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The Ifo DSGE Model for the German Economy

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The Ifo DSGE Model for the German Economy

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The Ifo DSGE Model for the German Economy

Abstract

This paper estimates a medium-scale open economy DSGE model for Germany and the rest of the Euro Area (REA). The parameter estimates indicate that there is a modest degree of structural heterogeneity between Germany and the rest of the Euro Area. In particular, (i) the private sector in Germany tends to adjust its capital stock faster than its counterpart in the REA, (ii) the innovations to government spending as well as those to the degree of competition in goods markets are relatively more volatile in Germany and (iii) nominal prices and wages appear to be slightly more flexible in Germany than in the REA. A comparison based on marginal likelihoods shows that the DSGE model fits the observable macroeconomic time series similarly well as unrestricted Bayesian VARs (BVARs) estimated on the same data set.

JEL Code: C11, C51, E17.

Keywords: DSGE model, German economy, monetary union, Bayesian estimation.

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1 Introduction

Over the past fifteen years, estimated medium scale Dynamic Stochastic General Equilibrium (DSGE) models have become increasingly popular as a laboratory for conducting macroeconomic research but also as a tool for every-day analysis and forecasting at large policy institutions like central banks, the European Commission or the International Monetary Fund (IMF). In these models, the majority of behavioral equations such as demand and supply functions possess an explicit microeconomic foundation since they are derived from first principles like the intertemporal maximization of a private agent's utility or profits subject to a bunch of budget, resource, technological and institutional constraints. Prices adjust to ensure an equilibrium in each market. Macroeconomic fluctuations are the result of real and nominal exogenous shocks, each of them having a clear economic interpretation. By inducing unexpected changes in e.g. consumer sentiment, average productivity or the market power of firms, these shocks force economic agents to optimally adjust their expenditure, production or price setting plans.

On the one hand, such models are well designed to simultaneously fit various properties of large macroeconomic data sets. For example, Christiano et al. (2005) and Altig et al. (2011) document the ability of DSGE models including several nominal and real rigidities to replicate the empirically observable reactions of the US economy to different types of nominal and real shocks, while Smets and Wouters (2002, 2007) and Del Negro et al. (2005) highlight the satisfactory forecasting performance of such models based on US and Euro Area data. On the other hand, DSGE models exhibit some clear advantages over purely statistical models like VARs and traditional Kynesian structural econometric models (Fernández-Villaverde, 2008; King and Rebelo, 1999). In particular, unlike VARs, the DSGE framework allows for a very detailed analysis of the precise economic mechanisms underlying the reactions to a given exogenous shock and the investigation of the effects of a much wider range of policy interventions or structural reforms (Favero, 2001). With respect to traditional Keynesian econometric models, the DSGE framework also represents a substantial improvement as it only relies upon a relatively small number of so called "deep parameters" which are widely believed to be time invariant. Accordingly, DSGE models are less prone to the Lucas critique. Finally, in contrast to VARs or the traditional Keynesian structural econometric set up, DSGE models are more suitable for welfare analyses.

This paper develops a medium scale DSGE model for a two-country monetary union. The latter consists of Germany and the Rest of the Euro Area (REA). In addition, our theoretical framework also contains a stylized description of the interactions of the two euro-area countries with the Rest of the World (ROW) via the world markets for goods, capital and crude oil/energy. We model the economic structure of both, Germany and the REA in a fairly detailed way in order to capture as many of the most important features of the two economies as possible. To the best of our knowledge, this is the first DSGE model which, albeit focusing

¹See Lucas (1976).

on Germany, also provides an equally detailed description of the REA's economy. In particular, unlike most existing frameworks assuming a homogeneous labor market, we account for heterogeneities among households with respect to their labor productivity. This facilitates the analysis of supply shifts in particular segments of the labor market, e.g. in that of low skilled workers, as experienced recently due to sizable immigration towards Germany and the Euro Area as a whole. For each economy, we also distinguish between a sector producing goods which are tradable across borders and are thus subject to international competition and a sector producing non-tradable products sold only domestically (e.g. the majority of services). This, for example, allows deeper insights in the mechanisms behind the divergence of relative competitiveness positions between Germany and the REA in the decade preceding the global financial crisis. We incorporate a very detailed government sector in the model as well. In particular, for both countries, we explicitly account for various direct and indirect taxes, different types of government expenditures like those on consumption, investment and transfers as well as the accumulation of government debt. Consequently, we are able to study the effects of various cyclical and structural interventions undertaken by the governments in Germany and the REA since the outbreak of the global financial crisis. In addition, like the majority of medium scale DSGE models, our theoretical framework also includes numerous frictions like imperfect competition in goods and labor markets, price and wage stickiness, incomplete financial markets. These features enable us to simulate the potential effects of several recent proposals for increasing competition and flexibility in European goods and labor markets. Finally, macroeconomic fluctuations in our model are driven by a bunch of exogenous shocks like sudden shifts in the monetary policy stance, oil prices, demand from the ROW or unexpected country specific changes in productivity, labor supply, the competitive pressure in different markets, government spending or investment or households demand attitude.

There are several related DSGE frameworks modeling Germany as a part of a currency union. Pytlarczyk (2005) also incorporates the economies of Germany and the REA in a relatively symmetric way. However, he completely abstracts from a sector producing non-tradables and the role played by oil demand and oil prices. Furthermore, he reduces the fiscal side to an exogenous process for government consumption and abstracts from the inclusion of any type of distortionary taxes or public debt. To analyse the evolution of the German current account, Kollmann et al. (2015) estimate a three country model comprising Germany, the REA and the ROW. They explicitly account for the presence of financial frictions and a housing market in Germany and also incorporate several important features of fiscal policy. However, while the structure of the German economy in their model is very rich, Kollmann et al. (2015) resort to a very stylized reduced-form representation of the REA consisting of a Phillips curve and a dynamic IS curve only. (Giesen et al., 2012) estimate a multi country model in which, however, Germany as well as the other economies are represented by simple reduced form equations. In addition, there are several estimated DSGE models for the Euro Area as whole which either assume that the latter is a closed economy (Smets and Wouters,

2002; Gerali et al., 2010) a small open economy (Ratto et al., 2009) or incorporate it in a two-country framework besides the US (Rabanal and Tuesta, 2013). From a technical point of view, our work is most closely related to that of Andrés et al. (2010) who estimate an open-economy DSGE model for Spain within the Euro Area.

Our main findings are twofold. First, there is a modest degree of heterogeneity between Germany and the Rest of the Euro Area with respect to the structural parameters shaping the business cycle of the two economies. In particular, the private sector in Germany adjusts its capital stock faster and at a lower cost than its counterpart in the REA, the shocks to government spending and to the degree of competition in goods markets seem to be relatively more volatile in Germany. Furthermore, nominal prices and especially wages appear to be slightly more flexible in Germany than in the REA. Finally, a comparison based on marginal likelihood (predictive densities) shows that the DSGE model fits the observable macroeconomic time series similarly well as unrestricted Bayesian VARs (BVARs) estimated on the same data set.

The paper is organized as follows. Section 2 presents the theoretical framework. Section 3 describes the data, the estimation procedure and discusses the results. Section 4 concludes.

2 The Model

The Euro Area consists of two countries - Germany and the rest of the Euro Area (REA) of size n and 1-n respectively. In each of them, there is a continuum of infinitely lived private households maximizing their utility functions subject to a budget and a capital accumulation constraint. On the supply side of each country, there are two sectors - one producing internationally tradable goods (e.g. manufacturing products) and one producing non-tradables which can be only sold in the domestic market (e.g. most services). The governments of Germany and the REA collect various types of taxes and issue public debt to finance several expenditure components as well as transfers to private households. The two euro-area countries face a common central bank conducting monetary policy. Each region of the currency union is also involved in trade via goods, capital and commodity markets with the rest of the world (ROW). Demand, prices and nominal interest rates in the ROW are assumed to be exogenous. Accordingly, in our model, the Euro Area is a small open economy with respect to the ROW. In what follows, we only derive the equations reflecting Germany's economic structure. The corresponding equations for the REA are not shown explicitly as they exhibit the same functional form. REA-specific variables and parameters are denoted by an asterisk, e.g. C_t^* for REA consumption. Variables corresponding to the ROW are denoted by a superscript W, e.g. C_t^W .

2.1 Households: Utility Maximization

Each household in the Home country (Germany) faces a standard optimization problem. Her goal is to maximize lifetime utility

$$E_{t} \sum_{s=0}^{\infty} \beta^{s} U(C_{j,t+s}, C_{j,t+s-1}, N_{T,j,t+s}, N_{N,j,t+s}, e_{u,t+s}, e_{n,t+s})$$

where $j \in [0, 1]$ is the household's index and $C_{j,t}$ and $N_{s,j,t}$ denote individual consumption and working hours in sector s, respectively. There are two sectors - one producing goods tradable across borders, s = T, and one producing non-tradable products, s = N. $e_{u,t}$ and $e_{n,t}$ are exogenous shocks to be described below.

The maximization problem is subject to two constraints. First, the flow budget constraint states that the sum of nominal expenditure on consumption and investment should be equal to the nominal income stream:

$$\mathcal{E}_{t} \frac{B_{j,t}^{w}}{i_{t}^{w} \Psi(\mathcal{E}_{t-1} b_{t-1}^{w})} + \frac{B_{j,t}}{i_{t} \Psi(b_{t-1})} + B_{j,g,t} + (1+\tau_{c,t}) P_{t} C_{j,t} + P_{H,t} I_{T,j,t} + P_{H,t} I_{N,j,t} =$$

$$= (1 - \tau_{w,t})W_{T,j,t}N_{T,j,t} + (1 - \tau_{w,t})W_{N,j,t}N_{N,j,t} + + (1 - \tau_{k,t})P_tR_{T,k,t}u_{T,j,t}K_{T,j,t} + (1 - \tau_{k,t})P_tR_{N,k,t}u_{N,j,t}K_{N,j,t} + + \mathcal{E}_tB_{j,t-1}^w + B_{j,t-1} + (1 - \tau_{g,t})r_{g,t-1}B_{j,g,t-1} + P_t\Upsilon_{j,t} + P_t\Gamma_{j,t} - P_tT_t,$$
(2.1)

where $I_{s,j,t}$ denotes real investment in physical capital in sector s, $K_{s,j,t}$ is the corresponding capital stock while $u_{s,j,t}$ denotes the degree of capital utilization in that sector. In each period, the effective capital $u_{s,j,t}K_{s,j,t-1}$ is rented out to goods producing firms at the real rental rate $R_{s,k,t}$. The household further invests in nominal bonds issued by euro-area households, $B_{j,t}$, nominal bonds issued by the domestic government, $B_{j,g,t}$, as well as international bonds, $B_{j,t}^w$. The corresponding gross nominal risk free interest rates are i_t , $r_{g,t}$ and i_t^w respectively. $W_{s,j,t}$ denotes the sector specific nominal wage received by household j. $\Gamma_{j,t}$ stands for dividends from the ownership of domestic firms. $\Upsilon_{j,t}$ denotes transfer payments between households designed to ensure that in each period the individual income level is equal across households. P_t denotes the nominal consumer price level to be defined below. $P_{H,t}$ is the nominal price of domestically produced goods. $\Psi(b_{t-1})$ and $\Psi(\mathcal{E}_{t-1}b_{t-1}^w)$ reflect the presence of costs of adjustment of the net foreign asset positions. P_t and P_t and P_t denote the real value of aggregate

²Apart from making the model more realistic, the assumption of costs for the adjustment of the net foreign asset position are necessary condition for the existence of a stable deterministic steady state. Alternatively, one can resort to the assumption of an endogenous subjective discount factor. See Schmitt–Grohe and Uribe (2003) for a discussion.

net euro-area and international assets, respectively.³ \mathcal{E}_t represents the nominal exchange rate (Euro/USD, quantity quotation) and, $\tau_{c,t}$, $\tau_{w,t}$, $\tau_{k,t}$, $\tau_{g,t}$ denote the rates at which consumption, labor income and the different types of capital income are taxed. Finally, T_t reflects net transfers from the domestic government.

