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## Endogenous divorce and human capital production

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University of Kent  
School of Economics Discussion Papers

# **Endogenous divorce and human capital production**

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# Endogenous divorce and human capital production

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## Abstract

This paper presents a model of parental decision making where parents care about consumption and the human capital of the children. Preferences over these goods can differ within households. Parents will agree to cooperate (stay married) if the utility they get from coordinating time inputs (i.e. child care or paid employment) is greater than they would get if they acted independently. The gain to cooperation arises because parental time inputs are not perfect substitutes in the production of the child's human capital, the cost is that when preferences differ, the chosen time allocations under cooperation may be very different to those chosen independently. Our model predicts that the human capital of children can both increase and fall after divorce. Divorce, if it occurs, will be instigated by the parent who cares most about the child, the parent that cares least about the child will never opt for divorce. This can explain the apparent contradiction that mothers are more likely than fathers to initiate divorce beyond infant age even though the traditional household literature presents women as home makers and ever devoted to household production.

JEL classification: C79, D19, J12 and J22

Keywords: collective model, human capital, divorce.

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## NON TECHNICAL SUMMARY

This paper presents a model that can explain why parents who care about their children may file for divorce. We show that the gains to cooperation (our definition of marriage) depend on any disparity in preference for children's human capital outcomes between the two parents and the production technology of such outcomes. In addition we show that the gains to marriage are asymmetric, the parent who cares less about the human capital of the children is always better off cooperating while the parent who cares most may be better off acting independently. This is because after divorce she has full control over her time use. We assume that both preferences and technology are only realised after the birth of the child. The arrival of a child is thus the source of an asymmetric shock to the gains to cooperation. We show further that the effect of divorce on children's outcomes is ambiguous.

We also simulate our model to understand better the source of differences in outcomes across families of different types. We show that about half the difference in outcomes between children across different family types is due to selection (what kind of family divorces) rather than the causal effect of divorce. Contrary to what is often thought this is not because people who file for divorce give a low weight to their children's human capital, it is because their (ex) *partners* have a lower weight than they do.

# 1 Introduction

There has been a growing interest from both academics and policy makers in the technology of child rearing and how this impacts on inequality and the inter-generational transmission of income and resources. A key aspect here is the trade off faced by altruistic parents between their own leisure and consumption and the future income/education/well being of their children. Early models (see e.g. Becker (1985)) focused very much on monetary inputs in households with one decision maker. This paper contributes to this field by considering the problem when there are two decision makers (or parents) who have different preferences. In such cases one parent may be better off acting independently (getting divorced). Thus this is a model of both child rearing and endogenous divorce. In this paper a key technological variable is the degree of complementarity between parental inputs in the production of children's human capital as this affects the gains from coordination or marriage.

This motivation for incorporating another decision maker is simple. During the last 40 years developed countries have seen large changes in marriage and divorce rates (Figure 1 provides an illustration for England and Wales). Currently about half of American children live or will live with a step family at some point in their lives (Halpern (2005)). In England and Wales, over half of all divorces involve one or more children under the age of 16. Of these, 20% are under the age of five and 63% under eleven. This makes it crucial to model parental interaction when parents act independently and to explain when and why parents may chose to cooperate (our definition of marriage) or not (our definition of divorce). This is the aim of this paper.

The model of Ermish (2003) also examines the trade off faced by two altruistic divorced parents. Here the issue is that the child's well-being is

a public good. Thus any transfer between one parent and the child has positive spillover on the other parent. Thus transfers to the child will be lower than under cooperation and utility of all three parties (mother, father, child) will be lower. In this world, therefore, no parent would choose divorce. Couples do make the active choice to split up, however, and this choice must be in part related to preferences over household level public goods (such as children). This is why econometricians worry about interpreting the difference in outcomes between children from divorced and intact families as being causal. Our model, on the other hand, endogenises the divorce choice. In the model the "cost of divorce" comes from the fact that the return to inputs is higher when parents coordinate.

In search and matching models (see for example Cornelius 2003) divorce can occur for two reasons. First if there are sufficient search frictions (so people do not meet Dr. Right straight away) and entry (or exit) costs to marriage are low then people may switch partners if they get a better offer.<sup>1</sup> These models may explain divorce that occurs before children are born. Secondly if marriage is an experience good then divorce may occur after the quality of the match is revealed. Our paper presents such a model. Like Fan (1999) and Weiss and Willis (1997)) we explain divorce after childbirth by arguing that match quality is only revealed after children are born.

In this paper, children's human capital is affected not by monetary investments but by time/effort spent by parents. In addition spent by the father with the child affects the marginal benefit of increased time spent by the mother. Support for this view is given in a companion paper (García-Alonso and Gosling (2014)). The intuition is simply that different people interact with children in different ways. Some parents are more active, some

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<sup>1</sup>This of course can generate a hold up problem in marital investments.

are more cerebral, some encourage risk taking, some provide security and structure. Even the same activity such as reading a story will be done with different voices. Thus rather than thinking of one parent as having an absolute or a comparative advantage in childcare it makes more sense to think of the production of human capital as having different types of inputs.

There are, of course two other aspects to divorce and its potential impact on children that we do not address. First is the direct effect of conflict and trauma that although important is beyond the scope of this economic analysis. Second is the decision to split up the household and hence sacrifice potential economies of scale (see Barham (2009)). In our baseline model children's human capital ( $H$ ) is the only public good and so actual living arrangements are irrelevant, we show that our model can be easily adjusted to take account of scale economies and that many of our major insights remain the same.

Some recent papers attempt to directly model conflict in intact marriages. Zhylyevskyy (2012) presents a multiple stage game describes a marriage interaction where the husband may decide to either divorce, offer some transfer to the wife or live in a state of conflict where utility shares are determined according to predetermined bargaining power. The impact of conflict on children is not specifically captured. The paper suggests that both eliminating separation periods and the implementation of perfect child support enforcement reduces conflict (with the latter also reducing the incidence of "inefficient" divorce itself). Anderberg and Rainer (2013) present a theoretical model that allows for males to sabotage the labour market efforts of their partners. It presents a nonlinear relation between relative wage of the female and *intra household* sabotage. Once more, the impact of conflict on children is not explicitly modelled. In our paper, we do not consider the impact of the

change in the mode of parental interaction may have on their psychological wellbeing, we focus on the impact that the change in interaction has on the children via the changes to the effort that the parents commit to the children.

The literature on parental decision in the presence of divorce laws is mainly empirical and focuses on the impact of divorce on the monetary investment of parents rather than other parental inputs such as the quality time that each parent devotes to the education of their children. When explaining the impact of the divorce, the focus seems to be on the monetary contribution of the father and the time the mother spends with the children. If the father has less access to the child after divorce that is seen as negative because it limits the property rights of the father over the child and this makes him want to invest less money on the child. Reinhold et al. (2013) argue that the very possibility of divorce, more specifically unilateral divorce, changes bargaining within couples in a way that is detrimental to children even of "intact" marriages. The positive impact that unilateral divorce seems to have on women's labour participation is indirectly blamed for the negative impact on children (Caceres-Delpiano and Giolito (2008), Reinhold et al (2013), Tartari (2006)). Tartari (2006) presents a dynamic model of human capital development where unilateral divorce is used to shield children from marital conflict (which is assumed to be an input in the child *quality*) but, deprives children of joint parental time and possibly affects parental investments, the father is seen as having control over money and the mother over child's time. Interestingly, these papers see unilateral divorce as something that would transfer bargaining power to the male.<sup>2</sup> The literature tends to present women as being victims rather than initiators of divorce, it is com-

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<sup>2</sup>Stevenson and Wolfers (2006) prove that unilateral divorce law has actually reduced domestic violence as unilateral divorce is said to transfer bargaining power towards the abused, this seems to contradict Tartari (2006)'s arguments.



mon to read statements such as "women being at risk of divorce" yet, an unexplained fact which they acknowledge and seems to contradict such view is that it is women who initiate the majority of unilateral divorce proceedings. It is mothers if anything who are more likely to initiate divorce beyond infancy (see e.g., Anderson (1997) or more recently Hewitt (2009)). Our model predicts that it is those who care most about children's outcomes that will opt for divorce and if these are most likely to be mothers their high propensity to file for divorce can be explained.

