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A Theory of Child Marriage

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Abstract

The practice of early marriage for women is prevalent in developing countries around the world today, and is believed to cause significant disruption in their accumulation of human capital. This paper develops an overlapping generations model of the marriage market to explain how the practice may be sustained in the absence of any intrinsic preference for young brides. We assume there is a desirable female attribute, relevant for the gains from marriage, that is only noisily observed before a marriage is contracted. We show that, in equilibrium, its prevalence declines with time spent on the marriage market and, thus, age can signal poorer quality and, consistent with the available evidence, require higher marriage payments. Model simulations for the case of Bangladesh show that (i) an intervention that raises the opportunity cost of marriage for adolescent girls can make it more and more attractive for future cohorts to postpone marriage such that its long-term impact on marriage and subsequent life choices may well exceed the impact on the first cohort which is exposed to it; (iii) a small-scale randomised control trial of the same intervention would significantly under-estimate its efficacy by failing to capture equilibrium effects.

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1 Introduction

Child marriage and the marriage of young adolescents remains prevalent in many parts of the world despite repeated efforts by national governments and international development agencies to discourage and end the practice. According to the State of World Population Report 2005, 48 per cent of women in Southern Asia, and 42 per cent of women in Africa in the age group 15-24 years had married before reaching the age of 18 (UNFPA 2005). Across all developing regions, one-third of women aged 20-24 were married or in a union before the age of 18 during the period 2000-2011 (UNFPA 2012). A large literature argues that early marriage is likely to cause disruption in the accumulation of human capital, due to early school drop-out, withdrawal from labour markets, and the adverse effects on health from early childbearing (literature reviewed by Jensen and Thornton 2003; UNFPA 2012; UNICEF 2001).

In recent years, there has been renewed efforts from national governments and transnational bodies to address the issue of child marriage. In July 2015, the United Nations Human Rights Council unanimously adopted a resolution to "eliminate child, early and forced marriage" and the Sustainable Development Goals specifically includes the elimination of child marriage as one of its targets (5.3) within the broader goal of gender equality.1 International organisations, and NGO’s have invested in developing interventions that raise awareness about the negative consequences of child marriage, that provide parents incentives to postpone marriage for their children, and that provide adolescents new opportunities to acquire skills and alternatives to a traditional path of early marriage and early motherhood.2

This paper develops an overlapping generations model of the marriage market to explain how the practice may be sustained in the absence of any intrinsic preference for young brides. We assume there is a desirable female attribute, relevant for the gains from marriage, that is only noisily observed before a marriage is contracted. We show that, in equilibrium, its prevalence declines with time spent on the marriage market and, thus, age can signal quality. Therefore, young potential brides, aware that they will be perceived as being of poorer quality the longer they remain on the marriage market, have an incentive to accept an offer of marriage sooner rather than later. The unobserved attribute for potential brides is a formalisation of the phenomenon that, in patriarchal societies, the 'honour' of a family

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1 http://www.reuters.com/article/2015/07/02/us-womensrights-un-resolution-idUSKCN0PC25O20150702


2 Notable examples include Brac’s Adolescent Development Programme in Bangladesh, which provides livelihood training courses, education to raise awareness on social and health issues, and clubs to foster socialisation and discussion among peers; and the Berhane Hewan project in Amhara, Ethiopia, a joint initiative between the New York based Population Council and the Amhara regional government, which uses community dialogue, and simple incentives involving school supplies to encourage delayed marriage and longer stay in school for girls. The World Bank, the UK Department for International Development, and the Nike Foundation are providing support to a number of similar projects around the world.
is strongly linked to the ‘purity’ of its female members, and experiences or associations that a girl may have outside of the paternal home can create uncertainty regarding her ‘purity’ (Otner 1978).

In the marriage market equilibrium, older brides have worse reputation and weaker outside options than young brides and therefore they pay a higher net marriage transfer. This is consistent with the empirical evidence that delaying marriage carries a penalty on the marriage market in the form of higher dowry payments at the time of marriage (Amin and Bhajracharya, 2011; Field and Ambrus, 2008).

Large-scale interventions of the kind discussed earlier provide adolescent girls with opportunities other than marriage. By improving their outside options, these interventions would impact upon the marriage market in two ways: (i) a girl (or her family) receiving a marriage offer can negotiate a more favourable marriage transfer (i.e. higher brideprice or lower dowry), which makes it less attractive for men to seek young brides in the first place; (ii) some adolescent girls may turn down marriage offers altogether to pursue these opportunities. Both effects would attenuate the signal of ‘bad quality’ associated with older women in the marriage market, causing more men to seek older brides and potentially more young women to decline offers of marriage in the next period. Consequently, the reputation of older women on the marriage market improves further the next period and the cycle continues.

Thus, expanding non-marriage related opportunities for adolescents can trigger a virtuous cycle of marriage postponement even if the original intervention is not sufficient, in itself, to persuade all women (and their families) to turn down offers of marriage. The fact that an intervention targeted at adolescent girls can make it more and more attractive for future cohorts to postpone marriage means that the long-term impact of such interventions on marriage and subsequent life choices may well exceed the impact on the first cohort which is exposed to it.

We illustrate this argument and provide a measure of its magnitude using the case of Bangladesh, which has one of the highest rates of female early marriage today. We solve for key parameters of the model using a number of moment-matching conditions derived from the demographic situation in the country during the 1970’s (before the country experienced dramatic changes in fertility and the education and labour force participation of women). The parameter values thus obtained suggest that the reputational effects described above play an important role in the marriage market in Bangladesh.

We show that an initiative that increased the opportunity cost of early marriage for women in Bangladesh would trigger a continuous decline in its incidence as per the reasoning above. An increase by half a standard deviation explains about 60% of the decline in early marriage experienced by cohorts of women born in the 1980’s. But the first cohort exposed to the initiative would experience only between one-third and two-third of this decline. Second, we show that a small-scale randomised control trial of the same initiative would fail
to achieve marriage market equilibrium changes of the full-scale intervention and, therefore, significantly under-estimate its potential for lowering the incidence of early marriage.

Within the field of economics, there is a rich literature explaining the phenomenon that in most societies, husbands are typically older than their wives. For example, Bergstrom and Bagnoli (1993) argue that the age gap in marriage is due, at least in part, to the fact that the individual characteristics which traditionally determine one’s desirability as a marriage partner are revealed or realised at a later age for men than for women. Coles and Francesco (2007) postulate that economic success (which increases with age) and physical health (which declines with age) are complementary for the gains realised from a marriage. This would lead to a pattern of age gap between marriage partners and – given that labour market opportunities have traditionally been more restricted for women – the tendency for older men to marry younger women.

Although child marriage may translate into a significant age gap between the marriage partners, it is important to recognise the two phenomena as being distinct. Unlike the marriage age gap, child marriage is specific to certain societies, varies across regions and can evolve quickly over time. Therefore, the phenomenon of child marriage merits a theory of its own, which is the aim of this paper.

A recent literature on marriage practices around the world has explored the possibility that certain, potentially harmful, practices can be maintained as self-sustaining equilibria. Mackie (1996) applies the concept of Shelling’s focal point to explain why the practice of footbinding in China had survived over a thousand years, and how it came to an end very quickly at the beginning of the 20th century. Mackie (2000) and Mackie and LeJeune (2009) applies the same game-theoretic framework to the practice of female circumcision and argues that it may be undergoing a similar transition in West Africa now, at the beginning of the 21st century. In a similar spirit, this paper investigates whether the practice of child marriage can be sustained as a self-sustaining equilibrium in the absence of intrinsic preferences for young brides; and if so, under what conditions could such an equilibrium unravel.

The remainder of the paper is organised as follows. Section 2 provides a brief discussion of sociological theories of early marriage and their relation to that proposed in this paper. The model of the marriage market is introduced and analysed in Section 3. Section 6 extends the model to incorporate heterogenous agents. The model is used to conduct numerical analysis and derive implications for the case of early marriage in Bangladesh in Section 7. Conclusions are provided in Section 8.

2 Sociological Explanations of Early Marriage

In his well-known study of marriage patterns across the world, Jack Goody has highlighted a number of reasons why young brides are preferred in traditional societies: they have a
longer period of fertility before them; and they are more likely to be obedient and docile, necessary qualities to learn and accept the rules and ways of her new household (Goody, 1990). Relatedly, Dixon (1971) attributed the historic practice of early marriage in China, India, Japan and Arabia to the prevalence of ‘clans and lineages’ which gave economic and social support to newly married couples, as well as pressures to produce children for strengthening and sustaining the clan. By contrast, the traditional emphasis on individual responsibility in ‘Western family systems’ meant that newly married couples were expected to be able to provide for themselves and their children, which ‘necessarily causes marital delays while the potential bride and groom acquire the needed skills, resources and maturity to manage an independent household’ (Dixon 1971).

A third explanation stems from the association made, in patriarchal societies, between family honour and female ‘purity’ (Otner 1978). Moghadam (2004) notes that, in these societies, the honor of women ‘and, by extension, the honor of their family depends in great measure on their virginity and good conduct’. Clans preserve the ‘honor’ of their women by keeping them in seclusion (the practice of purdah). as well as marrying young brides who are less likely to have made associations or had experiences that would cast doubt regarding their ‘purity’. Kandiyoti (1988) describes the early marriage of women as a phenomenon particular to ‘classic patriarchy’ with the husband or her natal family responsible for protecting her ‘honor’ depending on the extent to which the society practises endogamy.

The first two of the explanations of early marriage above do not, in themselves, explain why the age of marriage for women has remained low in some societies despite declines in fertility, and evolution towards a nuclear household model. The theoretical model in this paper provides a mechanism and rationale for the practice of early marriage which is linked to the third explanation above. We postulate that there is uncertainty about the ‘purity’ of an adolescent girl from the time that she reaches physical maturity and has increased mobility in her environment, and this creates informational asymmetry in the marriage market.

3 A Model of Marriage Timing

The model follows the literature on dynamic search and matching initiated by Diamond and Maskin (1979). The framework has previously been used to investigate a number of marriage-related phenomena. For example, Bergstrom and Bangoli (1993) and Coles and Francesco (2007) develop matching models to explain the age gap between husbands and wives; Anderson (2007) investigates the relationship between population growth and dowry prices and Bhasker (2015) looks at the implications of a demographic transition for the marriage market and, in particular, the age gap at marriage. Bhaskar (2011) shows how the possibility of sex selection at birth would lead to a congestion externality on the marriage market. Sautmann (2011) provides a characterisation of the marriage payoff functions which
would lead to commonly observed marriage age patterns around the world, including positive assortative matching in age and a positive age gap between grooms and brides.

Since our key interest is the question of marriage timing, we make certain simplifying assumptions vis-a-vis the existing literature on marriage markets; assumptions which, we will argue, are reasonable given contemporary marriage patterns in South Asia. We abstract away from any intrinsic age-related preferences on the marriage market and focus, instead, on the imperfect observability of the characteristics of potential marriage partners to explain the phenomenon of early marriage.

In our theoretical model, being ‘young’ corresponds to the period between the onset of physical maturity for girls and active employment for men, roughly between the ages of 14 and 18. We follow Bergstrom and Bagnoli (1993) in assuming that men’s economic capabilities are only revealed after they enter the workforce while women’s abilities in their traditional tasks – childbearing, child care, and household labour – are fully known by the time they reach physical maturity. But, unlike Bergstrom and Bagnoli (1993), we also assume that there is a perceived unobserved characteristic for girls while ‘young’ which the ethnographic literature calls ‘honour’ or ‘purity’. In the context of Bangladesh, the anthropologist Rozario (1992) notes that

"Many ... parents prefer to have their daughters marry as young as possible. About 15-16 years old is seen as ideal, while 18 years is considered too old, particularly if a girl begins to visit friends and neighbours outside the household and thereby caste doubt on her purity" p.140

The quotes indicates that ‘purity’ is seen as relevant for a potential bride’s suitability for marriage, and that it is regarded as an uncertain quality for ‘young’ women (as defined above). Dyson and Moore (1983) provides a stylised description of the north Indian kinship system which highlights similar concerns and the reasons behind it:

"Groups of patrilineally related males would have their honor, reputation, and consequently their power undermined should the chastity of their females – so crucial to the formation of alliances and the production of heirs – be subverted. Thus, the sexuality of females is very rigidly controlled." p. 44
In *Arguing with the Crocodile*, Sarah White provides a detailed ethnographic account of marriage practices in rural Bangladesh. She notes that it is customary for the groom’s family to initiate contact with the bride’s family, and that this contact is made through a ‘matchmaker’ who provides a communication line, and facilitates negotiations, between the two parties (White 1992). She notes that "the different parties manouvre and fight their own corners, aiming to achieve the best bargain they can" which suggests that bargaining is a key element of the negotiation process. Moreover "the many interests involved in the making of a marriage undoubtedly color the evidence given. Mismatches occur not only through lack of information, but also through deliberate deception", which suggests that each party has limited information about the potential match, and that they may choose not to disclose information about their daughter or son that may be regarded as a defect on the marriage market. Therefore, "... attention ... is often focused primarily on the wealth of the household, and the amount of dowry which is demanded or offered" (White 1992).