The second constraint is the law of motion for the individual capital stock in the two sectors respectively

$$K_{T,j,t} = (1 - \delta_{T,j,t}) K_{T,j,t-1} + e^{e_{T,q,t}} \phi \left(\frac{I_{T,j,t}}{K_{T,j,t-1}} \right) K_{T,j,t-1}, \tag{2.2}$$

and

$$K_{N,j,t} = (1 - \delta_{N,j,t}) K_{N,j,t-1} + e^{e_{N,q,t}} \phi \left(\frac{I_{N,j,t}}{K_{N,j,t-1}} \right) K_{N,j,t-1}, \tag{2.3}$$

where $\phi(.)$ is a convex function reflecting the presence of adjustment costs of capital. It is assumed that $\phi(\delta) = \delta$, $\phi'(\delta) = 1$ and $\frac{\phi''(\delta)}{\phi'(\delta)}\delta = -\xi_q$, where δ is the steady state value of the depreciation rate δ_t . According to these assumptions, adjustment costs play no role in the deterministic steady state but may affect the economy's reactions to exogenous disturbances.⁴ The investment specific technology shock evolves according to

$$e_{s,q,t} = \rho_{s,q} e_{s,q,t-1} + \epsilon_{s,q,t},$$
 (2.4)

where $\epsilon_{s,q,t}$ is a normally distributed white noise process. The time varying depreciation rate $\delta_{s,j,t}$ depends on the utilization rate of capital $u_{s,j,t}$ according to

$$\delta_{s,i,t} = \delta(u_{s,i,t}),\tag{2.5}$$

where we assume that u=1 in the steady state while $\delta(1)=\delta$, $\delta'(1)=\frac{1-\beta(1-\delta)}{\beta}$ and $\frac{\delta''(1)}{\delta'(1)}=\psi_u.^5$

There are two sector specific labor markets. Both are assumed to be monopolistically competitive. In particular, each household enjoys some monopoly

$$\Psi(b_t) = \exp(-\psi b_t)$$
 and $\Psi(\mathcal{E}_t b_t^w) = \exp(-\psi \mathcal{E}_t b_t^w)$

$$\phi\left(\frac{I_{s,t}}{K_{s,t-1}}\right) = \frac{a}{1-\xi_q} \left(\frac{I_{s,t}}{K_{s,t-1}}\right)^{1-\xi_q} + b,$$

where $a=\delta^{\xi_q}$ and $b=1-\frac{\delta}{1-\xi_q}$ and $s=\{T,N\}$.

⁵ $\delta(u_{s,t})$ can be specified as follows:

$$\delta(u_{s,t}) = \frac{a_u}{1 + \psi_u} (u_{s,t})^{1 + \psi_u} + b_u,$$

where $a_u=\frac{1-\beta(1-\delta)}{\beta}$ and $b_u=\delta-\frac{a_u}{1+\psi_u}$. See Greenwood et al. (1988) and Burnside and Eichenbaum (1996) for similar specifications.

³We assume

⁴In particular, following Jermann (1998) we assume

power and sets the nominal wage as a markup over the marginal rate of substitution between consumption and leisure. Furthermore, due to the assumption of a Calvo-type staggered wage setting, households are heterogeneous regarding their individual nominal wages and thus with respect to their labor supply. The presence of the transfer payment $\Upsilon_{j,t}$, however, ensures that for any given price vector in the economy, households are identical with regard to their consumption and investment plans. Accordingly, except for the wage setting decision, we can drop the household index and resort to the representative agent assumption.

The period utility function is assumed to be of the form

$$U(.) = e^{e_{u,t}} \left(\frac{1}{1-\sigma} \left(\frac{C_t}{C_{t-1}^{\gamma}} \right)^{1-\sigma} - \frac{\varsigma_T e^{e_{n,t}} (N_{T,t})^{1+\varphi}}{1+\varphi} - \frac{\varsigma_N e^{e_{n,t}} (N_{N,t})^{1+\varphi}}{1+\varphi} \right),$$

where $\gamma > 0$ measures the degree of *external* habit persistence in consumption.⁶ $\sigma > 0$ also affects the curvature of the utility function. φ denotes the inverse of the Frisch elasticity of labor supply. $e_{n,t}$ and $e_{u,t}$ denote a labor supply and a general preference shock, respectively. We assume

$$e_{n,t} = \rho_n e_{n,t-1} + \epsilon_{n,t} \tag{2.6}$$

and

$$e_{u,t} = \rho_u e_{u,t-1} + \epsilon_{u,t}, \tag{2.7}$$

where $\epsilon_{n,t}$ and $\epsilon_{u,t}$ are normally distributed White Noise processes.

Dividing both sides of (2.1) by the overall price level P_t and defining $\tilde{b}_t^w = \mathcal{E}_t b_t^w$, $\Delta e_t = \mathcal{E}_t / \mathcal{E}_{t-1}$ and $\Pi_t = P_t / P_{t-1}$ yields the budget constraint in real terms:

$$\frac{\tilde{b}_{j,t}^{w}}{i_{t}^{w}\Psi(\tilde{b}_{t-1}^{w})} + \frac{b_{t}}{i_{t}\Psi(b_{t-1})} + b_{g,t} + (1+\tau_{c,t})C_{t} + \frac{P_{H,t}}{P_{t}}I_{T,t} + \frac{P_{H,t}}{P_{t}}I_{N,t} =$$

$$= (1-\tau_{w,t})w_{T,j,t}N_{T,j,t} + (1-\tau_{w,t})w_{N,j,t}N_{N,j,t} +$$

$$+ (1-\tau_{k,t})R_{T,k,t}u_{T,t}K_{T,t-1} + (1-\tau_{k,t})R_{N,k,t}u_{N,t}K_{N,t-1} +$$

$$+ \frac{\Delta e_{t}\tilde{b}_{t-1}^{w}}{\Pi_{t}} + \frac{b_{t-1}}{\Pi_{t}} + (1-\tau_{g,t})\frac{r_{g,t-1}b_{g,t-1}}{\Pi_{t}} + \Gamma_{t} - T_{t} + \Upsilon_{j,t}.$$

The first order conditions to this problem read

$$(1 + \tau_{c,t})\mu_t = e^{e_{u,t}} \left(\frac{C_t}{C_{t-1}^{\gamma}}\right)^{-\sigma} C_{t-1}^{-\gamma} - \gamma \beta E_t e^{e_{u,t+1}} \left(\frac{C_{t+1}}{C_t^{\gamma}}\right)^{-\sigma} \frac{C_{t+1}}{C_t^{\gamma+1}}, \qquad (2.8)$$

$$\frac{P_{H,t}}{P_t}\mu_t = \omega_{T,t}e^{e_{T,q,t}}\phi'\left(\frac{I_{T,t}}{K_{T,t-1}}\right),\tag{2.9}$$

⁶See Ravn et al. (2006, 2010) for a discussion of the difference between external and internal habit formation.

$$\frac{P_{H,t}}{P_t}\mu_t = \omega_{N,t}e^{e_{N,q,t}}\phi'\left(\frac{I_{N,t}}{K_{N,t-1}}\right),\tag{2.10}$$

$$(1 - \tau_{k,t}) R_{T,k,t} \mu_t K_{T,t-1} = \omega_{T,t} \delta'(u_{T,t}) K_{T,t-1}, \qquad (2.11)$$

$$(1 - \tau_{k,t}) R_{N,k,t} \mu_t K_{N,t-1} = \omega_{N,t} \delta'(u_{N,t}) K_{N,t-1}, \qquad (2.12)$$

$$\omega_{T,t} = \beta E_{t} \mu_{t+1} (1 - \tau_{k,t+1}) R_{T,k,t+1} u_{T,t+1} + + \beta E_{t} \omega_{T,t+1} \left(1 - \delta_{T,t+1} + e^{e_{T,q,t+1}} \phi \left(\frac{I_{T,t+1}}{K_{T,t}} \right) - e^{e_{T,q,t+1}} \phi' \left(\frac{I_{T,t+1}}{K_{T,t}} \right) \frac{I_{T,t+1}}{K_{T,t}} \right),$$
(2.13)

$$\omega_{N,t} = \beta E_{t} \mu_{t+1} (1 - \tau_{k,t+1}) R_{N,k,t+1} u_{N,t+1} + + \beta E_{t} \omega_{N,t+1} \left(1 - \delta_{N,t+1} + e^{e_{N,q,t+1}} \phi \left(\frac{I_{N,t+1}}{K_{N,t}} \right) - e^{e_{N,q,t+1}} \phi' \left(\frac{I_{N,t+1}}{K_{N,t}} \right) \frac{I_{N,t+1}}{K_{N,t}} \right),$$
(2.14)

The derivatives with respect to domestic, Eurozone and international bonds yield:

$$\mu_t = \beta E_t \mu_{t+1} \frac{r_{g,t} (1 - \tau_{g,t+1})}{\Pi_{t+1}}, \qquad (2.15)$$

$$\mu_t = \beta E_t \frac{\mu_{t+1} i_t \Psi(b_{t-1})}{\Pi_{t+1}}, \tag{2.16}$$

$$\mu_{t} = \beta E_{t} \frac{\mu_{t+1} i_{t}^{w} \Psi(\tilde{b}_{t-1}^{w}) \Delta e_{t+1}}{\Pi_{t+1}}, \qquad (2.17)$$

where μ_t , $\omega_{T,t}$ and $\omega_{N,t}$ denote the Lagrangean multipliers for the budget constraint and the two laws of motion for capital, respectively. Furthermore, it is convenient to define the *average* marginal rate of substitution between consumption and working hours supplied to each sector

$$mrs_{T,t} = \varsigma_T e^{e_{n,t}} e^{e_{u,t}} \frac{N_{T,t}^{\varphi}}{(1+\tau_{c,t})\mu_t},$$
 (2.18)

and

$$mrs_{N,t} = \varsigma_N e^{e_{n,t}} e^{e_{u,t}} \frac{N_{N,t}^{\varphi}}{(1 + \tau_{c,t})\mu_t}.$$
 (2.19)

