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## Human capital risk, contract enforcement, and the macroeconomy

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Federal Reserve Bank of Chicago

**Human Capital Risk, Contract  
Enforcement, and the Macroeconomy**

*Tom Krebs, Moritz Kuhn, and Mark L. J. Wright*

October 2014

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# Human Capital Risk, Contract Enforcement, and the Macroeconomy\*

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## Abstract

We use data from the Survey of Consumer Finance and Survey of Income Program Participation to show that young households with children are under-insured against the risk that an adult member of the household dies. We develop a tractable macroeconomic model with human capital risk, age-dependent returns to human capital investment, and endogenous borrowing constraints due to the limited pledgeability of human capital (limited contract enforcement). We show analytically that, consistent with the life insurance data, in equilibrium young households are borrowing constrained and under-insured against human capital risk. A calibrated version of the model can quantitatively account for the life-cycle variation of life-insurance holdings, financial wealth, earnings, and consumption inequality observed in the US data. Our analysis implies that a reform that makes consumer bankruptcy more costly, like the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005, leads to a substantial increase in the volume of both credit *and* insurance.

**Keywords:** Human Capital Risk, Limited Enforcement, Life Insurance

**JEL Codes:** E21, E24, D52, J24

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# 1. Introduction

For many households, human capital comprises the largest component of their wealth. Human capital is subject to hard-to-diversify risk which is, in the case of disability or mortality risk, potentially catastrophic in magnitude. In this paper, we use micro-data from a range of sources to document that young married households with children are severely under-insured against one important type of human capital risk—the risk of the death of an adult household member—even though life insurance markets are fairly competitive. Specifically, the median young married household with children buys life insurance that covers between 10 and 40 percent of the net present value loss associated with the death of an adult family member, whereas older households are close to fully insured (see figure 1).<sup>1</sup> We then argue that the observed pattern of under-insurance is well explained by borrowing constraints that emerge endogenously due to the non-pledgeability of human capital and a bankruptcy code that limits wage garnishment (limited contract enforcement). Finally, we demonstrate that our approach has quantitatively important macroeconomic implications.

Our argument for the under-insurance of young households proceeds intuitively as follows. As is well known, labor earnings increase rapidly from the level achieved upon entering the job market before reaching a peak in late middle age (see figure 2). Following a long tradition in labor economics, we interpret the high earnings growth of young households as the result of investment in high-return post-schooling human capital (such as on-the-job training). Thus, young households have access to a risky investment opportunity with high expected returns, and also desire to smooth consumption over their life-cycle, but have little assets beyond their human capital. Consequently, young households have a strong motivation for borrowing, which is, however, tightly constrained by their lack of assets and their inability to pledge future earnings as collateral on their debts. On the margin, young households prefer to either consume or invest in human capital rather than purchase insurance, and hence under-insure against human capital risk even if insurance is priced in an actuarially fair manner.

To develop the argument more formally, we present a theory of the life-cycle accumulation of human capital and financial capital, the allocation of financial capital across assets, and the decision to purchase insurance against human capital risk. The enforcement of credit

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<sup>1</sup>Our measure of income losses takes into account social security survivor benefits, progressive income taxes, and implicit insurance from the possibility of re-marriage. See Section 2 for details.

contracts is limited by the fact that human capital is non-pledgeable and wage garnishment in the case of default is limited. Households in the model are heterogeneous, differing in their age, labor market status, marital status, number of dependents, and holdings of their human capital and financial assets. Household preferences are logarithmic in consumption and do not vary over the life-cycle.

We first show that the model is tractable, in the sense that individual decision rules are linear in household wealth (financial plus human) and that the infinite-dimensional wealth distribution is not a relevant state variable, and exploit the tractability of the model in both the theoretical and quantitative analyses conducted in this paper. In the theoretical analysis, we use a simplified version of the model with mortality risk as the only source of human capital risk to show analytically that young married households are under-insured against the risk of the death of an adult household member. We then provide a quantitative assessment of the theory in which we calibrate the general model to the U.S. bankruptcy code and to four features of the U.S. data: (i) the empirical life-cycle profile of median household earnings, (ii) estimates of labor market risk obtained by the empirical literature, (iii) the empirical life-cycle profile of mortality risk and rates of demographic transition (such as marriage and childbirth), and iv) the human capital losses associated with the death of a spouse estimated in this paper.

We emphasize three findings of our quantitative analysis. First, in equilibrium young households are severely under-insured against human capital risk, whereas older households are almost fully insured. In other words, our quantitative analysis suggests that realistic life-cycle variations in human capital returns combined with the basic features of the US bankruptcy code generate substantial under-insurance of young households through endogenous borrowing constraints. This under-insurance result holds when we consider insurance against mortality risk, in which case we measure the degree of insurance by the *life insurance coefficient* defined as the ratio of life insurance holdings (face value) over the human capital loss in the case of death of an adult family member. The under-insurance result also holds when we consider other forms of human capital risk and measure the degree of insurance by comparing the consumption volatility of households with access to insurance markets to the consumption volatility of households without access to insurance markets.

Our second finding is that the calibrated model economy can quantitatively account for the empirical life-cycle pattern of a number of important economic variables. Most importantly, the model provides an accurate account of the empirical life-cycle pattern of

life insurance holdings and the empirical life-cycle profile of under-insurance against the death of an adult family member. In other words, the under-insurance pattern observed in the life insurance data can be fully explained by borrowing constraints that emerge endogenously due to the limited enforcement of credit contracts. We further show that the model produces life-cycle profiles of financial wealth, human wealth, and consumption inequality that are in line with the data. We take these results as corroboration of our theory since the model has not been calibrated to match the corresponding targets.

Third, we find that limited contract enforcement has quantitatively important macroeconomic implications. There has been a long-standing debate among academic scholars and policy makers with regard to the relative merits of alternative consumer bankruptcy codes. In the US, this debate has led to legislation making it more costly to declare bankruptcy, such as the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005. The theoretical literature has argued that making it more costly to declare bankruptcy not only increases the volume of credit, but also the amount of insurance purchased by households. In addition, in our human capital model it further increases economic growth since it leads to more investment in the high-return asset. In this paper, we contribute to the debate by providing a quantitative analysis of the various channels in a macro model with physical capital and human capital. For the calibrated version of the model, we find that the insurance effect is substantial inducing a significant increase in human capital investment. Overall, the welfare gain from this reform for young households is around 0.5 percent of lifetime consumption.

In sum, in this paper we make an empirical contribution by providing evidence of under-insurance in the life insurance market, we make a theoretical contribution by showing analytically that a model with endogenous borrowing constraints due to limited contract enforcement can explain this empirical finding, and we make a contribution to the quantitative macro literature by demonstrating that a calibrated version of the model can account for the observed life-cycle patterns of insurance, earnings, financial wealth, and consumption. In addition to these substantive contributions, this paper also makes a methodological contribution by developing a tractable framework and demonstrating how to surmount the non-convexity in the household decision problem that is inherent in production economies with limited pledgeability and risk-averse agents. The tractability of the model is indispensable for providing analytical results regarding the relationship between age (expected human capital returns) and insurance. Further, it is essential for our quantitative analysis since our calibration approach and the general equilibrium analysis require that the computation of

the solution to the (highly complex) households decision problem is not too time-consuming.<sup>2</sup>

At this stage, a methodological comment seems in order. In this paper, we develop a general macroeconomic framework with limited contract enforcement and test one basic implication of the theory using data on life insurance holdings. The use of life insurance data has several advantages. First, these data are available for a representative sample of households. Second, the particular event in question (death of an adult household member) is clearly defined and the risk of this event is generally observable and verifiable through a detailed medical exam and questionnaire that, if falsified, can result in cancelation of the life insurance contract. This means that the market is likely to be less subject to concerns about adverse selection or moral hazard and hence a good basis for testing a theory of underinsurance based on liquidity constraints. Third, the existence of large number of competing providers, along with the fact that the industry is relatively lightly regulated, also suggest that the industry is quite competitive and therefore make the industry a relatively desirable candidate for analysis. Fourth, the particular test of underinsurance we devise is independent of the probability of the death event because it is only based on the payout in case the event occurs. Hence, mis-specification of (small) probabilities by individuals does not change our empirical results. Fifth and finally, as life insurance purchases typically have a degree of flexibility in choosing the size of the payout in the event of death, there exists an active intensive margin of life insurance choices that is either not available, or more limited, in other insurance contracts.

The rest of this paper is organized as follows. Following a review of the literature, Section 2 describes a number of aspects of the US data on life insurance and presents our measures of under-utilization of life insurance. Section 3 presents our theoretical model, and derives our underinsurance result analytically for a special case of the model. Section 4 describes our calibration while Section 5 describes our quantitative results. Section 6 establishes that the results of the model are robust to a number of changes to the model, while Section 7 concludes.

**Literature** Our paper is related to three strands of the literature. First, in common

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<sup>2</sup>The quantitative macro literature on limited commitment/enforcement with labor market risk has so far not analyzed models with life-cycle heterogeneity or endogenous human capital accumulation. There is, of course, quantitative work on incomplete-market models with life-cycle heterogeneity and human capital investment, but the computation of optimal household decision rules is much less complex in the incomplete-market case since there are fewer assets and therefore fewer portfolio choices.

with much of the literature on life insurance, we argue that the market for life insurance is relatively competitive and that issues of moral hazard and adverse selection are not of first-order importance, and hence abstract from them below (see Section 2.2 for a more detailed discussion). In contrast to our approach, most previous studies of life insurance demand have focused on variations in household preferences to explain empirical regularities. For example, Hong and Rios-Rull (2012) use data on life insurance holdings to estimate variation in preferences across households with different demographic characteristics, while Koijen, Nieuwerburgh, and Yogo (2012) analyze the effect of health status on preferences and hence on the life insurance demand of elderly men. One important exception is Hendel and Lizzeri (2003) who emphasize, as we do, commitment problems in the market for life insurance, but their focus is on the design of life insurance contracts (i.e. front-loading), whereas our interest lies in the effect of endogenous borrowing constraints on the purchase of life insurance contracts.

Second, our paper contributes to the literature on risk sharing in models with limited enforcement (Alvarez and Jermann, 2000, Kehoe and Levine, 1993, Kocherlakota, 1996, Thomas and Worrall, 1988). Our theoretical contribution is to develop a tractable model with human capital accumulation and to show how to avoid the non-convexity problem that arises in a class of limited enforcement production models.<sup>3</sup> Our substantive contribution is to show that a calibrated macro model with physical and human capital and limited contract enforcement generates a significant lack of consumption insurance and explains the observed life-cycle pattern of life insurance holdings. In contrast, the previous literature did not consider life-cycle variations and found that consumption insurance is almost perfect in calibrated models with physical capital (Cordoba, 2004, and Krueger and Perri, 2006).<sup>4</sup> Broer (2014) provides a detailed discussion of the quantitative implications of limited enforcement models without a life-cycle component. Finally, we share with Andolfatto and Gervais (2006) and Lochner and Monge (2011) the focus on human capital accumulation and endogenous borrowing constraints due to enforcement problems, but we go beyond their work by studying the interaction between borrowing constraints and insurance.

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<sup>3</sup>Wright (2001) has shown how to circumvent the non-convexity issue in linear production models (AK-model) with limited enforcement. See also Azariadis and Kaas (2008) for a contribution that exploits the linear production structure in limited enforcement models. The model structure we use in this paper is based on the human capital model with incomplete markets analyzed in Krebs (2003).

<sup>4</sup>Krueger and Perri (2006) match the cross-sectional distribution of consumption fairly well, but the implied volatility of individual consumption is negligible in their model.



Third, our paper is related to the voluminous literature on macroeconomic models with exogenously incomplete markets, and in particular studies of human capital accumulation (Krebs, 2003, Guvenen, Kuruscu, and Ozkan, 2011, and Huggett, Ventura, and Yaron, 2011) and the life-cycle profile of consumption (Storesletten, Telmer, and Yaron, 2004a, and Kaplan and Violante, 2010). This literature has shown that the incomplete-market framework is a powerful tool for understanding human capital investment choices and the observed life-cycle behavior of income, consumption, and human capital. In this paper, we show that a model with one financial friction, limited contract enforcement, explains equally well the life-cycle pattern of income, consumption, and human capital, while also explaining observed patterns in life insurance demand. Moreover, we show that, at least in the context of our discussion of the reform of the US bankruptcy code, the policy implication of these two classes of models differ significantly.

## 2. Empirical Evidence

In this section, we discuss the empirical evidence that motivates this paper. Specifically, we present the life-cycle profile of life insurance (Section 2.1), the life-cycle profile of mortality risk and under-insurance against this risk (Section 2.3), and the life-cycle variation of the relative importance of human capital and financial capital (Section 2.4).

### 2.1 Life Insurance

Our primary source of data on life insurance holdings is the Survey of Consumer Finance (SCF). Our data are drawn from the 6 surveys of the SCF conducted between 1992 and 2007. The unit of observation is the “family”, which corresponds to our concept of a household, and we measure the household’s age as the age of the household head. We focus our attention on married households with at least one child since they constitute a group of households that we can identify in our data as a group with a clear motive for purchasing life insurance (see also the discussion and references in Inkmann and Michaelides, 2012). We construct life-cycle profiles by computing median household values using five-year age bins for each survey removing possible time effects using time dummies as in Huggett et al. (2011).<sup>5</sup> Further details on the data, definition of variables, and sample selection are provided in the Appendix.

Life insurance contracts can be approximately divided into Term Insurance and Whole

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<sup>5</sup>We have also used cohort-dummies, with similar results.

Life Insurance.<sup>6</sup> Term insurance contracts only offer insurance against the death event, whereas whole life insurance contracts offer a combination of insurance and saving. We use the face value (amount of money paid in the case of death) of all insurance contracts, term insurance and whole life insurance, to construct the amount of insurance owned by a household, subtracting the savings component of whole life policies as reported in the SCF. The SCF presents total holdings for the household, and hence these data reflect the total payout from the death of both spouses (we present data on life insurance holdings by spouse from an alternative data source in Section 4 below).

Figure 3 shows the empirical life-cycle profile of life insurance purchases of married households with children. The blue diamonds show the median across all such households and shows that households in their early 20's have roughly \$15 thousand dollars of life insurance. This rises to about \$150 thousand by the time these households reach their 40's, and declines to \$50 thousand as households reach retirement age. The red dots show the median across only those households that have purchased some life insurance. Amongst these households, the young purchase around \$85 thousand in life insurance, rising quickly to \$200 thousand before declining slowly down to \$75 thousand in their early 60's.

The hump-shaped pattern in both series seems to indicate that young households are under-insured. However, it could also mean that young households simply need less insurance. To establish whether households are underinsured against the risk of death of a spouse, we need to take a stand on the “appropriate” level of insurance. One approach would be to use a model to deduce optimal holdings in the absence of market frictions, and use this as a measure of full insurance. We return to this approach later in the paper after presenting our model. In Section 2.3 below we turn to an alternative approach in which we proxy the size of the loss of human wealth upon death of a spouse by the present value of income losses taking into account the implicit insurance that results from the possibility of re-marriage, social security survivor benefits, and progressive income taxation. These two approaches yield almost identical answers for our baseline model (compare figures 7 and 8).

## 2.2 Life Insurance – Two Issues

Life insurance contracts can be divided into insurance that households purchase directly from insurance companies and insurance that is obtained through employment or membership in organizations (group insurance). If the amount of group insurance offered by the employer

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<sup>6</sup>Universal life insurance is grouped with whole life insurance.

exceeds the amount households want to hold, then these households are “involuntarily” over-insured and the insurance holdings observed in the data are not the outcome of the optimal insurance choice by households. Clearly, the phenomenon of involuntary over-insurance can only occur for households who have not purchased any individual life insurance from insurance companies. Although the SCF does not distinguish between group insurance and insurance purchased individually, data on employer provided life insurance are available from the Survey of Income and Program Participation (SIPP). Based on data drawn from the 2001 Panel of the SIPP, we find that for each age between 23 and 60, the median household with kids holds substantially more life insurance than the amount of insurance provided by the employer. Further, for the median household with children the amount of employer-provided life insurance is roughly constant over the life-cycle and the shape of the life-cycle profile of total (group plus individual) life insurance holdings is therefore not much affected by the presence of group life insurance. See the figure A6 and the Appendix for more details. Thus, we conclude that the consideration of insurance purchases as voluntary is appropriate to a first approximation. Hong and Rios-Rull (2012) come to a similar conclusion after analyzing data drawn from the International Survey of Consumer Financial Decisions.

In line with much of the previous literature on life insurance (e.g. Hendel and Lizzeri 2003, Hong and Rios-Rull 2012, and Koijen et al 2012), we model the market for life insurance as competitive with actuarially fair pricing. This seems reasonable given the large number of competing providers and the lack of regulatory inference, and has found support in the data (see, for example, Winter, 1981). We also follow the bulk of this literature in abstracting from considerations of asymmetric information. We argue that this is reasonable given that moral hazard problems appear small, and that adverse selection is limited by the requirement of a medical exam and the provision of a medical history with the risk that a policy will be voided if health information is not fully disclosed. Further, the available empirical evidence suggests that adverse selection is not of first-order importance in the market for life insurance (see, for example, Cawley and Philipson 1999, Hendel and Lizzeri 2003, and Koijen et al. 2012).<sup>7</sup>

### 2.3 Human Capital Risk and Underinsurance

Human capital is subject to a significant amount of idiosyncratic risk. In this paper, we

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<sup>7</sup>By contrast, there is considerable evidence of adverse selection in the market for annuities, where a medical exam is not required (Finkelstein and Poterba, 2004 and Friedman and Warshawski 1990). See also Society of Actuaries (2012) for details on the range of data collected by life insurers.

divide these risks into labor market risk and demographic risk, with a particular focus on the mortality risk of spouses. We view labor market risk as that risk which affects observed labor earnings, which includes job displacement risk and some forms of disability risk. We follow a substantial literature (e.g. Huggett et al. 2011, Krebs 2003) and set the parameters describing human capital risk so that the implied labor market earnings process is consistent with estimates of permanent labor market risk obtained by the empirical literature (Carroll and Samwick 1997, Meghir and Pistaferri 2004, Storesletten et al. 2004), and defer a discussion of the details until we calibrate the model in Section 4.

Demographic risk captures the effects of marriage, divorce, childbirth, and death of a spouse on household earnings. Rates of marriage and childbirth are calibrated using data from the SIPP. As a result of the small numbers of young widows and widowers in the SIPP, we cannot reliably estimate a life-cycle profile of re-marriage rates for widows. We therefore follow the macro literature on life insurance (Hong and Rios-Rull, 2012) and use re-marriage rates of divorcees as a first proxy for the re-marriage rates of widows/widowers, but in contrast to the previous literature we introduce an adjustment factor to take into account what is known about re-marriage rates of widows and widowers from the economics and sociology literatures.<sup>8</sup> Section 4.2 on calibration of the model discusses in detail our approach. Mortality risk is chosen to match the year-to-year average survival rates for the period 1991-2000 from the US life-tables for the respective group.

Computation of the size of the human capital loss associated with the death of a spouse is also inhibited by the relative paucity of data on young widows and widowers. We focus on households with median earnings and approximate the amount of household human capital lost in the case of death by the (expected) present value of the after-tax earnings differential between married households and the corresponding single household after including Social Security survivor benefits. We use the income tax and survivor benefit schedules from the year 2000, the mid point of our sample. See Section E of the Appendix for a detailed description of our approach and Section I of the Appendix for a robustness analysis.

Figure 4 plots the ratio of human capital losses in the event of death of a spouse to household labor earnings for the sample of married households with kids. To allow comparability with the life insurance data described above (which is aggregated for one household) the loss

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<sup>8</sup>Both widows and widowers have lower re-marriage rates than divorcees for each age group (Norton and Miller 1990, and Wilson and Clarke 1992).

associated with the death of the household head is added to the loss from the death of their spouse. The first line depicts the loss of labor pre-tax earnings without allowing for the possibility that a widow or widower can remarry and shows that young households with children lose roughly 30 years of earnings following the death of a spouse. The second line includes the effect of taxes and social security survivor benefits on lost earnings, but does not allow for re-marriage, and shows that the government provides a substantial amount of insurance against the death of a spouse: for young households with children, the loss has declined to 15 years of earnings after taxes and transfers. Finally, the third line, which also allows for remarriage as a kind of informal insurance against loss of a spouse, shows that the resulting income loss is reduced to between 8 and 9 years of annual earnings for young households, with smaller reductions for older households who face lower remarriage rates.<sup>9</sup> Overall, our results suggest that income losses in the case of death of a spouse are substantial, but much less than a simple calculation that does not take into account non-linear taxes, social security survivor benefits, and remarriage, would suggest. Further, human capital losses expressed as a fraction of household human capital decline with age, suggesting that younger households should purchase more life insurance than older households.

Although other sources of informal insurance are possible, we argue that, with one exception, they are likely to be insignificant. For example, it is possible that the surviving spouse increases their own labor supply. However, the substantial empirical literature examining the responses of spouses labor force participation and hours worked to shocks in earnings or disability typically finds little or no effect (Gallipoli and Turner 2011, Heckman and Macurdy 1980, Gruber and Cullen 1996). Similarly, private transfers from outside of the household appear insignificant following the disability of a spouse (Gallipoli and Turner 2009) and we argue are also likely insignificant following death. In Section I.3 of the Appendix we provide a detailed analysis that shows that the degree of informal insurance is relatively small for the affected group of households. One insurance mechanism that is plausibly significant is that the cost of living for a family is reduced following the death of a spouse, and we return to this issue in Section 4 below.

With our measure of the human capital loss in hand, we can now present our estimates of underinsurance. To this end, we introduce the *life insurance coefficient* defined as the

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<sup>9</sup>Our computed income losses without re-marriage are in line with the results in the literature (for example, Burkhauser et al. 2005 and Weaver 2010). This literature, however, has not computed effective income losses taking into account re-marriage.

ratio of the face value of life insurance holdings over the human capital loss in the case of death of an adult family member.<sup>10</sup> Figure 1 plots the life insurance coefficient for married households with children. The blue diamonds show insurance holdings for all households with kids and shows that the median household is insured against only one-tenth of the of the loss expected from the death of a spouse. This rises to roughly 50% by middle age, and to 75% by retirement. The red dots show the same data for the sample of married households with kids that purchase some life insurance. This figure begins at roughly 30% and rises to close to 100% only as households reach their late 50's. This is our main empirical result: there exists a positive correlation between age and the degree to which households insure against mortality risk by purchasing life insurance. Further, young households are severely under-insured, whereas older households are almost fully insured.<sup>11</sup>

## 2.4 Human Capital and Financial Capital

We now turn to a discussion of the relative importance of human capital and financial capital, and how this importance varies over the life-cycle. To this end, we use earnings (labor income) as a proxy for human capital and construct the life-cycle of the ratio of financial wealth to earnings. Data on earnings and financial wealth are also drawn from the 6 surveys of the Survey of Consumer Finance (SCF) conducted between 1992 and 2007 and life-cycle profiles are constructed in the way described in Section 2.1. We continue to focus on married households with children. The variable “financial wealth” is defined as “net worth” in the SCF, which is the value of all assets (including housing and excluding human capital) minus the value of all debt (including mortgage debt). Labor earnings are defined as wages and salaries plus two-third of business and farm income.

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<sup>10</sup>Our life insurance coefficient should not be confused with the insurance coefficient introduced by Blundell et al. (2008), which differs in three important respects. First, Blundell et al. (2008) consider the consumption response to all income shocks due to events that are not further specified, whereas we confine attention to one clearly defined event, namely the death of a spouse. Second, Blundell et al (2008) separate income shocks into a transitory and a permanent component and then separately define insurance coefficients for each component. In contrast, we focus on human capital losses that are permanent and therefore only consider an insurance coefficient with respect to permanent income shocks. Finally, Blundell et al. (2008) lump together different mechanisms of consumption insurance: insurance through friends and family, self-insurance through own saving, and insurance through the purchase of insurance contracts. In contrast, we focus on one type of insurance mechanism: insurance through the purchase of insurance contracts.

<sup>11</sup>The fact that households are underinsured even after conditioning only on those households that purchase life insurance suggests that fixed costs in the purchase of life insurance are not the only reason for the observed underinsurance.

Figure 5 plots the median ratio of net worth to labor earnings for married households with children of each age starting at 23 and ending at 60. As shown in the figure, households in their 20's and early 30's hold almost all of their wealth in human capital with the stock of net financial assets (including housing) less than one years flow of income from their human capital (i.e. labor earnings). By age 45, household net worth is roughly twice labor earnings, and it is not until households reach their 50's that net worth exceeds three time annual labor earnings.

The pattern in figure 5 is driven by the rapid accumulation of net financial assets, as labor earnings are also increasing in the early part of the life cycle. To illustrate this, figure 2 plots the lifecycle profile of labor earnings derived from our data for households from age 23, when many households have left college, to age 60.<sup>12</sup> As is well-known, labor earnings rapidly increase until age 35-40, after which they grow more modestly reaching a peak about age 50, and then declining as households approach retirement.

We follow a long tradition by interpreting these earnings profiles as the result of human capital accumulation decisions motivated by high returns to post-college education and on-the-job training (e.g. Becker 1964 and Ben-Porath 1967). There is a large literature estimating the returns to college-education, and some work on the returns to post-college education. Overall, the literature suggests a rate of return in the range of 8% – 10% (Krueger and Lindhal 2001), though individual estimates vary considerably and there is a large amount of heterogeneity due to differences in ability (Cunha, Heckman, and Navarro, 2005, and Taber, 2001). Estimates of wage gains from on-the-job training imply rate of returns that are even higher than 10% (Blundell et al. 1999, and Mincer 1994). Clearly, this evidence suggests that for many young households there is a strong incentive to invest in human capital. Moreover, this incentive exists regardless of whether returns are higher for the young because of decreasing returns to human capital accumulation, as in Ben-Porath (1967) and Huggett et al. (2011), or because human capital investment is less productive for older households, as in the model we describe next.

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<sup>12</sup>The age-earnings profile in figure 2 is computed from cross-sectional data, but a very similar concave life-cycle pattern emerges in studies that use panel data drawn from the PSID (Heathcote et al. 2010). Further, this concave pattern is also observed for the earnings or wages of individual workers, though the household earnings profile lies, of course, strictly above the individual earnings profile (Heathcote et al. 2010, Huggett et al. 2011).

### 3. Model

In this section, we develop a general version of our model and discuss two theoretical results. The model structure is similar to the incomplete-market model with human capital developed in Krebs (2003), but adds life-cycle considerations and limited contract enforcement. Our first theoretical result is a convenient characterization of equilibria (proposition 1 and proposition 2) that highlights the tractability of the model. Wright (2001) provides a characterization result similar to propositions 1 and 2 for a class of linear production models (AK models) with limited enforcement.<sup>13</sup> Proposition 3 is an analytical result and shows that, for a special case of the general model, age and insurance are positively correlated in equilibrium. Proofs are relegated to the Appendix.

#### 3.1. Goods Production

Time is discrete and open ended. There is no aggregate risk and we confine attention to stationary (balanced growth) equilibria. We assume that there is one good that can be consumed or invested in physical capital. Production of this one good is undertaken by one representative firm (equivalently, a large number of identical firms) that rents physical capital and human capital in competitive markets and uses these input factors to produce output,  $Y$ , according to the aggregate production function  $Y = F(K, H)$ , where  $K$  and  $H$  denote the aggregate levels of physical capital and human capital, respectively. The production function,  $F$ , has constant-returns-to-scale, satisfies a Inada condition, and is continuous, concave, and strictly increasing in each argument. This constant-returns-to-scale assumption in conjunction with the assumption that human capital is produced under constant-returns-to-scale (see below) implies that the model exhibits endogenous growth.

Given these assumptions on  $F$ , the derived intensive-form production function,  $f(\tilde{K}) = F(\tilde{K}, 1)$ , is continuous, strictly increasing, strictly concave, and satisfies a corresponding Inada condition, where we introduced the “capital-to-labor ratio”  $\tilde{K} = K/H$ . Given the assumption of perfectly competitive labor and capital markets, profit maximization implies

$$\begin{aligned} r_k &= f'(\tilde{K}) \\ r_h &= f(\tilde{K}) + f'(\tilde{K})\tilde{K}, \end{aligned} \tag{1}$$

where  $r_k$  is the rental rate of physical capital and  $r_h$  is the rental rate of human capital. Note

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<sup>13</sup>Angeletos (2007) and Moll (2012) develop tractable models of entrepreneurial activity in which individual consumption/saving policies are also linear in wealth. In all these approaches, tractability is achieved through the assumption that individual investment returns are independent of household wealth.



that  $r_h$  is simply the wage rate per unit of human capital and that we dropped the time index because of our stationarity assumption. Clearly, (1) defines rental rates as functions of the capital to labor ratio:  $r_k = r_k(\tilde{K})$  and  $r_h = r_h(\tilde{K})$ . Finally, physical capital depreciates at a constant rate,  $\delta_k$ , so that the (risk-free) return to physical capital investment is  $r_k - \delta_k$ .

### 3.2. Households

There are a continuum of households of mass one. Households are indexed by their age  $j$ , the exogenous state (shock)  $s_j$ , their human capital,  $h_j$ , and their asset holdings,  $a_j$ . In our quantitative analysis, the exogenous state has two components,  $s_j = (s_{1j}, s_{2j})$ , where  $s_{1j}$  refers to the family state of the household and  $s_{2j}$  describes idiosyncratic labor market risk. The family state  $s_{1j}$  is defined by the marital status (married, widowed, single-not-widowed), the number of kids, and the gender in the case of a single household, for a total of 17 different states. Note that mortality risk corresponds to the transition from married household to widowed household. The process  $\{s_j\}$  is Markov with stationary transition probabilities  $\pi_j(s_{j+1}|s_j)$ . We denote by  $s^j = (s_1, \dots, s_j)$  the history of exogenous states up to age  $j$  and let  $\pi_j(s^j|s_0) = \pi_j(s_j|s_{j-1}) \dots \pi_0(s_1|s_0)$  stand for the probability that  $s^j$  occurs given  $s_0$ . At age  $j = 0$ , a household begins life in the initial state  $(a_0, h_0, s_0)$ .

The life of a household is divided into three phases. The first phase runs from age  $j = 0, \dots, J$  and is the focus of our analysis. In the quantitative application, we identify  $j = 0$  with age 23 and  $j = J$  with age 60. Thus, with 17 different family states and 38 different age-groups, we have  $17 * 38 = 646$  different household types in the first phase of life. In this phase, households are working and married households face the (age-dependent) risk that an adult member of the household dies (mortality risk). For simplicity, we assume that the event that both adults die simultaneously has zero probability and, given our focus on married households with children, that single households do not face mortality risk. Married households also face the (age-dependent) risk of divorce. Single households meet with age-dependent probability to form a married household. Some households have children and the number of children in a household can increase or decrease by one. All transition probabilities over family states are exogenous. See Section 4 and the Appendix for more details about the specification of these transition probabilities.

The second phase of life,  $j = J + 1$ , is the pre-retirement stage. This phase is similar to the first phase, but now households do not age. However, they retire stochastically with fixed probability. The third and final phase of life is retirement. In the retirement phase, households receive no labor income and can only invest in a risk-free asset. Retired

households die with constant probability and are then replaced by a household age  $j = 0$  (age 23 in Section 4). Given that the focus of our analysis is on married households with kids in the first phase of life (i.e. with a household head not older than 60), we relegate to the Appendix a more detailed discussion of the decision problem in the second and third phase of life.

Households are risk-averse and have identical preferences that allow for a time-additive expected utility representation with logarithmic one-period utility function and pure discount factor  $\beta$ . For a household choosing the consumption plan  $\{c_j\}$ , expected life-time utility is given by

$$\begin{aligned} & \sum_{j=0}^J \beta^j \sum_{s^j} u(c_j, s_j) \pi_j(s^j | s_0) \\ & + \beta^{J+1} \sum_{s^{J+1}} V_{J+1}(h_{J+1}(s^J), a_{J+1}(s^{J+1}), s_{J+1}) \pi_{J+1}(s^{J+1} | s_0) \end{aligned} \quad (2)$$

where  $u(c_j, s_j) = \gamma_0(s_j) + \gamma_1(s_j) \ln c_j$  is the one-period utility function of the household,  $V_{J+1}$  is the value function in the pre-retirement stage and  $\gamma_0$  and  $\gamma_1$  are preference shocks that depend on the family state. For our baseline quantitative model we use  $u(c, s) = \ln(c)$  so that  $\gamma_1(s_j) = 1$  (state-independent marginal utility of consumption). We then relax this assumption in Section 6. Note that we have abstracted from the labor-leisure choice of households.

