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Good news is bad news: Leverage cycles and sudden stops

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Good News Is Bad News: Leverage Cycles and Sudden Stops

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Staff Report No. 738
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Good News Is Bad News: Leverage Cycles and Sudden Stops

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Abstract

We show that a model with imperfectly forecastable changes in future productivity and an occasionally binding collateral constraint can match a set of stylized facts about “sudden stop” events. “Good” news about future productivity raises leverage during times of expansion, increasing the probability that the constraint binds, and a sudden stop occurs, in future periods. The economy exhibits a boom period in the run-up to the sudden stop, with output, consumption, and investment all above trend, consistent with the data. During the sudden stop, the nonlinear effects of the constraint induce output, consumption, and investment to fall substantially below trend, as they do in the data.

Key words: news shocks, sudden stops, leverage, boom-bust cycle

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1 Introduction

In this paper, we show that an RBC-style model augmented with an occasionally binding collateral constraint and a predictable component in productivity can match the patterns observed in the data surrounding Sudden Stop events in small open economies. The facts we seek to match have been characterized by a recent empirical literature using data from emerging market crises of recent decades. First, such episodes are low-probability events that occur amid regular business cycle fluctuations. Second, financial crises are associated with deep recessions which are different from regular recessions both in terms of duration and magnitude. Third, Sudden Stops are almost always preceded by a substantial buildup of leverage. Fourth, these episodes typically occur after a period of expansion, with output, consumption and investment above trend, the trade balance below trend, and high asset prices.¹

In our model, agents faced with improving growth prospects optimally choose to borrow against their future income, increasing their leverage in good times and bringing them closer to an occasionally binding constraint on their debt holdings. On average, the good news is realized, leading to higher long-run consumption and output for the household. However, because good news also brings households closer to the constraint on their leverage, it exposes them to a greater risk that an unfavorable future shock will eventually lead the constraint to bind, thereby leading *ex post* to a worse outcome than they might otherwise have realized had all shocks arrived as surprises. In this sense, good news leads agents to engage in optimistic behavior that is both rational, since it is validated on average, but also risky, since it reduces the agent's ability to respond to negative shocks that might arrive in the future.

In this paper, we show that a reasonably calibrated model, which is an otherwise standard RBC-style model of a small open economy, matches all of the basic facts laid out in the opening paragraph. In particular, it predicts substantial booms in output, consumption, investment, asset prices and rising leverage whenever available information indicates high future growth rates for consumption, i.e. after positive news shocks. In most cases, such expectations are validated *ex-post*, and the risks associated with increased leverage are incurred rationally by private agents. However, in the

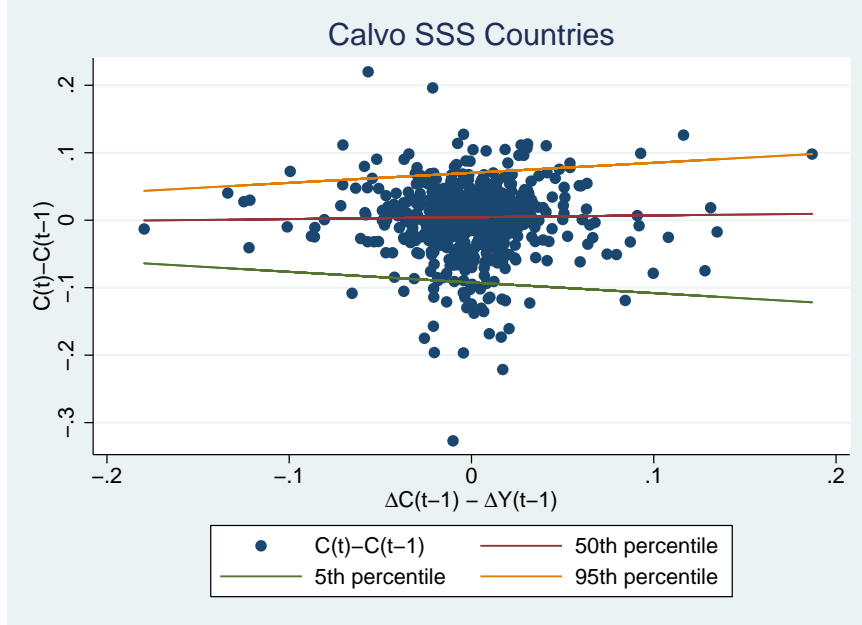
¹For more details on this empirical evidence, see [Calvo et al. \(2006\)](#), [Gourinchas and Obstfeld \(2012\)](#), [Mendoza and Terrones \(2012\)](#), [Korinek and Mendoza \(2014\)](#).

event of a sufficiently negative realization of actual productivity growth (or any other shock, in fact) the additional leverage accumulated by agents during the period of optimism causes the leverage constraint to bind, or bind more strongly, leading to a debt-deflation spiral that is strongly non-linear. Due to the non-linear effects of the binding credit constraint, the model delivers quantitatively realistic crashes in the event of a crisis, including a simultaneous and deep fall in consumption and borrowing that would otherwise be difficult to deliver in an open economy with access to international financial markets. In our economy, agents only partially internalize the risks generated by their own leverage choices so that such crisis, while rare, occur occasionally along an equilibrium path that in other ways resembles the standard small-open economy business cycle modeled in an extensive literature.

One implication of the story we describe above is that optimistic expectations should systematically coincide with *both* high current consumption relative to output and an increasing profile of *consumption risk*. Figure 1 plots the five, fifty, and ninety-five percent quantiles of one-year-ahead consumption growth conditional on the current level of consumption growth relative to output growth across a panel of emerging market economies. The data bear out the consumption risk implication of the news account: when consumption growth outpaces output growth in small open economies, the outer quantiles of future consumption spread out. Faced with improving growth prospects, agents borrowing against their future income to increase current consumption, thereby becoming more vulnerable to adverse realizations of future shocks and subsequent non-linear dynamics induced by the occasionally binding constraint. In the results section, we show our model also matches this stylized fact about conditional variances of consumption growth.

Existing models of small open economy business cycles, even those with credit market frictions, do not easily generate the set of facts cited above. These models typically require unusually large shocks to account for financial crises events and many are designed to study the financial crises in isolation.² Moreover, these models have a difficult time generating output and consumption booms in the period leading into the crisis. This is true because good times are usually associated with improved asset prices and, thus, improved net worth of the borrowers which relaxes borrowing constraints according to most common specifications. Thus financial crises in these models, if they occur, typically occur only after a series of bad realizations of shocks. News shocks address this challenge by introducing the possibility that borrowing and leverage rise in response to

²See, among others, Gertler et al. (2007), Gertler and Karadi (2011), Gertler and Kiyotaki (2010).



Note: The solid lines in the figure display the estimated quantile regression functions of consumption growth on the difference between the lag consumption growth and the lag income growth at different percentiles - $\Delta C_{i,t} = \eta_i + \beta(Q)(\Delta C_{i,t-1} - \Delta Y_{i,t-1}) + \epsilon_{i,t}$. The subscript t denotes time period covering annual data from 1970 to 2013, the subscript i denotes countries that experienced Systemic Sudden Stop (SSS) based on Calvo et al. (2006): Argentina, Colombia, Côte d'Ivoire, El Salvador, Indonesia, Malaysia, Mexico, Peru, Philippines, Korea, Russia, South Africa, Thailand, Turkey, Uruguay, Venezuela, and η_i denotes country fixed effect.

Figure 1: Consumption Risk in Small Open Economies

good shocks, and therefore increase during times of expansion. Crises in this case can be triggered by good news followed by a bad realization, and indeed even when no change in fundamental is finally observed, rather than following a sequence of purely negative shocks.

This paper is related to several strands of the literature. We draw many of our stylized facts from Mendoza (2010), who uses a similar model, augmented with shocks to imported intermediate inputs and a correlated shock structure, to deliver crisis of realistic magnitudes. Below we argue that news shocks have the potential to match the magnitude of Sudden Stop downturns as well as Mendoza (2010) and may do a better job at capturing the pre-bust boom period. Another related paper is that of Cao and L'Huillier (2014). They use a linearized open economy framework to think about medium-term business cycles caused by innovations that lead to increased expectations of future productivity, which are not always realized *ex post*. However, they do not consider the role of financing constraints and the associated non-linearities. Lorenzoni (2008) and Korinek (2010) also study more theoretical contexts where borrowing is collateralized by assets whose price agents take as given. Akinci and Queralto (2014) develops a model with banks, in which banks face an endogenous and occasionally binding leverage constraint, that matches a set of stylized facts around banking crisis

episodes in the advanced and emerging market economies. This paper explicitly models banks' *ex-ante* equity issuance behavior and shows that it plays an important role in generating the frequency and depth of banking crises *ex-post*. Other related papers include Uribe and Yue (2006); Bianchi and Mendoza (2010); Bianchi (2011); Benigno et al. (2012); Otrok et al. (2012), and Bianchi and Mendoza (2013).

