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## Foreign debt and capital accumulation

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# Foreign Debt and Capital Accumulation

By

**Horst Siebert**

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Contents: I. Introduction. – II. A Model of Borrowing and Accumulation. – III. Optimality Conditions. – IV. Optimal Capital Accumulation. – V. The Time Profile of Consumption and Debt. – VI. Capital Accumulation without Borrowing. – VII. Borrowing for Capital Accumulation Only. – VIII. What Can Go Wrong?

## I. Introduction

Capital accumulation is at the core of economic development. An increase in the capital stock allows a country to move upward along its production function or, if new technologies are incorporated into the new capital goods, to shift its production function to a higher level. Historical experience suggests that any process of industrialization must be driven by accumulating capital. This holds for production capital in the industrial base as well as for human capital. Moreover, social overhead capital like infrastructure in transportation is needed to make use of economies of scale and to increase the size of markets. In a closed economy, capital can only be accumulated by postponing consumption. In an open economy<sup>1</sup>, a country may borrow initially, however only at the opportunity cost of paying interest and repaying the debt in future periods.

A country will accumulate capital by postponing consumption if the benefit from not consuming, that is the marginal productivity of capital, exceeds the costs of waiting, that is the time preference rate. When foreign borrowing is introduced, it is worthwhile to finance the capital stock by debt if the marginal productivity of capital exceeds the interest rate, that is the cost of borrowing. Moreover, if the time preference rate is greater than the interest rate on debt, it is worthwhile to borrow for consumption and repay the debt later. Thus, borrowing for capital accumulation and for consumption will overlap. Compared with capital accumulation from postponing consumption, foreign borrowing will allow a higher consumption profile in the initial periods and hence a higher present value of consumption; capital accumulation via borrowing clearly is better than the postponement of consumption.

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<sup>1</sup> On the case in which an open economy transforms its initially given resource stock into capital compare Siebert [1985, Ch. VI].

However, in future periods the opportunity costs of borrowing become apparent, and future generations may have different perceptions on the benefits and the opportunity costs of borrowing.

In this paper, we compare the case of foreign borrowing with a scenario in which the country accumulates capital by postponing consumption. We study the optimal level of foreign debt and of the capital stock. We also point out the impact of debt on the time profile of consumption of the debtor country. In the usual two-period analysis [Chipman, 1985], the periods of debt repayment and debt service are collapsed into the second period only. In contrast to this approach, we use a continuous time model which allows to analyze the properties of the time profile of debt service and debt repayment [Bardhan, 1967; Sachs, 1984]. We show under which conditions borrowing yields a higher present value of welfare relative to the non-borrowing case. We establish conditions under which borrowing for capital accumulation and for consumption will occur simultaneously and under which debt is incurred for capital accumulation only. Moreover, we discuss some phenomena that may make it more difficult for the debtor country to pay interest and repay debt.

## II. A Model of Borrowing and Accumulation

Consider an economy that maximizes the present value of utility  $W$  from consumption  $C$

$$\omega = \int_0^T e^{-\delta t} W(C) dt \quad (1)$$

In order to simplify the notation, time indices of variables are omitted. In (1),  $\delta$  is the country's given time preference rate,  $T$  is terminal time and the period utility function has the usual properties

$$W_c > 0, W_{cc} < 0 \text{ and } n = -CW_{cc}/W_c > 0 \quad (2)$$

Terminal time  $T$  can be assumed to be given exogenously or it can be determined endogenously. Output  $Q$  is produced with capital  $K$  and labor  $L$  according to a neoclassical production function

$$Q \leq F(K, L) \text{ with } F_j > 0, F_{jj} < 0, F_{ij} > 0 \text{ for } i, j = K, L \quad (3)$$

The labor supply in each period is given and constant over time; the initial capital stock  $K_0$  is also given. The capital stock changes over time according to

$$\dot{K} = I - dK \quad (4)$$

where  $I$  is gross investment and  $d$  is the rate of depreciation. A dot over a variable denotes a change of a variable over time, a hat is the rate of change.

