrix, G is a 76×2 input matrix, and I and 0 are 2×2 identity and zero matrices.

The data vector is $\bar{y}_t = (y_t^T, \dots, y_{t-25}^T)^T$ and has observation equation

$$(6.3) \quad \bar{y}_t = H\mathbf{x}_t,$$

where $H = [I_{52 \times 52}, 0_{52 \times 24}]$ is the 52×76 observation matrix and $I_{52 \times 52}$ and $0_{52 \times 24}$ denote 52×52 identity and 52×24 zero matrices. An observation equation can also have an additive observation error, although this isn't needed here. Zero restrictions on Φ_i and Θ_i reduce (6.2)-(6.3) to an expanded real-time state-space representation of one of models (4.5)-(4.8) and account for up to 25-month delays in observing df_t .

Because models (4.5)-(4.8) have at most 12 AR lags, in the application we set $\Phi_{13}, \dots, \Phi_{26}$ to zero in the state equation, but keep lags 13-26 of y_t in the state vector and the state equation in order to account for 14-25 month delays in observing df_t . Complete state equation (6.2) needs to be used only for Kalman smoothing with VMA model UVMA12. Otherwise, for pure VAR models RVAR0, RVAR12, and UVAR12, we restricted \mathbf{x}_t to \bar{y}_t and used only the upper-left 52×52 quadrant of F and the 52×2 upper part of G .

Let $df_{t|t} = E[df_t | I_t]$ denote the Kalman-smoothed estimate of df_t , where $E[df_t | I_t]$ denotes the expectation of df_t conditional on current information I_t , which comprises current and past observations on di_t and df_t and the estimated model. For any $k = 1, \dots, 25$, $x_{2k+2,t}$ is element $2k+2$ of \mathbf{x}_t and equals df_{t-k} , so that

$$(6.4) \quad df_{t|t} = E[x_{2k+2,t+k} | I_t].$$

Thus, smoothed estimates $df_{t|t}$ can be computed, for any $k = 1, \dots, 25$, as k -month-ahead forecasts of element $2k+2$ of \mathbf{x}_t . Genuine forecasts, $df_{s|t}$, for $s > t$, and smoothed estimates, $df_{s|t}$, for $t - 25 \leq s < t$, can also be computed as forecasts of elements of \mathbf{x}_t . $RMSE_f$ in table 5 reflect Kalman-smoothed estimates of df_t made in out-of-sample months, but could alternatively reflect Kalman smoothed estimates of df_t made in the last out-of-sample month.

Anderson and Moore (1979) thoroughly discuss Kalman smoothing and smoothing in cases of no missing data. In the present real-time analysis, df_t

is missing in all non-February months, which requires using a missing-data extension of the Kalman smoother, which was first described for scalar time series by Jones (1980), was extended to vector time series by Ansley and Kohn (1983), and was described for and applied to mixed-frequency data by Zadrozny (1988, 1990).

In the application, we computed $df_{t|t} = E[x_{4,t+1}|I_t]$, using $k = 1$, and evaluated its accuracy in terms of standardized root mean-squared errors,

$$(6.5) \quad RMSE_f = \sigma_{f,\eta} / \sigma_{f,f},$$

where $\sigma_{f,f}$ and $\sigma_{f,\eta}$, respectively, denote out-of-sample standard deviations of df_t and of its estimation error, $\eta_t = df_t - df_{t|t}$.

We define $R_{e,f}^2$ and $R_{f,f}^2$ of df_t to reflect the percentage of df_t accounted for by current and lagged df_t and relate them to $RMSE_f$ in order to establish bounds for evaluating $RMSE_f$ in table 5. Thus, we define in-sample-estimation-period $R_{e,f}^2 = 1 - \sigma_{e,e_f}^2 / \sigma_{e,f}^2$, where first subscript "e" refers to "estimation," σ_{e,e_f}^2 denotes in-sample variance of residuals of df_t and $\sigma_{e,f}^2$ denotes in-sample variance of df_t ; and, we define out-of-sample-forecasting-period $R_{f,f}^2 = 1 - RMSE_f^2$, where first subscript "f" refers to "forecasting."