This paper proceeds as follows. Section 2 lays out the basic model used in our analysis. Section 3 summarizes our calibration and the solution method used for the non-linear model. In section 4, we highlight our main results regarding the drivers of Sudden Stops in the model economy, and examine the model's ability to match historical Sudden Stop experiences. Section 5 concludes.

2 Model

The model economy closely resembles the baseline RBC-style small open economy model of García-Cicco et al. (2010), with the addition of a collateral constraint as in Mendoza (2010). The economy is populated by a continuum of infinity-lived utility-maximizing consumer-workers. The representative consumer-worker chooses per-period consumption, hours, investment (c_t, h_t, i_t , respectively), the next-period capital stock, k_{t+1} , and the amount of debt, d_{t+1} , incurred in period t to be repaid in $t + 1$, to maximize the discounted expected future flow of utility

$$\max_{\{c_t, h_t, i_t, k_{t+1}, d_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, X_{t-1} h_t) \quad (1)$$

subject to the constraints

$$c_t + i_t = A_t F(k_t, X_t h_t) + \frac{d_{t+1}}{R_t} - d_t - k_t \phi(i_t/k_t) - \chi(R_t - 1)w_t h_t - g_t \quad (2)$$

$$k_{t+1} = (1 - \delta)k_t + i_t \quad (3)$$

$$\kappa \geq \frac{\frac{d_{t+1}}{R_t} + \chi R_t w_t h_t}{q_t k_{t+1}}. \quad (4)$$

Equation (2) is the household intertemporal budget constraint, which reflects that households must pay a cost of capital adjustment parameterized by the function $\phi(i/k)$ and also must finance a fraction χ of their wage bill with working capital. Unproductive government spending g_t is exogenous, and we assume it to be constant in the detrended model. Equation (3) represents a standard process for the evolution of capital.

The key equation for the questions of this paper is the occasionally binding collateral constraint given by (4). The right hand side of equation (4) defines leverage in the

economy as the ratio of total borrowing (including working capital required to hire labor) divided by the agent's net worth given by total capital times its price, which is exogenous to the agent but in equilibrium is given by Tobin's Q. Similar constraints have been used by many authors, including [Kiyotaki and Moore \(1997\)](#) and [Mendoza \(2010\)](#). In addition to the price of installed capital, q_t , consumers take the real wage, w_t , and the world interest rate on external borrowing, R_t , as given. In equilibrium, the real wage is given by $w_t = U_{h,t}/U_{c,t}$, which corresponds to the real wage that would be achieved under a standard decentralization of the economy.

In order to ensure that agents borrow in equilibrium, we calibrate the economy so that agents in the domestic economy exhibit a degree of "impatience" relative to the world investors. That is, in the long run,

$$\beta \bar{R}^* \gamma^{-\sigma} < 1 \quad (5)$$

where \bar{R}^* is the long-run world interest rate, γ is the long-run growth rate of technology of the domestic economy, and σ is the coefficient of relative risk aversion in preferences. In this framework, the interest rate faced by a small open economy on its external borrowing, R_t , is equal to the world interest rate, R_t^* .³

We note immediately that the environment incorporates two pecuniary externalities of the type emphasized by [Bianchi \(2011\)](#), and driven by the presence of the prices w_t and q_t in the collateral constraint. [Mendoza and Smith \(2006\)](#) and [Mendoza \(2010\)](#) argue that the quantitative effects of these externalities under the specification of the collateral constraint used here are rather small, and solve their model using a method that ignores them; whether this remains the case in our environment is not immediately clear and our solution method takes them into account. The first order conditions of the household's problem are presented in the [Appendix A](#).

3 Calibration and Solution

The functional forms of preferences and technology are the following:

$$U(c_t, X_{t-1}h_t) = \frac{(c_t - \theta\omega^{-1}X_{t-1}h_t^\omega)^{1-\sigma} - 1}{1-\sigma} \quad (6)$$

and

$$F(k_t, X_th_t) = k_t^\alpha (X_th_t)^{1-\alpha}. \quad (7)$$

³We have considered several different methods of stationarizing debt-to-GDP in the constrained economy; while each requires slightly different parameters to match the data, the consequences for the dynamics of the economy once it is calibrated to match the target moments are remarkably similar.

Table 1: Baseline parameterization of the model.

Parameter	Concept	Value
σ	Risk Aversion	2.000
ω	Labor Elasticity	1.900
\bar{R}^*	Long Run Interest Rate	1.040
α	Capital Share in Gross Output	0.306
δ	Capital Depreciation Rate	0.088
θ	Disutility of Labor	4.132
$\frac{g}{y}$	Government Exp.-to-GDP in unconstrained steady-state	0.110
γ	Long run productivity growth	1.010
χ	Share of Working Capital	0.258

We calibrate the parameters of preferences and production to match the values used in [Mendoza \(2010\)](#). The adjustment cost function is parameterized as

$$\phi(i/k) = \frac{\exp(\phi(i/k + 1 - \gamma - \delta))}{\phi} - (i/k + 1 - \gamma - \delta) - \frac{1}{\phi}.$$

The non-standard formulation of the adjustment cost is locally equivalent to the standard quadratic specification (it has the same level and slope) but ensures that, globally, Tobin's Q is never negative, a concern in some relatively unlikely regions of the state space.

We assume that the exogenous processes for the productivity shocks and the interest rate are given by

$$\log(\gamma_{t+1}/\gamma) = \rho_x \log(\gamma_t/\gamma) + \epsilon_{\gamma,t-1}^2 + \epsilon_{\gamma,t+1}^0 \quad (8)$$

$$\log(A_{t+1}) = \rho_a \log(A_t) + \epsilon_{A,t+1} \quad (9)$$

$$\log(R_{t+1}^*/\bar{R}^*) = \rho_r \log(R_t^*/\bar{R}^*) + \epsilon_{R,t+1} \quad (10)$$

where $\gamma_t \equiv \frac{X_t}{X_{t-1}}$, long-run gross productivity growth is given by the parameter γ , \bar{R}^* is long-run world interest rate, and the ϵ shocks terms are *iid* across time and variables.

We calibrate the set of preference and production parameters following [Mendoza \(2010\)](#) as closely as possible. We fix these parameters *a priori* and do not vary them in our calibration. We summarize these parameters in table 1. Similarly, the parameters ρ_a , ρ_x , and ρ_r are set to 0.6, 0.35, 0.5 respectively, which are roughly in the middle range of standard values for these parameters in the small open economy literature. There is little agreement on the relative importance of permanent versus stationary shocks in the open economy; our initial calibration puts relatively high weight (around 80 percent)

Table 2: Baseline parameterization of the model - exogenous processes.

Parameter	Concept	Value
σ_x^{news}	Std. Dev. of Trend News shock	0.020
σ_x^{surp}	Std. Dev. of Trend Surprise shock	0.020
σ_a	Std. Dev. of TFP shock	0.010
σ_r	Std. Dev. of Interest Rate shock	0.015
ρ_x	AR coeff. of Trend shock	0.350
ρ_a	AR coeff. of TFP shock	0.600
ρ_r	AR coeff. of Interest Rate	0.500

on non-stationary shocks. For the permanent component in productivity, we assume that one half is driven by news which is consistent with the findings of [Schmitt-Grohé and Uribe \(2012\)](#) of the news component of permanent productivity shocks. Finally, we set the long-run growth-rate of productivity to a conservative one percent annually, $\gamma = 1.01$. The parameters of the exogenous processes are gathered in table 2.