Households can invest in physical capital as well as human capital and they can buy and sell a complete set of financial assets (contracts) with state-contingent payoffs. More specifically, there is one asset (Arrow security) for each exogenous state  $s$ . We denote by  $a_{j+1}(s_{j+1})$  the quantity bought (sold) at age (in period)  $j$  of the asset that pays off one unit of the good if  $s_{j+1}$  occurs at age  $j+1$  (in the next period). Given an initial state,  $(h_0, a_0, s_0)$ , a household chooses a plan,  $\{c_j, h_{j+1}, \vec{a}_{j+1}\}$ , where the notation  $\vec{a}$  indicates that in each period the household chooses a vector of asset holdings. Further,  $c_j$  stands for the function mapping partial histories,  $s^j$ , into consumption levels,  $c_j(s^j)$ , with similar notation used for the other choice variables. A budget-feasible plan has to satisfy the sequential budget constraint, human capital evolution equation, and non-negativity constraints on total wealth (financial plus human), consumption and human capital

$$\begin{aligned} i) \quad & z(s_j) r_h h_j + a_j(s_j) = c_j + x_{h_j} + \sum_{s_{j+1}} a_{j+1}(s_{j+1}) q_j(s_{j+1}) \\ ii) \quad & h_{j+1} = (1 - \delta_h + \eta_j(s_j)) h_j + \varphi_j(s_j) h_j + \phi x_{h_j} \end{aligned} \quad (3)$$

$$\begin{aligned}
iii) \quad & h_j + \sum_{s_{j+1}} a_{j+1}(s_{j+1}) q_j(s_{j+1}) \geq 0 \\
iv) \quad & c_j \geq 0 \quad , \quad h_{j+1} \geq 0 \quad ,
\end{aligned}$$

where  $q_j(s_{j+1})$  is the price of a financial contract in period  $j$  that pays off if  $s_{j+1}$  occurs in  $j+1$ , which in our Markovian setting only depends on asset type  $s_{j+1}$  and current state  $s_j$ . Note that the equations in (3) have to hold in realization; that is, they hold for all  $j$  and all sequences  $s^j$ . Note also that (3iii) represents a debt constraint, and that (3iv) requires the stock of human capital,  $h_{j+1}$ , to be non-negative, which prevents elderly workers from shorting their human capital.

The variable  $x_{hj}$  captures the resource cost of (active) human capital investment measured in consumption units and  $\phi$  is a parameter describing the productivity of this investment. The term  $z_j$  in equation (3i) is a labor productivity shock that captures transitory movements in earnings and we normalize its mean to one:  $E[z_j] = 1$ . In the human capital evolution equation, the term  $\eta_j$  measures the loss/gain of household human capital when there is a transition from married household to single households and vice versa (death of spouse, divorce, marriage).<sup>14</sup> The term  $\varphi_j h_j$  represents increases in human capital that do not require an active input of resources, including returns to experience and learning-by-doing one's job, which are often referred to as experience capital. Note that this term has a random component so that  $\varphi_j$  also captures any labor market risk that is not part of the productivity shock  $z_j$ . The randomness in  $\varphi_j$  might describe variations in the return to experience that often occur when workers switch their employer and/or occupation. For our quantitative analysis, we assume  $\varphi_j(s_j) = \bar{\varphi}_j + \hat{\varphi}(s_j)$ , where the first component describes age-dependent learning-by-doing effects and the second component captures labor market risk. Finally,  $\delta_h$  is the depreciation rate of human capital.

Note that the budget constraint (3) assumes that physical capital and human capital are produced using similar technologies in the sense that one unit of physical capital can be transformed into  $\phi$  units of human capital. Thus, we assume constant returns to scale at the household level. This assumption, also made in Krebs (2003), implies that the household decision problem displays a certain linearity with respect to physical capital investment and human capital investment in the sense that goods invested in either human capital or physical capital generate returns that are independent of household size, where size is measured by

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<sup>14</sup>For notational ease, we expand the family state,  $s_{1j}$ , to include last period's marital status (married, single widowed, single not widowed) so that the  $\eta$ -shock only depends on the current  $s_{1j}$  and not on  $s_{1,j-1}$ .

total wealth (see below).

The assumptions we make in (3) have the advantage that they keep the model highly tractable, which, as we argued before, is essential for the theoretical and quantitative analysis conducted in this paper. Tractability in the general case requires that we do not impose a restriction on the ability of households to decumulate human capital. However, in the calibrated model economy used for our quantitative analysis, the restriction  $h_{j+1} \geq (1-\delta_{hj})h_j$  is always satisfied in equilibrium; that is, it holds for all household types at all ages and all realizations of uncertainty. Similarly, the restriction that human capital investment is non-negative,  $\varphi_j h_j + \phi x_{hj} \geq 0$ , is always satisfied in equilibrium. Thus, imposing these restrictions in (3) would not change the conclusions drawn in the quantitative analysis.<sup>15</sup>

In addition to the standard budget constraint, household's decisions have to satisfy a sequential enforcement (participation) constraint, which ensures that at no point in time individual households have an incentive to default on their financial obligations. More precisely, individual consumption plans have to satisfy

$$\begin{aligned} & \sum_{n=0}^{J-j} \beta^n \sum_{s^{j+n}|s^j} u(c_{j+n}, s_{j+n}) \pi_j(s^{j+n}|s^j) \\ & + \beta^{J+1-j} \sum_{s^{J+1}|s^j} V_{J+1}(h_{J+1}(s^J), a_{J+1}(s^{J+1}), s_{J+1}) \pi_{J+1}(s^{J+1}|s^j) \\ & \geq V_d(h_j(s^{j-1}), a_j(s^j), s_j) \end{aligned} \tag{4}$$

for all  $j$  and  $s^j$ , where  $V_d$  is the value function of a household who defaults. Note that the constraint set defined by (4) may not be convex since both the left-hand side and the right-hand side are concave functions of  $h$ . This is the non-convexity issue alluded to in the introduction; in proposition 1 we show how this problem is surmounted in the current setting.

The default value function  $V_d$ , is defined by the household decision problem in default. In this paper, we allow for different specifications of this default problem. In the baseline version of the model, we model default along the lines of Chapter 7 of the US bankruptcy code. More

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<sup>15</sup>Note that in (3) we have explicitly imposed a non-negativity constraint on the stock of human capital, and our general characterization of the household decision rule (proposition 1) holds with this constraint imposed. Of course, for a certain range of parameter values this constraint binds in equilibrium, but for the parameter values used in our quantitative analysis this constraints never binds (does not bind for all households types and uncertainty states).

precisely, we assume that upon default all debts of the household are canceled and all financial assets seized so that  $a_j(s_j) = 0$ . Following default, households are excluded from purchasing insurance contracts and borrowing (going short), but they can still save in a risk-free asset. We assume that exclusion continues until a stochastically determined future date that occurs with probability  $(1 - p)$  in each period; that is, the probability of remaining in (financial) autarky is  $p$ . Following a default, households retain their human capital and continue to earn the wage rate  $r_h$  per unit of human capital. After regaining access to financial markets, the households expected continuation value is  $V^e$ , which depends on  $h$  and  $s$  at the time of exiting default ( $a = 0$  at that point in time). For the individual household the function  $V^e$  is taken as given, but we will close the model and determine this function endogenously by requiring that  $V^e = V$ , where  $V$  is the equilibrium value function associated with the maximization problem of a household who participates in financial markets.<sup>16</sup> Details are found in the Appendix.

### 3.3 Equilibrium

We assume that insurance markets (financial markets) are perfectly competitive and abstract from transactions costs. Thus, insurance contracts (financial contracts) are priced in an actuarially fair manner (risk neutral pricing):

$$q_j(s_{j+1}; s_j) = \frac{\pi_j(s_{j+1}|s_j)}{1 + r_f} . \quad (5)$$

The pricing equation (5) can be interpreted as a zero-profit condition for financial intermediaries that can invest in physical capital at the risk-free rate of return  $r_f = r_k - \delta_k$  and can fully diversify idiosyncratic risk for each insurance contract  $s_{j+1}$ .

Below we show that the optimal plan for individual households is recursive; that is, the optimal plan is generated by a policy function,  $g$ . This household policy function in conjunction with the transition probabilities,  $\pi$ , define a transition function over states,  $(h, a, s)$ , in the canonical way. This transition function together with the initial distribution,  $\mu_0$ , and sequence of distributions for new-born households,  $\{\mu_{t,new}\}$ , induce a sequence of

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<sup>16</sup>The previous literature has usually assumed  $p = 1$  (permanent autarky). See, however, Krueger and Uhlig (2006) for a model with  $p = 0$  following a similar approach to ours. Note also that the credit (default) history of an individual household is not a state variable affecting the expected value function,  $V^e$ ; we assume that the credit (default) history of households is information that cannot be used for contracting purposes. This is in keeping with the U.S. bankruptcy code which limits the history of past behavior that can be retained in credit reports.

equilibrium distributions,  $\{\mu_t\}$ , of households over individual states. We assume that the financial capital of households who die is inherited by new-born households, which imposes a restriction on the mean of the marginal distribution  $\mu_{t,new}^m$  over  $a$ . Note that we allow the distribution  $\{\mu_{t,new}\}$  to have an explicit time-dependence since in our endogenous growth model the mean value of  $h$  and  $a$  will grow over time, and a stationary distribution over intensive-form or growth-adjusted variables can only be obtained if the mean value of the extensive-form variables also grows for new-born households. In our quantitative analysis, we directly specify the distribution of new-born households over growth-adjusted states.

Assuming a law of large numbers, aggregate variables can be found by taking expectation with respect to the induced equilibrium distribution. For example, the aggregate stock of human capital held by all households in period  $t$  is given by  $H_t = \int \sum_j h_j d\mu_{tj}(h_j)$ . A similar expression holds for the aggregate value of financial wealth. In equilibrium, human capital demanded by the firm must be equal to the corresponding aggregate stock of human capital supplied by households. Similarly, the physical capital demanded by the firm must equal the aggregate net financial wealth supplied by households. Because of the constant-returns-to-scale assumption, only the ratio of physical to human capital is pinned down by this market clearing condition. That is, in equilibrium we must have for all  $t$

$$\tilde{K} = \frac{\int \sum_j \sum_{s_{j+1}} q_j(s_{j+1}; s_j) a_{j+1}(s_{j+1}) d\mu_{jt}(h_j)}{\int \sum_j h_j d\mu_{tj}(h_j)}, \quad (6)$$

where  $\tilde{K}$  is the capital-to-labor ratio chosen by the firm that determines the equilibrium rental rates of physical capital and human capital,  $r_k$  and  $r_h$ , through (1).

To sum up, we have the following equilibrium definition:

**Definition** A stationary recursive equilibrium is a collection of rental rates  $(r_k, r_h)$ , an aggregate capital-to-labor ratio,  $\tilde{K}$ , a household value function,  $V$ , an expected household value function,  $V^e$ , a household policy function,  $g$ , and a sequence of distributions,  $\{\mu_t\}$ , of households over individual states,  $(h, a, s)$ , such that

- i) Utility maximization of households: for each initial state,  $(h_0, a_0, s_0)$ , and given prices, the household policy function,  $g$ , generates a household plan that maximizes expected lifetime utility (2) subject to the sequential budget constraint (3) and the sequential participation constraint (4).
- ii) Profit maximization of firms: the aggregate capital-to-labor ratio and rental rates satisfy the first-order conditions (1).

- iii) Profit maximization of financial intermediaries: financial contracts are priced according to (5)
- iv) Aggregate law of motion: the sequence of distributions,  $\{\mu_t\}$ , is generated by  $g$ ,  $\pi$ ,  $\mu_0$ , and  $\{\mu_{t,new}\}$ .
- v) Market clearing: equations (6) holds for all  $t$  when the expectation is taken with respect to the distribution  $\mu_t$ .
- vi) Expected household value function is identical to the household value function:  $V^e = V$ .

Note that the equilibrium value of  $\tilde{K}$  determines the equilibrium growth rate of the economy (see Appendix for details). Note also that in equilibrium the goods market clearing condition (aggregate resource constraint) automatically holds:

$$C_t + K_{t+1} + \frac{1}{\phi} H_{t+1} = (1 - \delta_k) K_t + \frac{1}{\phi} H_t + \frac{1}{\phi} \int \sum_j (\eta_j(s_j) + \varphi_j(s_j) - \delta_h) h_j d\mu_{tj}(h_j) + F(K_t, H_t) \quad (7)$$

### 3.4. Characterization of Household Problem

We next show that optimal consumption choices are linear in total wealth (human plus financial) and portfolio choices are independent of wealth. This property of the optimal policy function allows us to solve the quantitative model, which has a large number of household types and uncertainty states, without using approximation methods. The property also implies that the household decision problem is convex and the first-order approach can be utilized.

To state the characterization result, denote total wealth (human plus financial) of a household of age  $j$  at the beginning of the year by  $w_j = h_j/\phi + \sum_{s_j} a_j(s_j) q_{j-1}(s_j)$ . Note that  $\phi$  measures the productivity of goods investment in human capital and  $1/\phi$  is the shadow price of one unit of human capital in terms of the consumption/capital good. Denote the portfolio shares by  $\theta_{hj} = h_j/(\phi w_j)$  and  $\theta_{aj}(s_j) = a_j(s_j)/w_j$ . The sequential budget constraint (3) then reads:

$$\begin{aligned} w_{j+1} &= (1 + r_j(\theta_j, s_j)) w_j - c_j \\ 1 &= \theta_{h,j+1} + \sum_{s_{j+1}} q_j(s_{j+1}|s_j) \theta_{a,j+1}(s_{j+1}) \\ c_j &\geq 0, \quad w_{j+1} \geq 0, \quad \theta_{j+1} \geq 0 \end{aligned} \quad (8)$$

with

$$1 + r_j(\theta_j, s_j) \doteq [1 + \phi z(s_j) r_h - \delta_h + \eta_j(s_j) + \varphi_j(s_j)] \theta_{hj} + \theta_{aj}(s_j)$$

Clearly, this is the budget constraint corresponding to an inter-temporal portfolio choice problem with linear investment opportunities and no exogenous source of income. It also shows that  $(w, \theta, s)$  can be used as individual state variable for the recursive formulation of the utility maximization problem. Using this notation, we have the following result:

**Proposition 1.** The value function and the optimal policy function are given by

$$\begin{aligned} V_j(w_j, \theta_j, s_j) &= \tilde{V}_{0j}(s_j) + \tilde{V}_{1j}(s_j) [\ln w_j + \ln(1 + r_j(\theta_j, s_j))] \\ c_j(w_j, \theta_j, s_j) &= \tilde{c}_j(s_j) (1 + r_j(\theta_j, s_j)) w_j \\ \theta_{j+1}(w_j, \theta_j, s_j) &= \theta_{j+1} \\ w_{j+1}(w_j, \theta_j, s_j) &= (1 - \tilde{c}_j(s_j)) (1 + r_j(\theta_j, s_j)) w_j \end{aligned} \tag{9}$$

where the value function coefficients,  $\tilde{V}_{0j}(s_j)$ ,  $\tilde{V}_{d,0j}(s_j)$ , and  $\tilde{V}_{1j}(s_j)$  as well as the optimal consumption-to-wealth ratio,  $\tilde{c}$ , and the optimal portfolio choice,  $\theta_{j+1}^*$  are the solution to a maximization problem with linear constraints – see equation (A7) in the Appendix.<sup>17</sup>

*Proof:* See Appendix.

Proposition 1 provides a convenient characterization of the solution to the household decision problem for given investment returns (partial equilibrium). Importantly, it establishes that the policy functions are linear (and hence continuous) in wealth, so that concerns about discontinuities resulting from the non-convexity in the participation constraint (4) have been resolved. We next turn to the determination of investment returns (general equilibrium).

### 3.5. Equilibrium Characterization

Define the share of aggregate total wealth of households of age  $j$  and state  $s_j$  as

$$\Omega_j(s_j) \doteq \frac{E[(1 + r_j)w_j | s_j] \pi_j(s_j)}{\sum_j \sum_{s_j} E[(1 + r_j)w_j | s_j] \pi_j(s_j)}$$

Note that  $(1 + r_j)w_j$  is total wealth of an individual household after assets have paid off (after production and depreciation has been taken into account). Note also that  $\sum_j \sum_{s_{1j}} \Omega(s_{1j}) = 1$ . Further,  $\Omega$  is finite-dimensional, whereas the set of distributions over  $(w, s)$  is infinite-dimensional. Using the definition of wealth shares and the property that portfolio choices are wealth-independent, in the Appendix we show the following result:

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<sup>17</sup>The Appendix also contains the corresponding expressions for the default value function and default consumption policy.



**Proposition 2.** Suppose that  $(\theta, \tilde{c}, \tilde{V}, \tilde{K}, \Omega)$  solves the fixed-point problem defined by the equations (A4), (A10), and (A11) in the Appendix. Then  $(g, \tilde{V}, \tilde{K}, \{\mu_t\})$  is a stationary (balanced growth) equilibrium, where  $g$  is the individual policy function induced by  $(\tilde{c}, \theta)$  and  $\{\mu_t\}$  is the sequence of measures induced by the policy function  $g$ , the initial measure,  $\mu_0$ , and the transition matrix over demographic and labor market states,  $\pi$ .

*Proof.* See the Appendix.

Proposition 2 shows that the stationary equilibrium can be found without knowledge of the infinite-dimensional wealth distribution; only the finite dimensional distribution of wealth across family types  $\Omega$  matters. This is because the linearity of the policy functions in wealth make the infinite dimensional distribution of wealth across households of a given type irrelevant. Proposition 2 facilitates our quantitative analysis significantly since it implies that there is no need to approximate an infinite dimensional wealth distribution when computing equilibria.

### 3.6. Analytical Results

We now derive analytical results for a special case of the model. We use these results to discuss the main determinants of individual consumption, and to prove that in equilibrium there is a positive relationship between age and insurance.

We focus on the first phase of life,  $j = 1, \dots, J$ , and on households with two adult members (married households). We consider the case with only mortality risk so that  $s_j \in \{d, n\}$ , where  $s_j = d$  is the event that the death of an adult household member occurs and  $s_j = n$  is the event that death does not occur. Note that after the event “death of an adult household member” the household continues to exist. Mortality risk is an i.i.d. random variable,  $\eta$ , with age-independent probability  $\pi$  that death occurs and age-independent human capital loss  $\eta(d)$  in the death event. We normalize the mean of  $\eta$  to zero:  $\pi\eta(d) + (1 - \pi)\eta(n) = 0$ . Note that  $\eta(d)$  is the fraction of household human capital that is lost in the event that an adult member of the household dies. We also assume constant labor productivity  $z(s_j) = 1$  and no human capital risk beyond mortality risk:  $\varphi_j(s_j) = \bar{\varphi}_j$ .

We assume that young households have a higher rate at which they gain work experience on the job,  $\bar{\varphi}_j > \bar{\varphi}_{j+1}$ , which implies that expected human capital returns of young households are larger than the returns of older households. We choose state-independent preference parameters  $\gamma_0(s_j) = \gamma_1(s_j) = 1$ . Finally, we assume that defaulting households are not excluded from financial markets,  $p = 0$ , which rules out short positions in financial

assets (see Appendix, Proof of proposition 3).

Using the policy function (9) of our equilibrium characterization result, we find that in this example consumption growth is given by:

$$\begin{aligned}\frac{c_{j+1}}{c_j} &= \beta(1 + r_{j+1}(\theta_{j+1}, s_{j+1})) \\ &= \beta \{ (1 + \phi r_h - \delta_h + \varphi_{j+1} + \eta(s_{j+1})) \theta_{h,j+1} + \theta_{a,j+1}(s_{j+1}) \}\end{aligned}\tag{10}$$

Consumption growth depends on human capital choice,  $\theta_{h,j+1}$ , ex-ante human capital returns,  $\phi r_h - \delta_h + \varphi_{j+1}$ , ex-post shocks,  $\eta(s_{j+1})$ , and asset payoffs (insurance),  $\theta_{a,j+1}(s_{j+1})$ . From (10) we immediately conclude that consumption is independent of mortality shocks if  $\theta_{a,j+1}(d) - E[\theta_{a,j+1}] = \eta(d) \theta_{h,j+1}$ , where  $E[\theta_{a,j+1}] = \pi \theta_{a,j+1}(d) + (1 - \pi) \theta_{a,j+1}(n)$  is the fraction of total wealth the household is holding as financial wealth. This is intuitive since  $(\theta_{a,j+1}(d) - E[\theta_{a,j+1}]) w_{j+1}$  is the insurance pay-out in the case of death and  $\eta(d) \theta_{h,j+1} w_{j+1}$  is the human capital loss in the case of death, and when the two are equal we have full insurance and therefore deterministic consumption growth.

The above discussion demonstrates that in versions of our model of mortality risk without “preference shocks”, the full insurance outcome is achieved when the actual insurance payout,  $(\theta_{a,j+1}(d) - E[\theta_{a,j+1}]) w_{j+1}$ , is equal to the human capital loss in the case of death,  $\eta(d) \theta_{h,j+1} w_{j+1}$ . Thus, we can define the life insurance coefficient as the ratio:

$$I_{j+1} = \frac{\theta_{a,j+1}(d) - E[\theta_{a,j+1}]}{\eta(d) \theta_{h,j+1}}\tag{11}$$

Clearly, in any model without “preference shocks” the life insurance coefficient  $I$  varies between 0 (no insurance) and 1 (full insurance). For our example economy we have the following result:

**Proposition 3.** Suppose the economy is as described above. In equilibrium, young households are less insured than old households and a larger fraction of their total wealth is invested in human capital:

$$\begin{aligned}\theta_{h,j} &\geq \theta_{h,j+1} \\ I_j &\leq I_{j+1},\end{aligned}$$

where the inequalities are strict if in equilibrium there is some insurance, but not full insurance.

*Proof.* See the Appendix.

In the next section we establish that this qualitative result holds in a calibrated version of our general model, and assess the quantitative performance of this model.

## 4. Calibrating the Model

We now turn to the quantitative analysis. Section 4.1 lays out the model specification. Section 4.2 discusses our calibration strategy and the relevant empirical literature, while Section 4.3 discusses computation. All calibrated parameter values are collected in Table 1.

### 4.1 Model Specification

We set the length of a time period to one year and let  $j = 23, \dots, 60$  for the first phase of life. As in Huggett et al. (2011), we restrict attention to households up to age 60 for the following two reasons. First, the labor force participation of households falls as they approach the traditional retirement age. As these declines occur for reasons that are not modeled here, and as it results in a decline in the number of households with positive labor income in our SCF sample after age 60, we de-emphasize these ages. Second, the closer we get to the traditional retirement age, the more important non-negativity constraints on human capital investment become. By fitting the empirical life-cycle of earnings only up to age 60 and introducing a transition-group of households with stochastic retirement, we can ensure that for the calibrated model economy the rate of decumulation of human capital is bounded by the rate of depreciation over the entire life-cycle.

For our baseline model, we assume a one-period household utility function  $u(c, s) = \ln(c)$ . In other words, we assume a constant marginal utility of household consumption and normalize the constant to one:  $\gamma_1(s) = 1$ . Note that for this preference specification we have  $\tilde{c}_j = 1 - \beta$  in (9). Note also that with this specification utility is defined over household consumption without any adjustment for household size. In order to take into account the insurance effect of any reduction in living costs resulting from a smaller family size, below we scale the income losses associated with the death of the husband/wife according to a cost of living adjustment (see Section 4.2 below). We choose this simple preference specification to focus attention on our basic mechanism. Specifically, this preference specification ensures that the life cycle pattern of under-insurance generated by the model is solely due to the interaction of human capital risk with endogenous borrowing constraints. We discuss the possibility of life cycle variation in preferences in Section 6.3.

We assume that the exogenous state variable has two components. The first component describes the family state that is a pair  $(m_j, k_j)$ , where  $m_j$  is the marital state and  $k_j$  denotes the number of kids. We assume  $k_j \in \{0, 1, 2, 3\}$  and  $m_j \in \{ma, fw, fn, mw, mn\}$  corresponding to married (ma), female widowed (fw), female not widowed (fn), male widowed (mw), and male not widowed (mn). Thus, we have in total 17 family states (we do not have non-widowed single male households with children since we assume that after divorce children live with their mother). The transition matrix over family states is discussed in more detail in the Appendix. Transitions across marital states,  $m_j$ , lead to changes in the stock of human capital denoted by  $\eta_j$ . In particular,  $\eta_j$  captures the human capital losses in the case of the death of a spouse (transition from  $s_{1j} = ma$  to  $s_{1,j+1} = fw$  or  $s_{1,j+1} = mw$ ). We parameterize these human capital losses as follows.

We assume that for a fraction  $\pi_j$  of households we have  $\eta_j = \alpha \tilde{\eta}_j$  and for a fraction  $(1 - \pi_j)$  of households we have  $\eta_j = 0$ , where  $\alpha$  is distributed according to a generalized Pareto distribution with cumulative distribution function  $F(\alpha) = 1 - \left(1 + \frac{\psi(\alpha - \mu)}{\sigma_\alpha}\right)^{-\frac{1}{\psi}}$ . Households with  $\eta_j > 0$  buy life insurance and households with  $\eta_j = 0$  do not. Thus,  $\pi_j$  is the participation rate in the life insurance market. Below we calibrate  $\tilde{\eta}_j$  to the match the present value income losses of married households with median earnings as discussed in Section 2.3, and we use  $\alpha$  to capture the heterogeneity of life insurance holdings conditional on age  $j$  for those households who have purchased life insurance. We assume that households with  $\eta_j > 0$  keep their value until retirement and that households with  $\eta_j = 0$  draw a new value in the next year  $j + 1$ . This new value is drawn from the distribution of  $\alpha \tilde{\eta}_{j+1}$  with probability  $q_{j+1}$  and is equal to zero (no participation) with probability  $(1 - q_{j+1})$ . Thus, the participation rate evolves according to  $\pi_{j+1} = \pi_j + q_j(1 - \pi_j)$ . We assume that probability  $q_{j+1}$  has a linear trend,  $q_{j+1} = (1 - d)q_j$ , to capture the fact that more households purchase some life insurance as they age.

The second component of the exogenous state describes labor market risk specified by the two variables  $z_j$  and  $\varphi_j = \bar{\varphi}_j + \hat{\varphi}_j$ . We assume that  $\bar{\varphi}_j$  is deterministic and that productivity shocks,  $z_j$ , and human capital shocks,  $\hat{\varphi}_j$ , are i.i.d. with a finite, symmetric distribution that approximates a normal distribution. The assumption that human capital shocks are independently and (approximately) normally distributed is also made by Huggett et al. (2011) and Krebs (2003). We assume that  $z_j$  has mean 1 and  $\hat{\varphi}_j$  has mean 0 and denote variances of these two random variables by  $\sigma_z^2$  and  $\sigma_\varphi^2$ , respectively.

Households in pre-retirement age (age  $J = 61$ ) work and the duration of this phase of

life ends stochastically with retirement. Households age  $J = 61$  solve a recursive version of the household decision problem described in Section 3 (see also the Appendix). Upon retirement, the human capital of households becomes unproductive.<sup>18</sup> Retired households can save in a risk-free asset. Households age  $j = 23, \dots, 61$  do not die and retired households die stochastically, in which case they are replaced by a new-born household of age 23. The financial capital of deceased households is passed on to new-born households. New-born households also receive an initial endowment of human capital. The distribution of new-born households over human capital, physical capital, and family states is discussed in Section 4.2 below.

We assume a Cobb-Douglas aggregate production function,  $f(\tilde{k}) = A\tilde{K}^\alpha$ . The computation of equilibria exploits the characterization results in proposition 1 and proposition 2. See the Appendix for more details on our computational approach.

## 4.2 Model Calibration

### 4.2.1 Mortality Risk

Mortality risk is captured in the model by the transition from the marital state  $m_j = ma$  to the marital state  $m_{j+1} = mw$  (female spouse dies, producing a widower) or  $m_{j+1} = fw$  (male spouse dies). We choose the probability that a male or female spouse dies to match the year-to-year average survival rates for the period 1991-2000 for the US life-tables for the respective group (see figure A5 in the Appendix). We use the re-marriage rates of divorcees from the SIPP as a proxy for the re-marriage rates of widows/widowers, but introduce an adjustment to take into account of the evidence that indicates lower re-marriage rates for widows and widowers. Specifically, we compute the life-cycle profile of re-marriage rates of female/male divorcees age 30 to 50 from the SIPP and then scale down this life-cycle profile so that the average marriage rate corresponds to the average re-marriage rate of widows/widowers age 30 to 50 in the SIPP data. The result is depicted in figure A1 in the Appendix, and is in line with the evidence of re-marriage rates of widows and widowers presented in Norton and Miller (1990) and Wilson and Clarke (1992).

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The size of the negative human capital shock in the case of the death of an adult household

<sup>18</sup>Formally, we assume that the labor productivity of retired households drops to zero. To avoid that households sell their human capital upon retirement, we also assume 100 percent depreciation of existing human capital in the first period of retirement. Of course, in equilibrium households are almost fully insured against this retirement-shock. Alternatively, we can assume that upon retirement households receive a lump-sum payment from the government.

member,  $\eta_j$ , is calibrated as follows. Recall that we assume  $\eta_j = \alpha \tilde{\eta}_j$  with probability  $\pi_j$ , where  $\pi_j$  is the fraction of households who participate in the life insurance market and  $\alpha$  is distributed according to a generalized Pareto distribution. For married households of age  $j$  with number of kids  $k_j$ , we calibrate  $\tilde{\eta}_j$  to match the present value income losses as discussed in Section 2.3, but here we compute the losses separately for the case of death of the husband and death of the wife. The losses we compute are the losses for a married household with median earnings and they take into account the implicit insurance that arises from social security survivor benefits, progressive income taxation, and re-marriage. In addition, we make an adjustment to the human capital losses to take into account the insurance effect of any reduction in living costs resulting from a smaller family size, which we have ignored so far. We account for this effect by scaling the income losses according to the cost of living adjustment suggested by the consumption equivalence scale of Ruggles (1990). The resulting life-cycle profiles of human capital losses in case of death of the husband, respectively wife, are shown in figures A7 and A8 in the Appendix.<sup>19</sup>

We calibrate the remaining parameters  $\psi$ ,  $\mu$ ,  $\sigma_\alpha$ ,  $\pi_{23}$ ,  $q_{23}$  and  $d$  determining mortality risk as follows. We choose the values of the generalized Pareto distribution,  $\psi$ ,  $\mu$ , and  $\sigma_\alpha$ , to match three moments of the empirical distribution (not conditional on age) of life insurance holdings for married households with children: the median all households including those who did not purchase any life insurance, the median of households who purchased some life insurance ( $\alpha > 0$ ), which we normalize to one, and the mean of all households who have purchased some life insurance ( $\alpha > 0$ ). The resulting parameter values are  $\psi = 0.4807$ ,  $\mu = 0.5239$ , and  $\sigma_\alpha = 0.579$ . We set the value of  $\pi_{23}$  equal to the empirical participation rate of 23-year old married households with children and choose  $q_{23}$  and  $d$  so as to match the life cycle profile of the observed participation rates for this group (we minimize the squared distance between model and data).

#### 4.2.2 Divorce Risk

Divorce risk is captured in the model by the transition from the marital state  $m_j = ma$  to the marital state  $m_{j+1} = fn$  and  $m_{j+1} = mn$ . We choose the age-dependent probability of

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<sup>19</sup>Note that our approach to estimating the human capital loss,  $\eta$ , takes into account the implicit insurance provided by the possibility of re-marriage since re-marriage probabilities enter into our calculation of the expected income losses depicted in figures A7 and A8. Consequently, we do not incorporate into the model a positive human capital shock upon re-marriage to avoid double-counting. This approach also ensures that monetary gains from re-marriage are treated in the same way as monetary gains from the social security survivor benefits – both enter into the model only through the size of the human capital shock  $\eta$ .

divorce so that we match the corresponding separation rates in the SIPP data (see figure A2 in the Appendix). After divorce, the new single-female household receives  $x$  percent of the total household human capital and the new single-male household  $1 - x$  percent of the total household human capital. The number  $x$  is the ratio of median earnings of a single-female household over median earnings of a married household. We assume that after divorce the financial wealth is split equally between the man and the woman.