What values of $RMSE_f$ and $R_{f,f}^2$ are good or bad? We answer this question by stating formal (based on mathematics) and expected (based on heuristics) bounds on $RMSE_f$ and $R_{f,f}^2$. First, by construction, $RMSE_f \geq 0$ and $R_{f,f}^2 \leq 1$, which is neither good nor bad. We would like $RMSE_f \cong 0$ and $R_{f,f}^2 \cong 1$, which is very good, but, frequently, $RMSE_f > 1$ and $R_{f,f}^2 < 0$, which is bad. Second, df_t is estimated efficiently, which is good, only if all conditioning information is utilized fully. If df_t is estimated efficiently, then, $df_{t|t}$ and $\eta_t = df_t - df_{t|t}$ are uncorrelated, $RMSE_f < 1$, and $R_{f,f}^2 > 0$. Finally, if the data-generating process is constant over the whole sample period, then, we expect (heuristically) that $RMSE_f \cong \sqrt{1 - R_{e,f}^2}$ and that $R_{f,f}^2 \cong R_{e,f}^2$; otherwise, if the data-generating process changes from the in-sample to the out-of-sample periods, then, we expect that $RMSE_f > \sqrt{1 - R_{e,f}^2}$ and $R_{f,f}^2 < R_{e,f}^2$. Thus, we expect (heuristically) that

$$(6.6) \quad \text{RMSE}_f \geq \sqrt{1 - R_{e,f}^2}$$

or, equivalently, that $R_{f,f}^2 \leq R_{e,f}^2$. If $\text{RMSE}_f \cong$ or $< \sqrt{1 - R_{e,f}^2}$, then, df_t is estimated (in the out-of-sample period, using an in-sample estimated model), respectively, about as well as or better than we can expect.

7. Computing and Evaluating Real-Time Smoothed Estimates of df_t .

Table 4 reports statistics from estimating the models using 60 months of data for 1998:1 - 2002:12: in-sample or estimation-period R^2 of df_t ($R_{e,f}^2$), Akaike's (1973) information criterion (AIC), Schwarz's (1978) Bayesian information criterion (BIC), Ljung-Box Q statistics for testing serial correlations of residuals of di_t and df_t at lags 1-36, and their marginal significance levels or p values underneath. To be compatible with the $R_{e,f}^2$ of the regression models, the $R_{e,f}^2$ of a VARMA model was increased to include the explanatory effect of di_t on df_t , already in the $R_{e,f}^2$ of the regression models. This was done by taking the basic VARMA $R_{e,f}^2$, which accounts for variations in current df_t in terms of variations in lagged di_t and df_t and adding the effects of variations in current di_t , according to the estimated correlation between the di_t and df_t residuals and the transformation between the UREG0 and RVAR0 models discussed in section 4. Because of their equivalence, the UREG0 and RVAR0 models have identical IC and Q statistics.

The estimated model that minimizes AIC and BIC is considered the best one. Table 4 reports that RVAR12 has the lowest AIC and that RVAR0 has the lowest BIC. BIC has the more stringent penalty function and usually prefers the model with fewer parameters. Except for df_t in UREG12, di_t in UVAR12, and df_t in UVMA12, all residuals in table 4 have highly significant Q statistics. However, because the significant residual serial correlations do not match those of VARMA models, adding lags of variables or disturbances seems unlikely to produce lower Q statistics. Thus, we considered no other estimated models and picked RVAR0 and RVAR12 as the best models. However, any choice of best models based on in-sample AIC or BIC is tentative, because the final objective is to minimize out-of-sample RMSE_f .

Table 4: Regression and VARMA model summary estimation statistics.

| Model Number | Model Name | Estm Mthd | $R^2_{e,f}$ | | AIC | BIC | #Est Pars | Q_i | Q_f |
|--------------|------------|-----------|-------------|--|--------|--------|-----------|----------------|----------------|
| 1 | BENCH | --- | .9472 | | --- | --- | 0 | 178.3 .0000 | 138.9 .0000 |
| 2 | UREG0 | OLS | .9479 | | -143.4 | -139.2 | 2 | 178.3 .0000 | 138.9 .0000 |
| 3 | RREG12 | OLS | .9693 | | -160.5 | -151.0 | 5 | 178.3 .0000 | 62.83 .0002 |
| 4 | UREG12 | OLS | .9742 | | -151.2 | -121.8 | 14 | 178.3 .0000 | 77.54 .8027 |
| | | | | | | | | | |
| 5 | RVAR0 | MLE | .9576 | | -143.4 | -139.2 | 2 | 178.3 .0000 | 138.9 .0000 |
| 6 | RVAR12 | SUR | .9795 | | -164.1 | -134.8 | 14 | 65.79 .0001 | 72.80 .0000 |
| 7 | UVAR12 | OLS | .9845 | | -150.6 | -43.84 | 51 | 36.05 .1414 | 64.10 .0001 |
| 8 | UVMA12 | MLE | .9327 | | -99.62 | 7.192 | 51 | 61.63 .0003 | 20.15 .8591 |

The $RMSE_f$ results in table 5 lead to the following seven conclusions.