2.2 Households: The Consumption Bundle and the Price Index

Total consumption C_t is defined as a basket incorporating tradable and non-tradable goods

$$C_t = \left((1 - \Lambda)^{\frac{1}{z}} C_{T,t}^{\frac{z-1}{z}} + \Lambda^{\frac{1}{z}} C_{N,t}^{\frac{z-1}{z}} \right)^{\frac{z}{z-1}}, \qquad \Lambda \in (0,1), \quad z > 0,$$

where Λ is the so called distribution parameter and z denotes the elasticity of substitution between tradable and non-tradable goods. The sub-basket of tradables $C_{T,t}$, in turn, is itself composed of domestically produced goods, $C_{H,t}$, goods imported from the Eurozone, $C_{F,t}$, goods imported from the rest of the world, $C_{W,t}$, and the consumption of oil $C_{o,t}$:

$$C_{T,t} = \left(\alpha_1^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha_2^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} + \alpha_3^{\frac{1}{\eta}} C_{W,t}^{\frac{\eta-1}{\eta}} + \alpha_4^{\frac{1}{\eta}} C_{o,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}},$$

where α_1 measures the degree of home bias in tradables, $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$ and $\eta > 0$ is the elasticity of substitution between domestically produced and foreign goods. It is straightforward to derive the functions describing optimal overall demand for tradables and non-tradables, respectively:

$$C_{T,t} = (1 - \Lambda) \left(\frac{P_{T,t}}{P_t}\right)^{-z} C_t \tag{2.20}$$

and

$$C_{N,t} = \Lambda \left(\frac{P_{N,t}}{P_t}\right)^{-z} C_t \tag{2.21}$$

Analogously, the optimal demand for tradables produced at home and abroad satisfies:

$$C_{H,t} = \alpha_1 \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\eta} C_{T,t},$$
 (2.22)

$$C_{F,t} = \alpha_2 \left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\eta} C_{T,t},$$
 (2.23)

$$C_{W,t} = \alpha_3 \left(\frac{\mathcal{E}_t P_{W,t}}{P_{T,t}}\right)^{-\eta} C_{T,t}, \tag{2.24}$$

and

$$C_{o,t} = \alpha_4 \left(\frac{\mathcal{E}_t P_{o,t}}{P_{T,t}}\right)^{-\eta} C_{T,t}, \tag{2.25}$$

where $P_{o,t}$ is the oil price in US dollars.

The implied utility-based consumer price index reads

$$P_t = ((1 - \Lambda)P_{T,t}^{1-z} + \Lambda P_{N,t}^{1-z})^{\frac{1}{1-z}},$$
(2.26)

while the price index of tradable goods can be written as

$$P_{T,t} = \left(\alpha_1 P_{H,t}^{1-\eta} + \alpha_2 P_{F,t}^{1-\eta} + \alpha_3 (\mathcal{E}_t P_{W,t})^{1-\eta} + \alpha_4 (\mathcal{E}_t P_{o,t})^{1-\eta}\right)^{\frac{1}{1-\eta}}.$$
 (2.27)

As will become clear below, it is useful to define some relative prices. The prices of home, foreign and world tradables as well as oil relative to the average price of tradables read:

$$\frac{P_{H,t}}{P_{T,t}} = (\alpha_1 + \alpha_2 s_t^{1-\eta} + \alpha_3 s_{H,t}^{1-\eta} + \alpha_4 s_{o,H,t}^{1-\eta})^{\frac{1}{\eta-1}},$$
(2.28)

$$\frac{P_{F,t}}{P_{T,t}} = (\alpha_1 s_t^{\eta-1} + \alpha_2 + \alpha_3 s_{F,t}^{1-\eta} + \alpha_4 s_{o,F,t}^{1-\eta})^{\frac{1}{\eta-1}},$$
(2.29)

$$\mathcal{E}_{t} \frac{P_{W,t}}{P_{T,t}} = (\alpha_{1} s_{H,t}^{\eta-1} + \alpha_{2} s_{F,t}^{\eta-1} + \alpha_{3} + \alpha_{4} s_{o,W,t}^{1-\eta})^{\frac{1}{\eta-1}}, \tag{2.30}$$

and

$$\mathcal{E}_{t} \frac{P_{o,t}}{P_{T,t}} = (\alpha_{1} s_{o,H,t}^{\eta-1} + \alpha_{2} s_{o,F,t}^{\eta-1} + \alpha_{3} s_{o,W,t}^{\eta-1} + \alpha_{4})^{\frac{1}{\eta-1}}, \tag{2.31}$$

where $s_t = P_{F,t}/P_{H,t}$ denotes the terms of trade of the home country towards the rest of the Euro Area, $s_{H,t} = \mathcal{E}_t P_{W,t}/P_{H,t}$ denotes the terms of trade of the home country towards the rest of the world, $s_{F,t} = \mathcal{E}_t P_{W,t}/P_{F,t}$ denotes the REA terms of trade towards the rest of the world, $s_{o,H,t} = \mathcal{E}_t P_{o,t}/P_{H,t}$, $s_{o,F,t} = \mathcal{E}_t P_{o,t}/P_{F,t}$ and $s_{o,W,t} = P_{o,t}/P_{W,t}$. In addition, we define the relative prices of the composite tradable good as:

$$\frac{P_{T,t}}{P_t} = \left(1 - \Lambda + \Lambda \left(\frac{P_{N,t}}{P_{T,t}}\right)^{1-z}\right)^{\frac{1}{z-1}},\tag{2.32}$$

while the ratio of non-tradable goods prices to tradable goods prices is determined through the relationship

$$\frac{P_{N,t}}{P_{T,t}} = \frac{\prod_{N,t} P_{N,t-1}}{\prod_{T,t} P_{T,t-1}},$$
(2.33)

where $\Pi_{N,t}$ and $\Pi_{T,t}$ denote the gross rates of inflation for non-tradable and tradable consumption goods respectively. Note that a similar relation hold with respect to the various terms of trade, s_t , $s_{H,t}$, $s_{F,t}$, $s_{O,H,t}$, $s_{O,F,t}$ and $s_{O,W,t}$. In particular, we get:

$$s_t = \frac{\prod_{F,t}}{\prod_{H,t}} s_{t-1},\tag{2.34}$$

$$s_{H,t} = \Delta e_t \frac{\prod_{W,t}}{\prod_{H,t}} s_{H,t-1},$$
 (2.35)

$$s_{F,t} = \Delta e_t \frac{\Pi_{W,t}}{\Pi_{F,t}} s_{F,t-1},$$
 (2.36)

$$s_{o,H,t} = \Delta e_t \frac{\prod_{o,t} s_{o,H,t-1}}{\prod_{H,t} s_{o,H,t-1}},$$
 (2.37)

$$s_{o,F,t} = \Delta e_t \frac{\prod_{o,t}}{\prod_{F,t}} s_{o,F,t-1},$$
 (2.38)

and

$$s_{o,W,t} = \frac{\prod_{o,t} s_{o,W,t-1}}{\prod_{W,t} s_{o,W,t-1}},$$
(2.39)

where $\Pi_{W,t}$ and $\Pi_{o,t}$ are the gross rates of inflation in the prices for world goods and oil, both $\Delta e_t = \mathcal{E}_t/\mathcal{E}_{t-1}$. As shown below, $\Pi_{N,t}$, $\Pi_{H,t}$ and $\Pi_{F,t}$ are determined via the sector specific Phillips curves while $\Pi_{W,t}$ and $\Pi_{o,t}$ are assumed to follow exogenous stochastic processes.

 $C_{H,t}$ is an aggregator over a continuum of varieties of goods, defined as

$$C_{H,t} = \left(\int_0^1 C_{i,H,t}^{\frac{\epsilon_H - 1}{\epsilon_H}} di\right)^{\frac{\epsilon_H}{\epsilon_H - 1}}, \qquad \epsilon_H > 1, \tag{2.40}$$

where ϵ_H denotes the elasticity of substitution. $C_{N,t}$, $C_{H,t}$, $C_{F,t}$ and the components of government consumption, denoted by $G_{N,t}$ and $G_{H,t}$, are specified analogously and assumed to exhibit the same degree of substitutability between individual varieties.⁷ Note, that the government demands both, tradable and non-tradable goods produced in the Home country. Further, a part of domestically produced tradables are also used for investment by private agents $I_t = I_{T,t} + I_{N,t}$ and for building up government capital $I_{g,t} = I_{T,g,t} + I_{N,g,t}$. I_t and $I_{g,t}$ as well as world demand for German goods $C_{H,t}^W$ are also defined according to (2.40) with the same elasticity of substitution.

2.3 Households: Nominal Wage Setting

The aggregate nominal wage index in sector $s = \{T, N\}$ can be written as

$$W_{s,t} = \left(\int_0^1 W_{i,s,t}^{1-\epsilon_w} di\right)^{\frac{1}{1-\epsilon_w}} \tag{2.41}$$

$$= \left[(1 - \theta_{w,s}) \widetilde{W}_{s,t}^{1 - \epsilon_w} + \theta_{w,s} (W_{s,t-1} \prod_{t=1}^{\omega_{w,s}})^{1 - \epsilon_w} \right]^{\frac{1}{1 - \epsilon_w}}, \tag{2.42}$$

where ϵ_w denotes the elasticity of substitution between different types of labor, $\theta_{w,s}$ is the Calvo parameter for the sector specific labor market and $\omega_{w,s}$ denotes the degree of sector specific indexation to past inflation in the case a household is

$$C_{N,t} = \left(\int_0^1 C_{i,N,t}^{\frac{\epsilon_N-1}{\epsilon_N}} di\right)^{\frac{\epsilon_N}{\epsilon_N-1}}, \qquad \epsilon_N > 1.$$

⁷In particular, the consumption aggregator for non-tradable goods takes the form

not able to adjust its nominal wage in a particular quarter.

Each household faces the following sector specific labor demand function

$$N_{s,i,t} = \left(\frac{W_{i,s,t}}{W_{s,t}}\right)^{-\epsilon_w} N_{s,t}$$
 (2.43)

Assuming symmetry across households, we can neglect the household specific index i.

The nominal wage in period t + k which was last adjusted in period t is given by

$$W_{s,t,t+k} = \widetilde{W}_{s,t} \prod_{i=0}^{k-1} \prod_{s,t+i}^{\omega_{w,s}},$$
 (2.44)

where $\widetilde{W}_{s,t}$ is the nominal wage set in period t and $\omega_{w,s}$ denotes the degree of indexation.

