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A Model of Wage and Employment Effects of Service Offshoring*

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Abstract: This article shows that a skill-abundant country with a relatively high productivity has larger incentives to offshore unskilled than skilled intensive tasks (services), even though no assumption on the correlation between the degree of tradability and skill-intensity of the tasks is made. Assuming putty-clay technology that locks labor into tasks in the short run, it is shown that service offshoring yields wage and employment effects in the long run. These effects switch from negative to positive as the degree of tradability declines, being the switch for a large degree of tradability in the case of the skilled intensive tasks. The results are consistent with an emerging empirical literature that studies the effects of service offshoring on wages and employment.

Keywords: Service offshoring, Trade in Tasks, Skill-intensity

JEL Classification: J24, L24

Resumen: Este artículo muestra que un país abundante en mano de obra calificada con productividad relativamente alta tiene mayores incentivos a importar tareas (servicios) intensivas en mano de obra no calificada que calificada, a pesar de que no se realiza supuesto alguno sobre la correlación entre el grado de transabilidad y la intensidad en mano de obra calificada de las tareas. Suponiendo tecnología putty-clay que inmoviliza el trabajo entre tareas en el corto plazo se muestra que la importación de servicios tiene efectos sobre los salarios y el empleo en el largo plazo. Estos efectos cambian de negativos a positivos a medida que el grado de transabilidad disminuye, siendo el cambio para un mayor grado de transabilidad en el caso de los servicios intensivos en mano de obra calificada. Los resultados son consistentes con una literatura empírica emergente que estudia los efectos de la importación de servicios sobre los salarios y el empleo.

Palabras Clave: Importación de Servicios, Comercio en Tareas, Intensidad en Trabajo Calificado

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1. Introduction

The revolution of the Information and Communication Technologies (ICT) has allowed for the output of previously non-tradable labor tasks to be delivered electronically from overseas. The possibility of delivering output electronically has reduced the offshoring costs of the labor tasks, leading to an increase in offshoring (often referred to as trade in tasks). The offshoring of services has received particular attention in the media and academic research because it implies that, for the first time, white-collar employees are exposed to global competition (Mankiw and Swagel, 2006; Amiti and Wei, 2005; Trefler, 2006; Freund and Weinhold, 2002; Feenstra and Hanson, 2003 and Head et al., 2009).

Leading academic researchers have argued that understanding the effects of trade in tasks and, in particular, of service offshoring requires a new analytic framework (Grossman and Rossi-Hansberg, 2006; Grossman and Rossi-Hansberg, 2008; Baldwin, 2006; Blinder, 2006; Blinder, 2009; Krugman, 1996 and Krugman, 2011). In a nutshell, this new framework should account for three facts. First, tasks can be geographically separated from the production process and, therefore, global competition now occurs at the level of tasks, i.e. between workers performing similar services in different countries. Second, tasks with stronger tradability characteristics are more strongly exposed to global competition. Third, and foremost, the tradability of a task may not be correlated with its skill-intensity (and hence with the skill level of the worker fulfilling the job). This paper develops a model to show that, in a small skill-abundant country with high final goods productivity, the wage and employment effects of service offshoring vary across tasks depending on two characteristics that determine their exposure to global competition: Tradability and skill-intensity.¹ Appealingly, the model does not

¹ See Bhagwati et al. (2004); Deardorff, (2005); Markusen, (2005) and Markusen and Strand (2008) for papers arguing that skilled labor abundant countries specialize in skill-intensive activities.
make any assumption on the correlation between the tradability and skill-intensity of the tasks to obtain this result.

The model considers two sectors that have different skill-intensities and conceptualizes production in terms of tasks that vary in tradability. The costs of offshoring tasks are represented by a tradability index that varies smoothly across the tasks in the manner of Grossman and Rossi-Hansberg (2008) (henceforth GRH). In this setup a task's propensity to be offshored depends on tradability and skill-intensity. I consider skilled tasks and unskilled tasks and identify a cutoff traded task for each type of task, below which tasks are offshored due to high tradability and above which tasks are produced domestically due to low tradability. I show the cutoff traded task is greater for unskilled labor: Because the skill premium is lower in the skill-abundant country, this country reaps larger savings by offshoring the unskilled tasks.