1. BENCH is the best regression model, with the lowest $RMSE_f$ average and spread (over the twelve months) of .2167 and .3791; and, RVAR12 is the best VARMA model and best overall model, with the lowest $RMSE_f$ average and spread of .1746 and .3674. RVAR12 has a 19.4% lower $RMSE_f$ average and a 3.1% lower $RMSE_f$ spread than BENCH.

2. The close $RMSE_f$ of RVAR0 and RVAR12 suggest that accuracy of $df_{t|t}$ depends mostly on correlations between contemporaneous di_t and df_t and on delays in observations and less on correlations across current and lagged di_t and df_t as accounted for by RVAR12 but not by RVAR0.

3. Except for UVAR12, the VARMA models produce lower $RMSE_f$ average and spread than the regression models, either because the VARMA models are better models or because the Kalman smoother is applied to them. For the same model, the Kalman smoother should estimate df_t more accurately than a regression line, because it uses all current and past initial and final data, whereas a regression line uses only current and past initial data. Comparing $RMSE_f$ of

equivalent UREG0 and RVAR0 models illustrates this point: it seems to matter less which particular model is used than that the Kalman smoother is used.

Table 5: $RMSE_f$ of smoothed estimates of df_t occurring in 2003:1-2005:12.

| Month | BENCH | UREG0 | RREG12 | UREG12 | | RVAR0 | RVAR12 | UVAR12 | UVMA12 | Row Avrg |
|----------|-------|-------|--------|--------|--|-------|--------|--------|--------|----------|
| Jan | .1918 | .2076 | .1489 | .1423 | | .1856 | .0965 | .1958 | .1689 | .1672 |
| Feb | .0832 | .0745 | .1363 | .1797 | | .0609 | .0887 | .2215 | .0800 | .1156 |
| Mar | .1916 | .1801 | .2223 | .2515 | | .1478 | .1677 | .2457 | .1567 | .1954 |
| Apr | .1660 | .1892 | .3356 | .2835 | | .1637 | .2471 | .3409 | .1872 | .2392 |
| May | .0984 | .1083 | .0553 | .0559 | | .0914 | .1203 | .1735 | .0976 | .1001 |
| Jun | .1210 | .1157 | .1914 | .1557 | | .0984 | .0692 | .1362 | .1096 | .1247 |
| Jul | .2158 | .2186 | .1338 | .1681 | | .2039 | .1513 | .1824 | .1761 | .1813 |
| Aug | .3027 | .3130 | .2766 | .2355 | | .2994 | .2382 | .2777 | .2901 | .2792 |
| Sep | .4623 | .4921 | .4848 | .4962 | | .4430 | .4561 | .5204 | .4697 | .4781 |
| Oct | .2276 | .2311 | .1125 | .1258 | | .2008 | .1086 | .1683 | .1815 | .1695 |
| Nov | .3788 | .4030 | .4046 | .3995 | | .3479 | .2562 | .3133 | .3336 | .3546 |
| Dec | .0975 | .1023 | .1479 | .1334 | | .0837 | .0952 | .0952 | .0969 | .1065 |
| Col Avrg | .2167 | .2268 | .2259 | .2238 | | .1939 | .1746 | .2392 | .1957 | .2121 |
| Sprd | .3791 | .4176 | .4295 | .4403 | | .3821 | .3674 | .4252 | .3897 | |

Spread = maximum $RMSE_f$ - minimum $RMSE_f$.

4. Average $R_{e,f}^2$ in table 4 of the best df_t -estimating VARMA models, RVAR0, RVAR12, and UVMA12, is .9739, which implies $\sqrt{1 - R_{e,f}^2} = .1616$. RVAR12, RVAR0, and UVMA12 imply, respectively, column-average $RMSE_f$ of .1746, .1939, and .1957, which are not much above .1616 and, hence, suggest that di_t and df_t are generated by a relatively constant process over the whole sample period and that $df_{t|t}$ based on these models, especially on RVAR12, is approximately efficient.

5. The monthly pattern of $RMSE_f$ in table 5 could be seasonal, because the data are seasonal and releases of df_t are seasonal (only in February), or,

it could be random, because the short smoothing period covers only three years. $RMSE_f$ is lowest in May and highest in September and November, the latter because Hurricane Katrina struck in August 2005.