Accordingly, a household who has last adjusted her nominal wage in period t will face the following labor demand in period t + k:

$$N_{s,t,t+k} = \left(\frac{W_{s,t,t+k}}{W_{s,t}}\right)^{-\epsilon_w} N_{s,t+k}. \tag{2.45}$$

Accordingly, with respect to each of the two sectors, the typical household solves

$$E_{t} \sum_{k=0}^{\infty} (\beta \theta_{s,w})^{k} \left[e^{e_{u,t+k}} \left(\frac{1}{1-\sigma} \left(\frac{C_{t+k}}{C_{t+k-1}^{\gamma}} \right)^{1-\sigma} - \varsigma_{s} e^{e_{s,n,t+k}} \frac{N_{s,t,t+k}^{1+\varphi}}{1+\varphi} \right) \right], \quad (2.46)$$

subject to household budget constraint (2.1) and the labor demand function (2.45).

The first order condition with respect to adjusted nominal wage, $W_{s,t}$, can be written as

$$E_{t} \sum_{k=0}^{\infty} (\beta \theta_{w,s})^{k} N_{s,t,t+k} U_{C_{t+k}} \left(\frac{1 - \tau_{w,t+k}}{1 + \tau_{c,t+k}} \frac{\widetilde{W}_{s,t} \prod_{i=0}^{k-1} \prod_{t+i}^{\omega_{w,s}}}{P_{t+k}} - \frac{\epsilon_{w}}{\epsilon_{w} - 1} mrs_{s,t,t+k} \right) = 0,$$
(2.47)

where $mrs_{s,t,t+k} = -(U_{N_{s,t,t+k}}/U_{C_{t,t+k}})$ and assuming that $U_{C_{t,t+k}} = U_{C_{t+k}}$.

2.4 Firms: Production and Price Setting

Each firm in the home tradable sector produces according to a CES production technology, combining local inputs $Y_{i,t}$ and oil/energy inputs denoted by $O_{i,t}$:

$$Y_{i,T,t} = e^{A_{T,t}} \left(\wp^{\frac{1}{\xi}} Y_{i,t}^{\frac{\xi-1}{\xi}} + (1 - \wp)^{\frac{1}{\xi}} O_{i,t}^{\frac{\xi-1}{\xi}} \right)^{\frac{\xi}{\xi-1}}, \tag{2.48}$$

where the distribution parameter \wp measures the importance of energy within the production function and ξ reflects the elasticity of substitution between local inputs

and oil/energy. $Y_{i,t}$ is generated via a standard Cobb-Douglas function:

$$Y_{i,t} = N_{i,T,t}^{1-a_T} (u_{i,T,t} K_{i,T,t-1})^{a_T} (KG_{T,t-1})^{a_g},$$
 (2.49)

where a_T , $a_g \in (0, 1)$ and $KG_{T,t}$ reflects the stock of public capital, taken as given by the firm. The productivity shifter $A_{T,t}$ is exogenous and evolves according to

$$A_{T,t} = \rho_{a,T} A_{T,t-1} + \epsilon_{a,T,t}.$$

Cost minimization with respect to effective capital $u_{i,t}K_{i,t}$, labor input $N_{i,t}$ and oil input $O_{i,t}$ yields:

$$mc_{i,T,t}\wp^{\frac{1}{\xi}}(1-a_T)Y_{i,T,t}^{\frac{1}{\xi}}\frac{Y_{i,t}^{\frac{\xi-1}{\xi}}}{N_{i,T,t}} = (1+\tau_{wf,t})\frac{W_{T,t}}{P_t},$$
(2.50)

$$mc_{i,T,t} \wp^{\frac{1}{\xi}} a_T Y_{i,T,t}^{\frac{1}{\xi}} \frac{Y_{i,t}^{\frac{\xi-1}{\xi}}}{u_{i,T,t} K_{i,T,t-1}} = R_{k,T,t}$$
 (2.51)

and

$$mc_{i,T,t}(1-\wp)^{\frac{1}{\xi}}Y_{i,T,t}^{\frac{1}{\xi}}O_{i,t}^{-\frac{1}{\xi}} = \mathcal{E}_t \frac{P_t^o}{P_t},$$
 (2.52)

where $mc_{i,T,t}$ denotes real marginal costs in terms of the final consumption good. $\tau_{wf,t} \frac{W_{T,t}}{P_t}$ reflects the amount of workers' social contributions bared by employers. It is straightforward to show that due to constant returns to scale, firms are identical regarding their marginal costs. The latter are given by:

$$mc_{T,t} = e^{-A_{T,t}} \left[\wp \frac{\left(\left((1 + \tau_{wf,t}) \frac{W_{T,t}}{P_t} \right)^{1 - a_T} R_{k,T,t}^a \right)^{1 - \xi}}{(a_T^{a_T} (1 - a_T)^{1 - a_T})^{1 - \xi} (KG_{T,t-1}^{a_g})^{1 - \xi}} + (1 - \wp) \left(\frac{P_o}{P_t} \right)^{1 - \xi} \right]^{\frac{1}{1 - \xi}}.$$

Each firm in the tradables sector faces the following downward sloping demand function

$$Y_{i,T,t} = \left(\frac{P_{i,H,t}}{P_{H,t}}\right)^{-\epsilon_H} Y_{T,t} = \left(\frac{P_{i,H,t}}{P_{H,t}}\right)^{-\epsilon_H} (C_{H,t} + C_{H,t}^* + I_t + I_{g,t} + G_{T,t} + C_{H,t}^W).$$

The typical firm producing non-tradable goods operates subject to a standard Cobb-Douglas production function:

$$Y_{i,N,t} = e^{A_{N,t}} N_{i,N,t}^{1-a_N} (u_{i,N,t} K_{i,N,t-1})^{a_N} (KG_{N,t-1})^{a_g},$$
 (2.53)

where a_N , $a_g \in (0, 1)$ and $A_{N,t}$ evolves according to:

$$A_{N,t} = \rho_{a,N} A_{N,t-1} + \epsilon_{a,N,t}$$
.

Cost minimization yields

$$mc_{i,N,t}(1-a_N)\frac{Y_{i,N,t}}{N_{i,N,t}} = (1+\tau_{wf,t})\frac{W_{N,t}}{P_t}$$
 (2.54)

and

$$mc_{i,N,t}a_{N}\frac{Y_{i,N,t}}{u_{i,N,t}K_{i,N,t-1}} = R_{N,k,t},$$
 (2.55)

where $mc_{i,N,t}$ denotes real marginal costs in the non-tradable sector. Further, for each firm we have

$$mc_{N,t} = \frac{R_{N,k,t}^{a_N} \left((1 + \tau_{wf,t}) \frac{W_{N,t}}{P_t} \right)^{1 - a_N}}{a_N^{a_N} (1 - a_N)^{1 - a_N} e^{A_{N,t}} KG_{N,t-1}^{a_g}}.$$

Each firm producing non-tradables is confronted with the following demand function:

$$Y_{i,N,t} = \left(\frac{P_{i,N,t}}{P_{N,t}}\right)^{-\epsilon_N} Y_{N,t} = \left(\frac{P_{i,N,t}}{P_{N,t}}\right)^{-\epsilon_N} (C_{N,t} + G_{N,t}).$$

Both sectors are monopolistically competitive. The typical firm sets its optimal nominal price such that it maximizes the expected discounted present value of profits subject to the demand function and the technological restriction shaped by the production function. However, the price adjustment process is constrained by a Calvo-type friction - in each period a fraction $\theta_s \in (0,1)$, $s = \{T, N\}$, of randomly selected firms are not allowed to optimally change their prices. These firms rather partially index their prices to last-period's inflation. In particular, the price charged by a "non-adjuster" in period t in the tradables sector is given by

$$P_{i,H,t}^{non} = P_{i,H,t-1} \Pi_{H,t-1}^{\chi_T}, \qquad \chi_T \in (0,1),$$

while in the non-tradables sector we have

$$P_{i,N,t}^{non} = P_{i,N,t-1} \Pi_{N,t-1}^{\chi_N}, \qquad \chi_N \in (0,1),$$

where χ_T and χ_N denote the degree of indexation and $\Pi_{H,t}$ and $\Pi_{N,t}$ are the gross rates of producer price inflation in the two sectors. The remaining firms, constituting a fraction of $1-\theta_s$, optimally reset their prices. In particular, in the sector producing tradable goods, they solve the following problem

$$\max_{P_{i,H,t}} E_{t} \left(\sum_{q=0}^{\infty} \Upsilon_{q,t}^{T} \left(P_{i,H,t} \prod_{k=0}^{q-1} \Pi_{H,t+k}^{\chi_{T}} - P_{t+q} m c_{T,t+q} \right) \left(\frac{P_{i,H,t} \prod_{k=0}^{q-1} \Pi_{H,t+k}^{\chi_{T}}}{P_{H,t+q}} \right)^{-\epsilon_{H}} Y_{T,t+q} \right),$$

where the objective function is obtained after plugging the demand schedule into the profit function and observing that marginal costs are independent of the scale of production and identical across firms. $\Upsilon_{q,t}^T = \beta^q \theta_T^q \frac{\mu_{t+q}}{\mu_t} \frac{P_t}{P_{t+q}}$ is the effective stochastic discount factor. The corresponding maximization problem in the non-

tradables sector reads

$$\max_{P_{i,N,t}} E_t \left(\sum_{q=0}^{\infty} \Upsilon_{q,t}^{N} \left(P_{i,N,t} \prod_{k=0}^{q-1} \Pi_{N,t+k}^{\chi_N} - P_{t+q} m c_{N,t+q} \right) \left(\frac{P_{i,H,t} \prod_{k=0}^{q-1} \Pi_{H,t+k}^{\chi_T}}{P_{H,t+q}} \right)^{-\epsilon_N} Y_{N,t+q} \right),$$

where $\Upsilon^N_{q,t} = \beta^q \theta^q_N \frac{\mu_{t+q}}{\mu_t} \frac{P_t}{P_{t+q}}$ is the effective stochastic discount factor. Further, firms are assumed to be unable to price-discriminate across markets. Thus, at the individual good's level, the law of one price holds.

