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Exchange Rate Variations

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Banco de México

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## Endogenous Trade, Nontraded Goods and Real Exchange Rate Variations\*

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**Abstract:** This paper evaluates whether a macroeconomic trade model, where the decision of trade and the Balassa-Samuelson effect are endogenous, can explain recent empirical facts about the importance of nontraded goods prices in real exchange rate variations better than a standard Balassa-Samuelson model, where nontraded goods are exogenously determined. The model is modified and calibrated to an asymmetric equilibrium that allows the steady state of the model to match some of the US-Mexico relationships quite well. The results suggest an importance of nontraded goods in real exchange rate volatility closer to the empirical evidence. In addition, the model replicates the findings that nontraded prices exhibit higher volatility than the real exchange rate and that these prices are negatively correlated with traded goods prices.

**Keywords:** Real Exchange Rate, Endogenous Trade, Firm Heterogeneity, Firm Dynamics

**JEL Classification:** F12, F31, F41

**Resumen:** Este artículo evalúa si un modelo macroeconómico de comercio, donde la decisión de comercio y el efecto Balassa-Samuelson son endógenos, puede explicar hechos empíricos recientes acerca de la importancia de los precios de los bienes no comerciados en las variaciones del tipo de cambio real mejor que un modelo estándar de Balassa-Samuelson, donde los bienes no comerciados se determinan de forma exógena. El modelo es modificado y calibrado a un equilibrio asimétrico que permite al estado estacionario del modelo replicar algunas de las relaciones EU-México bastante bien. Los resultados sugieren una importancia de los bienes no comerciados en la volatilidad del tipo de cambio real más cercana a la evidencia empírica. Además, el modelo replica los hallazgos de que los precios no comerciados exhiben una mayor volatilidad que el tipo de cambio real y que estos precios están correlacionados negativamente con los precios de los bienes comerciados.

**Palabras Clave:** Tipo de Cambio Real, Comercio Endógeno, Firms Heterogéneas, Dinámica de la Firma

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## 1. Introduction

Recent empirical evidence provided by Hernandez (2012) shows that using current highly disaggregated data on prices and trade between Mexico and the US, nontraded goods prices can explain more than 60% of the real exchange rate volatility. The author also found that the relative price of traded goods (RERT) and the relative price of nontraded to traded goods (RERN) are negatively correlated at all frequencies of the data and in contrast with the findings of Mendoza (2000) and Naknoi (2008) that in flexible exchange rate regimes such correlation becomes positive.

In order to explain the relationship between nontraded goods prices and real exchange rate variations researchers have modified international macroeconomic models with sticky prices, export costs, distribution costs, imperfect competition and/or endogenous tradability. For example, Chari et al. (2002), confirming Engel (1999) results, develop a dynamic stochastic general equilibrium (DSGE) monetary model with sticky prices allowing them to replicate the high volatility of the real exchange rate (RER), but failing to replicate exchange rate persistence. Betts and Kehoe. (2001) proposed a multicountry-multisector general equilibrium model that matched the correlation between the relative price of nontraded to traded goods and the real exchange rate for the case of Mexico and the US. Naknoi (2008) proposed a two country DSGE monetary model with endogenous tradability and sticky wages and found that during exchange rate stabilization periods interest rate shocks create a negative correlation between RERT and RERN raising the importance of RERN by 31% in RER variations. Nevertheless, in a flexible exchange rate regime this correlation becomes positive so the same shocks deteriorate the contribution of RERN in RER volatility. Lastly, Drozd and Nosal (2010) adapted a standard international macroeconomic model, based on Stockman and Tesar (1995), that explicitly incorporates a nontraded goods sector.

This paper builds up on the endogenous tradability hypothesis suggested by Betts and Kehoe. (2001), Bergin and Glick. (2007); Bergin and Glick (2009) and Naknoi (2008) with the objective to evaluate how a business cycle model with endogenous tradability and entry and exit from both domestic and export markets, á la Ghironi and Melitz (2005), performs in explaining the results found in Hernandez (2012). Some of the main properties that make this model attractive to perform such task are that even though it generates PPP deviations through pricing to market behavior, it

is still possible for the model to imply a Balassa-Samuelson effect based in the microeconomic structure of the model.

The contributions of this paper to the literature are two fold. First, since in principle the model is not constructed to directly assess the impact of nontraded goods prices in real exchange rate variations, this paper suggest some changes in the model in order to perform such analysis. These changes, detailed below, will involve the differentiation between traded and nontraded goods prices, a different method to compute price indices closely to how is computed by statistical institutions, and an adjustment to the real exchange rate so that it can be expressed as a function of traded and nontraded goods prices. Second, the model is calibrated to an asymmetric equilibrium where two different economies interact, one large economy representing the US and one small economy representing Mexico.

As a result of the above the steady state of the model matches some of the US-Mexico relationships quite well. First, the steady state of the model implies that home (US) consumption is four times higher than foreign (Mexico) consumption, which is close to the one reported by Bergin and Glick (2009). Second, home average real wage is 1.4 times that of foreign country. Although it can be considered a good feature of the model that home wages are higher, this value is significantly smaller than the values reported in other studies where wages in the US are up to eight times higher than in Mexico. Third, the number of entry and exporting firms is also higher in home country. Lastly, in the steady state prices set by home firms are higher than those set by foreign firms.

The results show that the newly calibrated model implies an importance of nontraded goods in real exchange rate volatility closer to the empirical evidence in Hernandez (2012). In addition, the model also replicates the findings that the nontraded component is highly correlated with and more volatile than the real exchange rate. Another important feature of the model is that under the assumptions of an asymmetric equilibrium, different persistency and zero spillovers from productivity shocks the model can replicate the negative correlation between the relative price of traded goods and the relative price of nontraded to traded goods as found in Hernandez (2012). This does not occur in the symmetric version of the model where all variables synchronize and move in the same direction overestimating the results of the model. In particular, in the symmetric calibrated

model the components of the CPI base real exchange rate are positively correlated.

The paper is organized as follows. The next section provides the details of the model. The third section explains how nontraded goods prices are identified in the model and the modifications to the prices indices. The fourth section defines the real exchange rate and its decomposition into traded and nontraded goods prices and also discusses how the Balassa-Samuelson effect is endogenously generated by the model. The fifth section explains the calibration to an asymmetric equilibrium. Results are shown in the sixth section. The seventh section concludes.

## 2. The Model

Ghironi and Melitz (2005) specify a two country DSGE model with heterogeneous firms and entry and exit from both, domestic and the export markets with flexible prices. This model assumes that there exists a continuum of goods  $V$  ( $V^*$ ) in home (foreign) country. Hence, at any time  $t$  there is a variety of goods available to home (foreign) consumers  $V_t \subset V$  ( $V_t^* \subset V^*$ ). The set of varieties available at home may differ from the set of varieties available at foreign.<sup>1</sup>

Lastly, households and firms behave identically in both home and foreign countries. This assumption will simplify the exposition of the model since all the algebraical representation of home agents' problems and solutions are analogous to those of foreign agents' problems and solutions.