Skill-intensity and tradability are also relevant to understanding wage changes (the dependence on skill-intensity is explained below). To introduce a role for tradability the model considers human capital occupation-(task)-specific knowledge, which is relevant for understanding service offshoring effects because: (i) human capital is specific at the occupational level (Kambourov and Manovskii’s, 2009a and 2009b); (ii) even more specific in tradable occupations (Ritter, 2008) and (iii) occupation-specific exposure to offshoring is important (Ebenstein et al., 2013).²,³

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² I refer indistinguishably to “tasks” and “occupations” hereafter. Most empirical studies use data disaggregated at the occupational level. A simple way of reconciling these studies with this paper is to think about the task-content of the occupations. Thus, when referring to “the most tradable task”, the reader should think of occupations with a large content of tradable tasks.

³ Kambourov and Manovskii (2009a) employ worker-level data on wages and identify workers’ occupation and industry switches to show that “returns to occupational tenure are substantial”. In a companion paper, they relate the specificity of human capital to wage changes (Kambourov and Manovskii, 2009b). Ritter (2008) builds on their work to show that specific knowledge is more relevant in tradable occupations. I refer to Ebsetein et al. (2013) below.
In the model, the specificity of human capital locks labor into tasks, implying that supplies are inelastic in the short run and therefore the wage of tasks is fully determined by their demand. The ICT revolution decreases offshoring costs, causing a reduction in the demand for domestically produced tasks that varies according to tradability. The stronger a task’s tradability characteristics, the greater the demand reduction is, and therefore the higher the wage decrease is. Thus, the wage-reducing effect of service offshoring increases with tradability reflecting a stronger exposure to global competition.

Being consistent with the idea that knowledge is task-specific, I link wage and employment changes through retraining (the acquisition of further knowledge). Workers must go through a retraining process to transition from the most tradable to the least tradable tasks and thus receive a higher wage. The costs of retraining vary within tasks in which workers are distributed based on their ability to retrain according to a distribution. I assume the distribution is identical across tasks since my interest of study is the link between wage and employment changes. Under this assumption, the more service offshoring decreases the wage of a worker’s task, the more willing she is to retrain and transition to one of the least tradable tasks. In other words, the frequency of retraining is greater for tasks with strong tradability characteristics and, therefore, the decrease in employment increases with tradability as the wage-reducing effect does.\(^4\)

The findings differ from GRH’s outcomes in some important ways. In generating a greater cutoff task for unskilled labor, GRH rely on a potential negative correlation between the tradability and the skill-intensities of the tasks. In contrast, I rely on cross-country differences in factor proportions: the cutoff traded task is greater for the unskilled tasks because the relative supply of unskilled labor and

\(^4\) The model emphasizes the role of task-specific knowledge and of retraining as a mechanism lying behind the reallocation of employment. Its goal is not to neglect the relevance of other mechanisms such as transitions to unemployment, which may be complementary to retraining as suggested by Hummels et al. (2012).
therefore the skill premium are lower in the skill-abundant country. Their wage-reducing effect is isomorphic to a labor-supply increase and therefore takes place only when the number of factors is greater than the number of goods. Employing some features of their work and putty-clay technology, I prove the existence of a more general wage-reducing effect. Their effect reduces the wage of all tasks having the same skill-intensity by the same amount so that, in line with traditional theory, their model classifies workers into winners and losers only on the basis of skill-intensity. In contrast, my wage-reducing effect increases with tradability so that winners and losers are determined on the basis of the skill-intensities and the degree of tradability of their tasks. GRH show that the typical distributional conflict arising from reductions in trade costs does not take place if the wage-increasing (productivity) effect is greater than the wage-reducing effect. This result holds unambiguously in my model because the wage of the least tradable increases independent of their skill level. Furthermore, because my wage-reducing effect increases smoothly with tradability, some workers gain from offshoring even though their task types (the same skill-intensity and degree of tradability) are offshored. Finally, GRH demonstrate that service offshoring generates a reallocation of employment so that it goes to zero for all offshored. In my model, the employment responses to offshoring are different and vary smoothly with tradability within the set of offshored tasks. Smoothness represents an advantage when confronting the predictions of my model with data.

Another contribution to the literature is to run comparative statics that can be interpreted as a comparison among countries with different characteristics or as the impact of changes on a single country over time. I show an increase in trade costs on final goods magnifies the asymmetry of the wage effects across skill levels. That is, this increase makes the set of skilled losers smaller and the

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5 For these workers, the productivity effect is stronger than the foreign completion effect. See Kohler (2004) and GRH for other theoretical treatments of the productivity effect.

set of unskilled losers greater. On the contrary, increases in the final goods productivity and further ICT improvements primarily generate a redistribution of income from the most to the least tradable tasks.