The rest of the paper is organised as follows. The next section presents the model and draws out its implications for the impact of divorce on outcomes and what kind of matches will not be stable. Section 3 presents results of some model simulations that demonstrate the implications of our model for empirical researchers looking at the causal effect of divorce on outcomes. The last section concludes. Appendices at the end of the paper present a more general form of our model and also some analytical proofs.

## **2 The Model**

Our model starts at the point where a child is born, it is at that point that the weight each parent puts on children's human capital within their utility function is revealed. Parents then choose whether to "stay married" and make decisions within the household cooperatively (with exogenously given bargaining weights) or "divorce". If divorce happens, the Nash Rule will prevail. Divorce is one sided, it will occur if one partner prefers not to cooperate, indeed we show that if one parent wishes to separate the other wishes to stay together.

## 2.1 The individual utility function

We start by defining the objective functions of the two parents ( $h$  (high) stands for the parent who cares most about the human capital of the child and  $l$  (low) for parent who cares least) under the cooperative and the non cooperative setting. We assume no leisure and no savings so parents care only about their consumption, and the human capital of the child. Parent  $i$  ( $i = h, l$ ) devotes a  $\theta_i$  share of its available time to develop the human capital of the child (human capital production) and the rest of their time to do paid work at a real wage  $w$  (assumed to be the same for both parents)<sup>3</sup>.

$$U_i = H^{\alpha_i} C_i^{1-\alpha_i} \quad (1)$$

where  $H$  is the child's human capital and  $C_i$  is parent  $i$ 's consumption. We abstract from decisions about the types of good being consumed and questions of allocating expenditure between public and private goods by assuming a unique consumption good:

$$C = C_h + C_l = \Gamma w (2 - \theta_m - \theta_d) \quad (2)$$

$$C_i = s_i C$$

In the above equation,  $\Gamma$  is an scale economy variable normalised to equal one under divorce and to be greater or equal to one when they are married and  $s_i$  is the share of total consumption going to parent  $i$ .

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<sup>3</sup>We later discuss a more general form of this model that includes leisure. In this case parents allocate their time between childcare (producing  $H$ ) labour market time (producing consumption) and leisure (enjoyed directly)

We consider a Constant Elasticity of Substitution human capital production function in which we allow parental inputs to be anything from nearly perfect substitutes to perfect complements ( $\gamma < 1$ ).

$$H = \left( \frac{1}{2}\theta_h^\gamma + \frac{1}{2}\theta_l^\gamma \right)^{\frac{1}{\gamma}} \quad (3)$$

## 2.2 The cooperative equilibrium

Under cooperation the household problem is to find the optimal  $s_s$  and  $\theta_s$ . These are those which maximise

$$U_h^\omega U_l^{1-\omega} \quad (4)$$

where  $\omega$  is the exogenously given bargaining weight.

Substituting gives the following first order conditions:

$$\frac{\omega(1 - \alpha_h)}{s_h} = \frac{(1 - \omega)(1 - \alpha_l)}{1 - s_h} \quad (5)$$

$$\frac{\omega\alpha_h + (1 - \omega)\alpha_l}{(\theta_h^\gamma + \theta_l^\gamma)} \theta_h^{\gamma-1} = \frac{\omega\alpha_h + (1 - \omega)\alpha_l}{(\theta_h^\gamma + \theta_l^\gamma)} \theta_l^{\gamma-1} = \frac{\omega(1 - \alpha_h) + (1 - \omega)(1 - \alpha_l)}{(2 - \theta_h - \theta_l)} \quad (6)$$

These give us

$$\theta_h = \theta_l = \theta_c = \omega\alpha_h + (1 - \omega)\alpha_l \quad (7)$$

and

$$\frac{\omega(1 - \alpha_h)}{[(1 - \omega)(1 - \alpha_l) + \omega(1 - \alpha_h)]} = s_h \quad (8)$$

Hence, the optimal theta in the cooperative solution is just a weighted sum of both partners' preferences. Shifts in bargaining power matter more therefore for human capital when there is a big disparity in preferences. Increases in  $\omega$  make parent  $h$  better off, irrespective of the impact on children as they increase their private consumption.

### 2.3 The non-cooperative equilibrium

We now take a closer look at the properties of the non cooperative equilibrium. Under cooperation both partners share childcare and paid employment equally. After divorce this is no longer the case. We show that the partner who gives a higher weight to the child's human capital will do more childcare and spend less time in the labour market. We next show how these shares related to the desired inputs  $\alpha_h$  and  $\alpha_l$  and how this relationship is affected by the degree to which parental inputs are complements.  $\gamma$  (giving the elasticity of substitution of inputs) is a crucial parameter in our model; not only does this affect the *level* of investment by both parents but more importantly it determines the impact of differences in preferences on the nature of strategic interaction between the two parents.

In the non cooperative case, couples choose simultaneously and independently their own contributions to the child's human capital given their expectations about what the other parent is doing. We assume that an efficient maintenance child support system is in place and that the proportion of income going to parent  $h$  is  $p$ . She, given her expectation  $\theta_l^{e_h}$  of  $l$ 's investment, sets  $\theta_h$  to maximise:

$$\left( \frac{\theta_h^\gamma + [\theta_l^{e_h}]^\gamma}{2} \right)^{\frac{\alpha_h}{\gamma}} p^{1-\alpha_h} (2 - \theta_h - \theta_l^{e_h})^{1-\alpha_h} \quad (9)$$

giving the first order condition:<sup>4</sup>

$$\frac{\alpha_h}{[\theta_h^\gamma + [\theta_h^{e_l}]^\gamma]} \theta_h^{\gamma-1} = \frac{1 - \alpha_h}{[2 - \theta_h - \theta_l^{e_m}]} \quad (10)$$

In the appendix we show that a sufficient condition for the slope of this reaction function to be negative is that  $\gamma > 0$ . When inputs are complements the slope has an ambiguous sign. This point is illustrated graphically in the next section

### 2.3.1 Some illustrative examples

Figure 2 shows how these reaction functions (and hence the resultant equilibrium at the crossing point) depend on gamma and preferences. The preference weight of  $l$  is held fixed (at 0.2) but the elasticity of substitution and the preference weight of  $h$  is allowed to vary across panels. The coordinates of the equilibrium position and the resultant level of human capital are also given.

The first panel shows a case where inputs are substitutes ( $\gamma > 0$ ) and the  $\alpha^h$  is 0.4. The reaction functions are downward sloping and the equilibrium is where  $h$  does almost all the childcare. The resultant level of human capital is 0.295. This is slightly lower than under an egalitarian marriage (where it would be 0.3) but higher than if the parent  $l$  had only slightly more bargaining power than parent  $h$ <sup>5</sup>. The possibility that human capital can rise as well as fall after divorce is in fact one of the main findings of our paper.

Panel number 2 shows the case where inputs are complements ( $\gamma < 0$ ). Here there are actually two equilibria; one where both parents contribute

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<sup>4</sup>Note that  $p$  cancels out so that different assumptions on the share going to each parent will have no impact on predicted behaviour.

<sup>5</sup>The maximum level of  $\omega$  for which this is true is the solution to the following:  $2 + 0.2\omega < 0.295$  (i.e.  $0.475 = \omega$ )

nothing and another where they both contribute a positive amount. The intuition for the presence of the first "bad" equilibrium is that when inputs are complements both parents need to do some childcare for the child to have any human capital at all. In the remainder of the paper we assume that parents, although acting independently, can at least commit to a positive contribution in order to rule out the "bad" equilibrium. The good equilibrium contrasts with the equilibrium in panel 2 in two ways. First the level of human capital is slightly lower and second is the fact that childcare is still more evenly distributed.  $h$  still does more but she actually contributes less than her preference weight (0.327 rather than 0.4), while  $l$  contributes more than his preference weight. This convergence of inputs means that parents internalise in part the larger coordination problem after divorce when inputs are complements and is another finding of the paper

Inputs are again substitutes in panel number 3 but here  $\alpha^h$  is now 0.6. The interesting thing about panel 3 is that here time constraints are binding, the unconstrained Nash equilibrium would be at a point where  $h$ 's time input was greater than one. The intuition is that  $h$  would actually like to use some of the money given to her by  $l$  to buy in extra childcare. This possibility is ruled out by assumption<sup>6</sup>. Instead the constrained Nash point occurs at the top of the  $l$ 's reaction function. Human capital is again lower than under an egalitarian marriage but not by very much (0.360 rather than 0.4). Panel 4 shows the equilibrium with high  $\alpha_h$  and when  $\gamma < 0$ . Comparison of this with the other cases suggests differences in  $\alpha_h$  have less role when inputs are complements than when they are substitutes.