### 4.2.3 Labor Market Risk

We calibrate the two parameters  $\sigma_z^2$  and  $\sigma_\varphi^2$  as follows. Conditional on the family structure, our human capital model implies a labor income process that is consistent with the specification used by the empirical literature on labor market risk. Indeed, there is a one-to-one mapping between the two parameters  $\sigma_z^2$  and  $\sigma_\varphi^2$  and the labor income risk parameters estimated by the empirical literature. Specifically, a large literature (Carroll and Samwick 1997, Meghir and Pistaferri 2004, Storesletten et al. 2004b) has estimated transitory and permanent labor income risk as follows. Observed labor income,  $y_j$ , is decomposed into a transitory component,  $y_j^T$ , and a permanent/persistent component,  $y_j^P$ , with  $\ln y_j = \ln y_j^T + \ln y_j^P$ , where the transitory component is an i.i.d. process with  $\ln y_j^T \sim N(0, \sigma_T^2)$  and the permanent component is a logarithmic random walk with innovation term  $\epsilon \sim N(0, \sigma_\epsilon^2)$ . The two variances  $\sigma_T^2$  and  $\sigma_\epsilon^2$  can then be estimated separately using various moment conditions (Meghir and Pistaferri, 2004). We next show that our model specification leads to a labor income process with an i.i.d. component and a random walk component, and that estimates of  $\sigma_T^2$  and  $\sigma_\epsilon^2$  therefore provide us with estimates for  $\sigma_z^2$  and  $\sigma_\varphi^2$ .

Labor income in the model is given by  $y_j = z_j r_h h_j$  so that  $\ln y_j = \ln z_j + \ln r_h + \ln h_j$ . In equilibrium  $\ln h_j$  follows a random walk (see below) so that it can be identified with the permanent component,  $\ln y_j^P$ . Since  $z_j$  is i.i.d. it can be identified with the transitory component  $y_j^T$  ( $\ln r_h$  is a constant) and we therefore have  $\sigma_z^2 = \sigma_T^2$ . Given a value for  $\sigma_z^2$ , a value for  $\sigma_\varphi^2$  can be found as follows. Consider the evolution of the human capital stock  $h_j$

$$\begin{aligned} \ln h_{j+1} - \ln h_j &= \ln \theta_{h,j+1} - \ln \theta_{h,j} + \ln \beta + \ln(1 + r_j(\theta_h, s_j)) \\ &\approx \ln \theta_{h,j+1} - \ln \theta_{h,j} + \ln \beta + z(s_{2j})\phi r_h - \delta_h + \eta_j(s_{1j}) + \bar{\varphi}_j + \hat{\varphi}(s_{2j}) \end{aligned} \quad (12)$$

where we used the equilibrium policy for human capital and the approximation  $\ln(1+x) \approx x$ . Equation (12) shows that conditional on family structure, human capital in the model follow a random walk with age-dependent drift and innovation term that is approximately normally distributed with variance  $\sigma_\epsilon^2 = \phi^2 r_h^2 \sigma_z^2 + \sigma_\varphi^2$ . Given values for  $\phi$ ,  $r_h$ ,  $\sigma_\epsilon^2$ , and  $\sigma_z^2$  we find a

value for  $\sigma_\varphi^2$  using  $\sigma_\varphi^2 = \sigma_\epsilon^2 - \phi^2 r_h^2 \sigma_z^2$ .

The discussion shows how estimates of the transitory and permanent component of labor market risk,  $\sigma_T^2$  and  $\sigma_\epsilon^2$ , provide us with estimates of  $\sigma_z^2$  and  $\sigma_\varphi^2$  (for given values of  $\phi$  and  $r_h$ ). Estimates of  $\sigma_T^2$  and  $\sigma_\epsilon^2$  vary considerably, with a midpoint of around 0.0913 and 0.0225, respectively. For  $\sigma_T^2$  we choose the midpoint of 0.0913. For the permanent component we follow Huggett et al (2011) and choose a somewhat lower value, namely 0.0123. This choice is motivated by the fact that estimates of permanent labor income risk will overstate the true value of the variance if there is earnings profile heterogeneity in addition to stochastic shocks with a permanent component (Baker and Solon 2003 and Guvenen 2007).

#### 4.2.4 Investment Returns

We calibrate an annual risk-free rate of  $r_f = 3\%$ , in line with Kaplan and Violante (2010) and roughly in line with Huggett et al. (2011) and Krueger and Perri (2006) who use a 4% annual risk-free rate, but also deduct capital income taxes.

The depreciation rate of human capital,  $\delta_h$ , is chosen to match a target value for the ratio of human capital investment to GDP (see below). For given value of  $\delta_h$ , we choose the age-dependent learning-by-doing parameters  $\bar{\varphi}_j$  to match the life-cycle profile of earnings of the median household in our sample. Specifically, we assume that the age-dependence is described by an exponential function,  $\bar{\varphi}_j = A + Be^{-C \times j}$ , and choose the coefficients  $A$ ,  $B$ , and  $C$  in order to minimize the distance (L2-norm) between the empirical life-cycle of median earnings from age 23 to age 60 and the corresponding model prediction. The implied life-cycle profile of  $\bar{\varphi}_j$  is shown in figure A9 in the Appendix.

#### 4.2.5 Bankruptcy Code

We calibrate the costs of default to match the main features of Chapter 7 of the U.S. bankruptcy code before the reform in 2005 (i.e. before the implementation of the Bankruptcy Abuse Prevention and Consumer Protection Act). Specifically, we assume that households forfeit all financial assets, experience no garnishment of labor income, and are unable to borrow or buy insurance products for an average length of 7 years, so that the probability of re-establishing full financial market access is  $(1 - p) = 1/7$ .<sup>20</sup> Households in default may

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<sup>20</sup>Our parameterization is bracketed by Krueger and Perri (2006), who assume  $(1 - p) = 0$ , Chatterjee et al. (2007), who use  $(1 - p) = 1/10$ , and Livshits et al. (2007), who use  $(1 - p) = 1$  following the first period of default. The degree of variation in the parameter  $p$  reflects the fact that, as in our model, these papers abstract from a number of the costs of consumer default, and hence the calibration of the parameter



save in the risk-free asset, and may continue to rent their human capital to firms.

#### 4.2.6 Preferences, Endowment, and Production

We follow Huggett et al. (2011) and assume a capital share in output,  $\alpha$ , of .32, and target an aggregate capital to output ratio of 2.94. We also target an aggregate ratio of physical capital to human capital,  $\tilde{K}$ , of 0.4. This value lies somewhere in the middle of the range of estimates by the empirical literature, which suggests that the aggregate stock of human capital is 2 – 4 times larger than the aggregate stock of physical capital (Jorgenson and Fraumeni, 1993). Together with the interest rate target of 3 percent, these requirements pin down the parameter values  $\delta_k = 0.0785$  and  $A = 0.1818$ .

The retirement probability of households is chosen so that retirement occurs on average at age 65 and the death probability of retired households is chosen so that the expected age of death is 85. We choose an annual discount factor  $\beta = 0.95$  and the human capital productivity parameter  $\phi$  to match the average of the ratio of financial wealth to labor income, which yields  $\phi = 0.675$ . We choose the frequency distribution of newborn households (households age 23) over family types  $s_1$  in the model equal to the empirical distribution.

Newborn households inherit the financial capital of deceased retired households and receive an initial endowment of human capital. We assume that all newborn households of a particular family type  $s_1$  have the same endowment of financial and human capital, and assign initial endowments of human capital to different family types so that we match the empirical distribution of earnings across family type at age 23.

The calibration procedure described so far leaves free two parameter values: The human capital depreciation rate and the total human capital endowment of 23-year old households. We choose the values of these two parameters to ensure that the target values of 0.4 for the ratio  $\tilde{K}$  (see above) and 6 percent for the aggregate ratio of human capital investment to output,  $X_h/Y$ , are equilibrium outcomes. The value of 6 percent for  $X_h/Y$  is based on the work by Mincer about the cost of investment in on-the-job training. Specifically, using information on time spent in training and wages, Mincer (1989) estimates that the total volume of job training investment by US companies amounted to 296 billion in 1987, which results in a value of 6 percent for  $X_h/Y$ . Mincer (1989) also argues that this number is

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$p$  in part is a proxy for other default costs. In light of this, some authors have argued that the parameters governing the cost of default should be calibrated to match some aspect of the data, as in the choice of the level of wage garnishment in Livshits et al. (2007).

broadly consistent with a number of alternative empirical studies about on-the-job training.

The human capital depreciation rate,  $\delta_h$ , implied by our calibration is 4.29 percent annually. This value lies within the range of values used in the literature. Specifically, Mincer (1989) finds an annual depreciation rates for human capital accumulated on the job of about 4 percent. The literature on education usually finds somewhat lower values – see Browning, Hansen, and Heckman (1999) for a survey. However, macro work on human capital accumulation has often used calibrated models with annual human capital depreciation rates of 6 percent (Manuelli and Jones, 1990, and Krebs, 2003).

### 4.3 Computation of Equilibrium

The computation of equilibria is based on propositions 1 and 2. More specifically, we start with an aggregate capital-to-labor ratio,  $\tilde{K}$ , which defines the rental rates  $r_k$  and  $r_h$ , and solve the intensive-form household problem (proposition 1). Given the solution to the household problem, we compute a stationary relative wealth distribution,  $\Omega$ , using the law of motion described in the Appendix (proposition 2). We use this  $\Omega$  to compute a new  $\tilde{K}$  and iterate over  $\tilde{K}$  until the clearing holds. A detailed description of our solution method can be found in the Appendix.

Table 1 shows the parameter values of the calibrated model economy together with the targets used to calibrate the model. The model also has implications for additional aggregate macro statistics. Specifically, the calibrated model implies an annual aggregate consumption growth rate of 0.67 percent and an investment-to-output ratio,  $X_k/Y$  of 23 percent. These values are in line with the corresponding values for the US economy.

## 5. Results

We next present the results of our quantitative analysis of the model. Section 5.1 compares the models implications for insurance to the data, beginning with life insurance holdings and concluding with a description of consumption insurance. Section 5.2 discusses the models implications for wealth and the cross sectional variation in consumption levels. Section 5.3 presents the welfare consequences of mortality risk. Section 5.4 discusses our policy experiment: a reform of the bankruptcy code.

## 5.1 Insurance

### 5.1.1 Insurance of Median Household

In this subsection we assess the ability of the model to reproduce the life-cycle pattern of life insurance holdings. We begin the analysis on the median household conditional on age and the purchase of life insurance (intensive margin), and then turn to a discussion of heterogeneity within age groups in Section 5.1.2 below. Figure 6 shows the life-cycle profile of median life insurance holdings of married households with children who have purchased life insurance (intensive margin) in the data and as predicted by the model. As discussed before, the data display an inverted u-shape pattern: the median young married household holds around \$85 thousand in life insurance, rising quickly to \$200 thousand before declining slowly down to \$75 thousand in their early 60's. Figure 6 shows that the model is able to match these data both qualitatively and quantitatively.

We next evaluate the extent of underinsurance for the group of households depicted in figure 6 by analyzing the corresponding life insurance coefficient. Recall that in this paper we define the life insurance coefficient as the ratio of insurance payout in the case of death (actual insurance) over the human capital loss in the case of death (insurance need). In figure 7 we depict the life cycle profile of the life insurance coefficient for median married households with children who purchased some life insurance. In this figure, we proxy the human capital loss by the present value losses in the case of death, computed as described in Section 2.3. As in figure 6, the model provides an excellent quantitative account of the data, which is expected given that the fit between model and data in figure 6 is very good. Figure 7 shows that under-insurance for young households is severe and that insurance is strongly correlated with age: both the data and the model, young households are insured against roughly 30% of their potential loss, with the figure rising close to 100% only as households reach their late 50's.

The measure of underinsurance in figure 7 is based on estimated present value income losses. Alternatively, we can compute life insurance coefficients using the model-based measure of life insurance holdings required to generate full insurance against mortality risk. The result of this computation is depicted in figure 8 and confirms the result already shown in figure 7: both in the data and in the model, there exists a strong correlation between age and the degree to which households purchase insurance against mortality risk. Indeed, the correlation between age and life insurance coefficient is roughly the same in both figures; the only difference is that young households are insured against 30% of the losses implied by the

present value of income losses and against 40% of the losses according to the model-based calculation. This result is expected given that the human capital losses in the calibrated model economy are in line with the estimated net present value losses.

In the model, the human capital losses in the case of a husband's death are different from the human capital losses in the case of a wife's death. Consequently, the life insurance holdings for the two events differ. The SCF does not provide information about the split of insurance between husband and wife, but the SIPP data provide information about this split. In figure 9 we plot life insurance holdings separately for husband and wife, where we again focus on married households with children who have purchased some life insurance. The data show that in both cases there is an inverted u-shape, but this inverted u-shape is much more pronounced for insurance against the husband's death. Further, life insurance against the husband's death is about twice as much as life insurance against the wife's death. Figure 9 also shows that the model provides a good quantitative account of both life-cycle profiles, though on average the model slightly under-predicts holdings of insurance against the death of the husband, and slightly over-predicts the holdings of insurance against the death of the wife.

Figures 6-9 analyze to what extent households insure against mortality risk. Households in the model also face labor market risk and demographic risk in addition to mortality risk, and we next investigate the extent of insurance against all types of risk. To this end, we consider the model implication for the life-cycle variation of consumption insurance measured as lack of consumption volatility. More precisely, we define an insurance measure  $1 - \sigma_c / \sigma_{a,c}$ , where  $\sigma_c$  is the standard deviation of equilibrium consumption growth for households in the model and  $\sigma_{a,c}$  is the standard deviation of equilibrium consumption growth for hypothetical households with no access to credit and insurance markets (but with the ability to save in a risk-free asset). Figure 10 shows that consumption insurance increases substantially with age, beginning at 0.24 for households at age 23 and increases to 0.81 for households at age 60. This is in stark contrast to much of the previous quantitative literature which abstracts from life-cycle motives for borrowing and saving; in our model, a desire to smooth consumption over the life-cycle and to invest in human capital while young motivates young households to borrow as much as they can and purchase less insurance.

### 5.1.2 Heterogeneity in Insurance

In this subsection we move beyond the focus on the median married household with children who has purchased some life insurance and discuss additional moments of life insurance

holdings. We begin with a discussion of the extensive margin. Figure 11 depicts the life-cycle profile of the median life insurance holdings for all married households with children and those married households with children who have purchased some life insurance. Figure 11 shows that the model provides a good account of the empirical life-cycle profile of both the extensive margin and the intensive margin. This is confirmed by figure A10 in the Appendix, which shows the life-cycle profile of the participation rate in the life insurance market for married households with children. Recall that the evolution of participation in the model is controlled by the probability  $q_t$ . The evolution of  $q_t$  is controlled by the single parameter  $d$  which was chosen to match participation rates in an average sense. From figure A10 we conclude that the model with a single parameter governing the evolution of participation in life insurance markets produces an excellent match to the data on participation rates in the life insurance market over the entire life cycle.

Next we discuss the intensive margin. Figure 12 depicts the life-cycle profile of mean life insurance holdings for married households with children who have purchased some life insurance and figure 13 shows the corresponding life cycle profile for households in the highest decile of the distribution.<sup>21</sup> Figures 12 and 13 show that the model matches well the empirical life-cycle profile of these additional moments. Note that our calibration approach allows us to match by construction the mean and highest decile of life insurance holdings averaged over the life-cycle, but leaves no degree of freedom for matching the entire life-cycle profile. Note also that figures 11, 12 and 13 show that the shape of the life cycle profile of life insurance holdings is very similar for all three moments (median, mean, highest decile), both in the data as well as according to the model.

Figures A13 and A14 in the Appendix show the life-cycle profile of the life insurance coefficient for the median, mean, and highest decile of the distribution (the corresponding plot for the median is shown in figure 7). The figures show that for all three moments there exists a strong correlation between age and insurance. In sum, this paper's main quantitative result regarding the under-insurance of young households holds for all three moments both in the data and in the model.

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<sup>21</sup>We restrict attention to life insurance holdings up to the 99th percentile in both model and data to avoid problems caused by extreme observations. We always report the mean between the 90th and 99th percentile when we refer to the highest decile.

## 5.2 Wealth and Consumption

An essential feature of our mechanism generating under-insurance of young households is that young households have little financial wealth relative to their human wealth. In our model, the portfolio mix between human and financial capital is measured by  $\theta_h$ , the fraction of total wealth invested in human capital. Empirically, we construct a measure of portfolio holdings by taking the ratio of (net) financial wealth to labor earnings, and compare this to the model generated analog which is given by  $\frac{1-\theta_h}{\phi r_h \theta_h} \cdot (1 - \theta_h) / (\phi r_h \theta_h)$ . Figure 14 shows the life-cycle profile of this ratio in the SCF data and according to the model. Clearly, the model provides a very good account of this dimension of the data for young households, and matches the observed increase in financial wealth relative to human wealth through age 50, although it over-predicts wealth holdings for the oldest households. We view it as a success of the model that it captures well the shape of the life-cycle profile even though it has not been calibrated to match the target – there is only one free parameter, which is used to match the level. In other words, one basic prediction of the theory, namely that households with high expected human capital returns should be heavily invested in human capital, is qualitatively and quantitatively supported by the empirical evidence.

Another important dimension of the data is the consumption dispersion over the life-cycle. Figure 15 compares the variance of log adult-equivalent consumption in the US, estimated using data from the Consumer Expenditure Survey (CEX), from three different studies—Aguiar and Hurst (2008), Deaton and Paxson (1994), and Primiceri and van Rens (2009)—to the corresponding variance implied by the model. The figure shows that the model captures the increase in consumption dispersion observed in the data. Indeed, the model matches quite well the estimates of consumption dispersion reported by Aguilar and Hurst (2008), in particular the concave shape of the life-cycle profile of consumption dispersion. Note that these estimates are also very similar to the ones found in Heathcote et al. (2010).

Note that the consumption implications of our model differ substantially from the results obtained by the previous literature on limited enforcement (Krueger and Perri, 2006, and Boer, 2014) due to the life-cycle component and human capital investment. For example, in Section 5.1 we have shown that young households insure less than 40 percent of their human capital losses upon death of a spouse (see figure 7), which implies that these households experience a substantial (and permanent) drop in consumption levels in the event of the death of an adult family member relative to their expected consumption path. Further, in our calibrated model economy the participation constraints bind frequently for our youngest

households as they have an intense desire to borrow to invest in human capital and to smooth consumption over their lifecycle. This is why our measure of overall consumption insurance in Figure 10 shows that less than 25% of the volatility in the consumption of the young is insured in our benchmark model.

Though our model provides a better account of the consumption data than previous limited enforcement models analyzed in the literature, our baseline model still fails to match some dimensions of the consumption data. Specifically, the distribution of equilibrium consumption in the baseline model is rightward skewed since consumption growth is bounded from below by  $\beta(1 + r_f)$ . This property of the standard limited commitment model is well-known. For example, Broer (2014) argues that it is at odds with the CEX data. However, the distribution of equilibrium consumption growth becomes richer once we introduce state-dependent marginal utility of consumption,  $\gamma_1 = \gamma_1(s_1)$ , endogenous leisure choice with non-separable preferences, or unobserved heterogeneity in  $\beta$ . A deeper study of this interesting issue is beyond the scope of the current paper and constitutes an important topic for future research.

### 5.3 Welfare Cost of Mortality Risk

In this section, we turn to an analysis of the welfare effect of mortality risk and the welfare cost of lack of insurance against this type risk. To this end, we compute the welfare cost of mortality risk as the difference between welfare of a household who is fully insured against mortality risk and welfare of the same household in a situation with no access to insurance against mortality risk, where the no-insurance situation is defined by the absence of both private life insurance markets and public insurance through the social security system. To facilitate the interpretation of our results, we report all welfare changes in terms of the dollar value of the equivalent variation in the present value of consumption.<sup>22</sup> We focus on the group of 23-year old married households with children who face some mortality risk and therefore have purchased some life insurance, i.e. 23-year old married households with

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<sup>22</sup>Welfare for households who are fully insured is computed based on the equilibrium for a model with no  $\eta$ -shocks for all ages and family types. Welfare without insurance against mortality risk is computed using  $\eta$ -shocks that do not take into account social security survivor benefits (lack of public life insurance) and restrict asset payoffs to be constant across death states and no-death states (lack of private life insurance). However, even in the no-insurance case our  $\eta$ -shocks take into account the implicit insurance provided through re-marriage and through the reduction in living costs resulting from smaller family size. In all cases we are careful to make the appropriate mean adjustments in  $\eta$  so that we only consider mean-preserving spreads. We compute welfare costs as the product of the equivalent percentage change in consumption times the present value of the consumption loss of a 23-year old married households with children.

children and  $\alpha > 0$ .

Table 2 summarizes the results of our welfare calculations and shows that the welfare cost of mortality risk,  $\Delta$ , is \$15,089. This is a substantial amount given that mortality risk is quite low and reflects the very large income and consumption losses associated with mortality risk. This number hides a great deal of heterogeneity. The welfare cost for a 23-year old married household with median level of mortality risk is \$2,505, whereas this welfare cost is \$7,523 for the group of married households with average (mean) levels of mortality risk, and this cost increases to \$93,359 for the 10 percent of 23-year old married households who face the highest levels of mortality risk.<sup>23</sup> The welfare cost of mortality risk has two parts: the cost of moving from full insurance to imperfect insurance,  $\Delta_1$ , and the cost of moving from imperfect insurance to no insurance,  $\Delta_2$ . This decomposition of the welfare cost of mortality risk is shown in columns 2 and 3. A comparison of columns 2 and 3 shows that both components of the welfare costs are substantial, but the larger part of this welfare cost is the welfare difference between no insurance and imperfect insurance.

The above numbers capture the cost of removing access to both private and public insurance against mortality risk. The welfare cost of lacking only the private insurance market for a 23-year old married household,  $\Delta^p$ , is \$3,637. Further, this welfare cost is \$382 for a 23-year old married household with median level of mortality risk, \$1,015 for the group of married households with average (mean) levels of mortality risk, and \$15,682 for the 10 percent of 23-year old married households who face the highest levels of mortality risk. Columns 5 and 6 show the decomposition of these welfare cost into the two parts corresponding to moving from full insurance to imperfect insurance,  $\Delta_1^p$ , and from imperfect insurance to no insurance,  $\Delta_2^p$ . As in the previous case, the largest part of this welfare cost is the difference between no insurance and imperfect insurance.

#### 5.4 Reform of the Consumer Bankruptcy Code

There has been a long-standing debate among academic scholars and policy makers as to the relative merits of alternative consumer bankruptcy codes. For example, Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), and Livshits, MacGee, and Tertilt (2007) analyze the

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<sup>23</sup>Here median, mean, and upper 10-percentile refer to the moments of the distribution of mortality risk for households with  $\alpha > 0$ . Thus, these welfare cost are the costs for those households who have purchased some life insurance and whose life insurance demand is depicted in figures 6, 15, and 16. In our computation of the social welfare cost we average over all values of  $\alpha$  including  $\alpha = 0$  and therefore take into account households who have not purchased life insurance.



consequences of reforming the consumer bankruptcy code based on models with equilibrium default and no insurance markets. In these papers, an increase in the cost of bankruptcy increases borrowing and reduces default, which leads to a reduction in risk sharing since default is a means towards smoothing consumption across states of nature. In contrast, in our model an increase in the cost of bankruptcy increases borrowing *and* improves risk sharing since households can take better advantage of existing insurance markets, an effect that has also been studied in the theoretical contributions of Attanasio and Rios-Rull (2000) and Krueger and Perri (2010). In this section, we provide a quantitative evaluation of a change in the consumer bankruptcy code on household borrowing, insurance, and welfare.

Our experiment is motivated by the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) of 2005, which made it more difficult to file for bankruptcy under chapter 7—where debt is only repaid out of existing assets—and therefore forced more households to file under chapter 13 of the US bankruptcy code, where debts are repaid out of current earnings over a period of 3 to 5 years. See, for example, White (2007) for a detailed account of the US bankruptcy code before and after the reform. We implement this experiment by assuming that after implementation of the BAPCPA, in the event of bankruptcy, 30% of households are randomly assigned to file under chapter 13.<sup>24</sup> In line with the code, we model the consequence of filing for bankruptcy under chapter 13 as an exclusion from borrowing and insurance markets for an average of 4 years and a 25 percent garnishment of labor income during the period of exclusion. The BAPCPA also increased bankruptcy filing costs significantly, and we incorporate this change in legislation by introducing a one-off cost that is paid in the year of filing for bankruptcy by all defaulting households. We assume this cost to be proportional to the wealth of households and set the cost parameter so that it amounts to \$2,000 for the median wealth household. Finally, the BAPCPA increased the minimum number of years that have to pass until a consumer can file a second time under chapter 7 from 7 years to 8 years, and we incorporate this change in legislation by assuming that households filing for bankruptcy under chapter 7 are excluded for 8 years after the reform (instead of 7 years before the reform).

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<sup>24</sup>An important change the BAPCPA introduced was the "means test". This means test restricted filing under chapter 7 to those households with income below median income adjusted for family type, which suggest that after the reform 50 percent of all households are forced to file bankruptcy under chapter 13. However, defaulting households differ from non-defaulting households, and we take account of this fact by assuming that only 30 percent defaulting households are forced into chapter 13 after the reform. The number of 30 percent corresponds to the fraction of defaulting households in our SCF 2004 sample who have above median income.

We compute the welfare consequences of the reform by comparing the lifetime utility of new-born households (households age 23) in the two economies, before and after the reform. For this comparison, we compute the welfare change  $\Delta(s_1)$  as the equivalent variation of the bankruptcy reform, measured in units of lifetime consumption, and then average over family states  $s_1$  using the fixed stationary distribution over  $s_1$  (this distribution over exogenous shocks is not affected by the policy experiment). Note that the welfare change  $\Delta(s_1)$  is independent of the initial wealth level of a household so that in our case there is no need to average over wealth using an endogenous wealth distribution. Note further that we conduct a steady state comparison in the sense that we do not take into account the transition path of the aggregate capital-to-labor ratio  $\tilde{K}$  (and the corresponding transition path of investment returns).

We compute three different measures of the welfare effect of the reform of the bankruptcy code for new-born households. First, we keep human capital investment fixed and ask how much young households gain when additional borrowing enables them to buy more insurance. This welfare gain from better insurance is 0.5 percent of lifetime consumption. A welfare gain of half percent of lifetime consumption through improvements in risk sharing is substantially larger than any gain that the model of Krueger and Perri (2006) would predict, where households are almost fully insured even before the reform. Second, we allow households to adjust human capital choices but keep investment returns fixed (partial equilibrium with endogenous human capital accumulation). The welfare gain for households age 23 is 0.56 percent of lifetime consumption. Finally, we consider endogenous human capital accumulation in general equilibrium. In this case, the rate of return on human capital investment goes down and the return to physical capital investment goes up, which introduces a negative welfare effect for young households who are almost fully invested in human capital. The net welfare gain for newborn households is 0.25 percent when we take into account the general equilibrium adjustments of investment returns.

In figure A15 in the Appendix we plot the life-cycle profile of the ratio of financial wealth (net worth) over labor income before and after the reform and show in figure A16 the insurance measure  $1 - \sigma_c/\sigma_{a,c}$  before and after the reform. Figure A15 confirms that households borrow more after the reform – the youngest households have positive financial wealth before the reform and negative financial wealth after the reform. Figure A16 supports the idea that the reform improves insurance – the reform increases the insurance coefficient  $1 - \sigma_c/\sigma_{a,c}$ . Note that the reform improves insurance against all types of risk, including labor market

risk. Hence, the welfare gain of 0.43% is attributed to a gain in risk sharing in all insurance markets.

In general equilibrium the reform of the bankruptcy code decreases  $\tilde{K}$  since households invest more in human capital after the reform. We find that the reform of the bankruptcy code reduces  $\tilde{K}$  by 1.2 percent of its initial value of 0.4. In our endogenous growth model, any change in  $\tilde{K}$  also changes the aggregate growth rate of the economy. Specifically, the equilibrium value  $\tilde{K}$  is in general higher than the value of  $\tilde{K}$  that maximizes aggregate growth, and a reduction in  $\tilde{K}$  increases aggregate growth, an effect that is discussed in more detail in Krebs (2003). In our calibrated model economy, the growth gain is relatively modest: the annual growth rate increases by 0.02 percentage point.

## 6. Extensions and Robustness

In this section, we discuss several extensions of the baseline model that help us understand additional dimensions of the data. We also perform a battery of robustness checks with respect to our data analysis and changes in the parameter values of the calibrated model economy. Due to space limitation many of the details of the analysis and all figures are relegated to the Appendix.

### 6.1 Model Extensions

The baseline model used in Section 5 assumes that the marginal utility of consumption is independent of family structure and, in particular, independent of the death event. Motivated by two recent contributions (Kojien et al., 2012, and Hong and Rios-Rull, 2012), we have considered an extension of the baseline model that incorporates household preferences that depend on the number of children and the health status of the household. Details of the model extension and the analysis can be found in Section G.1 of the Appendix. The results can be summarized as follows. First, with the changes added to the model, the basic facts about life insurance and other asset holdings over the life-cycle for all married households with children are unchanged (see in particular figure A11 in the Appendix). Second, this extension improves the match between model and data in the sense that the extended model replicates additional cross-sectional facts. Third, if we interpret the change in marginal utility following the death of a parent as reflecting the consequent change in the cost of living, the resulting changes are relatively modest and increase in the number of kids.

In the baseline model, prior to retirement all agents can buy a complete set of insurance

products, including both life insurance and annuities. However, we constrain retirees to save in a risk-free asset with any wealth remaining at their death distributed to newborn households. In Section G.2 of the Appendix we also discuss an extension of the baseline model in which retirees have a bequest motive and can purchase annuities. We conclude this extension has little effect on the underinsurance choices of young households and hence that the restriction on retirees in the baseline model is relatively innocuous. Finally, in Section G3 of the Appendix we consider the implications of an incomplete market model with two financial assets, a risk-free asset and a life insurance contract, and ad-hoc borrowing constraints. We provide evidence that this type of model cannot explain the observed positive correlation between age and insurance without life-cycle variations in household preferences.

## 6.2 Sensitivity Analysis

We have conducted an extensive sensitivity analysis varying the main parameters of interest within a range of empirically plausible values. In particular, we considered realistic changes in the parameters controlling mortality risk and contract enforcement. Overall, we show that the main quantitative results of this paper are quite robust to the variations in parameter values produced by these variations in calibration targets. Details of our sensitivity analysis can be found in the Section H of the Appendix.

## 6.3 Empirical Robustness

We also conducted an extensive robustness analysis of our empirical results. The details of this analysis can be found in Section I of the Appendix. The results of this analysis can be summarized as follows.

We first analyzed to what extent our approach to estimating human capital losses upon the death of a spouse are biased. Specifically, we used data drawn from the SCF and the SIPP to investigate if there is evidence of selection bias when we estimate earnings losses by comparing the earnings of married households with the earnings of single households. Further, we considered if there are additional channels of insurance through inter vivo transfers using data drawn from the SCF and the PSID. We find little or no evidence of selection bias, and that inter vivos transfers are rare and small. Hence, our conclusion is that there is no evidence that our estimates of earnings losses have a substantial bias.

We also examined variation in our underinsurance measure by wealth level. As a result of assumptions made for analytical tractability (which are not central to the mechanism we propose) our model predicts that all households of a given demographic type should

hold levels of life insurance that are proportional to wealth. To assess to what extent this model implication is borne out in the data, we computed life insurance holdings and the life insurance coefficient over the life cycle for different quartiles of the wealth distribution. We find (see figure A24 and more generally Section I.2 of the Appendix for details) that patterns in underinsurance are broadly similar for all quartiles with two explicable exceptions: the richest households are better insured at old age, which probably reflects the use of life insurance to avoid bequest taxes; and the poorest households are more insured when young, which may reflect unobserved heterogeneity in human capital returns for the young or simple measurement issues since these households hold very little life insurance.

## 7. Conclusion

In this paper, we provided empirical evidence of under-insurance in the market for life insurance. We then developed a tractable macroeconomic model with risky human capital and used the model to provide an explanation of our empirical finding in terms of endogenous borrowing constraints due to limited enforcement. We also used the framework to analyze the possible consequences of under-insurance. The results of this paper suggest at least two lines of future research.