In the case of the tradables sector, the first order condition to this problem can be represented recursively as

$$\frac{\tilde{P}_{H,t}}{P_{H,t}} = \frac{\epsilon_H}{\epsilon_H - 1} \frac{\Im_{1,t}}{\Im_{2,t}} = mu_T \frac{\Im_{1,t}}{\Im_{2,t}},\tag{2.56}$$

where $\tilde{P}_{H,t}$ is the optimal reset price, while

$$\Im_{1,t} = \frac{mc_{T,t}}{P_{H,t}/P_t} Y_{T,t} + \beta \theta_T \frac{\mu_{t+1}}{\mu_t} \frac{\Pi_{H,t+1}}{\Pi_{t+1}} \left(\frac{\Pi_{H,t}^{\chi_T}}{\Pi_{H,t+1}} \right)^{-\epsilon_H} \Im_{1,t+1}$$

and

$$\Im_{2,t} = Y_{T,t} + \beta \theta_T \frac{\mu_{t+1}}{\mu_t} \frac{\Pi_{H,t+1}}{\Pi_{t+1}} \left(\frac{\Pi_{H,t}^{\chi_T}}{\Pi_{H,t+1}} \right)^{1-\epsilon_H} \Im_{2,t+1}.$$

 mu_T denotes the markup of prices over marginal costs in the stationary equilibrium. Note that each adjusting firm sets the same optimal price. The producer price index for the sector of tradables can be written as

$$1 = \left(\theta_T \left(\frac{\Pi_{H,t-1}^{\chi}}{\Pi_{H,t}}\right)^{1-\epsilon_H} + (1-\theta_T) \left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{1-\epsilon_H}\right). \tag{2.57}$$

Combining (2.56) and (2.57) yields an open economy version of the hybrid New Keynesian Phillips curve for domestic producer prices in the tradables sector

$$\hat{\Pi}_{H,t} - \chi_T \hat{\Pi}_{H,t-1} = \frac{(1 - \theta_T)(1 - \theta_T \beta)}{\theta_T} \left(\hat{mc}_{T,t} - \frac{\hat{P_{H,t}}}{P_t} \right) + \beta (\hat{\Pi}_{H,t+1} - \chi_T \hat{\Pi}_{H,t}).$$

For the sector producing non-tradable goods, we get the following recursive representation of the optimal pricing decision:

$$\frac{\tilde{P}_{N,t}}{P_{N,t}} = \frac{\epsilon_N}{\epsilon_N - 1} \frac{\Omega_{1,t}}{\Omega_{2,t}} = m u_N \frac{\Omega_{1,t}}{\Omega_{2,t}},\tag{2.58}$$

where $\hat{P}_{N,t}$ is the optimal reset price, while

$$\Omega_{1,t} = \frac{mc_{N,t}}{P_{N,t}/P_t} Y_{N,t} + \beta \theta_N \frac{\mu_{t+1}}{\mu_t} \frac{\Pi_{N,t+1}}{\Pi_{t+1}} \left(\frac{\Pi_{N,t}^{\chi_N}}{\Pi_{N,t+1}} \right)^{-\epsilon_N} \Omega_{1,t+1}$$

and

$$\Omega_{2,t} = Y_{N,t} + \beta \theta_N \frac{\mu_{t+1}}{\mu_t} \frac{\Pi_{N,t+1}}{\Pi_{t+1}} \left(\frac{\Pi_{N,t}^{\chi_N}}{\Pi_{N,t+1}} \right)^{1-\epsilon_N} \Omega_{2,t+1}.$$

 mu_N is the markup in the stationary equilibrium. Note that each adjusting firm sets the same optimal price. The corresponding producer price index can be written as

$$1 = \left(\theta_N \left(\frac{\prod_{N,t-1}^{\chi_N}}{\prod_{N,t}}\right)^{1-\epsilon_N} + (1-\theta_N) \left(\frac{\tilde{P}_{N,t}}{P_{N,t}}\right)^{1-\epsilon_N}\right). \tag{2.59}$$

Combining (2.58) and (2.59) yields an open economy version of the hybrid New Keynesian Phillips curve for domestic producer prices in the non-tradables sector

$$\hat{\Pi}_{N,t} - \chi_N \hat{\Pi}_{N,t-1} = \frac{(1 - \theta_N)(1 - \theta_N \beta)}{\theta_N} \left(\hat{mc}_{N,t} - \frac{\hat{P_{N,t}}}{P_t} \right) + \beta (\hat{\Pi}_{N,t+1} - \chi_N \hat{\Pi}_{N,t}).$$

The Phillips curves for the REA read

$$\hat{\Pi}_{F,t} - \chi_T^* \hat{\Pi}_{F,t-1} = \frac{(1 - \theta_T^*)(1 - \theta_T^* \beta)}{\theta_T^*} \left(\hat{mc}_{T,t}^* - \frac{\hat{P_{F,t}}}{P_t^*} \right) + \beta (\hat{\Pi}_{F,t+1} - \chi_T^* \hat{\Pi}_{F,t}),$$

and

$$\hat{\Pi}_{N,t}^* - \chi_N^* \hat{\Pi}_{N,t-1}^* = \frac{(1 - \theta_N^*)(1 - \theta_N^* \beta)}{\theta_N^*} \left(\hat{mc}_{N,t}^* - \frac{P_{N,t}^*}{P_t^*} \right) + \beta (\hat{\Pi}_{N,t+1}^* - \chi_N^* \hat{\Pi}_{N,t}^*),$$

2.5 The Government Sector

Public capital $KG_{s,t}$ evolves according to

$$KG_{s,t} = I_{g,s,t} + (1 - \delta_g)KG_{s,t-1},$$
 (2.60)

where $s = \{T, N\}$ denotes the sector for which the particular fraction of government capital is provided, $I_{g,s,t}$ is the fully home-biased public investment expenditure and δ_g is the constant depreciation rate of public capital. Government expenditure - the sum of public consumption and investment, interest payments on outstanding public debt and net transfers - is financed by several types of taxes (on consumption as well as on labor and capital income) and by issuing new government bonds. Accordingly, the government's budget constraint in real terms reads

$$b_{g,t} = \frac{(1 - \tau_{g,t})r_{g,t-1}b_{g,t-1}}{\Pi_t} + \frac{P_{H,t}}{P_t}(G_{T,t} + I_{g,T,t} + I_{g,N,t}) + \frac{P_{N,t}}{P_t}G_{N,t} - \tau_{c,t}C_t - (\tau_{w,t} + \tau_{wf,t})(w_{T,t}N_{T,t} + w_{N,t}N_{N,t}) - \tau_{k,t}(R_{T,k,t}u_{T,t}K_{T,t-1} + R_{N,k,t}u_{N,t}K_{N,t-1}) - T_t,$$
(2.61)

where $b_{g,t}=B_{g,t}/P_t$ is real public debt and T_t denotes the endogenous net lumpsum taxes/transfers. For simplicity, sector specific government consumption and investment are defined as $G_{T,t}=(1-\Lambda)G_t$, $G_{N,t}=\Lambda G_t$ and $I_{g,T,t}=(1-\Lambda)I_{g,t}$, $I_{g,N,t}=\Lambda I_{g,t}$ respectively. G_t and $I_{g,t}$ are exogenous and in log-linear terms evolve according to the stochastic processes:

$$\hat{G}_t = \rho_q \hat{G}_{t-1} + \epsilon_{q,t}$$

and

$$\hat{l}_{g,t} = \rho_{ig}\hat{l}_{g,t-1} + \epsilon_{ig,t},$$

where $\epsilon_{g,t}$ and $\epsilon_{ig,t}$ are normally distributed White Noise processes.

Transfer policy is endogenous and assumed to react to deviations of last period's ratio of government debt to gross domestic product (GDP) from its target level \bar{b}_g

$$T_t = T_0 + T_1 \left(\frac{B_{g,t-1}}{P_{Y,t-1} \bar{Y}_{t-1}} - \bar{b}_g \right), \qquad T_0, T_1 > 0$$

where \bar{Y}_t is real GDP and $P_{Y,t}$ is the corresponding GDP deflator to be discussed in Section 2.8 below. In real terms the last equation reads

$$T_{t} = T_{0} + T_{1} \left(\frac{b_{g,t-1}}{Y_{t-1}} \left(\frac{P_{Y,t-1}P_{T,t-1}}{P_{t-1}P_{T,t-1}} \right)^{-1} - \bar{b}_{g} \right). \tag{2.62}$$

The common central bank sets the area wide nominal short term policy rate according to a simple Taylor rule of the form:

$$i_{t} = \phi_{r} i_{t-1} + (1 - \phi_{r}) (\phi_{y} \hat{\overline{Y}}_{EA,t} + \phi_{\pi} (\Pi_{EA,t} - \Pi_{EA})) + v_{t}, \qquad (2.63)$$

whereas $\phi_r, \phi_y, \phi_\pi > 0$ and ϕ_r is the weight the central bank puts on smoothing the interest rate over time. $\widehat{Y}_{EA,t}$ is the log-deviation of area wide GDP from its steady state level and $\Pi_{EA,t} - \Pi_{EA}$ is the deviation of the area wide gross rate of consumer price inflation from its target value. Finally, v_t is the monetary policy shock following an AR(1) process:

$$v_t = \rho_v v_{t-1} + \epsilon_{v,t},$$

where $\epsilon_{v,t}$ is White Noise.

2.6 Rest of the World

Overall world demand for goods from outside Germany and the REA is follows an exogenous process given by:

$$C_t^W = (C_{t-1}^W)^{\rho_{c,w}} e^{\epsilon_{c,w,t}}$$
(2.64)

ROW demand for German and REA tradables is given by

$$C_H^W = \alpha_1^W \left(\frac{P_{H,t}}{\mathcal{E}_t P_{W,t}}\right)^{-\eta} C_t^W \tag{2.65}$$

and

$$C_F^W = \alpha_2^w \left(\frac{P_{F,t}}{\mathcal{E}_t P_{W,t}}\right)^{-\eta^*} C_t^w. \tag{2.66}$$

The gross rate of inflation in world tradables also follows an exogenous process given by

$$\Pi_{W,t} = (\Pi_{W,t-1})^{\rho_{\pi,w}} e^{\epsilon_{\pi,w,t}}.$$
 (2.67)

Finally, the nominal world interest rate and the gross rate of inflation in oil prices are assumed to evolve according to

$$i_t^W = (i_{t-1}^W)^{\rho_{i,w}} e^{\epsilon_{i,w,t}}$$
(2.68)

and

$$\Pi_{o,t} = (\Pi_{o,t-1})^{\rho_{\pi,o}} e^{\epsilon_{\pi,o,t}}.$$
 (2.69)

2.7 Aggregate Consistency and Equilibrium

The goods market equilibrium requires that the supply and demand are equal with respect to each good produced in the currency union. Accordingly, for German tradable goods we have

$$Y_{T,t} = C_{H,t} + \frac{1-n}{n} C_{H,t}^* + C_{H,t}^w + G_{T,t} + I_{T,t} + I_{N,t} + I_{g,T,t} + I_{g,N,t}.$$
 (2.70)

The corresponding aggregate consistency condition regarding goods produced in the REA reads

$$Y_{T,t}^* = \frac{n}{1-n} C_{F,t} + C_{F,t}^* + C_{F,t}^w + G_{T,t}^* + I_{T,t}^* + I_{N,t}^* + I_{g,T,t}^* + I_{g,N,t}^*.$$
 (2.71)