In line with this description, we model the matching process in the marriage market, as one that is initiated by the potential groom (or his family), via a match-maker, as discussed in the next section. Furthermore, we assume that both the potential bride and groom have unobserved characteristics relevant for the marriage which may be revealed through a ‘background check or inferred from observable characteristics such as their age. The marriage transfer is agreed upon through bargaining between the two parties. There is no remarriage market; i.e. once a bride and groom are married, they have no possibility of re-entering the marriage market to seek a new partner. And women remain on the marriage market – i.e. are considered eligible for marriage – during a finite number of years. These last two assumptions seem justified given that, in South Asia, the incidence of separation and divorce remains very low (Dommaraju and Jones, 2011) and that most women are married by their late twenties (Mensch, Singh and Casterline, 2005).

It is important to note that the modelling assumptions made in this paper are not appropriate in a context where there is a high incidence of polygamy, endogamy, or lack of emphasis on the bride’s ‘purity’. In the case of polygamy, married individuals have the option of re-entering the marriage market. Endogamy and/or absence of a notion of ‘purity’ would reduce the type of informational asymmetry discussed above. We leave the analysis of early marriage in these settings for future work.

### 3.1 Description of the Marriage Market

The marriage market consists of four types of individuals: ‘young’ men, ‘young’ women, ‘older’ men and ‘older’ women. The number of individuals of each type at time $t$ are given by $n_{m1}(t)$, $n_{m2}(t)$, $n_{f1}(t)$ and $n_{f2}(t)$ respectively.

Two individuals of the same type are exactly alike in terms of their observable charac-
teristics. They have an unobservable characteristic which we call ‘character’, which may be either ‘good’ or ‘bad’. The prior probability of ‘bad character’ among women is \( \varepsilon_f \in (0, 1) \), a constant that is exogenously given.

The utility from marriage depends on the marital transfers involved and the ‘character’ of the partners. If a marriage between a man and a woman involves a net pre-marital transfer \( \tau \) from the bride to the groom, and the probability that the bride has ‘bad character’ is \( \varepsilon_f \), then we represent the utility to the groom by the function \( u_m(\tau, \varepsilon_f) \). Similarly, if the probability that the groom has ‘bad character’ is \( \varepsilon_m \), then the utility to the bride is given by \( u_f(\tau, \varepsilon_m) \). Note that men are assumed not to have any intrinsic preference between ‘young’ and ‘older’ brides; and women are assumed not to have any intrinsic preference between ‘young’ and ‘older’ grooms.

The matching process between prospective grooms and prospective brides operates according to the following sequence of events:

1. Men state a preference as to whether they would like to be matched with a ‘young’ woman or an ‘older’ woman. A search is then initiated by the matchmaker.

2. The probability that the matchmaker finds a match for a man seeking a young bride is given by the function \( \mu \left( \frac{n_{f1}}{n_m} \right) \) where \( \theta \) is the proportion of all men who state a preference for a young bride at stage 1, and \( n_m = n_{m1} + n_{m2} \). Similarly, the probability that the matchmaker finds a match for a man seeking an older bride is given by \( \mu \left( \frac{n_{f2}}{\left(1 - \theta\right)n_m} \right) \).

3. If a match has been found, the prospective bride and groom decide whether they will enter into a discussion regarding marriage.

4. If marriage discussions are initiated, a ‘background check’ is performed on the prospective bride. For the sake of simplicity, we assume no such technology is available to perform a ‘background check’ on the prospective groom.

5. The background check can have two possible outcomes. It may reveal nothing or it may reveal that the prospective bride has ‘bad character’. If a prospective bride has ‘bad character’, then the background check reveals her true character with probability \( \pi \in (0, 1) \).

6. The two sides decide whether the marriage should go ahead. If they decide the marriage will not take place, they remain single for the remainder of the period. The utility levels

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\(^3\)Note that, if the marriage decisions are being made by the families of the bride and groom, then these functions should be regarded as the objective functions of the families. In the model, we do not differentiate between the two. The issue of agency of parents and children in the marriage decision, an important topic in its own right, is beyond the scope of this paper.
obtained from being single during one period are denoted by $u_m$ and $u_f$ for men and women respectively.

7. If they decide to marry, then a bargaining process determines the level of pre-marital transfers, if any. Specifically, if the probability that the prospective groom and bride have ‘bad’ character are given by $\varepsilon_m$ and $\varepsilon_f$ respectively, and the utility to the man and woman from their outside options are $v_m$ and $v_f$ respectively, then the net marital transfers (from the bride to the groom) agreed upon through bargaining is given by $\tau = \xi (v_m, v_f, \varepsilon_m, \varepsilon_f)$.

When any individual marries, he or she leaves the marriage market with no possibility of re-entry. Young women and young men who fail to/choose not to marry in a particular period, re-enter the marriage market as, respectively, older women and older men in the next period. Older women and older men who fail to/choose not to marry in a particular period must remain single thereafter (i.e. they leave the marriage market). In each period, a new cohort of $y$ young women and $y$ young men reach marriageable age. To simplify the dynamics, we further assume that young men prefer not to marry (perhaps because they do not have the means to support a family till they become ‘older’).

We make the following assumptions about the functions $u_m (\cdot)$, $u_f (\cdot)$, $\mu (\cdot)$ and $\xi (\cdot)$.

**Assumption 1** $u_m (\tau, \varepsilon_f)$ is strictly increasing in $\tau$ and strictly decreasing in $\varepsilon_f$.

**Assumption 2** $u_f (\tau, \varepsilon_m)$ is strictly decreasing in $\tau$ and strictly decreasing in $\varepsilon_m$.

**Assumption 3** $\mu (\cdot)$ is strictly increasing with a range of $[0, 1]$ and $\mu (0) = 0$.

**Assumption 4** $\theta \mu \left( \frac{\theta}{\beta} \right)$ is increasing in $\theta$ for all $x$.

**Assumption 5** $\xi (v_m, v_f, \varepsilon_m, \varepsilon_f)$ is (weakly) increasing in $v_m$, decreasing in $v_f$, decreasing in $\varepsilon_m$ and increasing in $\varepsilon_f$.

**Assumption 6** $u_m + u_f > u_m (\tau, 1) + u_f (\tau, \varepsilon_m)$ and $u_m + u_f > u_m (\tau, \varepsilon_f) + u_f (\tau, 1)$ for all $\tau$.

**Assumption 7** $u_m < u_m (\tau, \varepsilon_f)$ and $u_f < u_f (\tau, \varepsilon_m)$ for some $\tau$.

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4. Thus we assume a population growth rate of 0 and a sex ratio of 1. This is done for ease of notation and the main theoretical results do not depend on these assumptions. For the numerical analysis in Section 7, we allow the population growth rate to differ from zero and the sex ratio to differ from 1.

5. In the context of Bangladesh, there is strong evidence that men rarely marry before the age of 18 unless they are engaged in an income-generating activity. For example, we find from the Bangladesh Adolescent Survey (2005) that, for those below the age of 18, 27% of never-married boys and 100% of ever-married boys are engaged in an income-generating activity.
Assumption 8 \( u_m(\xi(v_m, v_f, \varepsilon_m, \varepsilon_f), \varepsilon_f) \) is decreasing in \( \varepsilon_f \) and \( u_f(\xi(v_m, v_f, \varepsilon_m, \varepsilon_f), \varepsilon_m) \) is decreasing in \( \varepsilon_m \).

Assumption 6 implies that if either the prospective bride or groom is found to have ‘bad character’ (with no new information obtained regarding the other party), then there is no level of pre-marital transfers such that both parties would prefer marriage to singlehood). By contrast, Assumption 7 implies that if no new information is obtained regarding the prospective bride or groom, then there exits a level of pre-marital transfers such that both parties would prefer marriage to singlehood.

Assumption 4 is a restriction imposed on the function \( \mu(\cdot) \) which ensures that as the proportion of men who state a preference for a particular type of bride (either ‘young’ or ‘older’) increases, so does the number of men who are matched with that type of bride.

Assumption 8 implies that as the reputation of a prospective partner improves, so does the utility from marrying him or her, even after taking into account that a partner with a better reputation will require a higher net marriage transfer.

Future utility is discounted by a factor \( \beta \in (0, 1) \) per period. For the sake of simplicity, we assume that men and women are infinitely lived (although they remain on the marriage market for only a finite number of periods).

We denote by \( \lambda_1 \) and \( \lambda_2 \), respectively, the probability of finding a match for young women and older women on the marriage market. By construction, we must have

\[
\lambda_1 n_f = \mu \left( \frac{n_{f1}}{\theta n_m} \right) \theta n_m \tag{1}
\]
\[
\lambda_2 n_f = \mu \left( \frac{n_{f2}}{(1 - \theta) n_m} \right) (1 - \theta) n_m \tag{2}
\]

3.2 Solving the Model

Whether men prefer young brides or older brides will depend on the ‘reputation’ (i.e. the probability of having bad moral characer) of each cohort.

Background Checks: Let us denote by \( \varepsilon_{f1} \) and \( \varepsilon_{f2} \) the probability of ‘bad character’ among young and older women respectively before any background check has been performed; and, by \( \hat{\varepsilon}_{f1} \) and \( \hat{\varepsilon}_{f2} \), the probability of ‘bad character’ among young and older women respectively after a background check has been performed and the check has revealed nothing.

As there is no information available on the character of young women when they first enter the marriage market, we must have \( \varepsilon_{f1} = \varepsilon \). For older women, the probability of bad character (before a background check has been performed) depends on the likelihood that
they were matched with a man when young. Specifically, using Bayes’ rule, we obtain

\[
\varepsilon_{f2} = \frac{\Pr(\text{bad}|\text{older})}{\Pr(\text{older}|\text{bad}) \Pr(\text{bad})} = \frac{[(1 - \lambda_1) + \lambda_1 \varepsilon_f]}{(1 - \lambda_1) + \lambda_1 \varepsilon_f}
\]

(3)

From (3), it is evident that if \( \lambda_1 > 0 \) and \( \varepsilon_f > 0 \), then \( \varepsilon_{f2} > \frac{[(1 - \lambda_1) + \lambda_1 \varepsilon_f]}{(1 - \lambda_1) + \lambda_1 \varepsilon_f} = \varepsilon_f \). Therefore, if there are some women who are getting married when young, the probability of bad character is higher among older women than among young women.

Given that a background check detects ‘bad character’ only with some probability \( \pi < 1 \), a woman for whom such a check has revealed nothing, will still be assigned a positive probability of having a ‘bad character’. We can determine this probability for young and older women using Bayes’ rule as follows:

\[
\hat{\varepsilon}_{f1} = \frac{\Pr(\text{bad}|\text{young+nothing})}{\Pr(\text{young+nothing}|\text{bad}) \Pr(\text{bad})} = \frac{\Pr(\text{young+nothing})}{(1 - \pi) \varepsilon_{f1}} = \frac{(1 - \varepsilon_{f1}) + (1 - \pi) \varepsilon_{f1}}{(1 - \pi) \varepsilon_{f1}} < \frac{(1 - \pi) (1 - \varepsilon_{f1}) + (1 - \pi) \varepsilon_{f1}}{\varepsilon_{f1}}
\]

(4)

\[
\hat{\varepsilon}_{f2} = \frac{\Pr(\text{bad}|\text{mature+nothing})}{\Pr(\text{mature+nothing}|\text{bad}) \Pr(\text{bad})} = \frac{\Pr(\text{mature+nothing})}{(1 - \pi) \varepsilon_{f2}} = \frac{(1 - \varepsilon_{f2}) + (1 - \pi) \varepsilon_{f2}}{(1 - \pi) \varepsilon_{f2}} < \frac{(1 - \pi) (1 - \varepsilon_{f2}) + (1 - \pi) \varepsilon_{f2}}{\varepsilon_{f2}}
\]

(5)

So, when a background check on a woman reveals nothing, the probability that she has ‘bad character’ declines but remains positive, in case of both young women and older women.

In summary, we have established the following results.

**Summary 1** \( \varepsilon_{f1} = \varepsilon_f \) and \( \varepsilon_{f2} > \varepsilon_f \)

\( \hat{\varepsilon}_{f1} < \varepsilon_{f1} \) and \( \hat{\varepsilon}_{f2} < \varepsilon_{f2} \)

\( \varepsilon_{f2} > \varepsilon_{f1} \) and \( \hat{\varepsilon}_{f2} > \hat{\varepsilon}_{f1} \)
Marriage Transfers: At stage 7 of the sequence of events in the game, matched men and women bargain over the level of pre-marital transfers. It should be evident that all young women have the same outside option as they are indistinguishable from one another on the marriage market. Similarly, all older men have the same outside option and all older women have the same outside option.

Therefore, the level of premarital transfers agreed upon through a process of bargaining in any marriage involving a young woman (for whom a background check has revealed nothing) and an older man should be the same. We denote this level of transfers by $\tau_1$.

Similarly, the level of premarital transfers agreed upon through a process of bargaining in any marriage involving an older woman (for whom a background check has revealed nothing) and an older man should be the same. We denote this level of transfers by $\tau_2$.