Consumption in any period is equal to production minus gross investment

minus interest payment on debt ( $rB$ ) plus borrowing ( $\dot{B}$ )

$$C = Q - rB - I + \dot{B}$$

Debt,  $B$ , and the change in debt,  $\dot{B}$ , are expressed in units of commodity  $Q$ , and  $r$  is the real interest rate assumed to be given. The country is small in the world capital market. Note that the increase in debt may be used for investment as well as for consumption. Rearranging, we have

$$\dot{B} = C + I - Q + rB \quad (5)$$

where the right-hand side denotes the deficit, (or, if negative, the surplus) in the balance of trade and services and the left-hand side indicates the balance in the capital account with  $\dot{B} > 0$  standing for capital imports (and  $\dot{B} < 0$  denoting capital exports, i.e. repayment of debt).

In terminal time  $T$ , debt must be repaid or financial assets are accumulated

$$B(T) \leq 0 \quad (6)$$

Eq. (5) is the budget constraint for each period; (5) and (6) together represent an intertemporal or dynamic budget constraint [Glick and Khavar, 1986].

In terminal time, the capital stock must be nonnegative

$$K(T) \geq 0 \quad (7)$$

Applying the classification of Glick and Khavar [1986], the model describes the problem of an optimizing borrower. Nonoptimizing behavior modelled by specific behavioral consumption functions including public choice approaches to government spending or rigidities preventing optimality of consumption and investment over time is not considered. Also, we do not study joint maximization problems of the borrower and the lender or strategic behavior [Sachs, 1984]. The borrower is assumed to be a small country having no impact on the interest rate of the world capital market; moreover, there is no feed-back of the amount of debt on the interest rate (small country assumption) or on additional funds available to the country. Default risk or country risk issues are not considered. Finally, the optimizing model does not explicitly analyze intertemporal market equilibria as in overlapping generation models [Buiter, 1981; Schmid, 1987].

### III. Optimality Conditions

The Lagrangean function  $Z$  is given by

$$\begin{aligned} Z = & W(C) + \lambda [I - dK] \\ & + \rho [C + I - Q + rB] \\ & + \mu [F(K, L) - Q] \end{aligned} \quad (8)$$

Optimality conditions are

$$\frac{\partial Z}{\partial C} = 0 \quad \Rightarrow W_c = -\rho \quad (8a)$$

$$\frac{\partial Z}{\partial I} = 0 \quad \Rightarrow \lambda = -\rho \quad (8b)$$

$$\frac{\partial Z}{\partial Q} = 0 \quad \Rightarrow \mu = -\rho \quad (8c)$$

$$\dot{\lambda} = \delta \lambda - \frac{\partial Z}{\partial K} \Rightarrow \dot{\lambda} = \delta \lambda + d\lambda - \mu F_k \quad (8d)$$

$$\dot{\rho} = \delta \rho - \frac{\partial Z}{\partial B} \Rightarrow \dot{\rho} = \delta \rho - r \quad (8e)$$

Defining a J-function according to van Long and Vousden [1977], we have

$$J = \xi B(T) + \Phi K(T) \quad (9)$$

with

$$\xi = e^{-\delta T} \rho(T) \quad (9a)$$

$$\xi B(T) = 0 \quad (9b)$$

$$\Phi = e^{-\delta T} \lambda(T) \quad (9c)$$

$$\Phi K(T) = 0 \quad (9d)$$

$$e^{-\delta T} H(T) = 0 \quad (9e)$$

where  $H = Z - \mu [F(K,L) - Q]$  is the Hamiltonian function.

