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## Comparing the effectiveness of employment subsidies

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## Comparing the Effectiveness of Employment Subsidies

by Alessio J. G. Brown, Christian Merkl  
and Dennis J. Snower

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Keywords: low wage subsidy; hiring voucher; targeting; employment; unemployment; duration; self-financing;

JEL classification: J23, J24, J38, J64, J68

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# Comparing the Effectiveness of Employment Subsidies

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## Abstract

This paper examines the implications of employment subsidies for employment, welfare, and inequality. In particular, it investigates how these effects depend on which groups these subsidies are targeted at. Our analysis focuses on policies that are "approximately welfare efficient" (AWE), i.e. policies that (a) improve employment and welfare, (b) do not raise earnings inequality and (c) are self-financing. We construct a microfounded, dynamic model of hiring and separations and calibrate it with German data. The calibration shows that hiring vouchers can be AWE, while low-wage subsidies are not AWE. Furthermore, hiring vouchers targeted at the long-term unemployed are more effective than those targeted at low-ability workers.

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# 1 Introduction

This paper compares the effectiveness of two types of employment subsidy policies: wage subsidies targeted at workers with low abilities (reflected in low wages) and hiring vouchers targeted at long-term unemployed workers, at workers with low abilities, or at both.

We make a number of contributions. First, we examine the policy effects in the context of a model that takes a variety of common labour market imperfections into account: insider wage bargaining, hiring and firing costs, and imperfections related to the tax and transfer system. These imperfections are responsible for unemployment that is inefficiently high. Accordingly, our analysis is appropriate to policy design in high-unemployment OECD countries. Second, our analysis allows us to compare the effects of different targeting schemes for employment subsidies. For this purpose, our model allows skills to depend both on heterogeneous abilities and heterogeneous durations of employment and unemployment.<sup>1</sup> Third, in accord with policy makers' actual concerns, we measure policy effectiveness in terms of employment and welfare, and also give explicit consideration to earnings inequality and government budgetary outlays.<sup>2</sup> Fourth, we derive the policy effects of employment subsidies with a purely microfounded model, without assuming a policy-invariant matching function. The reason for this approach is that the matching process may itself be influenced through employment subsidies, so that the use of an exogenously given matching function could run afoul of the Lucas critique.

Finally, since Pareto welfare-improving policies often insuperably difficult to identify in practice, we introduce a new criterion for evaluating policies: "approximate welfare efficiency" (AWE). A policy is approximately welfare efficient when it

1. improves aggregate employment and welfare (defined in terms of the utility functions of the households),
2. does not increase earnings inequality (measured in terms of the Gini coefficient), and
3. is self-financing (i.e. it does not require an additional government budgetary allocation).<sup>3</sup>

We argue that approximate welfare efficiency is a useful concept for policy making, since policies that are approximately welfare efficient are not only desirable for Benthamite reasons (the greatest happiness of the greatest number of people), but are unlikely to be blocked through the political process.<sup>4</sup>

We also investigate the employment and equity effects of implementing employment subsidies in excess of the magnitudes that are self-financing. Specifically, we examine how much employment could be created by each of the policy measures under consideration if the government's net budgetary allocation for this measures were increased by a specified amount.

We evaluate the effectiveness of alternative employment subsidy policies by calibrating the model for the German labour market and deriving the corresponding policy implications. This exercise is meant to be understood as illustrative of our novel approach, which is relevant to other high-unemployment OECD countries. We show that, for the calibrated model, hiring

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<sup>1</sup>We have sought a model that is rich enough to capture the various groups of workers relevant to the alternative targeting approaches, while at the same time being simple enough to generate straightforward policy guidelines.

<sup>2</sup>Previous literature has largely disregarded the complete impact of employment subsidies on the government budget constraint by requiring that aggregate payroll taxes finance aggregate employment subsidies and thereby ignoring the reduction of unemployment benefit payments, which result from reduced employment. (See Orszag and Snower (2003a and 2003b).)

<sup>3</sup>Clearly, approximate welfare efficiency is not equivalent to Pareto welfare efficiency, because an employment policy can obviously satisfy the three conditions above and still generate uncompensated losers.

<sup>4</sup>The reason is that the fear of rising earnings equality is the most common reason for blocking efficiency-improving employment reforms. See also Orszag and Snower (1998), Saint Paul (1995 and 1996).

vouchers targeted at the duration of unemployment, specifically at the long-term unemployed, is particularly effective in raising employment and welfare, without increasing earnings inequality or requiring an additional government budget outlay. Moreover, while low wage subsidies can also reduce earnings inequality, they are a relatively expensive and ineffective instrument for reducing unemployment and are not AWE at all.

Our results differ from the search and matching framework with endogenous job destruction, where a low wage subsidy would always have positive employment effects, while the effects are ambiguous for hiring vouchers (as they increase job creation and job destruction at the same time, see Pissarides, 2000, p. 217 f.). The reasons are the adoption of a new labour market model for our analysis (see Section 2) and the explicit incorporation of several continental European institutions (e.g., firing costs and insider bargaining). For example, hiring vouchers do not raise the wages of incumbent workers under insider bargaining, which is particularly realistic for continental European countries where union coverage is significant. Furthermore, since our set-up is richer than others in the literature (heterogeneous workers in terms of ability and duration), we are able to analyse targeted subsidies (e.g., to low-ability long-term unemployed), which may be particularly cost-effective. Interestingly, our positive results for targeted hiring vouchers are in line with the recent evaluations of two different hiring subsidy programmes in Germany<sup>5</sup> (see Jirjahn et al., 2009, and the Bernhard et al., 2008).

The paper is organised as follows. Section 2 relates our work to the existing literature. Section 3 presents the theoretical labour market model. Section 4 calibrates the model for Germany, shows the driving forces to make a policy effective and derives the policy implications. Finally, Section 5 concludes.

## 2 Relation to the Literature

Our paper is the first to examine the relative effectiveness of employment subsidies in context of a selection model (i.e., on employers' selection among a set of heterogeneous applicants) and not in context of a contact model (i.e., as typically done in the literature, the matching function and the free entry condition generate the number of contacts between employers and workers), see Brown et al. (2009) for details. Selection models do not suffer from the Shimer (2005) amplification puzzle, i.e., too low volatilities in the labour market.<sup>6</sup> As our model can generate strong amplification effects (i.e., appropriate macroeconomic volatilities, see Brown, Merkl and Snower (2009)), we consider it also as a reliable tool to analyse labour market policies.

There is a large theoretical and empirical literature on the impact and optimal design of employment subsidies, originating with the work by Pigou (1933) and Kaldor (1936).<sup>7</sup> The search and matching framework of Mortensen and Pissarides (1994) is frequently used to analyse the effect of employment subsidies (see e.g. Boone and van Ours (2004), Bovenberg et al. (2000), Cardullo and van der Linden (2006), Mortensen and Pissarides (2003), Pierrard (2005), and Vereshchagina (2002)). The matching technology - describing the relation between the inputs and output of the matching process - is assumed to be stable through time. This assumption is admissible provided that the matching technology (described by the functional form of the matching function) can be considered independent of the inputs and output of the matching process.<sup>8</sup> It is admissible to use the matching function to analyse labour market

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<sup>5</sup>One program was implemented on the federal level, the other one was limited to one particular "Land."

<sup>6</sup>See Lechthaler et al. (2010) for business cycle dynamics in general equilibrium and Snower and Merkl (2006) for the simplest prototype model in partial equilibrium.

<sup>7</sup>For a survey of the empirical literature, see for example Katz (1998). For US evidence, see Woodbury and Spiegelman (1987) and O'Leary et al. (2006). For international evidence, see for example N.E.R.A. (1995), and for British evidence, see Bell et al. (1999). As follows, we will focus on theoretical papers and the calibration thereof.

<sup>8</sup>A negative time trend is found when estimating the search and matching function, thus casting doubt on

policies, provided that these policies have no significant influence on the matching process itself. However, it seems implausible that active labour market policies should have no effect on the matching process.

Many theoretical analyses of employment policies are static and thus do not take into account the dynamic feedback effects of employment policies, as this paper does.<sup>9</sup> The existing literature considers only a small subset of possible targets for employment subsidies. A large part examines the rationale and economic effects of subsidies for the low skilled (e.g., Phelps (1994), Drèze and Snessens (1997), and Oskamp and Snower (2006))<sup>10</sup>, while less attention has been given to subsidies to long-term unemployed workers (Hui and Trivedi (1986), Snower (1994), Vereshchagina (2002)). Furthermore, there is a significant body of literature which proposes the introduction of a low-wage subsidy, either in order to stimulate employment (e.g. SVR (2006), Sinn et al. (2006)) or on grounds of equity considerations (e.g. Phelps (1994), (2003)).

### 3 The Model

We construct a Markov model of the labour market in which the dynamics of employment and unemployment is determined by transition probabilities among various labour market states. We derive these transition probabilities from optimisation principles.

Workers' productivities are *ability-dependent* and *duration-dependent*. This distinction is important for policy purposes. Whereas the duration of employment and unemployment is readily affected through the standard employment policy instruments, ability can be affected primarily through education and training policy and this latter influence generally takes much longer to manifest itself. Since our focus is on employment policy, we let the duration-dependent productivity differences be endogenous in our model (influenceable by the employment subsidies), whereas the ability-dependent productivity differences are defined as exogenous (not influenceable by the subsidies).<sup>11</sup>

Our model contains workers in three ability classes, the exogenous component of skill differences: low-ability, medium-ability and high-ability workers, denoted by  $\alpha = l, m, h$ , respectively. Within each ability class, workers' productivity depends on whether they are employed or unemployed and for how long. Specifically, there are five labour market states, two for the unemployed:

(i) the *long-term unemployed*  $U^L$ , who have been unemployed for more than a year (the period of analysis),

(ii) the *short-term unemployed*  $U^S$ , who have been unemployed up to one year, and three for the employed:

(iii) the *primary entrants*  $N^{E1}$ , who are short-term employed workers (employed up to one year) that were previously short-term unemployed,

(iv) the *secondary entrants*  $N^{E2}$ , who are short-term employed workers that were previously long-term unemployed, and

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the stability through time (Blanchard and Diamond (1989) for the United States, and Fahr and Sunde (2004) for Germany). In contrast to much of the search and matching literature, we use an endogenous job destruction rate. It can for example be expected that a wage subsidy reduces the firing rate, while a hiring subsidy does not do so. Omitting this feature would bias the results.

<sup>9</sup>See, e.g., Layard et al. (1991), pp. 490-492, and Snower (1994). Orszag and Snower (2000) have shown that the dynamic, long-run effects of employment subsidies, once the associated lagged adjustment processes have worked themselves out, differ from what may be expected in the short run.