Outside Options: Let us denote by $v_{f1}$ and $v_{f2}$ respectively the outside options of young women and older women on the marriage market. An older woman who does not marry in the current period will remain single hereafter. Therefore, her outside option is given by

$$v_{f2} = \zeta u_f$$

where $\zeta = \frac{1}{1-\beta}$. A young woman who does not marry in the current period will re-enter the marriage market as an older woman. She would need to consider not only whether she will find a partner when she is older, but also whether a second background check would reveal 'bad character'. Therefore, her probability of marrying as an older bride is given by $\lambda_2' = \lambda_2 (1 - \pi \varepsilon_{f2})$. Therefore, her outside option satisfies the following equation:

$$v_{f1} = u_f + \beta \zeta \left\{ \lambda_2' u_f (\tau_2, \varepsilon_m) + (1 - \lambda_2') u_f \right\} = \{1 + \beta \zeta (1 - \lambda_2')\} u_f + \beta \zeta \lambda_2' u_f (\tau_2, \varepsilon_m) \quad (6)$$

Let us denote by $v_m$ the outside option of older men on the marriage market (Since young men do not marry by assumption, we need not consider their outside options for the analysis). An older man who does not marry in the current period will remain single hereafter. Therefore, his outside option is given by

$$v_m = \zeta u_m$$

If an older man states a preference for a young bride, then a marriage with a young bride takes place with probability $\mu \left( \frac{n_{f1}}{\theta_{nm}} \right) (1 - \pi \varepsilon_{f1})$. From this outcome, he receives a continuation utility of $\zeta u_m (\tau_1, \varepsilon_{f1})$. Therefore, the expected utility to an older man from stating a preference for a young bride is given by

$$U_1 = \mu \left( \frac{n_{f1}}{\theta_{nm}} \right) (1 - \pi \varepsilon_{f1}) \zeta u_m (\tau_1, \varepsilon_{f1}) + \left[ 1 - \mu \left( \frac{n_{f1}}{\theta_{nm}} \right) (1 - \pi \varepsilon_{f1}) \right] v_m \quad (7)$$
If he states a preference for an older bride, then a marriage with an older bride takes place with probability $\mu \left( \frac{nf_2}{n_m} \right) (1 - \pi \varepsilon f_2)$. From this outcome, he receives a continuation utility of $\zeta u_m (\tau_2, \hat{\varepsilon} f_2)$. Therefore, the expected utility to an older man from stating a preference for an older bride is given by

$$U_2 = \mu \left( \frac{nf_2}{n_m} \right) (1 - \pi \varepsilon f_2) \zeta u_m (\tau_2, \hat{\varepsilon} f_2) + \left[ 1 - \mu \left( \frac{nf_2}{n_m} \right) (1 - \pi \varepsilon f_2) \right] v_m \tag{8}$$

Bargaining: Given the outside options of older men, young women and older women on the marriage market, we can write the marital transfers that would occur in the different types of marriages as follows:

$$\tau_1 = \xi (v_m, v_{f1}, \hat{\varepsilon} f_1) \tag{9}$$
$$\tau_2 = \xi (v_m, v_{f2}, \hat{\varepsilon} f_2) \tag{10}$$

### 3.3 Equilibrium in the Marriage Market

In any period, the state of the marriage market can be fully described by the number of individuals in each age-gender group and the reputation of each group. By assumption, there are $y$ young men and $y$ young women in the marriage market in each period. As we have assumed that young men do not marry, there are also $y$ older men in the marriage market each period. Therefore, the only group which can potentially vary in size over time is that of older women in the marriage market.

By assumption, the reputation of men and young women in each period are given by their prior probability of 'bad character', $\varepsilon$. Only the reputation of older women may vary over time, depending on the proportion of young women who received marriage offers in the preceding period. Therefore, the state of the marriage market can be summarised by the tuple $(nf_2, \varepsilon f_2)$.

Let us denote by $\theta_t$, the proportion of men who state a preference for young brides in period $t$. We can show that if the state of the marriage market in period $t$ is known, then $\theta_t$ suffices to calculate the state variable in the next period. We can reason as follows.

Letting $n_{f1} = n_{m2} = y$ and $\theta = \theta_t$ in (1), we obtain the fraction $\lambda_1 (t)$ of young women who are matched with men in period $t$.

$$\lambda_1 (t) y = \mu \left( \frac{y}{\theta_{ty}} \right) \theta_t y \tag{11}$$

Then, the number of older women in the marriage market in period $t + 1$ is given by

$$nf_2 (t + 1) = [1 - \lambda_1 (t) + \lambda_1 (t) \varepsilon \pi] y \tag{12}$$

The logic behind (12) is as follows: if a fraction $\lambda_1 (t)$ of young women are matched with men in period $t$, then a fraction $\lambda_1 (t) \varepsilon \pi$ will be discovered to have ‘bad character’. Therefore,
those who remain on the marriage market in the next period would include those who were not matched, numbering \((1 - \lambda_1(t))y\) and those who were found to have ‘bad character’, numbering \(\lambda_1(t)\varepsilon y\).

Similarly, using (3), the reputation of older women in period \(t + 1\) is given by

\[
\varepsilon_{f2}(t + 1) = \frac{[(1 - \lambda_1(t)) + \lambda_1(t)\pi]\varepsilon f}{(1 - \lambda_1(t)) + \lambda_1(t)\pi\varepsilon f}
\]

(13)

By Assumption 4 and (11), we see that \(\lambda_1(t)\) is increasing in \(\theta_t\). It follows, from (12) and (13), that \(n_{f2}(t + 1)\) and \(\varepsilon_{f2}(t + 1)\) are decreasing in \(\theta_t\); i.e. the number and reputation of older women in the marriage market is declining in the proportion of men who sought young brides in the preceding period.

We are now in a position to characterise equilibria in the marriage market. From equation (7), we can see that the expected utility to older men in some period \(t\), from stating a preference for young brides, \(U_1\), depends on \(\theta_t\). The outside option of young brides, and therefore the equilibrium marriage payments depend on the marriage prospects of older women in the next period. Thus, the value of \(U_1\) in period \(t\) also depends on \(\theta_{t+1}\). All other terms on the right-hand side of (7) are exogenously determined. Therefore, we can write the expected utility from stating a preference for a young bride as a function \(U_1(\theta_t, \theta_{t+1})\).

From equation (8), we can see that the expected utility to older men in period \(t\) from stating a preference for older brides, \(U_2\), also depends on \(\theta_t\). The reputation of older brides, and therefore the utility from marrying older brides depends on the proportion of young women who were married in the previous period. Thus, the value of \(U_2\) in period \(t\) also depends on \(\theta_{t-1}\). All other terms on the right-hand side of (8) are exogenously determined. Therefore, we can write the expected utility from stating a preference for an older bride as a function \(U_2(\theta_t, \theta_{t-1})\). It is straightforward to establish the following results.

**Lemma 1**

(i) Under Assumption 3, \(U_1(\theta_t, \theta_{t+1})\) is strictly decreasing in \(\theta_t\) and \(U_2(\theta_t, \theta_{t-1})\) is strictly increasing in \(\theta_t\).

(ii) Under Assumptions 1 and 5, \(U_1(\theta_t, \theta_{t+1})\) is strictly increasing in \(\theta_{t+1}\)

(iii) Under Assumption 8, \(U_2(\theta_t, \theta_{t-1})\) is strictly decreasing in \(\theta_{t-1}\)

**Proof.** See Appendix A. ■

The first part of Lemma 1 follows directly from the properties of the functions \(u_m(\cdot, \cdot)\) and \(\mu(\cdot)\). The second part of Lemma 1 has the following intuition: as the future marriage prospects of young women improve, they will require higher net marriage payments for marrying as young brides. Therefore, the expected utility to men from seeking young brides will decline. The third part of Lemma 1 has the following intuition: if a large fraction of men sought young brides in the preceding period, then the reputation of older women, and therefore the expected utility from seeking an older bride, is low in the current period.
If, in equilibrium, $\theta_t \in (0, 1)$, then we must have $U_1 (\theta_t, \theta_{t+1}) = U_2 (\theta_t, \theta_{t-1})$. If not, some men would be able to improve their expected utility by changing their age preference for prospective brides. From Lemma 1(i), we see that $U_1 (.)$ is monotonically decreasing, and $U_2 (.)$ is monotonically increasing, in $\theta_t$. Therefore there is, at most, one value of $\theta \in [0, 1]$ which satisfies the preceding equation, as in Figure 1.

Figure 1: Expected Utility to Men from Seeking Different Types of Brides

Similarly, we can reason that if $\theta_t = 0$, we must have $U_1 (0, \theta_{t+1}) \leq U_2 (0, \theta_{t-1})$ and if $\theta_t = 1$, then $U_1 (1, \theta_{t+1}) \geq U_2 (1, \theta_{t-1})$. These arguments enable us to compute the equilibrium value of $\theta_t$ whenever $\theta_{t-1}$ and $\theta_{t+1}$ are known. For this purpose, let us define function $I : [0, 1] \times [0, 1] \rightarrow [0, 1]$ as follows:
**Definition 2**

\[
I(\theta_{t-1}, \theta_{t+1}) =
\begin{cases} 
\theta & \text{if } U_1(\theta, \theta_{t+1}) = U_2(\theta, \theta_{t-1}) \text{ for some } \theta \in [0, 1] \\
0 & \text{if } U_1(\theta, \theta_{t+1}) < U_2(\theta, \theta_{t-1}) \text{ for each } \theta \in [0, 1] \\
1 & \text{if } U_1(\theta, \theta_{t+1}) > U_2(\theta, \theta_{t-1}) \text{ for each } \theta \in [0, 1]
\end{cases}
\]

Using this definition, we can establish the following result.

**Proposition 1** Given an initial value \( \theta_0 \), the sequence \( \{\theta_t\}_{t=1}^{\infty} \) constitutes a Perfect Baysian Equilibrium if and only if \( \theta_t = I(\theta_{t-1}, \theta_{t+1}) \) for \( t = 1, 2, \ldots \)

Proposition 1 provides us a convenient characterisation of all possible equilibria in the marriage market and the different ways in which the marriage age for women may evolve over time when all individuals in the marriage market are choosing their best response.

**4 Steady-State Equilibria**

Suppose there is some \( \theta \in [0, 1] \) which satisfies the following equation:

\[ I(\theta, \theta) = \theta \]

In this case, the strategy profile where a fraction \( \theta \) of men seek young brides in every period constitutes an equilibrium (when the initial value is also \( \theta \)).

**Lemma 2** There exists at least one steady-state value \( \theta \in [0, 1] \); i.e. \( I(\theta, \theta) = \theta \).

**Proof.** See Appendix A. ■

Figure 2 shows a possible plot of \( I(\theta, \theta) \) against \( \theta \). It follows from Lemma 1(i) that the curve is upward-sloping. At \( \theta = 0 \), it must lie on or above the 45-degree line and at \( \theta = 1 \), it must lie on or below the 45-degree line. From Lemma 2, we know that it must cross the 45-degree line at least once. Each value of \( \theta \) where the curve crosses the 45-degree line constitutes a steady-state equilibrium. Below we provide two examples of potential steady-state equilibria.
1. Suppose $I(1,1) = 1$. Then, there exists a steady-state equilibrium in which, in each period, all men seek young brides.

The probability that a young woman will find a match is given by $\lambda_1 = \mu \left( \frac{y}{y} \right) \frac{u}{y}$.

Older women do not receive any offers of marriage, and $\lambda_2 = 0$.

Therefore, $v_{f1} = v_{f2} = \zeta u_f$; i.e. both young and older women have the same outside option.

Older women have worse ‘reputation’ than young women: $\varepsilon_{f2} > \varepsilon_{f1}$.

Furthermore, we have

\[ U_1 = \mu \left( \frac{y}{y} \right) (1 - \varepsilon_{f1}) \zeta u_m (\tau_1, \hat{\varepsilon}_{f1}) + \left[ 1 - \mu \left( \frac{y}{y} \right) (1 - \varepsilon_{f1}) \right] v_m \]

\[ U_2 = \mu \left( \frac{\lambda y}{y} \right) (1 - \varepsilon_{f2}) \zeta u_m (\tau_2, \hat{\varepsilon}_{f2}) + \left[ 1 - \mu \left( \frac{\lambda y}{y} \right) (1 - \varepsilon_{f2}) \right] v_m \]

\[ \tau_1 = \xi (v_m, v_{f1}, \hat{\varepsilon}_{f1}) \]

\[ \tau_2 = \xi (v_m, v_{f2}, \hat{\varepsilon}_{f2}) \] (with $\hat{\varepsilon}_{f2}$ being derived from $\lambda_1$)

Since $\hat{\varepsilon}_{f2} > \hat{\varepsilon}_{f1}$, we have $\tau_1 < \tau_2$
Since $I(1, 1) = 1$, we have $U_1(1, 1) \geq U_2(1, 1)$; i.e. the men obtain a higher expected utility from seeking a young bride as opposed to an older bride.

From the point of view of potential grooms, young brides are harder to find and they are more ‘expensive’. But if the reputational effect offsets these disadvantages, we have a ‘child marriage’ equilibrium as described above. This ‘early marriage equilibrium’ is not feasible in the absence of the reputational mechanism that is central to the theoretical model. Alternatively, we could have the following situation.

2. Suppose $I(0, 0) = 0$. Then, there exists a steady-state equilibrium in which, in each period, all men seek older brides.

Young women do not receive any offers of marriage and $\lambda_1 = 0$. Therefore, $n_f = y$.

Older women receive offers of marriage with probability $\lambda_2 = \mu \left( \frac{y}{y} \right)$. Therefore, $v_f = \left\{ 1 + \beta \zeta (1 - \lambda_2) \right\} u_f + \beta \zeta_2 u_f (\tau_2, \hat{f}_2)$ and $v_f = \zeta u_f$.

Since $\lambda_2 > 0$, we have, $v_f > v_f$; i.e. younger women have a better outside option than older women.

Older women have the same reputation as young women: $\varepsilon f_2 = \varepsilon f_1$.