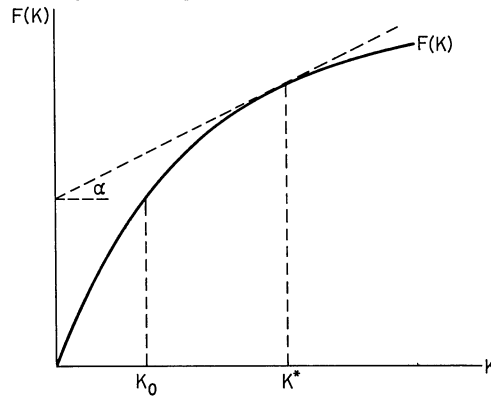
#### IV. Optimal Capital Accumulation

From (8b), we have for the rate of change in the shadow prices  $\hat{\lambda} = \dot{\rho}$  so that from (8c), (8d) and (8e) we have

$$r = F_k(K^*) - d \quad (10)$$

Eq. (10) defines the optimal capital stock,  $K^*$ . A capital-poor country<sup>2</sup>, with  $K_0 < K^*$  and  $F_k(K_0) > F_k(K^*)$ , will accumulate capital. Since (10) holds for any period, it must also hold for the first period. This implies that the optimal capital stock must be established right from the start. In the model, there is no restraint on accumulation except the balance of payments. For instance, there is no time involved in accumulating capital, and there are no physical restraints. Thus, accumulation is possible to happen instantaneously.

<sup>2</sup> A capital-rich country with  $K_0 > K^*$  will reduce its capital stock if capital can be used up for consumption.

Figure 1 – *The Optimal Capital Stock,  $K^*$ , for Capital Accumulation*

neously in the first period due to borrowing.

Eq. (10) is illustrated in Figure 1 where  $\tan \alpha$  is  $r + d$ ,  $K^*$  is the optimal capital stock and  $K_0$  is the initial capital stock<sup>3</sup>.  $K^* - K_0$  represents debt incurred for capital accumulation.

Once the optimal capital stock is reached, there is no net investment. Gross investment is a constant fraction of the constant (optimal) capital stock,  $dK^*$ . Output is completely specified by the production function, and it remains constant.

### V. The Time Profile of Consumption and Debt

Consumption in any period of time is optimal if the marginal utility of consumption is equal to the shadow price of debt or the user costs of borrowing,  $-\rho$ . Again, the user costs of borrowing must be equal to the shadow price of capital accumulation,  $\lambda$ , and the shadow price of output,  $\mu$ .

$$W_c(C) = -\rho = \lambda = \mu \quad (10a)$$

Consumption changes over time according to

$$\dot{C} = -\frac{1}{n} \hat{\rho} \quad (10b)$$

<sup>3</sup> The steady state properties with respect to the optimal capital stock can also be illustrated with the traditional C-K-phase diagram. The implications of the model can also be analyzed by the overall optimality condition  $\delta - \dot{W}_c = r = F_K(K) - d$  which follows from (8a) – (8e). The consumption rate of discount  $\delta - \dot{W}_c$  must be equal to the interest rate which must be equal to the net productivity of capital. The consumption profile over time can easily be derived from this overall optimality condition. For instance, let  $\delta > r$ . Then  $\dot{W}_c > 0$  which from  $\dot{W}_c = \dot{W}_{cc} \dot{C}$  implies  $\dot{C} < 0$ . The above condition can also be interpreted as a market equilibrium condition where the left-hand side models the saving behavior of households and the right-hand side represents the investment behavior of firms. An intertemporal equilibrium can be characterized by a time path of the interest rate driving the consumption and the investment decisions. I owe this view to H.H. Nachtamp.

Assume  $\delta > r$ . Then  $\hat{\rho} = \dot{\rho} / \rho > 0$ , that is  $\dot{\rho} < 0$  and  $-\dot{\rho} > 0$  so that  $\rho$  rises in absolute terms, and consumption is reduced over time.

Eq. (5) can be written as

$$\tilde{Q} = F(K^*, L) - dK^* = C + rB - \dot{B} = C + N \quad (11)$$

where net output  $\tilde{Q}$  on the left-hand side is a constant. With consumption falling over time due to (10b), the term  $rB - \dot{B} = N$  must rise over time, that is  $\dot{N} = -\dot{C} > 0$  (for  $\delta > r$ ).

With the capital stock being accumulated instantaneously in the first period, borrowing for capital accumulation will occur in the first period. In addition to borrowing for capital accumulation, the country also incurs debt for consumption if  $\delta > r$ . Borrowing for consumption, however, may be stretched over several periods initially. Consider the case, in which the initial capital stock  $K_0$  is identical to the optimal capital stock  $K^*$ . Then, with  $\delta > r$ , borrowing will occur for consumption only.