We then focus our calibration on three parameters and three moments from the data. First, we calibrate the discount factor to replicate the approximately two percent probability of a sudden stop event identified in the [Mendoza \(2010\)](#) data. This is a natural target for this parameter because the impatience of agents directly influences their incentives to remain close to the constraint, offsetting precautionary motives that otherwise would push agents further away from it. Second, we calibrate the adjustment cost parameter ϕ to match the approximately 8 percent fall in consumption below trend during Sudden Stop periods, also found by [Mendoza \(2010\)](#). The adjustment cost is the key parameter influencing the magnitude of non-linearities, and therefore the size of crashes, created by the “debt-deflation” spiral incurred whenever the constraint binds. Finally, we choose κ , the key parameter in the collateral constraint to match a long-run debt level of around 60 percent, which is within the range of typical values for emerging market economies. The parameter values required to match these moments are given in table 3.

In order to solve the model, we first stationarize the economy by dividing all trending variables by X_{t-1} . The resulting stationary first order conditions and corresponding balanced growth path are described the Appendix B. We then solve the model using a policy function iteration approach, approximating the policy functions with linear finite element basis functions in the seven-dimensional state space $x_t = [b_t, \log(k_t), \epsilon_{\gamma,t-2}^2, \epsilon_{\gamma,t-1}^2, \log(\gamma_t), \log(A_t), R_t^*]$. The solution procedure delivers piecewise

Table 3: Baseline parameterization of the model - key choices.

Parameter	Concept	Target	Value
β	Discount Factor	S.S. freq $\approx 2.0\%$	0.968
ϕ	Capital Adjustment Cost	S.S. C decline $\approx 8\%$	1.905
κ	Collateral Constraint	Debt/GDP $\approx 60\%$	0.360

linear policy functions $c(x_t)$, $h(x_t)$, and $k(x_t)$ denoting optimal consumption, hours, and capital which minimize mean-squared residuals to equations (11) - (16) as well as the constraints given in (2) - (4) over a finite grid.

4 Results

In this section, we first consider some features of an approximate version of the model in which the leverage constraint is *not* imposed, the interest rate is assumed to be slightly elastic with respect to aggregate debt, and the model is linearized around its stochastic steady-state.⁴ Much of the intuition for the results regarding the model with the constraint can be garnered by examining the correlations of consumption, investment, and leverage induced by the four shocks of the model. After showing in the unconstrained model that news shocks are a promising candidate for driving leverage and leverage-based crisis, we then turn towards a study of the non-linear model, showing that news shocks are indeed a key driver of Sudden Stop events in the model.

4.1 Model without the Leverage Constraint

Figure 2 plots impulse responses for a variety of variables to a one-standard deviation innovation for each shock in the unconstrained linearized model. The blue line represents the “news shock,” which is the key shock for our results. Given our preferences, labor supply in the unconstrained economy responds only to the marginal product of labor in the current period. Since capital is predetermined, output therefore cannot move

⁴With an exogenous world interest rate and no constraint on leverage, the ratio of debt-to-GDP in our economy would be effectively non-stationary. Thus, when we consider version of the model without the leverage constraint, we assume that R_t is related to the world interest rate according to the functional-form suggested by [Schmitt-Grohé and Uribe \(2003\)](#): $\log(R_t) = \log(R_t^*) + \psi \left(e^{\tilde{d}_t/y_t - \bar{d}/y} - 1 \right)$, where the parameter ψ measures the elasticity of the borrowing rate to the current debt-to-GDP ratio, and which we calibrate to be equal to a “low” value 0.01. The “target” long run debt-to-GDP ratio, \bar{d}/y , is chosen in a way that enables us to use the same discount factor for these two models.

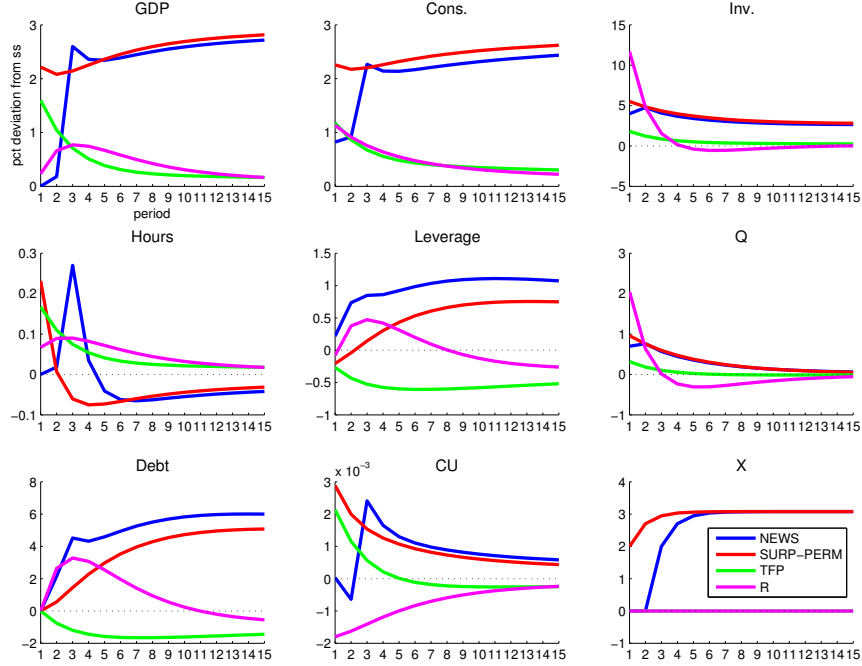


Figure 2: Impulse responses to all four shocks in the unconstrained economy, linearized around the steady-state.

on impact to the news shock. Yet, consumption-smoothing agents foresee high future consumption, and therefore increase consumption today. High consumption today, in turn, can be financed only by either increasing debt or by negative investment. The latter force, however, is countered by the desire to increase investment today in order to take advantage of the forecast future high productivity. Thus the current increase in consumption must be financed to some degree by an increase in debt. While the increase in debt is generic, the consequence for leverage is not. Since the measure of leverage also contains the price of installed capital, which rises on impact whenever investment adjustment costs are substantial, it is a matter of calibration as to whether the effect in the numerator or denominator dominates. According to our baseline calibration, however, the good news shock leads to a substantial increase in leverage, and thus tends to bring the economy closer to leverage levels that would bind if the constraint were imposed.

The responses to the news shock contrasts sharply with the responses to both the contemporaneous productivity growth shock and the temporary TFP shock. The productivity growth shock has a near-zero impact effect on leverage, followed by a positive but muted increase in leverage over subsequent periods. This persistent impact on

Table 4: Variance decomposition for linearized model.

	ΔY	ΔC	ΔI	Q	Leverage	TB/GDP
X_{news}	0.42	0.24	0.07	0.20	0.56	0.27
X_{surp}	0.35	0.48	0.12	0.26	0.27	0.09
A	0.21	0.15	0.02	0.02	0.12	0.03
R	0.02	0.13	0.79	0.52	0.04	0.61

leverage is driven primarily by the autocorrelation in the growth shocks, which causes even the surprise growth shock to contain a degree of news about future productivity. Contrasting even more starkly, a positive temporary TFP shocks in fact leads to both a high level of investment and an overall fall in leverage, a pattern that is not consistent either with the facts surrounding developing economies business cycles or the patterns surrounding Sudden Stop episodes. Finally, lower interest rates leads to relatively small changes in leverage, as investment, debt and Tobin's Q all increase simultaneously.

The different responses of leverage in the linear-unconstrained model are suggestive of our ultimate results in the leverage-constrained economy: since news shocks are the key driver of fluctuations in leverage, in the constrained economy they will also be the key force determining the risk of leverage-based crises. It is precisely the features of news shocks described above - that they tend to drive large pro-cyclical fluctuations in leverage - that make us view them as promising candidates as the shock primarily driving the boom-bust cycles associated with Sudden Stop events.

Table 4 confirms that, unconditionally, leverage in the unconstrained linearized economy is driven primarily by the news shocks and, to a lesser extent, by the surprise temporary and permanent shocks. This table also shows that, despite being temporary, the transient shock to TFP explains a sizable share of the variance in GDP growth rates. This is driven largely by the fact that, despite the autocorrelation in the growth process, shocks to the growth rate behave essentially like random walk shocks, leading to a large but then permanent change in the level of GDP.