The results of the model are consistent with recent findings in the empirical literature (see Section 5 for a detailed explanation). Crinó (2010) estimates the elasticity of occupational demands with respect to service offshoring. He finds a higher concentration of positive elasticities in the group of skilled occupations and of negative elasticities in the unskilled group. He also shows that the occupations with negative elasticities have stronger tradability characteristics so that service offshoring generates a reallocation of employment from the most to the least tradable tasks. This evidence is consistent with the model, where employment losses increase with tradability and the proportion of tasks in which employment falls is greater for the unskilled group. Furthermore, Crinó (2010) shows the probability of observing a positive elasticity decreases monotonically with tradability, even after controlling for skill-intensity. This result is also consistent with my setup because I do not rely on a potential correlation between tradability and skill-intensity to obtain the employment results. The model is also consistent with Liu and Trefler’s results (2011) that service offshoring increases occupational switching rates, with the increase being stronger on unskilled workers and workers fulfilling routine tasks. They also find that service offshoring generates a gradual adjustment in employment. As explained in Section 5, the model shows that switching rates are higher among the unskilled and the most tradable tasks and that the employment adjustment is gradual.

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7 To this purpose, he builds a continuous index of tradability. His index considers three tradability characteristics mentioned in the literature: how routine a job is (Autor et al. 2003 and Levy and Murname, 2006), whether it produces impersonal services (Blinder 2006) and whether it is ICT-enabled (Garner, 2004, Dossani and Kenney 2003). Jensen and Kletzer (2006 and 2010) provide an alternative measure of tradability based on the geographical concentration of services.

8 Crinò (2010) employs data at the occupational level and thus these statements require a mapping between tasks and occupations. This mapping is assumed throughout the paper. See footnote 2.
Finally, the paper is also related to a strand of studies showing that skill-intensity and tradability are relevant in explaining wage effects but do not focus explicitly on service offshoring. Hummels et al. (2011), for instance, show that offshoring increases the high-skilled wage and decreases the low-skilled wage of Danish workers. They find that, conditional on skill type, offshoring reduces the wage of workers whose occupations involve routine tasks (the most tradable occupations) and increases the wage of workers that use less tradable skills. The model is also consistent with Ebenstein et al.’s results (2013) that wage effects depend on occupational exposure to offshoring and that these effects are negative for workers who perform routine tasks. Finally, Firpo et al. (2011) demonstrate that tradability has become more relevant than other factors in explaining wage changes since the 2000s.

I obtain the wage impacts of service offshoring by comparing an offshoring regime to a non-offshoring regime, in which the offshoring costs of the labor tasks are lower. I present the former regime in Section 2 and the later regime in Section 3, in which I also show the wage results. Employing these results, the simple retraining model that I present in Section 4 yields the employment effects. I summarize the results and show they are consistent with facts documented by Crinó (2010) and Liu and Trefler (2011) in section 5. Section 6 concludes.

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9 Hummels et al. (2011) define broad and narrow measures of offshoring as based on the total value of imports.
10 Ebenstein et al. (2013) define offshoring as the total employment of foreign affiliates among multinational US firms in each industry.
11 Firpo et al. (2011) use CPS data to assess the contribution of de-unionization, technological change and tradability to changes in the distribution of wages.
12 See Arora and Gambardella (2005); Bardhan and Kroll (2003); Brainard and Litan (2004); Bronfenbrenner and Luce (2004); Kirkegaard (2004); and Schultze (2004) for other contributions to the literature.
2. Non-Offshoring Regime: Trade in Final Goods

2.1 Model Setup

I consider a world with two regions, Home and the Rest of the World (RW hereafter) and identify the variables concerning RW with a superscript asterisk (*). Labor is either skilled or unskilled and final goods have different skill-intensities: Production of the skilled-intensive good \( Y_s \) involves a continuum of skilled tasks and production of the unskilled-intensive good \( Y_u \) involves a continuum of unskilled tasks. Production of tasks of a given skill-intensity requires a unit of labor of that skill level and the measures of the continuums are normalized to one.