<sup>10</sup>Mortensen and Pissarides (2003) analyse low wage and hiring subsidies, but do not take different unemployment durations into account.

<sup>11</sup>Our analysis can be extended to education and training policy, then these ability classes would become endogenous; see, for example, Oskamp and Snower (2006).

(v) the *insiders*  $N^I$ , who are long-term employed, i.e. employed for more than a year.

The short-term unemployed workers are more productive than the long-term unemployed. Insiders are more productive than primary entrants who, in turn, are more productive than secondary entrants.

We assume constant returns to labour. Let  $a_{\alpha}^{d_n}$  be the labour productivity of an employee in duration class  $d_n$  and ability class  $\alpha$ ,<sup>12</sup> where  $d_n = I, E1, E2$  for employed workers  $d_u = S, L$  for unemployed workers. The firm faces a random cost  $\varepsilon_{\alpha,t}$ , which is iid across workers and time within the ability class  $\alpha$ . This cost may be interpreted as an operating cost or a negative productivity shock. The expected operating cost conditional on being retained or hired is normalised to zero and its cumulative distribution  $\Gamma_{\alpha}(\varepsilon_{\alpha})$  is time-invariant.

Agents in our model pursue the following sequence of decisions. First the government sets the income tax rate to ensure that its tax receipts are equal to its net budgetary outlays. Second, the random operating costs are revealed. Third, wages are determined through bargaining and then employment decisions are made.

### 3.1 The Government Budget Constraint

For simplicity, our model considers only four instruments of government policy: (i) a proportional payroll tax, with a tax rate  $\tau$ , (ii) an unemployment benefit  $b_{\alpha}$ , (iii) an employment subsidy, specifically a hiring voucher  $\sigma_{\alpha,t}^{d_u}$  targeted at workers of duration-dependent groups and ability-dependent groups or a wage subsidy  $\sigma_{\alpha,t}$  for employees of certain abilities  $\alpha$ , and (iv) the net allocation of government expenditures  $G_t$  to employment subsidies.

The government budget is given by<sup>13</sup>

$$G_t + \sum_{\alpha} N_{\alpha,t} w_{\alpha,t} \tau = \sum_{\alpha} \sum_{d_u} U_{\alpha,t}^{d_u} b_{\alpha} + \sum_{\alpha} \sum_{d_u} \sigma_{\alpha,t}^{d_u} U_{\alpha,t}^{d_u} \eta_{\alpha,t}^{d_u} + \sum_{\alpha} \sigma_{\alpha,t} N_{\alpha,t}, \quad (1)$$

where  $N_{\alpha,t}$  ( $U_{\alpha,t}^{d_u}$ ) is the number of (un)employed in  $t$  of ability class  $\alpha$  (and duration class  $d_u$ ),  $\eta_{\alpha,t}^{d_u}$  the hiring rate in  $t$  of ability class  $\alpha$  and duration  $d_u$ . I.e., the net government allocation  $G_t$  plus tax receipts is equal to government spending on unemployment benefits and the employment subsidies. For simplicity, we assume that the payroll tax is set so as to finance the unemployment benefits in the absence of subsidies and of a net government allocation (i.e.,  $\sigma_{\alpha,t} = \sigma_{\alpha,t}^{d_u} = 0$  and  $G_t = 0$ ).

### 3.2 Wage Bargaining

Let the wage  $w_{\alpha,t}$  for each ability class  $\alpha$  in period  $t$  be the outcome of a Nash bargain between the median insider of that ability class and her firm. Our wage bargaining model is analogous to the median voter model, where the utility of the median voter is maximised. The firm bargains with a union which maximises the utility of the median insider.<sup>14</sup> When the bargaining decision takes place, nobody has been fired yet. The median insider is the worker who is situated in the middle of the distribution and who is assumed to face no risk of dismissal at the negotiated wage. The wage is renegotiated in each period  $t$ . Thus, the present value in period  $t$  is independent of the present value in period  $t + 1$ .

Each worker has the following utility function:

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<sup>12</sup>We follow the notational convention that only those variables have time subscripts that actually vary through time in our model.

<sup>13</sup>As we calculate the long-run effects of different policies (i.e. the new steady states), the static budget constraint is relevant.

<sup>14</sup>Collective bargaining agreements cover more than two thirds of the contracts in most continental European countries (OECD, 2004).



$$v_t(c) = c_t, \quad (2)$$

which for simplicity and without loss of generality is a linear function of consumption  $c_t$ .<sup>15</sup>

Under bargaining agreement, the insider receives the wage  $w_{\alpha,t}(1-\tau)$ , where  $\tau$  is the payroll tax rate, and the firm receives the expected profit  $(a_\alpha^I - \varepsilon_\alpha^{MI} - w_{\alpha,t} + \sigma_{\alpha,t})$  in each period  $t$ , where  $\varepsilon_\alpha^{MI}$  is the operating cost of the median insider,  $\sigma_\alpha$  is a wage subsidy for workers of class  $\alpha$  and  $a_\alpha^I$  is the productivity of an insider of ability class  $\alpha$ . Thus the expected present value of the insider's utility  $V_{\alpha,t}^I$  under bargaining agreement is

$$V_{\alpha,t}^I = (w_{\alpha,t}(1-\tau)) + \delta \left( (1 - \phi_{\alpha,t+1}) V_{\alpha,t+1}^I + \phi_{\alpha,t+1} V_{\alpha,t+1}^S \right), \quad (3)$$

where  $\phi_{\alpha,t+1}$  is the firing rate, the time discount factor is  $\delta$ , and  $V_{\alpha,t+1}^S$  the expected present value of a short-term unemployed workers' returns. The expected present value of firm's returns under bargaining agreement are

$$\Pi_{\alpha,t}^I = (a_\alpha^I - \varepsilon_\alpha^{MI} - w_{\alpha,t} + \sigma_{\alpha,t}) + \delta \left( (1 - \phi_{\alpha,t+1}) \Pi_{\alpha,t+1}^I - \phi_{\alpha,t+1} f_{\alpha,t+1} \right), \quad (4)$$

where  $f_{\alpha,t+1}$  are firing costs.

Under disagreement, the insider receives a constant fallback income  $b_{\alpha,t}$ , which can be conceived as support received during the bargaining disagreement (such as support from family and friends, payments out of a strike fund, proceeds from temporary jobs, and other activities carried out during disagreement). It is assumed equal to the unemployment benefit. The firm's fallback profit represents the profits the firm receives during disagreement. In particular, we assume that during disagreement the insider engages in rent-seeking actions (such as strikes, work-to-rule, sabotage) that impose a cost on the firm that is marginally less than the cost of dismissing the insider ( $f_{\alpha,t}$ ).<sup>16</sup> Note that, in line with the literature on axiomatic and strategic bargaining<sup>17</sup> and recent contributions to the wage formation literature<sup>18</sup>, we distinguish carefully between the fallback positions and outside options of the bargaining parties.<sup>19</sup> These assumptions are in line with European labour legislations, where it is usually unlawful to fire workers during a strike.

Assuming that disagreement in the current period does not affect future returns, the present values of utility under disagreement for the insider are

$$V_{\alpha,t}^{II} = b_{\alpha,t} + \delta \left( (1 - \phi_{\alpha,t+1}) V_{\alpha,t+1}^I + \phi_{\alpha,t+1} V_{\alpha,t+1}^S \right) \quad (5)$$

and for the firm are

$$\Pi_{\alpha,t}^{II} = -f_{\alpha,t} + \delta \left( (1 - \phi_{\alpha,t+1}) \Pi_{\alpha,t+1}^I - \phi_{\alpha,t+1} f_{\alpha,t+1} \right). \quad (6)$$

Thus the insider's bargaining surplus is  $V_{\alpha,t}^I - V_{\alpha,t}^{II} = w_{\alpha,t}(1-\tau) - b_{\alpha,t}$ , and the firm's surplus is  $\Pi_{\alpha,t}^I - \Pi_{\alpha,t}^{II} = a_\alpha^I - \varepsilon_\alpha^{MI} - w_{\alpha,t} + \sigma_\alpha + f_{\alpha,t}$ .<sup>20</sup> The negotiated wage maximises the Nash product ( $\Lambda$ ):

$$\Lambda = (w_{\alpha,t}(1-\tau) - b_{\alpha,t})^\gamma (a_\alpha^I - \varepsilon_\alpha^{MI} - w_{\alpha,t} + \sigma_{\alpha,t} + f_{\alpha,t})^{1-\gamma}, \quad (7)$$

<sup>15</sup>In our model, for simplicity, workers consume all their income.

<sup>16</sup>See, for example, Lindbeck and Snower (1987).

<sup>17</sup>See Binmore, Rubinstein and Wolinsky (1986).

<sup>18</sup>See Hall and Milgrom (2008).

<sup>19</sup>This distinction is important in our analysis. After a bargaining disagreement is resolved, both parties return to their employment relationship without having to pay hiring and firing costs. If, however, the worker's and firm's fallback positions were their outside options, then we would implicitly be assuming that they must pay these labour turnover costs in order to reestablish the employment relationship. The latter, however, happens only when the relationship has been terminated, not when it has been put on hold during disagreement.

<sup>20</sup>See technical online appendix B.1 for the derivation of the negotiation surplus.

where  $\gamma$  represents the bargaining strength of the insider relative to the firm.

In the labour market equilibrium, let firing costs be proportional to the wage,  $f_{\alpha,t} = \rho w_{\alpha,t}$ , and the unemployment benefit be proportional to the wage as well,  $b_{\alpha,t} = \beta_{\alpha}(1 - \tau)w_{\alpha,t}$ , where  $\beta_{\alpha}$  is the net replacement ratio. Then the negotiated wage is

$$w_{\alpha,t} = \frac{\gamma}{[(1 - \gamma)(1 - \beta_{\alpha}) + \gamma(1 - \rho)]} (a_{\alpha}^I - \varepsilon_{\alpha}^{MI} + \sigma_{\alpha,t}). \quad (8)$$

In words, the wage depends positively on the median insider's productivity, the magnitude of the subsidy the insider receives, the replacement rate and the magnitude of firing costs relative to the wage.