4.2 Model with Leverage Constraint

Having made a set preliminary observations about the effects of news shocks in an unconstrained environment, we now turn to examining the non-linear model. Panels (a) and (b) of figure 3 display policy functions for debt accumulation and investment respec-

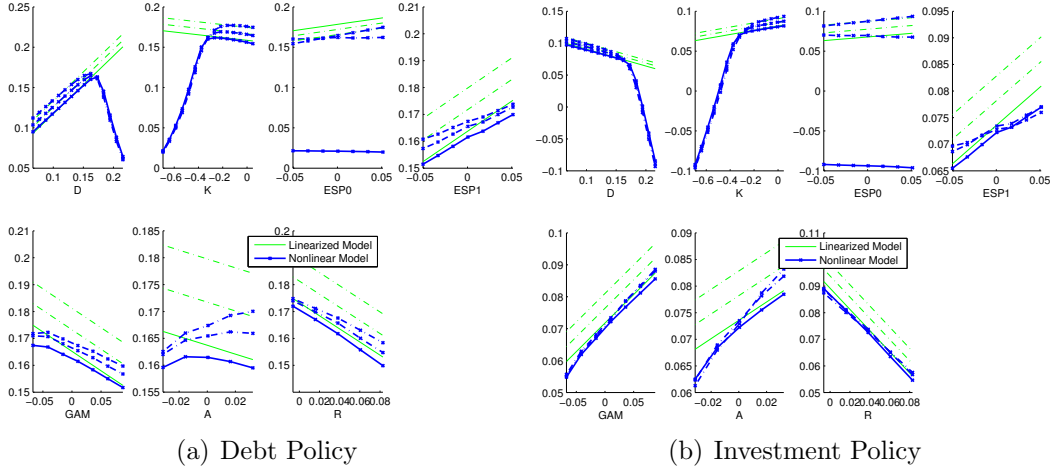


Figure 3: Non-linear policy functions for debt and investment. Lines represent policy as function of the x-axis state, taking as given one of three different levels (high, average, low) of the current “news” innovation, with all other states fixed at their unconditional means. In the third sub-panel of each figure, which plots policy as a function of the news shock, the three lines condition on different levels of the current capital stock.

tively.⁵ Each panel contains three lines, which differentiate between policy functions at low, average, and high realization of the current news shocks, as well as corresponding lines (in light green) for the same object computed in the unconstrained, linearized economy. The figures show that, for many regions of the state space, the policy functions delivered by the global and local solution procedures are quite similar. However, when debt is unusually high or capital unusually low, these functions also demonstrate strong non-linearities. In the first panel of both figures, in cases of extremely high debt, previously parallel lines in fact cross, suggesting that agents with better prospects for the future in fact do a larger degree of deleveraging than they would have otherwise done with lower expectations regarding future productivity growth.

Although some areas of the state-space clearly demonstrate strong non-linearities, in equilibrium these areas of the state-space may be visited infrequently; Sudden Stop events are by their definition infrequent events. In order to assess the importance of the constraint in the economy, we therefore solve fully non-linear versions of the economy both with and without the constraint imposed. For each version of the model, we generate a simulation of 100,000 years. To assess the average effects of the constraint, table 5

⁵Note that these are not the same objects solved for directly by the numerical procedure, and instead are derived as implications of the policy functions for consumption, hours, and capital. For numerical reasons, it is desirable to parameterize the most linear policies, while for exposition of course it informative to see the policies which exhibit the strongest non-linearities.

Table 5: Long-run values for unconstrained and constrained model.

	$\kappa = \infty$	$\kappa = 0.36$
Output	0.277	0.288
Hours	0.198	0.202
Consumption	0.178	0.188
Capital	0.585	0.634
$\frac{d}{y}$	0.669	0.603
leverage	0.387	0.344
p(bind)	0.668	0.190

compares the ergodic mean of the unconstrained economy with the same unconditional means generated by the constrained economy. With regard to unconditional means, the constraint causes only modest changes. Output, hours, and consumption are slightly higher in the model with the constraint, a difference generated by precautionary motives that lead agents to maintain higher average holdings of capital and lower average debt, both of which contribute to higher average consumption. Correspondingly, long run debt-to-GDP and leverage are both reduced to a substantial degree by the presence of the constraint. Overall, *unconditional* long-run average *flows* in the economy are mostly unaffected by the non-linearity in the economy, while *stock* variables such as capital and debt move in the expected direction given the cautionary motives of agents.

Tables 6 and 7 show unconditional second moments for the economy when the constraint does not and does bind respectively. Here, the constraint leads to a small increase in the variance of output, but a substantial increase in the unconditional variance of consumption and investment growth. Most notably, the standard deviation of investment growth increases by roughly 6 percent once the constraint is enforced. Moreover, while consumption growth is substantially less volatile than output in the unconstrained model, its variance increases once the constraint is imposed by around 0.6 percent, such that consumption variance surpasses output variance by a non-trivial amount. This finding echoes those of [Basu and Macchiavelli \(2014\)](#), who show that an always-binding collateral constraint can explain the excess volatility puzzle of consumption in developing economies and find cross-sectional empirical support for the mechanism.

In contrast, the standard deviation of leverage in the model with the binding constraint is roughly one-quarter of its value in the unconstrained model. With the addition of the constraint, the volatility of leverage declines for direct reasons, because the con-

Table 6: Unconditional moments for non-linear unconstrained model.

	ΔY	ΔC	ΔI	Q	Leverage	TB/GDP
Std. Dev.	3.76	3.26	15.11	3.16	7.06	3.86
Std. Dev./Std. Dev. (ΔY)	1.00	0.87	4.02	0.84	1.88	1.03
Auto-Correlation	-0.06	-0.06	-0.22	0.57	0.99	0.44
Corr. with ΔY	1.00	0.87	0.24	0.42	0.10	-0.08

Table 7: Unconditional moments for constrained model.

	ΔY	ΔC	ΔI	Q	Leverage	TB/GDP
Std. Dev.	3.81	4.00	20.87	3.31	1.73	3.89
Std. Dev./Std. Dev. (ΔY)	1.00	1.05	5.47	0.87	0.45	1.02
Auto-Correlation	-0.11	-0.26	-0.49	0.23	0.90	-0.04
Corr. with ΔY	1.00	0.83	0.35	0.44	0.04	-0.14

straint truncates the support of admissible values, and because precautionary motives lead to smaller leverage fluctuations conditional on being away from the constraint.

Figure 4 compares the shape of the equilibrium distributions of debt, investment, consumption, and output growth for the constrained and unconstrained economies. The figures show that, with the imposition of the constraint, investment growth and consumption growth display fat-tailed distributions, with excess kurtosis of 3.5 and 0.7 respectively. The presence of fat tails in these distributions are strong indicators that the non-linearity created by the constraint plays an important role along the equilibrium path of the economy. Given the one-sided nature of the constraint it is perhaps surprising that these distributions remain roughly symmetric. We provide some intuition for this finding in the following paragraphs.

Having established that the non-linearities induced by the leverage constraint matter for the basic unconditional moments of the economy, we now study the conditional moments of the economy in order to assess its ability to match the stylized facts regarding Sudden Stops and consumption and investment risk. Figure 5 offers a preliminary look at the dynamic effects of the constraint on the economy. This figure conditions on the leverage constraint binding in period zero, and plots the median value of various variables in the periods both before and after the identified binding episode.⁶ According

⁶In order to avoid “double counting” episodes, we take only the first instance of the constraint binding within a three period window.

