Offick, Sven; Wohltmann, Hans-Werner

Working Paper

Volatility effects of news shocks in New Keynesian models with optimal monetary policy

Economics Working Paper, No. 2016-06

Provided in Cooperation with:
Christian-Albrechts-University of Kiel, Department of Economics

Suggested Citation: Offick, Sven; Wohltmann, Hans-Werner (2016) : Volatility effects of news shocks in New Keynesian models with optimal monetary policy, Economics Working Paper, No. 2016-06, Kiel University, Department of Economics, Kiel

This Version is available at:
https://hdl.handle.net/10419/130566

C | A | U
Christian-Albrechts-Universität zu Kiel
Department of Economics
Economics Working Paper
No 2016-06

volatility effects of news shocks in new keynesian models with optimal monetary policy -updated version-

by Sven Offick and Hans-Werner Wohltmann

issn 2193-2476

Volatility effects of news shocks in New Keynesian models with optimal monetary policy

Sven Offick[†], *Christian-Albrechts-University Kiel*

Hans-Werner Wohltmann[‡], *Christian-Albrechts-University Kiel*

April 7, 2016

Abstract

This paper studies the volatility implications of anticipated cost-push shocks (i.e. news shocks) in a New Keynesian model with hybrid price setting both under optimal unrestricted and discretionary monetary policy with flexible inflation targeting. If the degree of backward-looking price setting behavior is sufficiently small (large), anticipated cost-push shocks lead in both policy regimes to a higher (lower) volatility in the output gap and in the central bank's loss than an unanticipated shock of the same size. This inversion of the volatility effects of news shocks follows from the inverse relation between the price-setting behavior and the optimal monetary policy. Under a fully microfounded hybrid New Keynesian Phillips curve with price indexation, this inversion of volatility results is not possible since the Phillips curve remains hybrid even in the limit case of full price indexation.

JEL classification: E32, E52

Keywords: Anticipated shocks, Optimal monetary policy, Volatility

[†]Department of Economics, Christian-Albrechts-University Kiel, Olshausenstr. 40, 24098 Kiel, Germany. Phone: ++49-431-880-1472, E-mail: offick@economics.uni-kiel.de

[‡]Department of Economics, Christian-Albrechts-University Kiel, Olshausenstr. 40, 24098 Kiel, Germany. Phone: ++49-431-880-1447, E-mail: wohltmann@economics.uni-kiel.de

1 Introduction

Several empirical studies emphasize the importance of news shocks for business cycle fluctuations. These shocks materialize in the future, but their size and maturity time is anticipated in advance by the agents. Most prominently, Schmitt-Grohé and Uribe (2012) find in an estimated real business cycle model that about 50 percent of economic fluctuations can be attributed to anticipated disturbances.[1]

A theoretical branch of the literature indicates that news shocks destabilize the economy, i.e. lead to a higher volatility than unanticipated shocks of the same form. Fève et al. (2009) demonstrate in a purely forward-looking rational expectations model that news shocks increase the volatility with increasing length of anticipation. With both backward- and forward-looking expectations, the volatility results are ambiguous as it is shown by Winkler and Wohltmann (2012) in an univariate model. However, they find that the anticipation of cost shocks – as considered here – greatly amplifies the volatility of all key macroeconomic variables in the estimated model of Smets and Wouters (2003).[2]

These (empirical and theoretical) findings rely on the assumption of forward-looking rational expectations. By contrast, under purely backward-looking expectations, the volatility is independent of the anticipation horizon.[3] Backward-looking expectations can be introduced via price indexation, rule-of-thumb behavior or bounded rationality.[4]

So far, optimal monetary policy has been studied almost exclusively in the presence of unanticipated disturbances.[5] One exception is the study of Winkler and Wohltmann (2011), who analyze optimal simple interest rules. They find that the inclusion of forward-looking

[1] Note that we limit our discussion to cost-push shocks for which the central bank faces a trade off between output and inflation stabilization even without instrument target as considered here. This type of shock is also found to be highly relevant for business cycle fluctuations, see e.g. Schmitt-Grohé and Uribe (2012).

[2] Further related to this branch of literature is the paper by Offick and Wohltmann (2013), who study the properties of the lag polynomial associated with news shocks.

[3] To see this, consider the model $y_t = \rho y_{t-1} + \varepsilon_{t-q}$, where $\varepsilon_{t-q} \sim N(0, \sigma^2)$ is an i.i.d. news shock that is anticipated q periods in advance. Assuming stationarity, the variance of this model is given by $Var(y_t) = \sigma^2/(1-\rho^2)$, i.e. independent of q.

[4] Bounded rationality assumes that agents have cognitive limitations and use simple heuristics (rule of thumbs) to guide their behavior and are recently under growing investigation, see e.g. De Grauwe (2012) and Lengnick and Wohltmann (2016).

[5] This includes Leitemo (2008), who finds an inverse relation between the private pricing behavior and the optimal monetary strategy. If the private sector is backward-looking, monetary policy should be forward-looking, and vice versa. This general result also holds for news shocks.

elements in an instrument rule is welfare enhancing in the case of anticipated shocks.[6] However, they focus on purely forward-looking private expectations and the resulting welfare effects. By contrast, we study the relation between news shocks, volatility, optimal monetary policy, and hybrid price setting.

In light of these findings, our paper contributes to the existing literature in three ways: First, we combine the theory of news shocks and optimal monetary policy in a New Keynesian framework. Second, we study the (de)stabilizing effects of anticipated cost shocks in a multivariate environment. Third, we analyze how the relative volatility results of news shocks change with increasing degree of backward-looking price setting behavior. We provide analytical results for the limit case of purely forward- and purely backward-looking price setting behavior.[7]

2 News shocks and optimal monetary policy

We assume that the inflation rate is governed by a standard hybrid New Keynesian Phillips curve of the form

$$\pi_t = \beta(1-\phi_\pi)E_t\pi_{t+1} + \beta\phi_\pi\pi_{t-1} + \kappa x_t + \varepsilon_{t-q} \tag{1}$$

where π_t and x_t are the inflation rate and the output gap measured as percentage deviations from the steady state, respectively. ϕ_π measures the degree to which the price setting behavior is backward-looking. For $\phi_\pi = 0$ ($\phi_\pi = 1$), the price-setting behavior is purely forward-looking (backward-looking).[8] ε_{t-q} is a white noise cost-push shock with unit variance which is anticipated q periods in advance. The shock is unanticipated for $q = 0$.

For convenience, we assume first that the central bank aims to minimize the weighted sum

[6] Further noteworthy is the paper by Winkler and Wohltmann (2009), who show how to solve rational expectations models with news shock under optimal unrestricted monetary policy.

[7] Details on the derivation of our results can be found in the Appendix.

[8] This general form is taken from Leitemo (2008). In Section 3, we consider a microfounded hybrid Phillips curve with partial price indexation, which does however not nest a purely backward-looking Phillips curve as special case.

of variance of the inflation rate and the output gap:[9]

$$Loss_q = Var_q(\pi_t) + \lambda Var_q(x_t) \tag{2}$$

We compute the optimal unrestricted monetary policy response under (timeless) commitment and the discretionary policy.[10] The optimal targeting rule under commitment, in which the central bank is able to commit to future policies, includes forward- and backward-looking elements:

$$\pi_t = -\frac{\lambda}{\kappa}(x_t - x_{t-1}) - \frac{\lambda}{\kappa}\phi_\pi x_{t-1} + \frac{\lambda}{\kappa}\beta^2\phi_\pi E_t x_{t+1} \tag{3}$$

Note that in both limit cases ($\phi_\pi = 0$ and $\phi_\pi = 1$) the system remains hybrid. This is due to the inverse relation between the price-setting behavior and the optimal monetary strategy.

Contrarily, the optimal discretionary policy is independent of backward-looking elements:

$$\pi_t = -\frac{\lambda}{\kappa}x_t + \frac{\lambda}{\kappa}\beta^2[(1-\phi_\pi)\rho_\pi^2 + \phi_\pi]E_t x_{t+1} \tag{4}$$

The undetermined coefficient ρ_π follows from the reduced form of inflation. ρ_π is independent from the anticipation horizon q and solves the following polynomial equation of order five:

$$\begin{aligned} 0 = {} & \beta^3(1-\phi_\pi)^2\rho_\pi^5 - \beta^2(1-\phi_\pi)\rho_\pi^4 + 2\phi_\pi\beta^3(1-\phi_\pi)\rho_\pi^3 - [\beta(1-\phi_\pi) + \beta^2\phi_\pi]\rho_\pi^2 \\ & + \left(1 + \frac{\kappa^2}{\lambda} + \beta^3\phi_\pi^2\right)\rho_\pi - \beta\phi_\pi \end{aligned} \tag{5}$$