Initially we have

$$\tilde{Q} < C \Leftrightarrow N < 0 \Leftrightarrow rB - \dot{B} < 0 \quad (11a)$$

and thus  $r < \dot{B}/B$ . This implies that debt increases in the initial period(s). The increase in debt is larger than the interest payment so that interest is paid through new debt.

Over time, consumption is reduced, and  $N$  is increased, that is  $N$  becomes smaller in absolute terms. Eventually, we must have

$$\tilde{Q} = C \Leftrightarrow N = 0 \Leftrightarrow rB - \dot{B} \quad (11b)$$

so that  $r = \dot{B}/B$  and debt rises with the interest rate. Due to the continuing reduction in consumption,  $N$  continues to rise, now becoming positive with

$$\tilde{Q} > C \Leftrightarrow N > 0 \Leftrightarrow rB - \dot{B} > 0 \quad (11c)$$

and  $r > \dot{B}/B$ . The increase in debt becomes smaller, and eventually in period  $t'$  we have

$$\dot{B} = 0 \Leftrightarrow \tilde{Q} = C + rB \quad (11d)$$

so that net output is sufficient for consumption and interest payment. From  $t'$  on,  $B$  must fall so that  $\dot{B} < 0$ .

Note that we have from (11)

$$\ddot{B} = r\dot{B} + \dot{C} < 0 \quad \text{for } t > t' \quad (11e)$$

so that  $B$  is a concave function and does not fall asymptotically to zero. In terminal time, the amount repaid is not infinitesimally small.

The consumption profile is steered by the shadow price  $\rho$ . We have  $\hat{\rho} = \hat{\lambda}$  and for  $\delta > r$  consumption is reduced over time. This decline in consumption

is engineered by the change in the shadow price  $\rho$  with the initial  $\rho_0$  determining the initial level of consumption. With  $\delta > r$  it is worthwhile to borrow initially in order to reach a high consumption level.

The shadow price  $\rho_0$  in the initial period plays a strategic role in determining the initial level of consumption. Start from the case in which the initially given capital stock is equal to the optimal capital stock and then reduce the initially given stock parametrically. The country has to borrow more for capital accumulation thus incurring a higher debt. Consequently,  $\rho_0$  must rise in absolute terms allowing a lower level of consumption initially and a lower consumption profile over time.

Assume a fixed finite planning horizon  $T < \infty$ . Then the transversality conditions (9a) and (9b) together with a rising  $-\rho$  and thus a positive  $-\rho(T) > 0$  imply that  $B(T) = 0$ . Debt has to be zero in  $T$ . As a matter of fact, the shadow price  $-\rho(T)$  in terminal time and the equation of motion of this shadow price determine the implicit value of borrowing in the initial period and thus specify the initial level of consumption and its time profile.

With  $\lambda(T) > 0$ , the capital stock must be zero in finite terminal time due to the transversality condition (9d). Due to the optimality condition (10), the capital stock remains constant up to terminal time  $T$ . But due to the monotonicity of the consumption profile a discontinuity in the consumption profile in  $T$  is ruled out. The capital stock is not used for consumption in  $T$ . It is used to repay debt. For finite terminal time, the capital stock and debt are zero.

Extending  $T$  further and further into the future, this result will hold. The shadow price  $-\rho(T)$  continues to drive the system. With  $B$  falling over time,  $rB$  will fall, but debt repayment will rise over time.

When  $T$  is determined endogenously, and if  $T < \infty$ , the transversality condition (9e) simplifies to<sup>4</sup>

$$e^{-\delta T} \{ W(C) - W_c C \} = 0 \Leftrightarrow W_c = W(C)/C \quad (12)$$

This implies that  $C(T) = 0$ . Condition 12 can only be reached in finite time if  $W_c(0) < \infty$ . With the usual assumption that the marginal utility of consumption is infinite with  $C$  approaching zero,  $C(T) = 0$  cannot be reached in finite time<sup>5</sup>.