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<sup>6</sup>but would actually be a good avenue for future research

### 2.3.2 General conclusions about the nature of the equilibrium point

The Nash equilibrium is defined where the reaction functions cross (i.e. when  $\theta_l^{e_h} = \theta_l$  and  $\theta_h^{e_l} = \theta_h$ ). At this point the relationship between the two equilibrium shares becomes:

$$\frac{\theta_h}{\theta_l} = \left( \frac{(1 - \alpha_l) \alpha_h}{(1 - \alpha_h) \alpha_l} \right)^{\frac{1}{1-\gamma}} \quad (11)$$

giving  $h$ 's contribution as:

$$\frac{2\alpha_h}{\left( 1 + \frac{\alpha_h}{\alpha_l} \left( \frac{(1-\alpha_l)\alpha_h}{(1-\alpha_h)\alpha_l} \right)^{\frac{1}{\gamma-1}} \right)} = \theta_h \quad (12)$$

and hence human capital  $H$  as:

$$\frac{2\alpha_h}{\left( 1 + \frac{\alpha_h}{\alpha_l} \left( \frac{(1-\alpha_l)\alpha_h}{(1-\alpha_h)\alpha_l} \right)^{\frac{1}{\gamma-1}} \right)} \left( \frac{\left( 1 + \left( \frac{(1-\alpha_l)\alpha_h}{(1-\alpha_h)\alpha_l} \right)^{\frac{\gamma}{\gamma-1}} \right)}{2} \right)^{\frac{1}{\gamma}} = H \quad (13)$$

**The relative contributions of  $h$  and  $l$**   $h$  always does more childcare than  $l$  as a sufficient and necessary condition for equation 11 to be greater than one is that  $\alpha^h > \alpha^l$  which is true by assumption. This is, of course, no great surprise. More interestingly it can be seen that the relative share  $\frac{\theta_h}{\theta_l}$  is always increasing in  $\gamma$ . Mathematically this is because the reaction functions are steeper. Intuitively this is because when the marginal effect of an extra hour of childcare depends more on the other parent's input, increases in  $H$  for parent  $h(l)$  have a higher (lower) opportunity cost. Indeed it is possible to show that there exists a threshold of  $\gamma$  (termed  $\gamma_{converge}$ ) below which  $h$  contributes less than  $\alpha^h$  and  $l$  contributes more than  $\alpha^l$

i.e.

$$\gamma_{converge} \equiv \frac{\ln(1 - \alpha_h) - \ln(1 - \alpha_l)}{\ln \alpha_h - \ln \alpha_l}$$

which is negative

The elasticity of substitution also affects the impact of differences in  $\alpha_l$ . There is again a threshold  $\gamma_{react}$  above which differences in  $\alpha_l$  will be negatively related to differences in  $\theta_h$ , holding constant  $\alpha_h$ . The threshold is given by  $\frac{-\alpha_l}{(1-\alpha_l)}$ . (see the appendix for proof)



**Specialisation** Figure 2 shows that it is possible that, for certain parameter values, the time constraint could be binding for parent  $h$ . At this point she spends all of her time in childcare while parent  $l$  contributes a small but still positive amount. Simple manipulation of equation (12) gives the two necessary conditions for a corner solution

$$\gamma > \frac{\ln(1 - \alpha_d) - \ln(1 - \alpha_h) + \ln(2\alpha_h - 1)}{\ln \alpha_d - \ln \alpha_h - \ln(2\alpha_h - 1)}, \alpha_h > \frac{1}{2} \quad (14)$$

$$\gamma_{corner} \equiv \frac{\ln(1 - \alpha_d) - \ln(1 - \alpha_h) + \ln(2\alpha_h - 1)}{\ln \alpha_d - \ln \alpha_h - \ln(2\alpha_h - 1)} \quad (15)$$

$\gamma_{corner}$  may lie below zero if  $\alpha_h$  is particularly high. Thus even when inputs are complements type  $h$ s may move out of the labour market following divorce if they care a lot about the human capital development of their children both absolutely and relative to parent  $l$ 's weighting. It is, however, always the case that  $\gamma_{corner} > \gamma_{converge}$  as

$$\frac{\ln(1 - \alpha_d) - \ln(1 - \alpha_h) + \ln(2\alpha_h - 1)}{\ln \alpha_d - \ln \alpha_h - \ln(2\alpha_h - 1)} > \frac{\ln(1 - \alpha_h) - \ln(1 - \alpha_l)}{\ln \alpha_h - \ln \alpha_l}$$

If the conditions in equation 14 hold, there is a unique equilibrium at the top of  $l$ 's reaction function. We can, however, derive no closed form results about this equilibrium given the non linearities of the reaction functions. In addition any comparative statics on technology or preferences will be contaminated by the discontinuity point given in equation 14. For the remainder of the paper, therefore we restrict  $\alpha_h$  to be less than  $\frac{1}{2}$  in order that such corner solutions can be ruled out.

**Impact of  $\gamma$  and hence divorce on human capital** To understand the impact of technology on the non-cooperative equilibrium it is helpful to

consider the extreme positions. What happens to  $H_d$  as  $\gamma$  gets close to 1 and  $-\infty$ ? When  $\theta_m$  and  $\theta_d$  are very close substitutes ( $\gamma$  is very close to 1),  $h$  can impose an outcome very close to her ideal share simply by contributing  $2\alpha_h$ <sup>7</sup> so long as  $\alpha_h \leq \frac{1}{2}$ . Conversely as  $\gamma$  goes to  $-\infty$ , the shares of both parents become  $\frac{2\alpha_h\alpha_l}{\alpha_h+\alpha_l}$ . As equation 13 is always increasing in  $\gamma$  for each  $(h,l)$  pair we can say that the non cooperative equilibrium human capital ( $H_d$ ) is in the following range:

$$\frac{2\alpha_h\alpha_l}{\alpha_h + \alpha_l} < H_d < \alpha_h$$

This is an important result. The minimum value that  $H_d$  can take ( $\frac{2\alpha_h\alpha_l}{\alpha_h+\alpha_l}$ ) lies strictly above the minimum value that the cooperative equilibrium can take ( $\alpha_l$ ). In fact we can define a threshold value of  $\omega$  below which human capital will be strictly higher following divorce whatever the value of  $\gamma$ . This is given by solving

$$\omega\alpha_h + (1 - \omega)\alpha_l < \frac{2\alpha_h\alpha_l}{\alpha_h + \alpha_l}$$

giving

$$\omega_T \equiv \frac{\alpha_l}{(\alpha_h + \alpha_l)}$$

When  $\omega > \omega_T$  the impact of divorce on human capital depends on the relative size of  $\omega$  and  $\gamma$ . For each value of  $\omega$ , we can define the minimum value of  $\gamma$  that is consistent with human capital rising after divorce. There are no closed form solutions for this curve but Figure 3 plots a few illustrative examples

The curves plotted in figure 3 have an exponential shape and demonstrate the  $H$  can increase following divorce even if  $h$  has a large fraction of the bargaining power. One interesting thing is that when  $\gamma$  is low the thresholds are falling in  $\alpha_h$  and rising in  $\alpha_d$ . This is because when inputs are complements

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<sup>7</sup> As  $\gamma \rightarrow 1$   $\left(\frac{\theta_m^\gamma + \theta_d^\gamma}{2}\right)^{\frac{1}{\gamma}} \rightarrow \left(\frac{\theta_m + \theta_d}{2}\right)$

parents converge towards the lower preference weight. The curves cross at some point so that when  $\gamma$  is close to 1 the thresholds are increasing in  $\alpha_h$  and  $\alpha_h - \alpha_l$ . All the functions tend to 1 as  $\gamma$  gets closer to 1.

