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Quadratic labor adjustment costs, business cycle dynamics and optimal monetary policy


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Quadratic Labor Adjustment Costs, Business Cycle Dynamics and Optimal Monetary Policy
by Wolfgang Lechthaler and Dennis Snower

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Quadratic Labor Adjustment Costs, Business Cycle Dynamics and Optimal Monetary Policy

Wolfgang Lechthaler and Dennis Snower

Abstract:

We build quadratic labor adjustment costs into an otherwise standard New-Keynesian model of the business cycle and show that this increases output persistence in a similar vein as other models of labor market frictions.

Furthermore, we demonstrate the implication of quadratic labor adjustment costs for monetary policy. We show that there is a simple rule determining whether quadratic labor adjustment costs imply a trade-off between stabilizing inflation and output.

Keywords: Monetary Persistence, Labor Adjustment Costs, Optimal Monetary Policy

JEL classification: E24, E32, E52, J23

The paper was previously circulated under the title “Quadratic Labor Adjustment Costs and the New-Keynesian Model”
1 Introduction

Recently, many attempts have been undertaken to incorporate labor market frictions into otherwise standard New-Keynesian models to improve the performance of the latter, e.g. to increase the persistence of output in response to monetary shocks. The approaches used range from linear labor turnover costs\(^1\) over fair wages\(^2\) to search and matching frictions.\(^3\) Recent attempts to use quadratic labor adjustment costs are Dib (2003) and Janko (2008), who show that this improves the performance of the model. Quadratic labor adjustment costs are also popular in large-scale DSGE-models.\(^4\)

In this paper we demonstrate that a model with simple quadratic labor adjustment costs yields very similar results as the more complicated models of labor market frictions. Of course, a model with quadratic labor adjustment costs cannot be used to analyze the dynamics of unemployment, but we demonstrate that they can be a useful and simple short-cut if one is more interested in the aggregate behavior of other variables.\(^5\)

As the main contribution of this paper, we demonstrate the consequences of quadratic labor adjustment costs for optimal monetary policy. In the standard New-Keynesian model the central bank does not face a trade off between stabilizing inflation and stabilizing output. If the central stabilizes one, it automatically stabilizes the other.\(^6\) Therefore it is optimal for the central bank to keep prices stable at any time. In this way it avoids price distortions and can replicate an economy with flexible prices.

Thomas (2008) and Blanchard and Gali (2010) show that this can still be the case if search and matching frictions are included and wages are flexible. However, they assume that employment is at the efficient level by using the Hosios (1990) condition.\(^7\) Faia (2008, 2009) demonstrates that relaxing this assumption implies a trade off between stabilizing output and inflation. It is no longer optimal to keep prices constant. Instead, a Ramsey planner allows for inflation in response to temporary shocks. Faia, Lechthaler and Merkl (2009) demonstrate that the same is true in a model of linear hiring and firing costs. Furthermore, they demonstrate that the optimal level of inflation depends on the magnitude of hiring and firing costs - the higher these costs are the higher is the optimal volatility of inflation.

\(^1\)See, e.g., Lechthaler, Merkl and Snower (2010).
\(^5\)While models of quadratic labor adjustment costs have been widely used in the literature, their empirical validity is still heavily debated. While Caballero and Engel (2004) and Caballero, Haltiwanger and Engel (1997) argue against quadratic adjustment costs, their approach has been criticized by Cooper and Willis (2004) for mismeasurement. Hamermesh (1989) finds evidence against quadratic adjustment costs at the firm level but not at the aggregate level. In a more recent study, Ejarque and Portugal (2007) find that quadratic adjustment costs are much more important than fixed adjustment costs.
\(^6\)See Gali (2008).
\(^7\)The Hosios condition is fulfilled if the elasticity of the matching function equals the bargaining power of workers.
In this paper we contribute to this literature by showing that similar conclusions can be derived in a much simpler framework, by incorporating quadratic labor adjustment costs. We demonstrate that, similar to the search and matching framework, there is a simple rule determining whether quadratic labor adjustment costs imply a trade-off between stabilizing inflation and output. If labor adjustment costs depend on aggregate variables (e.g., aggregate output), there is an externality calling for positive inflation after business-cycle shocks.

The paper proceeds as follows. First, we describe the underlying model. In section three we demonstrate some business cycle statistics and compare them to other models and the data. Section four analyzes optimal monetary policy, while section five concludes.