Furthermore, we have

\[
U_1 = \mu \left( \frac{y}{y} \right) (1 - \hat{f}_1) \zeta u_m (\tau_1, \hat{f}_1) + \left[ 1 - \mu \left( \frac{y}{y} \right) (1 - \hat{f}_1) \right] v_m
\]

\[
U_2 = \mu \left( \frac{y}{y} \right) (1 - \hat{f}_2) \zeta u_m (\tau_2, \hat{f}_2) + \left[ 1 - \mu \left( \frac{y}{y} \right) (1 - \hat{f}_2) \right] v_m
\]

\[
\tau_1 = \xi (v_m, v_f, \hat{f}_1)
\]

\[
\tau_2 = \xi (v_m, v_f, \hat{f}_2)
\]

Since $v_f > v_f$, we obtain $\tau_1 < \tau_2$.

Since $I(0, 0) = 0$, we have $U_2(0, 0) \geq U_1(0, 0)$; i.e. the men obtain a higher expected utility from seeking an older bride as opposed to a young bride.

From the point of view of potential grooms, older brides are harder to find. But if the lower expense of older brides (they require a lower net marital transfer) offsets this disadvantage, we have a delayed marriage equilibrium.

For both examples, we obtain the result that young brides make a smaller net marital transfer than older brides. Note that $U_2(0, 0) = U_1(1, 1)$. In other words, the men receive the same expected utility in the two potential steady-state equilibria. This is because, if $\theta_t = 0$, the reputation of older women in period $t$ is the same as that of young women; and, if $\theta_t = 1$, the outside option of young women in period $t$ is the same as that of older women. Therefore, from the point of view of potential grooms, older women in the $\theta = 0$
steady-state are ‘identical’ – i.e. they have the same reputation and require the same net marital transfers – as younger women in the $\theta = 1$ steady-state.

There are, potentially, other steady-state equilibria where $\theta$ takes some fixed value between 0 and 1 in each period. This would mean that a fixed proportion of men seek young brides and others seek older brides, and the expected utility from the two choices are the same: $U_2(\theta, \theta) = U_1(\theta, \theta)$. In all equilibria, young women have a better outside-option and/or a better reputation than older women. Therefore, young brides make a smaller net marital transfer than older brides. Formally, we have the following result.

**Proposition 2** In any steady-state equilibrium, $\tau_1 < \tau_2$.

**Proof.** See Appendix A. ■

Proposition 2 is consistent with the stylised fact that, for women, delaying marriage requires a higher dowry payment (Field and Ambrus 2008; Amin and Bhajracharya 2011).

5 Equilibria with Varying $\theta$

In the preceding section, we considered equilibria where a fixed proportion of men sought young brides in every period. Next, we consider a distinct class of equilibria; namely, those equilibria where this proportion changes over time. Note that although $\theta$ would change period by period, the marriage market would be in equilibrium in each period in the sense that all agents are choosing their best response, given the current state of the market and their expectation about the future. Furthermore, it is noteworthy that the marriage market can have such dynamics even though we have not, so far, introduced any external shocks or interventions.

The difficulty of analysing this class of equilibria is that they are driven by expectations about future values of $\theta$. A marriage market where there is an expectation of wide prevalence of early marriage in the future would have a different trajectory from another marriage market where there is an expectation of a wide prevalence of late marriage. In other words, we can have multiple equilibria for the same initial value of $\theta$.

Fortunately, there is one simple refinement that dramatically reduces the number of potential equilibria. The refinement is described in the following definition.

**Definition 3** Given an initial value $\theta_0$, the sequence $\{\theta_t\}_{t=1}^{\infty}$ constitutes a monotonic increasing equilibrium if $\theta_t = I(\theta_{t-1}, \theta_{t+1})$ and $\theta_t \geq \theta_{t-1}$ for $t = 1, 2, \ldots$. Given an initial value $\theta_0$, the sequence $\{\theta_t\}_{t=1}^{\infty}$ constitutes a monotonic decreasing equilibrium if $\theta_t = I(\theta_{t-1}, \theta_{t+1})$ and $\theta_t \leq \theta_{t-1}$ for $t = 1, 2, \ldots$.

This refinement provides us considerable predictive power about how the age of marriage will evolve over time, using just the initial value of $\theta_0$ and the fundamental parameters of the marriage market, as described in the following proposition.
Proposition 3  (i) Given an initial value $\theta_0$, there exists a monotonic increasing equilibrium if and only if there is a steady-state $\theta \in (\theta_0, 1]$. The equilibrium sequence $\{\theta_t\}_{t=1}^{\infty}$ converges to a steady-state $\theta \in (\theta_0, 1]$.

(ii) Given an initial value $\theta_0$, there exists a monotonic decreasing equilibrium if and only if there is a steady-state $\theta \in [0, \theta_0)$. The equilibrium sequence $\{\theta_t\}_{t=1}^{\infty}$ converges to a steady-state $\theta \in [0, \theta_0)$.

Proof. See the Appendix.

We can take an alternative approach based on the assumption that individuals are naive about their beliefs regarding the future marriage market; specifically, that their decisions in period $t$ rely on the assumption that $\theta_{t+1} = \theta_{t-1}$. This assumption is unsatisfying in the sense that people are repeatedly wrong about their future beliefs in the marriage market. However, as we shall show, their beliefs grow more and more accurate over time as $\theta$ approaches a steady-state value; moreover, it generates a unique and easily computable equilibrium path in the marriage market.

Definition 4  Given an initial value $\theta_0$, the sequence $\{\theta_\tau\}_{\tau=1}^{\infty}$ constitutes a Naive Expectations Equilibrium if, $\theta_t = I(\theta_{t-1}, \theta_{t-1})$ for $t = 1, 2, ...$

Note that a ‘Naive Expectations’ equilibrium does not satisfy the criteria for a Perfect Bayesian equilibrium. A steady-state value of $\theta$ satisfies the conditions for both the Perfect Bayesian equilibrium and the ‘Naive Expectations’ equilibrium. The advantage of a ‘Naive Expectations’ equilibrium is that the initial value $\theta_0$ uniquely determines the equilibrium path, as described in the following proposition.

Proposition 4  Given an initial value $\theta_0$, there is a unique sequence $\{\theta_\tau\}_{\tau=1}^{\infty}$ that satisfies the criteria of a ‘Naive Expectations’ equilibrium. Furthermore,

(i) if $I(\theta_0, \theta_0) > \theta_0$, then $\{\theta_\tau\}_{\tau=1}^{\infty}$ is an increasing sequence which converges to the smallest steady-state in the interval $(\theta_0, 1]$;

(ii) if $I(\theta_0, \theta_0) < \theta_0$, then $\{\theta_\tau\}_{\tau=1}^{\infty}$ is a decreasing sequence which converges to the largest steady-state in the interval $[0, \theta_0]$;

(iii) if $\theta_0 = I(\theta_0, \theta_0)$, then $\theta_\tau = \theta_0$ for $\tau = 1, 2, ...$

Proof. See the Appendix.

A particularly useful application of the Naive Expectations equilibrium is an algorithm of iterated future beliefs that generates a Perfect Bayesian equilibrium. In the first step, we construct an ‘equilibrium’ where, in period $t$, individuals behave as if the outcome in period $t+1$ would correspond to the Naive Expectations equilibrium: i.e. $E_t(\theta_{t+1}) = I(\theta_t, \theta_t)$. In the second step, we construct an ‘equilibrium’ where, in period $t+1$, individuals behave as if the outcome in period $t+1$ would correspond to that computed in the first step: i.e.
$E_t(\theta_{t+1}) = I(\theta_t, I(\theta_t, \theta_t))$. We can show that the sequence of equilibrium paths generated using this algorithm corresponds to a Perfect Bayesian equilibrium. Formally, we have the following proposition.

**Proposition 5** Suppose that the initial value $\theta_0$, together with the sequence $\{\theta_{t+1}^\alpha\}_{t=1}^\infty$ constitutes a Naive Expectations Equilibrium. Let $\theta_1^\alpha = I(\theta_0, \theta_2^\alpha - 1)$ and $\theta_{t}^\alpha = I(\theta_{t-1}^\alpha, \theta_{t+1}^\alpha)$ for $\alpha \in \{1, 2, \ldots\}$, $\tau \in \{2, 3, \ldots\}$. Let $\Theta = \{\lim_{\alpha \to \infty} \theta_{t+1}^\alpha\}_{t=1}^\infty$. The sequence $\Theta$ exists and, together with initial value $\theta_0$, constitutes a Perfect Bayesian equilibrium.

**Proof.** See the Appendix. □

Note that the Perfect Bayesian equilibrium generated via the algorithm of iterated future beliefs is not necessarily unique. But there is an intuitive appeal to it given that as people become more and more sophisticated in forming their future beliefs, the equilibrium path will converge to it. In the subsequent analysis, we use both the Naive Expectations equilibrium and the Perfect Bayesian equilibrium based on iterated future beliefs to examine how the marriage market may evolve in response to a shock.

### 6 Turning Down Marriage Offers and Agent Heterogeneity

In this section, we relax Assumption 7 and allow for the possibility that young women may turn down offers of marriage. To capture situations where some, but not all, young women may find it in their interest to turn down marriage offers, we allow the utility from remaining single while young to vary across individuals. This may be due to differences in opportunities or in preferences. Specifically, let $u_{f1} = y + \psi$ where $y$ is a constant common to all individuals and $\psi$ is distributed across individuals according to the cumulative distribution function $F(.)$.

We can define $x$ as the threshold value of the utility from singlehood at which a young woman is indifferent between accepting and refusing an offer of marriage. Then, $x$ must satisfy the following equation:

$$
(1 + \beta \zeta) u_f(\tau_1, \varepsilon_m) = x + \beta \zeta \left[ (1 - \lambda_2) u_{f2} + \lambda_2 u_f(\tau_2, \varepsilon_m) \right]
$$

where $\tau_1$ solves $u_m(\tau_1, \varepsilon_{f1}) = u_m$. Therefore, for $u_{f1} > x$, a woman would, in effect, choose to delay marriage. If $F(x - y) = 1$, then we obtain the situation described in Section 3.3. If $F(x - y) < 1$, some matches involving young women will not lead to marriage even when

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6In a similar fashion, we could extend the model to introduce heterogeneity in the wealth or preferences of men. This can lead to men with different characteristics opting for different types of brides but not impact upon the main dynamics related to early marriage. Therefore, for ease of exposition, we abstract away from groom heterogeneity in this paper.

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the background check has not signalled ‘bad character’. This has two implications for the marriage of young women on the marriage market. First, a potential groom is less likely to be matched with a young bride with whom a marriage contract can be negotiated. The probability of marriage for men who state a preference for young brides will be given by

\[ \mu \left( \frac{n_{f1}}{\theta n_m} \right) (1 - \pi \varepsilon_{f1}) F(x - y) \]

where \( \theta \), as before, is the proportion of men who seek young brides.

Second, if some women are postponing marriage when they are young, it provides an alternative reason why older women may be single (other than the two reasons discussed above: not being matched with a potential groom and being found to have ‘bad character’); and, consequently, the reputation of older women on the marriage market would improve. Specifically, it is given as follows:

\[ \varepsilon_{f2} = \frac{\Pr (\text{bad} | \text{older})}{\Pr (\text{older} | \text{bad}) \Pr (\text{bad})} \]

\[ = \frac{\Pr (\text{older} | \text{bad}) \Pr (\text{bad})}{\Pr (\text{older})} \]

\[ \implies \varepsilon_{f2} = \frac{\varepsilon_f [(1 - \lambda_1) + \lambda_1 \pi + \eta \lambda_1 (1 - \pi)]}{(1 - \varepsilon_f) [(1 - \lambda_1) + \eta \lambda_1] + \varepsilon_f [(1 - \lambda_1) + \lambda_1 \pi + \eta \lambda_1 (1 - \pi)]} \quad (15) \]

where \( \eta = 1 - F(x - y) \) denotes the proportion of young women who turned down offers of marriage in the preceding period. If \( \eta = 0 \), the expression in (15) is identical to that in (3). As \( \eta \) increases, \( \varepsilon_{f2} \) declines; i.e. there is a fall in the probability of ‘bad character’ among older women on the marriage market. In addition, the number of older women in the marriage market will go up with the proportion of young women who declined offers of marriage in the preceding period (represented by \( \eta' \) below):

\[ n_{f2} = [1 - \lambda_1 + \lambda_1 \varepsilon_f \pi + \eta' \lambda_1 (1 - \varepsilon_f \pi)] y \quad (16) \]

Equations (14) and (15) together yield, for given \( \theta_t \) and \( \theta_{t+1} \), the threshold value \( x_t = \hat{x} (y, \theta_t, \theta_{t+1}) \) above which young women turn down offers of marriage and the proportion of women who do so, \( \eta_t = 1 - F(x_t - y) \). (We suppress the parameter \( y \) in the function \( \hat{x}(.) \) hereafter in this section for ease of notation. The consequences of changing \( y \) are explored in the next section).