### **Concluding remarks about the impact of divorce on human capital**

In this model the "cost of divorce" arises from the complementarity between inputs making human capital more expensive when tastes differ. The impact of divorce on human capital is ambiguous, however, because divorce changes the relative power of the parents to affect outcomes. The degree to which this can happen depends crucially on the elasticity of substitution of inputs. Higher  $\gamma$  implies that divorce is more likely to raise human capital.

One issue is that this simple model excludes the "free rider" effect. Ermish (2003) presents a model where outcomes for children are worse following divorce because each parent does not consider the impact of happier children on the other parents utility. It is actually a simple matter to extend our model to include such a free rider effect. This can be done by introducing a private good (in our case example leisure)<sup>8</sup>. The full model is presented and discussed in the appendix but basic idea is that the allocation of time is between leisure, market work and childcare. Divorced parents may "over consume leisure" as they do not take into account the impact of changes in the public good (income and children's human capital) on the utility of the other parent. It can be shown, however that so long as preferences for leisure and  $\omega$  are sufficiently low it is still possible for human capital to rise following divorce. The effect of  $h$  having more say in outcomes can dominate the free-rider effect. It is, in fact, this model that we use when we conduct

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<sup>8</sup>As we discuss in the next subsection It is also possible to do this by making consumption in the non cooperative case not shared (so each parent simply consumes their own earnings).

the simulations.

All in all we have shown above that divorce has an ambiguous effect on outcomes for children. When inputs are complements and/or the bargaining power of  $h$  is high in marriage, outcomes are likely to be worse for children following a divorce. When inputs are substitutes outcomes are likely to be better.

## 2.4 When will divorce occur?

Divorce will occur if it can make one parent better off. Thus if  $h$ 's utility is higher following a divorce, then the marriage will dissolve. In order to derive the conditions under which this can occur we first need to specify the impact of divorce on consumption. Divorce impacts consumption in three ways. First by changing time spent in the labour market, second by affecting how total consumption is allocated between the two parents and lastly through the absence of scale economies.

We assume that each partner knows how the distribution of total income will be shared following a divorce with  $h$  getting  $pC_{DIV}$  and  $l$  getting  $(1 - p)C_{DIV}$ . Thus  $h$ 's consumption after divorce will be

$$2p(1 - \alpha_h) \left[ \frac{\left( 1 + \left( \frac{(1-\alpha_l)\alpha_h}{(1-\alpha_h)\alpha_l} \right)^{\frac{\gamma}{\gamma-1}} \right)}{\left( 1 + \frac{\alpha_h}{\alpha_l} \left( \frac{(1-\alpha_l)\alpha_h}{(1-\alpha_h)\alpha_l} \right)^{\frac{1}{\gamma-1}} \right)} \right]$$

and given her cooperative consumption of  $2\omega\Gamma(1 - \alpha_h)$  (where  $\Gamma$  denotes the degree of scale economies), her consumption will rise after marriage if

$$\left[ \frac{\left( 1 + \left( \frac{(1-\alpha_l)\alpha_h}{(1-\alpha_h)\alpha_l} \right)^{\frac{\gamma}{\gamma-1}} \right)}{\left( 1 + \frac{\alpha_h}{\alpha_l} \left( \frac{(1-\alpha_l)\alpha_h}{(1-\alpha_h)\alpha_l} \right)^{\frac{1}{\gamma-1}} \right)} \right] > \frac{\omega\Gamma}{p} \quad (16)$$

The ratio on the left hand side lies strictly above 1 if  $\alpha_h > \alpha_l$ . The size of  $\frac{\omega\Gamma}{p}$  is unknown but it is likely to be greater than 1 as  $\Gamma \geq 1$ . Thus the impact of divorce on  $h$  consumption is ambiguous but is negatively related to  $\gamma$ <sup>9</sup>. Total labour market time (and hence total consumption) falls with the elasticity of substitution, thus  $h$  needs to be able to negotiate a larger share if her consumption is to increase. In this vein we plot the threshold value of  $\frac{\omega\Gamma}{p}$  consistent with no change in consumption against  $\gamma$  for different combinations of preferences in figure 4 . Above the curves  $h$ 's consumption falls after divorce, below them it rises. Consider a benchmark value for  $\frac{\omega\Gamma}{p}$  of 1.2.<sup>10</sup> The curve for  $\alpha_h = 0.3, \alpha_l = 0.2$  never exceeds this value, the curve for  $\alpha_h = 0.3, \alpha_l = 0.1$  only exceeds this value when  $\gamma$  is very low, the curve for  $\alpha_h = 0.5, \alpha_l = 0.2$  only exceeds it when  $\gamma < 0.5$  while the curve for  $\alpha_h = 0.5, \alpha_l = 0.1$  only falls below 1.2 when  $\gamma$  is very high. Thus the impact of divorce on  $h$ 's consumption depends on both  $\gamma$  and preferences. When inputs are substitutes divorce is likely to reduce consumption, when they are complements it will increase providing there is a big difference between the two preference weights.

Given that the impact of divorce on both  $H$  and the consumption of parent  $h$  is ambiguous it is clear that the willingness of  $h$  to stay in the marriage will also depend on precise combination of parameters. The reasons for parent  $h$  to split will also vary. When  $\gamma$  is high we may see  $h$  opting for divorce because of the increase in  $H$  even if she will experience a fall in consumption. When  $\gamma$  is low she may chose divorce even if  $H$  is lower because her consumption will rise. In some cases (for example when  $\omega$  and  $\Gamma$  are both low) she will opt for divorce as her enjoyment of both goods will increase.

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<sup>9</sup>The ratio on the left hand side of equation 16 is strictly decreasing in  $\gamma$ .

<sup>10</sup>This is roughly in line with the McClements 1977 equivalence scale and  $\omega = p$ .

Figure 5 plots curves plotting the threshold value of  $\frac{\omega\Gamma}{p}$  consistent with  $h$  being indifferent between divorce and marriage for different combinations of preferences and  $\omega$ . In our model the divorce choice of  $h$  is a trade off between having more control over her time use and the need to coordinate when inputs are complements. This trade off improves when  $\gamma$  is close to 1 and figure 5 shows that all functions are increasing when  $\gamma$  is near 1. The functions are non monotonic though; when  $\gamma$  is very low increases in  $h$ 's consumption are able to compensate her for falls in  $H$ .

Figure 5 illustrates that  $\omega < \frac{1}{2}$  is neither a necessary or sufficient condition for  $h$  to prefer divorce. All the curves plotting the cases where  $\omega = 70\%$  have a range of gamma where the threshold is at or above 1 (the probable minimum value of  $\frac{\omega\Gamma}{p}$ ). Conversely the curves plotting the cases where  $\omega = 30\%$  all have ranges where the threshold is above 1.2 (the level given by the McClemmments equivalence scale). Nevertheless when  $\omega$  is higher divorce is less likely (ceteris paribus).

The thresholds are shown to be increasing in  $\alpha_h$  and falling in  $\alpha_l$ . Thus divorce is more likely when  $h$  cares more about  $H$  and  $l$  care less. Figure 5 shows  $\gamma$  to be crucial. Not only does the threshold change a lot with  $\gamma$  but these changes can and do override differences driven by tastes ( $\alpha_h$  and  $\alpha_l$ ) and differences in bargaining power ( $\omega$ ). All in all the model suggests that divorce is more likely when:

1. The bargaining weight of  $h$  is low.
2. The elasticity of substitution between inputs is either very high or very low.
3. Parent  $h$  cares more about  $H$ .
4. Parent  $l$  cares less about  $H$ .

We have shown above that  $h$  may opt for divorce in certain cases. We next move on to look at  $l$ 's choice. The first thing to note that if  $\frac{(1-\omega)\Gamma}{(1-p)} > 1$  then  $l$ 's consumption will unambiguously fall as the left hand side of equation 16 is strictly less than 1 when  $\alpha_h < \alpha_l$ . Indeed it is trivial to show that  $\frac{(1-\omega)\Gamma}{(1-p)} > 1$  is a sufficient condition for  $l$  to always prefer the cooperative equilibrium. This leads us to the key result of our model:

**Theorem 1** *Divorce is a possible equilibrium outcome when preferences are only revealed after the birth of the child. There exists a range of the parameter space where  $h$  may chose to separate. On the other hand,  $l$  will never choose to separate if  $\frac{(1-\omega)\Gamma}{(1-p)} > 1$ .*

The analysis above has demonstrated how and why divorce may arise endogenously. The mechanism is that preferences for a public good ( $H$ ) are not revealed until the match has been formed. This means that agents experience an asymmetric shock in the gains to marriage. The surplus for one parent will increase while the surplus for the other will fall and may in some cases become negative leading her to file for divorce. A key result is that it is the parent who cares most about  $H$ , if any, who will opt for divorce.