First, in the paper we restricted attention to insurance against one form of human capital risk, the death of a family member, and insurance against this type of risk that can be purchased in the market for life insurance. A promising avenue for future research is to investigate the extent to which limited contract enforcement helps us understand empirical patterns in other insurance markets. For example, one basic implication of the adverse selection and moral hazard approach to insurance is that households with higher risk exposure should buy more insurance (Chiappori and Salanie, 2000), and a number of empirical studies have found that this hypothesis is rejected in the data (Chiappori and Salanie, 2000, and Bernheim, Forni, Gokhale, and Kotlikoff, 2003). In contrast, the limited enforcement approach suggests a negative correlation between risk exposure and insurance if households with high risk exposure are also the households with the largest share of human capital in total wealth. Thus, theories of limited contract enforcement have the potential to explain the empirical findings of Chiappori and Salanie (2000) and Bernheim, Forni, Gokhale, and Kotlikoff (2003).<sup>25</sup>

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<sup>25</sup>Note that adverse selection, combined with unobserved preference heterogeneity, provides an alternative explanation of the observed inverse relationship between health insurance holdings and the exposure to

A second line of research would broaden the set of assets available to households. The most important alternative asset is housing, which is also risky and which is, to varying degrees, collateralizable. All else equal, the perceived (utility) rates of return to housing investment are large, so that access to this asset will further strengthen the results of this paper: households would like to borrow to invest in housing and human capital, and these investment opportunities will compete with the need to purchase insurance. To what extent this effect is offset by the fact that some housing wealth can be used as collateral against borrowing remains an open quantitative question.

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health shocks (Cutler, Finkelstein, and McGarry, 2008).

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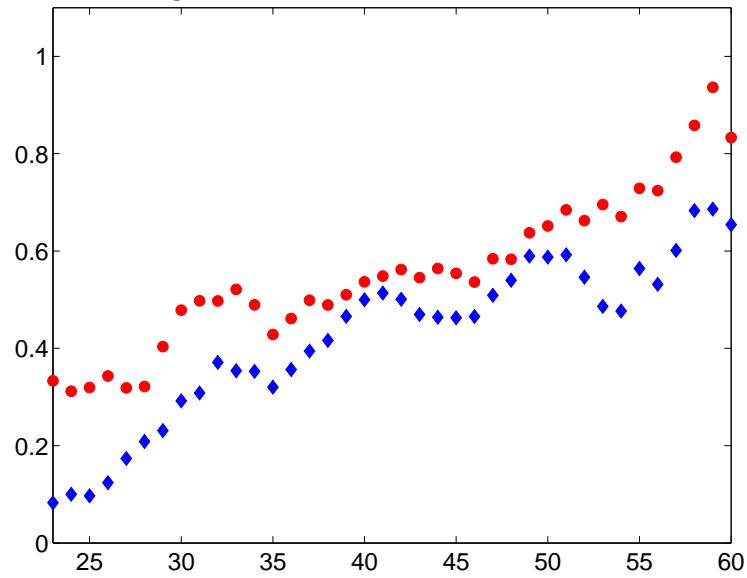
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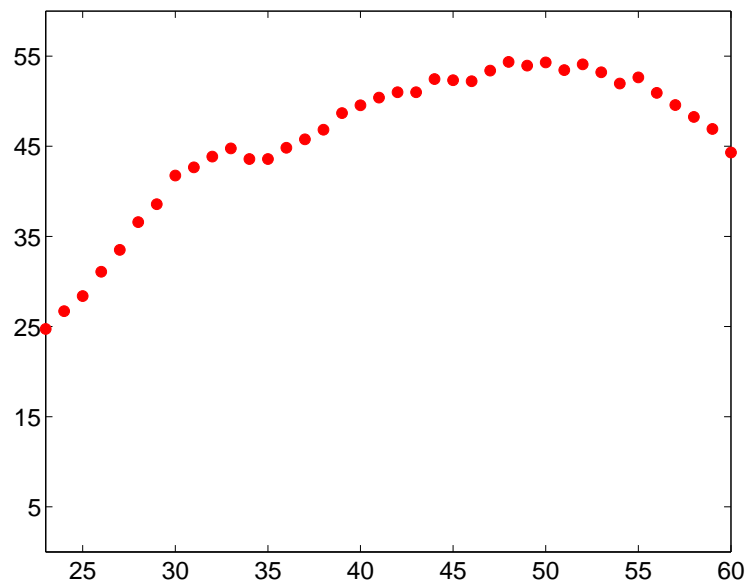
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Figure 1: Life Insurance Coefficient



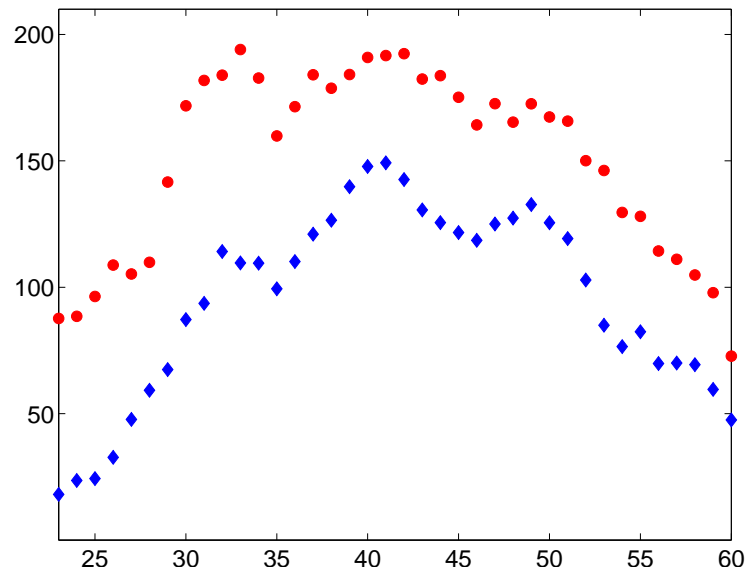
Notes: Life-cycle profile of life insurance coefficient. Life insurance coefficient is the ratio of life-insurance holdings to present value income loss in case of death. Red dots are for median married households with children who have purchased life insurance. Blue diamonds are for all married households with children. All data are from the SCF. See appendix for calculation of present value loss.

Figure 2: Labor income



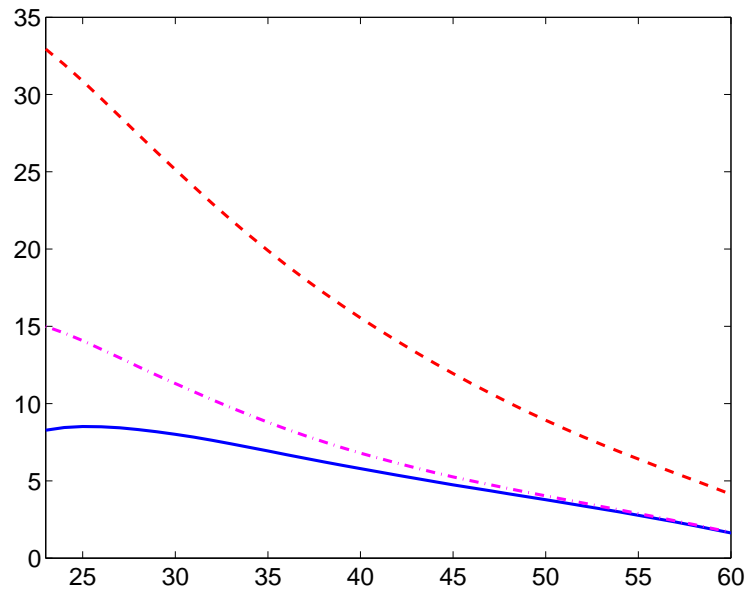
Notes: Life-cycle profile of median labor income for married households with children from the SCF (thousands of year 2000 dollars).

Figure 3: Life Insurance



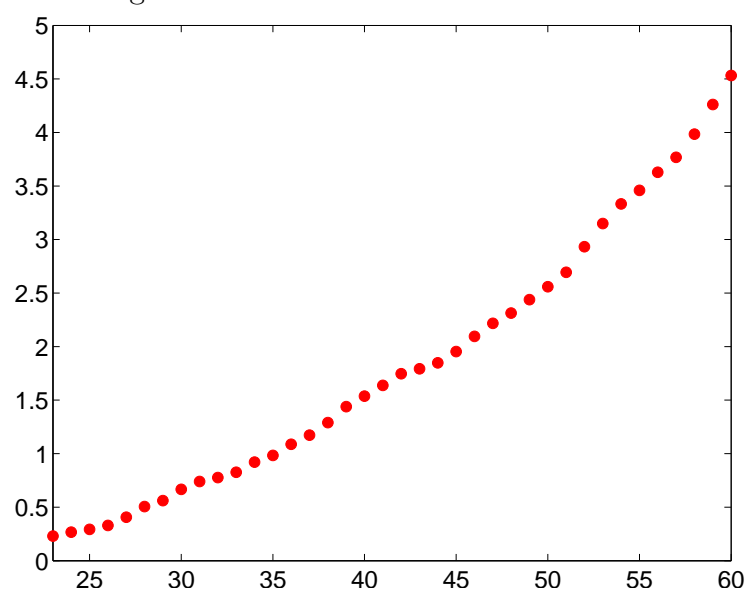
Notes: Life-cycle profile of median life insurance holdings. Red dots are for married households with children that have purchased life insurance. Blue diamonds are for all married households with children. All data are from the SCF (in thousands of year 2000 dollars).

Figure 4: Human capital loss



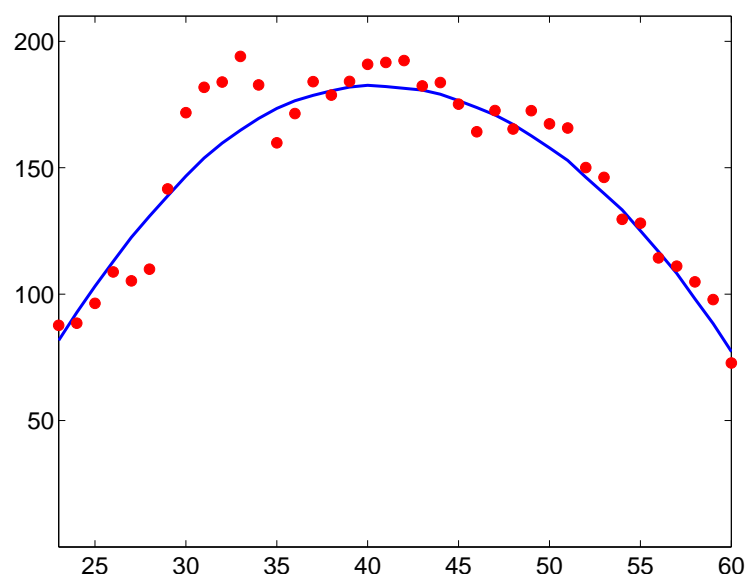
Notes: Life-cycle profile of sum of expected human capital loss in case of husband's and wife's death for all married households with children. Human capital loss is ratio of present value income loss over current labor income. Red dashed line: loss before transfers and taxes with zero probability to remarry. Pink dashed-dotted line: loss after transfers and taxes and zero probability to remarry. Blue solid line: loss after transfers and taxes and empirical remarriage rates. All data are from the SCF. See appendix for further details.

Figure 5: Networth to labor income ratio



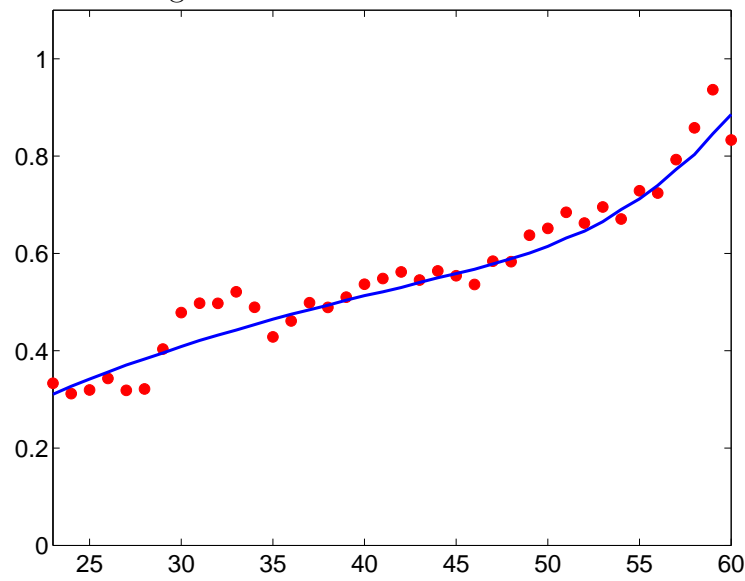
Notes: Life-cycle profile of the median ratio of networth to labor income for married households with children from the SCF.

Figure 6: Life insurance



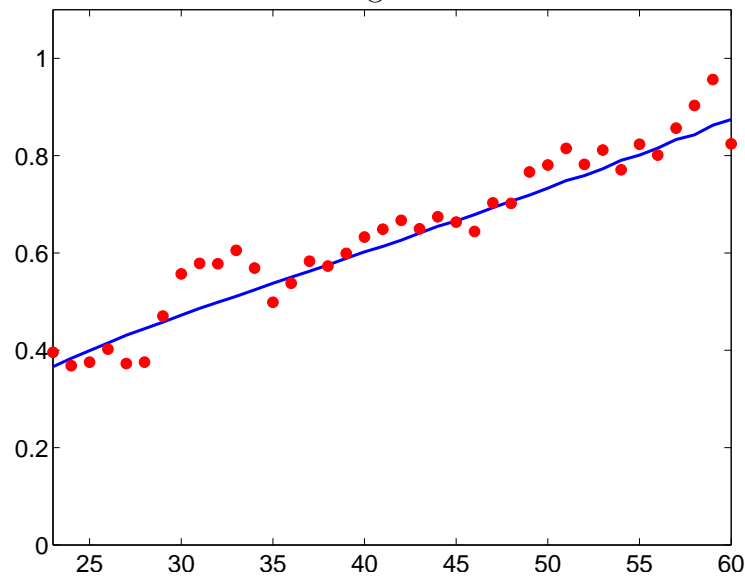
Notes: Life-cycle profile of median life insurance holdings for married households with children who have purchased some life insurance. Blue solid line shows model and red dots show SCF data (in thousands of year 2000 dollars).

Figure 7: Life Insurance Coefficient



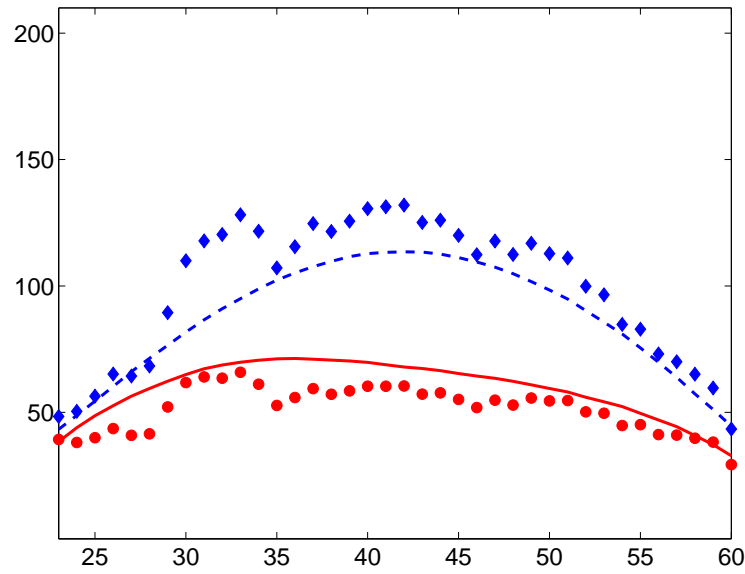
Notes: Life-cycle profile of life insurance coefficient for median married households with children who have purchased some life insurance. Life insurance coefficient is the ratio of life-insurance holdings to present value income loss in case of death. Blue solid line shows model and red dots show SCF data.

Figure 8: Actual life insurance holding over the models life insurance holding



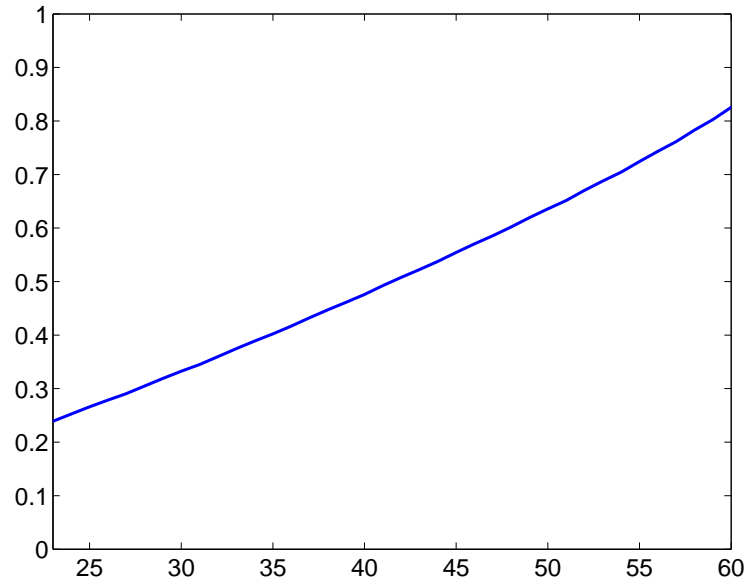
Notes: Life-cycle profile of the ratio of life insurance holding over full insurance for median married households with children who have purchased some life insurance. Blue solid line shows model and red dots show SCF data.

Figure 9: Life insurance for husband and wife



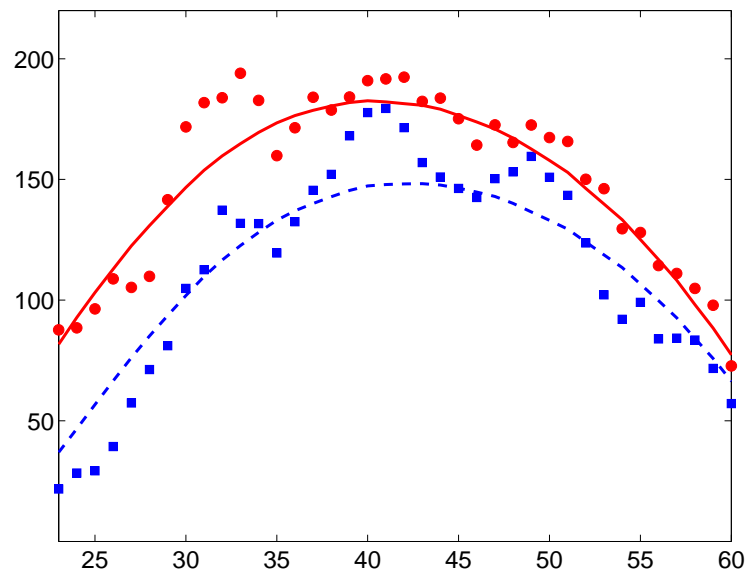
Notes: Life-cycle profile of median life insurance holdings for married households with children who have purchased some life insurance. Red solid line shows life insurance holdings for wife's death from model. Red dots show life insurance holdings for wife's death from data. Blue dashed line shows life insurance holdings for husband's death from model. Blue diamonds show life insurance holdings for husband's death from data. Data are from the SCF and the SIPP. See appendix for details of the construction of data profiles.

Figure 10: Consumption insurance



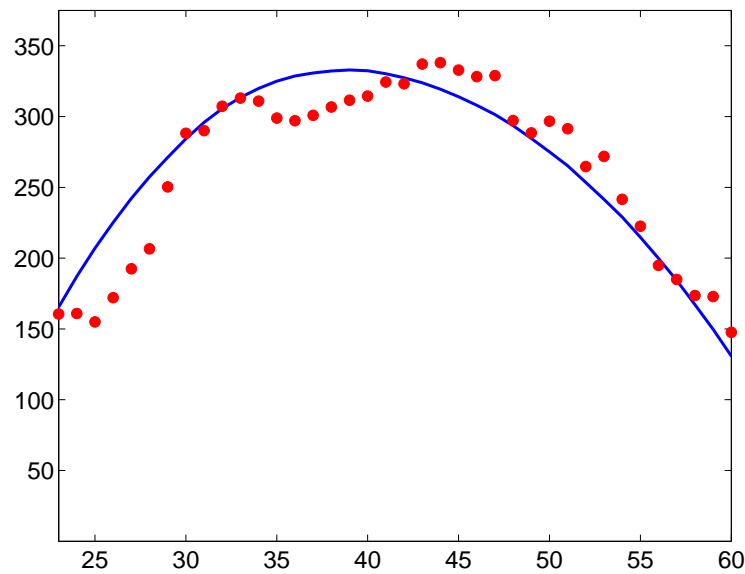
Notes: Consumption insurance in the model for married households with children. The insurance measure is one minus the ratio of the standard deviation of consumption in equilibrium relative to the standard deviation of consumption in financial autarky.

Figure 11: Life Insurance: Extensive and Intensive Margin



Notes: Life-cycle profile of median life insurance holdings for married households with children. Red solid line shows model prediction for all households that have purchased some life insurance. Red dots show all households that have purchased life insurance from the data. Blue dashed line shows model prediction for all households. Blue squares show all households from the data. All data are from the SCF (in thousands of year 2000 dollars).

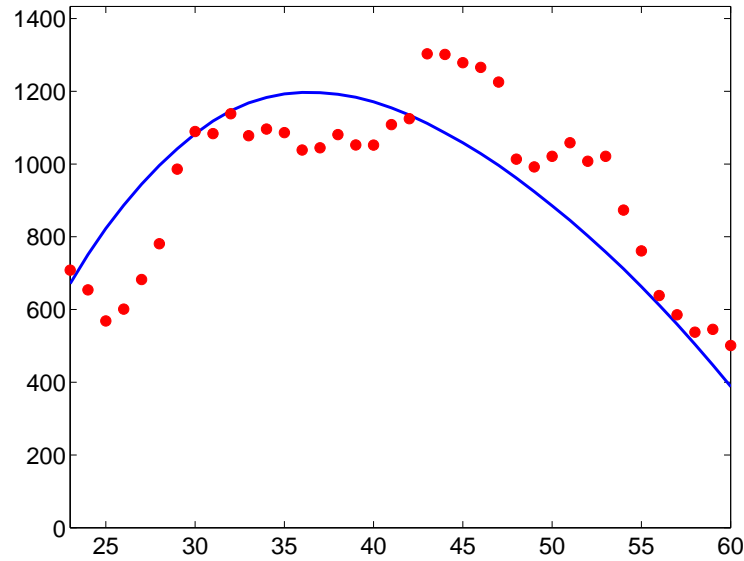
Figure 12: Mean Life Insurance



Notes: Life-cycle profile of mean life insurance holdings for married households with children who have purchased some life insurance. Blue solid line shows model and red dots show SCF data (in thousands of year 2000 dollars).

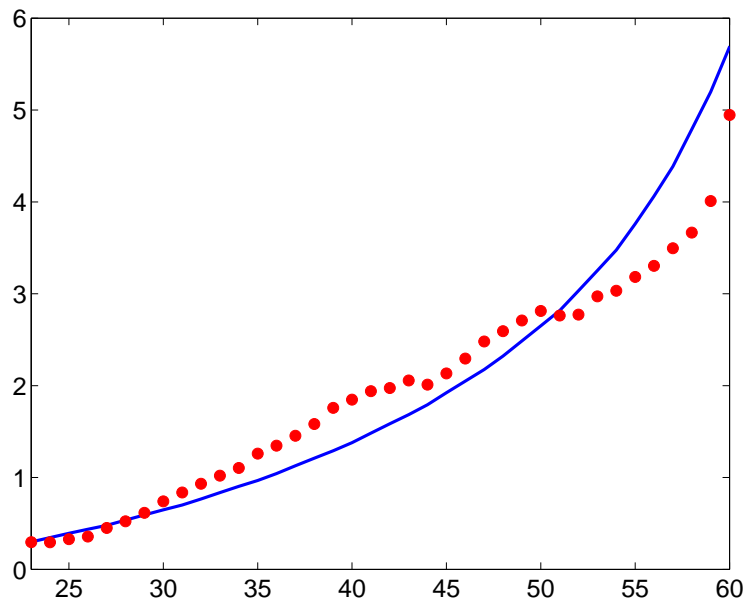


Figure 13: Top Decile of Life Insurance Holdings



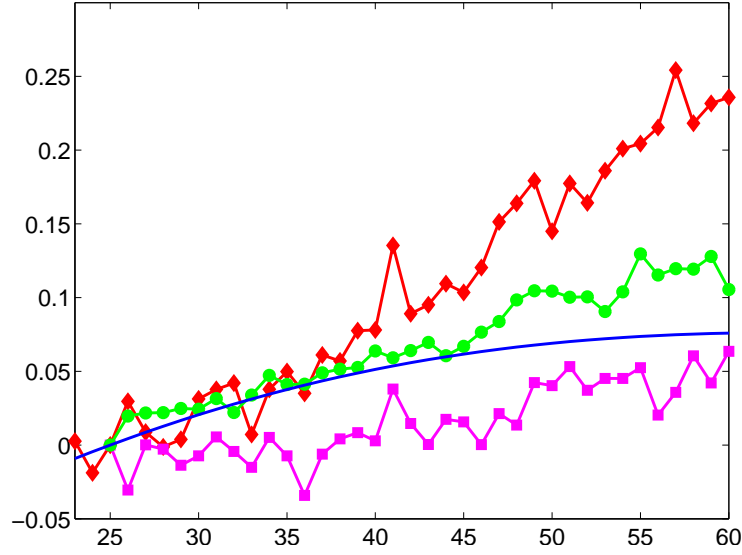
Notes: Life-cycle profile of top decile of life insurance holdings for married households with children who have purchased some life insurance. Red dots show data from the SCF in thousands of year 2000 dollars; to remove outliers, we calculate the mean life insurance holdings between the 90th and 99th percentile of life-insurance holdings. The blue solid line shows the model prediction which, for comparability, has been truncated at the 99th percentile of the empirical distribution.

Figure 14: Net Worth to Labor Income



Notes: Life-cycle profile of the median ratio of networth to labor income for married households with children. Blue solid line shows model and red dots SCF data.

Figure 15: Consumption Inequality



Notes: Life-cycle profile of the cross-sectional variance of consumption. The blue solid line shows the model prediction. The red diamonds show the profile estimated by Deaton and Paxson (1994), the green dots are the estimates of Aguiar and Hurst (2008), and the pink squares are the estimates of Primiceri and van Rens (2009). The data have been normalized to 0 at age 25.

Table 1: Calibration

parameter	value	description
$\beta$	0.95	discount factor
$\phi$	0.6749	(inverse of) price of human capital
$p_{ret}$	0.2	probability of retiring
$p_{death}$	0.05	probability of dying
$\sigma_{\eta}$	0.0913	standard deviation of permanent shocks
$\sigma_z$	0.3	standard deviation of transitory shocks
$p$	0.8571	probability of remaining in financial autarky
$\alpha$	0.32	capital share in output
$\delta_k$	0.0785	physical capital depreciation rate
$\delta_h$	0.0429	human capital depreciation rate
$A$	0.1818	total factor productivity
$\lambda$	1.6461	human capital endowment of young households

Table 2: Welfare Cost of Mortality Risk

	$\Delta$	$\Delta_1$	$\Delta_2$	$\Delta^p$	$\Delta_1^p$	$\Delta_2^p$
social	\$ 15,089	\$ 5,870	\$ 9,219	\$ 3,637	\$ 1,351	\$ 2,286
median	\$ 2,505	\$ 722	\$ 1,783	\$ 382	\$ 54	\$ 328
mean	\$ 7,523	\$ 2,550	\$ 4,973	\$ 1,015	\$ 252	\$ 763
top 10 %	\$ 93,359	\$ 37,634	\$ 55,725	\$ 15,682	\$ 6,137	\$ 9,545

Notes: All welfare changes are expressed as the dollar value of the equivalent variation in the present value of consumption for a 23-year old married household with children.  $\Delta$  is the welfare cost of no life insurance computed as the welfare difference between full insurance against mortality risk and no insurance against mortality risk.  $\Delta_1$  is the welfare cost of moving from full insurance to partial insurance against mortality risk.  $\Delta_2$  is the welfare cost of moving from partial insurance to no insurance against mortality risk.  $\Delta^p$  is the welfare cost of no private life insurance computed as the welfare difference between full insurance against mortality risk and no private insurance against mortality risk.  $\Delta_1^p$  is the welfare cost of moving from full insurance to partial private insurance against mortality risk.  $\Delta_2^p$  is the welfare cost of moving from partial private insurance to no private insurance against mortality risk. “Social” refers to the social welfare cost, that is, the welfare cost before the level of mortality risk is known. “Median”, “mean”, and “top 10” refer to the welfare cost for households with median, mean, and for the 10 % of households with the highest mortality risk, respectively. All welfare numbers are conditional on that it is known that  $\alpha$  is positive.