### 2.1. Intra-temporal Solution

At time  $t$ , households consume a basket of goods  $C_t$  defined over the available varieties  $V_t$  by constant elasticity of substitution composites with intertemporal elasticity of substitution  $\theta$  across varieties. Given the price of variety  $v$ ,  $p_t(v)$ , home representative household minimize the cost of consuming  $C_t$ , i.e.

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<sup>1</sup>As typical in the literature home variable are differentiated from foreign variables by denoting the last ones with an asterisk.

$$\begin{aligned} \min \quad & \int_{v \in V_t} p_t(v) c_t(v) dv \\ \text{s.t.} \quad & C_t = \left[ \int_{v \in V_t} c_t^{\frac{\theta-1}{\theta}} dv \right]^{\frac{\theta}{\theta-1}} \end{aligned}$$

and foreign representative households solve:

$$\begin{aligned} \min \quad & \int_{v^* \in V_t^*} p_t^*(v^*) c_t^*(v^*) dv^* \\ \text{s.t.} \quad & C_t^* = \left[ \int_{v^* \in V_t^*} c_t^{*\frac{\theta-1}{\theta}} dv^* \right]^{\frac{\theta}{\theta-1}} \end{aligned}$$

Denote home and foreign representative households' demand for variety  $v$  and  $v^*$  as  $c_t(v)$  and  $c_t^*(v^*)$  respectively. Then, from the solution of the minimization problem we get:

$$c_t(v) = \left( \frac{p_t(v)}{P_t} \right)^{-\theta} C_t \quad (1)$$

$$c_t^*(v^*) = \left( \frac{p_t^*(v^*)}{P_t^*} \right)^{-\theta} C_t^* \quad (2)$$

CES aggregation over varieties implies that the price index at home and foreign country can be expressed as:

$$P_t = \left[ \int_{v \in V_t} p_t(v)^{1-\theta} dv \right]^{\frac{1}{1-\theta}} \quad (3)$$

$$P_t^* = \left[ \int_{v^* \in V_t^*} p_t^*(v^*)^{1-\theta} dv^* \right]^{\frac{1}{1-\theta}} \quad (4)$$

Home and foreign firms behave monopolistically. Each firm produces one variety  $v$  ( $v^*$ ) with linear technology  $zZ_t L_t$  ( $z^* Z_t^* L_t^*$ ), where  $z$  ( $z^*$ ) is the firm relative productivity level,  $Z_t$  ( $Z_t^*$ ) indexes aggregate labor productivity, and  $L_t$  ( $L_t^*$ ) is firm's labor demand.

Given household demand for variety  $v$ , at any given  $t$  home firms selling in home market set prices so as to maximize profits  $\Pi_t$ , i.e. home firms solve:

$$\max \Pi_t = \left[ p_t - \frac{W_t}{zZ_t} \right] \left( \frac{p_t}{P_t} \right)^{-\theta} C_t \quad (5)$$

Analogously, foreign firms selling in foreign market solve:

$$\max \Pi_t^* = \left[ p_t^* - \frac{W_t^*}{z^*Z_t^*} \right] \left( \frac{p_t^*}{P_t^*} \right)^{-\theta} C_t^* \quad (6)$$

Profit maximization implies that the price set by each firm will be a function of its relative productivity level  $z$ . Let  $w_t = W_t/P_t$  be the real wage, then, optimal prices in real terms set by home and foreign firms in their own domestic market are:

$$\tilde{p}_t(z) = \frac{\theta}{\theta - 1} \frac{w_t}{zZ_t} \quad (7)$$

$$\tilde{p}_t^*(z^*) = \frac{\theta}{\theta - 1} \frac{w_t^*}{z^*Z_t^*} \quad (8)$$

This implies that total real profits of home and foreign firms selling in home and foreign market respectively are:

$$\tilde{\Pi}_{D,t}(z) = \frac{1}{\theta} \tilde{p}_t(z)^{1-\theta} C_t \quad (9)$$

$$\tilde{\Pi}_{D,t}^*(z^*) = \frac{1}{\theta} \tilde{p}_t^*(z^*)^{1-\theta} C_t^* \quad (10)$$

Firms desiring to enter the export market must face both a fixed cost  $w_t f_{X,t}/Z_t$  ( $w_t^* f_{X,t}^*/Z_t^*$ ) expressed in units of effective labor, and an iceberg-melting cost  $\tau_t$  ( $\tau_t^*$ ).<sup>2</sup> Once a firm has entered the export market it exhibits a pricing to market behavior in addition to behave monopolistically.

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<sup>2</sup>Where variables related to export goods are denoted by the subindex  $X$ .



Home exporting firms solve:

$$\max \tilde{\Pi}_{X,t} = Q_t \left[ \tilde{p}_{X,t} - Q_t^{-1} \frac{\tau_t w_t}{z Z_t} \right] \left( \frac{\tilde{p}_{X,t}}{P_t^*} \right)^{-\theta} C_t^* - \frac{w_t f_{X,t}}{Z_t} \quad (11)$$

and foreign exporting firms solve:

$$\max \tilde{\Pi}_{X,t}^* = Q_t^{-1} \left[ \tilde{p}_{X,t}^* - Q_t \frac{\tau_t^* w_t^*}{z^* Z_t^*} \right] \left( \frac{\tilde{p}_{X,t}^*}{P_t} \right)^{-\theta} C_t - \frac{w_t^* f_{X,t}^*}{Z_t^*} \quad (12)$$

where  $\tilde{p}_{X,t}$  ( $\tilde{p}_{X,t}^*$ ) is the real export price,  $Q_t = S_t \frac{P_t^*}{P_t}$  is the real exchange rate and  $S_t$  is the nominal exchange rate.<sup>3</sup>

Optimal prices set by the home and foreign exporting firms are:

$$\tilde{p}_{X,t}(z) = Q_t^{-1} \frac{\theta}{\theta - 1} \frac{\tau_t w_t}{z Z_t} \quad (13)$$

$$\tilde{p}_{X,t}^*(z^*) = Q_t \frac{\theta}{\theta - 1} \frac{\tau_t^* w_t^*}{z^* Z_t^*} \quad (14)$$

And real profits for each exporting firm are:

$$\tilde{\Pi}_{X,t}(z) = \frac{Q_t}{\theta} \tilde{p}_{X,t}(z)^{1-\theta} C_t^* - \frac{w_t f_{X,t}}{Z_t} \quad (15)$$

$$\tilde{\Pi}_{X,t}^*(z^*) = \frac{Q_t^{-1}}{\theta} \tilde{p}_{X,t}^*(z^*)^{1-\theta} C_t - \frac{w_t^* f_{X,t}^*}{Z_t^*} \quad (16)$$

## 2.2. Firm Dynamics, Average Prices and Average Profits

In home and foreign countries firms prices and profits depend on each firm specific productivity level  $z$ ,  $z^*$  respectively. However, how each firm makes the decision of whether or not to start activities and whether or not to enter the export market will be detailed below.