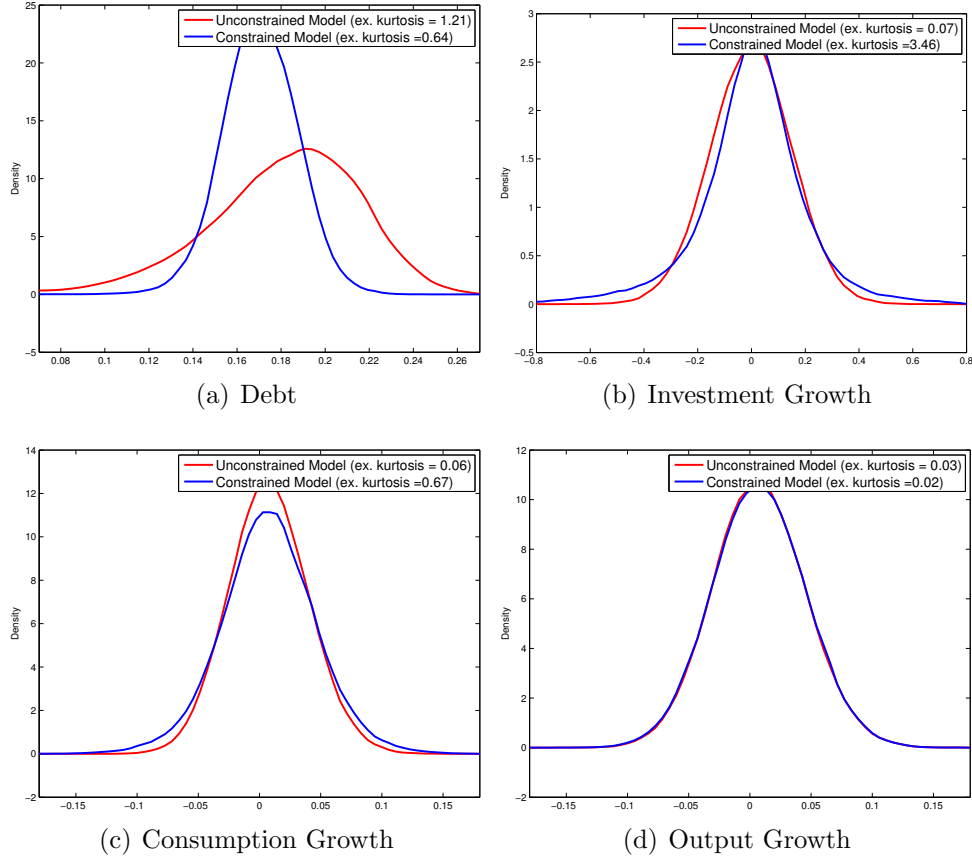


Figure 4: Ergodic distributions for the unconstrained and constrained economies. The distributions of debt is shifted downwards, while the growth rates of investment and consumption demonstrate substantially fatter tails (excess kurtosis) after the imposition of the constraint. The distribution of output growth is only slight changed, and demonstrates no corresponding excess kurtosis.

to our baseline calibration, the constraint binds around 19 percent of time. Included in this sample, therefore, are many instances when the constraint binds in a relatively benign fashion.

Qualitatively, figure 5 shows the set of patterns that might be anticipated by the impulse responses of the unconstrained model. Output, consumption, investment, and hours are all above trend prior to the constraint binding event, and all fall substantially once it binds. In the event period, the trade-balance shows a modest reversal, and leverage is seen to rise prior to the event, and then fall afterwards. The final row of the figure plots the realizations of the various exogenous process that underly the endogenous outcome around the constraint-binding episodes. The row shows that the economy has experienced, on average, a sequence of good news shocks leading up to the crisis and that, moreover, the level of permanent productivity growth in the economy is elevated leading into the event period. In short, good news is predicting future constraint-binding episodes. The rate of permanent productivity growth falls sharply in the period of the constraint binding, indicating on average, a negative contemporaneous growth shock offsetting the previous positive news shocks. Finally, notice that overall stationary TFP is low going into the crisis period, then rises back to its long-run level in subsequent periods. This is also unsurprising, given our observation in the linearized model that negative temporary TFP shocks lead to increased leverage.

The qualitative features of this simple event study demonstrate some of the patterns associated with Sudden Stop events but, at least on average, are not quantitatively consistent with the findings of [Mendoza \(2010\)](#) and others cited earlier. This is easiest to see from the response of Tobin’s Q which, though it falls, remains quite close to its long-run level. Similarly, while stationary TFP is low initially, non-stationary productivity growth is relatively high throughout the period, dropping only slightly below average in the crisis period. This suggests that a good proportion of binding events are relatively benign cases where a long series of good news is momentarily reversed, but not so far as to fully offset the “stock” of good news in the economy.

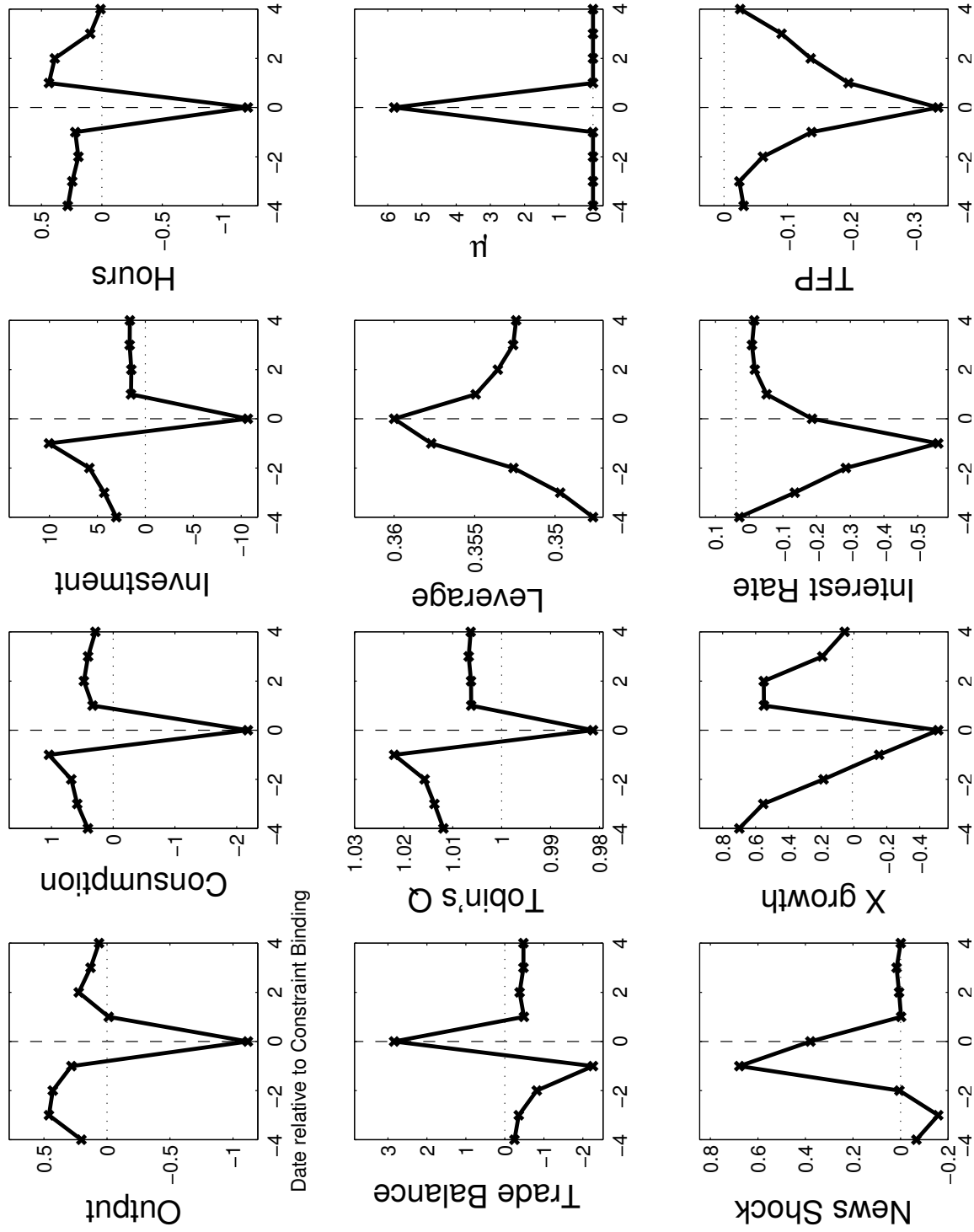


Figure 5: Constraint-binds event study. Panels show deviations from long-run level for each variable in the periods before and after the constraint binds. Output, consumption, investment, hours, and the trade balance plot median deviations from an HP trend, while all others variables plot median deviations from their ergodic means. The unconditional probability of the constraint binding is 19 percent.