# Online Appendix

## A Proof of Propositions

### A.1 Proof of proposition 1

Define total wealth (human plus financial) of a household of age  $j$ ,  $w_j$ , the portfolio choice,  $\theta_j$ , and the total investment return,  $r_j$  as in Section 3.4. Using this notation, the sequential budget constraint is given in (8). For age  $j = 1, \dots, J$ , the Bellman equation associated with the household utility maximization problem reads:

$$\begin{aligned}
 V_j(w_j, \theta_j, s_j) &= \max_{c_j, w_{j+1}, \theta_{j+1}} \left\{ \gamma_0(s_j) + \gamma_1(s_j) \ln c_j + \beta \sum_{s_{j+1}} V_{j+1}(w_{j+1}, \theta_{j+1}, s_{j+1}) \pi_j(s_{j+1} | s_j) \right\} \\
 \text{s.t.} \quad w_{j+1} &= (1 + r_j(\theta_j, s_j))w_j - c_j \\
 1 &= \theta_{h,j+1} + \sum_{s_{j+1}} q_j(s_{j+1} | s_j) \theta_{a,j+1}(s_{j+1}) \\
 c_j &\geq 0, \quad w_{j+1} \geq 0, \quad \theta_{h,j+1} \geq 0 \\
 V_{j+1}(w_{j+1}, \theta_{j+1}, s_{j+1}) &\geq V_{d,j+1}(w_{j+1}, \theta_{h,j+1}, s_{j+1}),
 \end{aligned} \tag{A1}$$

In default, a household who defaults at age  $j$  chooses a continuation plan,  $\{c_{j+n}, h_{j+n}\}$ , so as to maximize

$$\begin{aligned}
 &\sum_{n=0}^{J-j} (p\beta)^n \sum_{s^{j+n} | s^j} \left[ \gamma_0(s_{1,j+n}) + \gamma_1(s_{2,j+n}) \ln c_{j+n}(s^{j+n}) \right] \pi(s^{j+n} | s_0) \\
 &+ \sum_{n=0}^{\infty} (p\beta)^{J+1-j+n} \sum_{s^{J+1+n} | s^j} V_{J+1}(h_{J+1+n}(s^{J+n}), a_{J+1+n}(s^{J+1+n}), s_{J+1+n}) \pi(s^{J+1+n} | s_0) \\
 &+ \sum_{n=0}^{J-j} ((1-p)\beta)^n \sum_{s^{j+n} | s^j} V_{j+n}^e(h_{j+n}(s^{j+n-1}), s_{j+n}) \pi(s^{j+n} | s_j) \\
 &+ \sum_{n=0}^{\infty} (p\beta)^{J+1-j+n} \sum_{s^{J+1+n} | s^j} V_{J+1+n}^e(h_{J+1+n}(s^{J+n}), s_{J+1+n}) \pi(s^{J+1+n} | s_0)
 \end{aligned}$$

where  $\{c_{j+n}, h_{j+n}\}$  has to solve the sequential budget constraint (3) with  $a_j = 0$ . Define the investment return of a household in default as  $r_d(\theta_{hj}, s_j) = (1 + z_j(s_j)\phi r_h(s_j) - \delta_{hj} + \eta_j(s_j))\theta_{hj}$ , which is simply the human capital return times the fraction of wealth invested in human capital. In the period of default, we have in general  $\theta_{hj} \neq 1$ , but in all periods subsequent to default we have  $\theta_{h,j+n} = 1$ . In the period of re-gaining access to financial markets, a household in default has no financial assets, and we still have  $\theta_{h,j+n} = 1$ . The Bellman

equation of a household in default reads

$$\begin{aligned}
V_{dj}(w_j, \theta_{hj}, s_j) &= \max_{c_j, w_{j+1}} \left\{ \gamma_0(s_j) + \gamma_1(s_j) \ln c_j + p\beta \sum_{s_{j+1}} V_{d,j+1}(w_{j+1}, 1, s_{j+1}) \pi_j(s_{j+1}|s_j) \right. \\
&\quad \left. + (1-p)\beta \sum_{s_{j+1}} V_{j+1}^e(w_{j+1}, 1, s_{j+1}) \pi_j(s_{j+1}|s_j) \right\} \\
s.t. \quad w_{j+1} &= (1 + r_{dj}(1, s_j))w_j - c_j \\
c_j &\geq 0, \quad w_{j+1} \geq 0
\end{aligned} \tag{A2}$$

The Bellman equation (A2) for the default value function together with the Bellman equation (A1) and the condition  $V^e = V$  define a Bellman equation determining simultaneously the value function  $V$  and  $V_d$ . Suppose that the terminal value function  $V_{J+1}$  has the functional form (A7). Solving the problem backwards, guess-and-verify shows that the solution to this Bellman equation (A1) and (A2) for all  $j = 1, \dots, J$  is

$$\begin{aligned}
V_j(w_j, \theta_j, s_j) &= \tilde{V}_{0j}(s_j) + \tilde{V}_{1j}(s_j) [\ln w_j + \ln(1 + r_j(\theta_j, s_j))] \\
c_j(w_j, \theta_j, s_j) &= \tilde{c}_j (1 + r_j(\theta_j, s_j)) w_j \\
V_{dj}(w_j, \theta_j, s_j) &= \tilde{V}_{d,0j}(s_j) + \tilde{V}_{1j}(s_j) [\ln w_j + \ln(1 + r_{dj}(\theta_{hj}, s_j))] \\
c_j(w_j, \theta_j, s_j) &= \tilde{c}_j (1 + r_{dj}(\theta_{hj}, s_j)) w_j
\end{aligned} \tag{A3}$$

with

$$\tilde{c}_j(s_j) = \frac{\gamma_1(s_j)}{\tilde{V}_{1j}(s_j)}$$

The coefficients  $\tilde{V}_{1j}$  are determined recursively as the solution to

$$\tilde{V}_{1j}(s_j) = \gamma_1(s_j) + \beta \sum_{s_{j+1}} \tilde{V}_{1,j+1}(s_{j+1}) \pi_j(s_{j+1}|s_j)$$

and the coefficients  $\tilde{V}_{0j}$  and  $\tilde{V}_{d,0j}$  together with the optimal portfolio choices  $\theta_{j+1}^*$  are the solutions to the equation

$$\begin{aligned}
\theta_{j+1}^* &= \arg \max_{\theta_{j+1} \in \Gamma_{j+1}} \sum_{s_{j+1}} \tilde{V}_{1,j+1}(s_{j+1}) \ln(1 + r_{j+1}(\theta_{j+1}, s_{j+1})) \pi_j(s_{j+1}|s_j) \\
\Gamma_{j+1} &\doteq \left\{ \theta_{j+1} \left| \theta_{h,j+1} + \sum_{s_{j+1}} \frac{\theta_{a,j+1}(s_{j+1}) \pi_j(s_{j+1}|s_j)}{1 + r_f} = 1, \quad \theta_{h,j+1} \geq 0, \right. \right. \\
&\quad \left. \left. \frac{\tilde{V}_{0,j+1}(s_{j+1}) - \tilde{V}_{0d,j+1}(s_{j+1})}{\tilde{V}_{1,j+1}(s_{j+1})} \geq [\ln(1 + r_{d,j+1}(\theta_{h,j+1}, s_{j+1})) - \ln(1 + r_{j+1}(\theta_{j+1}, s_{j+1}))] \right\}.
\end{aligned} \tag{A4}$$

and

$$\begin{aligned}
\tilde{V}_{0j}(s_j) &= \gamma_0(s_j) + \gamma_1(s_j) \ln(\tilde{c}_j(s_j)) \\
&+ \beta \sum_{s_{j+1}} \tilde{V}_{0,j+1}(s_{j+1}) \pi_j(s_{j+1}|s_j) \\
&+ \beta \sum_{s_{j+1}} \tilde{V}_{1,j+1}(s_{j+1}) \ln\left(1 + r_{j+1}(\theta_{j+1}^*, s_{j+1})\right) \pi_j(s_{j+1}|s_j) \\
&+ \beta \ln(1 - \tilde{c}_j(s_j)) \sum_{s_{j+1}} \tilde{V}_{1,j+1}(s_{j+1}) \pi_j(s_{j+1}|s_j)
\end{aligned}$$

$$\begin{aligned}
\tilde{V}_{d,0j}(s_{1j}) &= \gamma_0(s_{1j}) + \gamma_1(s_j) \ln(\tilde{c}_j(s_j)) \\
&+ p\beta \sum_{s_{j+1}} \tilde{V}_{d,0,j+1}(s_{j+1}) \pi_j(s_{j+1}|s_j) \\
&+ (1-p)\beta \sum_{s_{j+1}} \tilde{V}_{0,j+1}(s_{j+1}) \pi_j(s_{j+1}|s_j) \\
&+ \beta \sum_{s_{j+1}} \tilde{V}_{1,j+1}(s_{j+1}) \log(1 + r_{d,j+1}(1, s_{j+1})) \pi_j(s_{j+1}|s_j) \\
&+ \beta \ln(1 - \tilde{c}_j(s_j)) \sum_{s_{j+1}} \tilde{V}_{1,j+1}(s_{j+1}) \pi_j(s_{j+1}|s_j)
\end{aligned}$$

This completes the proof for the case  $j = 1, \dots, J$ .

If  $j = J + 1$ , the household has entered a transition period from which retirement occurs stochastically at constant probability  $p_{ret}$ . In this case, the household problem is an infinite-horizon maximization problem with value function constraint, and the corresponding Bellman equation is a version of (A1) and (A2) in which the age-index is replaced by the constant  $J + 1$  (i.e. the index can be dropped) and there is a constant probability  $p_{ret}$  that the continuation utility is equal to a given continuation utility  $V_{ret}$ :

$$\begin{aligned}
V_{J+1}(w, \theta, s) &= \max_{c, w', \theta'} \left\{ \gamma_0(s) + \gamma_1(s) \ln c + (1 - p_{ret}) \beta \sum_{s'} V_{J+1}(w', \theta', s') \pi_{J+1}(s'|s) \right. \\
&\quad \left. + p_{ret} \beta \sum_{s'} V_{ret}(w', \theta', s') \right\} \\
s.t. \quad w' &= (1 + r_{J+1}(\theta, s))w - c \\
1 &= \theta'_h + \sum_{s'} q_{J+1}(s'|s) \theta'_a(s') \\
c &\geq 0, \quad w' \geq 0, \quad \theta'_h \geq 0 \\
V_{J+1}(w', \theta', s') &\geq V_{d,J+1}(w', \theta'_h, s'),
\end{aligned} \tag{A5}$$

and

$$\begin{aligned}
V_{d,J+1}(w, \theta_h, s) &= \max_{c, w'} \left\{ \gamma_0(s) + \gamma_1(s) \ln c + p\beta \sum_{s'} V_{d,J+1}(w', 1, s') \pi_{J+1}(s'|s) \right. \\
&\quad \left. + (1-p)\beta \sum_{s'} V_{J+1}^e(w', s') \pi_{J+1}(s'|s) \right\} \\
s.t. \quad w' &= (1 + r_{d,J+1}(1, s))w - c \\
c &\geq 0, \quad w' \geq 0
\end{aligned}$$

where we assumed that there is no retirement when the household is in default. We first discuss the retirement problem defining  $V_{ret}$  and then analyze the household problem in the pre-retirement phase (A5) determining  $V_{J+1}$  and  $V_{d,J+1}$ .

A household in retirement can only invest in the risk-free asset and the only source of income is capital income. Thus, there is no portfolio choice. We assume that retired households die with probability  $p_{death}$  and normalize the continuation utility after death to zero. Thus, the retirement value function for a household who retires in the current period has the functional form

$$V_{ret}(w, \theta, s) = \tilde{V}_{0,ret}(s) + \tilde{V}_{1,ret}(s) [\ln w + \ln(1 + r_{J+1}(\theta, s))] \quad (A6)$$

where we assumed that the household still works in the period in which the transition into retirement occurs. The coefficients  $\tilde{V}_{0,ret}$  and  $\tilde{V}_{1,ret}$  are given by

$$\tilde{V}_{1,ret}(s) = \gamma_1(s) + \beta(1 - p_{death}) \sum_{s'} \tilde{V}_{1,ret}(s') \pi_{ret}(s'|s)$$

and

$$\begin{aligned}
\tilde{V}_{0,ret}(s) &= \gamma_0(s) + \gamma_1(s) \ln(\tilde{c}_{ret}(s)) \\
&\quad + (1 - p_{death})\beta \sum_{s'} \tilde{V}_{0,ret}(s') \pi_{ret}(s'|s) \\
&\quad + (1 - p_{death})\beta \ln(1 + r_f) \sum_{s'} \tilde{V}_{1,ret}(s') \pi_{ret}(s'|s) \\
&\quad + (1 - p_{death})\beta \ln(1 - \tilde{c}_{ret}(s)) \sum_{s'} \tilde{V}_{1,ret}(s') \pi_{ret}(s'|s)
\end{aligned}$$

where  $\tilde{c}_{ret}(s) = \frac{\gamma_1(s)}{\tilde{V}_{1,ret}(s)}$ .

For the pre-retirement stage, we conjecture that the solution to (A5) is

$$V_{J+1}(w, \theta, s) = \tilde{V}_{0,J+1}(s) + \tilde{V}_{1,J+1}(s) [\ln w + \ln(1 + r_{J+1}, \theta_{J+1}, s_{J+1})] \quad (A7)$$

$$\begin{aligned}
c_{J+1}(w, \theta, s) &= \tilde{c}_{J+1}(1 + r_{J+1}(\theta, s)) w \\
V_{d,J+1}(w, \theta, s) &= \tilde{V}_{d,0,J+1}(s) + \tilde{V}_{1,J+1}(s) [\ln w + \ln(1 + r_{d,J+1}(\theta_h, s))] \\
c_{J+1}(w, \theta, s) &= \tilde{c}_{J+1}(1 + r_{d,J+1}(\theta_h, s)) w
\end{aligned}$$

where the coefficients  $\tilde{V}_{J+1}$  are determined by the recursive equation

$$\tilde{V}_{1,J+1}(s) = \gamma_1(s) + (1 - p_{ret})\beta \sum_{s'} \tilde{V}_{1,J+1}(s') \pi_{J+1}(s'|s) + p_{ret}\beta \sum_{s'} \tilde{V}_{1,ret}(s') \pi_{J+1}(s'|s)$$

and the coefficients  $\tilde{V}_{0,J+1}$  and  $\tilde{V}_{d,0,J+1}$  together with the optimal portfolio choices  $\theta_{J+1}^*$  are the solutions to the equation

$$\begin{aligned}
\theta_{J+1}^* &= \arg \max_{\theta_{J+1} \in \Gamma_{J+1}} \left\{ (1 - p_{ret}) \sum_{s'} \tilde{V}_{1,J+1}(s') \ln(1 + r_{J+1}(\theta_{J+1}, s')) \pi_{J+1}(s'|s) \right. \\
&\quad \left. + p_{ret} \sum_{s'} \tilde{V}_{1,ret}(s') \ln(1 + r_{J+1}(\theta_{J+1}, s')) \pi_{J+1}(s'|s) \right\} \quad (A8) \\
\Gamma_{J+1} &\doteq \left\{ \theta_{J+1} \left| \theta_{h,J+1} + \sum_{s'} \frac{\theta_{a,J+1}(s') \pi_{J+1}(s'|s)}{1 + r_f} = 1, \quad \theta_{h,J+1} \geq 0, \right. \right. \\
&\quad \frac{\tilde{V}_{0,J+1}(s') - \tilde{V}_{0d,J+1}(s')}{\tilde{V}_{1,J+1}(s')} \geq [\ln(1 + r_{d,J+1}(\theta_{J+1}, s')) - \ln(1 + r_{J+1}(\theta_{J+1}, s'))] \\
&\quad \left. \frac{\tilde{V}_{0,ret}(s') - \tilde{V}_{0d,ret}(s')}{\tilde{V}_{1,ret}(s')} \geq [\ln(1 + r_{d,ret}(\theta_{J+1}, s')) - \ln(1 + r_{ret}(\theta_{J+1}, s'))] \right\}
\end{aligned}$$

and

$$\begin{aligned}
\tilde{V}_{0,J+1}(s) &= \gamma_0(s) + \gamma_1(s) \ln(\tilde{c}_{J+1}(s)) \\
&\quad + (1 - p_{ret})\beta \sum_{s'} \tilde{V}_{0,J+1}(s') \pi_{J+1}(s'|s) \\
&\quad + (1 - p_{ret})\beta \sum_{s'} \tilde{V}_{1,J+1}(s') \ln(1 + r_{J+1}(\theta_{J+1}^*, s')) \pi_{J+1}(s'|s) \\
&\quad + (1 - p_{ret})\beta \ln(1 - \tilde{c}_{J+1}(s)) \sum_{s'} \tilde{V}_{1,J+1}(s') \pi_{J+1}(s'|s) \\
&\quad + p_{ret}\beta \sum_{s'} \tilde{V}_{0,ret}(s') \pi_{J+1}(s'|s) \\
&\quad + p_{ret}\beta \sum_{s'} \tilde{V}_{1,ret}(s') \ln(1 + r_{J+1}(\theta_{J+1}^*, s')) \pi_{J+1}(s'|s) \\
&\quad + p_{ret}\beta \ln(1 - \tilde{c}_{J+1}(s)) \sum_{s'} \tilde{V}_{1,ret}(s') \pi_{J+1}(s'|s)
\end{aligned}$$



$$\begin{aligned}
\tilde{V}_{d0,J+1}(s) &= \gamma_0(s) + \gamma_1(s) \log(\tilde{c}_{J+1}(s)) \\
&+ p\beta \sum_{s'} \tilde{V}_{d0,J+1}(s') \pi_{J+1}(s'|s) \\
&+ (1-p)\beta \sum_{s'} \tilde{V}_{0,J+1}(s') \pi_{J+1}(s'|s) \\
&+ \beta \sum_{s'} \tilde{V}_{1,J+1}(s') \log(1 + r_{d,J+1}(1, s')) \pi_{J+1}(s'|s) \\
&+ \beta \ln(1 - \tilde{c}_{J+1}(s)) \sum_{s'} \tilde{V}_{1,J+1}(s') \pi_{J+1}(s'|s)
\end{aligned}$$

We prove this conjecture as follows.

The sequential problem the household faces at the pre-retirement stage  $J+1$  is an infinite horizon problem with value function constraint, and the Bellman operator  $T$  associated with equation (A8) is monotone, but in general not a contraction mapping. However, adapting the argument made in Rusticchini (1998), the following result can be shown to hold in our setting:

**Lemma** Suppose that  $V_d$  and  $V^e$  are continuous functions. Suppose further that there is a unique continuous solution,  $V_0$ , to the Bellman equation without participation constraint. Let  $T$  stand for the operator associated with the Bellman equation. Consider the set of continuous functions  $B_W$  that are bounded in the weighted sup-norm  $\|V\| \doteq \sup_x |V(x)|/W(x)$ , where the weighting function  $W$  is given by  $W(x) = |L(x)| + |U(x)|$  with  $U$  an upper bound and  $L$  a lower bound, and endow this function space with the corresponding metric.<sup>1</sup> Then

- i)  $\lim_{n \rightarrow \infty} T^n V_0 = V_\infty$  exists and is the maximal solution to the Bellman equation (9)
- ii)  $V_\infty$  is the value function,  $V$ , of the sequential household maximization problem.

Notice first that a standard argument shows that the Bellman equation (A8) without participation constraint has a unique continuous solution,  $V_0$ . Guess-and-verify shows that this solution has the functional form (A7). Define  $V_n = T^n V_0$ . It is straightforward to show that if  $V_n$  has the functional form (A7), then the same is true for  $V_{n+1} = TV_n$ . From the lemma we know that  $V_\infty = \lim_{n \rightarrow \infty} T^n V_0$  exists and that it is the maximal solution to the Bellman equation (A8) as well as the value function of the corresponding sequential maximization problem (principle of optimality). Since the set of functions with this functional form is a closed subset of the set of continuous functions, we know that  $V_\infty$  has the functional form.

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<sup>1</sup>Thus,  $B_W$  is the set of all functions,  $V$ , with  $L(x) \leq V(x) \leq U(x)$  for all  $x \in X$ . For each particular application of the lemma, it has to be shown that this definition of the set of candidate value functions is without loss of generality for certain lower bound,  $L$ , and upper bound,  $U$ . In our case, the construction of the lower and upper bound is straightforward.

This proves that the conjecture is correct.

Finally, suppose that the exogenous state can be decomposed into two components,  $s = (s_1, s_2)$ , where  $s_1$  defines the family structure and  $s_2$  labor market risk. Assume further that  $s_2$  is i.i.d. It is straightforward to show from (A7) and (A8) that the i.i.d. component  $s_2$  does not affect choices  $\theta$  and  $\tilde{c}$  or value function coefficients  $\tilde{V}_0$  and  $\tilde{V}_1$ , that is, they are functions only of  $s_1$ . This completes the proof of proposition 1.

## A.2 Proof of proposition 2

From proposition 1 we know that individual households maximize utility subject to the budget constraint and participation constraint. Thus, it remains to derive the intensive-form market clearing condition and the stationarity condition determining  $\Omega$ .

Let  $\tilde{w}_j = (1 + r_j)w_j$  be the wealth of a household age  $j$  after all assets have paid off. The aggregate stock of human capital is

$$\begin{aligned}
H &= \sum_j E[\theta_{h,j+1}w_{j+1}]\pi_j & (A9) \\
&= \sum_j E[\theta_{h,j+1}(1 - \tilde{c}_j)(1 + r_j)w_j] \\
&= \sum_j \sum_{s_{1j}} E[\theta_{h,j+1}(1 - \tilde{c}_j)\tilde{w}_j|s_{1j}]\pi_j(s_{1j}) \\
&= \sum_j \sum_{s_{1j}} \theta_{h,j+1}(s_{1j})(1 - \tilde{c}_j(s_{1j}))E[\tilde{w}_j|s_{1j}]\pi_j(s_{1j}) \\
&= \tilde{W} \sum_{s_{1j}} \theta_{h,j+1}(s_{1j})(1 - \tilde{c}_j(s_{1j}))\Omega_j(s_{1j}) .
\end{aligned}$$

where  $\tilde{W} = \sum_j E[\tilde{w}_j]\pi_j$  is aggregate total wealth after assets have paid off. The second line in (A9) uses the equilibrium law of motion for the individual state variable  $w$ , the third line is simply the law of iterated expectations, the fourth line follows from the fact that the portfolio choices only depend on  $s_1$ , and the last line is a direct implication of the definition of  $\Omega$ . A similar expression holds for the aggregate stock of physical capital,  $K$ . Dividing the two expressions yields the intensive-form market clearing condition

$$\tilde{K} = \frac{\sum_{s_{1j}} (1 - \theta_{h,j+1}(s_{1j}))(1 - \tilde{c}_j(s_{1j}))\Omega_j(s_{1j})}{\phi \sum_{s_{1j}} \theta_{h,j+1}(s_{1j})(1 - \tilde{c}_j(s_{1j}))\Omega_j(s_{1j})} \quad (A10)$$

Define by  $\bar{r}_{j+1}(s_{1j}, s_{1,j+1})$  the expected investment return conditional on age and  $(s_{1j}, s_{1,j+1})$ . In stationary equilibrium the wealth distribution,  $\Omega$ , has to satisfy

$$\Omega_{j+1}(s_{1,j+1}) = \frac{E[\tilde{w}_{j+1}|s_{1,j+1}]\pi_{j+1}(s_{1,j+1})}{\sum_j \sum_{s_{1,j+1}} E[\tilde{w}_{j+1}|s_{1,j+1}]\pi_{j+1}(s_{1,j+1})} \quad (A11)$$

$$\begin{aligned}
&= \frac{E[(1+r_{j+1})(1-\tilde{c}_j)\tilde{w}_j|s_{1,j+1}]\pi_{j+1}(s_{1,j+1})}{\sum_j \sum_{s_{1,j+1}} E[(1+r_{j+1})(1-\tilde{c}_j)\tilde{w}_j|s_{1,j+1}]\pi_{j+1}(s_{1,j+1})} \\
&= \frac{\sum_{s_{1j}} E[(1+r_{j+1})(1-\tilde{c}_j)\tilde{w}_j|s_{1j}, s_{1,j+1}]\pi_j(s_{1j}|s_{1,j+1})\pi_{j+1}(s_{1,j+1})}{\sum_j \sum_{s_{1j}, s_{1,j+1}} E[(1+r_{j+1})(1-\tilde{c}_j)\tilde{w}_j|s_{1j}, s_{1,j+1}]\pi_j(s_{1j}|s_{1,j+1})\pi_{j+1}(s_{1,j+1})} \\
&= \frac{\sum_{s_{1j}} (1+\bar{r}_{j+1}(s_{1j}, s_{1,j+1}))\pi(s_{1,j+1}|s_{1j})(1-\tilde{c}_j(s_{1j}))E[\tilde{w}_j|s_{1j}]\pi_j(s_{1j})}{\sum_j \sum_{s_{1j}, s_{1,j+1}} \sum_{s_{1j}} (1+\bar{r}_{j+1}(s_{1j}, s_{1,j+1}))\pi(s_{1,j+1}|s_{1j})(1-\tilde{c}_j(s_{1j}))E[\tilde{w}_j|s_{1j}]\pi_j(s_{1j})} \\
&= \frac{\sum_{s_{1j}} (1+\bar{r}_{j+1}(s_{1j}, s_{1,j+1}))\pi(s_{1,j+1}|s_{1j})(1-\tilde{c}_j(s_{1j}))\Omega_j(s_{1j})}{\sum_j \sum_{s_{1j}, s_{1,j+1}} (1+\bar{r}_{j+1}(s_{1j}, s_{1,j+1}))\pi(s_{1,j+1}|s_{1j})(1-\tilde{c}_j(s_{1j}))\Omega_j(s_{1j})}
\end{aligned}$$

where the second line uses the equilibrium law of motion for the individual state variable  $x$ , the third line is simply the law of iterated expectations, the fourth line follows from the fact that portfolio choices only depend on  $s_1$  in conjunction with the definition of  $\bar{r}$ , and the last line is a direct implication of the definition of  $\Omega$ . This completes the proof of proposition 2.

### A.3 Proof of proposition 3

For each household age  $j$ , the solution of the household maximization problem determines the optimal portfolio choice  $\theta_j = (\theta_{hj}, \vec{\theta}_{aj})$ . Without loss of generality, assume that all households have some insurance in equilibrium, but not full insurance:  $\theta_{aj}(d) \neq \theta_{aj}(n)$  and  $\eta(d)\theta_{hj} \neq (\theta_{aj}(d) - E[\theta_{aj}])$ . In this case, for all age groups  $j$  the participation constraint binds if  $s = n$  and does not bind if  $s = d$ . If the participation does not bind, the consumption growth rate must be equal to  $1 + r_f$  with log-utility, which given the consumption rule (9) implies that the portfolio return in the bad state is equal to the risk-free rate. Adding the budget constraint, we find that the optimal portfolio choice,  $\theta_j$ , is determined by the following three equations:

$$\begin{aligned}
\theta_{hj} (1 + \phi r_h - \delta_h + \varphi_j - \eta(d)) + \theta_{aj}(d) &= 1 + r_f \quad (A12) \\
\theta_{hj} (1 + \phi r_h - \delta_h + \varphi_j - \eta(n)) + \theta_{aj}(n) &= e^{-(1-\beta)(\tilde{V}_j - \tilde{V}_{dj})} \theta_{hj} (1 + \phi r_h - \delta_h + \varphi_j - \eta(n)) \\
\theta_{hj} + \frac{\pi(d)\theta_{aj}(d)}{1 + r_f} + \frac{\pi(n)\theta_{aj}(n)}{1 + r_f} &= 1.
\end{aligned}$$

Suppose now that defaulting households keep access to financial markets:  $p = 0$ . In this case, we have  $\tilde{V}_j = \tilde{V}_{dj}$ , and from the third equation in (A12) it follows that  $\theta_{aj}(n) = 0$ . Further, solving for  $\theta_{hj}$  using  $\theta_{aj}(n) = 0$  yields:

$$\theta_{hj} = \frac{\pi(n)}{1 - \frac{\pi(d)}{1+r_f}(1 + \phi r_h - \delta_h + \varphi_j - \eta(d))} \quad (A13)$$

Clearly, equation (A13) shows that  $\theta_{hj} > \theta_{h,j+1}$  if  $\varphi_{hj} > \varphi_{h,j+1}$ . It further follows from

equation (A12) that the insurance pay-out is given by:

$$\theta_{aj}(b) - E[\theta_{aj}] = \pi(n) (1 + r_f - \theta_{hj}(1 + \phi r_h - \delta_h + \varphi_j - \eta(d))) . \quad (A14)$$

Using  $\theta_{hj} > \theta_{h,j+1}$ , it follows that  $\theta_{aj}(d) - E[\theta_{aj}] < \theta_{a,j+1}(d) - E[\theta_{a,j+1}]$ . This proves the first part of the proposition. A similar argument proves the second part of proposition 3.

## B Computation

For ages  $j = 1, 2, \dots, J$ , we solve the household problem backwards starting at  $j = J$ . The solution procedure is as follows:

**Step 1:** Find  $\tilde{V}_{1j}(\cdot)$  and  $\tilde{c}_j(\cdot)$  solving (A4)

**Step 2:** Find the optimal portfolio choice  $\theta_j$  for given  $\tilde{V}_{0,j+1}(\cdot)$  and  $\tilde{V}_{d0,j+1}(\cdot)$  using (A5)

1. Pick a current family structure  $s_{1j}$ .
2. Pick a human capital choice,  $\theta_{h,j+1}$ .
3. Pick a future family structure  $s_{1j+1}$ .
4. Order the states  $s_{2j+1}$  according to the size of the human capital shock  $\eta$ . Pick a partition  $S \equiv S_1 \cup S_2$ , where  $S_1 = \{1, \dots, n\}$  and  $S_2 = \{n+1, \dots, N\}$  with  $N$  being the number of states  $s_{2j+1}$ .
5. For given  $(s_{1j}, s_{1,j+1})$ , and human capital choice  $\theta_{h,j+1}$ , we find the asset portfolio,  $\theta_{a,j+1}(\cdot)$ , by

(a) Use participation constraint for all  $s_{2j+1} \in S_1$ :

$$\exp \left( \frac{1}{\tilde{V}_{1,j+1}(s_{1j+1})} \left( \tilde{V}_{0,j+1}(s_{1j+1}) - \tilde{V}_{d0,j+1}(s_{1j+1}) \right) \right) ((1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1}))\theta_{h,j+1} + \theta_{a,j+1}(s_{1j+1}, s_{2j+1})) \\ = (1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1}))\theta_{h,j+1}$$

(b) Equalization of IMRS for all  $s_{2j+1} \in S_2$  :  $\frac{u'(c_j, s_j)}{u'(c_{j+1}, s_{j+1})} = \beta(1 + r_f)$ .

Using our utility function this reads  $\frac{c_{j+1}}{c_j} = \frac{\gamma_1(s_{1j+1})}{\gamma_1(s_{1j})}\beta(1 + r_f)$ . Using our consumption policy function, we find  $\frac{c_{j+1}}{c_j} = \frac{\tilde{c}_{j+1}}{\tilde{c}_j}(1 - \tilde{c}_j)(1 + r_{j+1})$ . Further using  $\tilde{c}_j = \frac{\gamma_{1j}}{\tilde{V}_{1j}}$  we arrive at the following condition for all  $s_{2j+1} \in S_2$ :

$$\frac{\tilde{V}_{1,j}(s_{1j}) - \gamma_{1,j}(s_{1j})}{\tilde{V}_{1,j+1}(s_{1,j+1})} ((1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1}))\theta_{h,j+1} + \theta_{a,j+1}(s_{1j+1}, s_{2j+1})) = \beta(1 + r_f)$$

Thus, we have

$$\theta_{a,j+1}(s_{1j+1}, s_{2j+1}) = -(1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1}))\theta_{h,j+1} \left( 1 - \exp \left( \frac{\tilde{V}_{0d,j+1}(s_{1j+1}) - \tilde{V}_{0,j+1}(s_{1j+1})}{\tilde{V}_{1,j+1}(s_{1j+1})} \right) \right) \\ \forall s_{2j+1} \in S_1$$

$$\theta_{a,j+1}(s_{1j+1}, s_{2j+1}) = \frac{\tilde{V}_{1,j+1}(s_{1j+1})}{\tilde{V}_{1,j}(s_{1j}) - \gamma_{1,j}(s_{1j})} \beta(1 + r_f) - (1 + r_{hj}(s_{1j}, s_{1j+1}, s_{2j+1}))\theta_{h,j+1} \\ \forall s_{2j+1} \in S_2$$

6. Do this for all  $s_{1j+1}$

For given current family structure  $s_{1j}$ , find the portfolio vector  $(\theta_{h,j+1}, \theta_{a,j+1})$  that "solves" the portfolio constraint. This is our optimal portfolio for given  $s_{1j}$ .

7. Do this for all current family structures  $s_{1j}$ .

**Step 3:** Find  $\tilde{V}_{0j}(\cdot)$  and  $\tilde{V}_{d0,j}(\cdot)$  using (A5)

The household problem for  $j = J+1$  we solve as above, but now we drop the  $j$ -dependence and solve the corresponding fixed point problem.

## C Survey of Consumer Finance Data

The data are for the years 1992, 1995, 1998, 2001, 2004, and 2007 drawn from the Survey of Consumer Finances (SCF) provided by the Federal Reserve Board. The Survey collects information on a number of economic and financial variables of individual families through triennial interviews, where the definition of a "family" in the SCF comes close to the concept of a "household" used by the U.S. Census Bureau. See Kennickell and Starr-McCluer (1994) for details about the SCF.

For the sample selection, we follow as closely as possible Heathcote et al. (2010)<sup>2</sup>. We restrict the sample to households where the household head is between 23 and 60 years of age. We drop the wealthiest 1.46% of households in each year. Heathcote et al. (2010) show that this step makes the sample more comparable to that of the Panel Study of Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX). We drop all households that report negative labor income or that report positive hours worked but have missing labor income or that report positive labor income but zero or negative hours worked. We compute the average wage by dividing labor income by total hours worked, and drop in each year households with a wage that is below half the minimum wage of the respective year. For the data on life-insurance, we restrict the sample further to households that are married or live with a partner.

For the definition of variables we follow Kennickell and Starr-McCluer (1994). We only depart from their variable definitions when considering labor income, where we follow Heathcote et al. (2010) and add two-thirds of the farm and business income as additional labor income. As common in the literature, we associate financial wealth in the model with net worth in the SCF. Households' net worth includes the cash value of life-insurance as in Kennickell and Starr-McCluer (1994), but does not include the face value of insurance contracts. We associate life-insurance in the model with the face value of life-insurance from the data. All data has been deflated using the BLS consumer price index for urban consumers (CPI-U-RS). A detailed description of the relevant variables is as follows:

- **Assets** are the sum of financial and non-financial assets. The main categories of non-financial assets are cars, housing, real estate, and the net value of businesses where the household holds an active interest. Except for businesses all values are gross positions, i.e. before outstanding debt. The main categories of financial assets are liquid assets, CD, mutual funds, stocks, bonds, cash value of life-insurance, other managed investment, and assets in retirement accounts (e.g. IRAs, thrift accounts, and pensions accumulated in accounts.)
- **Debt** is the sum of housing debt (e.g. mortgages, home equity loans, home equity lines of credit), credit card debt, installment loans (e.g. cars, education, others), other residential debt, and other debt (e.g. pension loans).
- **Net-worth** is the sum of all assets minus all debt.

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<sup>2</sup>We use their Sample B for our analysis.

- **Labor income** is wages and salaries plus 2/3 of business and farm income.
- **Life-insurance** is the face value of all term life policies and the face value of all policies that build up a cash value. The cash value is not part of the life-insurance, but is part of the financial assets of an household.

## D Demographic Transitions

### D.1 Construction of Family Transition Matrix

We construct the stochastic matrix describing the transition of households over family states  $s_1$  as follows. We proceed in two steps. In the first step, we construct the transition function for marital states and in the second state we construct the transition matrix for the number of kids for each marital state. Age subscripts are dropped for convenience.