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<sup>3</sup>Note that the revenue for the home exporting firm is denominated in foreign consumption units but the costs are expressed in units of home consumption, hence the need to include  $Q_t^{-1}$ .

Before production starts all home and foreign firms are identical. They must draw their productivity level  $z$  and  $z^*$  from a standard distribution  $G(z)$ .<sup>4</sup> Once this productivity level is drawn it will remain fixed. This implies that firms will be heterogeneous in the sense that they will have different productivity levels.<sup>5</sup>

Producing implies that home and foreign firms must face a sunk entry cost measured in effective units of labor of  $w_t f_{E,t}/Z_t$  and  $w_t^* f_{E,t}^*/Z_t^*$ . These costs are paid period by period by the firms and hence excluded from the profit computation. Then, there exist productivity levels  $z_{min}$ ,  $z_{min}^*$  such that  $\tilde{\Pi}_{D,t}(z_{min}) = 0$  and  $\tilde{\Pi}_{D,t}^*(z_{min}^*) = 0$ . Therefore, any firm with productivity level below  $z_{min}$  will never produce. In the same fashion, since exporting firms must pay a fixed cost then this will define a cut-off productivity level  $z_{X,t}$ ,  $z_{X,t}^*$  such that  $\tilde{\Pi}_{X,t}(z_{X,t}) = 0$  and  $\tilde{\Pi}_{X,t}^*(z_{X,t}^*) = 0$ . These  $z_{X,t}$ ,  $z_{X,t}^*$  represent the minimum productivity level needed to enter the export market. Note that  $z_{min} < z_{X,t}$  and  $z_{min}^* < z_{X,t}^*$  so that exporting firms can sell in both domestic and export markets.

Once a firm has decided to start producing it will require one period to build up. Then, a firm initiating activities at time  $t$  will start producing at  $t + 1$ . At every  $t$  there exists an exogenous exit shock that occurs with fixed probability  $\delta$ . In addition, there is an unlimited number of firms expecting to enter the market, we denote these firms by  $N_{E,t}$  and  $N_{E,t}^*$  for home and foreign country respectively. These possible entrants are forward looking and correctly estimate the present discounted value of all their future profits.

There is a number of producing firms  $N_t$ ,  $N_t^*$  in every period at home and in the foreign country given by the number of firms that survive the previous period exit shock and the firms that entered in  $t - 1$ . This implies that:<sup>6</sup>

$$N_t = (1 - \delta)[N_{t-1} + N_{E,t-1}] \quad (17)$$

$$N_t^* = (1 - \delta)[N_{t-1}^* + N_{E,t-1}^*] \quad (18)$$

Now, as noted in Ghironi and Melitz (2005)  $G(z)$  can also be thought of as the productivity

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<sup>4</sup>Ghironi and Melitz (2005) provide an excellent description of such distribution.

<sup>5</sup>These differences also imply differences in the technology used by the firms.

<sup>6</sup>The probability of survival is given by  $1 - \delta$ .

distribution of all home (foreign) producing firms. Then, if the productivity level needed to enter the exporting market is known, it is possible to determine the number of exporting firms in each country; i.e. knowing  $z_{X,t}$  and  $z_{X,t}^*$  the number of exporting firms are  $N_{X,t} = [1 - G(z_{X,t})]N_t$  at home and  $N_{X,t}^* = [1 - G(z_{X,t}^*)]N_t^*$  at foreign.

Following Ghironi and Melitz (2005) two special average productivity levels are defined:

$$\tilde{z}_D \equiv \left[ \int_{z_{min}}^{\infty} z^{\theta-1} dG(z) \right]^{\frac{1}{\theta-1}} \quad (19)$$

$$\tilde{z}_{X,t} \equiv \left[ \frac{1}{1 - G(z_{X,t})} \int_{z_{X,t}}^{\infty} z^{\theta-1} dG(z) \right]^{\frac{1}{\theta-1}} \quad (20)$$

where  $\tilde{z}_D$  is the average productivity level of all home firms, and  $\tilde{z}_{X,t}$  is the average productivity level of all home exporting firms (analogous definitions apply for foreign country).

Then, define  $\hat{p}_{D,t} \equiv \tilde{p}_{D,t}(\tilde{z}_D)$  ( $\hat{p}_{D,t}^* \equiv \tilde{p}_{D,t}^*(\tilde{z}_D^*)$ ) as the real average price of home (foreign) goods sold in home (foreign) market and  $\hat{p}_{X,t} \equiv \tilde{p}_{X,t}(\tilde{z}_{X,t})$  ( $\hat{p}_{X,t}^* \equiv \tilde{p}_{X,t}^*(\tilde{z}_{X,t}^*)$ ) as the real average price of home (foreign) goods in the export market. Then, home price index must express the price of all home firms in the home market plus the price of all foreign exporters in home market. In real terms the home and foreign price index are:

$$1 = N_t(\tilde{p}_{D,t})^{1-\theta} + N_{X,t}^*(\tilde{p}_{X,t}^*)^{1-\theta} \quad (21)$$

$$1 = N_t^*(\tilde{p}_{D,t}^*)^{1-\theta} + N_{X,t}(\tilde{p}_{X,t})^{1-\theta} \quad (22)$$

In the same way we define  $\hat{\Pi}_{D,t} = \tilde{\Pi}_{D,t}(\tilde{z}_D)$  ( $\hat{\Pi}_{D,t}^* = \tilde{\Pi}_{D,t}^*(\tilde{z}_D^*)$ ) as home (foreign) average real domestic profits, and  $\hat{\Pi}_{X,t} = \tilde{\Pi}_{X,t}(\tilde{z}_{X,t})$  ( $\hat{\Pi}_{X,t}^* = \tilde{\Pi}_{X,t}^*(\tilde{z}_{X,t}^*)$ ) as home (foreign) average real export profits. Thus, we can represent the average total profits of home and foreign firms as:

$$\hat{\Pi}_t = \hat{\Pi}_{D,t} + \frac{N_{X,t}}{N_t} \hat{\Pi}_{X,t} \quad (23)$$

$$\hat{\Pi}_t^* = \hat{\Pi}_{D,t}^* + \frac{N_{X,t}^*}{N_t^*} \hat{\Pi}_{X,t}^* \quad (24)$$

where  $N_{X,t}/N_t = 1 - G(z_{X,t})$  and  $N_{X,t}^*/N_t^* = 1 - G(z_{X,t}^*)$  are the fraction of home and foreign exporting firms.

Firms will continue entering the domestic market up to the point where the average value of the firm,  $\tilde{v}_t$ , equals the entry cost providing us with the "free entry" conditions:<sup>7</sup>

$$\tilde{v}_t = \frac{w_t f_{E,t}}{Z_t} \quad (25)$$

$$\tilde{v}_t^* = \frac{w_t^* f_{E,t}^*}{Z_t^*} \quad (26)$$

Productivity levels  $z$  are parameterized as in Ghironi and Melitz (2005):  $G(z)$  is a Pareto distribution with lower bound  $z_{min}$  and shape parameter  $k > \theta - 1$ . Total average productivity  $\tilde{z}_D = [k/(k - \theta + 1)^{\frac{1}{\theta-1}}] z_{min}$ . Average productivity of exporters is  $\tilde{z}_{X,t} = [k/(k - \theta + 1)^{\frac{1}{\theta-1}}] z_{X,t}$ , and the share of exporting firms is  $N_{X,t}/N_t = \{[k/(k - \theta + 1)^{\frac{1}{\theta-1}}] z_{min}/\tilde{z}_{X,t}\}^k$  with analogous values for foreign country.