2 The Model

The model we use is closely related to Krause and Lubik (2007), but the search and matching labor market is replaced by quadratic labor adjustment costs as e.g. in Dib (2003).

2.1 Households

Households have a standard utility function of the form:

$$U = E \sum_{t=0}^{\infty} \beta^t \left( C_{t+1}^{1-\sigma} - \frac{1}{1-\sigma} \log \left( \frac{M_t}{P_t} \right) - \frac{L_{t+1}^{1+\phi}}{1+\phi} \right)$$ (1)

Utility depends positively on consumption $C$, real money balances $M/P$ (where $P$ is the price index) and negatively on labor input $L$. Households maximize utility with respect to the budget constraint:

$$B_t + C_t P_t = W_t L_t + (1+i_{t-1})B_{t-1} - \tau_t + \Pi_t$$ (2)

where $B$ are bond holdings, $W$ is the wage, $i$ is the interest rate, $\tau$ are lump-sum taxes and $\Pi$ are nominal profits. Utility maximization yields the standard consumption Euler equation, labor supply and money demand:

$$C_t = EC_{t+1} \left( (1+i_t)\beta \frac{P_t}{EP_{t+1}} \right)^{-\frac{1}{\phi}}$$ (3)

$$L_t^\varphi = C_t^{-\sigma} \frac{W_t}{P_t}$$ (4)

$$C_t = \frac{i_t M_t}{1+i_t P_t}$$ (5)
2.2 Production

We follow the recent literature in separating the markup pricing decision from the labor demand decision. This implies that there are three types of firms. Intermediate good producing firms employ labor to produce the intermediate good. Firms in the wholesale sector take the intermediate goods as input, and differentiate those. Subject to quadratic price adjustment costs, they sell to a final retail sector under monopolistic competition. Retailers bundle the differentiated goods to a consumption basket $C$, which is sold to households under perfect competition at the aggregate price level $P$.

2.2.1 Intermediate-good firms

Intermediate-good firms hire labor to produce the intermediate good $Z$. Their production function is: $Z = A \times L$. However, the labor input is subject to quadratic adjustments costs. Thus, profits in real terms are given by:

$$E \sum_{t=0}^{\beta^t} \left[ \frac{P_{z,t}}{P_t} A_t L_t - \frac{W_t}{P_t} L_t - \frac{\Psi}{2} \left( \frac{L_t}{L_{t-1}} - 1 \right)^2 Y_t \right]$$

where $P_z$ is the price of the intermediate good and the last term inside the brackets is the real adjustment cost expressed in units of the final good.

Maximizing profits with respect to $L_t$, we obtain the optimal labor input, which now depends on the labor input of the previous period and the the expected labor input of the following period:

$$\frac{P_{z,t}}{P_t} A_t = \frac{W_t}{P_t} + \Psi \left( \frac{L_t}{L_{t-1}} - 1 \right) Y_t - \beta \Psi \left( \frac{E L_{t+1}}{L_t} - 1 \right) \frac{E L_{t+1}}{L_t} Y_t$$

(6)

In this equation we have marginal returns on the left hand side and marginal costs on the right hand side. The marginal costs are no longer just made up by the wage as in the standard model, but include the costs of adjusting the workforce. Importantly, these costs depend on lagged and expected future levels of employment. Due to these costs, firms try to smoothen movements in labor and keep adjustments low.

2.2.2 Wholesale Sector

Firms in the wholesale sector are distributed on the unit interval and indexed by $i$. The homogenous intermediate good is the only input into wholesale production, being traded in a competitive market for price $P_z$ per unit. Wholesale firms produce a differentiated good $Y_i$ according to the production function: $Y_i = Z_i$ where $Z_i$ is their demand for intermediate goods. They sell the good to the final retail sector, according to the demand

$$Y_{i,t} = Y_t \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon}$$

(7)
where $Y$ is the final good, defined further below, and $\varepsilon$ is the elasticity of substitution between varieties.