#### 2.4.1 Renegotiation?

Separation is, of course, not Pareto efficient as a costless renegotiated settlement with a higher  $\omega$  could make  $h$  and  $l$  both prefer marriage to divorce. Our model can only explain divorce if such renegotiations can be ruled out. The key question is whether agents can commit to an allocation of time within marriage even though their outside option is only marginally worse. Future work will examine this question further. For the purposes of the present paper, therefore we assume that such renegotiations are either impossible or very costly.

### 2.4.2 Robustness checks - model without consumption sharing

This model assumes that divorced parents can and do share their income according to a certain and predictable sharing rule. This means that there is no divorce cost arising from the fact that labour market activity is a private good while investment in  $H$  is a public good. This next section examines whether a model that assumed that divorced parents could only consume their own earnings would have very different implications. In this model the first order conditions for parent  $h$  are given by

$$\frac{\alpha_h}{\theta_h \left(1 + \left[\frac{\theta_l}{\theta_h}\right]^\gamma\right)} = \frac{1 - \alpha_h}{1 - \theta_h}$$

There are no closed form solutions for this without specifying  $\gamma$ . Figure 6 therefore plots both the thresholds of  $\Gamma_p^\omega$  consistent with  $h$  being indifferent to marital status. It can be seen that when bargaining power is high,  $h$  is unlikely to file for divorce. Nevertheless the figure demonstrates that even in the presence of free-rider effects divorce may be an equilibrium outcome as the gains to cooperation are not symmetric.

We next move on to conduct some simulations to draw out the implications of our model for empirical researchers and for policy.

## 3 Implications for empirical research: Some simulations

In this section we are interested in examining the following question. What does our model imply for a researcher wishing to measure the impact of divorce on outcomes for children?

To motivate this we present some data from the millennium cohort survey. Figure 7 plots distributions of achievement at 5 (measured by total



foundation stage profile score) for two subsamples: those whose parents were living apart when the child was three and those who were living together. We assume that living together = cooperation (the model definition of marriage). The figure demonstrates that the distribution for children from intact marriages stochastically dominates those from divorced couples and that this difference is most marked at the bottom of the distribution. The question remains, however, whether how much of this gap is causal.

To conduct the simulations we use the extended version of our model reported in the appendix. This differs from the model discussed in the section above in that we include leisure as a separate good. This means that the potential for divorce to lower outcomes for children is increased as the model now includes a private good (leisure). We do this to replicate better the data.

In our simulations agents differ by:

- **Preferences.** These are now defined over three goods  $H$ , Consumption ( $C$ ) and Leisure ( $L$ ). Utility is Cobb Douglas and has the following form  $U = H^\alpha L^\beta C^{1-\alpha-\beta}$ . We assume that the quantity  $1 - \alpha - \beta$  is the same within each marriage but may differ across couples, this is because it is likely that preferences over the consumption can be observed before the match takes places. The relative size of  $\alpha$  and  $\beta$  is however only revealed after the child is born and may therefore differ between parents. We allocate these preference weights as transformation  $\frac{\exp(x)}{1+\exp(x)}(K)$  from a normal distribution. This constrains them to be a proportion of the constant  $K$ . The share  $1 - \alpha - \beta$  is constrained to be less than or equal to 0.6 and then  $\alpha$  is defined as a proportion of it.
- **Technology** ( $\gamma$ ). Rather than having a common production function we assume that the elasticity of substitution will vary across couples. The rationale for this is that the complementarity between parents in

the production of human capital will depend on their attributes. As we again assume that parenting styles and attributes are only revealed after the child is born,  $\gamma$  is another shock to marriage. We assume that  $\gamma$  is normally distributed with mean 0.2 and standard deviation 0.4<sup>11</sup>, values of  $\gamma$  above 1 are transformed by dividing them by the sample max

- **Relative bargaining weights ( $\omega$ )** As the identity of  $h$  and  $l$  is not known before the child is born we allow this to vary between 0.3 and 0.7, thus in some couples  $h$  will have more bargaining power. We assume that this distribution is again the transformation  $\frac{\exp(x)}{1+\exp(x)}$  of a normal distribution.
- **Economies of Scale ( $\Gamma$ )** These are allowed to vary between couples as it is likely that different preferences over consumptions, leisure and children implies that couples have different preferences over the type of goods they consume. Thus some couples may have larger economies of scale than others. Allowing  $\Gamma$  to differ across couples will also nest any differences in the ratio  $\frac{p}{\omega}$ . We assume that  $\Gamma$  is distributed  $N(1.2, 0.05)$ .
- All these random variables are assumed independently distributed and the sample size is 5000.

The graphs from these simulations are given in Figure 8. The first panel shows the equivalent to the densities plotted in figure 7. The graph shows a sizeable difference between the two distributions. In our sample outcomes are clearly worse for children of divorced couples. Thus our model is able to replicate what we saw in the data and what is often observed in other

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<sup>11</sup>These values were picked as they seemed to give the smoothest distribution once the positive values were transformed

empirical work. The second panel shows the difference between the two distributions that would be observed if there was no divorce. This is a simple way of illustrating the selection effect. We see quite clearly that about half the difference between the two distributions is driven by the fact that divorce is endogenous. Those couples who separate would have had worse outcomes anyway. The last panel presents, for information, the “actual” distribution of outcomes the children of divorced parents against the distribution that would be seen if their parents were still together. Divorce is still shown to have a negative impact but it is much much smaller than that seen by simply comparing outcomes across children from different family types.

One important point to stress is that the selection effects in our model are not driven by the fact that those who file for divorce care less about their children. In fact, within a couple, it will be those who care most about the children’s outcomes who will wish to separate. Divorce is the optimal strategy for a parent faced with a partner that cares much less than she (or he) does about the children’s human capital. Such families are likely to have poorer outcomes anyway.

## 4 Conclusions

In this paper we have presented a simple model that explains divorce as the result of a disparity in preferences. This disparity is only seen after the birth of a child (otherwise couples would sort on the basis of their preferences). This can reduce the gain from the coordination of time inputs to the parent who gives a higher weight to the child’s human capital and may prompt him or her to opt for divorce. We show:

1. A potential cost of divorce arises from the negative impact of any differences in time inputs on human capital when these are not perfect

substitutes.

2. The impact of divorce is asymmetric. Parents of type  $h$  (who place a higher weight on the child's human capital) may stand to gain from divorce while parents of type  $l$  stand to lose
3. The impact of divorce on children's outcomes is ambiguous. In some cases divorce can improve outcomes for children. This finding is robust (but made weaker) by the inclusion in the model of free rider effects.
4.  $H$  (the child's human capital) following divorce is strictly increasing in  $\gamma$ . Under marriage  $H$  is strictly increasing in  $\omega$  (the bargaining weight of type  $h$ ). This means that for each  $\omega$  there is a threshold value of  $\gamma$  (which may be  $-\infty$ ) above which divorce raises  $H$
5. Divorce can improve outcomes for type  $h$  even if it results in lower  $H$  because  $h$ 's consumption can increase even in the presence of scale economies. This is more likely when  $\gamma$  is low
6. When  $\gamma$  is low  $h$  will file for divorce because of the potential increase in consumption, when  $\gamma$  is high she files because of the potential increase in  $H$ . When  $\gamma$  is around 0 divorce is less likely
7. In our simulations about half the difference in outcomes between children across different family types is due to selection (what kind of family divorces) rather than the causal effect of divorce. Contrary to what is often thought this is not because people who file for divorce give a low weight to their children's human capital, it is because their (ex) *partners* have a lower weight than they do. Divorce is often the best strategy open to high  $\alpha$  parents married to low  $\alpha$  ones.