Then, the zero export profits condition becomes:

$$\hat{\Pi}_{X,t} = (\theta - 1) \frac{1}{k - \theta + 1} \frac{w_t f_{X,t}}{Z_t} \quad (27)$$

$$\hat{\Pi}_{X,t}^* = (\theta - 1) \frac{1}{k - \theta + 1} \frac{w_t^* f_{X,t}^*}{Z_t^*} \quad (28)$$

### 2.3. Intertemporal Solution

The representative household maximizes the expected discounted value of consumption given by:

$$\max E_t \left[ \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\gamma}}{(1-\gamma)} \right]$$

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<sup>7</sup> $\tilde{v}_t$  is equivalent to the present discounted value of all future profits.

where  $\gamma$  is the intertemporal elasticity of substitution.

Household's income come from wages ( $w$ ) and the returns obtained from two available types of assets: domestic risk-free bonds ( $B$ ) and shares in a mutual fund of home firms ( $\varrho$ ). The amount of shares purchased in  $t-1$  ( $\varrho_t$ ) pay a profit at the beginning of  $t$  equal to the total average profits of all producing home firms ( $P_t \hat{\Pi}_t N_t$ ), plus all the claims on future profits ( $P_t \tilde{v}_t$ ). In turn, the domestic risk-free bonds obtained in  $t-1$  ( $B_t$ ) yield a return of  $1 + r_t$ , where  $r_t$  is the consumption based interest rate known in  $t-1$ . During  $t$ , the representative household uses his income to consume and purchase bonds and shares. Note that the household does not have information about which firm will be affected by the exit inducing shock, forcing them to invest in all the surviving firms at the end of  $t-1$  and all the new entrants at the beginning of time  $t$ , i.e.  $N_{H,t} = N_t + N_{E,t}$ . Hence the budget constraint becomes:

$$B_{t+1} + \tilde{v}_t N_{H,t} \varrho_{t+1} + C_t = (1 + r_t) B_t + (\hat{\Pi}_t + \tilde{v}_t) N_t \varrho_t + w_t L$$

Similarly, foreign representative household solves:

$$\begin{aligned} \max \quad & E_t \left[ \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{*1-\gamma}}{(1-\gamma)} \right] \\ \text{s.t.} \quad & B_{t+1}^* + \tilde{v}_t^* N_{H,t}^* \varrho_{t+1}^* + C_t^* = (1 + r_t^*) B_t^* + (\hat{\Pi}_t^* + \tilde{v}_t^*) N_t^* \varrho_t^* + w_t^* L^* \end{aligned}$$

Solving forward the first order conditions an expression for the average value of firms is found (expected discounted value of all future profits). This implies that the average value of the firm is:

$$\begin{aligned} \tilde{v}_t &= E_t \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \hat{\Pi}_s \\ \tilde{v}_t^* &= E_t \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} \hat{\Pi}_s^* \end{aligned}$$

Assuming financial autarky ( $B_{t+1} = B_t = 0$ ;  $\varrho_{t+1} = \varrho_t = 1$ ) implies the following expressions for aggregate accounting:

$$C_t = w_t L + N_t \hat{\Pi}_t - N_{E,t} \tilde{v}_t \quad (29)$$

$$C_t^* = w_t^* L^* + N_t^* \hat{\Pi}_t^* - N_{E,t}^* \tilde{v}_t^* \quad (30)$$

Finally, balance trade implies that exports from home to foreign country must be equal to exports from foreign to home country when expressed in the same units of consumption, i.e.

$$Q_t N_{X,t} (\hat{p}_{X,t})^{1-\theta} C_t^* = N_{X,t}^* (\hat{p}_{X,t}^*)^{1-\theta} C_t \quad (31)$$

### 3. Nontraded Goods and Laspeyres Price Index

In principle the model is not constructed to directly assess the impact of nontraded goods' prices on real exchange rate variations. Hence, some changes in the model are necessary in order to perform such analysis. These changes will involve the differentiation between traded and nontraded goods prices, a different method to compute price indices, and an adjustment to the real exchange rate so that it can be expressed as a function of traded and nontraded goods prices.

The first step is to differentiate between traded and nontraded goods prices. Second, firms producing nontraded goods are characterized by having a productivity level below the cut-off export level  $z_{X,t}$ . Then, average real nontraded goods prices will depend on firms average productivity level  $z$  such that  $z < z_{X,t}$ .

Thus, lets define an additional average productivity level for all home and foreign firms producing nontraded goods as:

$$\tilde{z}_{N,t} \equiv \left[ \frac{1}{G(z_{X,t})} \int_{z_{min}}^{z_{X,t}} z^{\theta-1} dG(z) \right]^{\frac{1}{\theta-1}}$$

$$\tilde{z}_{N,t}^* \equiv \left[ \frac{1}{G(z_{X,t}^*)} \int_{z_{min}}^{z_{X,t}^*} z^{\theta-1} dG(z) \right]^{\frac{1}{\theta-1}}$$

where  $\tilde{z}_{N,t} < \tilde{z}_{D,t} < \tilde{z}_{X,t}$  and  $\tilde{z}_{N,t}^* < \tilde{z}_{D,t}^* < \tilde{z}_{X,t}^*$

With this new productivity level the model will include three different average prices in each country. Home firms producing nontraded goods setting an average real price of  $\tilde{p}_{N,t}$ .<sup>8</sup> Home exporting firms selling traded goods in their domestic market setting an average real price of  $\tilde{p}_{D,t}$ , and home exporting firms setting a average real price in the foreign market of  $\tilde{p}_{X,t}$ , with analogous prices in foreign country.

Having defined  $\tilde{z}_{N,t}$  and  $\tilde{z}_{N,t}^*$  it is possible to determine the fractions of home and foreign firms producing nontraded goods. These fractions are:

$$\frac{N_{N,t}}{N_t} = G(\tilde{z}_{X,t}) = 1 - \left[ \frac{k}{k - \theta + 1} \right]^{\frac{k}{\theta-1}} \left[ \frac{z_{min}}{\tilde{z}_{X,t}} \right]^k \quad (32)$$

$$\frac{N_{N,t}^*}{N_t^*} = G(\tilde{z}_{X,t}^*) = 1 - \left[ \frac{k}{k - \theta + 1} \right]^{\frac{k}{\theta-1}} \left[ \frac{z_{min}^*}{\tilde{z}_{X,t}^*} \right]^k \quad (33)$$

### 3.1. Laspeyres Price Index

Now, why should the price index be modified? The answer is that since preferences and the elasticity of substitution among varieties of goods are not observable in real life, statistical institutions do not compute the CPI based on the price index implied by the model. This explains why such price index is different from the empirical CPI data.<sup>9</sup>

In order to replicate the empirical evidence in Hernandez (2012) more accurately, the price index of nontraded and traded goods is computed following the methodology used by such statistical institutions.<sup>10</sup> This methodology consists on gathering data on prices from an ex-ante defined set of goods. Once these prices are obtained the goods are grouped in more general categories by averaging their prices. Finally, using previously defined weights and a base year the Laspeyres formula is applied.