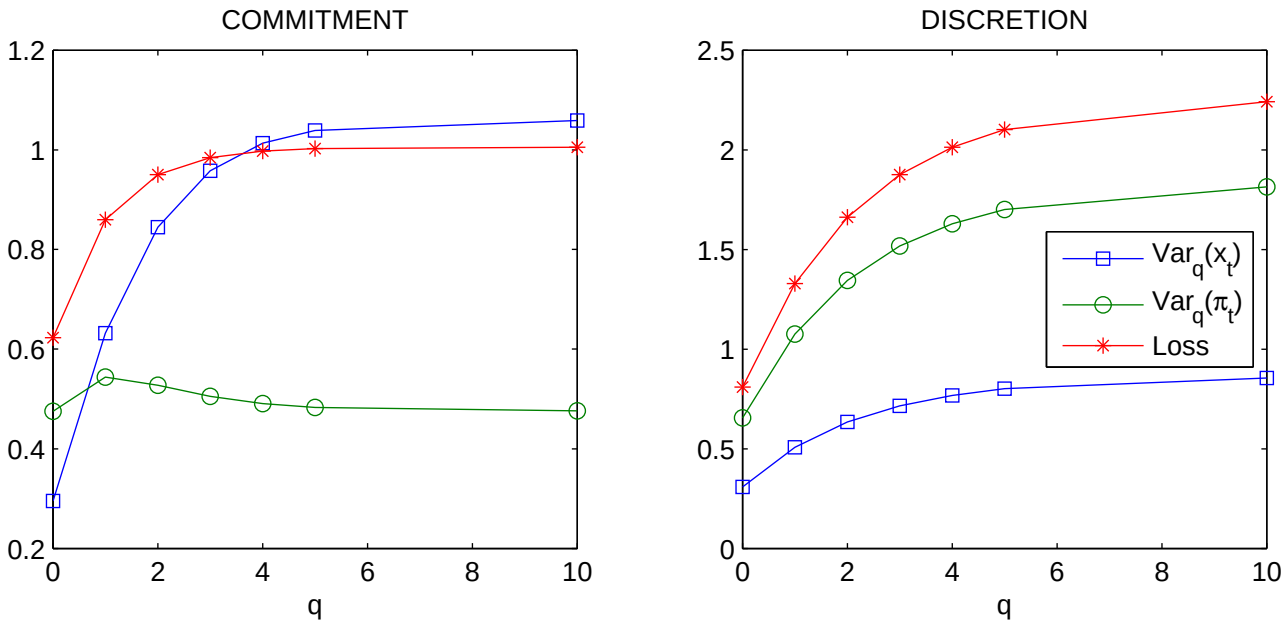
Note that under purely forward-looking price setting ($\phi_\pi = 0$), we obtain $\rho_\pi = 0$ and the discretionary policy is not forward-looking, but given by $\pi_t = -(\lambda/\kappa)x_t$. Under purely backward-looking price setting ($\phi_\pi = 1$), the discretionary policy is equivalent to the commitment policy.

Both targeting rules (3) and (4) are independent of the lead time q and, therefore, equivalent to the optimizations in Leitemo (2008). Equations (1) and (3) fully describe the dynamics of

[9] In Section 3, we also consider a micro-founded loss function which is a second order Taylor approximation of the household's welfare function.

[10] For simplicity, we refer in the following to the optimal unrestricted monetary policy as the policy under commitment.

Figure 1: Loss and variances in the case of purely forward-looking price setting



Note: Parameters are set to $\beta = 0.99$, $\kappa = 0.34$, $\lambda = 0.5$. Under commitment, the inflation variance peaks at $q^* = 0.74$ (assuming continuity of q).

the output gap and the inflation rate under commitment. Equations (1) and (4) fully describe the dynamics under discretionary policy.

Before we turn to the general case of hybrid private price-setting behavior, we discuss the limit case of purely forward-looking price setting.

2.1 Purely forward-looking price setters

We first discuss the volatility results in the regime commitment. For $\phi_\pi = 0$, the system can be reduced to an univariate hybrid equation of the form

$$x_t = aE_t x_{t+1} + bx_{t-1} + c\varepsilon_{t-q} \tag{6}$$

with $a = \beta b$, $b = \lambda/(\lambda(1+\beta) + \kappa^2)$, and $c = -\kappa/(\lambda(1+\beta) + \kappa^2)$. Since $1 > \beta > 0$, $sgn(a) = sgn(b)$. This implies that the variance of x_t is unambiguously increasing in q as it is shown by Winkler and Wohltmann (2012).

The volatility of the inflation rate, on the other hand, may also be decreasing in q. Its

variance is given by

$$Var(\pi_t) = \frac{2\beta_0^2}{(1+\alpha)(1+\delta)(1-\alpha\delta)} \left(\frac{\lambda}{\kappa}\right)^2 \left[1 - \frac{1-\alpha\delta}{\alpha-\delta}\delta^{2(q+1)} + \frac{(1-\alpha)(1+\delta)\delta\alpha}{\alpha-\delta}(\alpha\delta)^q\right] \quad (7)$$

where $|\alpha| < 1$ is the stable root of $\alpha_{1,2} = \left(1 \pm \sqrt{1-4ab}\right)/(2a)$, $\beta_0 = c/(1-a\alpha)$, and $\delta = a/(1-a\alpha)$.[11] An unanticipated shock may generate a higher inflation volatility than a cost-push shock that is anticipated in the infinite past:

$$Var_{q=0}(\pi_t) > Var_{q\to\infty}(\pi_t) \quad \text{if} \quad \frac{\kappa^2}{\lambda} > \sqrt{1+4\beta} - (1+\beta) \qquad (\lambda > 0) \quad (8)$$

The reason for the ambiguity in the inflation volatility are two opposing effects: On the one hand, the longer the length of anticipation, the higher is the variance of the output gap, which – in isolation – also leads to a higher variance in inflation. On the other hand, the response of the output gap becomes smoother, i.e. x_t is more autocorrelated, with increasing q. Since the inflation rate depends via the targeting rule on the change in the output gap, this reduces – in isolation – the variance of inflation.[12] Condition (8) does not imply that an anticipated cost shock gives a lower inflation volatility for all anticipation horizons. That is, the inflation variance may not be monotonic in q. For small values of q, $Var(\pi_t)$ may be increasing and the maximum is reached in $q = \max(q^*, 0)$ where

$$q^* = \frac{1}{\log\alpha - \log\delta}\left\{\log\frac{2\delta(1-\alpha\delta)}{(1-\alpha)\alpha(1+\delta)} + \log\frac{\log\delta}{\log\alpha\delta}\right\} \quad (9)$$

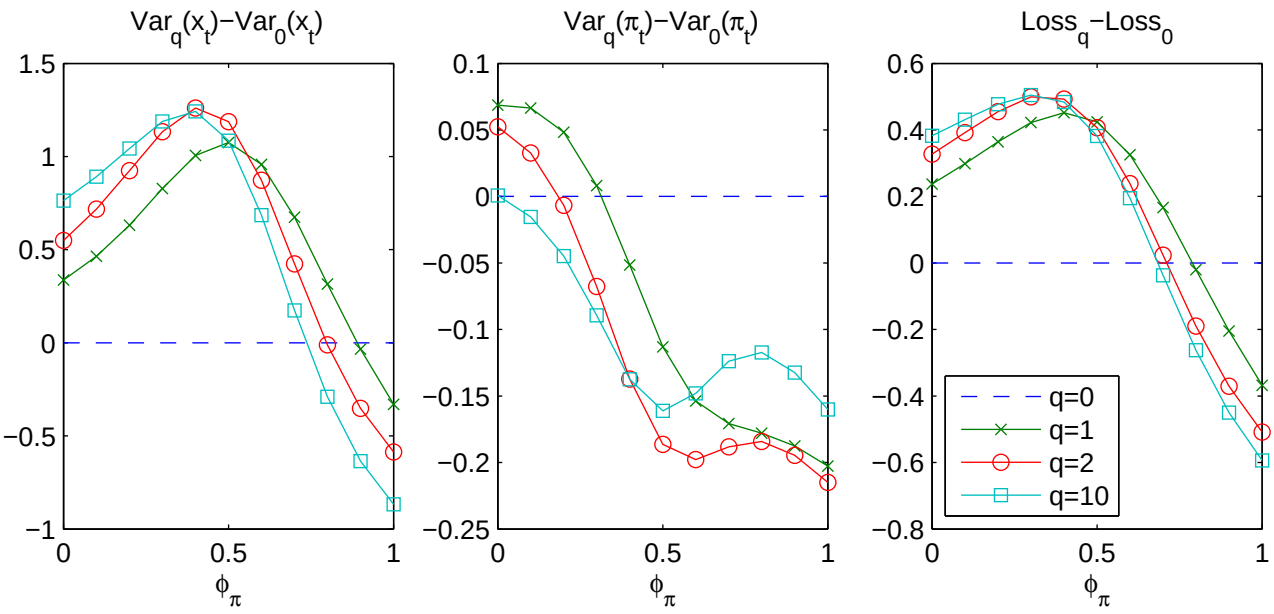
Despite the fact that the variance of inflation may be decreasing in q, the total loss (2) is always increasing in q. Only under strict inflation targeting ($\lambda = 0$) does the central bank perfectly stabilize the inflation rate and the loss is zero, independently of q.