The firms can change their price at any period but face quadratic price adjustment costs.\footnote{We stick to quadratic price adjustment costs to be comparable to Krause and Lubik (2007), but Calvo-staggering would yield similar results.} Noting that the production cost of a wholesale firm is the price of its input $(P_z)$, the problem of a price-resetting firm can be formulated as:

$$
\max_{P_{i,t}} \quad E \sum_{t=0}^{\infty} \Delta_t \left[ \frac{P_{i,t}}{P_t} Y_{i,t} - \frac{P_z}{P_t} Y_{i,t} - \frac{\Phi}{2} \left( \frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 Y_t \right]
$$

s.t. $Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} Y_t$

where $P_{i,t}$ denotes the new optimal price of producer $i$, $\Delta_t = \beta C_t^{-\sigma} / C_0^{-\sigma}$ is the stochastic discount factor from period $t$ to the current period and $\Phi$ is a parameter measuring the extent of price adjustment costs. Taking the derivative with respect to the price yields, after some manipulations, the following expectational Phillips curve:

$$
0 = (1 - \varepsilon) + \varepsilon \frac{P_z}{P_t} - \Phi (\pi_t - 1) \pi_t + E \{ \Delta_{t,t+1} \Phi (\pi_{t+1} - 1) \frac{Y_{t+1}}{\pi_t} \pi_{t+1} \}. \quad (8)
$$

### 2.2.3 Final Retail Sector

The final retailer operates in a competitive market and buys differentiated wholesale goods to arrange them into a representative basket, producing the final consumption bundle $Y$, according to

$$
Y_t = \left( \int Y_{i,t}^{-\frac{\varepsilon}{1-\varepsilon}} \, di \right)^{\frac{1}{1-\varepsilon}} \quad (9)
$$

which delivers the standard price index $P_t = \left( \int P_{i,t}^{1-\varepsilon} \, di \right)^{\frac{1}{1-\varepsilon}}$ from the cost minimization problem of the firm.

### 2.2.4 Monetary and Fiscal Policy

The policy instrument of the central bank is money growth which is defined by:

$$
\mu_t = \frac{M_t}{M_{t-1}} = \frac{M_t}{P_t} \frac{P_{t-1}}{M_{t-1}} = \frac{m_t \pi_t}{m_{t-1}} \quad (10)
$$

and follows an AR(1) process. Government consumption $G$ is financed by the lump-sum tax and also follows an AR(1) process.
2.2.5 Aggregation

Aggregate production is given by:

\[
Y_t = \int Y_{i,t} \, di = \int Z_{i,t} \, di = \int A_i L_{i,t} \, di = A_t L_t
\]  

(11)

Assuming that all profits are distributed to the households, aggregation of households’ income yields:

\[
C_t = Y_t - \frac{\Psi}{2} \left( \frac{L_t}{L_{t-1}} - 1 \right)^2 Y_t - \frac{\Phi}{2} (\pi - 1)^2 Y_t - G_t
\]  

(12)

Thus, the model consists of eight unknown variables: \(w, C, L, p, x, i, m, \pi\). The eight equations are the households optimality conditions (3, 4 and 5), labor demand of intermediate-good firms (6), the price set by a wholesale firm (8), the aggregation of output (11), the aggregation of income (12) and the definition of money growth (10).

3 Business Cycle Statistics

The model is calibrated in close consonance with Krause and Lubik (2007), since we want to compare our model to this benchmark. Thus the elasticity of substitution between intermediate products is set to \(\epsilon = 11\), while the elasticity of intertemporal substitution is set to \(\sigma = 2\). The subjective discount rate is set to \(\beta = 0.99\) and the parameter governing the cost of price adjustment is set to \(\phi = 40\). The parameter governing the disutility of labor is set to the standard value 1. Dib (2003) estimates the parameter for the labor adjustment costs to be \(\Psi = 1.85\).

In this section we compare the business cycle statistics of the model with quadratic labor adjustment costs with the results for search and matching frictions reported in Krause and Lubik (2007) and the results for linear labor turnover costs reported in Lechthalter, Merkl and Snower (2010). To this end, we use the exact same numbers for the shocks, i.e. the standard deviation of the money-supply shock is set to 0.00623, while the standard deviation of the productivity shock is set to 0.0049. The parameters of autocorrelation are set to 0.95 for the productivity and to 0.49 for the money shock.