### 3.3 Transitions Among Labour Market States

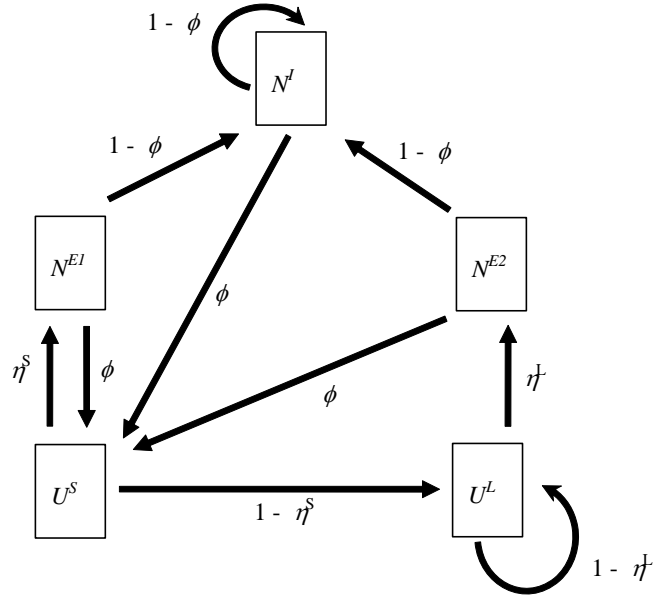


Fig. 1: Transitions among Labour Market States

The transitions<sup>21</sup> among the labour market states are summarised in Figure 1.<sup>22</sup> The short-term unemployed  $U^S$  are hired with probability  $\eta^S$  and then become primary entrants  $N^{E1}$ ; with probability  $(1 - \eta^S)$  they remain unemployed and then join the long-term unemployed  $U^L$ , thereby losing productivity. The long-term unemployed are hired with probability  $\eta^L$  and then become secondary entrants  $N^{E2}$ ; with probability  $(1 - \eta^L)$  they remain long-term unemployed.

Human capital not only depreciates with the duration of unemployment, but also appreciates with the duration of employment. At the end of a period, the primary entrants turn into insiders  $N^I$ , and thereby gain productivity. As insiders, they lose their jobs with probability  $\phi$  and then become short-term unemployed; with probability  $(1 - \phi)$  they are retrained.<sup>23</sup> The same holds

<sup>21</sup>For analytical simplicity, we choose to describe these transitions in terms of a small number of transition variables. To simplify notation in the figure, we suppress the subscripts referring to ability ( $\alpha$ ) and time ( $t$ ) in this figure, e.g. short-term unemployment ( $d_u = S$ ) is written as  $U^S$  rather than  $U_{\alpha,t}^S$ .

<sup>22</sup>The labour market system is summarised algebraically in the technical online appendix B.2.

<sup>23</sup>Entrants turn into insiders at the end of a period. In case they are fired at the beginning of the next period these entrants have been insiders just for an instant. That is the reason why for expositional convenience in figure 1 we let entrants become insiders only if retained.

for the secondary entrants: they, too, turn into insiders, who have a  $\phi$  chance of losing their jobs and a  $(1 - \phi)$  chance of retaining them.

We now proceed to derive the transition probabilities between the labour market states from microeconomic foundations.

### 3.4 Hiring and Firing

#### 3.4.1 The Firing Rate for Insiders

The expected present value of profit generated by an insider, after the random cost  $\varepsilon_{\alpha,t}$  at time  $t$  is observed, is:

$$\begin{aligned}\pi_{\alpha,t}^I &= (a_{\alpha}^I - w_{\alpha,t} - \varepsilon_{\alpha,t} + \sigma_{\alpha,t}) + \sum_{i=1}^{\infty} \delta^i \left[ (1 - \phi_{\alpha,t})^i (a_{\alpha}^I - w_{\alpha,t} + \sigma_{\alpha,t}) - \phi_{\alpha,t} f_{\alpha,t} (1 - \phi_{\alpha,t})^{i-1} \right] \\ &= \frac{a_{\alpha,t}^I - w_{\alpha,t} + \sigma_{\alpha,t} - \delta \phi_{\alpha,t} f_{\alpha,t}}{1 - \delta (1 - \phi_{\alpha,t})} - \varepsilon_{\alpha,t}.\end{aligned}\quad (10)$$

where  $\sigma_{\alpha,t}$  is the wage subsidy.

The expected incentive of retaining the insider,  $\nu_{\alpha,t}^I$ , is defined as the difference between the expected profit from a retained insider<sup>24</sup> and the expected profit from firing an insider ( $-f_{\alpha,t}$ ). Thus this *insider retention incentive* is

$$\nu_{\alpha,t}^I = \frac{a_{\alpha,t}^I - w_{\alpha,t} + \sigma_{\alpha,t} - \delta \phi_{\alpha,t} f_{\alpha,t}}{1 - \delta (1 - \phi_{\alpha,t})} + f_{\alpha,t}.\quad (11)$$

An insider is fired in period  $t$  when the realised value of the random cost  $\varepsilon_{\alpha,t}$  is greater than the insider employment incentive:<sup>25</sup>  $\varepsilon_{\alpha,t} > \nu_{\alpha,t}^I$ . Since the cumulative distribution of the operating cost is  $\Gamma_{\alpha}(\varepsilon_{\alpha,t})$ , the insider's firing rate is

$$\phi_{\alpha,t} = 1 - \Gamma_{\alpha}(\nu_{\alpha,t}^I).\quad (12)$$

#### 3.4.2 The Hiring Rate for Short-Term Unemployed Workers

The expected present value of profit generated by a *primary entrant* (a worker who has been hired after being short-term unemployed), after the random cost  $\varepsilon_{\alpha,t}$  at time  $t$  is observed, is

$$\pi_{\alpha,t}^{E1} = (a_{\alpha}^{E1} - w_{\alpha,t} - \varepsilon_{\alpha,t} - h_{\alpha,t} + \sigma_{\alpha,t} + \sigma_{\alpha,t}^S) + \sum_{i=1}^{\infty} \delta^i \left( (1 - \phi_{\alpha,t})^i (a_{\alpha}^I - w_{\alpha,t} + \sigma_{\alpha,t}) - \phi_{\alpha,t} f_{\alpha,t} (1 - \phi_{\alpha,t})^{i-1} \right),\quad (13)$$

where  $\sigma_{\alpha,t}^S$  is a hiring voucher for a short-term unemployed worker with ability  $\alpha$  and  $h_{\alpha,t}$  are hiring costs.

The expected incentive to hire a short-term unemployed worker,  $\nu_{\alpha,t}^S$ , is defined as the difference between the expected profit from an employed primary entrant<sup>26</sup> and the expected profit from not employing a short-term unemployed (i.e. zero). Thus, the *short-term unemployed hiring incentive* is

$$\nu_{\alpha,t}^S = a_{\alpha}^{E1} - w_{\alpha,t} - h_{\alpha,t} + \sigma_{\alpha,t} + \sigma_{\alpha,t}^S + \frac{\delta (1 - \phi_{\alpha,t}) (a_{\alpha}^I - w_{\alpha,t} + \sigma_{\alpha,t}) - \phi_{\alpha,t} f_{\alpha,t} \delta}{1 - \delta (1 - \phi_{\alpha,t})}.\quad (14)$$

<sup>24</sup>Recall that  $\varepsilon^I$  is normalised by  $E(\varepsilon|\varepsilon \leq \nu_{\alpha}^I) = 0$ .

<sup>25</sup>Equivalently, the insider is fired when the profit from retaining the insider is less than the firing cost.

<sup>26</sup>For simplicity we assume that  $E(\varepsilon|\varepsilon \leq \nu_{\alpha,t}^S) = 0$ .

A primary entrant is hired in period  $t$  when the realised value of the random cost  $\varepsilon_{\alpha,t}$  is less than the primary entrant hiring incentive:<sup>27</sup>  $\varepsilon_{\alpha,t} < \nu_{\alpha,t}^S$ . Thus the hiring rate for short-term unemployed is

$$\eta_{\alpha,t}^S = \Gamma_{\alpha}(\nu_{\alpha,t}^S). \quad (15)$$

### 3.4.3 The Hiring Rate for Long-Term Unemployed

The expected present value of profit generated by a *secondary entrant* (a worker who has been hired after being long-term unemployed),<sup>28</sup> after the random cost  $\varepsilon_{\alpha,t}$  at time  $t$  is observed, is

$$\pi_{\alpha,t}^{E2} = a_{\alpha}^{E2} - w_{\alpha,t} - \varepsilon_{\alpha,t} - h_{\alpha,t} + \sigma_{\alpha,t} + \sigma_{\alpha,t}^L + \frac{\delta(1 - \phi_{\alpha,t})(a_{\alpha}^I - w_{\alpha,t} + \sigma_{\alpha,t}) - \phi_{\alpha,t}f_{\alpha,t}\delta}{1 - \delta(1 - \phi_{\alpha,t})}, \quad (16)$$

where  $\sigma_{\alpha}^L$  is a hiring voucher for a long-term unemployed worker with ability  $\alpha$ . The expected incentive to hire a long-term unemployed ( $\nu_{\alpha,t}^L$ ) is defined as the difference between the expected profit from an employed secondary entrant ( $E(\pi_{\alpha,t}^{E2})$ ) and the expected profit from not employing a long-term unemployed (i.e. zero). The *long-term unemployed hiring incentive* is

$$\nu_{\alpha,t}^L = a_{\alpha}^{E2} - w_{\alpha,t} - h_{\alpha,t} + \sigma_{\alpha,t} + \sigma_{\alpha,t}^L + \frac{\delta(1 - \phi_{\alpha,t})(a_{\alpha}^I - w_{\alpha,t} + \sigma_{\alpha,t}) - \phi_{\alpha,t}f_{\alpha,t}\delta}{1 - \delta(1 - \phi_{\alpha,t})}. \quad (17)$$

A secondary entrant is hired in period  $t$  when the realised value of the random cost  $\varepsilon_{\alpha,t}$  is less than the secondary entrant hiring incentive:  $\varepsilon_{\alpha,t} < \nu_{\alpha,t}^L$ . Thus the hiring rate for long-term unemployed workers is

$$\eta_{\alpha,t}^L = \Gamma_{\alpha}(\nu_{\alpha,t}^L). \quad (18)$$

## 3.5 Employment, Unemployment and the Labour Market Equilibrium

The change in employment in each ability group ( $\Delta N_{\alpha,t}$ ) is the difference between the outflow from the unemployment pool ( $\eta_{\alpha,t}^S U_{\alpha,t-1}^S + \eta_{\alpha,t}^L U_{\alpha,t-1}^L$ ) and the outflow from the employment pool ( $\phi_{\alpha,t} N_{\alpha,t-1}$ ) of that ability group:  $\Delta N_{\alpha,t} = \eta_{\alpha,t}^S U_{\alpha,t-1}^S + \eta_{\alpha,t}^L U_{\alpha,t-1}^L - \phi_{\alpha,t} N_{\alpha,t-1}$ . Assuming a constant labour force  $M_{\alpha}$  in each ability class and defining the employment rate to be  $n_{\alpha,t} = N_{\alpha,t}/M_{\alpha,t}$ , we obtain the following *employment dynamics equation*:

$$n_{\alpha,t} = \eta_{\alpha,t}^S u_{\alpha,t-1}^S + \eta_{\alpha,t}^L u_{\alpha,t-1}^L + (1 - \phi_{\alpha,t}) n_{\alpha,t-1}. \quad (19)$$

The long-term unemployed comprise those workers who were either short- or long-term unemployed in the previous period and who have not been hired in the current period. Thus the *long-run unemployment dynamics equation* is

$$u_{\alpha,t}^L = (1 - \eta_{\alpha,t}^S) u_{\alpha,t-1}^S + (1 - \eta_{\alpha,t}^L) u_{\alpha,t-1}^L. \quad (20)$$

The short-term unemployment rate is the difference between the aggregate unemployment rate and the long-term unemployment rate:

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<sup>27</sup>Equivalently, the primary entrant is hired when the profit from employing this worker is greater than the hiring cost.