Note that under purely forward-looking price setting behavior, the welfare theoretic loss is of the same form as the assumed loss (2) with $\lambda = \kappa/\chi$, where χ denotes the elasticity of substitution between differentiated goods. The inequality (8) then reads $\kappa\chi > \sqrt{1+4\beta} - (1+\beta)$.

[11] Note that the output gap can be written as an ARMA(1,q) process of the form $x_t = \alpha x_{t-1} + \sum_{k=0}^{q} \delta^k \beta_0 \varepsilon_{t+k-q}$. A stable solution requires $|\alpha| < 1$.

[12] The two opposing effects can be directly seen by taking the variance of the targeting rule: $Var(\pi_t) = 2\lambda^2/\kappa^2[Var(x_t) - E(x_t x_{t-1})]$, where both $Var(x_t)$ and $E(x_t x_{t-1})$ are increasing in q.

Figure 2: Loss and variances for different degrees of hybridity under commitment



Parameter calibration: $\beta = 0.99$, $\kappa = 0.34$, $\lambda = 0.5$.

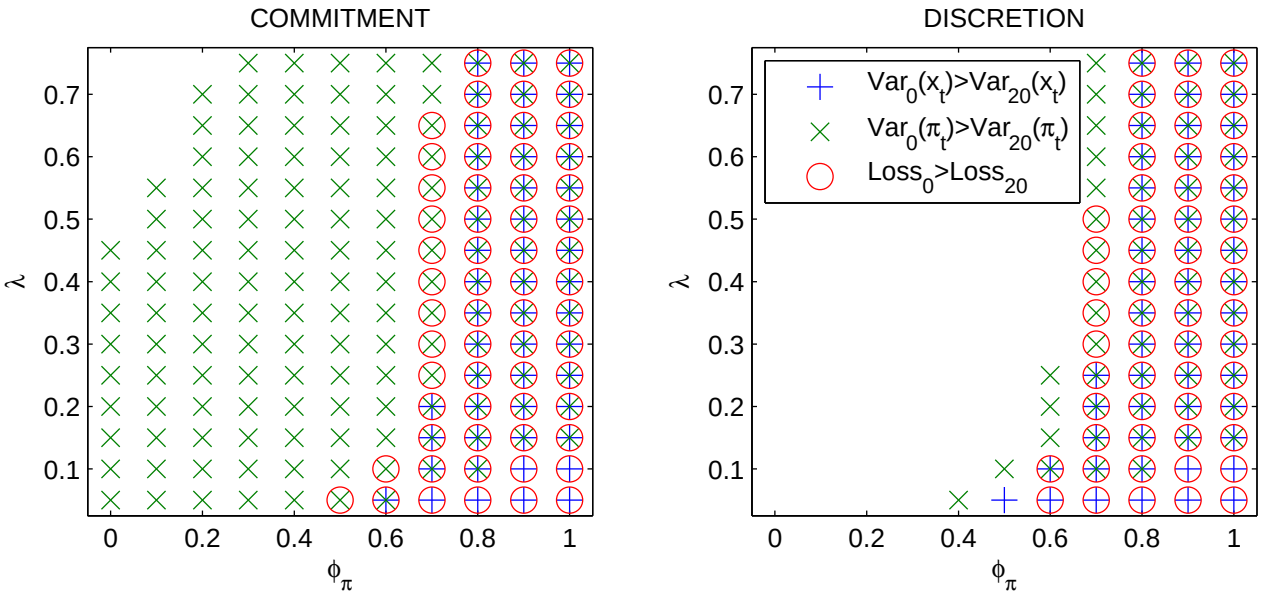
For $\chi = 8$ (implying a steady state mark-up of approximately 14 percent) and $\kappa = 0.34$, $\lambda \approx 0.05$ so that inequality (8) is satisfied as illustrated in figure 3.

In the regime discretion, the volatility of the output gap and the inflation rate is unambiguously increasing in q under purely forward-looking price setting. For $\phi_\pi = 0$, the model under discretion is purely forward-looking. This implies that the volatility of inflation and output gap is increasing in q as it is shown by Fève et al. (2009). These results are illustrated in figure 1.

2.2 Hybrid price-setting behavior

If we allow for backward-looking price-setting behavior (i.e. $\phi_\pi > 0$), the results under purely forward-looking price setting of the previous subsection may be reversed. We start with the regime commitment. For this regime, figure 2 shows the differences in the volatilities and the central bank's loss between an anticipated and an unanticipated cost shock for different degrees of hybridity and anticipation horizons. If ϕ_π – the degree of backward-lookingness – is sufficiently large, an anticipated cost shock leads simultaneously to a lower volatility in output and inflation and to a lower loss than an unanticipated shock of the same size. The reason for this inversion of volatility results (compared to $\phi_\pi = 0$) is the inverse relation between the

Figure 3: Parameter sensitivity



Note: The remaining parameters are calibrated as follows: $\beta = 0.99$, $\kappa = 0.34$. Note that $\oplus$ means that both $Var_{20}(x_t)$ and $Loss_{20}$ are smaller, $\otimes$ means that both $Var_{20}(\pi_t)$ and $Loss_{20}$ are smaller, and the combination of all three symbols means that both variances and the loss are smaller for $q = 20$ than for $q = 0$.

private pricing behavior and the optimal unrestricted monetary policy strategy.

Does this result also hold for lower weights λ the central bank puts on output stabilization? Figure 3 compares the volatilities and the loss of an unanticipated ($q = 0$) and an anticipated ($q = 20$) shock for different degrees of hybridity ϕ_π and for different weights λ. The left plot considers the regime commitment. For all values of $\lambda > 0$ (flexible inflation targeting), it holds: If the degree of backward-looking price setting behavior is sufficiently large (small), anticipated cost-push shocks lead to a lower (higher) volatility in the output gap and in the central bank's loss than a unanticipated shocks of the same size.

The left plot of figure 3 also shows that the volatility in inflation may decrease in q for a much larger range of hybridity ϕ_π (even for $\phi_\pi = 0$) than the volatility of output. However, the exact range depends on the weight λ: If λ is sufficiently small (in figure 3: $\lambda < 0.15$ suffices), the volatility in the inflation rate is increasing in q for sufficiently strong backward-looking price setting behavior.[13]

The volatility results under discretion are summarized in the right plot of figure 3. If the

[13]Strictly speaking, the volatility results of the inflation rate depend on the ratio λ/κ. Increasing κ has the same effect as reducing λ.

price setting is sufficiently backward-looking, discretion generates similar results to commitment. In the limit case $\phi_\pi = 1$, the two regimes are equivalent. Contrarily to commitment, the inflation volatility is only decreasing in q for sufficiently high degrees of backward-lookingness (in figure 3: $\phi_\pi \geq 0.4$).

3 Microfounded loss and Phillips curve

Until now, we have assumed a given central bank's loss which coefficients are not a function of structural parameters and a Phillips curve where the inflation expectations are a weighted average of forward-looking and backward-looking expectations. In this section, we check whether our above results also hold under (i) a central bank's loss that is a second order approximation of the household's welfare function and (ii) a hybrid New Keynesian Phillips curve that follows from partial price indexation. Let γ be the degree of price indexation, then the inflation dynamics are described by

$$\pi_t = \omega_1 E_t \pi_{t+1} + \omega_2 \pi_{t-1} + \omega_3 x_t + \varepsilon_{t-q} \tag{10}$$

with $\omega_1 = \beta/(1+\beta\gamma)$, $\omega_2 = \gamma/(1+\beta\gamma)$, and $\omega_3 = \kappa/(1+\beta\gamma)$.[14] Following Woodford (2003), the welfare theoretic loss function based on (A.21) is given by