Figure 6 seeks to focus attention on recognizable Sudden Stop episodes, and constitutes the main results of our paper. The figure plots the median path of different variables four years before and after Sudden Stop events, identified in the model-simulated data exactly as in [Mendoza \(2010\)](#).⁷ The figure shows that, in the four years prior to the crisis, the economy experiences a concurrent boom in output, consumption, investment, and hours, consistent with the facts he reports. Output falls just slightly in the period prior the Sudden Stop, then declines further to around four percent below trend in the event period. The total fall of output, from peak-to-trough, is just over five percent, slightly below what it is in the data. As a contrast, Mendoza’s calibrated model delivers a smaller initial boom and an overall peak-to-trough change of just slightly over four percent. For consumption, the peak-to-trough fall is over seven percent, which is very close to what is shown in the [Mendoza \(2010\)](#) data, and is substantially larger than the fall implied by his model. The model also delivers a quantitatively realistic investment boom prior to the Sudden Stop event, which is greater than thirty percent in both the model and the data. Finally, the trade balance shows a strong reversal as in the data, although the initial level prior to the Sudden Stop is not as far below trend as Mendoza’s data show.

While the model succeeds in matching the pre-crisis boom and the depth of Sudden Stop crashes, it substantially misses the rate of recovery in these variable post-crash. In particular, the model delivers a return of consumption, output and investment to pre-crisis levels in the year following the Sudden Stop, which is counter-factual. This observation is closely related to the earlier result that the fat-tails in consumptions and investment remained surprisingly symmetrical in the constrained model. We come back to this issue momentarily.

The final row of figure 6 sheds some light on the constellation of shocks leading up to sudden-stop events. The figure shows that news about future productivity is very positive in the period just prior to the Sudden Stop, but is in fact slightly negative in the two periods just prior to the event. In contrast, the level of productivity growth remains elevated in all periods before and after the event. This pattern can be explained upon realizing that, with several periods of delay, both good news and good surprise shocks to growth lead to increased leverage in the unconstrained model. After a sequence of such shocks, a combination of bad surprise shocks and muted news shocks is needed to deliver the below average productivity growth that is required to trigger a true overall

⁷[Mendoza \(2010\)](#) defines a Sudden Stop as a period in which the constraint binds, GDP is one standard deviation below its HP-trend level and the trade balance is one standard-deviation above it’s HP-trend level.

contraction in the economy, rather than a fast “correction” of the endogenous variables from elevated levels towards their long-run values. Although we have not reported it here, filtering Sudden Stop events using growth rates, rather than deviations from long-run trends, delivers the more intuitive result that news shocks are, on average, positive in the periods leading up to the event.

The process for TFP and the growth rate shock also shed light on the reasons for the quick recovery following the Sudden Stop. The figure show that Sudden Stops tend to occur in the economy when the stock of news, which is to be realized at future dates, is high. Since we have assumed a two period lag for news, there exists only one period during which the stock of unrealized news may remain high, and this news *must* be realized in the period following the Sudden Stop, leading to pattern of high productivity growth that systematically follows Sudden Stop events. Moreover, since the temporary TFP shock tends to be low in these periods, and is mean reverting, it also contributes to a fast increase in output and hours over the periods following the Sudden Stop. In principle, this excess speed of recovery could be mitigated by assuming longer horizons for news shocks, spreading out the eventual realization of news over potentially several periods post-crisis. While this is a natural avenue, numerical constraints on the size of state space constrain us to relatively short horizons for the news shocks.

Finally, the dashed-circle lines in figure 6 plot counterfactual outcomes in the *unconstrained* economy conditional on identical shock histories. The consumption fall in the same Sudden Stop periods are less than half that implied by the model with constraints, and the investment fall nearly an order of magnitude smaller. The consequences for hours are somewhat smaller, while the non-linearities induced on output are quite small. Overall, the figure demonstrates that for variables other than output, the non-linearities induced by the constraint are substantial.

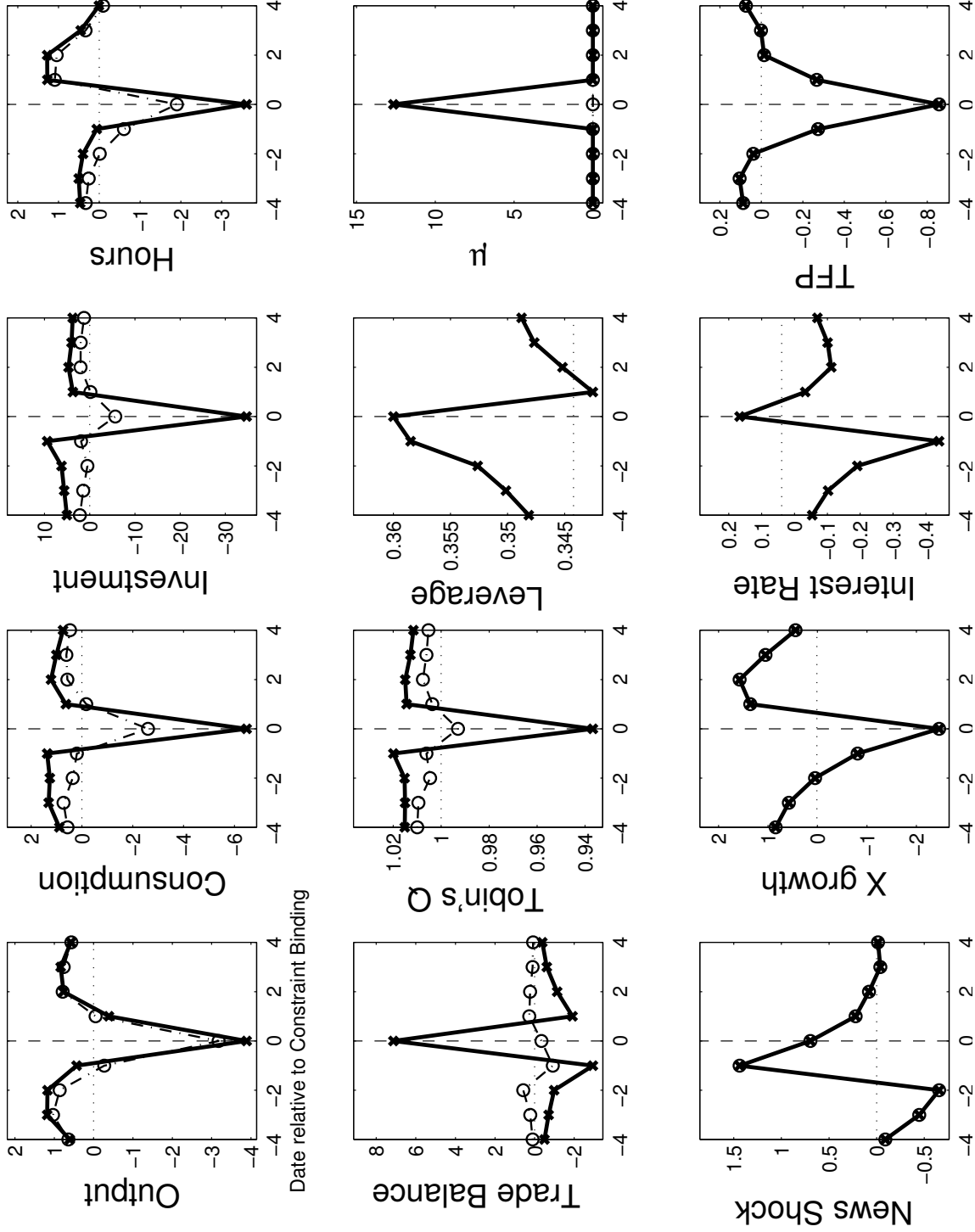


Figure 6: Sudden Stop event study. Solid lines plot median deviations from long-run level in the periods before and after a sudden stop event, identified as in [Mendoza \(2010\)](#) as periods in which the leverage constraint binds, output is at least one standard deviation below trend, and the trade balance is one standard deviation above trend. Output, consumption, investment, hours, and the trade balance plot median deviations from an HP trend, while all others variables plot median deviations from their ergodic means. For consecutive periods that fit this criterion, we plot based only the initial period. The unconditional probability of a sudden stop event in the model is 2.0 percent. Dashed lines plot counter-factual outcomes in model without leverage constraint.

4.3 News as a Risk Shock

To what degree does good news today predict future negative outcomes? Figure 7 plots the probability that the constraint binds in future periods as a function of the current percentile of the news shock hitting the economy. The first panel, which plots the probability of a binding constraint one period forward, demonstrates a strong upward slope: the constraint is roughly seven times more likely to bind tomorrow if the news arriving today is in the 95-percentile relative to the 5-th percentile. The second panel of figure 7 shows that the relationship between news and the constraint binding *in the period the news is forecasted to arrive* is far less straightforward. For low values of the news shock, the line is clearly upward sloping. However, for intermediate and high values of new, the relationship is non-monotonic and, indeed, non-concave: intermediate and very high values of news shocks are associated with binding episodes, while modestly high-value are not. Perhaps more importantly, the overall range of this conditional variation is far more narrow: between 15 and 22 percent, indicating that news today is not a very good indicator that the constraint may bind in the period that news is expected to arrive.