6. The narrow range of $RMSE_f$ in table 5 suggests that other models, such as structural models, are unlikely to produce lower $RMSE_f$ for this data.

7. The $df_{t|t}$ could be computed by Kalman smoothing as in Shumway and Katzoff (1991). Kalman smoothers are extensions of Kalman smoothers with numerous implementations (Anderson and Moore, 1979). Smoothed estimates can be computed using the more compact historical forms of the data and models.

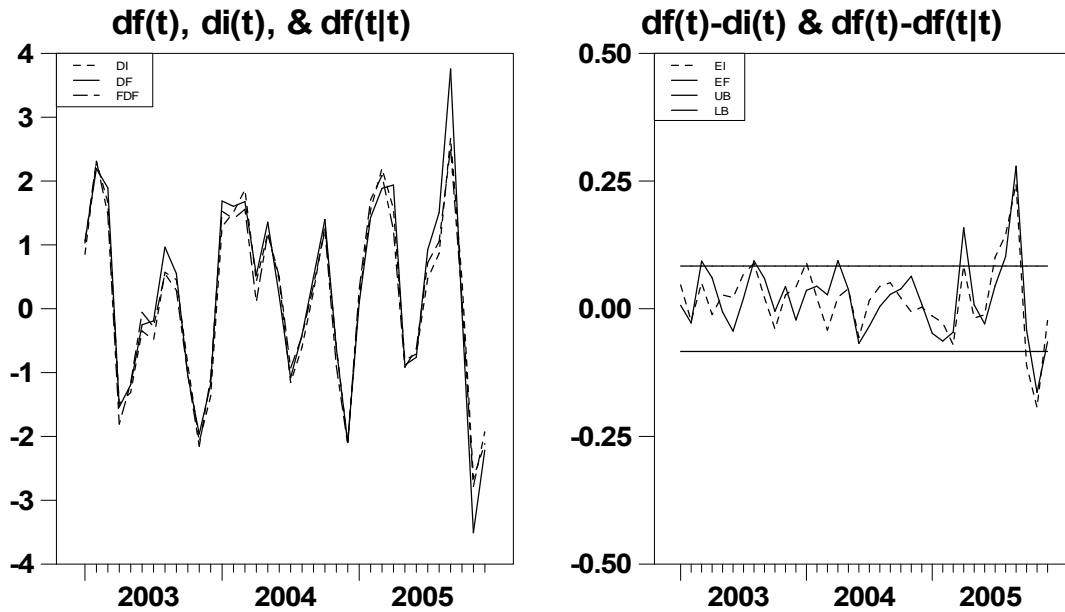
8. Conclusion.

The paper has described and applied regression and VARMA modeling methods for estimating the current value of a variable which is observed intermittently with delay and is contemporaneously and serially correlated with another variable which is observed every period without delay. In the regression branch of the method, the delayed variable is regressed on current and lagged values of the undelayed variable and the estimated regression line estimates the current value of the delayed variable. In the VARMA modeling branch of the method, a bivariate VARMA model is estimated for the variables, the Kalman smoother is applied to the estimated model, and an element of the one-step-ahead forecast of the state vector estimates the current value of the delayed variable. The methods were applied to U.S. monthly chained CPI of urban consumers (CCPIU) from January 1998 to December 2009, with initial and final CCPIU, respectively, being the undelayed and delayed variables. The results in table 5 show that estimated VARMA models and Kalman smoothing produced lower average root-mean-squared errors of estimates of final CCPIU than estimated regression models, namely, .2009 compared with .2233 or about 10% lower.

9. Supplemental Figure 2 and Table 6.

Figure 2 and table 6 supplement table 5 by graphing and tabulating df_t , di_t , $df_{t|t}$ based on RVAR12, errors $e_{i,t} = df_t - di_t$ and $e_{f,t} = df_t - df_{t|t}$, and 2-standard-error confidence bounds of $e_{f,t}$. To be consistent with table 5, before being graphed in figure 2, the errors in table 6 and standard-error bounds produced by the Kalman smoother were standardized by division by .2259.

Figure 2: df_t , di_t , $df_{t|t}$, $e_{i,t}$, $e_{f,t}$, and 2-standard-error confidence bounds.