$$Loss_q = Var_q(\pi_t - \gamma\pi_{t-1}) + \lambda Var_q(x_t) \tag{11}$$

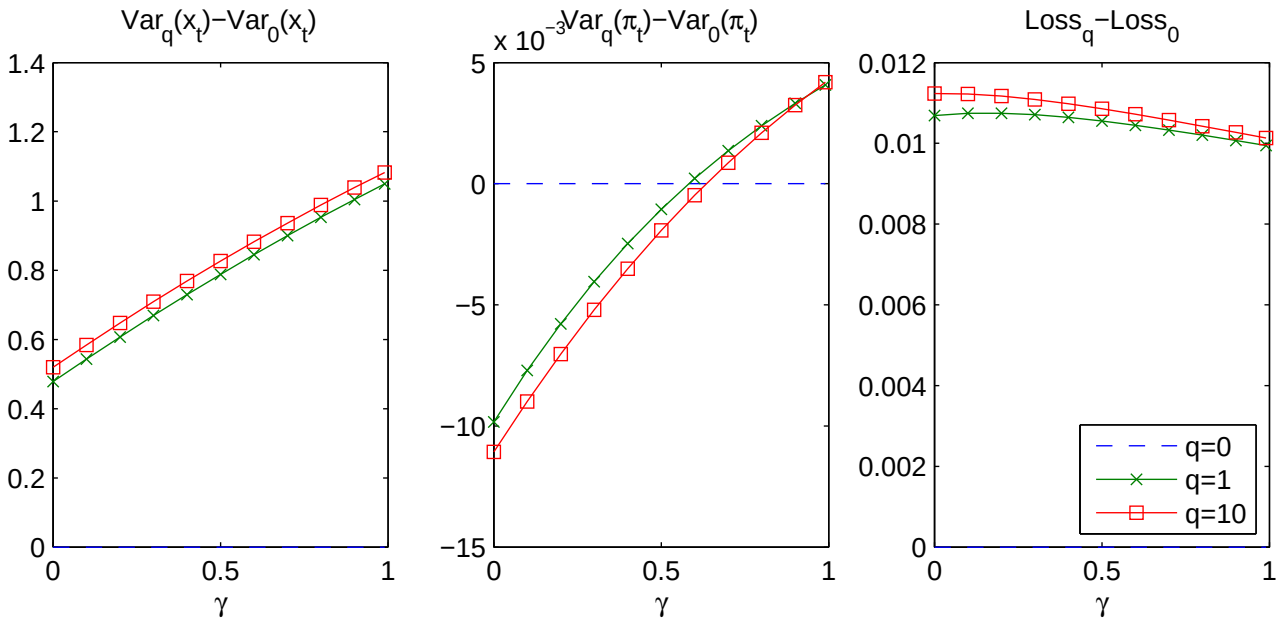
where $\lambda = \kappa/((1+\beta\gamma)\chi)$. χ denotes the elasticity of substitution between differentiated goods. Assuming separable preferences and neglecting capital, $\kappa = (\sigma+\eta)(1-\theta)(1-\beta\theta)/\theta$ where θ is the Calvo parameter, σ is the inverse of the intertemporal elasticity of substitution, and η is the inverse of the Frisch elasticity of labor supply.

Contrarily to the proposed loss (2), the loss function (A.22) has π_{t-1} as an additional argument if $\gamma > 0$.[15] Under the Phillips curve (A.21) and loss (A.22), the optimal unrestricted

[14] A similar Phillips curve is used by Smets and Wouters (2003).

[15] This is the reason why in figure 4 the loss difference $Loss_q - Loss_0$ is decreasing in γ, whereas both $Var_q(x_t) - Var_0(x_t)$ and $Var_q(\pi_t) - Var_0(\pi_t)$ are increasing in γ.

Figure 4: Loss and variances under commitment with microfounded loss and Phillips curve



Parameter calibration: $\beta = 0.99$, $\sigma = \eta = 2$, $\theta = 0.75$, $\kappa = (\sigma + \eta)(1 - \theta)(1 - \beta\theta)/\theta \approx 0.34$, $\chi = 8$.

monetary policy rule under commitment is given by

$$\beta\gamma E_t\pi_{t+1} - (1 + \beta\gamma^2)\pi_t + \gamma\pi_{t-1} = -\frac{\lambda}{\omega_3}\beta\omega_2 E_t x_{t+1} + \frac{\lambda}{\omega_3}x_t - \frac{\lambda}{\omega_3}\frac{\omega_1}{\beta}x_{t-1} \tag{12}$$

For $\gamma = 0$, the equations (A.21) to (12) collapse to the standard purely forward-looking case and are equivalent to our originally proposed model for $\phi_\pi = 0$. However, the hybrid New Keynesian Phillips curve (A.21) does not nest a purely backward-looking Phillips curve as special case in contrast to the originally proposed Phillips curve (1). In fact, the limit case of full price indexation ($\gamma = 1$) leads to a hybrid Phillips curve in which expected future and past inflation equally affect current inflation. This limit case roughly corresponds to $\phi_\pi = 0.5$. As a result, anticipated cost-push shocks in the model with price indexation generate a higher output volatility and central bank's loss than unanticipated shocks.[16] This is illustrated in figure 4 which shows the volatility and loss difference in the regime commitment.[17]

[16] Inflation volatility, on the other hand, may again also decrease in q if the model is forward-looking.
[17] Similar results can be obtained under discretionary policy.

4 Concluding remarks

This paper studies the volatility implications of anticipated cost-push shocks in a hybrid New Keynesian model with forward- and backward-looking price setting behavior both under optimal unrestricted monetary and discretionary policy. In both regimes, we find that an anticipated cost-push shock lead to a larger (smaller) volatility in the output gap and to a larger (smaller) central bank loss than an unanticipated shock of the same size if the degree of backward-looking price setting behavior is sufficiently small (large). By contrast, if the central bank follows an ad hoc or optimized contemporaneous Taylor-type rule and the price setters are purely (forward-) backward-looking, the volatility of the economy is (increasing with) independent of the anticipation horizon.

References

Blanchard, O.J. and C.M. Kahn (1980). The Solution of Linear Difference Models under Rational Expectations *Econometrica* 48, 267-270.

De Grauwe, P. (2012). Booms and busts in economic activity: A behavioral explanation. *Jounral of Economic Behavior and Organization* 83, 484-501.

Fève, P., Matheron, J. and Sahuc, J.-G. (2009). On the dynamic implications of news shocks. *Economics Letters* 102(2), 96-98.

Leitemo, K. (2008). Inflation-targeting rules: History-dependent or forward-looking? *Economics Letters* 100(2), 267-270.

Lengnick, M. and H.-W. Wohltmann (2016). Optimal monetary policy in a new Keynesian model with animal spirits and financial markets. *Journal of Economic Dynamics and Control* 64, pp. 148-165.

Offick, S. and H.-W. Wohltmann (2013). News shocks, nonfundamentalness and volatility. *Economics Letters* 119(1), 17-19.

Rudebusch, G. and L.E.O. Svensson (1999). Policy Rules for Inflation Targeting. Monetary Policy Rules, edited by J.B. Taylor. University of Chicago Press.

Schmitt-Grohé, S. and Uribe, M. (2012). What's News in Business Cycles. *Econometrica* 80(6), 2733-2764.

Smets, F. and R. Wouters (2003). An estimated dynamic stochastic general equilibrium model for the euro area. *Journal of the European Economic Association* 1, 1123-1175.

Söderlind, P. (1999). Solution and Estimation of RE Macromodels with Optimal Policy. *European Economic Review* 43, 813-823.

Winkler, R.C. and Wohltmann, H.-W. (2009). Rational Expectations Models with Anticipated Shocks and Optimal Policy: A General Solution Method and a New Keynesian Example. Economics Working Paper 2009-01, April 2009, Kiel University.

Winkler, R.C. and Wohltmann, H.-W. (2011). News Shocks and Optimal Simple Rules. *Review of Economics* 62, 1-11.

Winkler, R.C. and Wohltmann, H.-W. (2012). On the (de)stabilizing effects of news shocks. *Economics Letters* 114(3), 256-258.

Woodford, M. (2003). Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press.

Appendix

A Derivation of targeting rules

A.1 Ad hoc central bank loss and Phillips curve

In this section, we derive the targeting rules under optimal unrestricted monetary policy (commitment) and under optimal discretionary policy using the New Keynesian Phillips curve

$$\pi_t = \beta(1-\phi_\pi)E_t\pi_{t+1} + \beta\phi_\pi\pi_{t-1} + \kappa x_t + \varepsilon_{t-q} \tag{A.1}$$

and the central bank loss

$$Loss_q = Var_q(\pi_t) + \lambda Var_q(x_t) \tag{A.2}$$

where $0 < \beta < 1$, $0 \leq \phi_\pi \leq 1$, $\kappa > 0$, $q \in N_0$, $\lambda > 0$.

A.1.1 Commitment

The central bank minimizes the loss function (A.2) subject to the Phillips curve (A.1). The Lagrange function to this problem is given by[18]

$$J_{t_0} = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\pi_t^2 + \lambda x_t^2 + \mu_t[\beta(1-\phi_\pi)E_t\pi_{t+1} - \pi_t + \beta\phi_\pi\pi_{t-1} + \kappa x_t + \varepsilon_{t-q}]\right\} \tag{A.3}$$

with the first-order conditions for $t > t_0$ (timeless perspective solutions)

$$\frac{\partial L_t}{\partial x_t} = 2\lambda x_t + \mu_t\kappa = 0 \quad \Leftrightarrow \quad \mu_t = -2\frac{\lambda}{\kappa}x_t \tag{A.4}$$

$$\frac{\partial L_t}{\partial \pi_t} = 2\pi_t - \mu_t + \beta^2\phi_\pi E_t\mu_{t+1} + (1-\phi_\pi)\mu_{t-1} = 0 \tag{A.5}$$

[18] Note that, strictly speaking, the central bank loss (A.2) is equivalent to $\lim_{\beta\to 1}(1-\beta)\sum_{t=t_0}^{\infty}\beta^{t-t_0}(\pi_t^2 + \lambda x_t^2)$, see Rudebusch and Svensson (1999). The constant $(1-\beta)$ however does not affect the first-order conditions and we set the discount factor close to unity, i.e. $\beta = 0.99$.