We have chosen to keep the model as simple as possible to highlight the source of the incentive to stop cooperation and in order to obtain analytical results. Our model is a static, one off decision taken by parents when the child is born. We want to capture the impact that newly acquired knowledge about the quality of the match has on the couple and therefore, on the child. We do not focus on the decisions to have further children as we do not believe that would add any further information to the quality of the match. This very simple model generates some useful insights into why couples may separate and the source of achievement gaps by family status.

The main weakness of the simple model is, however, that we do not incorporate any role for free riding in the non cooperative model. In the appendix and in the simulations we discuss a model in which parents enjoy leisure as well as consumption and the child's human capital. Following a divorce leisure would obviously be "over consumed" as parents would not take into account the impact of increases in their leisure on the utility of the other parent. We show that when demand for leisure is not too high the utility of one parent can still rise after divorce even in the presence of a private good.

The aim of our paper was to link the literature on human capital development, the literature on impact of divorce on children outcomes and the literature on divorce and conflict. The latter link is implicit in our modelling assumptions. More research is required to fully understand the impacts of interactions between parents on human capital development. The simple unitary model of parental investment is becoming increasingly less relevant as more and more children are being bought up by parents living apart.

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## A Graphs

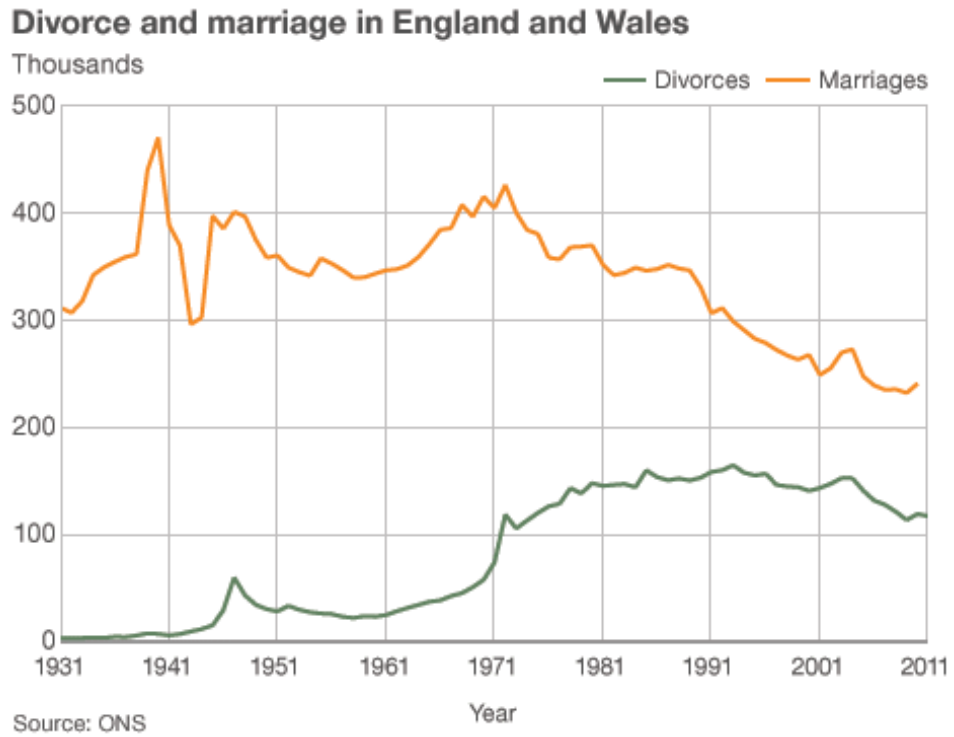


Figure 1: Trends in divorce and marriage.



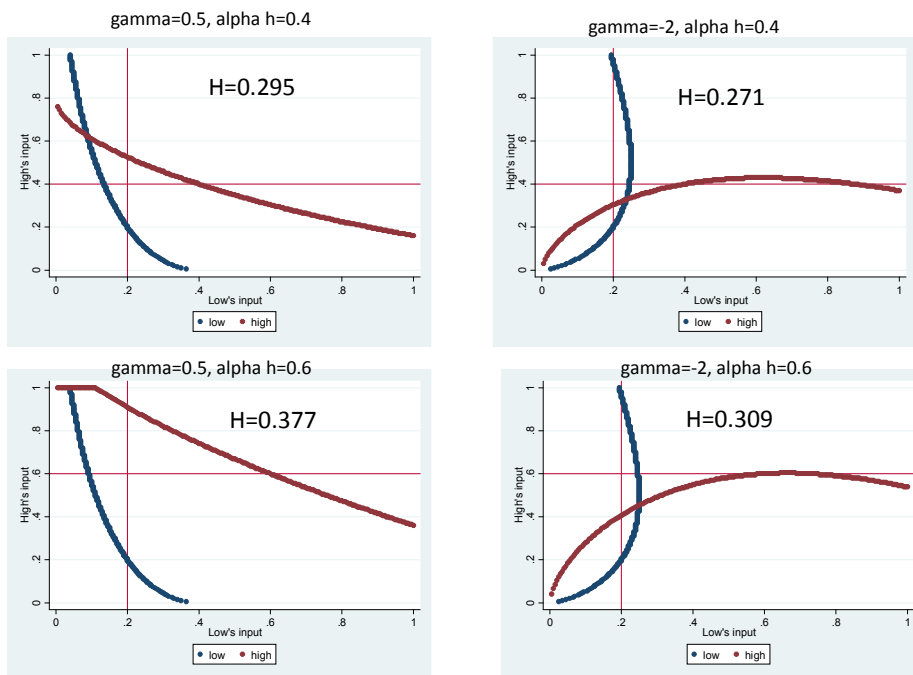


Figure 2: Simulated reaction functions for the case where  $\alpha_l = 0.2$ .

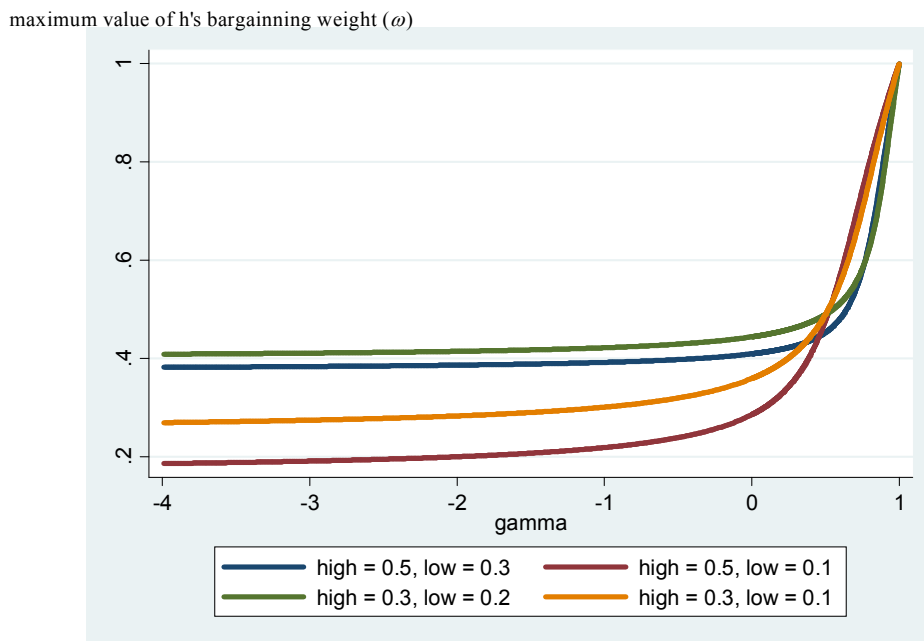


Figure 3: Simulated threshold values for  $\omega$  above which  $H$  is higher under marriage.

maximum value of  $h$ 's relative consumption shares  $\left[ \Gamma \frac{\omega}{p} \right]$