<sup>4</sup> The transversality condition for optimal  $T$  is

$$e^{-\delta T} \{ W(C) + \rho (dK + C - Q + rB) \} = 0$$

which together with conditions (8a), (9b) and (9d) implies (12).

<sup>5</sup> Allowing terminal time to approach infinity, the transversality condition for optimal time  $T$  is

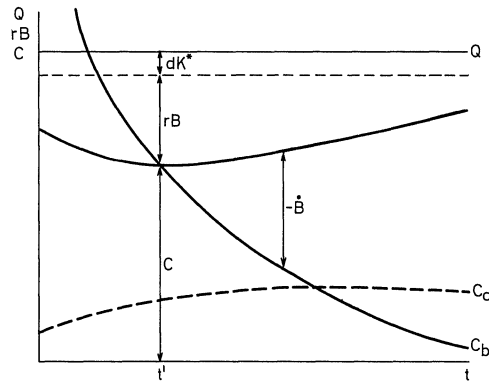
$$\lim_{T \rightarrow \infty} \{ e^{-\delta T} [W(C) - CW_c] + re^{-\delta T} \rho(T) B(T) + \rho_0 e^{-rT} (dK - Q) \} = 0$$

With  $\rho_0 = e^{-(\delta-r)T} \rho(T)$ . The third term approaches zero since  $dK - Q$  is a constant. Moreover, we have  $W/C > W_c$  for  $C > 0$  in the case of a concave utility function. Since  $CW_c > 0$ , we have  $W > W_c - CW_c > 0$ .



Figure 2 spells out the long-run balance-of-payments implications of foreign borrowing. Consumption  $C_b$  declines<sup>6</sup> due to a relatively high time preference rate, that is  $\delta > r$ .

Figure 2 – Long-Run Balance-of-Payments Implications of Foreign Borrowing



Capital accumulation occurs instantaneously in the first period; and debt rises for some periods to finance consumption. Borrowing is reduced, and it stops in  $t'$ . From then on, debt is repaid and an increasingly larger part of output is being used up for interest payment and the repayment of debt.

The term  $N = rB - \dot{B}$  in (11), that is interest payment and repayment of debt, can also be interpreted as the trade balance. Prior to  $t'$ ,  $N < 0$  is the import deficit; after  $t'$ ,  $N > 0$  denotes the export surplus. Since the term  $N$  rises over time the country has to reduce its import deficit over time prior to  $t'$ ; after  $t'$ , the country has to generate a rising export surplus over time. The model thus specifies the time profile of the balance of trade being driven by the shadow price of debt. The model implies a debt cycle over time.

For  $C \rightarrow 0$ ,  $W \rightarrow 0$  and thus  $W - CW_c \rightarrow 0$ . Therefore the transversality condition is

$$\lim_{T \rightarrow \infty} e^{-\delta T} H(T) = \lim_{T \rightarrow \infty} e^{-\delta T} \rho(T) rB(T) = 0$$

which implies  $r \rho_0 \lim_{T \rightarrow \infty} e^{-rT} B(T) = 0$ . This can only be satisfied if the term converges so we

must have  $\dot{B} < r$ . This is consistent with  $B(T) = 0$ , but for  $T \rightarrow \infty$  we may also have  $B(T) > 0$ . With  $T \rightarrow \infty$ , the penalty  $\rho(T) < 0$  loses its bite. Note that the transversality condition is consistent with  $W_c(0) \rightarrow \infty$ . For  $T \rightarrow \infty$ , the transversality condition for the capital stock is  $\lim_{T \rightarrow \infty} \lambda_0 e^{-rT} K(T) = 0$ . Eq. (10b) continues

to require a constant optimal capital stock so that  $K(T) = K^*(T) > 0$  is consistent with the transversality condition. For  $T \rightarrow \infty$ , the capital stock is not used to repay debt.