The optimal monetary response of the central bank can be summarized by the following targeting rule:[19]

$$\pi_t = -\frac{\lambda}{\kappa}(x_t - x_{t-1}) - \frac{\lambda}{\kappa}\phi_\pi x_{t-1} + \frac{\lambda}{\kappa}\beta^2\phi_\pi E_t x_{t+1} \tag{A.6}$$

A.1.2 Discretion

Under discretionary policy, the central bank is not able to commit to future policies. It can, therefore, not directly affect private expectations, but only indirectly through the persistence of the inflation rate given by the hybrid structure of the New Keynesian Phillips curve. To account for this, we replace the expected future inflation rate in the central bank's optimization using the reduced-form solution of the inflation rate and the output gap. The guessed reduced-form solutions are given by

$$\pi_t = \rho_\pi \pi_{t-1} + \rho_0 \varepsilon_t + \rho_1 \varepsilon_{t-1} + \ldots + \rho_q \varepsilon_{t-q} \tag{A.7}$$

$$x_t = \eta_\pi \pi_{t-1} + \eta_0 \varepsilon_t + \eta_1 \varepsilon_{t-1} + \ldots + \eta_q \varepsilon_{t-q} \tag{A.8}$$

with the undetermined coefficients $\rho_\pi, \rho_0, \ldots, \rho_q$ and $\eta_\pi, \eta_0, \ldots, \eta_q$. The reduced-form solutions imply

$$E_t\pi_{t+1} = \rho_\pi^2 \pi_{t-1} + (\rho_\pi\rho_0 + \rho_1)\varepsilon_t + \ldots + (\rho_\pi\rho_{q-1} + \rho_q)\varepsilon_{t-q+1} + \rho_\pi\rho_q\varepsilon_{t-q} \tag{A.9}$$

$$E_t x_{t+1} = \eta_\pi\rho_\pi \pi_{t-1} + (\eta_\pi\rho_0 + \eta_1)\varepsilon_t + \ldots + (\eta_\pi\rho_{q-1} + \eta_q)\varepsilon_{t-q+1} + \eta_\pi\rho_q\varepsilon_{t-q} \tag{A.10}$$

Replacing $E_t\pi_{t+1}$ in the Phillips curve (A.1) gives

$$\pi_t = \beta[(1-\phi_\pi)\rho_\pi^2 + \phi_\pi]\pi_{t-1} + \kappa x_t + k_t \tag{A.11}$$

[19] Since the targeting rule is independent of q, it is equivalent to the one derived in Leitemo (2008).

where $k_t = \beta(1-\phi_\pi)[(\rho_\pi\rho_0+\rho_1)\varepsilon_t + \ldots + (\rho_\pi\rho_{q-1}+\rho_q)\varepsilon_{t-q+1} + \rho_\pi\rho_q\varepsilon_{t-q}] + \varepsilon_{t-q}$. The Lagrange function to the discretionary optimization problem is then given by

$$J_{t_0} = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\pi_t^2 + \lambda x_t^2 + \mu_t\{\beta[(1-\phi_\pi)\rho_\pi^2+\phi_\pi]\pi_{t-1} - \pi_t + \kappa x_t + k_t\}\right\} \tag{A.12}$$

with the first-order conditions

$$\frac{\partial L_t}{\partial x_t} = 2\lambda x_t + \kappa\mu_t = 0 \quad \Leftrightarrow \quad \mu_t = -2\frac{\lambda}{\kappa}x_t \tag{A.13}$$

$$\frac{\partial L_t}{\partial \pi_t} = 2\pi_t - \mu_t + \beta^2[(1-\phi_\pi)\rho_\pi^2+\phi_\pi]E_t\mu_{t+1} = 0 \tag{A.14}$$

Thus, the optimal discretionary monetary response reads

$$\pi_t = -\frac{\lambda}{\kappa}x_t + \beta^2[(1-\phi_\pi)\rho_\pi^2+\phi_\pi]\frac{\lambda}{\kappa}E_t x_{t+1} \tag{A.15}$$

which is independent of the anticipation horizon q (holds $\forall q \geq 0$).[20]

To determine the unknown coefficient ρ_π, we replace x_t, π_t, $E_t\pi_{t+1}$, and $E_t x_{t+1}$ in the Philips curve (A.1) and targeting rule (A.15):

$$\begin{aligned}
0 = &\left[\beta^2[(1-\phi_\pi)\rho_\pi^2+\phi_\pi]\eta_\pi\rho_\pi - \frac{\kappa}{\lambda}\rho_\pi - \eta_\pi\right]\pi_{t-1} \\
&+ \sum_{j=0}^{q-1}\left[\beta^2[(1-\phi_\pi)\rho_\pi^2+\phi_\pi](\eta_\pi\rho_j+\eta_{j+1}) - \frac{\kappa}{\lambda}\rho_j - \eta_j\right]\varepsilon_{t-j} \\
&+ \left[(1+\phi_\pi-\phi_\pi\rho_\pi^2)\beta^2\eta_\pi\rho_q - \frac{\kappa}{\lambda}\rho_q - \eta_q\right]\varepsilon_{t-q}
\end{aligned} \tag{A.16}$$

$$\begin{aligned}
0 = &\left[\beta(1-\phi_\pi)\rho_\pi^2 + \beta\phi_\pi + \kappa\eta_\pi - \rho_\pi\right]\pi_{t-1} \\
&+ \sum_{j=0}^{q-1}\left[\beta(1-\phi_\pi)(\rho_\pi\rho_j+\rho_{j+1}) + \kappa\eta_j - \rho_j\right]\varepsilon_{t-j} \\
&+ \left[\beta(1-\phi_\pi)\rho_\pi\rho_q + \kappa\eta_q + 1 - \rho_q\right]\varepsilon_{t-q}
\end{aligned} \tag{A.17}$$

For $\{\rho_\pi, \rho_0, \rho_1, \rho_2, \eta_\pi, \eta_0, \eta_1, \eta_2\}$ to be solutions, equations (A.16) and (A.17) should hold for any

[20] The targeting rule (A.15) is equivalent to the one derived in Leitemo (2008) for $q = 0$.

$\{\pi_t, \varepsilon_t\}$. Thus, the unknown coefficient ρ_π is determined by the system

$$\beta^2[(1-\phi_\pi)\rho_\pi^2 + \phi_\pi]\eta_\pi\rho_\pi - \frac{\kappa}{\lambda}\rho_\pi - \eta_\pi = 0 \tag{A.18}$$

$$\beta(1-\phi_\pi)\rho_\pi^2 + \beta\phi_\pi + \kappa\eta_\pi - \rho_\pi = 0 \tag{A.19}$$

Solving (A.19) for ρ_π and substituting into (A.18), ρ_π is the stable solution of the following polynomial equation of order five:

$$\begin{aligned} 0 = {} & \beta^3(1-\phi_\pi)^2\rho_\pi^5 - \beta^2(1-\phi_\pi)\rho_\pi^4 + 2\phi_\pi\beta^3(1-\phi_\pi)\rho_\pi^3 - \left(\beta(1-\phi_\pi) + \beta^2\phi_\pi\right)\rho_\pi^2 \\ & + \left(1 + \frac{\kappa^2}{\lambda} + \beta^3\phi_\pi^2\right)\rho_\pi - \beta\phi_\pi \end{aligned} \tag{A.20}$$

Note that the limit case $\phi_\pi = 0$ implies $\rho_\pi = 0$ so that the targeting rule (A.15) collapses to $\pi_t = -(\lambda/\kappa)x_t$. In the limit case $\phi_\pi = 1$, the targeting rules under optimal unrestricted monetary and discretionary policy are equivalent.

A.2 Microfounded central bank loss and Phillips curve

In this section, we derive the targeting rules under optimal unrestricted monetary policy (commitment) and under optimal discretionary policy using the microfounded hybrid New Keynesian Phillips curve that follows from partial price indexation:

$$\pi_t = \omega_1 E_t\pi_{t+1} + \omega_2\pi_{t-1} + \omega_3 x_t + \varepsilon_{t-q} \tag{A.21}$$

with $\omega_1 = \beta/(1+\beta\gamma)$, $\omega_2 = \gamma/(1+\beta\gamma)$, and $\omega_3 = \kappa/(1+\beta\gamma)$.[21] γ is the degree of price indexation. Following Woodford (2003), the welfare theoretic loss function based on (A.21) is given by

$$Loss_q = Var_q(\pi_t - \gamma\pi_{t-1}) + \lambda Var_q(x_t) \tag{A.22}$$

[21] A similar Phillips curve is used by Smets and Wouters (2003).

with

$$\lambda = \frac{\kappa}{(1+\beta\gamma)\chi} \tag{A.23}$$

where χ is the elasticity of substitution between differentiated goods. κ can be presented as a function of structural parameters. Assuming separable preferences and neglecting capital, $\kappa = (\sigma+\eta)\frac{(1-\theta)(1-\beta\theta)}{\theta}$ where θ is the Calvo parameter, σ is the inverse of the intertemporal elasticity of substitution, and η is the inverse of the Frisch elasticity of labor supply.[22]