The results of our simulations are reported in table 1. It can be seen that all three models report relatively similar numbers. The model of Krause and Lubik (2007) fairs a bit better than the other two models with respect to the autocorrelation of inflation, while the model with quadratic labor adjustment costs yields better numbers with respect to the volatility of inflation and the correlation between output and inflation. The important thing to note is that the model with quadratic labor adjustment costs does not perform worse - with respect to output and inflation - than the more complex models. But of course the simple model cannot say anything about fluctuations in unemployment and the flow-rates of workers between unemployment and employment.
### Table 1: Business cycle statistics

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<td>KL</td>
<td>QAC</td>
<td>LMS</td>
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<td>0.98</td>
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<td>0.25</td>
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<td>0.01</td>
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</tbody>
</table>


4 Optimal Monetary Policy

4.1 Flexible Price Allocation

Before describing the optimal policy plan, we ask the question whether it is feasible to implement the flexible price allocation. The answer is: it depends. For the specification of quadratic labor adjustment costs used in this paper, where the adjustment costs depend on the level of aggregate output, like, e.g., in Dib (2003), the flexible price allocation is not feasible. If adjustment costs do not depend on aggregate output, like, e.g., in Pesenti (2008), the flexible price allocation is indeed feasible.

The central planner seeks to maximize utility subject to the resource constraint:

$$U_t = E \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi} \right)$$

s.t. $$C_t = A_t L_t - \frac{\Psi}{2} \left( \frac{L_t}{L_{t-1}} - 1 \right)^2 Y_t$$

After some manipulations the first order condition reads

$$A_t = \frac{L_t^2}{C_t^{1-\sigma}} + \Psi \left( \frac{L_t}{L_{t-1}} - 1 \right) Y_t \frac{C_{t-1}^{1-\sigma}}{L_{t-1}} - \beta \frac{C_{t+1}^{1-\sigma}}{C_t^{1-\sigma}} \psi \left( \frac{EL_{t+1}}{L_t} - 1 \right) \frac{EL_t}{L_t^2} Y_t - \frac{E_{t+1}}{L_t} Y_t$$

This outcome has to be compared with the solution of the competitive economy. Under flexible prices the price distortion through Rotemberg adjustment costs vanishes. Furthermore, the distortion through monopolistic competition can be tackled through the appropriate choice of a subsidy ($1/\varepsilon$) on the marginal costs of monopolistic firms, i.e. on the price $P_z$ of the intermediate firm.  

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9Since we are looking at a flexible price economy we ignore money in the utility function.

10See Gali (2008) for more details.
optimal price of the intermediate good is still be described by equation 6. Plugging in equation 4 for the wage and substituting the stochastic discount factor yields:

\[ A_t = \frac{L_t^\phi}{C_t^{1-\sigma}} + \Psi \left( \frac{L_t}{L_{t-1}} - 1 \right) \frac{Y_t}{L_{t-1}} - \frac{C_{t+1}^{1-\sigma}}{C_t^{1-\sigma}} \psi \left( \frac{EL_{t}+1}{L_t} - 1 \right) \frac{EL_{t+1}}{L_t^2} Y_t \]  

(15)

It is immediately clear that this equation does not coincide with the solution of the central planner given in equation 14. The equation of the central planner exhibits an additional term, driving a wedge \( \Gamma \) between the two equations:

\[ \Gamma_t = -C_t^{1-\sigma} \frac{\Psi}{2} \left( \frac{L_t}{L_{t-1}} - 1 \right)^2 A_t \]  

(16)

Basically, through the dependence of adjustment costs on aggregate output, an individual firm poses an externality on other firms. In other words, it ignores socially relevant costs that the central planner (of course) includes. This implies that the central planner wants to further smooth employment adjustments over the business cycle, or put differently, the competitive economy exhibits fluctuations too large. In a world with sticky prices the planner would want to use inflation to smooth fluctuations in employment and output.

However, from the analysis above it is also clear, that this externality depends crucially on the exact specification of labor adjustment costs. If labor adjustment costs took the form \( \Psi / 2 (L_t/L_{t-1} - 1)^2 \), i.e. just ignoring the dependence on output, would make the flexible price allocation feasible. In such a case, no trade-off between stabilizing inflation and output would arise.