The analogous equilibrium conditions in the sector of non-tradable goods take the form:

$$Y_{N,t} = C_{N,t} + G_{N,t} (2.72)$$

and

$$Y_{N,t}^* = C_{N,t}^* + G_{N,t}^*. (2.73)$$

The real net foreign asset position of the home country evolves according to

$$\frac{b_{t}}{i_{t}\Psi(b_{t-1})} + \frac{\tilde{b}_{t}^{w}}{i_{t}^{w}\Psi(\tilde{b}_{t-1}^{w})} = \frac{b_{t-1}}{\Pi_{t}} + \frac{\Delta e_{t}\tilde{b}_{t-1}^{w}}{\Pi_{t}} + \frac{P_{H,t}}{P_{t}}Y_{T,t}
- \frac{P_{T,t}}{P_{t}}C_{T,t} - \frac{P_{H,t}}{P_{t}}(G_{T,t} + I_{T,t} + I_{N,t} + I_{g,T,t} + I_{g,N,t}) - \mathcal{E}_{t}\frac{P_{o,t}}{P_{t}}O_{t},$$
(2.74)

where $b_t = B_t/P_t$, i.e. B_t is deflated by the consumer price level. Note that due to the relationship (2.72) non-tradable goods are completely absent in (2.74). The real net foreign asset position of the foreign country evolves according to

$$\frac{b_{t}^{*}}{i_{t}\Psi(b_{t-1}^{*})} + \frac{\tilde{b}_{t}^{w*}}{i_{t}^{w}\Psi(\tilde{b}_{t-1}^{w*})} = \frac{b_{t-1}^{*}}{\Pi_{t}^{*}} + \frac{\Delta e_{t}\tilde{b}_{t-1}^{w*}}{\Pi_{t}^{*}} + \frac{P_{F,t}}{P_{t}^{*}}Y_{T,t}^{*}
- \frac{P_{T,t}}{P_{t}}C_{T,t}^{*} - \frac{P_{F,t}}{P_{t}^{*}}(G_{T,t}^{*} + I_{T,t}^{*} + I_{N,t}^{*} + I_{g,T,t}^{*} + I_{g,N,t}^{*}) - \mathcal{E}_{t}\frac{P_{o,t}}{P_{t}^{*}}O_{t}^{*}.$$
(2.75)

Finally, international bonds are in zero net supply which implies

$$nB_t = -(1-n)B_t^*,$$

or expressed in real terms

$$nb_t = -(1-n)b_t^* \frac{P_t^*}{P_t}. (2.76)$$

2.8 Further Model Equations

Nominal value added in the tradables sector, $V_{T,t}$ is defined as

$$V_{T,t} = P_{H,t}Y_{T,t} - \mathcal{E}_t P_{o,t} O_t$$

while in the non-tradables sector the corresponding relationship reads

$$V_{N,t} = P_{N,t} Y_{N,t}$$

Let us denote real gross domestic product by \bar{Y}_t and its deflator by $P_{Y,t}$. Then, by using the last two equations, we can write nominal GDP as:

$$P_{Y,t}\bar{Y}_t = V_{T,t} + V_{Nt} = P_{H,t}Y_{T,t} - \mathcal{E}_t P_{o,t}O_t + P_{N,t}Y_{N,t}. \tag{2.77}$$

Dividing both sides of equation (2.78) by the average price of home tradable goods yields

$$\frac{P_{Y,t}}{P_{H,t}}\bar{Y}_t = Y_{T,t} - \frac{\mathcal{E}_t P_{o,t}}{P_{H,t}} O_t + \frac{P_{N,t}}{P_{H,t}} Y_{N,t}.$$
 (2.78)

Note that, as explained above, $s_{o,H,t} = \frac{\mathcal{E}_t P_{o,t}}{P_{H,t}}$ while

$$\frac{P_{N,t}}{P_{H,t}} = \frac{\prod_{H,t} P_{N,t-1}}{\prod_{N,t} P_{H,t-1}}.$$

We still need to determine the evolution of the price ratio $P_{Y,t}/P_{H,t}$. For this, we define the GDP deflator as follows

$$P_{Y,t} = \underbrace{\frac{P_{H,t}Y_{T,t} - \mathcal{E}_t P_{o,t} O_t}{P_{Y,t} \bar{Y}_t}}_{\text{share of tradables value added in GDP}} \cdot P_{H,t} + \underbrace{\frac{P_{N,t}Y_{N,t}}{P_{Y,t} \bar{Y}_t}}_{\text{share of non-tradables value added in GDP}} \cdot P_{N,t}.$$

After a straightforward rearrangement we obtain

$$\frac{P_{Y,t}}{P_{H,t}} = (Y_{T,t} - s_{o,H,t}O_t) \left(\frac{P_{H,t}}{P_{Y,t}\bar{Y}_t}\right) + \left(\frac{P_{N,t}}{P_{H,t}}\right) Y_{N,t} \left(\frac{P_{H,t}}{P_{Y,t}\bar{Y}_t}\right) \left(\frac{P_{N,t}}{P_{H,t}}\right). \tag{2.79}$$

The deviations of the area wide GDP and consumer price inflation from their respective steady statess are constructed as weighted averages with weights equal to the corresponding country size:

$$\widehat{\bar{Y}}_{EA,t} = n\widehat{\bar{Y}}_t + (1-n)\widehat{\bar{Y}}_t^*,$$

$$\widehat{\Pi}_{EA,t} = n\widehat{\Pi}_t + (1-n)\widehat{\Pi}_t^*,$$

where "hatted" variables denote log-deviations from the deterministic stationary equilibrium.

Next, we define aggregate employment and aggregate real wages. Aggregate employment is simply

$$N_t = N_{T,t} + N_{N,t}$$
.

The average real wage in the economy is

$$w_t = \frac{N_{T,t}}{N_t} w_{T,t} + \frac{N_{N,t}}{N_t} w_{N,t}.$$

3 Data and Estimation

In this section we, first, discuss how we calibrate a subset of the structural parameters of the model. Next, we lay out our estimation strategy with respect to the remaining parameters and describe the data. Finally, we discuss the main empirical results.

To be able to conduct the analysis in this section, one needs a precise definition of a *tradable* and a *non-tradable* production sector. There is an ongoing debate in the literature on which branches of the economy should be classified as producing internationally tradable products and which not and whether this distinction is country-specific or not.⁸ Here we follow Andrés et al. (2010) and Gaechter et al. (2013) and assume that the tradables sector comprises industry excluding construction and agriculture while the rest of aggregate output is produced in the non-tradables sector.⁹

⁸See for example Gaechter et al. (2013) and the literature cited there.

⁹In particular, we resort to the sectoral breakdowns of main GDP aggregates and employment

3.1 Calibrated Parameters

It has become standard in the related literature dealing with the estimation of DSGE models to calibrate a subset of the structural parameters describing the evolution of the economy. This is done for reasons of dimensionality and because some parameters can be barely identified based on the typical set of observable macroeconomic aggregates. ¹⁰ In this paper, we follow the same strategy and calibrate several parameters which have been shown to be difficult to estimate properly.

In particular, six parameters are set to the values used in the bulk of the DSGE literature. These are the subjective discount factor, β (β^*), the inverse of the intertemporal elasticity with respect to consumption, σ (σ^*), the habit persistence parameter, γ (γ^*), the inverse of the Frish elasticity of labor supply, φ (φ^*), the depreciation rate of capital, δ (δ^*), and the parameter governing the adjustment costs of net foreign asset positions, ψ (ψ^*) (see Table 1). The elasticity of substitution between differentiated labor varieties φ (φ^*) is calibrated as in Smets and Wouters (2002).

To calibrate the distribution parameters appearing in the various CES aggregators of the model, we borrow from existing empirical work using data for Germany, the Euro Area or other advanced economies. In particular, the share of non-tradables in overall consumption Λ (Λ *) is taken form Gomes et al. (2012) and Kolasa and Lombardo (2014) who calculate it for the Euro Area as a whole. Rudolf and Zurlinden (2014) arrive at a very similar value for Switzerland. We estimate the weights within the aggregator of tradable consumption goods α_1 (α_1^*), α_2 (α_2^*) , α_3 (α_3^*) and α_4 (α_4^*) by resorting to national accounts data for Germany and the Euro Area regarding the shares in overall consumption of imports from the REA, imports from the ROW as well as oil consumption. Kollmann et al. (2015), Forni et al. (2012), Gomes et al. (2012) and Kolasa and Lombardo (2014) obtains similar estimates. The value of the distribution parameter in the production function for tradables $\wp(\wp^*)$ is taken from Bodenstein et al. (2011). The elasticity of substitution between tradables and non-tradables $z(z^*)$ is set to the value suggested in Gomes et al. (2012), Andrés et al. (2010) and Kolasa and Lombardo (2014). The elasticity of substitution between the different types of tradable goods η (η^*) is calibrated as in Forni et al. (2012) and Kolasa and Lombardo (2014). Finally, the elasticity of substitution between oil and the other input factors in the production function is taken from Forni et al. (2012) and Bodenstein et al. (2011).

The parameters of the production functions a_T (a_N^*) and a_N (a_N^*) are cali-

provided by EUROSTAT. The database contains disaggeregated quarterly time series on nominal and real value added, gross wages and salaries, employment, working hours, sectors-specific deflators and other variables for 10 sectors. The sectors are: (1) agriculture and (2) industry excluding construction, which are assumed to constitute the tradables sector, (3) construction, (4) trade transport and accommodation, (5) information and communication, (6) financial services, (7) real estate activities, (8) public sector, defence and health activities, (9) arts and entertainment and (10) professional, scientific and technical activities.

¹⁰See for example Pytlarczyk (2005), Fernández-Villaverde et al. (2016) and many others.

brated based on the disaggregated sectoral data provided by EUROSTAT in its sectoral breakdowns of main GDP aggregates and employment. In particular, we compute the average labor shares in value added for both, the tradables and the non-tradables sectors and account for the presence of monopoly profits. The values of $a_{g,T}$, $a_{g,N}$ and $a_{g,T}^*$, $a_{g,N}^*$ are borrowed from Kollmann et al. (2015). The elasticity of the depreciation rate with respect to capital utilization ψ_u (ψ_u^*) is set to the value advocated by Burnside and Eichenbaum (1996).