A.2.1 Commitment

The central bank minimizes the loss function (A.22) subject to the Phillips curve (A.21). The Lagrange function to this problem is given by

$$J_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ (\pi_t - \gamma\pi_{t-1})^2 + \lambda x_t^2 + \mu_t[\omega_1 E_t\pi_{t+1} - \pi_t + \omega_2\pi_{t-1} + \omega_3 x_t + \varepsilon_{t-q}] \right\} \tag{A.24}$$

with the first-order conditions

$$\frac{\partial L_t}{\partial x_t} = 2\lambda x_t + \omega_3\mu_t = 0 \quad \Leftrightarrow \quad \mu_t = -2\frac{\lambda}{\omega_3}x_t \tag{A.25}$$

$$\frac{\partial L_t}{\partial \pi_t} = 2(\pi_t - \gamma\pi_{t-1}) - \mu_t - 2\beta\gamma(E_t\pi_{t+1} - \gamma\pi_t) + \beta\omega_2 E_t\mu_{t+1} + \frac{\omega_1}{\beta}\mu_{t-1} = 0 \tag{A.26}$$

$$\Leftrightarrow \quad 2\beta\gamma E_t\pi_{t+1} - 2(1+\beta\gamma^2)\pi_t + 2\gamma\pi_{t-1} = \beta\omega_2 E_t\mu_{t+1} - \mu_t + \frac{\omega_1}{\beta}\mu_{t-1} \tag{A.27}$$

The optimal monetary response of the central bank can be summarized by the following targeting rule:

$$\beta\gamma E_t\pi_{t+1} - (1+\beta\gamma^2)\pi_t + \gamma\pi_{t-1} = -\frac{\lambda}{\omega_3}\beta\omega_2 E_t x_{t+1} + \frac{\lambda}{\omega_3}x_t - \frac{\lambda}{\omega_3}\frac{\omega_1}{\beta}x_{t-1} \tag{A.28}$$

[22] χ can bet set to 8 implying a steady state price mark-up on the goods market of approximately 14 percent. Setting $\sigma = \eta = 2$, $\beta = 0.99$ and $\theta = 0.75$ gives $\kappa = 0.34$.

A.2.2 Discretion

To derive the discretionary policy, we again use the following reduced-form solutions:

$$\pi_t = \rho_\pi \pi_{t-1} + \rho_0 \varepsilon_t + \rho_1 \varepsilon_{t-1} + \ldots + \rho_q \varepsilon_{t-q} \tag{A.29}$$

$$x_t = \eta_\pi \pi_{t-1} + \eta_0 \varepsilon_t + \eta_1 \varepsilon_{t-1} + \ldots + \eta_q \varepsilon_{t-q} \tag{A.30}$$

implying

$$E_t \pi_{t+1} = \rho_\pi^2 \pi_{t-1} + (\rho_\pi \rho_0 + \rho_1)\varepsilon_t + \ldots + (\rho_\pi \rho_{q-1} + \rho_q)\varepsilon_{t-q+1} + \rho_\pi \rho_q \varepsilon_{t-q} \tag{A.31}$$

$$E_t x_{t+1} = \eta_\pi \rho_\pi \pi_{t-1} + (\eta_\pi \rho_0 + \eta_1)\varepsilon_t + \ldots + (\eta_\pi \rho_{q-1} + \eta_q)\varepsilon_{t-q+1} + \eta_\pi \rho_q \varepsilon_{t-q} \tag{A.32}$$

Replacing $E_t \pi_{t+1}$ in the Phillips curve (A.21) gives

$$\pi_t = (\omega_1 \rho_\pi^2 + \omega_2)\pi_{t-1} + \omega_3 x_t + k_t \tag{A.33}$$

where $k_t = \omega_1[(\rho_\pi \rho_0 + \rho_1)\varepsilon_t + \ldots + (\rho_\pi \rho_{q-1} + \rho_q)\varepsilon_{t-q+1} + \rho_\pi \rho_q \varepsilon_{t-q}] + \varepsilon_{t-q}$. The Lagrange function to the discretionary optimization problem is then given by

$$J_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ (\pi_t - \gamma \pi_{t-1})^2 + \lambda x_t^2 + \mu_t [\omega_1 \rho_\pi^2 \pi_{t-1} - \pi_t + \omega_2 \pi_{t-1} + \omega_3 x_t + k_t] \right\} \tag{A.34}$$

with the first-order conditions

$$\frac{\partial L_t}{\partial x_t} = 2\lambda x_t + \omega_3 \mu_t = 0 \quad \Leftrightarrow \quad \mu_t = -2\frac{\lambda}{\omega_3} x_t \tag{A.35}$$

$$\frac{\partial L_t}{\partial \pi_t} = 2(\pi_t - \gamma \pi_{t-1}) - \mu_t - 2\beta\gamma(E_t \pi_{t+1} - \gamma \pi_t) + \beta(\omega_1 \rho_\pi^2 + \omega_2) E_t \mu_{t+1} = 0 \tag{A.36}$$

$$\Leftrightarrow \quad 2\beta\gamma E_t \pi_{t+1} - 2(1 + \beta\gamma^2)\pi_t + 2\gamma \pi_{t-1} = \beta(\omega_1 \rho_\pi^2 + \omega_2) E_t \mu_{t+1} - \mu_t \tag{A.37}$$

Thus, the optimal discretionary monetary response reads

$$\beta\gamma E_t \pi_{t+1} - (1 + \beta\gamma^2)\pi_t + \gamma \pi_{t-1} = -\frac{\lambda}{\omega_3} \beta(\omega_1 \rho_\pi^2 + \omega_2) E_t x_{t+1} + \frac{\lambda}{\omega_3} x_t \tag{A.38}$$

which is independent of the anticipation horizon q.

To determine the unknown coefficient ρ_π, we replace x_t, π_t, $E_t\pi_{t+1}$, and $E_t x_{t+1}$ in the Philips curve (A.21) and targeting rule (A.38):

$$0 = \left[\omega_1\rho_\pi^2 - \rho_\pi + \omega_3\eta_\pi + \omega_2\right]\pi_{t-1} + \sum_{j=0}^{q}\psi_{1,j}\varepsilon_{t-j} \tag{A.39}$$

$$0 = \left[\frac{\omega_1}{\omega_3}\beta\lambda\eta_\pi\rho_\pi^3 + \beta\gamma\rho_\pi^2 + \left(\frac{\omega_2}{\omega_3}\beta\lambda\eta_\pi - 1 - \beta\gamma^2\right)\rho_\pi - \frac{\lambda}{\omega_3}\eta_\pi + \gamma\right]\pi_{t-1} + \sum_{j=0}^{q}\psi_{2,j}\varepsilon_{t-j} \tag{A.40}$$

where $\psi_{1,j}$ and $\psi_{2,j}$ are constants which are not relevant for the determination of ρ_π and η_π. ρ_π and η_π follow from the solution of

$$\omega_1\rho_\pi^2 - \rho_\pi + \omega_3\eta_\pi + \omega_2 = 0 \tag{A.41}$$

$$\frac{\omega_1}{\omega_3}\beta\lambda\eta_\pi\rho_\pi^3 + \beta\gamma\rho_\pi^2 + \left(\frac{\omega_2}{\omega_3}\beta\lambda\eta_\pi - 1 - \beta\gamma^2\right)\rho_\pi - \frac{\lambda}{\omega_3}\eta_\pi + \gamma = 0 \tag{A.42}$$

The two equations can be reduced to a single equation of order five:

$$\begin{aligned} -\beta\lambda\frac{\omega_1^2}{\omega_3^2}\rho_\pi^5 + \beta\lambda\frac{\omega_1}{\omega_3^2}\rho_\pi^4 + \left(-2\beta\lambda\frac{\omega_1\omega_2}{\omega_3^2}\right)\rho_\pi^3 + \left(\beta\gamma + \beta\lambda\frac{\omega_2}{\omega_3^2} + \lambda\frac{\omega_1}{\omega_3^2}\right)\rho_\pi^2 \\ + \left(-\beta\lambda\frac{\omega_2^2}{\omega_3^2} - 1 - \beta\gamma^2 - \frac{\lambda}{\omega_3^2}\right)\rho_\pi + \lambda\frac{\omega_2}{\omega_3^2} + \gamma = 0 \end{aligned} \tag{A.43}$$

$$\begin{aligned} \Leftrightarrow \quad -\beta\lambda\omega_1^2\rho_\pi^5 + \beta\lambda\omega_1\rho_\pi^4 - 2\beta\lambda\omega_1\omega_2\rho_\pi^3 + \left(\beta\gamma\omega_3^2 + \beta\lambda\omega_2 + \lambda\omega_1\right)\rho_\pi^2 \\ - \left(\beta\lambda\omega_2^2 + \omega_3^2 + \beta\gamma^2\omega_3^2 + \lambda\right)\rho_\pi + \lambda\omega_2 + \gamma = 0 \end{aligned} \tag{A.44}$$

The limit case $\gamma = 0$ implies $\omega_2 = 0$ and $\rho_\pi = 0$.