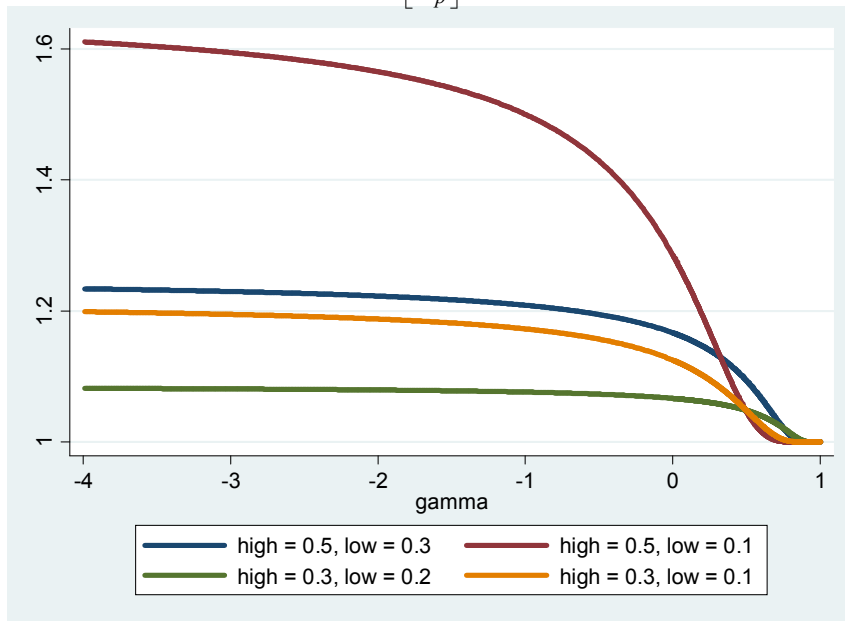


Figure 4: Thresholds above which  $h$ 's consumption is higher under marriage.

maximum value of  $h$ 's relative consumption shares  $\left[ \Gamma \frac{\omega}{p} \right]$

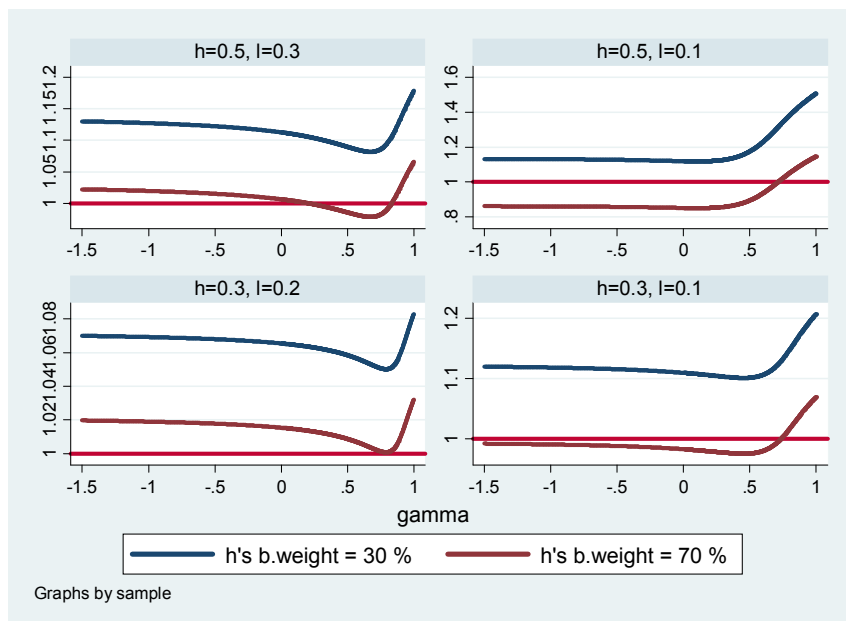


Figure 5: Thresholds above which  $h$  is better off under marriage.

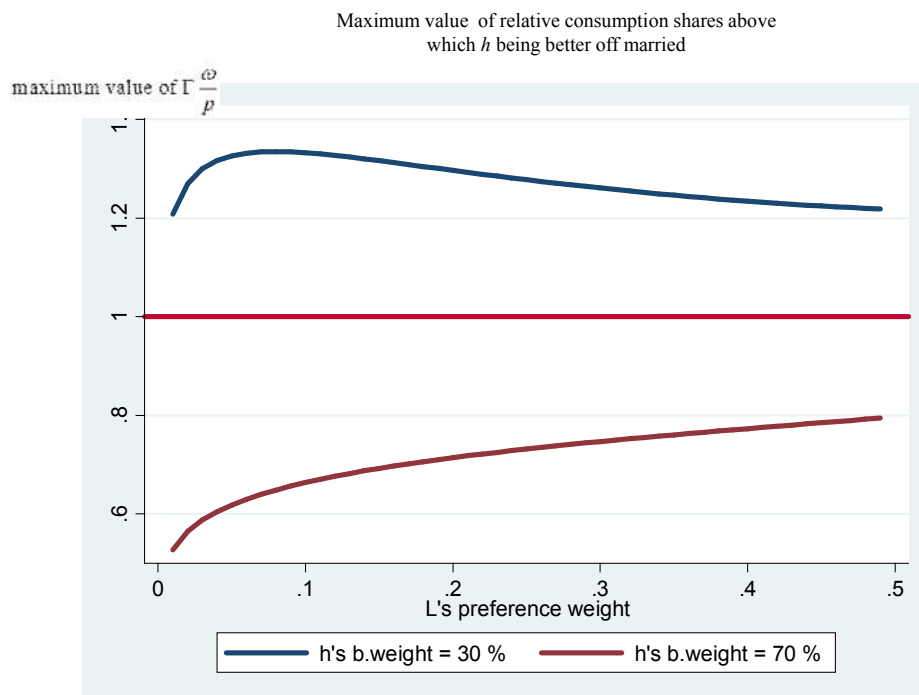
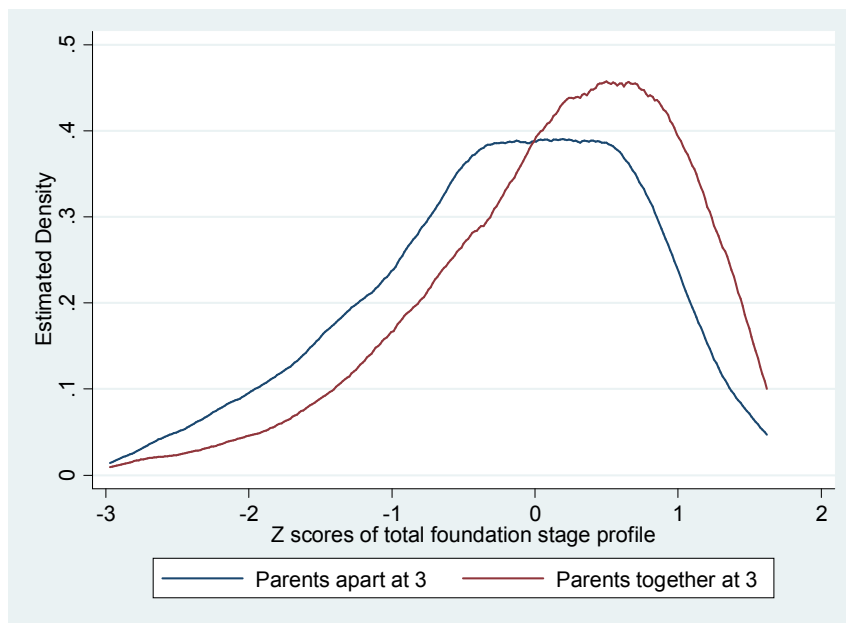


Figure 6: Model with no consumption sharing after divorce



Source: authors' own estimation from the Millennium Cohort Study

Figure 7: Distribution of achievement at 5 by parental marital status

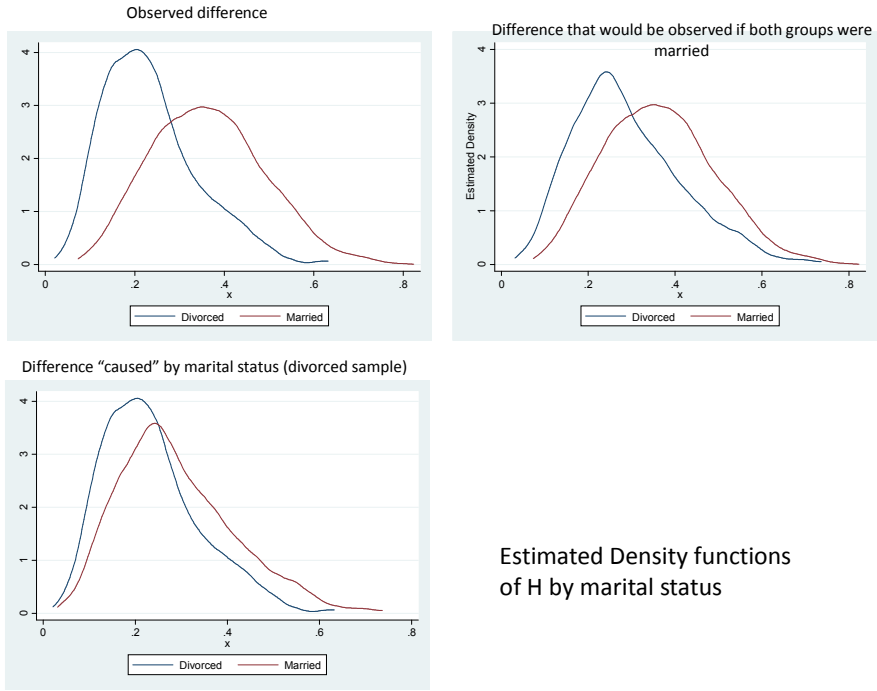


Figure 8: Simulated Distributions of human capital by marital status.