<sup>28</sup>For simplicity we assume that  $E(\varepsilon|\varepsilon \leq \nu_{\alpha}^L) = 0$ .

$$u_{\alpha,t}^S = 1 - n_{\alpha,t} - u_{\alpha,t}^L. \quad (21)$$

The labour market equilibrium is the solution of the system comprising

- employment and unemployment dynamic equations (19), (20) and (21),
- the government budget constraint, equation (1),
- the firing and hiring rates, equations (12), (15), (18), and
- the wage equation (8).

## 4 Evaluation Of Employment Subsidies

We now proceed to calibrate the model above for German data and compare the effectiveness of alternatively targeted employment subsidies. First we describe the calibration; second we provide an intuitive account of what determines policy effectiveness. Third, we turn to the numerical results when the subsidies are self-financing. Finally, we relax the self-financing constraint.

### 4.1 Calibration

Naturally, in calibrating our model, we take data commonly used in the literature. Although we performed numerous robustness checks which did not affect the qualitative ranking of hiring vouchers versus low-wage subsidies,<sup>30</sup> our quantitative results should nevertheless be interpreted with caution. The model is, after all, very simple.

The period of analysis is one year. The real interest rate  $r$  is set at 4% per year, which corresponds to the average real interest rate in the OECD over the last four decades, and we set  $\delta = \frac{1}{1+r}$ .

Firing costs and hiring costs are set proportional to 60% ( $f_\alpha = \rho w_\alpha$  with  $\rho = 0.6$ ) and 10% ( $h_\alpha = \mu w_\alpha$  with  $\mu = 0.1$ ) of the labour costs respectively (Chen and Funke (2005)). The net replacement rates  $\beta_\alpha$  are set to 78.25% for low-ability, 68.25% for medium-ability, and 64.67% for high-ability workers (OECD (2006)), respectively. The tax-rate  $\tau$  that balances the government's budget in the absence of subsidies amounts to 6.9%. Table (1) summarises the relevant calibration values.<sup>31</sup>

Keane and Wolpin (1997) estimated rates of skill depreciation during unemployment: white collar workers lose about 30% of their skills after being unemployed for one year, whereas the number is about 10% for blue collar workers (see Keane and Wolpin (1997), p. 500). In Ljungqvist and Sargent (1998, p. 527) the rate of depreciation of skills during unemployment is twice the rate of accumulation. In line with these studies we assume an insider productivity advantage is 10% and a skill depreciation of 20% of the respective productivity due to long-term unemployment.

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<sup>30</sup>In other words, we varied the parameters below within sensible parameter ranges. See Appendix A.6 for details.

<sup>31</sup>See the technical online appendix B.3 for the calculation of the ability specific labour costs and wages as well as for the bargaining power.

parameter		value
$r$	real interest rate	0.04
$\delta$	discount factor	$\frac{1}{1+r}$
$\rho$	firing costs relative to the wage	0.6
$\mu$	hiring costs relative to the wage	0.1
$\beta_l$	net replacement rate for low-skilled	0.7825
$\beta_m$	net replacement rate for low-skilled	0.6825
$\beta_h$	net replacement rate for low-skilled	0.6467
$\tau$	payroll tax rate	0.069
$\gamma$	bargaining power	0.2134

Table 1: Calibration parameter values.

	low-skilled	medium-skilled	high-skilled	aggregate
$m_s$	16.6	59.4	24	100
$u_{s,0}$	18	10.2	5.2	10.3
$u_{s,0}^S$	7.7	5.1	2.8	5
$u_{s,0}^L$	10.3	5	2.4	5.3
$u_{s,0}^L/u_{s,0}$	57	49	46	50
$\eta_{s,0}^S$	49	59	55	56.4
$\eta_{s,0}^L$	38	42	51	43.5
$\phi_{s,0}$	9.4	5.7	2.9	5.6

Table 2: Steady state values of the labour share, unemployment, hiring and firing rates for each skill class and in aggregate in percentage for Germany.

Table (2)<sup>32</sup> shows the percentage values for Germany for the three ability classes of the relevant variables of the employment dynamics equations. The percentage share of the labour force  $m_\alpha$  for each ability class for Germany (2002) is taken from OECD (2005a), the respective aggregate unemployment rates for Germany (2002)  $u_{\alpha,0}$  from OECD (2005b, 2005c). The actual hiring rates for 1996 of each ability and duration group  $\eta_{\alpha,0}^S$  and  $\eta_{\alpha,0}^L$  are taken from Wilke's (2005) Kaplan-Meier functions for Germany. According to OECD (2005b, 2005d) the average share of long-term unemployment ( $u_{\alpha,0}^L/u_{\alpha,0}$ ) is around 50 percent and similar across all ability classes. The firing rates  $\phi_{\alpha,0}$  are assigned the values necessary for the model to reproduce the unemployment rates of the respective ability classes<sup>33</sup>. We interpret these numbers as steady state values.

<sup>32</sup>Note that variables with subscript "0" denote the value at the "old" steady state, i.e. before any policy exercise.

<sup>33</sup>The firing rate of 9% for low-ability employees is close to what can be found in the literature (e.g. Brüssig and Erlinghagen (2005), Fitzenberger et al. (2003) and Wilke (2005)). The firing rate for high-ability is somewhat lower than in reality since many high-ability workers rotate back into work quickly. This phenomenon cannot be captured by our model since it is calibrated on a yearly basis and workers stay unemployment for at least a year. However, this property does not affect the model outcomes for the performed exercises.

We linearise the model around the old steady state<sup>34</sup> and calculate the long-run effects of the policy exercises (new steady state) as permanent deviations from the old steady state. Thus, we have to choose the first derivative of the cumulative density functions in our model ( $\Gamma'_{\alpha,0}$ ), which determines the hiring and firing elasticities. For this purpose, we use empirical estimates, as summarised in Orszag and Snower (1999, p. 208). The first derivative of the cumulative function for the hiring rate ( ${}_{\eta}\Gamma'_{\alpha,0}$ ) (denoted with subscript  ${}_{\eta}$ ) is set in such a way that the hiring elasticity with respect to a hiring voucher is equal to 0.5.<sup>35</sup> Also in line with the aforementioned empirical literature, we set the first derivative of the cumulative function ( ${}_{\phi}\Gamma'_{\alpha,0}$ ) for the firing rate (denoted with subscript  ${}_{\phi}$ ) in such a way that a one-period reduction of the wage has an elasticity of 0.125.

Furthermore we evaluate the relative magnitudes of the hiring vis-à-vis the firing elasticity by comparing our model with the business cycle dynamics in Germany. In our model low wage subsidies have a similar effect as an increase in productivity, except that they have to be financed by the government. Therefore, the relative contributions of the job-finding and the separation rate to empirical unemployment fluctuations could give us a hint on the relative importance of these two margins. Bachmann, 2005, and Gartner et al., 2009, show that separations do not seem to matter a lot along the German business cycle. Furthermore, it can be seen in Gartner et al., 2009, that productivity and separations have a correlation close to zero in Germany (while the job-finding rate is clearly pro-cyclical). Thus, we conclude that our relatively larger hiring elasticities seem plausible from the business cycle perspective.

To double check that we have chosen appropriate hiring and firing elasticities, we compare the endogenous reactions of our model to the empirical labour demand literature. A permanent 10 percent wage cut (*ceteris paribus*) for low-ability workers generates for example an increase in the employment rate of 8.7 percent in the long-run, which yields a long-term labour demand elasticity of -0.87.<sup>36</sup>

Empirical labour demand elasticities for Germany are generally estimated in a range from -0.3 to -0.9. (See Riphahn et al. (1999).) Sinn et al. (2006, p. 10) point out that these estimation results rather reflect short-term than long-term elasticities, refer to studies where considerable higher estimates have been found for the low wage sector and consider an elasticity of -1 as realistic. Thus, we see ourselves well in line with the empirical labour demand literature for Germany. Finally, we perform a sensitivity analysis and conclude that the qualitative conclusions are robust to variations in the elasticities of the hiring and firing rate, see Section 4.5 for a summary and Appendix A.1 for details.

## 4.2 *Determinants of Policy Effectiveness*

Recalling that our notion of approximate welfare efficiency involves the satisfaction of three constraints - an employment and welfare constraint, an earnings inequality constraint, and a self-financing constraint - we find, in our calibration exercises, that as each subsidy is increased, the self-financing constraint is reached first. Thus the self-financing constraint determines the magnitude of each policy intervention that is compatible with approximate welfare efficiency.

We begin by examining the potential of alternative policies to be self-financing.

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<sup>34</sup>See technical online appendix B.4.1.

<sup>35</sup>The hiring elasticity is defined as the reaction of the hiring rate to a hiring voucher for short-term unemployed, which is permanently paid during the first year of the employment spell ( $\chi_{\alpha} = \frac{\partial \eta_{\alpha}^S}{\eta_{\alpha}^S} / \frac{\partial \sigma_{\alpha}^S}{w_{\alpha}}$ ). For simplicity, we choose the same  ${}_{\eta}\Gamma'_{0,\alpha}$  for short-term and long-term unemployed in each ability group.

<sup>36</sup>Note that the endogenous labour demand elasticity in our model varies with the size of the wage movement. The bigger the change in the wage, the smaller is the labour demand elasticity (in absolute terms).

### 4.2.1 Wage Subsidies versus Hiring Vouchers

Let us start with comparing the two most general types of subsidies:

- a wage subsidy,  $\sigma_{\alpha,t}$ , paid to the firm for each employed worker of a specific ability class,
- 1-period hiring voucher,  $\sigma_{\alpha,t}^{du}$ , paid to the firm for hiring a worker of a specific target group (duration and ability).

Intuitively, hiring vouchers are easier to self-finance than wage subsidies, for the following reasons:<sup>37</sup>

**Deadweight Effect** The deadweight effect - defined as the amount of subsidy payments which are paid to workers who would have been employed in absence of the subsidy - is much larger for wage subsidies than for hiring vouchers. For any particular ability group, the proportion of unemployed workers who would have been hired without the subsidy is clearly smaller than the proportion of all employed workers who would have been retained without the subsidy. For example, 82% of the low-skilled workers in Germany are already employed in absence of a low-wage subsidy, while this group contains 16.6% of all workers. Thus, 13.6% of the workforce would receive a low wage subsidy, although these people would be employed without a subsidy. For hiring vouchers deadweight subsidy payments cover only about 5% of the workforce.