As per the reasoning above, the expected utility to potential grooms from seeking a young bride, in a given period \( t \), will depend on \( x_t \) while the expected utility from seeking an older bride will depend on \( x_{t-1} \) since it determines, as per (15), the reputation of older women in the current period. We can therefore represent the expected utilities by \( \hat{U}_1 (\theta_t, \theta_{t+1}, x_t) \) and \( \hat{U}_2 (\theta_t, \theta_{t-1}, x_{t-1}) \) respectively. Using these expected utility functions, we can define a function \( \hat{I}(.) \) akin to the function \( I(.) \) introduced in Section 3.3:
Definition 5

\[ \hat{I}(\theta_{t-1}, x_{t-1}, \theta_{t+1}) = \begin{cases} 
\theta & \text{if } \hat{U}_1(\theta, \theta_{t+1}, x_t) = \hat{U}_2(\theta, \theta_{t-1}, x_{t-1}) \text{ for some } \theta \in [0,1] \\
0 & \text{if } \hat{U}_1(\theta, \theta_{t+1}, x_t) < \hat{U}_2(\theta, \theta_{t-1}, x_{t-1}) \text{ for each } \theta \in [0,1] \\
1 & \text{if } \hat{U}_1(\theta, \theta_{t+1}, x_t) > \hat{U}_2(\theta, \theta_{t-1}, x_{t-1}) \text{ for each } \theta \in [0,1] 
\end{cases} \]

where \( x_t = \hat{x}(\theta, \theta_{t+1}) \).

Using the definition of \( \hat{I}(\theta_{t-1}, x_{t-1}, \theta_{t+1}) \), we can provide a characterisation of equilibria as follows.

Proposition 6 Given initial values \( \theta_0 \) and \( x_0 \), a sequence \( \{\theta_T, x_T\}_{T=1}^{\infty} \) constitutes a Perfect Bayesian Equilibrium if and only if, we have \( \theta_t = \hat{I}(\theta_{t-1}, x_{t-1}, \theta_{t+1}) \), and \( x_t = \hat{x}(\theta_t, \theta_{t+1}) \), for \( t = 1, 2, \ldots \).

There exists a steady-state equilibrium at \( \theta \in [0,1] \) if and only if \( \theta = \hat{I}(\theta, x, \theta) \) where \( x = \hat{x}(\theta, \theta) \). In other words, if the proportion of men seeking young brides, and the threshold value above which women decline offers of marriage, are equal to \( \theta \) and \( \hat{x}(\theta, \theta) \) respectively in some period, and \( \theta = \hat{I}(\theta, \hat{x}(\theta, \theta), \theta) \), then there exists an equilibrium where these values remain the same in all future periods.

We can use this framework to examine how the marriage market responds to policy changes and economic changes. An interesting and important result we can show theoretically is that a marriage market, which is initially in a steady-state, will require multiple periods to adjust to a shock – such as an unexpected change in policy or a demographic change. This is because the reputation and number of older women on the marriage market in period \( t \) are determined by decisions in period \( t - 1 \). Therefore, these variables are unaffected by a shock in period \( t \). However, if men and young women on the marriage market make different choices in period \( t \) in response to the shock, this will potentially affect the reputation and number of older women on the marriage market in period \( t + 1 \). In other words, the state variables will have evolved. Therefore, the marriage market cannot move to a new steady-state equilibrium immediately in response to the shock in period \( t \). Formally, we can establish the following:

Proposition 7 Suppose the marriage market is initially in a steady-state equilibrium when it experiences a shock. If it moves to a new steady-state in response to the shock, then it will require more than one period to reach the new steady-state.

Proof. See Appendix A. \( \blacksquare \)

An implication of Proposition 7 is that a policy or programme evaluation carried out on the first cohort subject to a policy change or initiative has been introduced would not capture the full extent of its impact on the marriage market because of the dynamics inherent in it.
7 An Application of the Theoretical Model to Early Marriage in Bangladesh

In the preceding sections, we developed an overlapping-generations model of marriage timing and provided a theoretical characterisation of the marriage market equilibrium within this model. In this section, we explore how well such a model can explain phenomena related to early marriage, in particular changes in the incidence of early marriage over time. We also use the model to provide insights about how the incidence of early marriage would respond to government interventions and demographic shocks.

For the numerical analysis, we calibrate the model using the context of Bangladesh. The assumptions of the model are, arguably, a good approximation of the key marriage patterns in Bangladesh, which is characterised by low incidence of divorce and polygamy, and high rates of exogamy. According to the figures available from the latest Demographic and Health Survey, about 65% of women aged 20-24 years were married by the age of 18 (NIPORT 2013) and Bangladesh has one of the highest rates of early marriage for women in the world today (UNFPA 2012). The 1929 Child Marriage Restraint Act imposed a minimum age of marriage, which is presently 18 for women and 21 for men. However, this law is frequently ignored and rarely enforced. For adolescent girls in Bangladesh, marriage brings about a sudden change in roles and responsibilities. It typically involves leaving school and withdrawing from the labour market to undertake household duties (Amin et al. 2014; Amin, Mahmud & Huq, 2002).

Despite the high incidence of early marriage among women in Bangladesh today, it is important to recognise that the average age of marriage has not been stationary. The following figures show how the incidence of early marriage among girls in Bangladesh has evolved over a twenty-year period, using data from the Bangladesh Women’s Life Choices and Attitudes Survey (WiLCAS 2014). We see that for cohorts born between 1975 and 1981, the proportions marrying early were more or less constant, but it has declined for each of the subsequent cohorts. This decline has been attributed to both decreased cost of schooling for girls as a result of government initiatives on tuition and stipend programmes (Schurmann 2009, Asadullah and Chaudhury 2009) as well as increased labour market opportunities for women, in particular in the ready-made garments sector (Heath and Mobarak 2015).

---

7In the 2011 Bangladesh Demographic and Health Survey, among women aged 15-49 years, 1% are divorced. Among married men in the same age group, less than 1% are in polygynous marriages (NIPORT 2013). As for exogamy, we find among married female respondents (aged 20-39 years) in the 2014 Bangladesh WiLCAS that 88% had no family connection with their husband prior to marriage.

8The Bangladesh WiLCAS 2014 is a nationally representative survey of women aged 20-39 years with information on education, employment, marriage, childbirth, and beliefs and attitudes funded by Australian Aid through the Australian Development Research Awards Scheme. Further information about the survey can be obtained from Asadullah and Wahhaj (2015).
We calibrate the model based on the marriage market conditions for women born between 1975 and 1982. Since the incidence of early marriage was relatively stable during this period we assume, for the purpose of this calibration, that the marriage market is in steady-state. Table 1 shows the functional forms used for the numerical analysis: the utility function is quasi-linear, the matching function takes a Cobb-Douglas form, and the marriage transfers are given by the symmetric Nash bargaining solution. It is straightforward to verify that these functional forms satisfy Assumptions 1-5 and 8. The functional form for the matching function is motivated by the use of log-linear functions for the estimation of matching functions, and evidence found in favour of constant returns to scale, in a wide range of studies (see Mortensen and Pissarides 1999; and Petrongolo and Pissarides 2001).

We let $m_0 = 0.90$ which implies that when there are equal number of men and women in one segment of the marriage market, 90% of them find matches. The assumption of a
high match probability is reasonable given that each search period in the model corresponds to 5 years. Furthermore, we let $\alpha = 2$, which implies that the utility cost of a spouse with poor reputation is convex, which is a reasonable in the context of a traditional society where social capital is important. The qualitative findings from simulating the model, discussed below, are not sensitive to the precise values assumed for $m_0$ and $\alpha$.

The values of the functional parameters $\pi$, $\sigma$ and $\gamma$ are chosen to match the steady-state values to key moments in the data; specifically, the proportion of women who have early marriages, the ratio of equilibrium marriage transfers for older brides to that for younger brides, and the proportion of women who remain unmarried at the end of their second period in the marriage market. The full set of data moments that enter the numerical analysis, as well as the data sources, are shown in Table 2.

<table>
<thead>
<tr>
<th>Function</th>
<th>Functional Form Used</th>
<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility</td>
<td>$u_m(\tau, \varepsilon_f) = \tau + \sigma (1 - \varepsilon_f)\alpha$</td>
<td>$\alpha = 2, \sigma$ estimated</td>
</tr>
<tr>
<td></td>
<td>$u_m(\tau, \varepsilon_f) = -\tau + \sigma (1 - \varepsilon_m)\alpha$</td>
<td></td>
</tr>
<tr>
<td>Matching</td>
<td>$f(n_m, n_f) = m_0 (\theta n_m)^{\gamma} (n_f)^{1-\gamma}$</td>
<td>$m_0 = 0.9, \gamma$ estimated</td>
</tr>
<tr>
<td>Function</td>
<td>$\theta n_m \mu \left( \frac{n_f}{\theta n_m} \right) = f(n_m, n_f)$</td>
<td></td>
</tr>
<tr>
<td>Marriage</td>
<td>$\xi(v_m, v_f, \varepsilon_m, \varepsilon_f) = \arg \max [u_m(\tau, \varepsilon_f) - v_m][u_f(\tau, \varepsilon_m) - v_f]$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Data Moment</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Married before 18, Females born 1975-82</td>
<td>72.05%</td>
<td>2014 WiLCAS</td>
</tr>
<tr>
<td>Mean Dowry for Brides &lt; 18, Females born 1975-82</td>
<td>Tk 7,131</td>
<td>2014 WiLCAS</td>
</tr>
<tr>
<td>Mean Dowry for Brides &gt;= 18, Females born 1975-82</td>
<td>Tk 14,887</td>
<td>2014 WiLCAS</td>
</tr>
<tr>
<td>Std. dev. of Dowry, Brides &lt; 18, born 1975-82</td>
<td>Tk 36,713</td>
<td>2014 WiLCAS</td>
</tr>
<tr>
<td>Males 20-24 yrs, 1980</td>
<td>5.701m</td>
<td>UN Pop. Div.</td>
</tr>
<tr>
<td>% Never Married, Female 25-29 yrs</td>
<td>2.2%</td>
<td>1993/94 DHS</td>
</tr>
</tbody>
</table>

*World Population Prospects: The 2012 Revision*
Recall that the utility obtained from being single during one period, represented by \( u_m \), \( u_{f1} \) and \( u_{f2} \) in the model, are exogenously given. For older women, we set the utility from remaining single equal to that derived from marriage to a man of ‘bad character’ with zero transfers. This assumption is intended to capture the strong social pressures of marriage for women and can be backed by anecdotal evidence on women unwilling to leave husbands who are prone to abuse and violence.\(^{10}\) For men, we set the utility from remaining single equal to that derived from marriage to a woman of ‘bad character’ and a positive transfer equal to 1 unit. This is intended to capture the asymmetry in social pressures and economic opportunities between men and women, as well as provide a precise meaning for a 1 unit transfer. Finally, given that educational and economic opportunities were limited for both young and older women during the period under consideration, we fix \( u_{f1} = u_{f2} \): i.e. the utility from singlehood is the same for young women and older women (Note that, where there is a positive incidence of early marriage, young women on the marriage market will, nevertheless, have better outside options then older women because they have higher chances of being married). We also fix \( \eta = 0 \) for the moment-matching exercise – i.e. we assume that no marriage offers are refused in the initial steady-state but we allow \( \eta \) to adjust for the subsequent simulations.

In addition, we set \( \beta = 0.77 \) and \( \varepsilon = 0.05 \). Given that each period in the model corresponds to 5 years, the value assumed for \( \beta \) represents an annual discount factor of 0.95, which is broadly in line with assumptions made in the macroeconomics literature. The value chosen for \( \varepsilon \) is guided by: (i) the fact that a lower value causes the estimated \( \pi \) to hit its constraint at 1 and (ii) a significantly higher value seems improbable as it would translate into a significant pattern of marriage partners of bad character being ‘found-out’ following marriage, and there is limited evidence of this in the context of Bangladesh.

For the moment-matching exercise, we solve a system of 11 equations in 11 unknowns listed in Appendix B. The values of the parameters \( \gamma \), \( \sigma \) and \( \pi \), and the steady-state values for variables in the system thus obtained are reported in Table 2B.

Table 2B: Parameters and Steady-State Values from Moment-Matching Exercise

<table>
<thead>
<tr>
<th>variable</th>
<th>( \theta )</th>
<th>( \lambda_1 )</th>
<th>( \varepsilon_{f2} )</th>
<th>( \lambda_2 )</th>
<th>( n_{f2} )</th>
<th>( \tau_2 )</th>
<th>( \tau_1 )</th>
<th>( \sigma_{f1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>steady-state value</td>
<td>0.716</td>
<td>0.7486</td>
<td>0.1456</td>
<td>0.922</td>
<td>0.243</td>
<td>0.489</td>
<td>0.234</td>
<td>1.928</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>parameter</th>
<th>( \gamma )</th>
<th>( \sigma )</th>
<th>( \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>steady-state value</td>
<td>0.5747</td>
<td>1.2339</td>
<td>0.7515</td>
</tr>
</tbody>
</table>

Note that the theory nests model in which the age of potential brides is not associated with ‘quality’: this is obtained when \( \pi = 0 \): which would mean that there is, effectively, no mechanism to detect the quality of potential brides. The fact that we obtain a high value for \( \pi \) in the moment-matching exercise, and consequently a significantly worse reputation

\(^{10}\)See, for example, Bloch and Rao (2002).
for older women ($\varepsilon_{f2} = 0.1456$ while $\varepsilon_{f1} = 0.05$) suggests that the hypothesized reputational effects are important for the marriage market in Bangladesh.