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<sup>8</sup>By optimal pricing  $\tilde{p}_{N,t} = \frac{\theta}{\theta-1} \frac{w_t}{\tilde{z}_{N,t} Z_t}$  and  $\tilde{p}_{N,t}^* = \frac{\theta}{\theta-1} \frac{w_t^*}{\tilde{z}_{N,t}^* Z_t}$ .

<sup>9</sup>Ghironi and Melitz (2005) adjusted the model price indices in order to reflect average prices and product variety and hence obtain price indices that are close to the empirical CPI data

<sup>10</sup>Bureau of Labor Statistics in the US, Banco de Mexico up to June 2011 and from July 2011 Instituto Nacional de Estadística, Geografía e Informática (INEGI).

Hence, the price index could be defined as:

$$P_t = \sum_{i=1}^I \omega_i \frac{p_t(i)}{p_0(i)}$$

where  $p_0(i)$  is the price of good  $i$  in the base year zero,  $p_t(i)$  is the price of good  $i$  in time  $t$ , and  $\omega_i$  is the weight that good  $i$  has in the price index.

Note that the model already provides average prices for three different goods: nontraded goods, traded goods sold domestically, and imported goods. This can be thought of as the prices that are observed by the statistical institutions that elaborate the CPI. Then, the Laspeyres index can be easily computed. Moreover, given that the Laspeyres index is a weighted average it can be expressed as a function of two other price indices: nontraded goods price index and traded goods price index.

Then, we can define the home (foreign) price index respectively for nontraded goods as:

$$\hat{P}_{N,t} = \frac{\hat{P}_{N,t}}{\hat{P}_{N,0}} \quad (34)$$

$$\hat{P}_{N,t}^* = \frac{\hat{P}_{N,t}^*}{\hat{P}_{N,0}^*} \quad (35)$$

Traded goods are composed by goods produced by exporting domestic firms and sold in the domestic market, plus the imported goods produce by foreign exporting firms. Applying the Laspeyres formula again we can define the price index of traded goods as:

$$\hat{P}_{T,t} = \left( \frac{N_X}{N_X + N_X^*} \right) \frac{\hat{P}_{D,t}}{\hat{P}_{D,0}} + \left( \frac{N_X^*}{N_X + N_X^*} \right) \frac{\hat{P}_{X,t}^*}{\hat{P}_{X,0}^*} \quad (36)$$

$$\hat{P}_{T,t}^* = \left( \frac{N_X^*}{N_X + N_X^*} \right) \frac{\hat{P}_{D,t}^*}{\hat{P}_{D,0}^*} + \left( \frac{N_X}{N_X + N_X^*} \right) \frac{\hat{P}_{X,t}}{\hat{P}_{X,0}} \quad (37)$$

Then, the price index can be expressed as:  $\hat{P}_t = \omega_N \hat{P}_{N,t} + \omega_T \hat{P}_{T,t}$  in home country and  $\hat{P}_t^* = \omega_N^* \hat{P}_{N,t}^* + \omega_T^* \hat{P}_{T,t}^*$  in foreign country.<sup>11</sup> The weights will be given by the number of firms producing

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<sup>11</sup>Note that exporting firms set a price equal to  $\hat{p}_{D,t}$  ( $\hat{p}_{D,t}^*$ ) in their domestic market respectively.



nontraded (traded) goods relative to the total number of firms selling in each country and the base year prices will be the prices at the steady state of the model.<sup>12</sup>

#### 4. The Real Exchange Rate

As stated by Ghironi and Melitz (2005) the real exchange rate ( $Q$ ) in the model measures whether households' welfare increases by consuming home or foreign goods. On the other hand the real exchange rate obtained using CPI data measures the cost of a foreign basket of goods in terms of home goods, or in other words whether average prices are higher or lower in home than in foreign. By using their modified price indices, These authors define  $\tilde{Q} = (\frac{N_t^*}{N_t})^{\frac{1}{\theta-1}} Q_t$  as a theoretical counter part to the empirical real exchange rate.

However, to analyze the importance of nontraded goods prices in explaining real exchange rate variations it is necessary to use a definition of the real exchange rate that allows it to be decomposed into nontraded and traded components. Hence, the following modification is suggested:

Let this new real exchange rate be denoted by  $RER_t$ , then using the Laspeyres base price indices mentioned above and the model implied exchange rate  $Q_t$  we have:

$$RER_t = Q_t \frac{\omega_N^* \hat{P}_{N,t}^* + \omega_T^* \hat{P}_{T,t}^*}{\omega_N \hat{P}_{N,t} + \omega_T \hat{P}_{T,t}} \quad (38)$$

Then,  $RER_t$  can be decomposed into the relative price of traded goods ( $RERT_t$ ) and the relative price of nontraded to traded good ( $RERN_t$ ):

$$RER_t = (RERT_t) * (RERN_t) \quad (39)$$

where  $RERT_t = Q_t \frac{\hat{P}_{T,t}^*}{\hat{P}_{T,t}}$  and  $RERN_t = \frac{\omega_T^* + \omega_N^* \frac{\hat{P}_N^*}{\hat{P}_T^*}}{\omega_T + \omega_N \frac{\hat{P}_N}{\hat{P}_T}}$  (see Hernandez (2012) for details).

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<sup>12</sup>A detailed derivation of these weights and the price indices is given in Appendix A.

When comparing  $RER_t$  against the real exchange rate defined by Ghironi and Melitz (2005) ( $\tilde{Q}_t$ ), the simulations suggested that both variables exhibit similar dynamics which leads to conclude that the new defined  $RER_t$  is also close to the empirical real exchange rate (see Appendix B).

Simulations from the model were transformed by Christiano and Fitzgerald asymmetric filter (CF) and HP filter. Then, to analyze the importance of nontraded good prices to explain real exchange rate variations implied by the model and contrast the results against those in Hernandez (2012) the sample correlation and the ratio of standard deviations between RERN and the real exchange rate were computed. In addition, the following variance decomposition is used:<sup>13</sup>

$$vardec(rer, rerN) = \frac{var(rerN)}{var(rerN) + var(rerT)}$$

#### 4.1. *The Balassa-Samuelson Effect*

The model endogenously generates a Balassa-Samuelson effect via the endogenous mechanism of entry of firms. Before a raise of home aggregate productivity, a higher number of firms will want to enter into home market whereas in the now relative less productive foreign market there will be no entry of new firms. This will increase home effective labor costs raising nontraded goods prices relative to those of foreign leading to an appreciation of the real exchange rate.