**Wage Effect** The wage-effect is defined as the proportion of the subsidy that goes into wage increases and thus does not encourage employment creation. This wage effect is stronger for wage subsidies than for hiring vouchers, since newly hired workers generally have less market power than incumbents (including those that would qualify for wage subsidies).

The underlying institutional implications are particularly relevant to continental European countries, where high labour turnover costs exist and most contracts are bargained by unions, who care about insiders. Kohaut and Schnabel (2004) and Schnabel (2005) show that collective wage agreements<sup>38</sup> also anchor wage agreements for many firms that are not covered by collective agreements<sup>39</sup>.

But even if a certain share of wages is negotiated individualistically, so that both wage subsidies and the hiring subsidies affect the negotiated wage, wage subsidies are expected to have a larger wage effect than hiring subsidies. The reason is that, in practice, the positions of newly hired workers are less protected by labour turnover costs than those of the insiders. Thus the latter will be able to capture more of the bargaining surplus than newly hired workers.

Recent microeconomic evidence for Germany by Stephan (2010) strongly supports the model's implication that entrants' wages are not affected by hiring vouchers. It shows that there are no significant wage effects for workers who obtain a hiring subsidy (vis-à-vis those who do not). Stephan (2010) does not discriminate whether this is due to insider bargaining or the low bargaining power of entrants. However, the underlying reason for the lack of wage effects for hiring subsidies does not matter for our model analysis.

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<sup>37</sup>Since the following effects are strongly interrelated, we will not try to disentangle them in the numerical exercise below.

<sup>38</sup>Approx. 70 percent of the workers in West Germany are covered by wage agreements either on the firm or the sector level.

<sup>39</sup>This anchoring comprises the wages of 50 percent of the remaining non-covered workers (see Kohaut and Schnabel, 2004, for details).



### 4.2.2 Targeting Hiring Vouchers at Duration versus Ability

The choice of targeting scheme (focused on unemployment duration or ability groups) depends on the following effects. Overall, the net effect of these is ambiguous. Therefore, we will proceed to analyse the effectiveness quantitatively.

**Deadweight Effect** Low-ability workers and the long-term unemployed have the lowest hiring rates and thus the lowest deadweight.

**Replacement Rate Effect** Since unemployed, low-ability workers have the highest replacement rates (as shown in section 4.1), hiring vouchers for these groups are more likely to be self-financing than wage subsidies.

**Transition Effect** When hiring vouchers bring workers back to work, their human capital appreciates. This effect is strongest for long-term unemployed since they have been most affected by human capital loss.

## 4.3 Numerical Results: AWE Subsidies

We solve our above model for Germany, and compare the effectiveness of the following employment subsidy policies:

(i) A low-wage subsidy ( $\sigma_l$ ) which is paid (each period) for each low-wage/ability employee. It will reduce the firing rate, by making employees more profitable for the firm. Thus, it raises the insider retention incentive, whereby the firm retains more workers with high operating costs (low productivity).

At the same time the hiring rate will increase since the subsidy provides the incentive to hire more low productivity workers, who would not have been hired otherwise.

(ii) A hiring voucher targeted at low-ability workers ( $\sigma_l^{du}$ ), which is paid for hiring unemployed, low-ability workers. Following the same rationale as above, the firm will hire more workers than without a voucher. In contrast to the first policy, the firing rate will not be affected since the voucher is only paid for new hires and not for the entire employment stock.<sup>40</sup>

(iii) A hiring voucher targeted at long-term unemployed workers ( $\sigma_\alpha^L$ ), which is paid if a long-term unemployed worker is hired.

(iv) A hiring voucher targeted at the low-ability, long-term unemployed workers ( $\sigma_l^L$ ).

Our exercise reveals that a *low-wage subsidy* is not an AWE (approximately welfare efficient) policy for Germany. While a low-wage subsidy creates employment and reduces inequity, it *is not self-financing*. This result is driven by the *deadweight effect* and the *wage effect* above. (Thus low wage subsidies can only be implemented if the government is willing to provide extra resources permanently.) Furthermore, our results show that *hiring vouchers* for Germany can be self-financing and thereby AWE, depending on the target group.

To determine the most effective employment subsidy, we examine the approximately welfare efficiency of hiring vouchers targeted at the low-productivity groups, namely at long-term unemployed as well as at the low-ability unemployed, and compare their employment, welfare and equity effects.

For both groups there are two possible options for hiring vouchers (HV):

**Option 1:** the same lump-sum voucher is paid for hiring a long-term unemployed worker (low-ability worker) irrespectively of his ability class (unemployment duration),

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<sup>40</sup>This holds for constant returns to labour; we will discuss the effect of decreasing returns and displacement effect in section 4.4.

**Option 2:** a different voucher is paid for hiring a long-term unemployed worker (low-ability worker) depending on his ability class (unemployment duration).

While option 1 implies a voucher which is self-financing across ability classes (unemployment duration), option 2 is determined to be self-financing within each ability class (unemployment duration), thereby, preventing cross-subsidisation across ability classes (unemployment duration).

**Targeting Long-Term Unemployed:** Vouchers targeted at long-term unemployed (LTU) workers are AWE for Germany. Table (3) compares the effectiveness of the two design options by describing their unemployment, welfare and equity implications, the latter given by the Gini coefficient.<sup>41</sup>

If a same lump sum hiring voucher is paid for all long-term unemployed compared to an ability specific payment, the self-financing restriction is hit much earlier. While only 947 € per worker are AWE in the former case, up to 4390 € (2503 €) can be paid for low-ability (medium-ability) workers in the latter. The intuition is straightforward: option 2 fully exploits the larger self-financing areas for long-term unemployed workers in the low-ability and medium-ability class, thereby, it prevents costly cross-subsidisation. The self-financing AWE subsidy decreases with productivity due to a smaller *deadweight effect* and the bigger *replacement rate effect*.

By comparing the results of these two exercises, we see that hiring vouchers of different magnitudes for each ability group are most effective in terms of unemployment reduction, welfare improvement and earnings inequality reduction.<sup>42</sup>

		HV for LTU <sub>(Option 1)</sub>	HV for LTU <sub>(Option 2)</sub>	HV for LAU <sub>(Option 2)</sub>
<b>1</b>	<b>Subsidy</b>	<b>947</b>	<b>4390 / 2503 / 0</b>	<b>4390 / 0</b>
<b>2</b>	<b>Subsidy in % of respective wage</b>	<b>3.7 / 3.1 / 2.2</b>	<b>16.9/8.4/0</b>	<b>16.9</b>
3	% Change of Low-Ability Long-Term Unemployment	-2.1	-8.7	-8.7
4	% Change of Low-Ability Unemployment	-0.8	-4.4	-4.4
<b>5</b>	<b>% Change of Long-Term Unemployment</b>	<b>-2.0</b>	<b>-6.2</b>	<b>-2.9</b>
<b>6</b>	<b>% Change of Overall Unemployment</b>	<b>-0.9</b>	<b>-2.9</b>	<b>-1.4</b>
<b>7</b>	<b>% Change of Welfare</b>	<b>0.03</b>	<b>0.07</b>	<b>0.02</b>
<b>8</b>	<b>Gini Coefficient (old Steady State 11.47)</b>	<b>11.45</b>	<b>11.41</b>	<b>11.45</b>

Table 3: Approximately welfare efficient hiring vouchers (HV) for long-term unemployed (LTU) workers in design options 1 and 2, for low-ability unemployed (LAU) workers in design option 2 and the resulting unemployment, welfare and equity implications. [Option 2 vouchers are differentiated in those for low-, medium- and high-ability (LTU) or a in those for long-term and short-term unemployed (LAU)]

<sup>41</sup>The welfare of the workforce is calculated as the sum of the utility of the workers over the various labour market states. See technical online appendix B.5. The cross-policy ranking of changes in welfare corresponds to the ranking of changes in overall unemployment. Furthermore, note that the Gini coefficient generated by our model is lower than in reality, as our model does not generate income differentials within ability groups and it does not take non-wage related inequalities into account (e.g., due to a wealth distribution).

<sup>42</sup>We also checked the robustness of these results, by setting the firing costs for entrants to zero in a simplified nonlinear setting. We compared the effects of hiring vouchers with and without firing costs for entrants. The reduction of unemployment due to hiring vouchers is only marginally smaller in the case without firing costs for entrants compared to the case with firing costs for entrants. Thus, our results are not driven by the assumption that entrants are protected by firing costs.

**Targeting Low-Ability Unemployed:** If a lump sum hiring voucher is targeted at all low-ability unemployed (LAU) (option 1), there is no self-financing area at all. But as shown in table (3) differentiating the vouchers for short-term and long-term unemployed workers reveals an approximately welfare efficient hiring voucher for low-ability workers (4390 €), which though is present only for long-term unemployed workers.<sup>43</sup> Thus, targeting vouchers at long-term unemployed workers (targeted at the low- and medium-ability workers) is more effective than targeting low-ability workers (roughly 3% unemployment reduction vis-à-vis 1.4 percent reduction).<sup>44</sup> This is due to a strong transition effect and a weaker deadweight effect of hiring vouchers for long-term unemployed workers. This quantitative result depends on our calibration for Germany and the relative ranking of these two targeting approaches could be different in other continental European countries. It is though a robust result that hiring vouchers are more likely to be self-financing than low-wage subsidies, see Appendix A.1 for robustness checks.

**Employment-Equity Trade-Off:** Interestingly, the self-financing hiring voucher reduces unemployment and inequality at the same time.

#### 4.4 Numerical Results: Relaxing the Self-Financing Constraint

We now examine the effectiveness of these policies when we relax the self-financing condition, by allowing a small increase in government spending.<sup>45</sup> Specifically, we assume that in the long-run the government allocates a net expenditure of € 50 in real terms ( $G_t/M_t = 50$ , per year and per person of the workforce) for active labour market policies, which corresponds to roughly €2 billion for Germany. These expenditures are allocated to the targeted groups by increasing the subsidy (in equal Euro steps for all targeted groups) until the (new steady state's) budget constraint is reached.

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<sup>43</sup>The reason is that short-term unemployed workers have a higher productivity than the long-term unemployed, thereby a higher hiring rate, which implies a higher *deadweight effect* and a smaller *transition effect*. Both impede a self-financing areas for these workers.

<sup>44</sup>Overall, it has to be mentioned that the size of the approximately welfare efficient subsidy depends crucially on the hiring elasticities, which can be influenced substantially by policy makers. Designing a successful subsidy system should include complementary measures, such as improving job placement or increasing the pressure to accept job offers, to ensure the aforementioned simulated or even better long run effects. (Such complementarities are discussed, for example, in Coe and Snower (1997) and Orszag and Snower (1998).)