Home is a small skill-abundant country and as such is assumed to import the unskilled-intensive good.\(^{13,14}\) This country has a technological advantage in the production of final goods, which is given by the following Cobb-Douglas functions employed by Acemoglu et al. (2007)

\[
Y_f = A \exp \left\{ \int_0^1 \ln(z_{ij}) \, di \right\}, \quad i \in [0,1], \quad j = s, u, \tag{1}
\]

\[
Y_f^* = \exp \left\{ \int_0^1 \ln(z_{ij}^*) \, di \right\}, \quad i \in [0,1], \quad j = s, u, \tag{2}
\]

where \( z_{ij} \) denotes task \( ij \)'s usage, and \( A > 1 \) is Home's Hicks neutral technological parameter. This technology differs from GRH’s in that tasks that use the same labor-type are substitutes. I will assume task-substitutability hereafter in the main body of the paper and, thus, be able to define a specific

\(^{13}\) This assumption allows me to solve the equilibrium in terms of foreign wages and to consider the existence of separate task-markets in Section 3. It implies that GRH’s relative price effect is absent. Note, however, this effect harms unskilled workers and benefits skilled employees because offshoring increases the relative supply of the unskilled-intensive good. In this paper, costs savings are also greater in the production of the unskilled-intensive good and offshoring also harms the unskilled workers by a greater amount.

\(^{14}\) See Hummels et al. (2001); Olsen (2006); Trefler, 2006; UNCTAD (2004); Michel and Rycx (2012); Daveri and Jona-Lasinio (2008); Gorg and Hanley (2005); Crinó (2008) and Crinó (2012) for offshoring in economies that could hardly have an impact in foreign wages.
output-constrained demand for each task as a function of its price (wage). In the Appendix Section, I show the results are not sensitive to the substitutability assumption by using a Leontief production function in the manner of GRH.

The goods market is perfectly competitive, and trade costs, which are of the Samuelson-Bergson iceberg type, apply to both final goods. For one unit of a product to arrive in the other region $\tau > 1$ units must be shipped.

2.2 Equilibrium in the Non-Offshoring Regime

A set of equilibrium wages $w_{ij} i \in [0,1], j = s, u$, fulfills two requirements: The task markets clearing and the zero-profit conditions.

I will begin by addressing clearing in the task markets. The output-constrained demands for tasks are given by cost minimization and written as follows

$$z_{ij}^d = (Y_j \exp\left(\int_0^1 \ln(w_{ij}) \, dt\right))/Aw_{ij}, \ i \in [0,1], \ j = s, u,$$

(3)

$$z_{ij}^*d = (Y_j^* \exp\left(\int_0^1 \ln(w_{ij}^*) \, dt\right))/w_{ij}^*, \ i \in [0,1], \ j = s, u,$$

(4)

where $z_{ij}^d$ is the output-constrained demand for task $ij$ and $w_{ij}$ denotes its price.

Every task is produced in equilibrium because the demand and supply of no task can be jointly equal to zero.\(^\text{15}^\) Since workers supply the task with the highest wage at their skill level, the price of all tasks having the same skill-intensity must be the same in equilibrium (so that every task’s supply is positive). More formally, the tasks markets clearing conditions are written as follows

$$w_{is} = w_s, \ \forall \ i \in [0,1], \ w_{iu} = w_u, \ \forall \ i \in [0,1],$$

(5)

\(^{15}\) The demand for a task goes to zero when its price goes to infinity and its supply goes to zero only when another task has a strictly higher price. Thus, demand and supply can never equal to zero jointly because no task has a price that is strictly higher price than infinity.
\[ w_i^s = w_s^*, \forall i \in [0,1], \quad w_i^u = w_u^*, \forall i \in [0,1]. \]  

(6)

Plugging these conditions in the output-constrained demands shows that labor of type \( j \) is evenly allocated across the \( ij \) tasks.\(^1\) Since all tasks are produced in equilibrium, each region produces the two goods (incomplete specialization holds).

Equations (5)-(6) and the zero-profit conditions determine domestic wages. In a competitive economy with incomplete specialization, these conditions hold when the effective price of every good equals its unit production costs. These costs equal marginal costs under constant returns to scale technologies and are written as follows

\[
MC_j = \exp \left\{ \int_0^1 \ln(w_{ij}) \, di \right\} / A, \quad j = s, u, \tag{7}
\]

\[
MC_j^* = \exp \left\{ \int_0^1 \ln(w_{ij}^*) \, di \right\}, \quad j = s, u, \tag{8}
\]

where \( MC_j \) denotes the marginal cost of producing good \( Y_j \). Equations (7)-(8) yield the following zero-profit conditions \(^2\)

\[
p^T = w_s / A, \quad \tau = w_u / A; \tag{9}
\]

\[
\tau p^T = w_s^*, \quad 1 = w_u^*, \tag{10}
\]

where \( \tau \) denotes iceberg costs and the number 1 indicates that the relative price of the unskilled-intensive good has been chosen as the numeraire. Simple algebra on Equations (9)-(10) shows that \( w_s / w_u = w_s^*/(w_u^* \tau^2) < w_s^*/w_u^* \); since domestic consumers face iceberg costs, the skill premium is lower in the skill-abundant country (Home). This result is consistent with Acemoglu’s result (2003) that unskilled to skilled wages and the supply of skills are positively correlated across countries. Alternatively, this result can be expressed in terms of Home-to-RW wages as follows

\(^1\) All tasks are produced in the same amount because their demands are symmetric and have the same price.