#### D.1.1 Marital States

There are in total 5 marital states: Married ( $ma$ ), female widowed ( $fw$ ), female single and not widowed ( $fn$ ), male widowed ( $mw$ ), and male single and not widowed ( $mn$ ). We stack family states in a vector  $x = \{ma, fw, fn, mw, mn\}$  and construct transition matrix  $\Pi$ . The transition matrix follows the conventional structure with initial states in rows and terminal states in columns. The order of states is given by the order of  $x$ . We set all transition rates between sexes to zero.

$$\Pi = \begin{pmatrix} \pi(ma, ma) & \pi(ma, fw) & \pi(ma, fn) & \pi(ma, mw) & \pi(ma, mn) \\ \pi(fw, ma) & \pi(fw, fw) & 0 & 0 & 0 \\ \pi(fn, ma) & 0 & \pi(fn, fn) & 0 & 0 \\ \pi(mw, ma) & 0 & 0 & \pi(mw, mw) & 0 \\ \pi(mn, ma) & 0 & 0 & 0 & \pi(mn, mn) \end{pmatrix}$$

For a married household, the transition probabilities  $\pi(ma, fw)$  and  $\pi(ma, mw)$  are computed using the life tables for males, respectively females. We interpret the transition from married household to female single non-widowed, respectively male single non-widowed, as divorce. We assume that the female is the decision maker in a married household and that after divorce the woman does not care about the well-being of the male, which is equivalent to setting transition probability from married to single male non-widowed to zero in the household decision problem:  $\pi(ma, mn) = 0$ .<sup>3</sup> The probability to stay married is determined as the residual  $\pi(ma, ma) = 1 - \pi(ma, fw) - \pi(ma, mw) - \pi(ma, fn)$ .

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<sup>3</sup>For the law of motion of the model distribution over family states, we adjust these transition probabilities to account for the fact that there are two new households, one  $fn$  and one  $mn$ .

For male and female widowed household, we assume that they either re-marry with probability  $\pi(mw, ma)$  and  $\pi(fw, ma)$ , respectively, or stay widowed with probability  $\pi(mw, mw)$ , respectively  $\pi(fw, fw)$ . Similarly, male and female single, non-widowed households can either marry with probability  $\pi(mn, ma)$  and  $\pi(fn, ma)$ , respectively, or stay single with probabilities  $\pi(mn, mn)$  and  $\pi(fn, fn)$ .

### D.1.2 Children

We consider 4 different states for the number of kids in the household: no kids, 1 kid, 2, kids, 3 kids (or more). The number of kids increases by one in the case of the birth of a child and decreases by one in the case that a child leaves the household (moves out). The number of children also changes if households marry, in which case the kids of the two marrying households are combined.

We distinguish between the fertility rate of a married woman and the fertility rate of a single woman, but because of data scarcity assume that widowed woman and non-widowed women have the same fertility rates. Similarly, we distinguish between moving-out rates of children for married households and moving-out rates for single households. Denote the probability that a married household increases/decreases the number of kids by one by  $\pi(ma, +1)$  and  $\pi(ma, -1)$  and the corresponding transition probability for a female single household by  $\pi(f, +1)$  and  $\pi(f, -1)$ . For married households, the transition rates for the number of kids are then summarized by the transition matrix

$$T_{ma} = \begin{pmatrix} 1 - \pi(ma, +1) & \pi(ma, +1) & 0 & 0 \\ \pi(ma, -1) & 1 - \pi(ma, +1) - \pi(ma, -1) & \pi(ma, +1) & 0 \\ 0 & \pi_m(m) & 1 - \pi_f(m) - \pi_m(m) & \pi_f(m) \\ 0 & 0 & \pi(ma, -1) & 1 - \pi(ma, +1) \end{pmatrix}$$

Similarly, for single female households who do not re-marry the transition rates for the number of kids are summarized by the transition matrix

$$T_f = \begin{pmatrix} 1 - \pi(f, +1) & \pi(f, +1) & 0 & 0 \\ \pi(f, -1) & 1 - \pi(f, +1) - \pi(f, -1) & \pi(f, +1) & 0 \\ 0 & \pi(f, -1) & 1 - \pi(f, +1) - \pi(f, -1) & \pi(f, +1) \\ 0 & 0 & \pi(f, -1) & 1 - \pi(f, -1) \end{pmatrix}$$

For male single households who do not marry the number of kids cannot increase, but can decrease by one due to moving out. If we denote the moving out rate by  $\pi(m, -1)$ , the



transition matrix for male single households who do not marry reads:

$$T_m = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \pi(m, -1) & 1 - \pi(m, -1) & 0 & 0 \\ 0 & \pi(m, -1) & 1 - \pi(m, -1) & 0 \\ 0 & 0 & \pi(m, -1) & 1 - \pi(m, -1) \end{pmatrix}$$

Finally, there is the event that a single female household and a single male household get married and the kids are combined. In this case, the transition matrix is for female single households is

$$T_{f,ma} = \begin{pmatrix} \mu_0 & \mu_{1f} & \mu_{2f} & 1 - \mu_0 - \mu_{1f} - \mu_{2f} \\ 0 & \mu_0 & \mu_1 & 1 - \mu_0 - \mu_1 \\ 0 & 0 & \mu_{0f} & 1 - \mu_{0f} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where  $\mu_{if}$  denotes the probability that a female and a male marry and the combined number of kids is  $i$ . A similar transition matrix describes the transition rates for male single households, which we denote by  $T_{m,ma}$

Combining the transition matrices for marital status and the number of kids results in the joint transition matrix for family states:

$$\Pi \otimes T = \begin{pmatrix} \pi(ma, ma)T_1 & \pi(ma, fw)T_1 & \pi(ma, fn)T_1 & \pi(ma, mw)T_1 & \pi(ma, mn)T_0 \\ \pi(fw, ma)T_{f,ma} & \pi(fw, fw)T_2 & \pi(fw, fw)T_2 & 0 & 0 \\ \pi(fn, ma)T_{f,ma} & \pi(fn, fn)T_2 & \pi(fn, fw)T_2 & 0 & 0 \\ \pi(mw, ma)T_{f,ma} & 0 & 0 & \pi(mw, mw)T_3 & \pi(mw, mw)T_0 \\ \pi(mn, ma)T_{f,ma} & 0 & 0 & \pi(mn, mn)T_0 & \pi(mn, mn)T_0 \end{pmatrix}$$

where  $T_0$  is the transition matrix to zero kids in the next period independent of the current number of kids today and  $T_{4,f}$  and  $T_{4,m}$  denote the respective transition matrices for females and males.

## D.2 Calibration of Family Transition Matrix

In this section, we describe how we estimate transition probabilities between family states from the data. The data are the core files of waves 1 to 9 and the wave 2 fertility history topical module from the 2001 panel of the Survey of Income and Program Participation (SIPP). The death probabilities for males and females are constructed using death probabilities from the life tables published by the Human Mortality Database (HMD).

### D.2.1 SIPP Data

We use data from the 2001 Panel of the Survey of Income and Program Participation (SIPP) to get estimates of transition probabilities between family states, as well as for information on employer-provided life insurance and the within household split of life insurance between husband and wife. After the National Center for Health Statistics (NCHS) stopped publishing detailed data on marriage and divorce in 1990, the SIPP has become the primary data source for marital history information (See Kreider and Fields 2001 for details). Death probabilities for males and females are not derived using the SIPP but are taken from the life tables published by the Human Mortality Database (HMD 2011).

The SIPP is conducted by the Census Bureau. The 2001 Panel collects data on roughly 35,100 households that are representative of the U.S. non-institutionalized population. It collects information on demographic characteristics, marital status, household relationship, and education. It also collects data on labor market activity, income, and participation in benefit programs. In addition, there are topical modules that provide information on specific topics. We use data from interviews conducted between February 2001 and January 2004. A household in the panel is interviewed every 4 months and each household has 9 interviews in total over the survey period. At each interview, information for the 4 months preceding the interview is collected. If household members leave the household, they stay in the sample. Each interview is referred to as a wave. Each wave is divided into 4 rotation groups so that each month roughly a fourth of the households are interviewed. There is a set of questions that is asked for each month covered by the survey. This is the information contained in the core modules. This data is supplemented by data from topical modules. We rely on wave 3 topical module for the data on life-insurance and on wave 2 fertility history topical module for transitions in the number of children in the household. We merge data from the core files of waves 1 to 9 to create a panel of marital status histories.

We restrict the sample to reference persons and their spouses to get a sample of household heads comparable to the Survey of Consumer Finances. We label persons as married that report being married with the spouse present or absent.<sup>4</sup> We label persons as widowed following the coding in the data, and label all other single persons as not widowed. This last status includes divorced, separated, and never married. For each individual, we assign an age-specific marital status using the marital status the person had for the longest period of each age. We derive age-specific transition rates by computing the share of individuals who

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<sup>4</sup>The SIPP does not have a marital state “living with partner” as in the Survey of Consumer Finances (SCF).

change their marital status with age using the panel dimension of the data. The transition rates are computed for 5-year age bins. The first bin covers ages 21–25 and the last bin ages 58–62. The mid point of the bin is taken as point in the age profile to which the transition rate is assigned. We regress the raw data on a fourth order polynomial in age. We use the estimated profile as input to our model. If estimated transition rates are negative, we set them to zero.

### D.2.2 Remarriage Rates

We derive remarriage rates for divorced households and widowed households separately. The small sample size of widows at young ages leads to noisy estimates. We therefore impute remarriage rates for young widows by scaling the remarriage profile of divorced households. We use a pooled sample of widows from age 30 – 50 and compare it to a sample of divorcees. We find that remarriage rates for widows are 44% of remarriage rates of divorcees at these ages. We use this scaling factor to impute remarriage rates for widows.<sup>5</sup> This shifting factor is very close to a corresponding shifting factor found in the NCHS data for 1988 by Wilson and Clarke (1992). Their data does not suffer from a small sample problem as they observe 77,000 widowed women who remarry out of a population of 12.3 million female widows.<sup>6</sup> Using Table 2 from their paper, we find that female widows aged 25 – 54 have remarriage rates that are 47% of that of divorcees of the same age.<sup>7</sup> The difference in remarriage rates is not driven by a different age composition of the two samples. Wilson and Clarke (1992) report remarriage rates broken down to smaller age groups and the pattern is very stable across these groups. For age group 25 – 29 remarriage rates for widows are 44.9% of the remarriage rates of divorcees, for age group 30–34 the number is 46.5%, for age group 35–44 it is 44.4%, and for age group 45 – 54 it is 50.7%.<sup>8</sup> Similar results can be found in the report by Norton and Miller (1990) that uses the 1985 marriage and fertility history supplement to the Current Population Survey (CPS). They report median duration completed time in divorce and widowhood for persons who remarry. Although this is a selected subsample of

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<sup>5</sup>If we look over the age range 23 – 61 remarriage rates for widows are 42% of the average rate of divorcees and for the a pooled sample from age 40 – 60 remarriage rates for widows are 63% of the rates for divorcees.

<sup>6</sup>They also report numbers for males but male widows are only a small fraction of all widows (17 %).

<sup>7</sup>The number for male widows is 68%.

<sup>8</sup>They also report data for 1980 and very similar pattern persist. However, comparing the remarriage rates from the two years shows that there are strong trends in remarriage rates over time so that the remarriage rates from their paper would substantially overstate remarriage rates in the period to which our model is calibrated to.

widows and divorcees, they report similar differences. The median duration of widows is almost twice as large as for divorcees for persons 45 years and younger.<sup>9</sup> Hong and Ríos-Rull (2012) also find lower remarriage rates for widows using data from the Panel Study of Income Dynamics (PSID) but do not report specific figures in their online appendix.

Figure A1 shows remarriage rates calculated using data from the SIPP for both divorcees and widows (male and female). The remarriage rates of widows are depicted by blue diamonds. As shown in the figure, the rates for widows aged less than 30 are missing, due to their absence from the sample. Even after age 30, the rates fluctuate wildly, at around 4% at the beginning of the 30's and rising higher than the levels observed for divorcees in their late 30s. This fluctuation is, however, driven by very few observations. The red dots show the rates for divorcees, and the red dotted line shows the smoothed version of these data that we use in the model. The blue solid line is our adjusted remarriage rate for widows. As can be seen, this line does a good job matching the remarriage rates of people in the 40s, for which we have more data.

### D.2.3 Fertility Rates

We use the wave 2 fertility history topical module to derive fertility rates by age. This module has information on the year of birth of the last child.<sup>10</sup> We assign a birth event to a women if there is at most one year difference between the current calendar year (2001 in our case) and the year of birth of the child. We adjust the age of the mother by one year if the year of birth was in the previous calendar year. The age-specific fertility rate is the share of females at each age that had a birth event. Given that the period during which the child could have been born covers two calendar years, we adjust rates to a one-year lifespan. The fertility rates for each age are computed for centered 5-year age bins. We regress the transition rate data on a fourth order polynomial in age and use the estimated profile as the input to our model. In line with observed fertility rates, we set fertility from age 45 onwards to zero. We derive separate fertility rates for single  $\pi_f(s)$  and married woman  $\pi_f(m)$ . We use marital status information from the fourth interview of the second wave when the question of the topical module are asked. The results are depicted in figure A3.

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<sup>9</sup>They only report the median time to remarriage for the pooled group of widows younger than 45 years (approximately 3.9 years). For divorcés of the different subgroups the duration below age 45 is very similar (approximately 2.3 years.)

<sup>10</sup>The month information is suppressed for confidentiality in the public use files.

### D.2.4 Moving Out Rates

For the calibration of the probability that children move out of the household, we restrict the sample to those households who have at least one household member who has information at all 9 waves. We do this to avoid the underestimation of moving out rates due to sample attrition. In line with our model, we consider as children those children of the reference person that are less than 23 years of age.<sup>11</sup> A child “moves out” of the household if a person that has been a child of the reference person is no longer reported as residing at home, or if that child turns 23 of age. The moving out rates are computed for 5-year age bins using the age of the mother. We use the mother’s age to be consistent across married and single divorced households. We regress the transition rate data on a fourth order polynomial in age. We use the estimated profile as input to our model.<sup>12</sup> We derive separate moving out rates for single  $\pi_m(s)$  and married households  $\pi_m(m)$ . As there are very few single fathers in the sample, moving out rates for single families are not distinguished by the sex of the household head. The results are shown in figure A4.

### D.2.5 Death probabilities

The probability that a household member dies is taken from the life tables of the Human Mortality Database (HMD 2011). We use averages of death probabilities separately for males and females for the period 1990 - 2007. Figure A5 shows the life-cycle profile of the death probabilities for males and females.

### D.2.6 Initial Distribution

To derive the initial distribution over family states, we use reference persons and their spouses. We assign each person the family status from the fourth interview in wave 2 (see fertility rates above). The definition for children is as in the case of the moving out rates. We consider all persons age 21 to 25 for the initial distribution (the 5-year bin around age 23).

### D.2.7 Consistency

To check the consistency of the estimated family transition matrix with the observed cross-sectional distribution over family states, we have computed various life-cycle profiles derived from the estimated transition matrix and initial distribution. Overall, the deviations between

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<sup>11</sup>In contrast, the SIPP counts as children (variable RFNKIDS) all children in the household under age 18 including grandchildren or children of household members other than the reference person and its spouse.

<sup>12</sup>If estimated transition rates are negative, we set them to zero.

implied cross-sectional distributions and empirical distributions are small. This provides evidence in support for our calibration strategy.

## E Survivor Benefits and Taxes

### E.1 Survivor Benefits

Suppose death of an adult household member occurs at age  $j$ . For each age  $k > j$ , we can compute a social security survivor benefit for a median-income widowed household,  $B_{j,k}$  that depends on the number of children,  $n$ . We compute this benefit as follows:

- **Step 1:** For each  $j$  and  $n$ , compute Average Indexed Monthly Earnings (AIME) as

$$AIME_{j,n} = \frac{1}{j-20} \sum_{i=20}^j \nu_{i,n} y_{i,n}^m$$

where  $y_{i,n}^m$  is the median labor income of a married household with  $n$  kids age  $i$  and  $0 < \nu_{i,n} < 1$  is a weight that measures the fraction of household earnings that has been generated by the deceased household member. We use the difference for households with  $n$  kids between married and a single households of the respective gender to proxy this fraction. We assume that households' first year of full earnings is at age 20 and further assume  $y_{n,20}^m = y_{21,n}^m = y_{22,n}^m = y_{23,n}^m$ .

- **Step 2:** Compute the Primary Insurance Amount (PIA) as

$$PIA_{j,n} = 0.9 * \min\{b_1, AIME_{j,n}\} + 0.32 * \min\{b_2, \max\{AIME_{j,n} - b_1, 0\}\} + 0.15 * \max\{AIME_{j,n} - b_2, 0\}$$

For the “bend points”  $b_1$ ,  $b_2$ , and  $b_3$  we use the official bend points in the year 2000.

- **Step 3:** Compute the maximum family benefit,  $B_{j,n,max}$

We have

$$\begin{aligned} B_{j,n,max} = & 1.5 * \min\{b_1^f, PIA_{j,n}\} + 2.72 * \min\{b_2^f, \max\{PIA_{j,n} - b_1^f, 0\}\} \\ & + 1.34 * \min\{b_3^f, \max\{PIA_{j,n} - b_2^f, 0\}\} + 1.75 * \max\{PIA_{j,n} - b_3^f, 0\} \end{aligned}$$

As bend points  $b_1^f$ ,  $b_2^f$ , and  $b_3^f$  we use again the official bend points in the year 2000.

- **Step 4:** Compute potential benefits,  $\tilde{B}_{j,n,k}$ :

Table A1: Tax rates for 2000

Marginal Tax Rate	Married Filing Jointly		Single	
	Tax Brackets		Tax Brackets	
	Over	Below	Over	Below
15.0%	\$0	\$43,850	\$0	\$26,250
28.0%	\$43,850	\$105,950	\$26,250	\$63,550
31.0%	\$105,950	\$161,450	\$63,550	\$132,600
36.0%	\$161,450	\$288,350	\$132,600	\$288,350
39.6%	\$288,350	—	\$288,350	—

The amount of benefits the surviving household members can potentially receive is

$$\tilde{B}_{j,n,k} = \max(0, I(n > 0) * 0.75 * PIA_{j,n} - \max\{0.5(y_{j,n,k}^s - \tau), 0\} + 0.75 * n * PIA_{j,n})$$

where  $y_{j,n,k}^s$  is the labor income of the surviving spouse at age  $k$ ,  $n$  is the number of surviving children, and  $\tau$  is fixed threshold for income from where on deduction lead to a phase out of benefits.  $I(n > 0)$  denotes an indicator if there are children in the household. We set the value  $\tau$  equal to the official threshold for the year 2000.

- **Step 5:** Compute the actual benefit  $\hat{B}_{j,k}$

The actual benefit for the surviving family members are

$$\hat{B}_{j,n,k} = \min(\tilde{B}_{j,n,k}, B_{j,n,max})$$

- **Step 6:** To get the benefits  $B_{j,n,k}$  paid out to households, we subtract income taxes that have to be paid on benefits  $\tau_{j,n,k}^B$  by comparing income taxes of a household with benefits to a household without benefits. We include the income tax advantage of benefits that benefits are only taxed to 50%. We subtract the additional taxes that have to be paid on benefits from the benefits. The benefits paid out to the surviving family members are

$$B_{j,n,k} = \max(0, \hat{B}_{j,n,k} - \tau_{j,n,k}^B)$$

## E.2 Payroll and Social Security Taxes

We compute the average tax rate for a median-income household using estimated earnings profiles for married households and single households. We compute federal taxes with standard deductions taking into account deductions and tax credits for children. We use nominal

tax brackets for the year 2000 (which is consistent with using real data in year 2000 dollars) to compute average tax rates.<sup>13</sup> The rates vary according to the filing status of the household. For 2000, the U.S. income tax brackets and marginal tax rates are given in Table A1.

The child tax credit was introduced in 1997 for tax year 1998. In 2000, it was \$500 per qualifying child (under age 17). There is a means test for the credit. From the 1997 law, which was in force in 2000, the reduction was \$50 per \$1000 over the threshold of \$110,000 for married filing jointly, and \$75,000 for non married individuals.

The numbers for the personal exemption for married couples, single people, and per dependent for 2000 are 5,600, 2,800, and 2,800. That is, in 2000, a married household filing jointly could claim \$5600 plus an extra \$2800 per dependent.

The social security tax and medicare tax paid by the employee was 6.2% and 1.45%, respectively. We add these taxes to the federal income tax to arrive at a total average tax rate.

## F Employer-Provided Life Insurance

Here we address the issue to what extent the existence of employer-provided group insurance has the potential to distort our results. If the amount of group insurance offered by the employer exceeds the amount households want to hold, then these households are “involuntarily” over-insured and the insurance holdings observed in the data are not the outcome of the optimal insurance choice by households. Clearly, the phenomenon of involuntary over-insurance can only occur for households who have not purchased any individual life insurance from insurance companies. Although the SCF does not distinguish between group insurance and insurance purchased individually, we can use data on employer provided life insurance from the Survey of Income and Program Participation (SIPP) to analyze this issue. Figure A6 shows the median life insurance holding of married households with children who have purchased some life insurance, and also the holdings of employer-provided life insurance for the same group of households. The figure shows that for each age between 23 and 60, the median household with kids holds substantially more life insurance than the amount of insurance provided by the employer. Further, for the median household the amount of employer-provided life insurance is roughly constant over the life-cycle and the shape of the life-cycle profile of total (group plus individual) life insurance holdings is therefore not much

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<sup>13</sup>Using data from The Tax Foundation: <http://taxfoundation.org/article/us-federal-individual-income-tax-rates-history-1913-2011-nominal-and-inflation-adjusted-brackets>



affected by the presence of group life insurance. Thus, we conclude that the consideration of insurance purchases as voluntary is appropriate to a first approximation. Hong and Rios-Rull (2012) come to a similar conclusion after analyzing data drawn from the International Survey of Consumer Financial Decisions.

## G Model Extensions

### G.1 Child- and Health-Dependent Preferences

Two recent papers (Kojen et al. 2012 and Hong and Rios-Rull 2012) have used data on life insurance holdings, and holdings of other assets, to estimate the evolution of household preferences as they age and decline in health, the strength of the bequest motive, as well as the effect of changes in household size on the cost of living. In both of these papers, patterns in life insurance data are assumed to be driven by variation in preferences and cost of living parameters, in contrast to our paper where under-insurance of young households is generated through borrowing constraints.<sup>14</sup> Motivated by these papers, in this section we consider an extension of our model that incorporates household preferences that depend on the number of children and the health status of the household. To simplify the discussion, we focus here on a model without the additional mortality heterogeneity, that is, we focus on households with median level of mortality risk.

We introduce two changes to our baseline model. First, we parameterize the change in the marginal utility of consumption of a household following the death of a spouse, with the parameter varying with the number of children in the household. This may be interpreted as capturing changes in the cost of living (for example, if it is cheaper to live with a smaller household, the marginal utility should decline) beyond the simple insurance component accounted for in our baseline model, or as capturing the strength of the bequest motive for younger households. In terms of the model, we allow the marginal utility of consumption,  $\gamma_1$ , to change following the death of an adult household member or divorce, and assume that the size of the change may vary with the number of children. To reduce the number of free parameters, we assume that  $\gamma_1$  is the same for all married households independently of the number of children and that for single households  $\gamma_1$  is independent of sex (male/female) and marital status (divorcee/widow). We normalize  $\gamma_1$  of married households to one and choose the value of the remaining parameters to match the life-cycle average of life insurance

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<sup>14</sup>Kojen et al. (2012) study a complete-market model without financial frictions, which implies that under-insurance cannot occur. Hong and Rios-Rull (2012) use an incomplete-market model with ad-hoc borrowing constraint, but neither missing insurance markets nor binding borrowing constraints play an important role in their analysis.

holdings of married households with different number of children separately.

The second change to the model specification is the introduction of a health state. Following Koijen et al (2012), we assume that households can be either in good health or in bad health and that households in bad health have lower marginal utility of consumption,  $\gamma_1$ . Further, a married household in bad health who experiences the death of an adult household member becomes a healthy single household. This assumption captures the idea that it is the sick member of the household who dies, an assumption that seems plausible especially for older households. Finally, we assume that up to age 35 all households are in good health and that starting age 35 the probability of becoming sick (moving into bad health) increases linearly. Thus, we have parameterized the health process by two parameters, and we calibrate these two parameters to match two targets taken from Koijen et al. (2012): (i) the relative number of households who move from self-reported good health into self-reported bad health in the age group 50 – 60; and, (ii) the difference in the demand for life insurance between bad health and good health households ages 50 – 60.<sup>15</sup>

Our results can be summarized as follows. First, with these changes to the model, the basic facts about life insurance and other asset holdings over the life-cycle for all married households with children are unchanged. For example, the model's prediction for the median life insurance holdings of households with children is barely affected by this change in model specification. In figure A11 we plot the life-cycle profile of life insurance holdings for all married household with children, and find that it is very close to the plot for our baseline model (figure 6). Second, this extension improves the match between model and data in the sense that the extended model replicates additional cross-sectional facts. Specifically, households in bad health demand more life insurance than households in good health and households with two and three children hold substantially more life insurance than households with one child. In particular, the extended model implies that, consistent with the data, moving from one child to two children increases the bequest motive by an amount that is equal to \$25,000 of life insurance, while moving from two children to three children increases life insurance holdings by \$10,000. Third, if we interpret the change in marginal utility following the death of a parent as reflecting the consequent change in the cost of living, the resulting changes are relatively modest and increase in the number of kids: the cost of living falls following the death of a spouse by roughly 4% for households with either no kids or 1 kid, rises 2%

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<sup>15</sup>Koijen et al. 2012 report that 20 percent of all households age 50 – 60 move from good health to bad health. In an early working paper version, they also report the results of a regression that shows that moving from good health to poor health adds about 50,000 of life insurance controlling for age and other explanatory variables.

of households with 2 kids, and rises by 3% for households with four kids. Equivalently, this may be interpreted as the strength of the bequest motive for young households rising with the number of kids.

## G.2 Annuities, Life Insurance and Bequests

In our baseline model, prior to retirement all agents can buy a complete set of insurance products, including both life insurance and annuities. However, we constrain retirees to save in a risk free security with any wealth remaining at their death distributed to newborn households. In the absence of this constraint, and without a bequest motive, retirees would only purchase annuities. We briefly discuss a variant of our model in which retirees have a bequest motive and are able to buy annuities.

Suppose that retired households preferences are augmented with a bequest motive in the form of an additive utility term of the form  $v(b, s) = \kappa * u(b, s)$  where  $u(b, s)$  is the utility function of a household,  $b$  are bequests and  $\kappa$  governs the strength of the bequest motive. Note that under this assumption, the homotheticity properties of the model are preserved. If annuities are priced in an actuarially fair manner, and if  $\kappa$  is chosen so that the marginal utility of a unit of bequests equals the marginal utility of a unit of annuity wealth, then retirees will choose a level of bequests that equals their annuity wealth. This may be implemented by a portfolio with equal holdings of annuities and life insurance, which is equivalent to the restriction imposed in our baseline model<sup>16</sup>.

This turns out to be a not unreasonable description of the data. Although Johnson, Burman and Kobes (2004) estimate that people aged 65 and older hold on average just 1% of their wealth in *private* annuities, Gustman, Mitchell, Samwick and Steinmeier (1997) estimate that people aged 51-61 hold between one quarter and one half of their wealth in annuity-like pensions and social security. Thus, we conclude that our restriction on retirees portfolio choices is relatively innocuous.

## G.3 Comparison with Incomplete Market Model

In this section, we consider an incomplete-market model in which households may borrow and save using a risk-free asset subject to an exogenously imposed borrowing constraint, and may purchase life insurance, but are exogenously prohibited from accessing other financial assets. In other words, we consider the standard incomplete market model augmented by a

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<sup>16</sup>These decisions may also be implemented by holding equal amounts of life insurance and annuities with the remainder of their wealth in a risk free asset. Note that we are abstracting from the fact that life insurance can be used to avoid gift and inheritance taxes.

life insurance contract. This class of models has been used in Hong and Rios-Rull (2012) to analyze how age-dependent household preferences shape the pattern of life insurance holdings over the life cycle. In principle this class of models can also provide an explanation of the observed positive correlation between age and insurance that is solely based on binding borrowing constraints. In this model, young households expect higher future earnings growth than older households and therefore want to borrow more than older households to smooth consumption, but might be prohibited from borrowing if the exogenous borrowing constraints are too tight. In this case, younger households also buy less life insurance than older households – a positive correlation between age and insurance emerges in equilibrium.

We now provide evidence that the incomplete-market model with two assets we described above cannot explain the empirical pattern of under-insurance without age-dependent preferences or some friction in addition to borrowing constraints. To see this, note that in this model the ad hoc borrowing constraints only generates a positive correlation between age and insurance for households with negative net financial wealth. By contrast, in our theory, households may hold positive net financial assets and yet still be constrained in their ability to borrow against some subset of the possible future states of nature. This stark prediction of the incomplete markets model is strongly rejected by the data. Figure A12 plots the life insurance coefficient  $I$  using SCF data on only those married households with children that have positive net worth, where the human capital loss is based on the present value of income losses as described in Section 2.3. As in figure 1, the under-insurance of young families with positive net worth depicted in figure A12 is severe, while older households are almost fully insured. Indeed, there is almost no difference between the life-cycle profiles of insurance conditional and unconditional on positive financial wealth.

There are, of course, a continuum of incomplete markets models that differ in the restrictions on financial markets that are exogenously imposed. Indeed, by allowing a complete set of assets and carefully choosing exogenous borrowing constraints, it is possible to construct a variant of the incomplete markets model that exactly replicates the equilibrium of our baseline model. More generally, our findings suggest that for any incomplete-market model to match the data on underinsurance it must allow agents to purchase a sufficiently rich array of financial assets so that they can be constrained in their borrowing against income earned in some state tomorrow, while still holding positive net financial assets on average. Of course, an incomplete-market model with sufficiently many assets is observationally very similar to our model, except that our modeling approach is more tractable and determines borrowing constraints endogenously.

## H Sensitivity Analysis

We have conducted an extensive sensitivity analysis varying the main parameters of interest within a range of empirically plausible values. Overall, the main quantitative results of this paper have shown to be quite robust to these variations in parameter values (targets). For the two most important parameter dimensions, namely human capital loss upon death and contract enforcement, the results are as follows.

The analysis conducted in the paper shows how variations in mortality risk (the size of the human capital loss in the case of death of a family member) affect our conclusions regarding the life-cycle profile of insurance and under-insurance. Figures 12 and 13 in the paper demonstrate that substantial variations in the level of mortality risk induce significant shifts in the life-cycle profile of life insurance holdings, but have only small effect on the general shape of the life-cycle profile. In addition, we show in figures A13 and A14 that the life-cycle profile of the insurance coefficient is very similar across a wide range of levels of mortality risk. Thus, the model’s main implication regarding the link between age and under-insurance holds across a wide range of parameter values, that is, regardless of the location of the life-cycle profile of human capital losses.

Our discussion of the reform of the Consumer Bankruptcy Code in Section 5.4 provides an example of a substantial change in contract enforcement.<sup>17</sup> The results in Section 5.4 show that even though the improvement in contract enforcement has a sizable effect on the relationship between age and under-insurance, the link between age and under-insurance is still very strong after the reform. Moreover, the life-cycle profile of under-insurance implied by the model provides a reasonable good fit of the data even after the reform – see figure A17. Thus, we conclude that this paper’s main results is robust to substantial changes in the enforcement parameter. Note that this result does not rule out that very large changes in credit enforcement, as they occur in a cross-country comparison, have very large effects on the relationship between age and under-insurance.

## I Empirical Robustness

### I.1 Earnings losses

In this section we investigate to what extent our estimates of the earnings losses upon death

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<sup>17</sup>In Section 5.4 we conduct a policy experiment and therefore do not re-calibrate the model. We have also conducted the analysis re-adjusting all parameter values, and the results are almost identical to the ones shown in Section 5.4.

or divorce could be biased. We consider two issues.

First, in the paper we estimate the earnings losses by comparing the household income of married couples with children to the earnings of single households (male, respectively female) with children. We use all single male/female households as a comparison group, instead of using only widows or divorcees, in order to obtain more precise estimates of the life-cycle profile of earnings (the SCF only contains a limited number of observations on divorcees and/or widows). Clearly, our estimated earnings losses are incorrect if the earnings of all single households substantially differ from the earnings of all divorcees/widows/widowers. In figures A18 and A19 we show that there is no evidence for this view. Specifically, figure A18 shows that the life-cycle profile of median earnings of single, female households with children is very similar to the corresponding life-cycle profile for single, female households with children who are divorcees or widows. Similarly, figure A19 shows that the life-cycle profile of median earnings of single, male households with children is very similar to the corresponding life-cycle profile for single, male households with children who are divorcees or widowers.

Second, our use of cross-sectional data (SCF) to estimate earnings losses upon divorce and/or death can potentially lead to selection bias. For example, if high-income people are more likely to stay married (i.e. less likely to divorce), then the pool of married households will have more high-income people than the group of single households. Further, single high-income people might have different re-marriage rates than all single households. To investigate these and related issues, we next use panel data on divorce rates and re-marriage rates drawn from the SIPP.