Figure 4 shows a plot of $I(\theta, \theta)$ against $\theta$ for the model calibrated to the marriage market in Bangladesh. The curve crosses the 45-degree line at just one point. This occurs at $\theta = 0.7160$. Therefore, the model implies that given the sex ratios, population growth rate and outside options for men and women relative to marriage during the late 1980’s in Bangladesh, there was only one feasible steady-state equilibrium, with a high proportion of men seeking young brides.

In the following sections, we use the calibrated model to explore how the marriage market would evolve in response to demographic shocks and policy changes and compare the model dynamics to that observed in the data. For the simulations, we assume that the outside options of young women are normally distributed and we choose the standard deviation of the distribution to equate the standard deviation of $\tau_1$ to that of dowries for young brides in the data.

7.1 Demographic Changes and the Incidence of Child Marriage

Bangladesh experienced a sharp decline in fertility between the 1970’s and the 1990’s, a phenomenon which is commonly attributed to family planning programmes launched in the late 1970’s (see, for example, Joshi and Schultz 2007). The total fertility rate fell from 6.3 in 1975 to 3.4 in 1994 (NIPORT 2013). The rate of population growth can have important
implications for the marriage market due to a phenomenon known as the ‘marriage squeeze’ (Rao 1993; Bhat and Halli 1999). Within the framework of the theory presented above, a growing population makes the cohort of young women larger than that of older men, which increases the chances of success of potential grooms seeking young brides. Conversely, a sudden decline in fertility will translate into increased difficulty in finding young brides 14-18 years later which could explain, in principle, why early marriage in Bangladesh began to decline for cohorts born after 1982. To explore this possibility, we introduce a demographic shock to the model, assuming it is initially in a steady-state, and examine how the incidence of early marriage evolves in subsequent years. Specifically, we reduce the cohort growth rate from 15% to 5% (recall that each cohort in the model corresponds to a 5-year age group), keeping all other parameter values as discussed in the preceding section.

Table 3 below compares the proportion of women married before age 18 in the data, and according to the calibrated model following the demographic shock for the cohorts born in 1975-82 and subsequent 5-year age groups. The data for the 1993-97 birth cohort is missing as some of these women were below 18 at the time of the survey.

Table 3: % Married before Age 18, Data and Simulation of Demographic Shock

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1975-82</td>
<td>72.05</td>
<td>72.05</td>
<td>72.05</td>
</tr>
<tr>
<td>1983-87</td>
<td>65.12</td>
<td>73.47</td>
<td>73.43</td>
</tr>
<tr>
<td>1988-92</td>
<td>59.10</td>
<td>73.85</td>
<td>73.90</td>
</tr>
<tr>
<td>1993-97</td>
<td>42.90\textsuperscript{11}</td>
<td>73.87</td>
<td>74.33</td>
</tr>
<tr>
<td>New Steady-State</td>
<td>-</td>
<td>74.33</td>
<td>74.33</td>
</tr>
</tbody>
</table>

Table 4: Simulated Values for Demographic Shock (Naive Expectations)

<table>
<thead>
<tr>
<th>Cohort (Year-of-Birth)</th>
<th>$\theta$</th>
<th>$\lambda_1$</th>
<th>$\varepsilon_{f2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Steady-State</td>
<td>0.716</td>
<td>0.7486</td>
<td>0.1456</td>
</tr>
<tr>
<td>1983-87</td>
<td>0.69</td>
<td>0.7660</td>
<td>0.1456</td>
</tr>
<tr>
<td>1988-92</td>
<td>0.70</td>
<td>0.7707</td>
<td>0.1527</td>
</tr>
<tr>
<td>1993-97</td>
<td>0.71</td>
<td>0.7754</td>
<td>0.1548</td>
</tr>
<tr>
<td>New Steady-State</td>
<td>0.71</td>
<td>0.7754</td>
<td>0.1574</td>
</tr>
</tbody>
</table>

We consider two scenarios for the model simulations: (i) agents have naive expectations (as defined in Section 5) and (ii) agents have rational expectations. The proportion married before 18 for birth cohorts 1972-1982 is identical in the simulations and the data as the parameter values $\gamma$, $\sigma$ and $\pi$ are chosen to match data-moments including $\theta$. For subsequent birth cohorts, the simulations show an increase in early marriage, contrary to the data which shows a sharp decline (Table 3).

\textsuperscript{11}This figure understates the actual proportion marrying before 18 in this group as the youngest among them were 17 at the time of the survey and, therefore, could potentially be married before they reach 18.
The reason for the counter-intuitive increase in early marriage in the simulations is as follows. As per our intuition, the proportion of men who seek young brides ($\theta$) initially decline due to the demographic shock; but they have relatively fewer young women to choose from and this results in an *increase* in the proportion of women who receive offers of early marriage ($\lambda_1$) as shown in Table 4 (We provide only the calculated values for naive expectations as the case of forward-looking expectations path is very similar in this instance). The increase in early marriage causes the reputation of older women on the marriage market to worsen ($\varepsilon f_2$), and therefore, the proportion of men seeking young brides increase again in subsequent periods. Thus, a decline in population growth rate is, at least in this instance, *conducive* to early marriage. We conclude that the decline in population growth rate in Bangladesh cannot, on its own, explain the decline in early marriage since the 1990’s.

It is significant that when the market has reached a new steady-state, following the demographic shock, the reputation of older women in the marriage market is *worse* than what it had been in the initial equilibrium. This is because in the new demographic phase following the fertility decline, where there are more potential grooms for every potential young bride, the presence of an older woman in the marriage market is even *more* ‘suspect’ than it had previously been. Thus, the demographic transition from high population growth to low population growth aggravates the position of women. This effect, due to the reputational effects related to the marriage timing of women, contrasts with the finding by Bhaskar (2015) that declining population growth rate – by creating a shortage of potential brides in the marriage market – will improve the situation of women in developing countries.

### 7.2 Changes in the Opportunity Cost of Early Marriage

The opportunities available to adolescent girls in Bangladesh have expanded significantly since the 1980’s. First, the Bangladesh government introduced a number of educational reforms and initiatives that lowered the cost – and improved access – to schooling for Bangladeshi women: in 1990, free tuition was introduced for all girls enrolled in classes VI-VIII, and in 1994, a female secondary school stipend programme was launched, extending to all non-metropolitan areas in the country by 2000 (Schurmann 2009). In 1993, a food-for-education programme – whereby households received grain for the government if they had one or more children enrolled in primary school – was introduced in rural areas (Ahmed and Nino 2002), and subsequently this initiative was converted to a cash-for-education programme. Second, the labour market opportunities improved significantly, most notably because of the dramatic growth of the ready-made garments sector which today employs more than three million women (Heath and Mobarak 2015).

Marriage for Bangladeshi women has traditionally meant the termination of schooling and withdrawal from the labour market, as this is the moment when they begin to live with
their in-laws, take on new domestic responsibilities and and practise a certain degree of social seclusion (Amin and Suran 2009). Therefore, the initiatives in education and the changes in the labour market highlighted above expanded the opportunities for single women to a much greater extent than they did for married women. Thus, arguably, these initiatives and changes increased the opportunity cost of early marriage for Bangladeshi women.

If so, it is reasonable to model these changes as an increase in the utility that young women derive from singlehood.12 Such an increase would raise the probability that a young woman declines an offer of marriage, and decreases the net marriage transfer that can be obtained from her (because her outside option during the bargaining process is higher). Consequently, it will be less attractive for a potential groom to seek a young bride, and this could result in a decline in early marriage.

To explore this possibility, we introduce an increase in the utility from singlehood for young women, assuming it is initially in steady-state, and investigate how the marriage market evolves in subsequent periods. Specifically, we increase the utility from singlehood for all young women by half the standard deviation of the utility distribution.13

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12 We assume, implicitly, that the intervention does not impact upon the outside option of older women on the marriage market or their contributions to marriage. In the context of Bangladesh, these effects are likely to be small as there is a strong social stigma associated with spinsterhood (documented, for example, by Rozario 1992) and labour force participation among married women remains low (NIPORT 2013).

13 For a sense of the magnitude of this change in the context of Bangladesh, we can compare with the impact of exposure to ready-made garments factories on the marriage rate of girls estimated by Health and Mobarak (2015): a 28% decrease in the hazard of getting married for girls in garments-proximate villages (with sample mean exposure). If this change was due entirely to an increase in the opportunity cost of early marriage, and we assume that 10% of marriage offers are declined in control areas, it would require a shift of 0.75 sd in the distribution.
Table 5: Simulated Values for Opportunity Cost Increase (Naive Expectations)

<table>
<thead>
<tr>
<th>Cohort (Year-of-Birth)</th>
<th>Marriage before 18 %</th>
<th>θ</th>
<th>λ₁</th>
<th>ε_{f2}</th>
<th>η</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady-State</td>
<td>72.05</td>
<td>0.716</td>
<td>0.7486</td>
<td>0.1456</td>
<td>0.0031</td>
</tr>
<tr>
<td>Period 1</td>
<td>68.42</td>
<td>0.65</td>
<td>0.7185</td>
<td>0.1456</td>
<td>0.0105</td>
</tr>
<tr>
<td>Period 2</td>
<td>65.93</td>
<td>0.60</td>
<td>0.6944</td>
<td>0.1304</td>
<td>0.0160</td>
</tr>
<tr>
<td>Period 3</td>
<td>64.34</td>
<td>0.57</td>
<td>0.6794</td>
<td>0.1218</td>
<td>0.0174</td>
</tr>
<tr>
<td>Period 4</td>
<td>63.28</td>
<td>0.55</td>
<td>0.6692</td>
<td>0.1169</td>
<td>0.0180</td>
</tr>
<tr>
<td>New Steady-State</td>
<td>61.11</td>
<td>0.51</td>
<td>0.6480</td>
<td>0.1083</td>
<td>0.0201</td>
</tr>
</tbody>
</table>

Table 6: Simulated Values for Opportunity Cost Increase (Forward-Looking Expectations)

<table>
<thead>
<tr>
<th>Cohort (Year-of-Birth)</th>
<th>Marriage before 18 %</th>
<th>θ</th>
<th>λ₁</th>
<th>ε_{f2}</th>
<th>η</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady-State</td>
<td>72.05</td>
<td>0.716</td>
<td>0.7486</td>
<td>0.1456</td>
<td>0.0031</td>
</tr>
<tr>
<td>Period 1</td>
<td>65.32</td>
<td>0.60</td>
<td>0.6944</td>
<td>0.1456</td>
<td>0.0227</td>
</tr>
<tr>
<td>Period 2</td>
<td>62.56</td>
<td>0.54</td>
<td>0.6640</td>
<td>0.1198</td>
<td>0.0209</td>
</tr>
<tr>
<td>Period 3</td>
<td>61.59</td>
<td>0.52</td>
<td>0.6534</td>
<td>0.1120</td>
<td>0.0207</td>
</tr>
<tr>
<td>Period 4</td>
<td>61.11</td>
<td>0.51</td>
<td>0.6480</td>
<td>0.1095</td>
<td>0.0201</td>
</tr>
<tr>
<td>New Steady-State</td>
<td>61.11</td>
<td>0.51</td>
<td>0.6480</td>
<td>0.1083</td>
<td>0.0201</td>
</tr>
</tbody>
</table>

In Table 5, we see that, in the case of naive expectations, the increase in the utility from singlehood results in a 7 percentage point decline in the % of men seeking young brides (θ) for the first cohort exposed to the ‘shock’. The value of θ declines further for subsequent cohorts, reaching a new steady-state value of 0.51 after 8 periods. We also see that the reputation of older women in the marriage market (ε_{f2}) improves over time after the shock and the proportion of young women declining offers of early marriage (η), albeit very small, is increasing over time. The co-evolution of θ, ε_{f2} and η captures a virtuous cycle as follows. As a result of increased utility from singlehood from the intervention, young women have greater bargaining power and can negotiate a lower dowry. Therefore, fewer men make marriage offers to them, and this improves the reputation of older women on the marriage market the following period. This causes more men to seek older brides and more women to decline offers of early marriage which leads to a further increase in the reputation of older women next period. And thus the cycle continues. The evolution of the marriage market is similar in the case of forward-looking expectations (Table 6) but the steady-state is reached more quickly.\(^\text{14}\)

The incidence of early marriage declines by about 11 percentage points when the new

\(^{14}\text{One difference of note in the case of forward-looking expectations is that the marriage refusal rate initially jumps and then declines to the new steady-state value. This is because young women anticipate that they will have better reputation than their elders if they re-enter the marriage market next period and, therefore, are willing to decline current marriage offers. However, as the marriage market evolves towards late marriage, their outside options improve, and they are able to negotiate better terms of marriage (lower dowry) in the current period and so the refusal rate starts to fall.}\)
steady-state is reached, which is roughly half the magnitude of the decline in $\theta$.$^{15}$ But the first cohort which exposed to the shock experiences a decline in the early marriage rate in the range of 3.6 to 6.7 percentage points (depending on whether expectations are naive or forward-looking). Therefore, the effect of the shock on the first cohort only captures about one-third to two-third of the total effect on early marriage by the time that the new steady-state is reached.

### 7.3 Randomised Control Trials relating to Child Marriage

The current gold standard for testing the efficacy of development interventions, including those targeted at adolescents, is to run small-scale randomised control trials (RCT’s). It is well-known that the potential effect that a scaled-up intervention may have on market equilibria or on social norms may be missed in a small scale RCT as the trial may be insufficient to achieve the hypothesized equilibrium shifts (see, for example, Duflo et al. 2007, Banerjee and Duflo 2009). In the case of programmes aimed at reducing early marriage, the equilibrium effects are likely to be important as discussed above, and therefore a small-scale trial may significantly under-estimate the efficacy of the intervention under consideration. The theory proposed in this paper – where the marriage market and social norms play important roles – provides a good setting to evaluate how significant this under-estimation is likely to be.