Higher home effective labor costs will reduce profits for domestic exporting firms. However, relatively cheaper labor costs in foreign will increase profits of foreign exporting firms. This will imply that for home firms to successfully enter the export market they have to become more productive than before. In turn, foreign firms which were not productive enough before are now able to enter the export market. As a result, imports at home are now more expensive than imports in foreign leading to an increase in home demand for domestically produce goods reinforcing the real exchange rate appreciation.<sup>14</sup>

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<sup>13</sup>Unfortunately, the volatility of RERN implied by the model is too high with respect to that of the real exchange rate, causing the second type of variance decomposition used in Hernandez (2012) to attain values significantly higher than one, making this statistic inadequate.

<sup>14</sup>For a more detailed discussion see Ghironi and Melitz (2005).

## 5. Asymmetric Calibration

The model is calibrated around an asymmetric steady state so as to replicate US and Mexico economies in 2002.<sup>15</sup> The time frequency of the model is quarterly.

### 5.1. Parameter Values

Following the business cycle literature, the discount parameter ( $\beta$ ) is equal to 0.99 and the intertemporal elasticity of substitution ( $\gamma$ ) is equal to 2. The probability of a firm exit ( $\delta$ ) is kept at 0.025 as in Ghironi and Melitz (2005). Also, the value of the elasticity of substitution across varieties of goods is 3.8 in line with Bernard et al. (2003) and Ghironi and Melitz (2005).

### 5.2. Calibration of the Pareto Distribution

Using data for over 5.5 million US firms, Luttmer (2007) shows that firm size distribution in the US tends to a Pareto distribution with shape parameter  $k = 1.06$ . However, for the variance of firm size to be finite it is required that  $k > \theta - 1$ . Given that  $\theta = 3.8$   $k$  has to be bigger than 2.8.

Ghironi and Melitz (2005) deals with this issue by using a value of  $k = 3.4$  to match the standard deviation of the log of plant sales reported by Bernard et al. (2003) of 1.67. More recently, Demidova (2008) having identified a minor algebraic mistake in Ghironi and Melitz (2005) used a value of  $k = 3.3$ .<sup>16</sup> In this analysis, a range of values for  $k \in [3, 3.4]$  interval were used with no significant impact in the results of the model. Hence,  $k$  was kept at 3.3 as suggested by Demidova (2008).

Lastly, the minimum (and necessarily positive) value of firm productivity ( $z_{min}$ ) is normalized to 1.

### 5.3. Exogenous Variables

Labor supply values are approximated by the relative population between US and Mexico in 2002. Obtaining data on US population from the Census Bureau and data on Mexico's population

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<sup>15</sup>Since the data in Hernandez (2012) start in 2002 this year will be taken as representative of the steady state of the model.

<sup>16</sup>This value was choose so as to match the standard deviation of the log plant sales obtained in the simulations of Bernard et al. (2003) which was of 0.84 instead of the value of 1.67 found in the data.

from Instituto Nacional de Estadística, Geografía e Informática (INEGI) it was found that Mexico's relative population to US population was around 35%. Hence, a value of 3 is assigned to home's labor supply and, for computational convenience, the value of 1.1 is assigned to foreign's labor supply.<sup>17</sup>

The steady state values of home and foreign aggregate productivity ( $Z$ ,  $Z^*$ ) were obtained from Veloso et al. (2006). These authors obtained total factor productivity values for seven Latin American economies. Their results show that from 1960 to 1980 Mexico's TFP was, in average, 12% higher than US TFP. However, after 1980 these Latin American economies suffered a fall in their TFP. For the case of Mexico, its TFP became only 77% of US TFP in the year 2000. Thus, keeping the value of home aggregate productivity ( $Z$ ) equal to 1 the value assigned to foreign aggregate productivity ( $Z^*$ ) is 0.77.

To calibrate the fixed export costs it is necessary to calculate the ratio of exporting firms in US and Mexico in the year 2002. For consistency with Ghironi and Melitz (2005) calibration, this paper follows Bernard et al. (2003)<sup>18</sup> defining the ratio of exporting firms equal to *value of total manufacturing exports* divided by *the Value of all manufacturer's shipments*.<sup>19</sup>

For the case of Mexico data on value of total manufacturing production from INEGI Censos Economicos 2004 were obtained.<sup>20</sup> With respect to Mexico's value of exports it is important to note that the value of total exports reported in Mexico's official publications is overstated due to offshoring and processing trade. de la Cruz et al. (2011) estimate that Mexico's foreign value added is around 66.2% of total manufacturing exports, using this estimate combined with export data obtained from Anuario Estadístico de Comercio Exterior 2003 it was found that the exporting firms represent 18.7% of total manufacturing production. In the other hand, US exporting firms are 17.6% (see Table 1).

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<sup>17</sup>When assigning the value between (1, 1.09) to  $L^*$  the solution of the steady state includes some imaginary values. This gets corrected when 1.1 is used.

<sup>18</sup>These authors found that the ratio of exporting firms in the manufacturing sector was 21% in 1999.

<sup>19</sup>Data on the value of all manufacturer's shipments were obtained from 2002 Economic Census Manufacturing and the value of total manufacturing exports were obtained from 2002 Foreign Trade Statistics.

<sup>20</sup>This value is reported in Mexican pesos hence it was converted to US dollars using the average annual exchange rate in 2002 which was 9.66.

Finally, home and foreign interest rates as well as the iceberg-melting costs are symmetric across countries with  $r = r^* = (1/\beta) - 1$  and  $\tau = \tau^* = 1.3$ . Table 2 below summarizes the calibration of the model.

#### 5.4. Productivity Shocks

A more recent study developed by Mandelman and Zlate. (2008) shows that US productivity shocks are more persistent than Mexico's productivity shocks. The authors also found that the spillover parameter from US to Mexico is higher than the spillover parameter from Mexico to US, but that both parameters are not statistically different from zero.

Hence the productivity shocks are given by:

$$\begin{bmatrix} Z_t \\ Z_t^* \end{bmatrix} = \begin{bmatrix} \rho_Z & \rho_{Z,Z^*} \\ \rho_{Z^*,Z} & \rho_Z^* \end{bmatrix} \times \begin{bmatrix} Z_{t-1} \\ Z_{t-1}^* \end{bmatrix} + \begin{bmatrix} \epsilon_t^Z \\ \epsilon_t^{Z^*} \end{bmatrix} \quad (40)$$

where

$$\begin{bmatrix} \rho_Z & \rho_{Z,Z^*} \\ \rho_{Z^*,Z} & \rho_Z^* \end{bmatrix} = \begin{bmatrix} 0.996 & 0.003 \\ 0.049 & 0.951 \end{bmatrix} \quad (41)$$

and the variance covariance matrix becomes:

$$\Sigma = \begin{bmatrix} 0.0050939^2 & 0.00001898 \\ 0.00001898 & 0.013957^2 \end{bmatrix} \quad (42)$$

#### 5.5. Steady State

The steady state of the model is able to match some of the stylized facts between US and Mexico's economies. For example, in the steady state home consumption is four times higher than foreign consumption, which is close to the one reported by Bergin and Glick (2009). Home real wage is 1.4 times that of foreign country. This value is significantly smaller than the values reported in other studies where wages in the US are up to eight times higher than in Mexico. Nevertheless, it can be considered a good feature of the model than home wages are higher.