\(^2\) Note that I have imposed the task market clearing conditions in Equations (9)-(10).
\[
\frac{w_s}{w_{s'}} = \frac{A}{\tau}, \hspace{1cm} (11)
\]
\[
\frac{w_u}{w_{u'}} = A\tau. \hspace{1cm} (12)
\]

Equations (11)-(12) state that the Home-to-RW wage is greater for the unskilled workers. The following section employs this result to show that service offshoring exerts a stronger reducing effect on the wage of the unskilled workers.

3. Offshoring Regime: Trade in Final Goods and Intermediate Tasks

3.1 Setup of the Service Offshoring Model

Offshoring costs are expressed in terms of foreign labor and increase with the \(i\) index: A firm performing task \(i\) abroad requires \(\beta t(i)\) units of foreign labor. The \(\beta\) parameter is a shift parameter which is assumed to fall as a result of the ICT revolution. The offshoring cost schedule \(t(i)\) is assumed to be continuously differentiable and the task ordering implies that \(t'(i) > 0\).

To ensure that Home offshores skilled tasks I assume that its technological advantage (and hence the skilled wage shown in (11)) is sufficiently large that it fulfills the following condition

\[
A > \tau \beta t(0). \hspace{1cm} (13)
\]

Finally, I assume that human capital is task-specific so that it cannot be reallocated across tasks in the short-run (the specificity of human capital is often referred to as putty-clay technology). Under this assumption, the supply of offshored tasks is the sum of the domestic labor employed in these tasks in the non-offshoring regime and the imports of the tasks. As mentioned in the introduction, Kambourov and Manovskii (2009) provide empirical support for the putty-clay assumption and Ritter (2008) shows that human capital is more specific for tradable occupations. In my model, the putty-clay technology will imply that exposure to offshoring is relevant at the task (occupation) level. This insight is consistent with the evidence provided by Ebenstein et al. (2013).
3.2 Equilibrium in the Offshoring Regime

A set of wages $w_{ij}$ $i \in [0,1]$, $j = s,u$ fulfills three requirements in equilibrium: besides the task markets clearing and the zero-profit conditions, offshoring decisions must be cost-minimizing.

I will begin by addressing cost-minimization. This condition implies that firms choose a cutoff traded task $I$, below which task are offshored due to high tradability and above which tasks are produced domestically due to low tradability. Thus, cost-minimizing decisions are summarized by

\[
\min_{I_j} MC_j(I_j) = \exp\left((1 - J_j) \ln(w_{ntj}) + \int_0^{J_j} \ln \left(w^*_j \beta t(i)\right) di\right)/A, \ j = s,u, \tag{14}
\]

where $J_j \in [0,1]$ denotes a feasible choice of the cutoff traded task, $w^*_j \beta t(i)$ is the effective import price of task $ij$, and $w_{ntj}$ is the price of a non-offshored task. Equation (14) assumes that the cutoff traded task differs across skill levels and the wage of every offshored task equals its effective import price. These conditions are fulfilled when the unskilled to skilled wage is greater in Home and the task markets clear, as shown below. Solving the optimization problem shown in (14) leads to the following condition

\[
w^*_j \beta t(I_j) = w_{ntj}, \ j = s,u, \tag{15}
\]

where $I_j$ is the choice of the cutoff traded task that minimizes costs among all feasible choices $J_j \in [0,1]$. Offshoring decisions are cost-minimizing when the cutoff traded task is such that firms are indifferent between offshoring the task and purchasing it in the domestic market.

The second equilibrium condition, task markets clearing, determines the wage of the offshored tasks. The market of an offshored task clears as its wage equals its effective import price: The demand for domestically produced tasks equals zero at higher prices and is such that there is an
excess demand at any lower price.\textsuperscript{18} Intuitively, the import price of an offshored task summarizes the extent to which it is exposed to global competition and thus sets an upper bound to its wage.