In the following, we simulate a number of scenarios where $x\%$ of young women in the population benefit from an intervention that increase their opportunity cost of child marriage by half a standard deviation of the utility distribution. The marriage market is initially in the steady-state computed above. We assume that the programme beneficiaries are identifiable if they re-enter the marriage market as older women and, therefore, they can potentially have a different reputation from non-beneficiaries (if, for example, they had a different marriage refusal rate). To abstract away from differences arising from the speed at which the market adjusts to the intervention, we focus on the new steady-state values following the intervention.

First, we consider the case where $x = 10\%$; i.e. where one in every ten young women in the marriage market, chosen at random, are exposed to the programme.$^{16}$ From Table

---

$^{15}$The reason for the disparity between the change in $\theta$ and $\lambda_1$ is that as fewer men seek young brides, the chances of success increase for those who continue to do so; this is evident in the fact that, for a young woman, the probability of receiving a marriage offer ($\lambda_1$) falls at about half the rate of $\theta$ following the shock.

$^{16}$To have a sense of the scale of an RCT with $x = 10\%$ in the context of Bangladesh, we can proceed as follows. According to the 2005 Bangladesh Adolescent Survey, 63% of married adolescents have a marriage partner from the same sub-district and, therefore, we can take the sub-district to be a very rough approximation of the typical size of the marriage market. According to the 2011 Population Census of Bangladesh, the average size of a sub-district is about 250,000 and the number of female adolescents aged 15-19 within the sub-district about 13,000. Therefore, the intervention would have to target 1,300 adolescent girls in the sub-district.
7, we see that the rate of child marriage – for programme beneficiaries – falls by about 1.6 percentage points. For \( x = 20\% \), the corresponding figure is about 2.5 percentage points. By contrast, when all women in the population are beneficiaries of the intervention, the rate of child marriage declines by over 10 percentage points.

The main channel through which these effects occur is by changing the expected utility to a potential groom from seeking young brides. A programme beneficiary is more likely to decline a marriage offer or negotiate a more favourable marriage transfer for herself and this makes her less attractive from the perspective of the man. However, in the case of a small-scale intervention, the chances that the potential groom seeking a young bride is matched with a programme beneficiary remains low and therefore the impact on the equilibrium is small. In particular, the figures in Table 7 suggest that a trial targeting \( x\% \) of adolescents in the marriage market will shift \( \theta \) – the proportion of men seeking young brides – by roughly \( x\% \) of that of the full-scale intervention. On the other hand, the small-scale trials are able to capture reasonably well the direct effect on early marriage refusals: \( \eta = 0.0130 \) for the 10% intervention as opposed to 0.0195 for the fully scaled-up programme.

<table>
<thead>
<tr>
<th>Cohort (Year-of-Birth)</th>
<th>Marriage before 18 %</th>
<th>( \theta )</th>
<th>( \lambda_1 )</th>
<th>( \varepsilon_{f2} )</th>
<th>( \eta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Steady-State</td>
<td>72.05</td>
<td>0.716</td>
<td>0.7486</td>
<td>0.1456</td>
<td>0.0031</td>
</tr>
<tr>
<td>10%</td>
<td>70.43</td>
<td>0.70</td>
<td>0.7415</td>
<td>0.1384</td>
<td>0.0130</td>
</tr>
<tr>
<td>20%</td>
<td>69.51</td>
<td>0.68</td>
<td>0.7324</td>
<td>0.1346</td>
<td>0.0138</td>
</tr>
<tr>
<td>50%</td>
<td>67.15</td>
<td>0.63</td>
<td>0.7090</td>
<td>0.1258</td>
<td>0.0158</td>
</tr>
<tr>
<td>100%</td>
<td>61.66</td>
<td>0.52</td>
<td>0.6534</td>
<td>0.1097</td>
<td>0.0195</td>
</tr>
</tbody>
</table>

The numerical analysis suggests the following procedure as a rough way to incorporate the equilibrium effects when estimating the potential impact of a scaled-up intervention on early marriage on the basis of a small-scale trial covering \( x\% \) of the marriage market:

\[
\text{marriage rate} = \lambda_1 (1 - \pi \varepsilon_{f1}) (1 - \eta)
\]

\[
\Rightarrow \text{\Delta marriage rate} = (1 - \pi \varepsilon_{f1}) \left[ \Delta \lambda_1 (1 - \eta) + \lambda_1 \Delta (1 - \eta) \right]
\]

\[
\Rightarrow \text{\Delta marriage rate} \approx (1 - \pi \varepsilon_{f1}) \left[ \frac{\Delta \lambda_1 \lambda_1}{(x/100)} (1 - \eta) + \lambda_1 \Delta_x (1 - \eta) \right]
\]

\[
\Rightarrow \text{\Delta marriage rate} \approx (1 - \pi \varepsilon_{f1}) \left[ \frac{\Delta_x \theta}{2 (x/100)} (1 - \eta) + \lambda_1 \Delta_x (1 - \eta) \right]
\]

where \( \Delta_x \) denotes the change resulting from an \( x\% \) trial and \( \Delta \) the corresponding change resulting from the scaled-up intervention.
8 Discussion

In recent years, international donors and development agencies have designed and implemented a range of interventions aimed at expanding the economic and educational opportunities of adolescents, particularly girls, in developing countries; with the reduction of the incidence of child marriage and postponement of childbirth among their intended goals.

In this context, it is important to realise that decisions regarding the timing of marriage of a son or daughter, although they are made within a single household, may be influenced by the choices made by other households in the community or region. In this paper, we investigated a particular mechanism through which these individual choices can influence the costs and benefits of early marriage versus late marriage for everyone in a marriage market: if some qualities of prospective brides are not fully observable at the time of marriage, then the incidence of child marriage will influence the perceived quality of older women on the marriage market.

One of the key innovations in the theory proposed in this paper is the signalling and reputational effect associated with marriage timing for women: marriage delays signal ‘bad quality’ and this imposes a pressure on women to accept early offers of marriage and a preference for potential grooms to seek young brides. It is important to note that the theory also nests a model in which the age of potential brides is not associated with ‘quality’. This is equivalent to the case where $\pi = 0$: which would mean that there is, effectively, no mechanism to detect the quality of potential brides and, therefore, if an older woman is still on the marriage market it is only because she did not find a match, or refused a marriage offer, in her youth.

In calibrating the model to the case of Bangladesh, we allowed $\pi$ to be computed so that the steady-state variables match the data-moments. We obtained $\pi = 0.7515$, which means that in about 3 out of 4 instances, the background check at Step 4 in the game can reveal the quality of a potential bride with ‘bad character’. Thus, the moment-matching exercise indicates that the reputational concerns are important for explaining the marriage patterns in Bangladesh.

In this situation, expanding opportunities for some adolescent girls – to the extent that they or their parents choose to postpone their marriage – can trigger a virtuous cycle whereby the perceived quality of older brides improve, which in turn makes it more attractive for other girls to postpone marriage, and so on. In a study of the timing of marriage and childbearing in rural Bangladesh (Schuler et al., 2006), this idea is succinctly captured in a reported interview with an 18-year-old girl:

‘... when asked why her parents had delayed her marriage while her younger sisters had married at ages 12-15 ... [she replied] "My father thought it was unnecessary for girls to read and write but in my case he did not object ... None
of my peers were sitting idle at home so I also went to school. Now it is better for girls. They don’t have to pay school fees — the government finances it ... Everyone has had some schooling, at least up to the eighth or ninth grade. No one would want to marry an illiterate girl so they are sent to school." p. 2831

The fact that an intervention targeted at adolescent girls can make it more and more attractive for future cohorts to postpone marriage means that the long-term impact of such interventions on marriage and subsequent life choices may well exceed the impact on the first cohort which is exposed to it.

9 Appendix A

Proof. of Lemma 1: (i) By Assumption 3, \( \mu \left( \frac{\theta_{t+1}}{\theta_{m}} \right) \) is strictly decreasing in \( \theta_t \). Therefore, using the definitions of \( U_1(\theta_t, \theta_{t+1}) \) and \( U_2(\theta_t, \theta_{t-1}) \) in (7) and (8), we have \( U_1(\theta_t, \theta_{t+1}) \) strictly decreasing in \( \theta_t \) and \( U_2(\theta_t, \theta_{t-1}) \) is strictly increasing in \( \theta_t \).

(ii) Using (6), we see that the outside option of young women in period \( t \), \( v_{f1} \), is increasing in the value of \( \lambda_2 \) in period \( t + 1 \). Using (2), \( \lambda_2 \) is decreasing in \( \theta_{t+1} \). Therefore, \( v_{f1} \) is decreasing in \( \theta_{t+1} \). Then, using (9) and Assumption 5, \( \tau_1 \) is increasing in \( v_{f1} \). It follows from (7) and Assumption 1 that \( U_1 \) is increasing in \( \theta_{t+1} \).

(iii) Using (3), \( \varepsilon_{f2} \) in period \( t \) is increasing in \( \lambda_1 \) in period \( t - 1 \). Using (1), \( \lambda_1 \) is increasing in \( \theta_{t-1} \). Therefore, \( \varepsilon_{f2} \) is increasing in \( \theta_{t-1} \). Then, using (8) and (10) and Assumption 8, \( U_2 \) is decreasing in \( \theta_{t-1} \). ■

Proof. of Lemma 2: Define \( J(\theta) = I(\theta, \theta) - \theta \). Given that \( u_m(\cdot) \), \( \mu(\cdot) \) and \( \xi(\cdot) \) are continuous functions, so is \( J(\cdot) \). By construction, \( J(\theta) \geq 0 \) at \( \theta = 0 \) and \( J(\theta) \leq 0 \) at \( \theta = 1 \). If \( J(0) = 0 \) or \( J(1) = 0 \), then the corresponding \( \theta \) value would constitute a steady-state. If \( J(0) > 0 \) and \( J(1) < 0 \), then it follows from the Intermediate Value Theorem that there exists some \( \theta \in [0, 1] \) such that \( J(\theta) = 0 \). Therefore, \( I(\theta, \theta) = \theta \). ■

Proof. of Proposition 2: Consider a steady-state equilibrium in which a proportion \( \theta \) of men seek young brides. If \( \theta \in (0, 1) \), then \( \lambda_1, \lambda_2 > 0 \). Then, using (3)-(5), we have \( \hat{\varepsilon}_{f2} > \hat{\varepsilon}_{f1} \) and using (6), \( v_{f1} > v_{f2} \). It follows from (9),(10) and Assumption 5 that \( \tau_1 < \tau_2 \). If \( \theta = 0 \), then \( \lambda_1 = 0 \) and \( \lambda_2 > 0 \). Then, \( \hat{\varepsilon}_{f2} = \hat{\varepsilon}_{f1} \) and \( v_{f1} > v_{f2} \). Therefore, we have \( \tau_1 < \tau_2 \). If \( \theta = 1 \), then \( \lambda_1 > 0 \) and \( \lambda_2 = 0 \). Therefore, \( \hat{\varepsilon}_{f2} < \hat{\varepsilon}_{f1} \) and \( v_{f1} = v_{f2} \). Once again, we have \( \tau_1 < \tau_2 \). ■

Proof. of Proposition 3: (i) Consider a monotonic increasing sequence \( \{\theta_t\}_{t=0}^\infty \) where \( \theta_t \in [0, 1] \) for all \( t \). First, we show that this sequence has a point of convergence in the interval \( (\theta_0, 1] \). Let \( y = \lim_{t \to \infty} \theta_t \). It follows that \( y = \sup(\theta_t) = x \).

Next, we show that \( y \) corresponds to a steady-state equilibrium, i.e. \( I(y, y) = y \). We prove this by contradiction. Suppose \( I(y, y) > y \). Then, as \( I(\cdot) \) is continuous in both
arguments, we can find $\varepsilon$ sufficiently small such that
\[ I(y - \varepsilon, y - \varepsilon) > y \]

Given the convergence property of $\{\theta_t\}_{t=0}^{\infty}$, there exists $T$ sufficiently large such that for $t \geq T$, we have $\theta_t > y - \varepsilon$. Therefore,
\[ I(\theta_T, \theta_{T+2}) > y \]
\[ \implies \theta_{T+1} > y \]

This contradicts the earlier statement that $y$ is the supremum of the sequence.

Now suppose that $I(y, y) < y$ and let $z = y - I(y, y) > 0$. Consider some $\delta > 0$. By Lemma 1 and continuity of $I(.,.)$, we have, for each $\theta_1, \theta_2 \in (y - \delta, y)$
\[ I(\theta_1, \theta_2) \leq y - z \]

Given the convergence property of $\{\theta_t\}_{t=0}^{\infty}$, there exists $T$ sufficiently large such that for $t \geq T$, we have $\theta_t \in (y - \delta, y)$. Therefore for each $r \geq T + 1$, we have
\[ \theta_r = I(\theta_{r-1}, \theta_{r+1}) \leq y - z \]

Therefore,
\[ \sup \{\theta_t\}_{t=T+1}^{\infty} \leq y - z \]

Given that the sequence $\{\theta_t\}_{t=0}^{\infty}$ is monotonic, we have
\[ \sup \{\theta_t\}_{t=0}^{\infty} \leq y - z < y \]

which contradicts the earlier statement that $y$ is the supremum of the sequence. Therefore, it must be that $y$ corresponds to a steady-state equilibrium.