In the left graph, df_t , di_t , $df_{t|t}$ are depicted, respectively, by solid, evenly-dashed, and unevenly-dashed lines; in the right graph, df_t -estimation errors, $e_{i,t} = df_t - di_t$ and $e_{f,t} = df_t - df_{t|t}$, and 2-standard-error confidence bounds of $e_{f,t}$ are, respectively, depicted by evenly-dashed and solid lines.

Table 6: Numbers Underlying Table 5 and Figure 2.

| Month | df_t | di_t | $df_{t t}$ | $df_t - di_t$ | $df_t - df_{t t}$ |
|---------|-----------|-----------|------------|---------------|-------------------|
| 2003:1 | 1.059426 | 0.852800 | 1.029448 | 0.206626 | 0.029978 |
| 2003:2 | 2.195151 | 2.302536 | 2.317998 | -0.107385 | -0.122847 |
| 2003:3 | 1.893475 | 1.668810 | 1.480396 | 0.224665 | 0.413079 |
| 2003:4 | -1.539022 | -1.488333 | -1.806657 | -0.050689 | 0.267635 |
| 2003:5 | -1.190376 | -1.310190 | -1.162496 | 0.119814 | -0.027880 |
| 2003:6 | -0.250663 | -0.347280 | -0.056592 | 0.096617 | -0.194071 |
| 2003:7 | -0.190663 | -0.491805 | -0.281821 | 0.301142 | 0.091158 |
| 2003:8 | 0.964597 | 0.571549 | 0.548911 | 0.393048 | 0.415686 |
| 2003:9 | 0.551247 | 0.456338 | 0.289162 | 0.094909 | 0.262085 |
| 2003:10 | -1.052098 | -0.876101 | -1.030397 | -0.175997 | -0.021701 |
| 2003:11 | -1.961222 | -2.079998 | -2.154754 | 0.118776 | 0.193532 |
| 2003:12 | -1.182770 | -1.371769 | -1.085642 | 0.188999 | -0.097128 |
| 2004:1 | 1.687414 | 1.286495 | 1.527973 | 0.400919 | 0.159441 |
| 2004:2 | 1.604391 | 1.510979 | 1.408610 | 0.093412 | 0.195781 |
| 2004:3 | 1.678370 | 1.864005 | 1.558038 | -0.185635 | 0.120332 |
| 2004:4 | 0.527502 | 0.427789 | 0.110470 | 0.099713 | 0.417032 |
| 2004:5 | 1.355959 | 1.182507 | 1.188051 | 0.173452 | 0.167908 |
| 2004:6 | 0.201992 | 0.451038 | 0.502721 | -0.249046 | -0.300729 |
| 2004:7 | -1.081764 | -1.153462 | -0.931304 | 0.071698 | -0.150460 |
| 2004:8 | -0.408177 | -0.601900 | -0.427486 | 0.193723 | 0.019309 |
| 2004:9 | 0.490708 | 0.264952 | 0.366244 | 0.225756 | 0.124464 |
| 2004:10 | 1.399156 | 1.304695 | 1.227766 | 0.094461 | 0.171390 |
| 2004:11 | -0.613947 | -0.586540 | -0.895612 | -0.027407 | 0.281665 |
| 2004:12 | -2.090220 | -2.104176 | -2.130870 | 0.013956 | 0.040650 |
| 2005:1 | 0.085752 | 0.148536 | 0.298636 | -0.062784 | -0.212884 |
| 2005:2 | 1.427234 | 1.547942 | 1.707116 | -0.120708 | -0.279882 |
| 2005:3 | 1.885741 | 2.197780 | 2.088557 | -0.312049 | -0.202816 |
| 2005:4 | 1.937359 | 1.566135 | 1.235226 | 0.371224 | 0.702133 |
| 2005:5 | -0.882269 | -0.801716 | -0.916265 | -0.080553 | 0.033996 |
| 2005:6 | -0.760514 | -0.708369 | -0.629521 | -0.052145 | -0.130993 |
| 2005:7 | 0.920044 | 0.473169 | 0.721203 | 0.446875 | 0.198841 |
| 2005:8 | 1.506380 | 0.866402 | 1.053808 | 0.639977 | 0.452571 |
| 2005:9 | 3.755989 | 2.673409 | 2.520455 | 1.082580 | 1.235534 |
| 2005:10 | -0.037918 | 0.454298 | 0.148269 | -0.492216 | -0.186187 |
| 2005:11 | -3.507909 | -2.653479 | -2.782071 | -0.854430 | -0.725838 |
| 2005:12 | -2.213681 | -2.112154 | -1.927204 | -0.101527 | -0.286477 |

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