<sup>6</sup> A sufficient condition for a convex function  $C(t)$  is  $\dot{C} < 0$  and  $\ddot{C} > 0$ . From (8a) we have

$$\dot{C} = \frac{W_c}{W_{cc}} (\delta - r) \text{ and } \ddot{C} = (\delta - r) \frac{W_{cc}^2 - W_c W_{ccc}}{W_{cc}^2} \dot{C}$$

For  $\delta > r$ , we have  $W_c W_{ccc} / W_{cc}^2 > 1 \iff \ddot{C} > 0$ . This condition is satisfied, for instance, for an isoelastic utility function.

### VI. Capital Accumulation without Borrowing

Why does a country borrow in the first place? Can it improve its welfare by only relying on capital accumulation from internal saving? If we study capital accumulation in a closed economy, (5) simplifies to  $C=Q-I$ , that is, consumption is given by output minus gross investment, and the equation of motion for the capital stock changes into

$$\dot{K} = Q - C - dK \quad (13)$$

The Lagrangean then is given by

$$Z = W(C) + \lambda [Q - C - dK] + \mu [F(K, L) - Q] \quad (14)$$

with the optimality conditions

$$\frac{\partial Z}{\partial C} = 0 \quad \Rightarrow \quad W_c = \lambda$$

$$\frac{\partial Z}{\partial Q} = 0 \quad \Rightarrow \quad \lambda = \mu$$

$$\dot{\lambda} = \delta \lambda - \frac{\partial Z}{\partial K} \Rightarrow \hat{\lambda} = \delta + d - F_K$$

For a capital-poor country, that is  $F_K(K_0) > \delta + d$ ,  $\lambda$  falls, that is consumption increases over time. The country has to put a high penalty on consumption initially, allowing more consumption with capital being accumulated.

The optimal capital stock now has to be interpreted as the steady state capital stock being reached in the long run when  $\delta + d = F_K(\tilde{K}^*)$ . Without borrowing, the capital-poor country will accumulate a smaller capital stock if we assume  $\delta > r$ . Now the time preference rate (assumed to be identical in the borrowing and in the non-borrowing case) determines the long-run capital stock. Moreover, the capital-poor country cannot accumulate the (smaller) capital stock instantaneously as in the borrowing case, but it needs time. Finally, it must start at a lower consumption level initially, using a higher penalty on consumption in order to induce internal capital accumulation. Its consumption path ( $C_a$  in Figure 2) is on a lower level for quite a few of the earlier periods.

The advantage of borrowing is to allow a higher present value of welfare from consumption<sup>7</sup>. The higher time profile of consumption in the initial periods outweighs the reduced consumption possibilities of future periods.

<sup>7</sup> A lower present value of welfare in the borrowing case (for  $\delta > r$ ) relative to the non-borrowing case clearly would contradict optimality because in the borrowing case the country can decide not to borrow. If it decides to borrow as it will for  $\delta > r$ , borrowing increases its welfare.

### VII. Borrowing for Capital Accumulation Only

We have so far discussed an open economy that incurs debt for capital accumulation as well as for consumption.  $F_K(K_0) - d > r$  implies that it is optimal to borrow for investment;  $\delta > r$  implies that it is worthwhile to borrow for consumption. However, the country will not borrow for consumption if the interest rate is higher than the time preference rate, i.e.  $r > \delta$ . Then  $\hat{\rho} < 0$ , and consumption increases over time. This implies a high  $\rho$  in absolute terms in the initial situation in order to keep initial consumption down. If we have  $F_K(K_0) - d > r > \delta$ , capital will be accumulated because the benefit of accumulating capital outweighs its opportunity costs. The accumulation occurs instantaneously due to condition (10b). This implies borrowing for capital accumulation<sup>8</sup> only. Due to the low consumption profile initially, the country actively contributes to the accumulation of capital by postponing consumption. This consumption profile reduces the amount to be borrowed.

### VIII. What Can Go Wrong?

Borrowing implies restraints for future periods in terms of giving up goods (or generating a balance-of-trade surplus) in order to pay interest and repay debt. In our model of optimal borrowing, the opportunity costs of borrowing are, of course, taken into account. A country will only incur a debt, if the marginal benefit from a unit borrowed will be (greater than or) equal to the user cost of borrowing.