Finally, I address the zero-profit conditions. I write these conditions by equating unit production costs to the effective final goods prices. Using Equation (10) to substitute for the final goods price in terms of RW’s wages the zero-profits conditions can be written as follows

\[
w_{nts}(J_s) = \exp \left\{ \ln(w^*_s) + \frac{\ln(A/\tau) - J_s \ln(\beta) - \int_0^{J_s} \ln(t(i)) \, di}{1 - J_s} \right\}, \quad (16)
\]

\[
w_{ntu}(J_u) = \exp \left\{ \ln(w^*_u) + \frac{\ln(A\tau) - J_u \ln(\beta) - \int_0^{J_u} \ln(t(i)) \, di}{1 - J_u} \right\}. \quad (17)
\]

I will refer to the wages that appear on the left-hand-sides of equations (16) and (17) as the “zero-profit wages” hereafter. Note in these equations that as the choice of the cutoff traded equal zero \((J_j = 0)\), the zero-profit wages collapse to their values from the non-offshoring regime shown in Equation (9). Note also that these wages are increasing in the parameters whose increase reduces costs (e.g. \(A\) increases): If the zero-profit conditions hold, the zero-profit wages must increase to restore marginal cost to its original value. The next subsection employs the zero-profit wages and the offshoring minimizing rule shown in (15) to derive \(w_{ntj}\) and \(J_j\) in equilibrium.

### 3.3 Offshoring Implications and Predictions

The convex and heavily weighted curve that appears in Figure 1 with a vertical intercept equal to \(\beta_1\) represents the offshoring cost schedule for the offshoring regime (and that whose intercept is \(\beta_o\) represents the non-offshoring regime).\textsuperscript{19} The super-index \(\tau_0\) denotes that we study a case in which trade costs on final goods are low. The square-dotted curve represents the relationship between the zero-profit-to-RW wage and every feasible choice for the cutoff traded task implied by Equation (17),

\textsuperscript{18} The demand for domestically produced tasks equals zero at higher prices because firms can always offshore the tasks.

\textsuperscript{19} Convexity eases the exposition but the only requirements are that \(t(i)\) is continuously differentiable and \(t'(i) > 0\).
and its vertical intercept is the Home-to-RW wage from the non-offshoring regime. The circle-dotted curve shows the same relationship for the skilled tasks. The two dotted curves slope upward indicating that greater choices for the cutoff traded tasks generate cost savings that must be compensated for with increases in the wages of the non-offshored tasks (so that profits are zero).

The equilibrium for the unskilled tasks lies at the intersection of the square-dotted and the offshoring cost curves, where the choice of the cutoff traded task $J_u$ equals the equilibrium choice $J_u = I_u^\tau$. The intersection of the circle-dotted and the offshoring cost curves denotes the equilibrium for the skilled tasks. Note that the cutoff traded is greater for the unskilled tasks because the vertical intercept of the square-dotted curves is higher. That is, because the Home-to-RW wage from the non-offshoring regime is greater for the unskilled tasks (i.e. the Home’s unskilled to skilled wage is higher), this country reaps larger savings by offshoring these tasks and therefore the cutoff is higher for unskilled labor. It is important to note the model does not rely on any sort of correlation between tradability and skill-intensity to generate this result. Since the sign and the extent of this correlation are unknown, this feature of the model represents an advantage.\(^\text{20}\)

\(^{20}\) GRH consider a case in which the cutoff traded tasks are different whenever $\beta$ differs across skill groups. However, this case, which they consider implausible, would imply a correlation between the tradability and the skill-intensity of a task.
Figure 2 depicts equilibrium for a skill-abundant country whose trade costs are greater ($\tau_1 > \tau_0$). Comparing Figure 1 to Figure 2 shows the differential impact of service offshoring on two countries with different levels of trade costs. The square-dotted curve shifts upward and the circle-dotted curve shifts downward: The RW-to-Home unskilled wage is smaller in Figure 2, and therefore the difference in the savings the country reaps by offshoring the unskilled and the skilled tasks is greater in comparison with Figure 1. Thus, an increase in trade costs makes the set of offshored unskilled tasks greater and the set of offshored skilled tasks smaller, magnifying the differential impact of service offshoring across skill levels.