We first group married households into four bins divided by the quartiles of the married household earnings distribution and examined divorce rates for these groups. The quartiles vary with age, which we compute by pooling all married households within a 5-year-window centered at each age. As the quartiles vary with age, married households are “dynamically reclassified” according to their current place in the age specific earnings distribution. The result of this analysis is plotted by age in figure A20. As can be seen from the figure, the patterns are not monotonic in income. The lowest earning groups (denoted by the red circles) tend to have the lowest divorce rates, especially at younger ages. The highest income group (the pink asterisks) tended to have *higher* divorce rates at younger ages, while their divorce rates at older ages were broadly similar to the lowest income group. The second highest income group tends to have the highest divorce rates, while the second lowest group has higher divorce rates when young but when old has divorce rates roughly comparable to those for the highest and lowest income groups. Thus, we conclude that there is no clear

relationship between earnings and divorce rates.

We next examine remarriage rates of divorcees and widows grouped similarly according to their individual incomes. Specifically, the groupings are done using age varying quartiles of the distribution of *all* single households calculated analogously to the divorce rate calculations above. These remarriage rates are plotted in Figure A21. The Figure shows that the remarriage rates for the lowest income group are indeed lower than for the higher income group and in some cases significantly so, although we should also note that these calculations are based on a relatively small number of observations (broken into four groups by income, there are roughly 20 observations on divorced and widowed people in each income class for each age in the early and mid 20's). There is no stable relationship between the remarriage rates of the second lowest and second highest groups, with the second lowest group having higher remarriage rates than the second highest group in their late 20s and thirties, but roughly equal or lower rates the rest of the time.

Finally, we investigate if previously married people are more likely to remarry into lower income married households. Some evidence on this is collected in the table A2, which examines the transition rates (in percentage points) across four income groups associated with marriage of previously married single households. As can be seen in the table, there is a very strong tendency for the single households in the middle income categories to marry into lower income households. Specifically, almost 45% of previously married singles in the second lowest income category marry into the lowest income category of all married households, while 76% are in the lower half of the distribution. Likewise, almost 70% of previously married singles in the second highest income category marry into households in the lower half of the distribution of married household incomes. This pattern is also preserved after conditioning on age, although the data is very choppy due to the small number of observations: for almost all ages from 23 to 55, singles in the second lowest income grouping are most likely to transition to the lowest married income group, and stay in the bottom of the distribution in excess of 70% of the time; the same is true for the second highest income grouping of previously married people, with the exception of ages 28 to 32 where they are most likely to stay in (that is, transition into) the second highest married income group.

## **I.2 Heterogeneity in Life Insurance Holding by Wealth**

For tractability, the model embodies a number of assumptions designed to generate linear homogeneous policy functions. As a result, the model makes the strong prediction that, conditional on demographic type, all households make the same portfolio choices and hold, relative to wealth, the same amount of life insurance. Although this prediction is a result of

Table A2: Income transitions at remarriage

	1	2	3	4
1	73.4	20.3	5.2	1.1
2	44.7	31.3	15.3	8.7
3	28.6	38.6	24.6	8.3
4	3.4	14.8	39.1	42.7

assumptions made for tractability, and does not necessarily result from the main mechanism we emphasize, it is nonetheless of interest to examine how far the model strays from the data in this respect. In this section we construct measures of underinsurance by wealth level in order to investigate this phenomenon. Our general finding is that differences in underinsurance across different subsets of the wealth distribution are small for most ages, and that the remaining small differences are most likely due to unobserved differences in human capital returns or small sample size.

To investigate this question, we partition the sample at each age in four wealth (net worth) groups. We apply the same sample selection criteria as in the main part of the paper: In particular, we look at married households with children. For each age-wealth group, we first determine the group specific median life-insurance holdings conditional on having life-insurance. The data are in 2000 Dollars, and the profiles are shown in Figure A22 above.

Next, we turn to the heterogeneity of the human capital loss in case of death. To do this, we construct age-dependent wealth distributions. We do this separately for married and single households. We group single and married households according to their current wealth positions in four age groups using wealth quartiles as boundaries.<sup>18</sup> We then derive life-cycle labor income profiles for each of these wealth groups. We use conditional median income for each wealth group as our measure of labor income.<sup>19</sup> We use these income profiles by wealth groups to derive estimates of the human capital loss comparable to Figures A7 and A8.<sup>20</sup> Recall, the human capital loss is the present value of the labor income loss over current

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<sup>18</sup>Specifically, to group households of age  $j$  in wealth groups, we look at all married households in a 5-year age window centered at  $j$ . We use 5-year age windows throughout the analysis to avoid too small sample sizes at each age observation. We then derive the wealth quartiles for these households and group households accordingly.

<sup>19</sup>Using the group specific mean rather than the median does not change the results much. Human capital losses are slightly higher.

<sup>20</sup>In figures A7 and A8, we report measures of the human capital loss separately for husband's and wife's death and decompose it for different assumptions on remarriage rates and availability of social insurance.



labor income. To ease the exposition, we show here the sum of the human capital losses after transfers and taxes and use family transition rates as in the paper (see the discussion above on the dependence of transition rates on income). Note also that we are assuming that a widow with children from a second quartile married household by wealth transitions upon the death of their spouse to a second quartile single parent household and then, if and when they remarry, transition back to a second quartile married household.

Figure A23 shows the human capital loss for the four wealth quartiles. We also show the profile for the median household as used in the calibration of the benchmark model. Note that the median refers to the median of labor income and not wealth. The median corresponds to the sum of the human capital loss from Figures A7 and A8 above.

Finally, we combine the information from these figures to construct the measure of underinsurance separately for each wealth group of married households with kids. The results are collected in Figure A24. The figure shows that for the second, third and fourth quartiles, the life insurance coefficients are surprisingly similar. In particular, for all three of these quartiles, the young appear to hold insurance against roughly 20% of the human capital loss in the event of the death of spouse, rising to roughly 80% or more in their late 50s. The fourth quartile does show somewhat more insurance at older ages, which may reflect the small sample, but the second and third quartiles are very similar throughout the entire life cycle.

The first quartile is also similar throughout the middle years of the life cycle, but is an exception at both the youngest and oldest ages. In fact, for the lowest quartile the measure of underinsurance is “U-shaped” starting at in excess of 80% before falling to 40% by age 30, and then rising back to 80% by their late 50s.

The differences exhibited by both the youngest and oldest lowest wealth quartile families may also be the result of a small sample problem. However, an alternative explanation is that there is an additional factor correlated with low wealth that leads to low returns to human capital investment at young ages and hence results in less binding borrowing constraints. One obvious candidate is that there is an unmeasured and un-modeled ability difference, or alternatively that the difference reflects differences in their education before age 23. When education directly affects the return to human capital investment, or whether it is simply correlated with unmeasured differences in ability, this suggests that splitting the sample by education level would also be informative.

To assess this possibility we also examine differences in the life insurance coefficient by wealth quartile conditional on a households level of education. After conditioning on

demographic characteristics and wealth, further conditioning on education levels results in even smaller samples, and so we limit our analysis to two education groups: a “high” (at least some college education) level and a “low” (no more than high school education) level. The resulting underinsurance measures for the high education group are shown in Figure A25. As displayed in the Figure, the life insurance coefficients for all four wealth quartiles with a high level of education are very similar throughout almost all of the life cycle. This suggests that the difference in underinsurance for the youngest low wealth households may be due to differences in education or to unmeasured differences in ability.

In summary, we find that differences in underinsurance across different subsets of the wealth distribution are small for most ages, with any differences occurring at both ends of the age distribution where small sample sizes are a concern. We take this to be confirming evidence for our homogeneity generating assumptions. It is also possible that the high levels of insurance observed for the youngest low wealth households are driven by differences in their underlying return to human capital accumulation as a result of differences in ability and/or education, which is not a feature of the benchmark model.

### **I.3 Additional Insurance Through Inter Vivos Transfers**

An alternative explanation for the underinsurance patterns we document in the data is that we have omitted some other form of insurance against the risk of loss of ones spouse. One obvious candidate source for additional insurance are inter vivos transfers from family members: transfers that are not bequests (they occur during the givers lifetime). In this section, we present new data on the size of inter vivos transfers and review the secondary literature on the subject. In summary, looking across a broad range of studies and data sources, inter vivos transfers appear to be relatively uncommon and are typically small. More importantly, there is little or no evidence that inter vivos transfers to widows and widowers (or, in the absence of data on widows, inter vivos transfers to previously married single parents) are more common or larger than transfers to “in tact” or “still married” families. Finally, there is little evidence to suggest that transfers to young single parents are larger than to older single parents. Hence, we conclude that any insurance provided by inter vivos transfers is small and likely negligible, and moreover that it is unable to explain the pattern of underinsurance of younger households found in the data.

In coming to this conclusion, we examined several sources. We first looked at data from surveys of US households. The first data source we considered was the Survey of Consumer Finance (SCF). It is important to stress that there [ARE] very few young widows and widowers in the SCF. Specifically, pooling across all waves from 1995 to 2007 there are 2012

widows in the data set, of which 751 have children living with them.<sup>21</sup> However, of these, only 65 are under the age of 40, and none of them are under the age of 30. Hence, there is little we can say about young widows with children in general and so we must often look at the group of previously married—widowed or divorced—single parents as a proxy for widows.

There are two questions in the SCF that pertain to inter vivos gifts. First, there is a retrospective question about the reception of gifts or transfers that reads “Including any gifts or inheritances you may have already told me about, have you or your husband/wife/partner ever received an inheritance, or been given substantial assets in a trust or in some other form?” We further restrict attention to transfers, and further to those transfers received from parents, grandparents, or aunts and uncles (that is, we exclude inheritances and trusts as well as transfers from outside the family). A second question relates to recent income and asks whether the household has received an “inheritance/gift”, “other help/support from relatives”, or a “gift or support n.e.c.” Thus the second set of figures include bequests as well as transfers from outside the extended family. We refer to the first measure as *asset transfers* and the second as *income transfers* to capture the fact that the second includes support that is potentially ongoing.

Looking first at widows and widowers of all ages, we find that of the 2012 widows in the sample, only 15 report receiving income from these sources, and only 41 report receiving assets in this way. Hence, not only is the median transfer zero, transfers are also zero at the 90 percentile. If we restrict attention to widows with kids, 20 receive an asset transfer but none report receipt of an income transfer.

Given the small number of widows we are unable to break these numbers down by age with any confidence and so we next turned to the sample of all previously married single parents (widows and divorcees). Once again, looking across all such households, we find that such gifts and transfers are uncommon and small. If we look at all previously married households, less than one per-cent (0.9%) report receiving income from these sources while only 4% report receiving assets. Averaging across the entire sample, the mean income transfer was roughly \$100 while the mean asset transfer was \$1500 (all numbers have been converted to year 2000 dollars) reflecting the fact that a small number of households received very large transfers. If we further restrict attention to previously married households with children, the numbers are almost exactly the same, although the mean transfer of income rises slightly to \$120. Strikingly, these numbers are quite similar to those for transfers to intact families. Among intact families with kids, less than one per-cent (0.6%) report an income transfer and roughly

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<sup>21</sup>We cannot use 1992 data because marital history information is only available from 1995 onwards.

4% report an asset transfer while the mean amounts were roughly \$80 of income and \$1900 of assets. Thus, there is little evidence that transfers to single parents are greater than those given to intact families.

There is also little evidence to suggest that these transfers are larger for younger families. Looking across all previously married single households with kids, none of the families under age 30 report an income transfer while only 2% report an asset transfer with a mean value of \$600. This compares to similarly aged intact families of which 0.3% report an income transfer (a mean of \$7) and 5% report an asset transfer (a mean of roughly \$3000). If we turn to families under age 40, 0.6% of previously married single parents report an income transfer (mean of \$167) while 3.5% report an asset transfer (mean of \$1100). The numbers for married families with kids are very similar: 0.6% report an income transfer (mean of \$100) while 4% report an asset transfer (mean of \$2000).

In summary, the SCF data does not provide support for the idea that widows receive significant inter vivos transfers from their extended families above and beyond what extended families provide for their children and grandchildren in general. Nor is there any sense that transfers are larger for young households. Hence, we do not believe they can explain the patterns of underinsurance observed in the data.

Next we examined the Panel Study of Income Dynamics (PSID) and, in particular, responses to the 1988 supplement on *Time and Money Transfers* which has been used previously to study inter vivos transfers by Altonji, Hayashi and Kotlikoff (1997) amongst others<sup>22</sup>. Respondents were asked if they had received “money help” from a parent or someone else outside the family, and whether they had received “time help”. Answers to these questions were merged with demographic information from the PSID individual and family files.

Once again, the relative scarcity of widows in the data—there are 152 of them in our sample—limits analysis, as does the fact that they are typically quite elderly (their median age is 54). Of the 152 widows, only 53 have children at home, 18 of which the parents are under age 40 and 3 of which the parents are under age 30. As a result we will again also look at the sample of previously married single parents as well as the sample of widows alone.

Looking across the sample of all widows, roughly 9% received monetary transfers (from either family members or from outside the extended family) with an average transfer of roughly \$220. Likewise, 20% of widows received time transfers which averaged 47 hours per

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<sup>22</sup>This literature argues that inter vivos transfers between parents and children are driven *not* by altruism or a desire to insure children against adverse outcomes, but rather in exchange for expected future transfers from the children (e.g. Cox 1987, Altonji, Hayashi, and Kotlikoff. 1997, Cox and Rank 1992).

year. Restricting attention to widows with children (and bearing in mind the small sample size problem), the proportion receiving a monetary transfer rose to 15% with a mean of \$276, although the fraction receiving time help fell slightly to 19% with the mean amount of help also falling to 34 hours.

In order to obtain more data, we then turned to the subsample of all previously married single households of which there are 516 in the sample, and of which 214 have children. Of these 112 are under age 40 and 39 are under age 30. Thus there is still a relatively small sample of young households to work with. For this subsample as a whole, 15% received monetary transfers which averaged \$460 per year while 27% received time transfers averaging 94 hours per year. Restricting attention to previously married single parents, 22% reported receiving a monetary transfer averaging \$696 per year, while 32% report receiving time help in an amount averaging 151 hours per year. Looking at younger previously married single parents, and again bearing in mind the small sample problem, the receipt of transfers is more common but they tend to be no larger in size. Specifically, 25% of previously married single parents under age 40 receive money transfers averaging \$450, while 31% of previously married single parents under 30 received money averaging \$356 per year. As for time transfers, 37% of the under 40 sample received time help averaging 149 hours per week, while for the under 30 sample the corresponding numbers were 41% and 182 hours.

Finally, in order to assess whether these transfers are a form of insurance against losing a spouse, we look at the sample of intact households (that is, those households that are still married. We find that, if anything, transfers to intact households are no less common or smaller (and in some cases are larger) than to divorced and widowed households. Looking at all married households, 22% receive a money transfer averaging \$901 per year while 32% receive a time transfer averaging 11 hours per year. Restricting attention to married households with children, 23% receive a money transfer averaging \$683 while 37% receive a time transfer averaging 138 hours per year. For younger households with children, 26% of those under 40 receive a money transfer averaging \$737 while 43% receive a time transfer averaging 167 hours, while 32% of those under 30 receive a money transfer averaging \$672 and 52% receive a time transfer averaging 212 hours per year.

To summarize, like the SCF data, the PSID data reveal that: transfers of either time or money are uncommon and small; provide no evidence that they are larger for widows and other single parents than for intact families; and, are no larger for the youngest widows and single parents. Hence, we conclude that they cannot explain the pattern of underinsurance that we document in the paper.

We also reviewed the literature on inter vivos transfers and found that it comes to similar

conclusions. For the USA, as noted above, there is very little evidence on widows and widowers due to the lack of data and most results are derived for previously married single parents including both widows and divorcees. The best study is probably Hao (1996) who uses 1980s data from the National Survey of Families and Households (NSFH 1987-88), a dataset which contains 68 widows and roughly 1000 previously married women with children living at home (no results for widows alone are presented). Hao also finds that inter vivos transfers are rare: only 15% of previously married single mothers have received a monetary transfer from family in the past five years (the number rises closer to 20% if we include transfers from non-family members such as neighbors and friends). Conditional on receiving a transfer, the mean transfer was about ten thousand dollars, although this is driven entirely by a small number of very large transfers (Hao does not report the median transfer conditional on receiving a transfer, or the unconditional mean transfer, but the standard deviation is reported to be roughly eighty (80) thousand dollars). By contrast, previously married single fathers were more likely to receive a transfer (27%) but the transfers were smaller (conditional on receiving a transfer, the mean was under five thousand dollars). Importantly, the frequency of transfers to single household heads was roughly similar to observed transfers to intact families (20% of all intact families received a transfer from family in the past 5 years, rising to 30% if we include transfers from non-family members), and while the mean transfer was lower (roughly seven thousand) the difference was not statistically significant. No results are presented by the age of the parent.

Another possibility is that parents provide other forms of non-financial transfers such as free child care above and beyond what would have been provided should both parents have survived. We know of little direct evidence on this question. One exception is Marks and McLanahan (1993) who also use the NSFH to look at the provision of instrumental support (child care, transport, and repairs to house or car) and emotional support of single and married parents with young children provided by their own parents. Respondents are asked only if they received support, and not about the quantity of support provided. They find that previously married mothers are slightly more likely to receive instrumental support (42%) than married women (30%), as well as emotional support (40% to 31%) from their parents. However, single mothers were only slightly more likely to receive support from siblings and other family members, while single fathers were less likely to receive such support than their married counterparts. In addition, single mothers and fathers were no more likely to receive support from friends, with instrumental support for both mothers and fathers especially less likely.

Further evidence can be drawn from studies using foreign datasets, although it must be

acknowledged that it can be difficult to interpret results from these data due to the presence of differences in tax and transfer systems that, amongst other things, favor bequests relative to inter vivos gifts or vice versa. Halvorsen and Thoresen (2011) study the inter vivos gifts of roughly two thousand Norwegian households from 2000 to 2001 where inter vivos gifts are defined to include “any money transfer, payment of regular or extraordinary expenses, payment of travels/holidays, interest on loans or down payments on loans, and financial support through transferring cars/housing or in other ways allowing the children to make free use of cars/housing.” They find that, even using this expansive definition, only 18% of parent-adult child pairs experienced an inter vivos gift, with a conditional average of less than five thousand dollars. They find that parental inter vivos transfers to unmarried children were somewhat more likely than to married children. This is somewhat surprising given that Norwegian institutions place strict limits on the ability of parents to leave unequal bequests to their children, and hence inter vivos gifts are the primary way through which unequal transfers can be made. They interpret their results as suggesting that households have a very strong desire to make equal bequests and gifts (in addition, between two thirds and three quarters of parents state that their aim is to give equal transfers) as opposed to making transfers and bequests based on the “needs” of their children, which includes presumably whether or not they have been widowed.

Similar evidence on the lack of inter vivos transfers to widows and widowers comes from French data. Arrondel and Masson (2001) use data from the INSEE “Actifs financiers 1992” survey and find that transfers from parents and grandparents to children appear unrelated to whether or not their child was a widow or widower. Specifically, whether or not an adult child is a widow or widower had no significant effect on the likelihood that a parent makes an inter vivos gift (and the point estimates were that it had a negative effect). By contrast, if one of the *parents* is a widow or widower the probability of inter vivos gifts to adult children increases significantly (possibly reflecting an early distribution of higher anticipated bequests). Likewise, a child that is a widow or widower is no more likely to receive free housing, a regular monetary stipend, or a monetary loan from their parents nor were their parents more likely to act as cosigner on a mortgage.

Suggestive evidence may also be drawn from the literature looking at monetary transfers following other important life events such as the occurrence of disability of a primary wage earner. Gallipoli and Turner (2009) examine data from the 1999-2007 waves of the Canadian Survey of Labor and Income Dynamics following an occurrence of disability. They find that “there is a small, marginally significant difference in the amount of, and likelihood of receiving, transfers from individuals outside the household in the initial years following

onset but that effect appears to peter out at longer durations” (they do not report numerical results).

Suggestive evidence on non-monetary transfers may also be drawn from the literature examining spousal labor supply responses following job displacement and disability shocks. If time transfers such as child care were large following such a shock this should allow spouses to significantly increase their labor supply following a shock. However, the literature on spousal labor supply in response to job displacement shocks find either no effect on labor supply (Layard, Barton and Zabalza 1980, Maloney 1987, 1991) or only small effects (Mincer 1962, Bowen and Finegan 1968, Heckman and Macurdy 1980, 1982, Lundberg 1985, Spletzer 1992, Gruber and Cullen 2000; for a contrary finding, see Stephens 2002). Likewise, Gallipoli and Turner (2011) find no evidence of increased participation or increased hours worked by spouses following disability of husband, while Coile (2004) finds that women decrease their labor supply when their husbands experience a health shock like a heart attack or a cancer diagnosis (contrast the findings of Charles 1999). There is also some evidence that labor supply responses are particularly small for families with children (see Juhn and Potter 2007, Gong 2010 and Lundberg 1981 on a wife’s labor supply following a husband’s job loss; and Reis 2010 following health shocks).

In summary, the evidence that we have obtained combined with the results from the literature leave us confident in asserting that inter vivos transfers do not provide much insurance against the risk of death of a spouse, and moreover are no greater for young families and hence cannot explain the pattern of underinsurance of the young that we document.

#### **I.4 Additional Insurance Through Remarriage**

It possible that young widows remarry into households that have higher than median married incomes, at least relative to older widows. If so, remarriage provides more insurance to young widows than to older widows and hence might explain the pattern of underinsurance for the young. To investigate this we used our data from the SCF from 1995-2007. As noted above, there are very few young widows in the SCF and so we must necessarily look at data on divorced families for illumination on this question, under the assumption that the remarriage process for divorcees is not too different than the process for widows. The SCF contains information on a limited number of marital history items including whether or not respondents have been married before or if they are currently in their first marriage. It does not report the age of remarriage.<sup>23</sup>

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<sup>23</sup>We cannot use 1992 data because marital history information is only available from 1995 onwards.



Bearing in mind these limitations, we computed the median labor income of households in which both spouses were in their first marriage with those of married households in which one of the spouses has been previously married. In interpreting the results, it is important to stress that even after adding divorcees to our sample, there are still very few young married households in which one spouse has been previously married: there are only 30 married households in our data in which the household head was aged between 23 and 25 and was previously married, and only 55 married households in which the spouse of the household head was aged between 23 and 25 and was previously married.

The results are plotted in Figures A26 and A27. The red circles correspond to first marriages while the blue squares correspond to married households in which one spouse was previously married. The first figure compares the earnings of first time marriages plotted against the *age of the household head*, against the earnings of married couples in which the *household head was previously married*. The second figure compares the earnings of first time marriages plotted against the *age of the spouse of the household head*, against the earnings of married couples in which the *spouse of the household head was previously married*. If both the household head and their spouse have been remarried, we include them in both comparison groups. Two things stand out in the Figures. The first is that the household earnings of 23 and 24 year old remarried people are somewhat larger than for similarly aged households on their first marriage. Note that it is important to keep in mind the relatively small number of data points generating this result. The second is that by ages 25-30, any difference in earnings has roughly disappeared, while after age 30 the households of remarried people tend to have less income than the households of people on their first marriage.

We take three things away from these plots. First, over the lifecycle, divorced and widowed spouses tend to marry into families that have slightly lower incomes than first time married households. This means that our estimates of the degree of insurance provided by remarriage are generous and hence that our measures of underinsurance are conservative. Second, there is evidence that the very youngest (ages 23 and 24) divorced and widowed households remarry into higher income households, although the difference in earnings is small (roughly between one and two years of earnings for these households) relative to the amount of underinsurance documented. However, we must be careful in pushing these results too hard given the relatively small samples involved. Third, any difference in remarried earnings has been eliminated by the time the household reaches their late 20s meaning that these forces cannot explain the underinsurance we observe across all younger households.

In summary, we believe that the evidence is inconsistent with remarriage providing substantially more insurance against the loss of a spouse than the figures we present in the

paper, and that conceivably we are slightly underestimating the degree of underinsurance.

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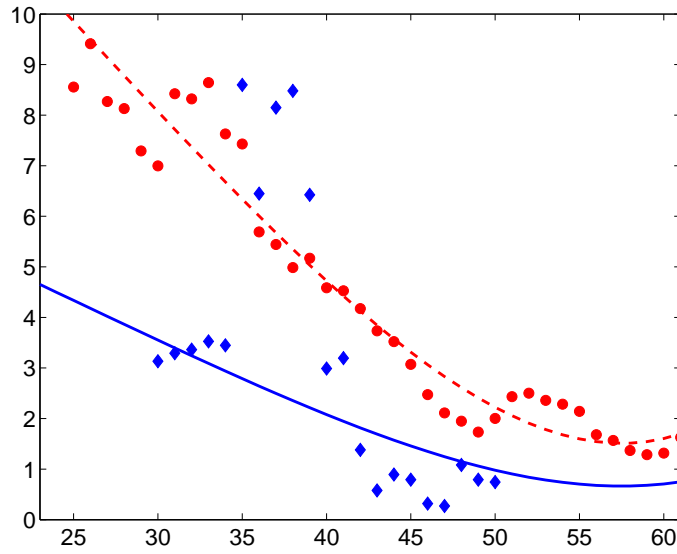
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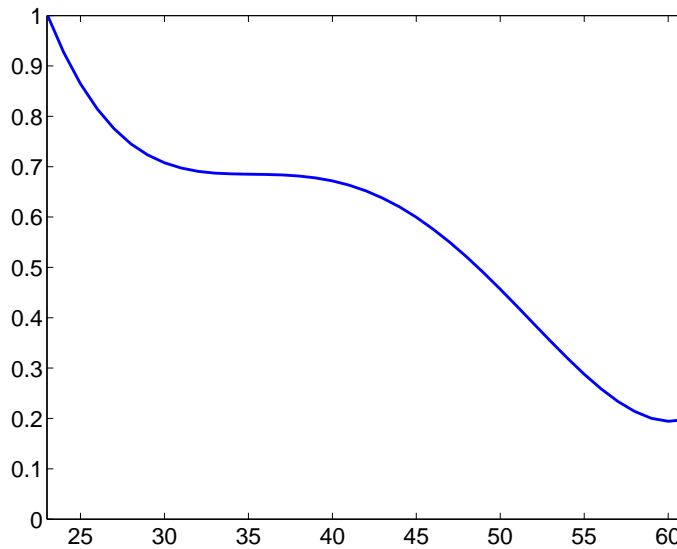
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Figure A1: Remarriage rates



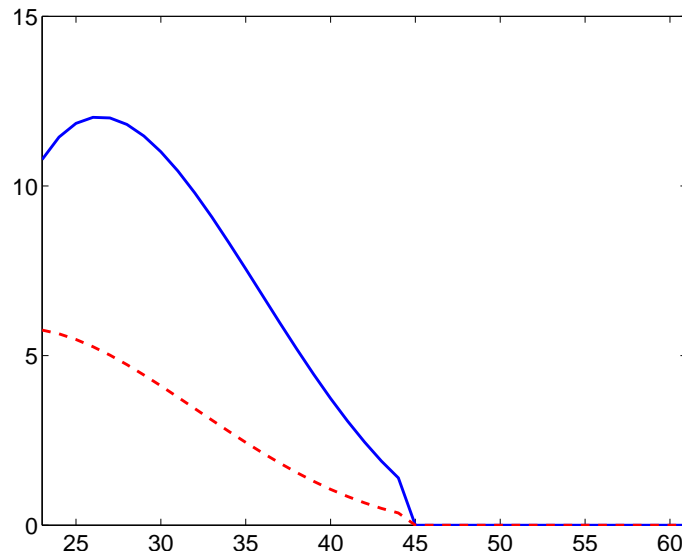
Notes: Life-cycle profile of remarriage rates (percentage points). Red dots show remarriage rates for divorced singles. Remarriage rates for non-widowed singles in the model correspond to the smoothed life-cycle profile (red dashed line). Blue diamonds show remarriage rates for widowed singles age 30-50 in the data. Remarriage rates for widowed singles in the model correspond to the (adjusted) life-cycle profile (blue solid line). The adjustment of the life-cycle profile of remarriage rates for widows is derived as the mean ratio of the blue diamonds to the red dots for ages 30-50. All remarriage rates are derived using 2001 SIPP data.

Figure A2: Divorce rates



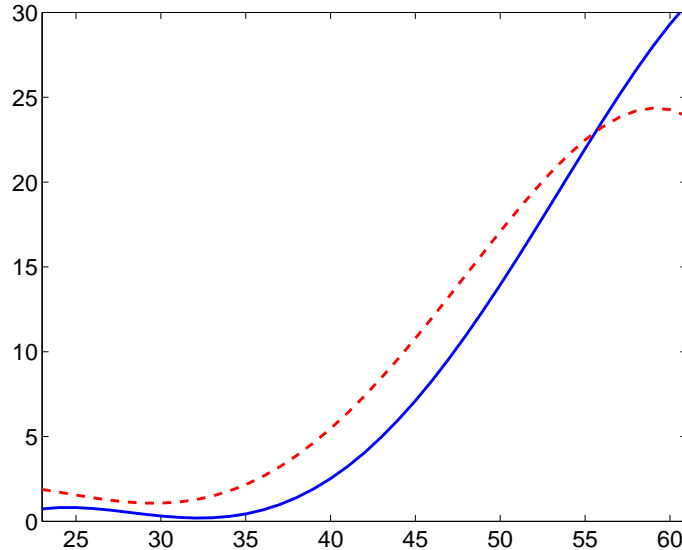
Notes: Smoothed life-cycle profile of divorce rates (percentage points). Divorce rates are derived for all married households using 2001 SIPP data. Smoothed profiles come from a regression on a fourth order polynomial in age and a constant.

Figure A3: Fertility rates



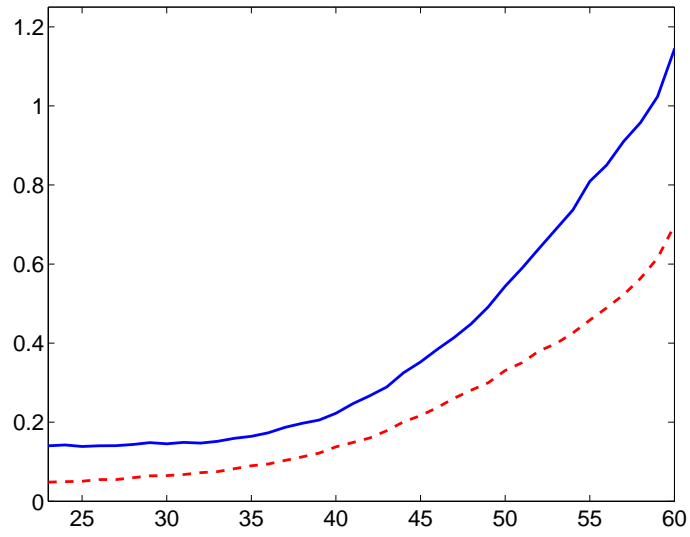
Notes: Smoothed life-cycle profile of fertility rates (percentage points). Red dashed line shows singles and blue solid line married females. Fertility rates are derived using wave 2 topical module to the 2001 SIPP. Smoothed profiles come from a regression on a fourth order polynomial in age and a constant.

Figure A4: Moving out rates



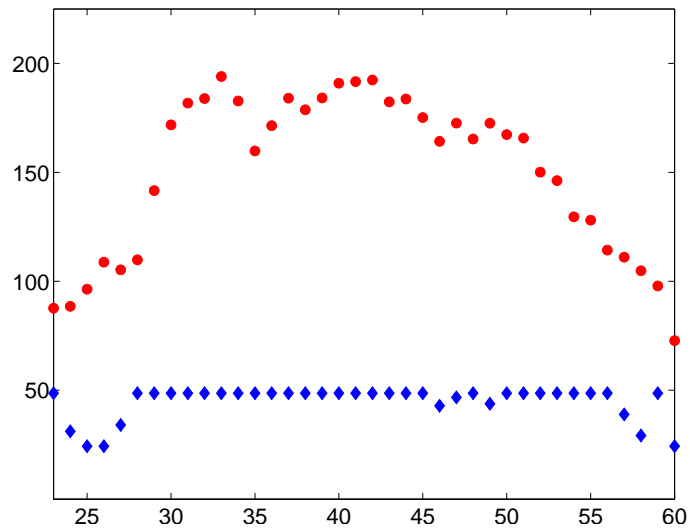
Notes: Smoothed life-cycle profile of moving out rates for single and married households (percentage points). Red dashed line shows single parent households. Blue solid line married households. Moving out rates are derived using 2001 SIPP data. Smoothed profiles come from a regression on a fourth order polynomial in age and a constant.