In the real world, quite a few phenomena can affect this intertemporal optimality condition so that initially chosen time profiles of consumption and debt repayment will no longer be followed. Including such phenomena into our framework allows a richer analysis.

A high time preference rate increases the divergence between the time profile of consumption in the borrowing and the non-borrowing case. A high discount rate is the expression of the country's impatience, and impatience may be caused by a high population pressure, by high aspiration levels and by political demands of voters or groups of society. Such an impatience implies a

<sup>8</sup> In this case we have from (11) that

$$C < \bar{Q} \Leftrightarrow N > 0 \Leftrightarrow r > \dot{B}/B \quad (i)$$

because the consumption level is low initially. From (11) we also have  $\dot{N} = -\dot{C}$  and consequently

$$\dot{B} = rB + \dot{C} \quad (ii)$$

We can rule out  $\dot{B} = 0$  because this implies  $\bar{Q} = C + rB$ . Over time  $C$  rises, so that with  $\bar{Q}$  constant,  $B$  has to fall which is impossible for  $B = 0$ . We can also rule out  $\dot{B} > 0$ . Then (ii) implies  $\dot{B} > 0$  for rising  $C$ . Debt would rise progressively over time, but the transversality condition implies  $B(T) = 0$ . Consequently  $\dot{B} < 0$ . Note that this case does not reach a steady state for consumption due to the repayment of debt.  $\rho$  falls in absolute value over time so that consumption can be increased from period to period.

low weight on user costs, and it thus allows a high level of consumption initially, which is a relatively low incentive for capital accumulation. In the borrowing case,  $-\rho$  has to rise more quickly over time. Of course, the burden for future periods will be increased. In the case of internal capital accumulation, a smaller capital stock will be accumulated. The  $C_a$ -curve will shift upwards for initial periods, but the consumption level will be lower later on<sup>9</sup>.

A low stock of capital initially increases the amount to be borrowed, and a higher borrowing implies a higher penalty on consumption both initially and in later periods. The country will be on a lower consumption profile.

In the optimization model, the marginal productivity of capital is known, but in reality, it is an expected variable. If expectations on the marginal productivity of capital do not materialize as in Poland in the 1970s, the country is stuck with interest and principal payments as shown in Figure 2, but it cannot generate the output  $Q$ . Thus, either the consumption path must be suppressed or debt service will be curtailed. Of course, the strategy of capital accumulation as a means for economic development is not too promising if the country's technology is characterized by a production function on a low(er) level (than in Figure 2) suggesting only a small accumulation opportunity.

A similar argument relates to the terms of trade. In the model, the country borrows in real terms and repays in real terms, so that the terms of trade (in an intertemporal sense) are set equal to unity remaining constant over time. The analysis can be made richer by assuming that the country borrows foreign currency and by introducing a price for its export commodity  $Q$  in terms of foreign currency. If this price is reduced over time, lower foreign currency earnings are available to finance debt service ( $rB - \dot{B}$ ).

In the model the amount borrowed is used instantaneously for accumulating capital (and for consumption). This transfer of debt into physical capital may be interrupted at several instances, and funds borrowed for investment may be used for consumption. This occurs for instance when the time preference rate increases. Funds intended for capital accumulation may even be used for capital exports. Moreover, accumulating capital may not occur instantaneously and may be time consuming. In these cases, not enough real output is generated in the future necessary for the debt service.

If contrary to the model, the country has borrowed at a floating rate, and unexpectedly the interest rate rises, the country will be unable to service the debt from current output.

<sup>9</sup> Note that if we reduce the time preference rate parametrically and if we make it sufficiently low, we may have  $\delta < r$ , and the time profile of consumption in the borrowing case, i.e. the  $C_b$ -curve, will have a similar shape as the  $C_a$ -curve.  $\hat{\rho} < 0$ , and consumption increases. From  $\hat{\rho} = \hat{\lambda}$  we have  $\delta - r = (\delta + d) - F_K(K^*)$  which implies  $\delta + d > F_K(K_0)$  initially so that by the parametric variation of the time preference rate the country has become a capital-rich country.