The average markups in the tradables and the non-tradables sector correspond to the estimates provided by Christopoulou and Vermeulen (2012) for Germany and the REA. The calibration of the various tax rates is based on the values provided by Kollmann et al. (2015) and Andrés et al. (2010). We fix the steady state levels of labor in both sectors N_t (N_T^*) and N_N (N_N^*) by calculating the sample averages of the overall fraction of time devoted to work as well as the share of working hours employed by the tradables sector in Germany and the REA. Correspondingly, the parameters measuring the disutility of labor ς_T (ς_T^*) and ς_N (ς_N^*) are set such that they are consistent with N_t (N_T^*) and N_N (N_N^*) . Finally, we fix the target level of the government-debt-to-GDP ratio, \bar{b}_g (\bar{b}_g) at 60% which corresponds to the value envisaged in the Treaty of Maastricht on the European union. 11 We follow Kollmann et al. (2015) and Andrés et al. (2010) and set the parameter measuring the responsiveness of government transfers to deviations from the public debt target, T_1 (T_1^*), to 0.01. This value allows for sufficiently persistent fluctuations in the public-debt-to-GDP ratio while, at the same time, retaining the local stability and uniqueness of the equilibrium in our model. Table 1 summarizes our calibration.

3.2 Data and Priors

In order to estimate the remaining free parameters of the model, we use data for the period 1995:Q1-2015:Q2 on 22 macro-economic variables. In particular, we resort to the following series for both, Germany and the Euro Area as a whole: (1) log of real GDP, (2) log of real value added in the tradables sector, (3) log of real private consumption, (4) log of real gross fixed capital formation, (5) log of real government consumption, (6) log of real government investment, (7) log of overall working hours, (8) log of working hours in the tradables sector, (9) consumer price inflation, (10) aggregate inflation in hourly wages. Further, our data set also employs several variables which are not specific to any of the subregions of the currency union: (11) a short term nominal interest rate in the Euro Area, (12) nominal 3-month Treasury Bill rate in the US.

Variables (1), (3)-(5), (7) and (9) are taken from the quarterly national accounts provided by EUROSTAT. To compute the growth rate of the overall hourly wage, (10), we first divide nominal gross wages and salaries by aggregate working hours and compute the growth rate of the resulting series. (2) and (8) stem from EUROSTAT's sectoral data which can be found in the section "Basic breakdowns

¹¹See for example http://eur-lex.europa.eu/legal-content/EN/TXT/?uri=uriserv:xy0026

 Table 1: Calibrated Parameters

	Home		Foreign	
Meaning	Symbol	Value	Symbol	Value
Discount factor	β	0.998	eta^*	0.998
Intertemporal elast. of substitution	σ	2	σ^*	2
Consumption Habits	γ	0.7	γ^*	0.7
Inverse of Frisch elasticity of labor supply	arphi	1.5	$oldsymbol{arphi}^*$	1.5
Capital depreciation rate in steady state	δ	0.025	δ	0.025
Foreign assets: adjustment costs	ψ	0.0019	ψ^*	0.0019
Labor demand elasticity	ϕ	3	ϕ^*	3
Share of non-tradables in consumption	Λ	0.56	\wedge^*	0.56
Share in tradables bundle (domestic goods)	$lpha_1$	0.6	$lpha_1^*$	0.6
Share in tradables bundle (REA/Germany)	$lpha_2$	0.15	$lpha_2^*$	0.09
Share in tradables bundle (ROW goods)	$lpha_3$	0.22	$lpha_3^*$	0.28
Share in tradables bundle (oil)	$lpha_4$	0.03	$lpha_3^*$	0.03
Share of oil in production	$1-\wp$	0.024	$1-\wp^*$	0.024
Subst. between tradables and non-tradables	Z	0.5	z^*	0.5
Subst. between home and foreign goods	η	1.1	η^*	1.1
Subst. btw. oil and other input factors	ξ	0.2	ξ*	0.2
Elast. in production function	a_T	0.43	a_T^*	0.43
Elast. in production function	a_N	0.29	a_N^*	0.29
Elast. in production function	$a_{g,T}, a_{g,N}$	0.1	$a_{g,T}^*$, $a_{g,N}^*$	0.1
Elasticity of capital depreciation rate	ψ_u	1.3	ψ_u^*	1.3
Markup tradables	mu_T	1.16	mu_T^*	1.18
Markup non-tradables	mu_N	1.53	mu_N^*	1.55
Tax rate: capital	$ au_k$	0.2	$ au_k^*$	0.2
Tax rate: consumption	$ au_{\scriptscriptstyle C}$	0.19	$ au_k^* \ au_c^* \ au_g^* \ au_w^*$	0.22
	$ au_g$	0.2	$ au_g^*$	0.2
Tax rate: labor	$ au_{\scriptscriptstyle W}$	0.37		0.41
Tax rate: employer	$ au_{wf}$	0.25	$ au_{wf}^*$	0.38
Government debt to GDP ratio	b_g	0.6	b_g^*	0.6
Labor in steady state (tradables)	N_T	0.13	N_T^*	0.10
Labor in steady state (non-tradables)	N_N	0.20	N_N^*	0.23

of main GDP aggregates and employment" and particularly in the sub-sections "Gross value added and income A10 breakdowns" and "Employment A10 breakdowns". The series for real value added in the tradables sector, (2), is constructed as follows. First, we compute the growth rate of tradables value added as a weighted average over the corresponding growth rates in agriculture and industry (excluding construction). The weighting factors are based on previous quarter's nominal value added. Then the resulting growth rate is used to construct an index of real value added in the tradables sector. Working hours in this sector, (8) are obtained by simply adding up working hours in industry and agriculture. In the case of government investment, (6), we proceed as follows: data on nominal gross fixed capital formation by the general government is taken from EUROSTAT's section "Quarterly Government Statistics/ Quarterly Nonfinancial Accounts for General Government". Then, for both, Germany and the Euro Area, the nominal series are divided by the corresponding country-specific deflator of aggregate gross fixed capital formation. We proxy the monetary policy rate in the Euro Area (variable (11)) by the so called "shadow rate" provided by Leo Krippner (Krippner, 2013, 2014). The shadow rate is derived from a term structure model. It is designed to better capture the true stance of monetary policy, especially in periods characterized by a combination of a (nearly) binding zero lower bound constraint on nominal short term interest rates on the one hand, and sizable unconventional quantitative measures adopted by the central bank on the other. The extraordinary expansionary stance of monetary policy in such episodes is most likely not properly reflected in standard policy or short term rates due to the binding zero lower bound. The nominal 3-moth Treasury Bill rate in the United States (variable (12)) is taken from the International Finance Statistics database of the IMF.

Since the model is stationary in the relative deviations form the respective steady state levels, all observable variables need to be transformed accordingly. In particular, all real variables are detrended by a linear trend while the various inflation and nominal interest rates are simply demeaned. The estimation procedure is standard and can be summarized as follows. First, the solution of the linearized model is written in state space form which, in turn, is extended by including an observation equation linking the predetermined state variables to observable variables. Second, the Kalman filter is used to construct the likelihood function. Third, based on Bayes' theorem, the posterior kernel function is set up by combining the likelihood function with the prior distributions of the model parameters. Fourth, the posterior kernel is maximized with respect to the parameters. Finally, to numerically evaluate the posterior distributions of the parameters as well as other moments of interest, we resort to the Metropolis Hastings (MH) sampling algorithm with four Markov-Chains. Herman deviations are developed to the parameters as well as other moments of interest, we resort to the Metropolis Hastings (MH) sampling algorithm with four Markov-Chains.

¹²See Smets and Wouters (2002, 2007) for a very similar approach regarding data transformations.

¹³The particular optimization algorithm we employ is the Covariance Matrix Adaptation Evolution Strategy included in DYNARE.

¹⁴The MH method belongs to a broader class of Monte-Carlo-Marcov-Chain (MCMC) algo-

We apply Bayesian techniques for mainly two reasons. First, the specification of prior distributions over the parameters makes the optimization algorithm more stable. This is especially important when working with relatively short samples of data as in the current paper (Fernandez-Villaverde and Rubio-Ramirez, 2004; Fernández-Villaverde et al., 2016). Second, the Bayesian approach provides a framework for model evaluation and comparison even in the case of misspecified and/or non-nested models (Schorfheide, 2000; Rabanal and Rubio-Ramirez, 2005). This is done on the basis of the marginal likelihood. An additional advantage of the Bayesian approach is that the posterior distribution provides a complete characterisation of shock and parameter uncertainty (Fernández-Villaverde et al., 2016).

Tables 2, 3 and 4 give an overview of our assumptions regarding the prior distribution of the non-calibrated model parameters. Our choice of priors strictly follows the standards in the related DSGE literature. In particular, the variances of the shocks are assumed to have an Inverse-Gamma distribution with a degree of freedom equal to 2. This distribution guarantees a positive variance with a sufficiently large domain. The parameters which should only take values between 0 and 1, like the autocorrelation coefficients of the shock processes and the various price and wage stickiness parameters are assumed to follow a Beta distribution. The latter covers the range between 0 and 1. The remaining parameters, i.e. those appearing in the monetary policy rule and the adjustment-cost function for capital are assumed to be normally distributed. The standard errors were set such that the domain covers a reasonable range of possible values. Moreover, the relatively large number of shocks helps to avoid singularity problems and allows for a better fit between the theoretical and the empirical unconditional data moments.

3.3 Results

Overall, the observable series turn to be quite informative with respect to the estimated parameters as suggested by the substantial and in many cases significant deviations between the respective prior and the posterior distributions (see Tables 2, 3 and 4). Moreover, as measured by the marginal likelihood (predictive data density), the DSGE model seems to explain the evolution of the observable series similarly well as Bayesian vector autoregressive (BVAR) models run on the same data set and based on a dummy observations prior. Such models are frequently used in forecasting as numerous studies have documented their desirable properties in terms of data fit and short run forecasting power. In particular, the DSGE model, although being outperformed by the BVAR (2), BVAR (3) and BVAR(4), does an equally good job as the BVAR(1) model (see Table 5). The better forecasting

rithms used to numerically recover posterior distributions. We parameterize the MH with a scale factor equal to 0.35 which ensures an acceptance rate of around 0.25. Further, we simulate four Markov-Chains, each consisting of 100 000 draws. This turns to be sufficient to ensure convergence of the Markov-Chains according to the univariate and multivariate statistics proposed by Brooks and Gelman (1998).