The steady state also implies that the number of existing firms in the home country are six times those existing in Mexico. This number may be considered as extremely high, but remember than in this model each firm produces one variety and that the set of available varieties ( $V$ ) in the US is also significantly bigger than the set of varieties available in Mexico ( $V^*$ ). In the same fashion, the number of exporting firms and the number of entry firms in home country are around five times higher than the number of exporting and entry firms in Mexico. Finally, in the steady state prices set by home firms are higher than prices set by foreign firms.

Finally, balance trade is not guaranteed due to the asymmetric calibration of the model. A simple approach is to use a variation of the risk sharing condition. Hence, to assure balance trade at every  $t$  equation (31) is modified as follows:

$$A_t Q_t N_{X,t} (\hat{p}_{X,t})^{1-\theta} C_t^* = N_{X,t}^* (\hat{p}_{X,t}^*)^{1-\theta} C_t \quad (43)$$

where  $A_t$  is endogenously determined at every  $t$ .

## 6. Results

The model is log-linearized around the asymmetric steady state. Table 3 below shows the results of the model and contrast them against the empirical evidence provided in Hernandez (2012) using quarterly frequency data.

The calibrated model is able to closely match the correlation between the nontraded component and the real exchange rate. For example, in the case of the HP-filtered data the correlation implied by the model is 0.59 which is not that far apart from the 0.64 in the data. Similarly, looking at the CF-filtered transformation, the correlation between the nontraded component and the real exchange rate is 0.66 in the model which is somewhat smaller than the 0.74 in the data.

In turn, as in most macro models the volatility of the real exchange rate implied by the model falls somewhat short from the empirical evidence. As a consequence, the ratio of standard deviations between nontraded goods prices and the real exchange rate are overestimated. For instance, looking at CF-filtered data the value of this ratio is 1.12 whereas in the simulated model this ratio

is 1.98. The same occurs for the HP-filtered data with a ratio of standard deviations of 1.05 while the simulated models implies a ratio of 1.72.

When analyzing the contribution of the nontraded component in the real exchange rate by using the variance decomposition method, it is found that the model performs quite well. First, for the simulations of the model show a value of such statistic to be 0.62 which is very close to the 0.68 for the CF-filtered data. Similarly, in the case of the HP-filtered data the value of the variance decomposition is 0.59 and in the simulated model the value is 0.56.

Table 3 also shows the correlation between the relative price of nontraded goods and the relative price of traded goods for the calibrated model and contrast them against the correlations in Hernandez (2012). The model is successful in replicating the negative correlation between the two components of the real exchange rate, albeit it overstates the value of such correlation.

The model is able to replicate the empirical results in the sense that once the aggregate productivity shocks take place, this increases the number of firms entering home and foreign market. As more firms are able to export, this pushes down the cut-off export productivity level allowing less productive firms to enter the export market. Furthermore, higher aggregate productivity increases wages in both countries. Then, the raise in wages and the fall in the mentioned cut-off lead to higher export prices in general.

Since home firms always set prices higher than foreign firms (asymmetric calibration) then imports are more expensive for foreign than for home. Hence, the tradable price index in foreign increases more than in home leading to a depreciation of the relative price of nontraded goods (traded component).

In turn, with more exporting firms the number of producers in the nontraded sectors decreases having only a very small negative effect on nontraded average productivity in both countries. Given that aggregate productivity and wages raise at the time of the shock, the impact on nontraded goods prices is very small keeping the price index of these goods virtually unchanged in both countries. Hence, nontraded goods are now relatively cheaper than traded, particularly in home country, implying an appreciation of the relative price of nontraded to traded goods (nontraded component). Since the depreciation of the traded component is bigger in magnitude than the appreciation of the nontraded component this lead to an initial depreciation of the real exchange rate.

In the long-run, as the aggregate productivity gradually return to its steady state level, the number of firms entering home and foreign falls as well as real wages. However, given that home shock is more persistent entering firms and wages fall faster at foreign. This drives up cut-off export productivity in both countries, but differences in persistency between shocks also produce different dynamics across countries. In one hand, cut-off export productivity increases persistently at home and ends up in an upper level than its steady state. In the other hand, at foreign this cut-off raises for some time but then it comes down to a level slightly below the steady state.

The above results into persistent higher prices at home, whilst at foreign prices decrease at a faster rate leading to a gradual appreciation of the traded component and, at the same time, to a marginal depreciation of the nontraded component. However, this component remains significantly appreciated when compared to its steady state level. When combined, these tow effects imply a strong appreciation of the real exchange rate generated mainly by the also strong appreciation of the nontraded component.

### 6.1. Sensitivity Analysis

In this section, the asymmetric calibrated model is contrasted against the symmetric calibration used by Ghironi and Melitz (2005) in which the two countries are identical with exactly the same parameters. Moreover, both productivity shocks are also identical and there is now productivity spillovers from one country to another, i.e. let the innovations to shocks be normally distributed with zero mean, but now assume a standard deviation is equal to 0.00852 and a productivity correlation innovation of 0.258. Then, the bivariate autoregressive process is of the form:

$$\begin{bmatrix} \rho_Z & \rho_{Z,Z^*} \\ \rho_{Z^*,Z} & \rho_{Z^*}^* \end{bmatrix} = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix} \quad (44)$$

and the variance covariance matrix becomes:

$$\Sigma = \begin{bmatrix} 0.00852^2 & 0.258 \\ 0.258 & 0.000852^2 \end{bmatrix} \quad (45)$$



Column 3 in Table 3 shows the results. It can be seen that the symmetric calibration overestimates the value of the statistics. For example, in the case of CF-filtered data the value for the variance decomposition is 0.68 but the symmetric simulated model give a value of 0.89. When compared with a variance decomposition of 0.62 in the asymmetric model one can argue that the asymmetric calibration improves the performance of the model. A similar behavior is observed for the correlation between the nontraded component and the real exchange rate where the symmetric model suggest a correlation of 0.97 which is way higher than the correlation in the data and in the asymmetric model.

The results for the ratio of standard deviations are mixed. In the asymmetric case, the nontraded component is more volatile than the CPI based real exchange rate for both transformations (CF and HP filtered simulations). In contrast, in the symmetric model the ratio of standard deviations is smaller than one implying that the nontraded component is not as volatile. However, even though in the data the ratio of standard deviations is also smaller than one, it is not as big as the asymmetric model implies.

Finally, the symmetric model fails to replicate the negative correlation between the traded and nontraded component observed in the data. The cause of such shortcoming is that under a symmetric assumption once the productivity shocks take place, all prices at home and foreign synchronize leading to a positive correlation between the traded and nontraded component. Furthermore, such synchronization is being reinforced by the positive correlated shocks and the presence of productivity spillovers.