<sup>45</sup>Since the government outlay is small, we could reinterpret "approximately welfare efficient" policies to be ones that increase employment and welfare, do not increase earnings inequality, and are "approximately" self-financing. We do not calculate a welfare measure for this exercise since the government either has to cut other spending positions (which may reduce the agents' utility) or create government debt (which has to be paid by future taxes).

		<b>HV for LTU<sub>(Option 2)</sub></b>	<b>HV for LAU<sub>(Option 2)</sub></b>	<b>Low-Wage Subsidy</b>
<b>1</b>	<b>Total Subsidy (% of repective wage)</b>	<b>51.7/38.5/20.5</b>	<b>64.6 / 47.7</b>	<b>1.5</b>
<b>2</b>	<b>Additional Subsidy</b>	<b>9033</b>	<b>12363</b>	<b>403</b>
3	% Marginal Reduction of Low-Ability Unemployment	-7.6	-18.7	-1.1
4	% Marginal Reduction of Long-Term Unemployment	-14.1	-11.4	-0.6
5	% Marginal Reduction of Overall Unemployment	-7	-5.3	-1.0
<b>6</b>	<b>% Total Reduction of Low-Ability Unemployment</b>	<b>-11.7</b>	<b>-22.3</b>	<b>-1.1</b>
<b>7</b>	<b>% Total Reduction of Long-Term Unemployment</b>	<b>-19.5</b>	<b>-14</b>	<b>-0.6</b>
<b>8</b>	<b>% Total Reduction of Total Unemployment</b>	<b>-9.7</b>	<b>-6.6</b>	<b>-1.0</b>
<b>9</b>	<b>Gini Coefficient</b>	<b>11.29</b>	<b>11.38</b>	<b>11.32</b>

Table 4: The Effects on Increased Government Spending on Hiring Vouchers (HV) for Long-term Unemployed (LTU) and Low-Ability Unemployed (LAU) Workers.

Table (4) compares the effects of hiring vouchers for long-term unemployed (LTU) and for low-ability (LAU) workers and low-wage subsidies, with regard to unemployment and inequality. The "marginal unemployment reduction" refers to the changes beyond the self-financing subsidy, whereas the "total reduction" is calculated relative to the steady state before the policy was applied.<sup>46</sup>

**Employment Effect** Low wage subsidies perform worse than hiring vouchers in stimulating employment, due to the deadweight effect<sup>47</sup> and the wage effect. The above net expenditure on hiring subsidies should be spent for hiring vouchers targeted at long-term unemployed workers in different ability classes and could cut long-term unemployment by 20% and overall unemployment by roughly 10%.

**Displacement Effect** To introduce the displacement effect into our model – viz., hiring vouchers induce firms to substitute the subsidised workers for existing employees – we assume decreasing returns to labour in the short run, under a constant stock of physical capital.<sup>48</sup> We assume a Cobb-Douglas function  $Y = AN^{1-\alpha}\bar{K}^\alpha$ , where capital is a constant  $\bar{K}$ . If additional workers are hired, the marginal product of labour of each of the workers is reduced. Thus, the difference between marginal productivity and the wage falls and firms will start firing more workers and hiring fewer workers (i.e., there is a countervailing effect to the stimulus from hiring subsidies). Thus, the diminishing returns generate substitutability between workers.

When we assume that the capital share for Germany is 33% (i.e.  $\alpha = 1/3$ , see Statistisches Bundesamt (2006)), we find that the displacement effect is weak: the size of AWE hiring vouchers for low-ability, long-term unemployed workers is reduced from 17 to 14% of the wage. The reason is the transition effect.

<sup>46</sup>Naturally, as low-wage subsidies are not approximately efficient, the marginal reduction is equal to the overall reduction.

<sup>47</sup>While the hiring voucher targeted at the long-term unemployed is paid to roughly 2.2% of the overall working population, it is almost 13.6% for low-wage subsidies. The deadweight is substantial for wage subsidies as 99.7% of the recipients would also have been employed in the absence of a subsidy. This number is considerably smaller for hiring vouchers targeted at long-term unemployed (68.5%).

<sup>48</sup>Clearly, the more sharply decreasing the returns to labour are, the more the employment of subsidized workers will reduce the marginal product of the existing workers, and thus the greater the incentive to dismiss these existing workers.

If the hiring voucher pushes a long-term unemployed worker back to work, her human capital appreciates. Thus when the voucher expires, former long-term unemployed workers face a retention probability which is considerably higher than their initial hiring probability. Furthermore, even if workers are displaced or the worker is fired once the voucher expires, a long-term unemployed worker is exchanged for a short-term unemployed worker with higher human capital and thereby a higher reemployment probability. This transition effect has been shown to be both statistically and economically significant in different German labour market programmes. See, for example, the evaluation of the "Hamburger Modell," a lump-sum hiring voucher adopted in a model experiment in Hamburg (Jirjahn et al. (2009)), and the evaluation of the so-called "Eingliederungszuschuss," which is a limited hiring subsidy (Bernhard et al. (2008)).

In the long run, of course, the capital adjusts to the larger workforce with higher human capital, and this adjustment naturally reduces, and possibly eliminates, displacement.<sup>49</sup> Thus we conclude that displacement effects have no substantial effect on our results.

**Equity Effect** Whereas low-wage subsidies improve equity by raising the wages of low-ability workers, hiring subsidies improve equity by bringing the long-term unemployed back to work. Our calibration shows that the hiring subsidies have a stronger equity-reduction effect.<sup>50</sup>

## 4.5 Robustness of the Results

To check the robustness of our results, we varied the hiring and firing elasticity within reasonable bounds. Specifically, we provided a sensitivity analysis for hiring elasticity values of 0.25 and 0.75 (instead of 0.5 in the benchmark exercise) and firing elasticity values of 0.05 and 0.2 (instead of 0.125 in the benchmark exercise) and furthermore, we determined which elasticities are necessary to change the ranking of our results, see Appendix A.1 for the numerical results. In none of these exercises, low wage subsidies are self-financing. Thus, they always fail to be AWE. In most of the cases, there exist hiring vouchers for LTU that are self-financing (except for very low hiring elasticities). In all these exercises, subsidies that are targeted at long-term unemployed are the more effective measure to reduce unemployment than low-wage subsidies. This is also true when relaxing the self-financing constraint. Unreasonable values for the hiring elasticity and for the firing elasticity, namely 0.1 and 0.6, respectively, are needed to make low-wage subsidies more effective in reducing unemployment. Therefore, we conclude that our main results are very robust about a reasonable parameter space.

## 5 Concluding Thoughts

In the context of a new labour market model, which is illustratively calibrated for Germany, we have shown that low-wage subsidies (targeted at low-income/ability workers) are not approximately welfare efficient (AWE), i.e. no positive low-wage subsidies are self-financing. By contrast, hiring vouchers can be AWE. Our calibrated model suggests that hiring subsidies for the long-term unemployed are more effective than hiring vouchers for low-income/ability

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<sup>49</sup>Although our quantitative analysis above omits displacement effects, we consider our calibration to be rather conservative (with respect to the size of the approximately efficient subsidy). While we used a tax rate (6.9 percent) to balance the budget (financing unemployment related expenses), in reality other tax revenues would also increase with the employment rate. This would raise the self-financing, and thereby approximately efficient subsidy and thus lead to a higher effectiveness (in terms of employment, welfare and inequality).

<sup>50</sup>To gain some perspective on our analysis in the appendix A.2 we discuss some important effects - lying beyond the scope of our model (namely, substitution effects between skill classes, asymmetric information, skill acquisition effects, households' job acceptance incentives) - some of which may be expected to weaken or even reverse our qualitative results.

workers. The same ranking, in terms of employment, holds for employment subsidies financed through government expenditures extending beyond the AWE limit.

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# A Appendix

## A.1 Sensitivity Analysis

In the following we provide a sensitivity analysis to assess the robustness of our results with respect to the hiring and firing elasticities. Specifically, we have varied the first derivative of the cumulative function for the hiring rate as well as for the firing rate, which we had set to match empirical estimates, as summarized in Orszag and Snower (1999, p. 208), such that the hiring elasticity with respect to a hiring voucher is equal to 0.5 and that a one-period reduction of the wage has an firing elasticity of 0.125. Specifically, we provided a sensitivity analysis for hiring elasticity values of 0.25 and 0.75 as well as firing elasticity values of 0.05 and 0.2.

### A.1.1 Hiring Elasticity

**Self-financing Subsidy** As shown in Tables 5 and 6, the hiring elasticity is crucial for the effectiveness and especially the self-financing character of the subsidies under inspection. Even with a high elasticity of 0.75 the low-wage subsidy is not self-financing. In contrast, the hiring vouchers for long-term unemployed workers cease to being self-financing with a very low elasticity of 0.25. Under our calibration and under any hiring elasticity, self-financing low-wage subsidies fail to be more effective in reducing unemployment relative to hiring vouchers for long-term unemployed workers.

	<i>Hiring Elasticity of 0.75</i>	<b>HV for LTU<sub>(Option 2)</sub></b>	<b>Low-Wage Subsidy</b>
<b>1</b>	<b>Subsidy</b>	<b>17802 / 16714/ 529</b>	<b>0</b>
<b>2</b>	<b>Subsidy in % of respective wage</b>	<b>68.6/55.8/1.2</b>	<b>0</b>
3	% Change of Low-Ability Long-Term Unemployment	-36.9	0
4	% Change of Low-Ability Unemployment	-19.4	0
<b>5</b>	<b>% Change of Long-Term Unemployment</b>	<b>-32.3</b>	<b>0</b>
<b>6</b>	<b>% Change of Overall Unemployment</b>	<b>-15.5</b>	<b>0</b>
<b>7</b>	<b>Gini Coefficient (old Steady State 11.47)</b>	<b>11.139</b>	<b>11.47</b>

Table 5: AWE hiring vouchers (HV) for long-term unemployed (LTU) in option 2 and the resulting unemployment, welfare and equity implications for a hiring elasticity of 0.75.

	<i>Hiring Elasticity 0.25</i>	<b>HV for LTU<sub>(Option 2)</sub></b>	<b>Low-Wage Subsidy</b>
<b>1</b>	<b>Subsidy</b>	<b>0</b>	<b>0</b>
<b>2</b>	<b>Subsidy in % of respective wage</b>	<b>0</b>	<b>0</b>
3	% Change of Low-Ability Long-Term Unemployment	0	0
4	% Change of Low-Ability Unemployment	0	0
<b>5</b>	<b>% Change of Long-Term Unemployment</b>	<b>0</b>	<b>0</b>
<b>6</b>	<b>% Change of Overall Unemployment</b>	<b>0</b>	<b>0</b>
<b>7</b>	<b>Gini Coefficient (old Steady State 11.47)</b>	<b>11.47</b>	<b>11.47</b>

Table 6: AWE hiring vouchers (HV) for long-term unemployed (LTU) in option 2 and the resulting unemployment, welfare and equity implications for a hiring elasticity of 0.25.