Figure A5: Death probabilities



Notes: Death probabilities for males and females from the life tables of the Human Mortality Database (percentage points). Blue solid line shows death probability of males. Red dashed line shows death probability of females.

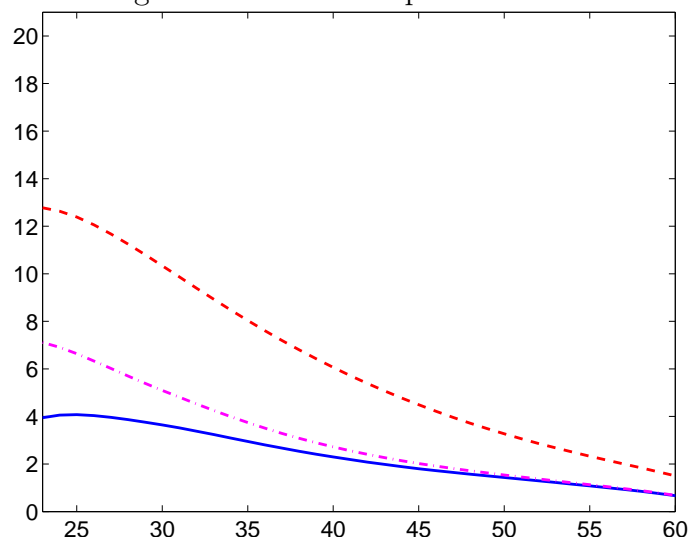
Figure A6: Employer-provided life insurance



Notes: Life-cycle profile of median life insurance holdings for married households with children who have purchased some life insurance. Red dots show all holdings. Blue diamonds show holdings of employer-provided insurance. All data are from wave 3 topical module to the 2001 SIPP (in thousands of year 2000 dollars).

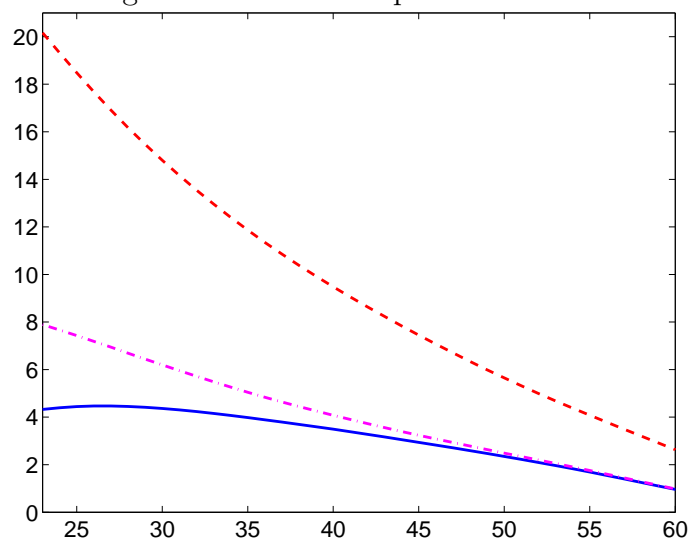


Figure A7: Human capital loss male



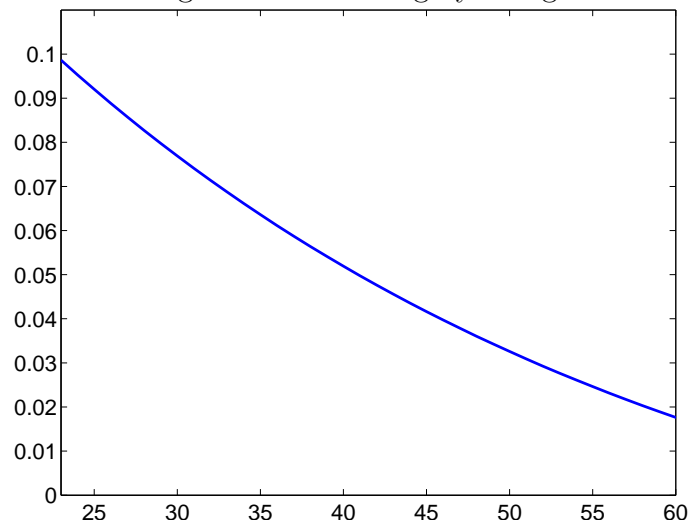
Notes: Life-cycle profile of expected human capital loss in case of wife's death for all married households with children. Human capital loss is ratio of present value labor income loss over current labor income. Red dashed line: loss before transfers and taxes with zero probability to remarry. Pink dashed-dotted line: loss after transfers and taxes and zero probability to remarry. Blue solid line: loss after transfers and taxes and empirical remarriage rates. All data are from the SCF.

Figure A8: Human capital loss female



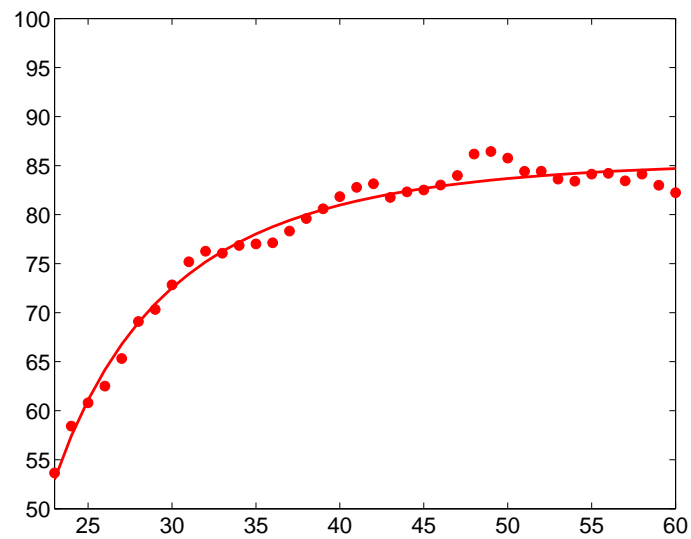
Notes: Life-cycle profile of expected human capital loss in case of husband's death for all married households with children. Human capital loss is ratio of present value labor income loss over current labor income. Red dashed line: loss before transfers and taxes with zero probability to remarry. Pink dashed-dotted line: loss after transfers and taxes and zero probability to remarry. Blue solid line: loss after transfers and taxes and empirical remarriage rates. All data are from the SCF.

Figure A9: Learning-by-doing



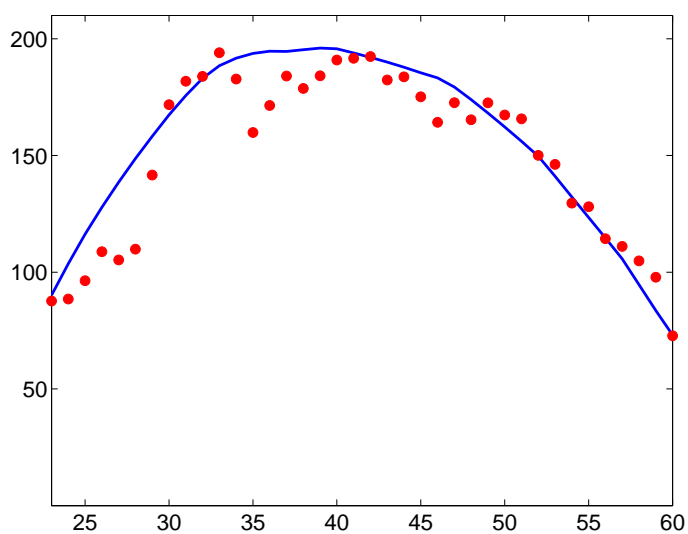
Notes: Life-cycle profile of learning-by-doing  $\bar{\varphi}_j$  from calibrated model.

Figure A10: Participation rate



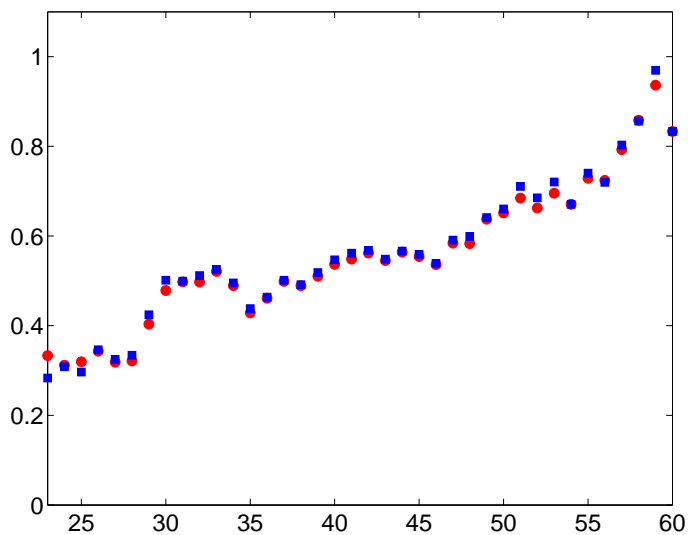
Notes: Life-cycle profile of participation rate in the life-insurance market for married households with children. Red solid line shows the model's prediction. Red dots show the share of households that report having purchased some life-insurance from the SCF.

Figure A11: Extended model with health- and child-dependent preferences



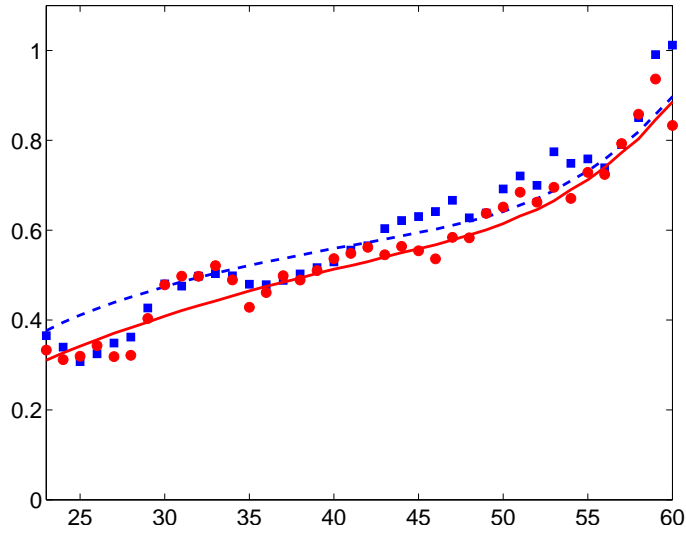
Notes: Life-cycle profile of median life insurance holdings for married households with children who have purchased some life insurance. Blue solid line shows model. Marginal utility from consumption is different for households in poor and good health and differs across single households with different number of kids. Red dots show SCF data (in thousands of year 2000 dollars).

Figure A12: Life Insurance Coefficient with Positive Network



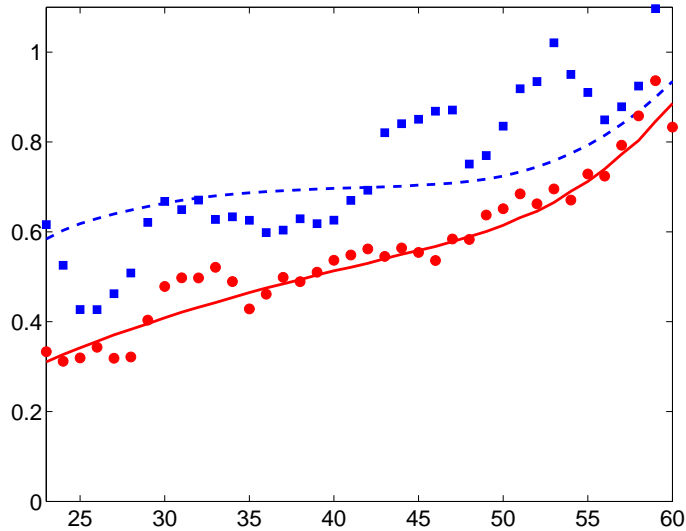
Notes: Life-cycle profile of life insurance coefficient for median married households with children who have purchased some life insurance. Red dots show data for all households. Blue squares show data for all households with positive network. Data is from SCF.

Figure A13: Life Insurance Coefficients for Mean and Median Households



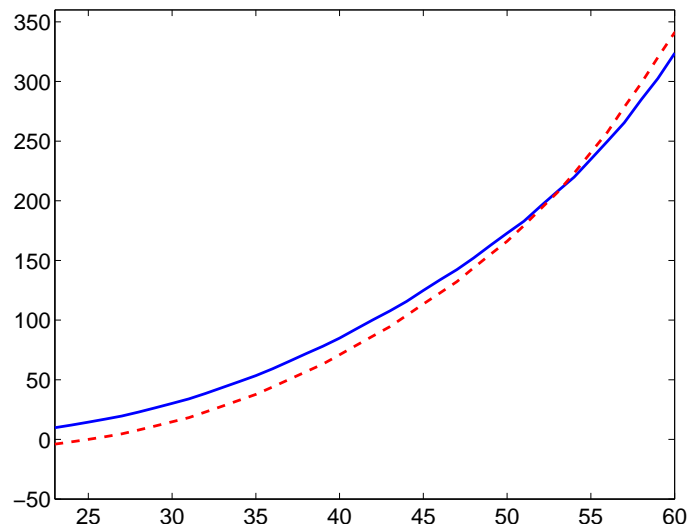
Notes: Life-cycle profile of life insurance coefficient with different levels of life insurance for married households with children who have purchased some life insurance. The red solid line shows the coefficient for households with median levels of life insurance holdings from the model. The blue dashed line shows households with mean life insurance from the model. Red dots show the life insurance coefficient for median life-insurance holdings from the data. Blue squares show the mean life-insurance holdings from data.

Figure A14: Life Insurance Coefficients for Median and Top Decile Households



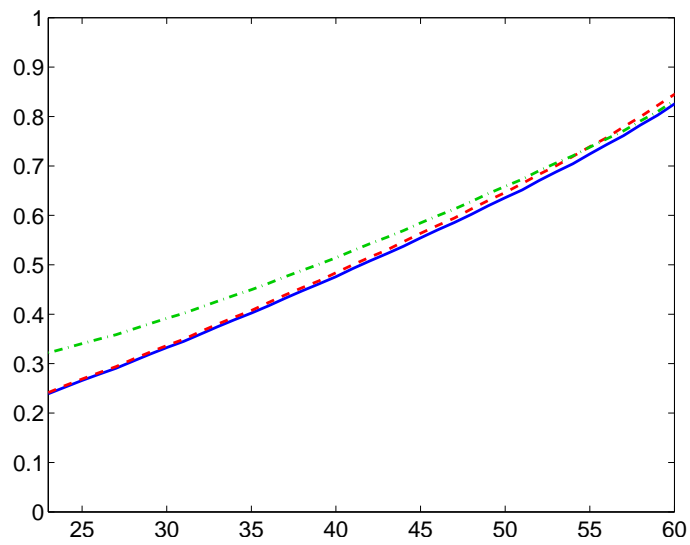
Notes: Life-cycle profile of life insurance coefficient with different levels of life insurance for married households with children who have purchased some life insurance. The red solid line shows the life insurance coefficient for households with median levels of life insurance holdings from the model. The blue dashed line shows the life insurance coefficient for households in the top decile of life insurance holdings from the model. Red dots show the life insurance coefficient for median life-insurance holdings from the data. Blue squares show households in the top decile of life insurance holdings from the data; to remove outliers, we calculate the mean life insurance holdings between the 90th and 99th percentile of life-insurance holdings; for comparability, model output has been truncated at the 99th percentile of the empirical distribution.

Figure A15: Networth to labor income ratio



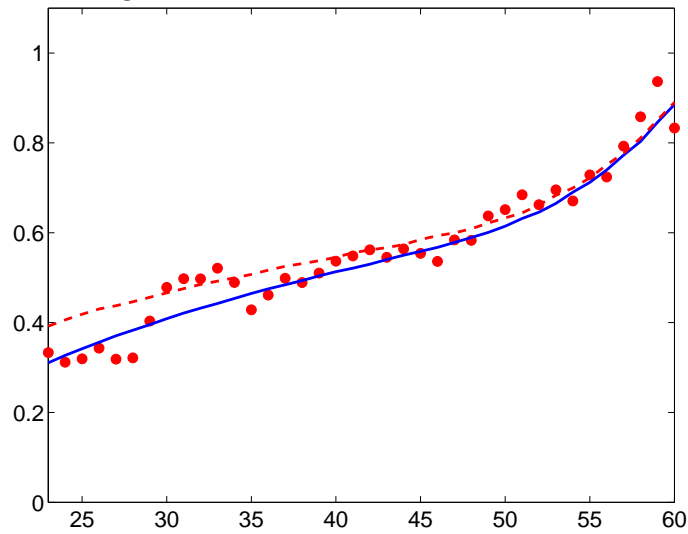
Notes: Life-cycle profile of the median ratio of networth to labor income for married households with children before the reform of the bankruptcy code (solid line) and after the reform (red dashed line).

Figure A16: Consumption insurance



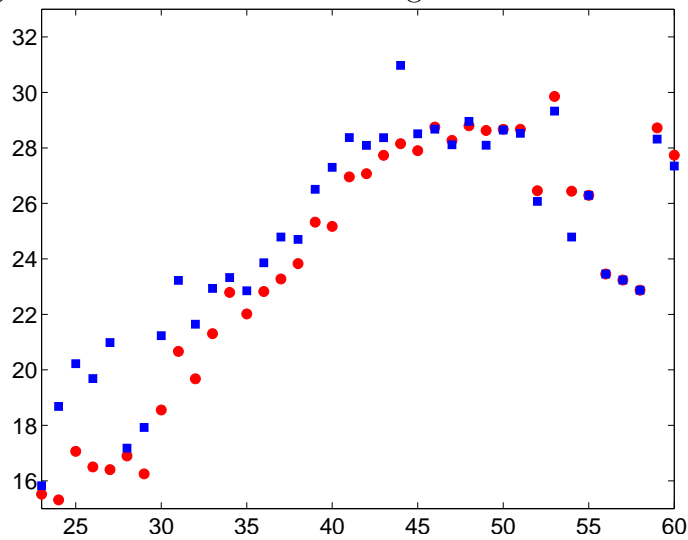
Notes: Life-cycle profile of consumption insurance before the reform of the bankruptcy code (solid line), after the reform with endogenous human capital allocation (red dashed line), and after the reform with fixed human capital allocation (green dashed dotted line).

Figure A17: Life Insurance Coefficient



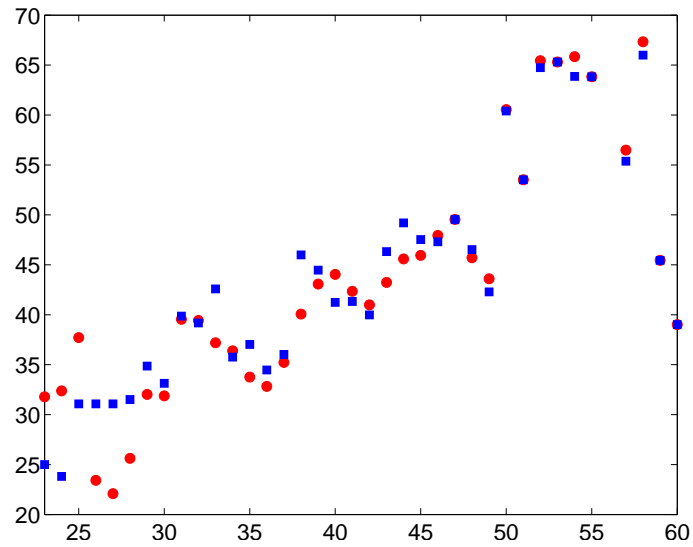
Notes: Life-cycle profile of life insurance coefficient before and after the reform of the bankruptcy code for median married households with children who have purchased some life insurance. Life insurance coefficient is the ratio of life-insurance holdings to present value income loss in case of death. Blue solid line shows benchmark model before the reform of the bankruptcy code. The red dashed line shows model with fixed human capital allocation after the reform of the bankruptcy code. Red dots show SCF data. See appendix for calculation of present value loss.

Figure A18: Labor income of single females with children



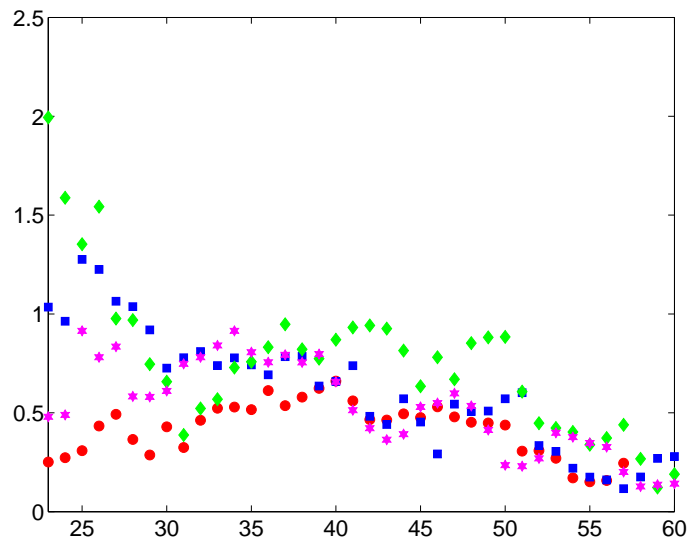
Notes: Life-cycle profile of median labor income for single female-headed households with children and different marital histories. Blue squares show all single households, red dots show all single households who are divorced or widowed. All data are from the SCF (in thousands of year 2000 dollars).

Figure A19: Labor income of single males with children



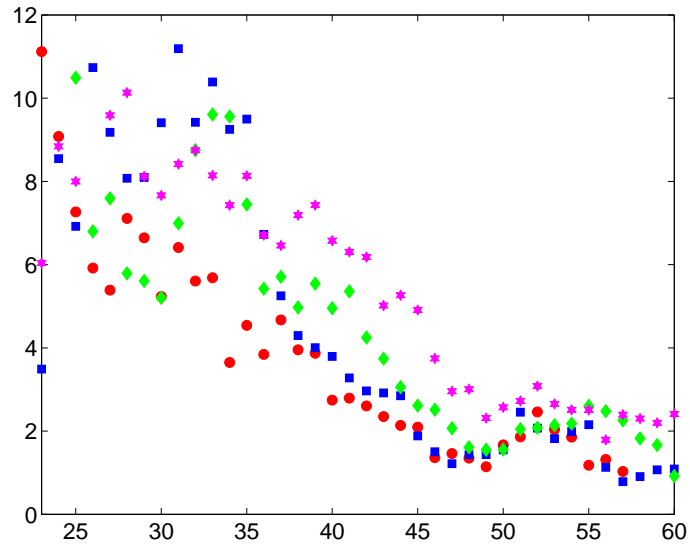
Notes: Life-cycle profile of median labor income for single male-headed households with children and different marital histories. Blue squares show all single households, red dots show all single households who are divorced or widowed. All data are from the SCF (in thousands of year 2000 dollars).

Figure A20: Divorce rates by income



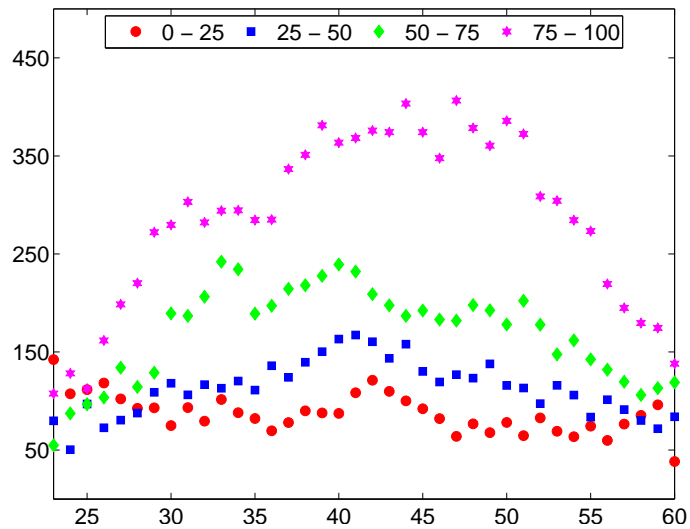
Notes: Life-cycle profile of divorce rates for households conditional on household income quartile (percentage points). Red circles show first, blue squares second, green diamonds third, and pink stars fourth income quartile. Divorce rates are derived for all married households using 2001 SIPP data.

Figure A21: Remarriage rates by income



Notes: Life-cycle profile of remarriage rates for all single households conditional on individual income quartile (percentage points). Red circles show first, blue squares second, green diamonds third, and pink stars fourth income quartile. Remarriage rates are derived using 2001 SIPP data.

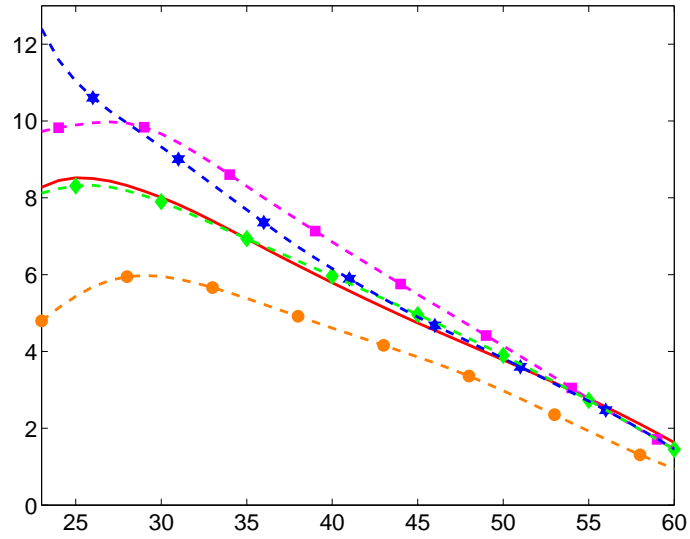
Figure A22: Face value of life-insurance by wealth quartile



Notes: Life-cycle profile of life insurance holdings conditional on wealth quartile for married households with children who have purchased some life insurance. Red dots show first, blue squares second, green diamonds third, and pink stars fourth wealth quartile. All data are from the SCF (in thousands of year 2000 dollars).

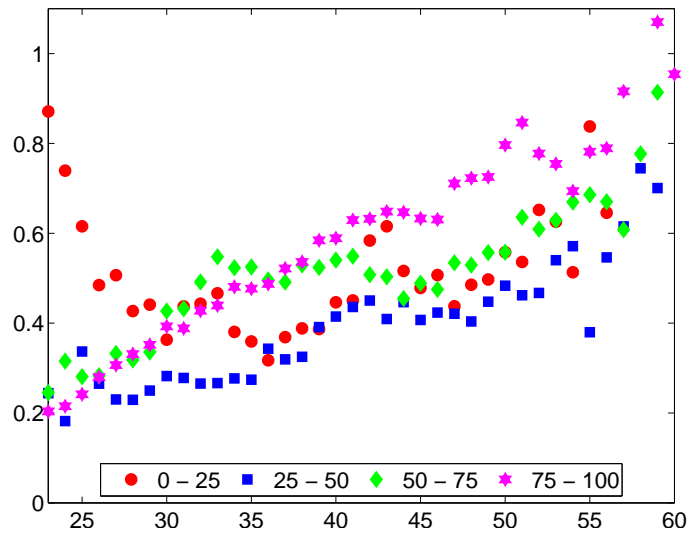


Figure A23: Human capital loss by wealth quartile



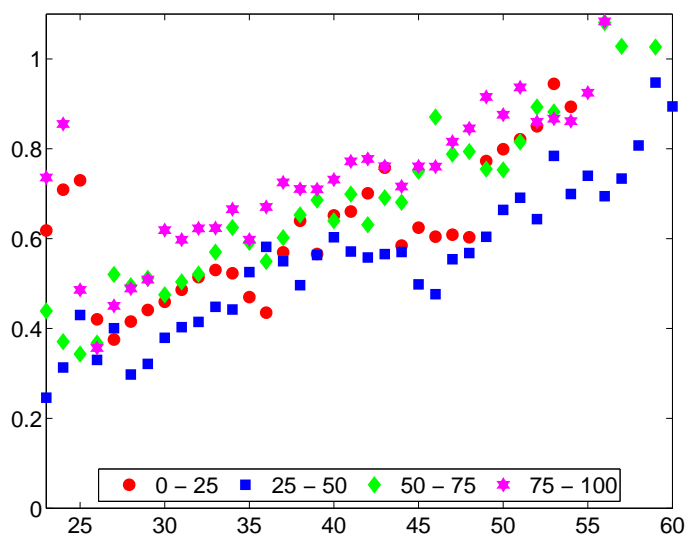
Notes: Life-cycle profile of sum of expected human capital loss in case of husband's and wife's death for all married households with children conditional on wealth quartile. Human capital loss is ratio of present value labor income loss over current labor income. Red solid line shows the median household as used in the calibration. Orange dots show data for first, pink squares second, green diamonds third, and blue stars fourth wealth quartile. All data are from the SCF. See appendix for further details.

Figure A24: Life Insurance Coefficient by wealth quartile



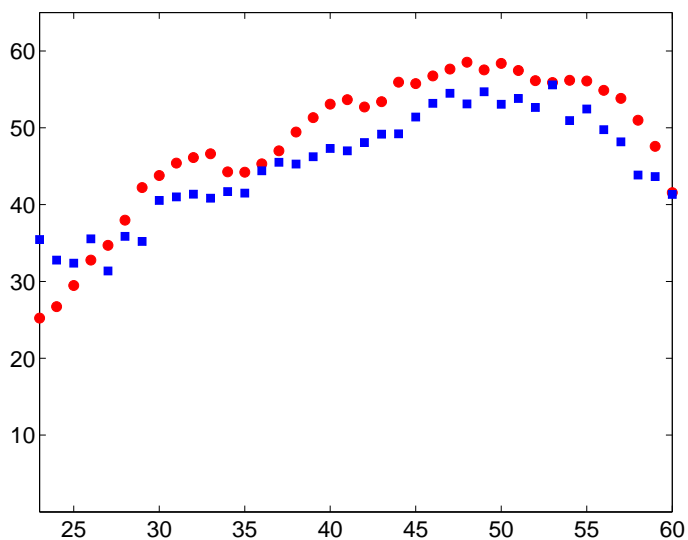
Notes: Life-cycle profile of life insurance coefficient by wealth quartiles for married households with children who have purchased some life insurance. Life insurance coefficient is the ratio of life-insurance holdings to present value income loss in case of death. Red dots show data for first, blue squares second, green diamonds third, and pink stars fourth wealth quartile. All data are from the SCF. See appendix for calculation of present value loss.

Figure A25: Life Insurance Coefficient for high education households by wealth quartile



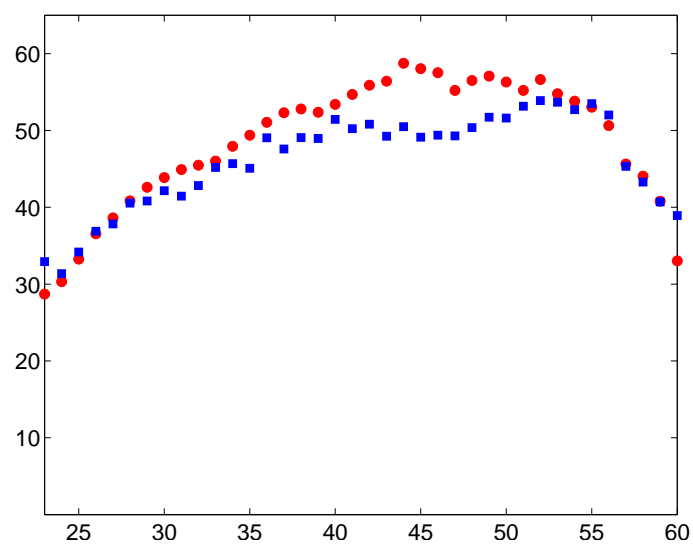
Notes: Life-cycle profile of life insurance coefficient by wealth quartiles for married households with children who have purchased some life insurance. Life insurance coefficient is the ratio of life-insurance holdings to present value income loss in case of death. Red dots show data for first, blue squares second, green diamonds third, and pink stars fourth wealth quartile. All data are from the SCF and for households where the head has at least some college education. See appendix for calculation of present value loss.

Figure A26: Labor income of first- and second-time married households



Notes: Life-cycle profile of median labor income for married households. Age is age of head. Red circles show first-time married households, blue squares show households where one of the spouses is in his/her second marriage (remarried households). All data are from the SCF (in thousands of year 2000 dollars).

Figure A27: Labor income of first- and second-time married households



Notes: Life-cycle profile of median labor income for married households. Age is age of spouse. Red circles show first-time married households, blue squares show households where one of the spouses is in his/her second marriage (remarried households). All data are from the SCF (in thousands of year 2000 dollars).

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