Figure 8 plots the analogous probability that an identified Sudden Stop episode occurs one and two periods forward as a function of the current news shock hitting the economy. The function in first panel is again clearly upward sloping, with probability of a Sudden Stop nearly zero when the “stock” of news is bad, reaching nearly five percent when news is very good. The strong convexity of this figure shows that very high values of the news shock are especially good predictors of Sudden Stop risk, even though of course they always remain unexpected events. In terms of the risk of Sudden Stops, good news is indeed bad news. The downward slope of the second panel, however, demonstrates the importance of the requirement that output is below the HP-trend level in the period of the sudden stop; identifying Sudden Stop with a reversal in output growth, rather than a low level relative to trend, would deliver a figure more similar to the second panel of Figure 7.

Finally, motivated by the stylized fact in figure 1, we examine the implication of news for the conditional variance of consumption. The panels in figure 9 show that Sudden Stop risk, and risk to consumption growth, are not perfectly correlated. Panel (a) shows the median and 95 percent confident bands of one-period ahead consumption growth conditional on the percentile of the current news shock. The figure shows that risk to consumption growth increases monotonically with “better” news, with the width of the 95 percent forecasting interval rising from around 13 to 20 percent, as the news shock

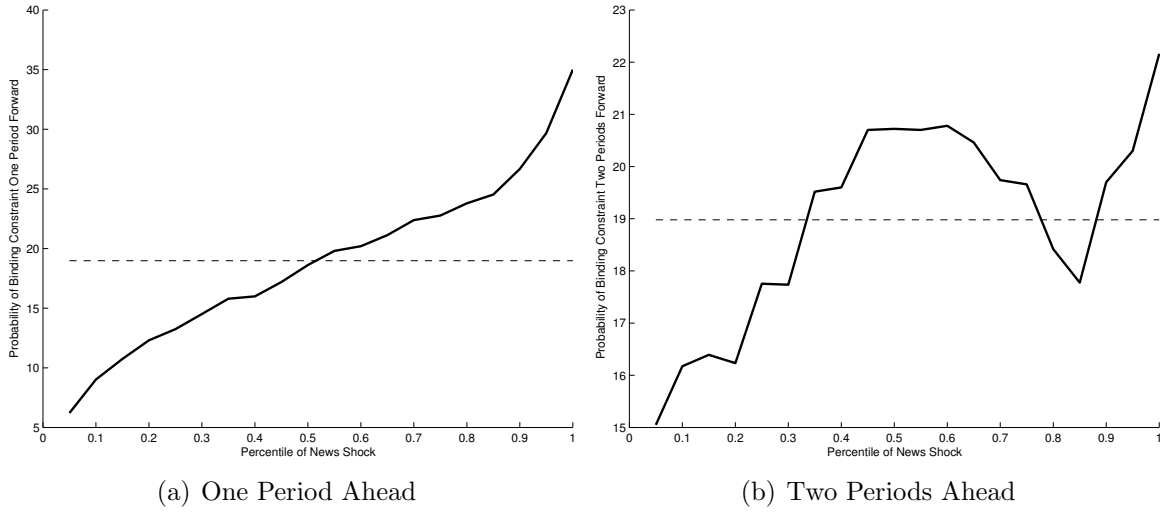


Figure 7: The figure plots the probability of a constraint-binding event occurring two periods hence conditional on the percentile of the trend news shock today.

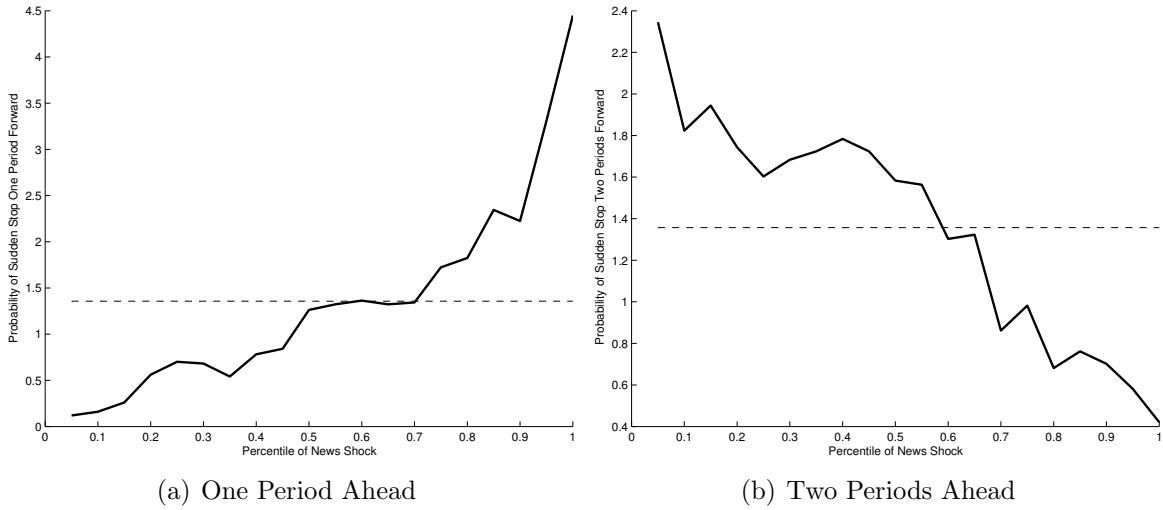


Figure 8: The figure plots the probability of a sudden stop event occurring two periods hence conditional on the percentile of the trend news shock today.

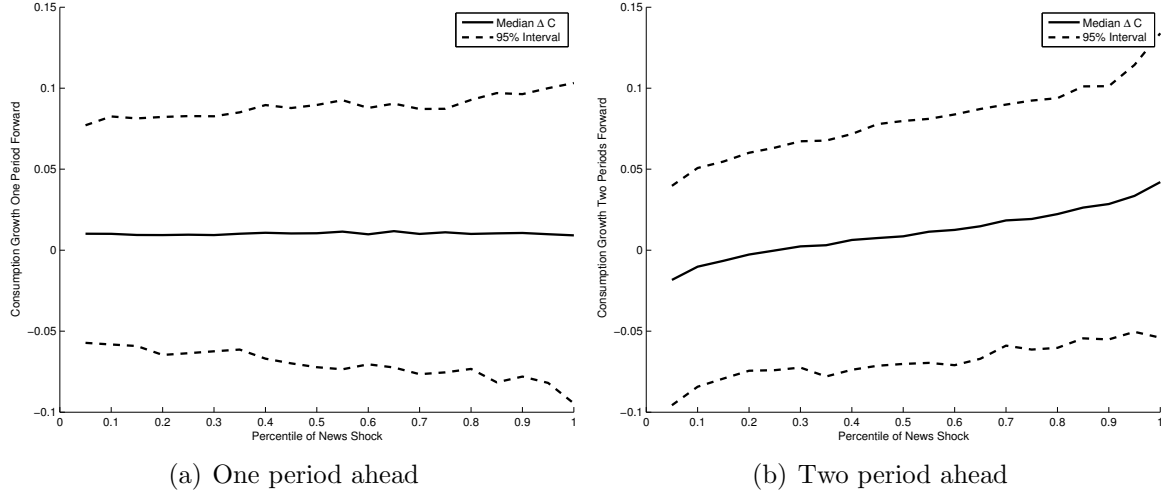


Figure 9: Good news leads to greater tail risk in later periods. The figure plots the conditional 99 percent confidence bands of consumption growth subsequent to different levels of the current realization of the news shock. News today leads to higher consumption growth variance in the following period. Two periods ahead, goods news lead to high average growth, but again a larger distribution of possible growth outcomes.

increases from its first decile to the the last. The result in this panel is consistent with the finding that good news increases Sudden Stop risk, since such events are associated with large moments in consumption, but it also suggests that criteria used to identify Sudden Stops is missing important aspect of consumption risk. The near linearity of the conditional quantiles indicates, instead, that moderately good news shocks lead to nontrivial increases in consumption risk. Panel (b) shows the same average and 95 percent confidence bands for two-period ahead consumption growth. Since news affects productivity two periods ahead, average consumption growth increases with higher news shocks as this horizon as expect. Yet, the width the of the distributions also increases with news, going from 13 to around 18 percent, indicating that *even in the period of realization* good news increases consumption risk; this increase in risk, however, is also associated with shift in the average that decreases the likelihood that any particular outcome is identified as a Sudden Stop according to standard definitions.