Equity capital is an alternative to foreign borrowing. Of course, equity capital implies dividend payments, and in the 1970s developing countries did not want to see this transfer occur. But at the same time, dividend payments have the advantage of not implying a constant obligation to pay. They allow more flexibility with respect to some of the unfavorable events discussed above. With equity capital, the country can shift the risk of unfavorable events to the foreign firm. Note, however, that foreign firms will anticipate these risks, and if the risks are sufficiently high, the country may not be able to attract foreign direct investment. Moreover, in addition to economic risks, the foreign firms will consider political risks such as changes in taxation, regulation and property rights. In addition to this property of risk shifting, equity capital alleviates the transversality condition in the sense that only debt incurred for consumption has to be paid back<sup>10</sup>.

Finally, the phenomena that make it more difficult for the debtor country to pay interest and repay debt may lead to a breach of contracts [Sachs, 1984]. As is illustrated in Figure 2, with the passage of time, the utility derived from borrowing will be reduced and the opportunity costs will rise. At a moment of time  $t$  sufficiently far away from the initial period the burden of borrowing,  $rB - \dot{B}$ , may seem high whereas utility derived from consumption may appear low. The political process tends to have a short memory, and in the perceptions of the debtor country the advantage of the time profile  $C_b$  over  $C_a$  is forgotten.

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<sup>10</sup> The transversality condition (9b) only relates to debt and not to equity capital. Eq. (5) has to be revised to accommodate dividend payments. This idea was pointed out by an anonymous referee.

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Zusammenfassung: Devisenschuld und Kapitalakkumulation. – Eine offene Volkswirtschaft kann Kredit aufnehmen, um Kapital zu akkumulieren. In dem Beitrag wird der Fall der Auslandsverschuldung mit einem Szenario verglichen, in dem ein Land seinen Kapitalstock durch Konsumverzicht aufbaut. Die Inzidenz der Verschuldung auf das Zeitprofil des Konsums und der Verschuldung wird analysiert. Das Modell spezifiziert auch das Zeitprofil der Handelsbilanz. Die intertemporalen Opportunitätskosten der Kreditaufnahme werden erörtert, und es werden Bedingungen abgeleitet, unter denen sich die Verschuldung für Kapitalbildung und Konsum oder nur für die Kapitalbildung lohnt. Im Paradigma des Modells werden Konstellationen diskutiert, unter denen es für das verschuldete Land schwieriger wird, Zinsen und Tilgung zu zahlen.

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Résumé: Dette étrangère et accumulation de capital. – Une économie ouverte peut emprunter à l'étranger pour accumuler du capital. L'auteur compare le cas d'un emprunt étranger avec un scénario dans lequel un pays accumule du capital en remettant la consommation locale. Il analyse l'effet de l'emprunt sur le profil de temps de la consommation et de la dette. Le modèle spécifie aussi le profil de temps de la balance commerciale. Les coûts d'opportunité d'emprunter sont analysés et les conditions sont spécifiées sous lesquelles il est profitable d'emprunter pour l'accumulation de capital et/ou pour la consommation. Dans le contexte du modèle l'auteur discute quelques phénomènes qui le rendent plus difficile au pays débiteur de payer les intérêts et de rembourser les dettes et même de rompre le contrat de crédit.

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Resumen: Deuda externa y acumulación de capital. – En una economía abierta un país puede endeudarse a los efectos de acumular capital. En este trabajo se compara el endeudamiento externo con una situación en la cual un país acumula capital internamente posponiendo el consumo. Se analiza el impacto del endeudamiento sobre el perfil temporal del consumo y de la deuda. El modelo también especifica el perfil temporal del saldo comercial. Se estudian los costos de oportunidad del endeudamiento y se derivan las condiciones bajo las cuales resulta beneficioso endeudarse para acumular capital y/o para consumir. En el contexto del modelo se discuten algunos fenómenos que podrían dificultar al país deudor el pago del servicio de la deuda y de la deuda misma, y que podrían inducir a cuestionar el contrato de crédito.