## Relaxing the *Self-financing Constraint*

	<i>Hiring Elasticity of 0.75</i>	<b>HV for LTU</b>	<b>Low-Wage Subsidy</b>
<b>1</b>	<b>Total Subsidy (% of repective wage)</b>	<b>89.8/74.2/13.7</b>	<b>1.7</b>
<b>2</b>	<b>Additional Subsidy</b>	<b>5504</b>	<b>434</b>
3	% Marginal Reduction of Low-Ability Unemployment	-4.1	-2.8
4	% Marginal Reduction of Long-Term Unemployment	-9.6	-1.3
5	% Marginal Reduction of Overall Unemployment	-4.6	-1.0
<b>6</b>	<b>% Total Reduction of Low-Ability Unemployment</b>	<b>-22.8</b>	<b>-2.8</b>
<b>7</b>	<b>% Total Reduction of Long-Term Unemployment</b>	<b>-38.9</b>	<b>-1.3</b>
<b>8</b>	<b>% Total Reduction of Total Unemployment</b>	<b>-19.4</b>	<b>-1.0</b>
<b>9</b>	<b>Gini Coefficient</b>	<b>11.09</b>	<b>11.31</b>

Table 7: The Effects on Increased Government Spending on Hiring Vouchers (HV) for Long-term Unemployed (LTU) and Low-Wage Subsidies for a Hiring Elasticity of 0.75.

	<i>Hiring Elasticity of 0.25</i>	<b>HV for LTU</b>	<b>Low-Wage Subsidy</b>
<b>1</b>	<b>Total Subsidy (% of repective wage)</b>	<b>17.0 / 0 / 0</b>	<b>1.4</b>
<b>2</b>	<b>Additional Subsidy</b>	<b>4415</b>	<b>376</b>
3	% Marginal Reduction of Low-Ability Unemployment	-2.2	-0.6
4	% Marginal Reduction of Long-Term Unemployment	-3.5	-0.3
5	% Marginal Reduction of Overall Unemployment	-2.9	-1.0
<b>6</b>	<b>% Total Reduction of Low-Ability Unemployment</b>	<b>-2.2</b>	<b>-0.6</b>
<b>7</b>	<b>% Total Reduction of Long-Term Unemployment</b>	<b>-3.5</b>	<b>-0.3</b>
<b>8</b>	<b>% Total Reduction of Total Unemployment</b>	<b>-2.9</b>	<b>-1.0</b>
<b>9</b>	<b>Gini Coefficient</b>	<b>11.43</b>	<b>11.33</b>

Table 8: The Effects on Increased Government Spending on Hiring Vouchers (HV) for Long-term Unemployed (LTU) and Low-Wage Subsidies for a Hiring Elasticity of 0.25.

The sensitivity analysis for the exercise of relaxing the self-financing constraint shows similar results – while hiring vouchers are more sensitive than low-wage subsidies to the elasticity, the qualitative statement is not affected by reasonable values for the elasticities. Hiring vouchers for long-term unemployed workers are able to provide stronger employment effects. With an elasticity of 0.25, hiring subsidies for long-term unemployed workers are still three times more effective in terms of a reduction of the unemployment rate than low-wage subsidies. The relative ranking is inverted once a unreasonably low value for the hiring elasticity of 0.1 is applied.

### A.1.2 *Firing Elasticity*

The firing elasticity plays only a role for the low-wage subsidies and as can be seen in the following Tables 9 and 10, , varying it from 0.05 to 0.20 does not make any big difference and leaves our results unaffected.

## *Self-financing Subsidy*

	<i>Low-Wage Subsidy for a Firing Elasticity of</i>	<b>0.2</b>	<b>0.05</b>
<b>1</b>	<b>Subsidy</b>	<b>0</b>	<b>0</b>
<b>2</b>	<b>Subsidy in % of respective wage</b>	<b>0</b>	<b>0</b>
3	% Change of Low-Ability Long-Term Unemployment	0	0
4	% Change of Low-Ability Unemployment	0	0
<b>5</b>	<b>% Change of Long-Term Unemployment</b>	<b>0</b>	<b>0</b>
<b>6</b>	<b>% Change of Overall Unemployment</b>	<b>0</b>	<b>0</b>
<b>7</b>	<b>Gini Coefficient (old Steady State 11.47)</b>	<b>11.47</b>	<b>11.47</b>

Table 9: The effect of the firing elasticity on the approximate welfare efficiency of low-wage subsidies.

Low-wage subsidies remain non-self-financing even with a firing elasticity of 0.2. For any firing elasticity<sup>51</sup> low-wage subsidies fail to be more effective in reducing unemployment relative to hiring vouchers for long-term unemployed workers.

## *Relaxing the Self-financing Constraint*

	<i>Low-Wage Subsidy for a Firing Elasticity of</i>	<b>0.2</b>	<b>0.05</b>
<b>1</b>	<b>Total Subsidy (% of respective wage)</b>	<b>1.6</b>	<b>1.5</b>
<b>2</b>	<b>Additional Subsidy</b>	<b>423</b>	<b>403</b>
3	% Marginal Reduction of Low-Ability Unemployment	-1.7	-1.1
4	% Marginal Reduction of Long-Term Unemployment	-0.9	-0.6
5	% Marginal Reduction of Overall Unemployment	-1.0	-1.0
<b>6</b>	<b>% Total Reduction of Low-Ability Unemployment</b>	<b>-1.7</b>	<b>-1.1</b>
<b>7</b>	<b>% Total Reduction of Long-Term Unemployment</b>	<b>-0.9</b>	<b>-0.6</b>
<b>8</b>	<b>% Total Reduction of Total Unemployment</b>	<b>-1.0</b>	<b>-1.0</b>
<b>9</b>	<b>Gini Coefficient</b>	<b>11.31</b>	<b>11.33</b>

Table 10: The effect of the firing elasticity on the employment and equity effects of low-wage subsidies when relaxing the self-financing constraint.

When relaxing the self-financing constraint a firing elasticity of 0.2 does not make much difference to the employment and equity effects of low-wage subsidies. Low-wage subsidies are more effective in reducing unemployment once a unreasonably high value for the firing elasticity of 0.6 is applied.

## **A.2 Extensions**

In this section we discuss various other effects - lying beyond the scope of our model - some of which may be expected to weaken or even reverse our qualitative results.<sup>52</sup>

### **A.2.1 Substitution Effects**

In our setting workers of different ability groups are not substitutable. A strong substitutability would make it profitable for firms to substitute medium and high ability workers with low

<sup>51</sup>Our robustness analysis considered firing elasticities up to a value of 1.

<sup>52</sup>For additional effects see the working paper version of this paper.

ability workers. First, this problem would be more severe for low wage subsidies as they are paid permanently and as they are exclusively targeted at low wages. Second, the empirical literature delivers evidence that the substitutability between different ability groups is quite low (see e.g. Buslei and Steiner, 1999 and SVR, 2006, for German evidence as well as Kremer and Maskin, 1996, for cross country evidence). Thus, substitution effects are not strong, but would reinforce our ranking of policies.

### **A.2.2 *Asymmetric Information***

Subsidies, which are targeted at low wage workers, may provide an incentive for households to shift from a full-time to a part-time position in order to cash the subsidy. If the subsidy is restricted to full time employees only, there may be an incentive for firms and workers to take advantage of asymmetric information to collude and cheat the government to qualify for the subsidy (i.e. falsely claim a lower hourly wage and more hours; and maybe follow a black market activity at the same time). While this problem may not be entirely absent for hiring vouchers which are targeted at long-term unemployed workers, for low wage subsidies it is a lot more difficult to abuse such a scheme. Low-wage workers are only a subset of the target group and they are only eligible if they were unemployed for longer than one year. Furthermore, the voucher is only paid out for a limited period of time. This problem is though particularly relevant for low-wage subsidies, as these are exclusively targeted at low wages and paid permanently. Thus, we conclude that asymmetric information problems again reinforce the ranking in favor of hiring vouchers for long-term unemployed.

### **A.2.3 *Households' Job Acceptance Incentives***

In our model above, as noted, hiring and firing decisions are made by firms, and the households' employment incentives enter the model only through wage bargaining. We have argued that this is reasonable in countries with high unemployment (like Germany), for then labour demand is generally the short side of the labour market, so that firms' labour demands play a leading role in determining employment, while most unemployed households may be expected to accept jobs willingly. The critical reader however may wonder if our ranking of policies (hiring subsidies for long-term unemployed outperform low wage subsidies) would be overturned if we took account of households' job acceptance incentives. Then hiring vouchers may reduce short-term unemployed households' job acceptance rate, since they increase the households' present value of income from unemployment. On the other hand, hiring vouchers raise firms' job offer rate and this may also raise long-term unemployed households' job acceptance rate due to strategic complementarities.

To assess the potential importance of these possibilities, consider the following thought experiment. Let us interpret our hiring rate as a matching rate, i.e. as the product of the job offer rate (i.e. the probability that a firm offers a job to an outsider) and the job acceptance rate (i.e. the probability that the outsider accepts the job offer). Assume that the government allocates a net expenditure of € 50 (per year and per person of the workforce) for employment subsidies. Then assume, that the new matching rate under low-wage subsidies of 50% (39%) for short-term (long-term) low-ability consists of a job offer rate of 66%(52%) and a job acceptance rate of 75% (in both cases).<sup>53</sup> To make the low wage subsidy equivalent with the hiring voucher for long-term unemployed (in terms of job creation) an increase of the job acceptance rate for the former case to 100%,<sup>54</sup> while the job acceptance rate stays constant for the latter, would not be sufficient.<sup>55</sup>

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<sup>53</sup>These matching rates correspond to the hiring rates which result from the policy exercise in section 4.3.3.

<sup>54</sup>For short-term and long-term unemployed.

<sup>55</sup>With different numerical values it would be mathematically feasible to change the ranking. However, such

#### A.2.4 *Skill-Acquisition Incentives*

Low-wage subsidies may create disincentives to acquire human capital . While we assume that the composition of different ability groups is exogenously given, in the long-run employment subsidies may affect workers' incentives to acquire human capital. Oskamp and Snower (2006) show that the positive short-run employment effects of a low wage subsidy may be undone by negative long-run employment effects, as fewer people choose to invest in skills and thus the more unemployment-prone low ability group becomes larger. This effect is particularly strong for low wage subsidies and much less severe with hiring vouchers for long-term unemployed, as they are non-permanent and not exclusively targeted at the lowest ability group.