(ii) The proof for a monotonic decreasing sequence proceeds in the same manner as above, such that we let $x = \inf \{\theta_t\}$ and show that the sequence converges to $x$. Using the reasoning by contradiction used above, we then show that $x$ is a steady-state value, i.e. $I(x, x) = x$.

\textbf{Proof.} of Proposition 4: Given $\theta_0$, we can construct a sequence $\{\theta_t\}_{t=1}^{\infty}$ as follows. Let $\theta_{t+1} = I(\theta_t, \theta_t)$ for $t = 1, 2, \ldots$. By construction, the sequence $\{\theta_t\}_{t=1}^{\infty}$, together with the initial value $\theta_0$, satisfies the definition of a ‘Naive Expectations’ equilibrium in 4. Furthermore, as $I(.,.)$ is increasing in both arguments by Lemma 1, given $\theta_0$, no other sequence $\{\theta'_t\}_{t=1}^{\infty}$ can satisfy the definition in 4. Therefore, given the initial value $\theta_0$, there is a unique sequence that satisfies the criteria of the ‘Naive Expectations’ Equilibrium.

(i) Suppose $I(\theta_0, \theta_0) \neq \theta_0$. As $I(1, 1) \leq 1$, and $I(.,.)$ is a continuous function, it follows that there exists at least one steady-state value in the interval $(\theta_0, 1]$. Let us denote the first
(i.e. smallest) of these steady-state values as \( \theta_s \); i.e. \( I(\theta_s, \theta_s) = \theta_s \). By continuity of \( I(\ldots) \), we have \( \theta < I(\theta, \theta) \) for \( \theta \in [0, \theta_s) \).

By construction, \( \theta_1 = I(\theta_0, \theta_0) \). Since \( \theta_0 < I(\theta_0, \theta_0) \), we have \( \theta_1 > \theta_0 \). We can show by contradiction that \( \theta_1 < \theta_s \). Define \( K(\theta) = I(\theta, \theta) \). By Lemma 1, \( K(\theta) \) is increasing in \( \theta \). Therefore, \( K^{-1}(\theta) \) is an increasing function. Therefore, if \( \theta_1 < \theta_s \), then \( K^{-1}(\theta_1) > K^{-1}(\theta_s) \).

By construction, \( K(\theta_0) = I(\theta_0, \theta_0) = \theta_1 \). Therefore \( K^{-1}(\theta_0) = \theta_1 \). Therefore \( \theta_0 > \theta_s \), which is a contradiction of our initial premise.

By the same reasoning, \( \theta_2, \theta_3, \ldots < \theta_s \). Since \( I(\theta, \theta) > \theta \) for \( \theta < \theta_s \), it follows that \( \{\theta_\tau\}_{\tau=1}^\infty \) is an increasing sequence. Following the proof of Proposition 3, we can show that the sequence converges to \( y = \sup \{\theta_\tau\} \) and that \( y \) corresponds to a steady-state. As all elements of the sequence are smaller than \( \theta_s \), it follows that the steady-state must be \( \theta_s \).

(ii) Suppose \( I(\theta_0, \theta_0) < \theta_0 \). As \( I(0,0) \geq 1 \) and \( I(\ldots) \) is a continuous function, it follows that there is at least one steady-state value in the interval \([0, \theta_0)\). Let us denote the largest of these steady-state values as \( \theta'_s \); i.e. \( I(\theta'_s, \theta'_s) = \theta'_s \). Following the reasoning in part (i) above, we can show that sequence \( \{\theta_\tau\}_{\tau=1}^\infty \) is a decreasing sequence that converges to \( \theta'_s \).

(iii) Suppose \( I(\theta_0, \theta_0) = \theta_0 \). Since \( \theta_{t+1} = I(\theta_t, \theta_t) \) for \( t = 1, 2, \ldots \), it follows that \( \theta_1 = I(\theta_0, \theta_0) = \theta_0 \), and \( \theta_2 = I(\theta_1, \theta_1) = I(\theta_0, \theta_0) = \theta_0 \), etc. Therefore, each element of the sequence \( \{\theta_\tau\}_{\tau=1}^\infty \) is equal to \( \theta_0 \).

First, we show that \( \lim_{\alpha \to \infty} \theta^\alpha_\tau \) indeed exists.

**Proof.** of Proposition 5: By Proposition 4, \( \{\theta^0_\tau\}_{\tau=1}^\infty \) is either an increasing sequence or a decreasing sequence. Without loss of generality, let us assume that it is an increasing sequence.

Then, by assumption, \( \theta^0_2 > \theta_0 \). It follows from the properties of function \( I(\ldots) \) that \( \theta^1_1 = I(\theta_0, \theta^0_2) \geq I(\theta_0, \theta_0) = \theta^0_1 \).

Similarly, \( \theta^1_2 = I(\theta^1_1, \theta^0_3) \geq I(\theta^0_1, \theta^0_1) = \theta^0_2 \). Following the same reasoning, \( \theta^1_\tau \geq \theta^0_\tau \) for all \( \tau \in \{3, 4, \ldots\} \).

Similarly, \( \theta^2_1 = I(\theta_0, \theta^0_2) \geq I(\theta_0, \theta^0_2) = \theta^1_1 \). Following the same reasoning, \( \theta^\alpha_\tau \geq \theta^{\alpha-1}_\tau \) for all \( \alpha \in \{3, 4, \ldots\} \) and each \( \tau \).

Therefore, \( \{\theta^\alpha_\tau\}_{\alpha=1}^\infty \) is an increasing sequence for each \( \tau \). Moreover, by construction, each \( \theta^\alpha_\tau \in [0, 1] \); i.e. the sequence is bounded. Let \( x_\tau = \sup \{\theta^\alpha_\tau\} \). By definition, for each \( \varepsilon > 0 \), \( \exists \alpha \in \mathbb{N} \) such that \( |\theta^\alpha_\tau - x_\tau| < \varepsilon \) (if not, \( x_\tau \) would not be the supremum of the sequence). Given that \( \{\theta^\alpha_\tau\}_{\alpha=1}^\infty \) is a monotonic sequence, it follows that \( |x_\tau - \theta^\alpha_\tau| < \varepsilon \) for \( \alpha \geq \alpha \). Therefore, for \( r, s \geq \alpha \), we obtain

\[
|\theta^r_\tau - \theta^s_\tau| \leq |x_\tau - \theta^r_\tau| + |x_\tau - \theta^s_\tau| \leq 2\varepsilon
\]

Therefore, \( \{\theta^2_\tau\}_{\alpha=1}^\infty \) constitutes a Cauchy sequence. Every Cauchy sequence of real numbers has a limit (see, for example, Lang, p.143). Therefore, \( \{\theta^\alpha_\tau\}_{\alpha=1}^\infty \) has a limit. Similarly, we can
show that if \( \{ \theta_\tau^0 \}_{\tau=1}^{\infty} \) is a decreasing sequence, then so is \( \{ \theta_\tau^\alpha \}_{\alpha=1}^{\infty} \) and that it has a limit. Let \( \theta_\tau^\infty = \lim_{\alpha \to \infty} \{ \theta_\tau^\alpha \} \). By construction, \[
\theta_\tau^\infty = \lim_{\alpha \to \infty} \theta_\tau^\alpha = \lim_{\alpha \to \infty} I (\theta_{\tau-1}^\alpha, \theta_{\tau+1}^{\alpha-1}) = I (\theta_{\tau-1}^\infty, \theta_{\tau+1}^\infty)
\]
The last equality follows from the continuity of the function \( I (., .) \). Then, using Proposition 1, the initial value \( \theta_0 \), together with the sequence \( \{ \theta_\tau^\infty \}_{\tau=1}^{\infty} \), satisfies the criteria for Perfect Bayesian Equilibrium. ■

**Proof.** of Proposition 7: Let us denote by \( \{ \theta_s, x_s \} \) the initial steady-state equilibrium, and by \( \{ \theta_t, x_t \} \) the state of the marriage market in period \( t \) following the shock. Suppose that \( \{ \theta_s', x_s' \} \) denotes the new steady-state. Therefore, if the new steady-state is reached in the period following the shock, we can write
\[
\hat{U}_2 (\theta_{t+1}, \theta_t, x_t) = \hat{U}_2 (\theta_{s}', \theta_{s}', x_{s}')
\]
\[
\hat{U}_2 (\theta_t, \theta_{t-1}, x_{t-1}) = \hat{U}_2 (\theta_{s}', \theta_s, x_s)
\]
By assumption, \( \theta_s' \neq \theta_s \) and \( x_s' = \hat{x} (\theta_{s}', \theta_{s}') \neq \hat{x} (\theta_s, \theta_s) = x_s \). It follows that
\[
\hat{U}_2 (\theta_t, \theta_{t-1}, x_{t-1}) \neq \hat{U}_2 (\theta_{t+1}, \theta_t, x_t)
\]
(17)
Using the properties of a marriage market equilibrium, we have
\[
\hat{U}_2 (\theta_t, \theta_{t-1}, x_{t-1}) = \hat{U}_1 (\theta_t, \theta_{t+1}, x_t)
\]
(18)
\[
\hat{U}_2 (\theta_{t+1}, \theta_t, x_t) = \hat{U}_1 (\theta_{t+1}, \theta_{t+2}, x_{t+1})
\]
(19)
and, by assumption,
\[
\hat{U}_1 (\theta_t, \theta_{t+1}, x_t) = \hat{U}_1 (\theta_{s}', \theta_{s}', x_{s}') = \hat{U}_1 (\theta_{t+1}, \theta_{t+2}, x_{t+1})
\]
(20)
Using (18)-(20), we have
\[
\hat{U}_2 (\theta_t, \theta_{t-1}, x_{t-1}) = \hat{U}_2 (\theta_{t+1}, \theta_t, x_t)
\]
which contradicts (17). Therefore, the original postulate that a new steady-state is reached in period \( t \) and that it differs from the initial steady-state cannot be true. ■

10 Appendix B

The following table lists the system of equations used to compute the steady-state values and parameters \( \gamma, \sigma \) and \( \pi \).
Table B1: System of Equations for Computing the Steady-State and Parameter Values

<table>
<thead>
<tr>
<th>#</th>
<th>Equation</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\lambda_1 n_{f1} - \mu \left( \frac{n_{f1}}{\theta m_m} \right) \theta n_m = 0$</td>
<td>equilibrium</td>
</tr>
<tr>
<td>2</td>
<td>$n_{f2} - \left( \frac{n_{f2}}{\text{cap}} \left[ 1 - \lambda_1 \left(1 - \pi \varepsilon_f \right) \right] \right) = 0$</td>
<td>equilibrium</td>
</tr>
<tr>
<td>3</td>
<td>$\varepsilon_{f2} - \frac{\left((1-\lambda_1) + \lambda_1 \pi \varepsilon_f \right)}{(1-\lambda_1) + \lambda_1 \pi \varepsilon_f} = 0$</td>
<td>equilibrium</td>
</tr>
<tr>
<td>4</td>
<td>$\lambda_2 n_{f2} - \mu \left( \frac{n_{f2}}{(1-\theta) n_m} \right) (1 - \theta) n_m = 0$</td>
<td>equilibrium</td>
</tr>
<tr>
<td>5</td>
<td>$\nu_{f1} - \left(1 + \beta \xi \left( 1 - \lambda_2' \right) \right) u_f + \beta \varepsilon_{f1} { -\tau_2 + (1 - \varepsilon_m) } = 0$</td>
<td>equilibrium</td>
</tr>
<tr>
<td>6</td>
<td>$\tau_1 - \frac{1}{2} \left( (1 - \varepsilon_m)^{\alpha} - (1 - \varepsilon_{f1})^{\alpha} - \nu_{f1} + \nu_m \right) = 0$</td>
<td>equilibrium</td>
</tr>
<tr>
<td>7</td>
<td>$\tau_2 - \frac{1}{2} \left( (1 - \varepsilon_m)^{\alpha} - (1 - \varepsilon_{f2})^{\alpha} - \nu_{f2} + \nu_m \right) = 0$</td>
<td>equilibrium</td>
</tr>
<tr>
<td>8</td>
<td>$U_1 \left( \theta, n_{f1}, n_m, \tau_1, \varepsilon_{f1} \right) - U_2 \left( \theta, n_{f2}, n_m, \tau_2, \varepsilon_{f2} \right) = 0$</td>
<td>equilibrium</td>
</tr>
<tr>
<td>9</td>
<td>$\tau_1/\tau_2 - 2.09 = 0$</td>
<td>moment-matching</td>
</tr>
<tr>
<td>10</td>
<td>$\lambda_1 \left(1 - \pi \varepsilon_{f2}\right) - 0.7205 = 0$</td>
<td>moment-matching</td>
</tr>
<tr>
<td>11</td>
<td>$(1 - \lambda_1 + \lambda_1 \pi \varepsilon_f) \left(1 - \lambda_2 + \lambda_2 \pi \varepsilon_{f2}\right) - 0.05 = 0$</td>
<td>moment-matching</td>
</tr>
</tbody>
</table>

Furthermore, $\lambda_2' = \lambda_2 \left(1 - \pi \varepsilon_{f2}\right)$, $\varepsilon_{f1}$ is given by (4), and $\varepsilon_{f2}$ by (5).

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