B Derivation of variances and loss

B.1 Hybrid univariate model

In this section, we derive the variance of a hybrid univariate model of the form

$$y_t = aE_t y_{t+1} + by_{t-1} + c\varepsilon_{t-q} \tag{B.1}$$

with $\varepsilon_t \overset{i.i.d.}{\sim} N(0,\sigma^2)$ can be written as MA(∞) of the form

$$y_t = \sum_{s=0}^{\infty} \alpha^s \sum_{k=0}^{q} \delta^k \beta_0 \varepsilon_{t-s+k-q} = \sum_{s=0}^{\infty} \alpha^s h_{t-s} \quad \text{with} \quad h_t = \sum_{k=0}^{q} \delta^k \beta_0 \varepsilon_{t+k-q} \tag{B.2}$$

where $\alpha = \left(1 - \sqrt{1-4ab}\right)/(2a)$, $\beta_0 = c/(1-a\alpha)$, and $\delta = a/(1-a\alpha)$. The variance of y_t can be derived as follows:

$$Var(y_t) = \sum_{s=0}^{\infty}\sum_{\tilde{s}=0}^{\infty} \alpha^s \alpha^{\tilde{s}} \sum_{k=0}^{q}\sum_{\tilde{k}=0}^{q} \beta_0^2 \delta^k \delta^{\tilde{k}} E(\varepsilon_{t-s+k-q}\varepsilon_{t-\tilde{s}+\tilde{k}-q}) \tag{B.3}$$

$$= \beta_0^2 \sum_{s=0}^{\infty} \alpha^{2s} \sum_{k=0}^{q} \delta^{2k}\sigma^2 + 2\beta_0^2 \sum_{s=0}^{\infty}\sum_{j=0}^{q-1}\sum_{k=0}^{q-1-j} \alpha^{2s+j+1}\delta^{2k+j+1}\sigma^2 \tag{B.4}$$

$$= \beta_0^2 v_t \sigma^2 + 2\beta_0^2 w_t \sigma^2 \tag{B.5}$$

v_t and w_t can be simplified to

$$v_t = \sum_{s=0}^{\infty} \alpha^{2s} \sum_{k=0}^{q} \delta^{2k} = \frac{1}{1-\alpha^2}\frac{1-\delta^{2(q+1)}}{1-\delta^2} \tag{B.6}$$

$$w_t = \sum_{s=0}^{\infty}\sum_{j=0}^{q-1}\sum_{k=0}^{q-1-j} \alpha^{2s+j+1}\delta^{2k+j+1} \tag{B.7}$$

$$= \frac{\alpha\delta}{1-\delta^2}\sum_{s=0}^{\infty}\alpha^{2s}\sum_{j=0}^{q-1}(\alpha\delta)^j - \frac{\delta^{2(q+1)}}{1-\delta^2}\sum_{s=0}^{\infty}\alpha^{2s}\sum_{j=0}^{q-1}\left(\frac{\alpha}{\delta}\right)^{j+1} \tag{B.8}$$

$$= \frac{\alpha\delta}{1-\delta^2}\frac{1}{1-\alpha^2}\frac{1-(\alpha\delta)^q}{1-\alpha\delta} - \frac{\alpha\delta^{2(q+1)}}{1-\delta^2}\frac{1}{1-\alpha^2}\frac{1-\left(\frac{\alpha}{\delta}\right)^q}{\delta-\alpha} \tag{B.9}$$

In summary, the variance of y_t is given by $Var(y_t) = V(q)$ where

$$V(q) = \frac{\beta_0^2}{(1-\alpha^2)(1-\delta^2)}\left\{1 - \delta^{2(q+1)} + 2\frac{\alpha\delta}{1-\alpha\delta}\left[1-(\alpha\delta)^q\right] + 2\frac{\alpha}{\alpha-\delta}\left[\delta^{2(q+1)} - \delta^2(\alpha\delta)^q\right]\right\}\sigma^2 \tag{B.10}$$

Note that $V(q)$ can also be written as

$$V(q) = \frac{1}{1-\alpha^2}[Var(h_t) + 2\alpha Cov(x_{t-1}, h_t)] \tag{B.11}$$

where

$$Var(h_t) = \frac{\beta_0^2}{1-\delta^2}(1-\delta^{2(q+1)})\sigma^2 = \beta_0^2\sigma^2\sum_{k=0}^{q}\delta^{2k} \tag{B.12}$$

$$Cov(x_{t-1}, h_t) = \frac{\beta_0^2}{1-\delta^2}\left\{\frac{\delta}{1-\alpha\delta}\left[1-(\alpha\delta)^q\right] + \frac{1}{\alpha-\delta}\left[\delta^{2(q+1)} - \delta^2(\alpha\delta)^q\right]\right\}\sigma^2 \tag{B.13}$$

$$= \beta_0^2\sigma^2\delta\sum_{j=0}^{q-1}(\alpha\delta)^j\sum_{k=0}^{q-1-j}\delta^{2k} \tag{B.14}$$

B.2 Purely forward-looking price setting

In this section, we discuss the variance of the output gap, the inflation rate and the central bank's loss in the limit case of purely forward-looking price setting behavior. In this limit case ($\phi_\pi = 0$), the New Keynesian model under optimal unrestricted monetary policy given by (A.1) and (A.6) reduces to

$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t + \varepsilon_{t-q} \tag{B.15}$$

$$\pi_t = -\frac{\lambda}{\kappa}(x_t - x_{t-1}) \tag{B.16}$$

The output gap x_t can be written as hybrid univariate model equation of the form (B.1) with

$$a = \beta b \tag{B.17}$$

$$b = \frac{\lambda}{\lambda(1+\beta)+\kappa^2} \tag{B.18}$$

$$c = -\frac{\kappa}{\lambda(1+\beta)+\kappa^2} \tag{B.19}$$

Hence, the variance of x_t is given by $Var(x_t) = V(q)$, where

$$\delta = \frac{2a}{1+\sqrt{1-4ab}} \tag{B.20}$$

$$\alpha\delta = \frac{1-\sqrt{1-4ab}}{1+\sqrt{1-4ab}} \tag{B.21}$$

$$1-4ab = \frac{(1-\beta)^2 + 2(1+\beta)z + z^2}{(1+\beta+z)^2} \tag{B.22}$$

$$z = \frac{\kappa^2}{\lambda} \tag{B.23}$$

Since $dVar_q(h_t)/dq > 0$ and $dCov_q(x_{t-1}, h_t)/dq > 0$, it holds $dVar_q(x_t)/dq > 0$.

The variance of the inflation rate can be deduced from the targeting rule (B.16):

$$Var(\pi_t) = 2\left(\frac{\lambda}{\kappa}\right)^2 (Var(x_t) - E(x_t x_{t-1})) \tag{B.24}$$

$$= \frac{2}{1+\alpha}\left(\frac{\lambda}{\kappa}\right)^2 [Var_q(h_t) - (1-\alpha)Cov_q(x_{t-1}, h_t)] \tag{B.25}$$

$$= \frac{2}{1+\alpha}\left(\frac{\lambda}{\kappa}\right)^2 \beta_0^2\sigma^2 \left[\sum_{k=0}^{q}\delta^{2k} - (1-\alpha)\delta\sum_{j=0}^{q-1}(\alpha\delta)^j \sum_{k=0}^{q-1-j}\delta^{2k}\right] \tag{B.26}$$

To derive the condition for $Var_{q=0}(\pi_t) > Var_{q\to\infty}(\pi_t)$, note that

$$Var_{q=0}(h_t) = \beta_0^2\sigma^2 \tag{B.27}$$

$$Cov_{q=0}(x_{t-1}, h_t) = 0 \tag{B.28}$$

$$Var_{q\to\infty}(h_t) = \frac{\beta_0^2}{1-\delta^2}\sigma^2 \tag{B.29}$$

$$Cov_{q\to\infty}(x_{t-1}, h_t) = \frac{\beta_0^2}{1-\delta^2}\frac{\delta}{1-\alpha\delta}\sigma^2 \tag{B.30}$$

Using the definitions (B.17) to (B.19), $Var_{q=0}(\pi_t) \overset{!}{>} Var_{q\to\infty}(\pi_t)$ is equivalent to

$$(1-\alpha)Cov_{q\to\infty}(x_{t-1}, h_t) > Var_{q\to\infty}h_t - Var_{q=0}h_t \quad \Leftrightarrow \quad \frac{1-\alpha}{1-\alpha\delta} > \delta \quad \Leftrightarrow \tag{B.31}$$

$$1 - \beta^{-1}a - 2\beta^{-1}a^2 > [(2+\beta^{-1})a - 1]\sqrt{1 - 4\beta^{-1}a^2} \quad \Leftrightarrow \tag{B.32}$$

$$(\beta^2 - \beta) + (1+2\beta)\frac{\delta^2}{\lambda} + \frac{\kappa^4}{\lambda^2} > \left(\beta - \frac{\kappa^2}{\lambda}\right)\sqrt{(1-\beta)^2 + 2(1+\beta)\frac{\kappa^2}{\lambda} + \frac{\kappa^4}{\lambda^2}} \tag{B.33}$$