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an exercise remains economically highly implausible. With a job acceptance rate of 50%, an increase of this rate by roughly 50% would be necessary (for a low wage subsidy) in order to obtain equivalence between a low wage subsidy and a hiring voucher for long-term unemployed. Note that the job acceptance rate after the introduction of a hiring voucher for long-term unemployed would in addition have to stay unaffected.

## B Technical Appendix

### B.1 The Negotiation Surplus

The insider's bargaining surplus is

$$\begin{aligned} V_{\alpha,t}^I - V_{\alpha,t}^{II} &= w_{\alpha,t}(1 - \tau) + \delta \left( (1 - \phi_{\alpha,t+1}) V_{\alpha,t+1}^I + \phi_{\alpha,t+1} V_{\alpha,t+1}^S \right) \\ &\quad - b_{\alpha,t} - \delta \left( (1 - \phi_{\alpha,t+1}) V_{\alpha,t+1}^I + \phi_{\alpha,t+1} V_{\alpha,t+1}^S \right) \\ &= w_{\alpha,t}(1 - \tau) - b_{\alpha,t}, \end{aligned} \quad (22)$$

and the firm's surplus is

$$\begin{aligned} \Pi_{\alpha,t}^I - \Pi_{\alpha,t}^{II} &= (a_{\alpha}^I - \varepsilon_{\alpha}^{MI} - w_{\alpha,t} + \sigma_{\alpha,t}) + \delta \left( (1 - \phi_{\alpha,t+1}) \Pi_{\alpha,t+1}^I - \phi_{\alpha,t+1} f_{\alpha,t+1} \right) - \\ &\quad (-f_{\alpha,t} + \delta \left( (1 - \phi_{\alpha,t+1}) \Pi_{\alpha,t+1}^I - \phi_{\alpha,t+1} f_{\alpha,t+1} \right)) \\ &= a_{\alpha}^I - \varepsilon_{\alpha}^{MI} - w_{\alpha,t} + \sigma_{\alpha} + f_{\alpha,t}. \end{aligned} \quad (23)$$

For the wage bargain, the following relationship holds:

$$(1 - \gamma) (w_{\alpha,t} (1 - \tau) - b_{\alpha,t}) = \gamma (a_{\alpha}^I - \varepsilon_{\alpha}^{MI} - w_{\alpha,t} + \sigma_{\alpha} + f_{\alpha,t}) (1 - \tau). \quad (24)$$

### B.2 The Labour Market System

The labour market system for each ability group  $a$  in period  $t$  may be described as follows:

$$S_{\alpha,t} = T_{\alpha,t} S_{\alpha,t-1}, \quad (25)$$

where  $S_t$  is a vector of the labour market states:

$$S_{\alpha,t} = (N_{\alpha,t}^I, N_{\alpha,t}^{E1}, N_{\alpha,t}^{E2}, U_{\alpha,t}^S, U_{\alpha,t}^L)', \quad (26)$$

and  $T_{\alpha,t}$  is a Markov matrix of transition probabilities:

$$T_{\alpha,t} = \begin{pmatrix} (1 - \phi_{\alpha,t}) & (1 - \phi_{\alpha,t}) & (1 - \phi_{\alpha,t}) & 0 & 0 \\ 0 & 0 & 0 & \eta_{\alpha,t}^S & 0 \\ 0 & 0 & 0 & 0 & \eta_{\alpha,t}^L \\ \phi_{\alpha,t} & \phi_{\alpha,t} & \phi_{\alpha,t} & 0 & 0 \\ 0 & 0 & 0 & (1 - \eta_{\alpha,t}^S) & (1 - \eta_{\alpha,t}^L) \end{pmatrix} \quad (27)$$

### B.3 Labour Costs and Wages

The different abilities' labour costs and wages are calculated as follows: The aggregate producer wage and gross value added per worker can be obtained from Statistische Ämter des Bundes und der Länder (2006). The aggregate producer wage is defined as the average real gross wage per employee plus social security payments. We took the 2003 values for gross value added<sup>56</sup> (50334 Euros) and real labour costs (32672 Euros) since the OECD numbers which we used for further calculations were only available until this point in time.

Using the wage equation (8), we calculated the average bargaining power in the economy, where the variables denote aggregate values:

$$w = (1 - \gamma) \beta w + \gamma ((a - \varepsilon^{MI}) + \rho w). \quad (28)$$

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<sup>56</sup>We interpret this as the productivity of the median insider  $(a^I - \varepsilon)$ .

$$\gamma = \frac{w - \beta w}{(a - \varepsilon^{MI}) + \rho w - w\beta}. \quad (29)$$

We obtain  $\gamma = 0.2134$ .

Ability group specific relative labour costs for Germany are calculated as follows (OECD (2005c)): High-ability workers earn 148% of their medium-ability counter-parts' wage and low-ability 87%, respectively. Low-ability workers' highest education level is lower secondary education, whereas it is upper secondary education or post-secondary non-tertiary education for medium-ability and tertiary education for high-ability. Assuming that the bargaining power is the same in all ability groups and using the respective replacement rates<sup>57</sup> we get for each ability group  $\alpha$

$$(a_\alpha^I - \varepsilon^{MI}) = \frac{w_\alpha - (1 - \gamma_\alpha) \beta_\alpha w_\alpha - \gamma_\alpha \rho w_\alpha}{\gamma_\alpha}. \quad (30)$$

	low-skilled	medium-skilled	high-skilled	aggregate
$m_\alpha$	16.6	59.4	24	100
$w_\alpha$	25948	29940	44100	32672
$(a_\alpha^I - \varepsilon^{MI})$	31179	47012	75069	51109

Table 4: Relevant Labor Cost Values

Table 4 summarises the relevant values.<sup>58</sup> Starting from this steady state we will perform policy exercises and compare the resulting new steady states.<sup>59</sup>

## B.4 Linearisation

### B.4.1 Firing Rate

Non-linear equation:

$$\phi_{\alpha,t} = 1 - \Gamma_\alpha \left( \frac{a_\alpha^I - w_{\alpha,t} + \sigma_{\alpha,t} - \phi_{\alpha,t} f_{\alpha,t} \delta}{1 - \delta (1 - \phi_{\alpha,t})} + f_{\alpha,t} \right), \quad (31)$$

where  $\sigma_\alpha$  ist a wage subsidy for ability class  $\alpha$ . Linearisation:

$$\begin{aligned} \phi_{\alpha,new} &= \phi_{\alpha,0} - \phi \Gamma'_{\alpha,0} \left[ \frac{1}{1 - \delta (1 - \phi_\alpha)} \right]_0 \frac{1}{1 + V_\alpha} \left[ \begin{array}{c} (a_{\alpha,new}^I - w_{\alpha,new} + \sigma_\alpha) \\ - (a_{\alpha,0}^I - w_{\alpha,0}) \end{array} \right] \\ &\quad - \phi \Gamma'_{\alpha,0} \left[ \frac{-\phi_\alpha \delta}{(1 - \delta (1 - \phi_\alpha))} + 1 \right]_0 \frac{1}{1 + V_\alpha} (f_{\alpha,new} - f_{\alpha,0}), \end{aligned} \quad (32)$$

with

$$V_\alpha = \phi \Gamma'_{\alpha,0} \left[ \frac{\delta (f_\alpha (\delta - 1) - (a_\alpha^I - w_\alpha))}{(1 - \delta (1 - \phi_\alpha))^2} \right]_0,$$

where variables with subscript "0" are at the old steady and variables with subscript "new" are at the new steady state.<sup>60</sup>

<sup>57</sup>Furthermore, we assumed that the firing costs are 60% of the labor costs, see Chen and Funke (2003).

<sup>58</sup>Due to the aggregation the value for the aggregate labor cost is not equal to the original value for real labor costs (50334), which we used to compile the bargaining strength and the ability group specific relative labor cost.

<sup>59</sup>See Appendix B.4.

<sup>60</sup>In the calibration, we assume for simplicity that  $\frac{\delta E(\varepsilon|(1-\phi))}{\delta \phi} = 0$ .



### B.4.2 Hiring Rates

Non-linear equation:

$$\eta_{\alpha,t}^S = \Gamma_{\alpha} \left( a_{\alpha}^{E1} - w_{\alpha,t} + \sigma_{\alpha,t}^S + \sigma_{\alpha,t} + \frac{\delta (1 - \phi_{\alpha,t}) (a_{\alpha}^I - w_{\alpha,t} + \sigma_{\alpha,t}) - \phi_{\alpha,t} f_{\alpha,t} \delta}{1 - \delta (1 - \phi_{\alpha,t})} - h_{\alpha,t} \right), \quad (33)$$

where  $\sigma_{\alpha}^S$  is the hiring voucher for short-term unemployed workers of ability class  $\alpha$ .

Linearisation:

$$\begin{aligned} \eta_{\alpha,new}^S &= \eta_{\alpha,0}^S +_{\eta} \Gamma'_{\alpha,0} [(a_{\alpha,new}^{E1} - w_{\alpha,new} + \sigma_{\alpha}^S + \sigma_{\alpha}) - (a_{\alpha}^{E1} - w_{\alpha,0})] \\ &\quad +_{\eta} \Gamma'_{\alpha,0} \left[ \frac{\delta (1 - \phi_{\alpha})}{1 - \delta (1 - \phi_{\alpha})} \right]_0 [(a_{\alpha,new}^I - w_{\alpha,new} + \sigma_{\alpha}) - (a_{\alpha,0}^I - w_{\alpha,0})] \\ &\quad -_{\eta} \Gamma'_{\alpha,0} \left[ \frac{\phi_2 \delta}{1 - \delta (1 - \phi_2)} \right]_0 (f_{\alpha,new} - f_{\alpha,0}) -_{\eta} \Gamma'_{\alpha,0} (h_{\alpha,new} - h_{\alpha,0}) \\ &\quad +_{\eta} \Gamma'_{\alpha,0} \left[ \frac{-\delta ((a_{\alpha}^I - w_{\alpha}) + f_{\alpha} (1 - \delta))}{[1 - \delta (1 - \phi_{\alpha})]^2} \right]_0 (\phi_{\alpha,new} - \phi_{\alpha,0}). \end{aligned} \quad (34)$$

And equivalently for the second unemployment duration group. Ability Group Specific Numbers.

### B.5 The welfare of the workforce

The welfare ( $\Omega$ ) of the workforce is calculated as the sum of the utility of the workers over the various labour market states.

$$\Omega_t = \sum_{\alpha} w_{\alpha,t} (1 - \tau) m_{\alpha} n_{\alpha,t} + \sum_{\alpha} \sum_{d_u} b_{\alpha,t} u_{\alpha,t}^{d_u} m_{\alpha}. \quad (35)$$