Table 2: Priors and Posteriors: Price and Wage Stickiness, Capital Accumulation and Monetary Policy

	Prior			Posterior		
Parameter	Shape	Mean	SD ¹ or DF ²	Mean	Lower ³	Upper ³
Calvo (Prices):						
$ heta_{\mathcal{T}}$	Beta	0.7	0.05	0.65	0.60	0.69
$ heta_{\mathcal{T}}^*$	Beta	0.7	0.05	0.63	0.59	0.66
$ heta_N$	Beta	0.7	0.05	0.67	0.62	0.70
$ heta_{N}^{*}$	Beta	0.7	0.05	0.74	0.70	0.81
Indexation (Prices):						
χ_{T}	Beta	0.5	0.15	0.27	0.12	0.41
$\chi_{\mathcal{T}}^*$	Beta	0.5	0.15	0.16	0.07	0.26
XN	Beta	0.5	0.15	0.31	0.17	0.52
χ_N^*	Beta	0.5	0.15	0.22	0.11	0.32
Calvo (Wages):						
$\theta_{w,T}$	Beta	0.7	0.05	0.49	0.40	0.52
$ heta_{w, au}^*$	Beta	0.7	0.05	0.63	0.57	0.73
$\theta_{w,N}$	Beta	0.7	0.05	0.69	0.61	0.74
$\theta_{w,N}^*$	Beta	0.7	0.05	0.75	0.70	0.82
Indexation (Wages):						
$\omega_{w,T}$	Beta	0.5	0.15	0.41	0.22	0.64
$\omega_{w.T}^*$	Beta	0.5	0.15	0.22	0.10	0.35
$\omega_{w,N}$	Beta	0.5	0.15	0.24	0.10	0.38
$\omega_{w,N}^*$	Beta	0.5	0.15	0.16	0.07	0.23
Adjustment Costs (Capital):						
ξ_q	Normal	3	0.9	5.61	4.02	6.04
ξ_q^*	Normal	3	0.9	6.46	6.16	8.83
Monetary Policy:						
ϕ_r	Beta	8.0	0.1	0.87	0.79	0.91
ϕ_{π}	Normal	1.5	0.2	1.42	1.31	1.52
$\phi_{\scriptscriptstyle \mathcal{Y}}$	Beta	0.2	0.15	0.15	0.08	0.24

¹⁾ SD - standard deviation; 2) DF - degrees of freeedom; 3) Upper and Lower - 90% bounds of the posterior distribution

Table 3: Priors and Posteriors: Autocorrelation of Shocks

	Prior			Posterior			
Parameter	Shape	Mean	SD ¹ or DF ²	Mean	Lower ³	Upper ³	
Preference:							
$ ho_u$	Beta	0.85	0.1	0.85	0.71	0.95	
$ ho_u^*$	Beta	0.85	0.1	0.89	0.78	0.99	
Labor Supply:							
	Beta	0.85	0.1	0.91	0.82	0.97	
$ ho_n \ ho_n^*$	Beta	0.85	0.1	0.91	0.78	0.92	
۲n	Deta	0.00	0.1	0.00	0.10	0.52	
Capital Accumulation:							
$ ho_{T,q}$	Beta	0.85	0.1	0.92	0.83	0.98	
$ ho_{T,q}^*$	Beta	0.85	0.1	0.90	0.79	0.98	
$ ho_{N,q}$	Beta	0.85	0.1	0.80	0.71	0.89	
$ ho_{N,q}^*$	Beta	0.85	0.1	0.73	0.61	0.81	
Technology:							
$ ho_{T,a}$	Beta	0.85	0.1	0.87	0.78	0.97	
$ ho_{T,a}^*$	Beta	0.85	0.1	0.98	0.97	0.99	
$ ho_{N,a}$	Beta	0.85	0.1	0.76	0.68	0.87	
$ ho_{N,a}^*$	Beta	0.85	0.1	0.86	0.75	0.97	
Cost Push (Prices):							
$\rho_{T,p}$	Beta	0.85	0.1	0.52	0.36	0.60	
$ ho_{T,p}^{*}$	Beta	0.85	0.1	0.72	0.61	0.85	
$ ho_{N,p}$	Beta	0.85	0.1	0.46	0.35	0.60	
$ ho_{N,p}^*$	Beta	0.85	0.1	0.68	0.59	0.77	
<i>™ N</i> , <i>p</i>	Deta	0.00	0.1	0.00	0.03	0.11	
Government Consumption:							
$ ho_g$	Beta	0.85	0.1	0.70	0.61	0.79	
$ ho_g^*$	Beta	0.85	0.1	0.77	0.65	0.87	
Government Investment:							
$ ho_{i,g}$	Beta	0.85	0.1	0.87	0.80	0.96	
$ ho_{i,g}^*$	Beta	0.85	0.1	0.80	0.73	0.88	
Monetary Policy:							
,	Beta	0.85	0.1	0.75	0.64	0.85	
$ ho_{ m extsf{v}}$	Deta	0.03	0.1	0.75	0.04	0.03	
Rest of the World:							
$ ho_{\pi,o}$	Beta	0.85	0.1	0.18	0.11	0.29	
$ ho_{i,w}$	Beta	0.85	0.1	0.93	0.86	0.97	
$ ho_{\pi,w}$	Beta	0.85	0.1	0.71	0.62	0.82	
$\rho_{c,w}$	Beta	0.85	0.1	0.83	0.78	0.92	

¹⁾ SD - standard deviation; 2) DF - degrees of freeedom; 3) Upper and Lower - 90% bounds of the posterior distribution

Table 4: Priors and Posteriors: Standard Errors of Shocks

		Prior			Posterio	 r
Parameter	Shape	Mean	SD ¹ or DF ²	Mean	Lower ³	Upper ³
Preference:						
σ_u	Inv Gamma	0.02	2	0.11	0.04	0.16
σ_u^*	Inv Gamma	0.02	2	0.22	0.017	0.28
Laban Complex						
Labor Supply:	Inv. Commo	1	2	1.06	0.60	1 17
σ_n	Inv Gamma Inv Gamma	1 1	2 2	1.06 0.44	0.68 0.26	1.47 0.69
σ_n^*	IIIV Gallilla	1	2	0.44	0.20	0.09
Capital Accumulation:						
$\sigma_{T,q}$	Inv Gamma	0.03	2	0.27	0.16	0.38
$\sigma_{T,q}^*$	Inv Gamma	0.03	2	0.18	0.12	0.21
$\sigma_{N,q}$	Inv Gamma	0.03	2	0.11	0.06	0.21
$\sigma_{N,q}^*$	Inv Gamma	0.03	2	0.07	0.02	0.15
Technology:						
$\sigma_{T,a}$	Inv Gamma	0.4	2	0.68	0.50	0.86
$\sigma_{T,a}^*$	Inv Gamma	0.4	2	0.56	0.47	0.62
$\sigma_{N,a}$	Inv Gamma	0.4	2	0.47	0.40	0.54
$\sigma_{N,a}^*$	Inv Gamma	0.4	2	0.15	0.10	0.21
N,a	iiiv Gaiiiiia	0.1	2	0.15	0.10	0.21
Cost Push (Prices):						
$\sigma_{T,p}$	Inv Gamma	0.03	2	0.61	0.52	0.70
$\sigma_{T,p}^*$	Inv Gamma	0.03	2	0.31	0.26	0.35
$\sigma_{N,p}$	Inv Gamma	0.03	2	0.30	0.24	0.35
$\sigma_{N,p}^*$	Inv Gamma	0.03	2	0.18	0.12	0.21
Government Consumption:	Inv Gamma	0.15	2	0.39	0.35	0.45
$\sigma_g \ \sigma_a^*$	Inv Gamma	0.15	2 2	0.39	0.33	0.43
σ_g	IIIV Gaiiiiia	0.13	2	0.17	0.13	0.19
Government Investment:						
$\sigma_{i,q}$	Inv Gamma	0.15	2	0.39	0.33	0.44
$\sigma_{i,g}^*$	Inv Gamma	0.15	2	0.18	0.11	0.24
Monetary Policy						
Monetary Policy:	Inv Gamma	0.015	2	0.02	0.01	0.03
$\sigma_{\scriptscriptstyle V}$	IIIV Gallilla	0.013	2	0.02	0.01	0.03
Rest of the World:						
$\sigma_{\pi,o}$	Inv Gamma	0.15	2	2.12	1.71	2.64
$\sigma_{i,w}$	Inv Gamma	0.15	2	0.42	0.37	0.52
$\sigma_{\pi,w}$	Inv Gamma	0.15	2	0.31	0.22	0.42
$\sigma_{c,w}$	Inv Gamma	0.15	2	0.87	0.74	1.03

¹⁾ SD - standard deviation; 2) DF - degrees of freeedom; 3) Upper and Lower - 90% bounds of the posterior distribution

Table 5: Model Comparison

Model	Log Marginal Likelihood
DSGE (Laplace approximation)	2242.15
DSGE (Harmonic Mean)	2248.31
BVAR(1)	2139.03
BVAR(2)	2573.18
BVAR(3)	2788.81
BVAR(4)	2915.47

ability of the BVARs with more than one lag is barely surprising as documented in the literature. 15

Furthermore, our posterior estimates point towards a very modest structural heterogeneity between Germany and the REA. Strictly speaking, the difference between the two economies is statistically significant only with respect to a handful of parameters. First, the posterior distributions of ξ_q and ξ_q^* suggest that in the REA it is relatively more difficult and costly to adjust the capital stock. Second, the shocks to the degree of competitiveness (cost-push shock) in German goods markets seem to be significantly less persistent than their respective counterparts in the rest of the Euro Area (see block "Cost Push (Prices)" in Table 3). Finally the volatility of cost-push shocks in the tradables sector $\sigma_{T,p}$ ($\sigma_{T,p}^*$), the volatilities in the TFP process in the non-tradables sector $\sigma_{N,a}$ $(\sigma_{N,a}^*)$ and the volatility of government spending shocks $\sigma_{T,g}$ ($\sigma_{T,g}^*$) and $\sigma_{i,g}$ ($\sigma_{i,g}^*$) turn to be relatively higher in Germany. The cross country differences with respect to the remaining parameters are not statistically significant. Nevertheless, there is some tendency for German nominal prices and wages to be slightly more flexible than those in he REA. In particular, the frequency of nominal wage adjustments and the degree of indexation to past inflation is relatively higher in Germany (see blocks "Indexation (Prices)" and "Indexation (Wages)" in Table 2).

4 Conclusion

The main objective of this paper is to estimate a medium scale DSGE model for a union consisting of Germany and the Rest of the Euro Area (REA). The model can be used as a tool for the conduct of forecasts as well as policy simulations and evaluations. The theoretical framework developed here has several advantages relative to purely statistical approaches: In particular, the DSGE model uncovers much more details about the precise economic mechanisms underlying a particular

¹⁵See for example Schorfheide (2000), Fernandez-Villaverde and Rubio-Ramirez (2004).

forecast or the aggregate effects of a policy intervention. In addition, policy simulation or conditional forecasts undertaken within a DSGE set-up are relatively less prone to the Lucas critique and thus, more reliable.

The model is estimated based on aggregate date for Germany and the Euro Area as a whole by using Bayesian techniques. A comparison based on marginal likelihoods (predictive densities) shows that the DSGE model fits the observable macroeconomic time series similarly well as unrestricted Bayesian VARs (BVARs) estimated on the same data set. Furthermore, the parameter estimates indicate that there is a modest degree of structural heterogeneity between Germany and the Rest of the Euro Area. In particular, Germany is relatively more flexible in adjusting its capital stock and faces more volatile shocks to government spending and to the degree of competition in goods markets than the REA. Furthermore, nominal prices wages appear to be slightly more flexible in Germany than in the REA.

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