## B Proofs referred to in text

### B.1 The slope of the reaction functions

Let us start from a point  $\theta_h = M$  and  $\theta_l = D$  where parent  $h$  has no incentive to reduce or increase her time inputs. Thus we must have

$$\frac{\alpha_h}{(M^\gamma + D^\gamma)} M^{\gamma-1} = \frac{1 - \alpha_h}{2 - M - D}$$

Let us now increase  $D$  by  $x$ . If the reaction function has a negative slope then

$$\frac{\alpha_h}{(M^\gamma + (D+x)^\gamma)} M^{\gamma-1} < \frac{1 - \alpha_h}{2 - M - D - x}$$

substitution gives

$$\frac{(M^\gamma + D^\gamma)}{2 - M - D} < \frac{(M^\gamma + (D+x)^\gamma)}{2 - M - D - x}$$

if  $\gamma$  and  $x$  are both positive then  $D^\gamma < (D+x)^\gamma$  and  $-D > -D - x$  so this expression is always true. For  $\gamma < 0$  then  $D^\gamma > (D+x)^\gamma$  and we cannot say for sure.

### B.1.1 $l$ response to shifts in $\alpha_h$

Note that we can write the  $l$ 's input as

$$\frac{2\alpha_l}{\left(1 + \frac{\alpha_l}{\alpha_h} \left(\frac{(1-\alpha_h)\alpha_l}{(1-\alpha_l)\alpha_h}\right)^{\frac{1}{\gamma-1}}\right)} = \theta_l$$

Differentiating with respect to  $\alpha_h$  gives:

$$\frac{2\alpha_l}{\left(1 + \frac{\alpha_l}{\alpha_h} \left(\frac{(1-\alpha_h)\alpha_l}{(1-\alpha_l)\alpha_h}\right)^{\frac{1}{\gamma-1}}\right)^2} \left(\frac{\alpha_l}{\alpha_h^2} \left(\frac{(1-\alpha_h)\alpha_l}{(1-\alpha_l)\alpha_h}\right)^{\frac{1}{\gamma-1}} + \frac{1}{\gamma-1} \left(\frac{\alpha_l}{\alpha_h} \left(\frac{(1-\alpha_h)\alpha_l}{(1-\alpha_l)\alpha_h}\right)^{\frac{1}{\gamma-1}} \left(\frac{1}{(1-\alpha_l)}\right)\right)\right)$$

and hence  $l$ 's response will be positive if

$$\gamma < \frac{-\alpha_l}{1 - \alpha_l}$$

## C Model with leisure

Here we derive an extended version of the model where parents allocate time between market work ( $\pi$ ), development of their child's human capital ( $\theta$ ) and leisure ( $1 - \pi - \theta$ )



The utility functions become

$$U_h = H^{\alpha_h} L_h^{\beta_h} C_h^{1-\alpha_h-\beta_h} \text{ and } U_l = H^{\alpha_l} L_l^{\beta_l} C_l^{1-\alpha_l-\beta_l}$$

Under marriage the first order conditions are

$$\frac{\alpha_l + \omega(\alpha_h - \alpha_l)}{\theta_h^\gamma + \theta_l^\gamma} \theta_h^{\gamma-1} = \frac{\omega\beta_h}{1 - \theta_h - \pi_h} = \frac{1 - \alpha_l - \omega(\alpha_h - \alpha_l) - \omega\beta_h - (1 - \omega)\beta_l}{\pi_h + \pi_d}$$

and

$$\frac{\alpha_l + \omega(\alpha_h - \alpha_l)}{\theta_h^\gamma + \theta_l^\gamma} \theta_l^{\gamma-1} = \frac{(1 - \omega)\beta_l}{1 - \theta_l - \pi_l} = \frac{1 - \alpha_l - \omega(\alpha_h - \alpha_l) - \omega\beta_h - (1 - \omega)\beta_l}{\pi_h + \pi_d}$$

and

$$\omega \frac{1 - \alpha_h - \beta_h}{s} = (1 - \omega) \frac{1 - \alpha_l - \beta_l}{1 - s}$$

giving

$$\omega \frac{1 - \alpha_h - \beta_h}{[(1 - \omega)(1 - \alpha_l - \beta_l) + \omega[1 - \alpha_h - \beta_h]]} = s$$

and

$$\pi_h = 1 - \theta_h \left[ \frac{2\omega\beta_h}{\alpha_l + \omega(\alpha_h - \alpha_l)} + 1 \right]$$

$$\pi_l + \pi_h = 2 - 2\theta_h \left[ \frac{[\omega\beta_h + (1 - \omega)\beta_l] + \alpha_l + \omega(\alpha_h - \alpha_l)}{\alpha_l + \omega(\alpha_h - \alpha_l)} \right]$$

$$\alpha_l + \omega(\alpha_h - \alpha_l) = \theta_h$$

$$1 - 2\omega\beta_h - [\alpha_l + \omega(\alpha_h - \alpha_l)] = \pi_h$$

Thus just as in the baseline model outcomes are basically the preference weights. After divorce the first order conditions become

$$\frac{\alpha_h}{\theta_h^\gamma + \theta_l^\gamma} \theta_h^{\gamma-1} = \frac{\beta_h}{1 - \theta_h - \pi_h} = \frac{1 - \alpha_h - \beta_h}{\pi_h + \pi_d}$$

and

$$\frac{\alpha_l}{\theta_h^\gamma + \theta_l^\gamma} \theta_l^{\gamma-1} = \frac{\beta_l}{1 - \theta_l - \pi_l} = \frac{1 - \alpha_l - \beta_l}{\pi_h + \pi_d}$$

We now make the simplification that

$$1 - \alpha_h - \beta_h = 1 - \alpha_l - \beta_l$$

which eases the algebra considerably. Thus we have

$$\theta_h \left[ \frac{\alpha_h}{\alpha_l} \right]^{\frac{1}{\gamma-1}} = \theta_l$$

and this can be used to solve the system giving

$$\frac{2\alpha_h}{\left[ [1 + \beta_l] + [1 + \beta_h] \left( \frac{\alpha_h}{\alpha_l} \right)^{\frac{\gamma}{\gamma-1}} \right]} = \theta_h$$

$H$  then becomes

$$\frac{2\alpha_h}{\left[ [1 + \beta_l] + [1 + \beta_h] \left( \frac{\alpha_h}{\alpha_l} \right)^{\frac{\gamma}{\gamma-1}} \right]} \left[ 1 + \left( \frac{\alpha_h}{\alpha_l} \right)^{\frac{\gamma}{\gamma-1}} \right]^{\frac{1}{\gamma}}$$

If we take the limit as  $\gamma$  becomes we get  $\frac{2\alpha_h}{[1+\beta_l]}$ . Thus to demonstrate that  $H$  could rise after divorce we need to show that

$$\alpha_l + \omega(\alpha_h - \alpha_l) < \frac{2\alpha_h}{[1 + \beta_l]}$$

is a possible inequality. Solving we get

$$\frac{\alpha_l}{\alpha_h} < \left[ 1 - \frac{\beta_l}{(1-\omega)(1+\beta_l)} \right]$$

and as  $\frac{\beta_l}{(1-\omega)(1+\beta_l)}$  is always  $> 1$ , then a higher  $H$  after divorce is a possible outcome.

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