An increase in final goods productivity reduces RW-to-Home wages. Thus, Home reaps larger savings by offshoring both skilled and unskilled tasks, and therefore the set offshored tasks becomes larger for both skill groups. The key to this result is that an increase in the technological advantage is not diminished by importing tasks. Note that a reduction in $\beta$ generates a similar result. A reduction in this parameter makes the sets of unskilled and skilled tasks greater.
3.4 Implications for Wages and Predictions

The possibility of undertaking cheaper labor services abroad reduces costs and therefore service offshoring is isomorphic to a productivity increase. The output expansion caused by this increase raises the demand for domestic labor exerting a wage-increasing effect; I refer to this effect hereafter as the productivity effect.\footnote{Over the set of tasks located to the left of the equilibria in Figure 1, the zero-profit wage of a non-offshored task is greater than its effective import price; thus, firms reduce their marginal costs by offshoring these tasks.} On the other hand, the exposure to foreign competition increases because service offshoring makes domestically produced tasks available in the international market at their effective import price. The wage of an offshored task is never greater than this price in equilibrium and thus offshoring exerts a wage-reducing effect; this is the foreign competition effect.

The balance between the productivity and the foreign competition effects determines the set of workers whose wage decreases as a result of service offshoring (the set of losers) and the set of workers whose wage increases (the set of winners). Figure 3 depicts these sets for the skilled tasks.

\footnote{Heshmati (2003), Olsen (2006) and Amiti and Wei (2009) approach this effect from an empirical perspective. More recently, Hummels et al. (2011) and Sethupathy, G. (2013) provides evidence for the existence of the productivity effect.}
The square-dotted curves denote the wage schedule in the offshoring regime and the horizontal line indicates the wage that skilled workers receive in the non-offshoring regime. These two curve types intersect at the solid vertical line that identifies the indifferent skilled task $i^0_s$, for which the wage is the same in the offshoring and the non-offshoring regimes. Figure 4 shows the corresponding sets of losers and winners for the unskilled tasks.

**FIGURE 3. SCHEDULE OF SKILLED WAGES**

NOTES: Losers, winners fulfilling offshored tasks and winners fulfilling non-offshored tasks.

Figures 3 and 4 distinguish three types of workers and regions. Workers fulfilling the non-offshored tasks are located to the right on the x-axis. These workers gain from service offshoring because their wage increases thanks to the productivity effect. Workers that fulfill the offshored tasks with the lowest offshoring costs are located to the left on the x-axis. The ICT revolution increases the exposure to foreign competition of these tasks by a great amount and therefore the wage of these workers decreases. Finally, workers fulfilling the offshored tasks with the highest offshoring costs are located in the middle regions of the figures. Surprisingly, their wage increases even though their types of tasks (the same skill-intensity and tradability) are offshored. This result shows that when the wage-reducing effect increases with tradability workers performing offshored tasks with sufficiently weak tradability characteristics gain from service offshoring.
Figures 3-4 also show that skill levels are also relevant to determining losers and winners. Since the unskilled to skilled wage from the non-offshoring regime is greater in Home, the unskilled workers are more exposed to foreign competition. Comparing Figure 3 to Figure 4 shows the differential impact of service offshoring across skill levels. The proportion of tasks whose wage decreases (“loser tasks”) is greater for the unskilled tasks, and the wage loss of an unskilled loser is greater (as it is her wage decrease as a proportion of her wage from the non-offshoring regime).

The findings differ from GRH’s seminal paper in some important ways (see footnote 22 for how the papers differ in terms of the results about the cutoff traded tasks). Their wage-reducing effect is isomorphic to a labor-supply increase and therefore takes place when the number of factors is greater than the number of goods. The wage-reducing effect that I present reflects the exposure to global competition of each task and its existence does not depend on the number of factors or goods. Their effect reduces the wage of all tasks having the same skill-intensity by the same amount so that workers are classified into winners and losers only on the basis of skill levels. In contrast, my wage-reducing effect increases with tradability so that winners and losers are determined on the basis of the skill-intensities and the degree of tradability of their tasks. GRH show that the typical distributional conflict
arising from reductions in trade costs does not take place if the wage-increasing (productivity) effect is greater than the wage-reducing effect. This result holds unambiguously in my model because the wage of the least tradable increases independent of their skill level. Furthermore, because my wage-reducing effect increases smoothly with tradability, some workers gain from offshoring even though their task types are offshored.

Turning back to Figures 3 and 4, I perform comparative statics exercise. Comparative statics on $A$ show that as Home’s technological advantage increases, domestic firms reap greater savings by offshoring both the unskilled and the skilled tasks. Thus, skilled and unskilled workers become more exposed to global competition, and consequently the set of losers and a loser’s loss at a given tradability level increase for both skill groups. Yet, a higher technological advantage also enhances the productivity effect and therefore increases the offshoring gains for workers performing the least tradable tasks. That is, an increase in final goods productivity generates a redistribution of income from the most to the least tradable tasks.