Finally, figure 10 reproduces the reduced-form evidence from figure 1 in the context of the model. It plots consumption growth one period forward as a function of current consumption growth less current output growth. Above the 20 percentile, the figure reproduces the increasing dispersion and flat mean found in the data. Quantitatively, the degree of dispersion and its increase with higher current levels of excess consumption are quite realistic. The model delivers counterfactual predictions for the lowest levels of

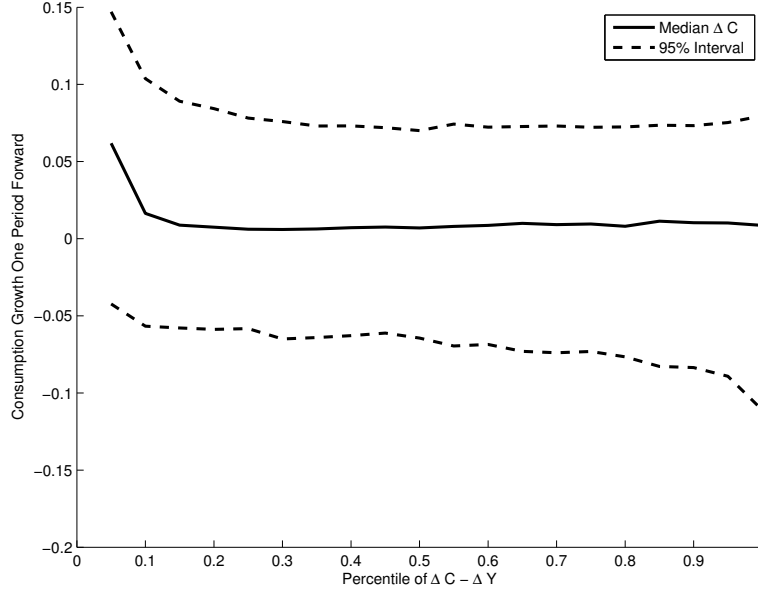


Figure 10: Excess consumption growth and risk.

excess consumption growth, however. This results is due directly to the unrealistically quick recoveries we noted earlier: the non-linear aspects of the model imply that crisis periods are periods with very low consumption growth relative to output, and quick recoveries implies high consumption growth in the period subsequent to such events.

5 Conclusions

This paper has shown that shocks to expectations about future productivity growth are a good candidate for explaining the observed patterns in developing economies of concurrent growth and leverage expansion followed by occasional reversals in both leverage and real quantities. The presence of an occasionally binding collateral constraint amplifies these reversals substantially, yielding the characteristic features of Sudden Stops. The simple model presented here does a remarkably good job at matching the stylized facts about Sudden Stops both qualitatively and, for many variables, quantitatively. Moreover, the arrival of good news leads to a high-probability of “tail” outcomes, including large decreases in consumption. The presence of externalities in this context, and consequently the insufficiently strong precautionary motives faced by agents, suggests the possibility that the information contained in news shocks could, in fact, be detrimental to welfare. We plan to examine this possibility in future work.

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A Household's Optimality Conditions

Let the multipliers on the constraints in equation (2) through (4) be given by λ_t , $\lambda_t q_t$, and $\lambda_t \mu_t$ respectively. Then the first order conditions of the household problem are

$$U_{c,t} = \lambda_t \quad (11)$$

$$-\frac{U_{c,t}}{U_{h,t}} = A_t F_{h,t} - \chi(R_t - 1)w_t - \mu_t \chi R_t w_t \quad (12)$$

$$q_t = 1 + \phi' \left(\frac{i_{t+1}}{k_{t+1}} \right) \quad (13)$$

$$q_t(1 - \mu_t \kappa) = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(A_t F_{k,t+1} + q_{t+1}(1 - \delta) + \phi' \left(\frac{i_{t+1}}{k_{t+1}} \right) \left(\frac{i_{t+1}}{k_{t+1}} \right) - \phi \left(\frac{i_{t+1}}{k_{t+1}} \right) \right) \right] \quad (14)$$

$$(1 - \mu_t) = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} R_t \right], \quad (15)$$

the constraints in equations (2) - (4), as well as the complementary slackness conditions $\mu_t \geq 0$ and

$$\mu_t \left(\kappa - \frac{\frac{d_{t+1}}{R_t} + \chi R_t (w_t h_t)}{q_t k_{t+1}} \right) = 0. \quad (16)$$

B Stationary Equilibrium and Model Steady-state

Let $\tilde{z}_t \equiv z_t/X_{t-1}$, for $z_t \in \{c_t, k_t, w_t, b_t\}$, and let $\tilde{\lambda}_t \equiv \frac{\lambda_t}{X_{t-1}^{1-\sigma}}$. Then, given the functional forms provided above, the stationary first order conditions of the economy are given as follows.

$$\tilde{\lambda}_t = \left(\tilde{c}_t - \theta \frac{h_t^\omega}{\omega} \right)^{-\sigma} \quad (17)$$

$$\theta h_t^{\omega-1} = A_t(1-\alpha)\tilde{k}_t^\alpha h_t^{1-\alpha} \gamma_t^{1-\alpha} - \chi(R_t - 1)\tilde{w}_t - \mu_t \chi R_t \tilde{w}_t \quad (18)$$

$$q_t = 1 + \phi \left(\frac{\tilde{i}_t}{\tilde{k}_t} + 1 - \gamma - \delta \right) \quad (19)$$

$$q_t(1 - \mu_t \kappa) = \beta E_t \left[\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \gamma_t^{-\sigma} \left(A_{t+1} \alpha \tilde{k}_{t+1}^{\alpha-1} (\gamma_{t+1} h_{t+1})^{1-\alpha} + q_{t+1}(1 - \delta) + \dots \right. \right. \\ \left. \left. \phi' \left(\frac{\tilde{i}_{t+1}}{\tilde{k}_{t+1}} \right) \left(\frac{\tilde{i}_{t+1}}{\tilde{k}_{t+1}} \right) - \phi \left(\frac{\tilde{i}_{t+1}}{\tilde{k}_{t+1}} \right) \right] \right] \quad (20)$$

$$(1 - \mu_t) = \beta E_t \left[\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \gamma_t^{-\sigma} R_t \right] \quad (21)$$

$$\tilde{k}_{t+1} \gamma_t = (1 - \delta) \tilde{k}_t + \tilde{i} \quad (22)$$

$$\tilde{c}_t = A_t \tilde{k}_t^\alpha (h_t \gamma_t)^{1-\alpha} - \tilde{d}_t + \tilde{d}_{t+1} \gamma_t / R_t - \chi(R_t - 1)\tilde{w}_t - \tilde{i}_t - \tilde{k}_t \phi \left(\frac{\tilde{i}_t}{\tilde{k}_t} \right) \quad (23)$$

$$\log(R_t) = \log(R_t^*) \quad (24)$$

as well as the complementary slackness conditions $\mu_t \geq 0$ and

$$\mu_t \left(\kappa - \frac{\frac{\tilde{d}_{t+1}}{R_t} + \chi R_t \tilde{w}_t \tilde{h}_t}{q_t \tilde{k}_{t+1}} \right) = 0, \quad (25)$$

and the equilibrium definition $\tilde{w}_t = \theta h_t^{\omega-1}$.

Linearization requires that we solve the for non-stochastic steady of the economy. To do this, we assume values for \bar{h} and \bar{d}/y , and then find the values of θ and long-run debt that are consistent with our assumptions. Rearranging equation (20) and imposing steady-state implies that

$$\frac{k}{h} = \left[\frac{\frac{\gamma^\sigma}{\beta} - 1 + \delta}{\alpha} \right]^{\frac{1}{\alpha-1}} \quad (26)$$

Given our assumption for \bar{h} , the long run capital level follows immediately. From there, the resource constraint and the production function can be used to determine consumption, and equation (17) can be solved for θ .