Let $z = \kappa^2/\lambda$, then inequality B.33 can be simplified to

$$z^2 + 2(1+\beta)z + \beta(\beta - 2) > 0 \tag{B.34}$$

and holds if

$$z = \frac{\kappa^2}{\lambda} > \sqrt{1+4\beta} - (1+\beta) \tag{B.35}$$

Although the variance of the inflation rate may decrease with increasing anticipation horizon

q, it can be shown that the loss

$$Loss_q = Var_q(\pi_t) + \lambda Var_q(x_t) \tag{B.36}$$

is always smaller for $q = 0$ than for $q \to \infty$. It holds:

$$Loss_{q\to\infty} = \left\{ \frac{2}{1+\alpha} \left(\frac{\lambda}{\kappa}\right)^2 \frac{1-\delta}{1-\alpha\delta} \frac{1}{1-\delta^2} + \lambda \frac{1}{1-\alpha^2} \frac{1}{1-\delta^2} \frac{1+\alpha\delta}{1-\alpha\delta} \right\} \beta_0^2 \sigma^2 \tag{B.37}$$

$$Loss_{q=0} = \left\{ \frac{2}{1+\alpha} \left(\frac{\lambda}{\kappa}\right)^2 + \lambda \frac{1}{1-\alpha^2} \right\} \beta_0^2 \sigma^2 \tag{B.38}$$

Then $J_{q\to\infty} \overset{!}{>} J_{q=0}$ is equivalent to

$$\frac{2\lambda}{\kappa}[1-\alpha(1+\delta)] < \frac{2\alpha+\delta(1-\alpha\delta)}{(1-\alpha)(1-\delta)} \quad \Leftrightarrow \quad 2b[\sqrt{1-4ab}-b] < b + b(1+\beta)\sqrt{1-4ab} \tag{B.39}$$

Since $b = 1/[1+\beta+z]$, $1-4ab = 1-4\beta/[1+\beta+z]^2$, (B.39) is equivalent to

$$0 < 4\beta(1-\beta)^2 + \beta(2-\beta)(1+\beta+z) + 4(1+\beta+z) \tag{B.40}$$

This inequality is always satisfied since $z = \kappa^2/\lambda > 0$.

B.3 Purely backward-looking price setting

In this section, we discuss volatility results in the opposite limit case of purely backward-looking price setting behavior. In this limit case ($\phi_\pi = 1$), the New Keynesian model under optimal unrestricted monetary policy given by (A.1) and (A.6) reduces to[23]

$$\pi_t = \beta\pi_{t-1} + \kappa x_t + \varepsilon_{t-q} \tag{B.41}$$

$$\pi_t = -\frac{\lambda}{\kappa}(x_t - \beta^2 E_t x_{t+1}) \tag{B.42}$$

[23] Note that under purely backward-looking price setting behavior, the model under discretionary policy is equivalent to the model under unrestricted monetary policy. Thus, the results in this section also hold in the regime discretion.

The inflation rate can be written as a hybrid univariate equation of the form

$$\pi_t = aE_t\pi_{t+1} + b\pi_{t-1} + c(\varepsilon_{t-q} - \beta^2 E_t\varepsilon_{t-q+1}) \tag{B.43}$$

with $c = \varphi/(1+\varphi+\varphi\beta^3)$, $b = \beta c$, $a = \beta^2 c$, and $\varphi = \lambda/(\kappa^2)$. The system can again be written as

$$\pi_t = \sum_{s=0}^{\infty} \alpha^s h_{t-s} \tag{B.44}$$

where $\alpha = (1 - \sqrt{1-4ab})/2a$ and

$$h_t = \sum_{k=0}^{q} \delta^k \beta_0 \varepsilon_{t+k-q} - \beta^2 \sum_{k=0}^{q-1} \delta^k \beta_0 \varepsilon_{t+k-q+1} \tag{B.45}$$

The variance of the inflation rate can be derived as follows:

$$Var(\pi_t) = E\left(\sum_{s=0}^{\infty}\alpha^s \sum_{k=0}^{q} \delta^k\beta_0\varepsilon_{t-s+k-q} - \beta^2 \sum_{s=0}^{\infty}\alpha^s \sum_{k=0}^{q-1} \delta^k\beta_0\varepsilon_{t-s+k-q+1}\right)^2 \tag{B.46}$$

$$= V(q) - 2\beta^2 Z + \beta^4 V(q-1) \tag{B.47}$$

where $V(\cdot)$ is given by (B.10) and

$$Z = E\left[\left(\sum_{s=0}^{\infty}\alpha^s \sum_{k=0}^{q} \delta^k\beta_0\varepsilon_{t-s+k-q}\right)\left(\sum_{s=0}^{\infty}\alpha^s \sum_{k=0}^{q-1} \delta^k\beta_0\varepsilon_{t-s+k-q+1}\right)\right] \tag{B.48}$$

$$= E\left[\left(\sum_{s=0}^{\infty}\alpha^s \beta_0\varepsilon_{t-s-q}\right)\left(\sum_{s=0}^{\infty}\alpha^s \sum_{k=0}^{q-1} \delta^k\beta_0\varepsilon_{t-s+k-q+1}\right)\right] + \delta V(q-1) \tag{B.49}$$

$$= \frac{\alpha}{1-\alpha^2}\frac{1-(\alpha\delta)^q}{1-\alpha\delta}\beta_0^2\sigma^2 + \delta V(q-1) = \frac{\alpha}{1-\alpha^2}\beta_0^2\sigma^2 \sum_{j=0}^{q-1}(\alpha\delta)^j + \delta V(q-1) \tag{B.50}$$

Then $Var_q(\pi_t)$ can be written as

$$Var(\pi_t) = V(q) - 2\beta^2\frac{\alpha}{1-\alpha^2}\frac{1-(\alpha\delta)^q}{1-\alpha\delta}\beta_0^2\sigma^2 + (\beta^4 - 2\beta^2\delta)V(q-1) \tag{B.51}$$

C Solution method for rational expectations models with news shocks under optimal unrestricted monetary policy

The hybrid New Keynesian model under optimal unrestricted monetary policy given by equations (A.1) and (A.6) can be written in matrix form

$$\Phi s_{t+1} = \Psi s_t + g\varepsilon_{t+1} \tag{C.1}$$

where $s_{t+1} = (\tilde{\eta}_{t+1}^{(q)}, \tilde{x}_{t+1}, \tilde{\pi}_{t+1}, E_t x_{t+1}, E_t \pi_{t+1})'$, $\tilde{\eta}_{t+1}^{(q)} = (\eta_{t+1}^{(0)}, \eta_{t+1}^{(1)}, \ldots, \eta_{t+1}^{(q-1)}, \eta_{t+1}^{(q)})'$ with $\tilde{x}_t = x_{t-1}$, $\tilde{\pi}_t = \pi_{t-1}$, $\eta_t^{(j)} = \varepsilon_{t-j} \, \forall \, j = 0, \ldots, q$ and $g = (1, 0, \ldots, 0)'$, and

$$\Phi = \begin{pmatrix} I_{q+3} & 0_{2\times(q+3)} \\ 0_{(q+3)\times 2} & \Phi_{22} \end{pmatrix} \tag{C.2}$$

$$\Psi = \begin{pmatrix} \Psi_{11} & 0_{(q+1)\times 5} \\ 0_{4\times q} & \Psi_{22} \end{pmatrix} \tag{C.3}$$

with $0_{n\times m}$ as $(n \times m)$-dimensional zero matrix, I_n as n-dimensional identity matrix, and

$$\Phi_{22} = \begin{pmatrix} \frac{\lambda\beta^2\phi_\pi}{\kappa} & 0 \\ 0 & \beta(1-\phi_\pi) \end{pmatrix} \tag{C.4}$$

$$\Psi_{11} = \begin{pmatrix} 0_{1\times q} \\ I_q \end{pmatrix} \tag{C.5}$$

$$\Psi_{22} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -(1-\phi_\pi)\frac{\lambda}{\kappa} & 0 & \frac{\lambda}{\kappa} & 1 \\ -1 & 0 & -\beta\phi_\pi & -\kappa & 1 \end{pmatrix} \tag{C.6}$$

Let $w_{t+1} = (\tilde{\eta}_{t+1}^{(q)}, \tilde{x}_{t+1}, \tilde{\pi}_{t+1})'$ contain the backward-looking variables. The variance-covariance matrix $Cov(w_t) = \Sigma_w$ in vectorized form is given by

$$vec(\Sigma_w) = (I_{(q+3)^2} - M \otimes M)^{-1} vec(gg')\sigma^2 \tag{C.7}$$

where $M = Z_{11}S_{11}^{-1}T_{11}Z_{11}^{-1}$.[24] According to Söderlind (1999), Z_{11}, S_{11}, and T_{11} follow from the Generalized Schur decomposition $\Phi = \overline{Q}'S\overline{Z}'$ and $\Psi = \overline{Q}'T\overline{Z}'$ with

$$S = \begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix}, \quad T = \begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix}, \quad Z = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \tag{C.8}$$

$\overline{Q}$ and $\overline{Z}$ are the complex-conjugates of Q and Z, respectively. The $(q+3 \times q+3)$-dimensional submatrices S_{11} and T_{11} contain the stable eigenvalues of the system (C.1).

[24] Note that S_{11}^{-1} and Z_{11}^{-1} exist if the conditions of Blanchard and Kahn (1980) hold.