## **7. Conclusion**

This paper evaluates whether a "new macro-trade" model à la Ghironi and Melitz (2005), where entry of heterogeneous firms into international trade is endogenous, can explain recent empirical facts about the importance of nontraded goods prices to explain real exchange rate variations. The model is calibrated to resemble US and Mexico's economies. This calibration allows the steady state of the model to match some stylized facts regarding such economies. For instance, in the steady state consumption in home country is four times higher than consumption in foreign which is close to the findings in the literature. The number of exporting and entry firms is

significantly higher in home. Also, prices and real wages are also higher in home than in foreign.

The results show that the newly calibrated model implies an importance of nontraded goods in real exchange rate volatility close to the empirical evidence in Hernandez (2012). Also, as found by this author, in the model the nontraded component is more volatile than the CPI based real exchange rate. Another important feature of the model is that under the assumptions of an asymmetric equilibrium, different persistency and zero spillover from productivity shocks, the model can replicate the negative correlation between the relative price of traded goods and the relative price of nontraded to traded goods.

Finally, we can conclude that the asymmetric calibration of the Ghironi and Melitz (2005) model allows it to replicate most of the stylized facts reported in Hernandez (2012) when using quarterly frequency data, although with some caveats regarding the magnitudes of some of the correlations.

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Table 1: Ratio of Exporting Firms in US and Mexico 2002

US Dollars

**United States**

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Value of all Manufacturer's Shipments	3,916,136.7
Direct Exports	487,275.5
Export Supporting Shipments	200,472.2
US Total Exports	687,747.7
Percentage of Exporting Firms	17.6%

**Mexico**

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Total Manufacturing Production	248,798.6
Total Manufacturing Exports	137,712.0
Value of Intermediate Goods Imports	91,165.3
Mexico Total Exports	46,546.7
Percentage of Exporting Firms	18.7%

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Sources: 2002 Economic Census Manufacturing, 2002 Foreign Trade Statistics, INEGI Censos Economicos 2004, de la Cruz et al. (2011), Anuario Estadístico de Comercio Exterior 2003.

Table 2: Steady State Values and Parameters for the Calibrated Model

	<i>Asymmetric Model</i>	<i>Symmetric Model</i>
Exogenous Variables		
$Z$	1.00	1.00
$Z^*$	0.77	1.00
$f_E = F_E^*$	1.00	1.00
$f_X$	0.0012	0.0084
$f_X^*$	0.0072	0.0084
$L$	3.00	1.00
$L^*$	1.10	1.00
$r = r^*$	$\frac{1}{\beta} - 1$	1.3
$\tau = \tau^*$	1.3	
Parameter Values		
$\beta$	0.99	0.99
$\gamma$	2.00	2.00
$\delta$	0.025	0.025
$\theta$	3.80	3.80
$k$	2.968	3.40
$z_{min}$	1.00	1.00

Table 3: Asymmetric and Symmetric Model vs the Empirical Evidence

	<i>Hernandez (2012)<sup>a</sup></i>	<i>Asymmetric Calibration</i>	<i>Symmetric Calibration</i>
<b>CF-filtered</b>			
Correlation of nontraded and RER	0.74	0.66	0.97
Stdev nontraded/Stdev RER	1.12	1.98	0.83
Contribution of nontraded component vardec <sup>b</sup>	0.68	0.62	0.89
Correlation of nontraded and traded	-0.50	-0.87	0.48
<b>HP-filtered</b>			
Correlation of nontraded and RER	0.64	0.59	0.96
Stdev nontraded/Stdev RER	1.05	1.72	0.80
Contribution of nontraded component vardec	0.59	0.56	0.85
Correlation of nontraded and traded	-0.48	-0.82	0.47

<sup>a</sup> Corresponds to the results for quarterly data reported in Table 3. of the cited article.

<sup>b</sup> The statistic "vardec" represents the proportion of the variance of the nontraded component relative to the overall variance of the real exchange rate.

## Appendix A.

Statistical institution in the US and Mexico produce the CPI using the Laspeyres formula. First, they reduce the whole sample of goods  $\Phi$  to  $I$  categories (305 in the US and 315 in Mexico). Each category  $I$  contains a number of goods  $\phi \subset \Phi$ . For each category, the relative price, to a base year, of all  $\phi$  goods is added together and averaged. For example, the CPI of the category "new vehicles" may be formed by the average price of  $N$  new vehicles, i.e.

$$P_{t,vehicle} = \frac{\frac{1}{N} \sum_{n=1}^N P_{t,n}}{\frac{1}{N} \sum_{n=1}^N P_{0,n}}$$

Then, the Laspeyres formula is applied so that:

$$CPI_t = \sum_{i=1}^I \omega_i p_{t,i}$$

where  $\omega_i$  is the weight of category  $i$  in the CPI.

In the model there are three categories of goods: nontraded goods, traded goods produced and sold domestically and imported goods. Since all the goods within each category have the same price, the Laspeyres price index then becomes:

$$\tilde{P}_t = \frac{N_{N,0}}{N_{N,0} + N_{X,0} + N_{X,0}^*} \frac{\tilde{\rho}_{N,t}}{\tilde{\rho}_{N,0}} + \frac{N_{X,0}}{N_{N,0} + N_{X,0} + N_{X,0}^*} \frac{\tilde{\rho}_{D,t}}{\tilde{\rho}_{D,0}} + \frac{N_{X,0}^*}{N_{N,0} + N_{X,0} + N_{X,0}^*} \frac{\tilde{\rho}_{X,t}^*}{\tilde{\rho}_{X,0}^*}$$

where each price is weighted by the share of firms within each category of goods in the base year 0.

To express the Laspeyres price index as a function of nontraded and traded goods price indices we need to re-scale the weights of each category so they add up to one (where the subscript 0 was omitted for simplicity):



$$\tilde{P}_t = \frac{N_N}{N_N + N_X + N_X^*} \left[ \left( \frac{N_N}{N_N + N_X + N_X^*} \right) \tilde{\rho}_{N,t} \right] + \frac{N_X + N_X^*}{N_N + N_X + N_X^*} \left[ \left( \frac{N_X}{N_X + N_X^*} \right) \tilde{\rho}_{D,t} + \left( \frac{N_X^*}{N_X + N_X^*} \right) \tilde{\rho}_{X,t}^* \right]$$

where the first term in brackets in the right hand side is the price index of nontraded good  $\tilde{P}_{N,t}$  and the second term in brackets is the price index of traded goods  $\tilde{P}_{T,t}$ . This results into equations (13) to (16) in section 4.

Hence, the price index becomes:

$$\tilde{P}_t = \frac{N_N}{N_N + N_X + N_X^*} \tilde{P}_{N,t} + \frac{N_X + N_X^*}{N_N + N_X + N_X^*} \tilde{P}_{T,t}$$

If we let be  $\omega_N = \frac{N_N}{N_N + N_X + N_X^*}$  and  $\omega_T = \frac{N_X + N_X^*}{N_N + N_X + N_X^*}$  then the price index can also be expressed as  $\tilde{P}_t = \omega_N \tilde{P}_{N,t} + \omega_T \tilde{P}_{T,t}$

## Appendix B.