### 4.2 The Ramsey Planner

The optimal policy plan is determined by a monetary authority that maximizes the discounted sum of utilities of all agents given the constraints of the competitive economy.\(^{11}\) Of course the money growth rule given in equation 10 is no longer valid. To be able to concentrate on the distortions through price and labor adjustment costs, we assume that the distortion through monopolistic competition is offset by an appropriate subsidy. The solution algorithm used is the one of the dynare-package.\(^{12}\)

As in Faia (2008,2009) we determine optimal monetary policy in an economy that is hit by shocks to aggregate productivity and shocks to aggregate demand (government spending).\(^{13}\) We parameterize the shock processes in line with these papers and the evidence for industrialized countries. Productivity shocks follow an AR(1) process. The autocorrelation is set to \( \rho_a = 0.95 \) and the


\(^{12}\)See Juillard (1996).

\(^{13}\)Since we analyze optimal monetary policy here we can no longer use monetary shocks.
Figure 1: Ramsey policy after productivity shock

standard error of the shock is $\sigma_a = 0.008$, while government consumption follows an AR(1) with $\sigma_g = 0.0074$ and $\rho_g = 0.9$ (see, e.g., Perotti (2004)).

Figure 1 shows impulse response functions of the Ramsey plan after positive productivity shocks, comparing different values of adjustment costs. As is standard, in the model without any adjustment costs the Ramsey plan stabilizes prices at any time, setting the interest rate in such a way that agents have no desire to change the price.\(^{14}\) In this way the price-distortion can be avoided and the economy replicates an economy with flexible prices.

However, in line with Faia (2008, 2009) and Faia, Lechthaler and Merkl (2009), this is no longer the case, once there are labor market frictions.\(^{15}\) These frictions make output adjustments costly and thus the Ramsey planner tries to smoothen employment fluctuations. This can only be achieved by allowing for fluctuations in the price level. Thus, under quadratic labor adjustment costs the Ramsey planner can no longer achieve the first-best outcome but has to trade off the effects of the nominal distortion (price rigidities) against the effects of the real distortion (labor adjustment costs).

After a positive productivity shock labor supply and therefore labor input is

\(^{14}\)See e.g. Gali (2008).

\(^{15}\)Note that Blanchard and Gali (2010) arrive at a different result. This is due to their assumption that unemployment is at the efficient level.
reduced immediately (because the increase in productivity increases consumption and therefore decreases marginal utility of consumption). In the presence of labor adjustment costs it is costly to reduce labor input and therefore the Ramsey planner tries to smoothen adjustments and counteracts by stimulating demand even further. It can do so by reducing prices resulting in deflation on impact of the shock. The reduction in labor input is not avoided completely but only smoothened and therefore inflation becomes positive in later periods. As in Faia, Lechthaler and Merkl (2009) the fluctuations in the price level become larger with the level of labor adjustment costs.

The effects of government spending shocks are illustrated in figure 2. As common in these kind of models, government spending increases output but partially crowds out private consumption. Since productivity is constant, the increase in output can only be accomplished by increasing labor input. Again, in the presence of labor adjustment costs the Ramsey planner tries to smoothen labor adjustments. In this case this means counteracting the increase in output, by lowering consumption even further. This is accomplished by increasing prices on impact of the shock. In later periods the planner tries to push back consumption and therefore causes deflation.

The important conclusion so far is that the inclusion of quadratic labor adjustment costs is sufficient to introduce a trade-off for monetary policy. It is
no longer optimal for the Ramsey-planner to keep prices absolutely stable. In that sense we can replicate the results in Faia (2008, 2009) and Faia, Lechthaler and Merkl (2009), but in a much simpler framework. Similar to models with matching frictions, however, the trade-off is quantitatively not very large.

In a last exercise we demonstrate the dependence of optimal inflation volatility on the size of labor-adjustment costs in figure 3. It can be seen that optimal inflation volatility increases with the level of labor adjustment costs. But the relationship is not linear so that the effects becomes smaller with higher labor adjustment costs.

5 Conclusion

Labor market frictions have been at the heart of recent developments in the New-Keynesian literature. Incorporating labor market frictions can increase the persistence of a model’s reaction in response to temporary shocks and can lead to trade-offs between output and inflation stabilization.

In this paper we demonstrate that these goals can be achieved with a simpler framework than the search and matching approach, dominant in the literature. A model with quadratic labor adjustment costs yields similar business cycle statistics as a model with search and matching frictions, thus suggesting quadratic labor adjustment costs as a simple and useful shortcut. Furthermore, quadratic labor adjustment costs are sufficient to break down the divine coincidence, the result that stable prices imply a stable output gap in the standard New-Keynesian model. However, this last result depends heavily on the exact specification of labor adjustment costs.
References