The comparative statics provide a framework to analyze the effects of further ICT improvements. It has been argued that the full effects of service offshoring have not been felt yet because offshoring costs will fall by an even greater extent in the future (Blinder, 2006). As mentioned above, further ICT improvements (reductions in $\beta$) have similar effects as an increase in final goods productivity. That is, these improvements are expected to generate a redistribution of income according to the tradability of workers’ tasks.

Finally, a trade costs increase also magnify the effects across skill groups in terms of wages. This increase reduces the RW-to-Home unskilled wage so that unskilled workers in Home become more exposed to global competition. This implies the foreign competition effect is stronger for these workers and, therefore, the set of unskilled losers and the wage loss of unskilled loser at a given tradability level increases (Figure 6). By the same token, the set of skilled losers and the wage of a skilled loser at a given tradability level become smaller (Figure 5).
FIGURE 5. CHANGE IN THE SCHEDULE OF SKILLED WAGES AS TRADE COSTS INCREASE

FIGURE 6. CHANGE IN THE SCHEDULE OF UNSKILLED WAGES AS TRADE COSTS INCREASE
4. The Retraining Process

4.1 Retraining Model Setup

Workers that transition to a non-offshored task benefit from a wage increase but must retrain to obtain task-specific knowledge. Employment decreases in tasks where retraining occurs and it increases in the non-offshored tasks. This employment reallocation has no impact on wages since the wage of the non-offshored tasks and the wage of an offshored task are determined by the task market clearing and by the zero-profit conditions, respectively.

Completion of the retraining process requires “production” of $\theta$ hours of effective learning. The production of effective learning is modelled by a C.E.S production function whose are arguments are the amount of hours allocated to retraining in each period ($h_t$) and the amount of periods the retraining program lasts ($R$). In the spirit of Ben-Porath’s seminal paper (1967), workers can allocate time to retraining by splitting a fixed amount of time per period between working and retraining. Therefore, retraining is costly because workers give away working income during the retraining process. Workers also choose the duration of their retraining program $R$; shorter retraining programs are associated with larger retraining costs because the learning technology is modelled by C.E.S function.\(^23\)

The rate at which time is traded for effective learning depends on a worker’s ability to retrain and is summarized by a parameter $a$. Abilities to retrain differ across workers: Workers fulfilling task $ij$ are distributed according to a c.d.f. $G(.)$ with support ($a, \bar{a}$), where $g(a)$ denotes the proportion of workers whose rate is lower than $a$. As noted in the introduction, I assume that $G(.)$ is identical across tasks because my interest of study is the link between wage and employment changes.

\(^23\) The C.E.S. technology implies that it is more costly to learn when the learning is concentrated over a small amount of periods. This fact can be interpreted as s “crammers’ assumption” so that it takes much for a crammer to learn.
There exists a financial market and a utility function which are also modelled in the spirit of Ben Porath’s model (1967). Borrowing and lending happen at a rate $r$ and utility is independent of other time consuming activities. These assumptions ensure that workers make retraining decisions by maximizing their lifetime income. Without loss of generality, their work lives are assumed to last $T$ periods and the amount of time per period is normalized to 1.

4.2 Equilibrium

A worker will retrain if and only if conditional on the fact that she retrained, she chooses a sufficiently short retraining program. Only in this way her post-retraining life is sufficiently long that she recovers the retraining costs. Thus, I will first study the choice of the program’s duration and then investigate whether, given this choice, the worker retrains. For a worker fulfilling task $ij$ whose ability to retrain equals $a$, the optimization problem can be written as follows

$$\max_{R_{ij}^a(h_{ij,t})_0} I_{ij}^{a, Ret}$$

$$S.T. \quad Q_{ij} = a\left[\int_0^{R_{ij}^a} (h_{ij,t}^a)^\rho \, dt\right]^{\frac{1}{\rho}} = \theta,$$

$$R_{ij}^a \leq T$$

where

$$I_{ij}^{a,R} = \int_0^{R_{ij}^a} (1 - h_{ij,t}^a) \omega_j^* \beta t(i) \exp\{-rt\} \, dt + \int_{R_{ij}^a}^T \omega_{ntj} \exp\{-rt\} \, dt.$$  

is the worker's lifetime income under the retraining option, $Q_{ij}$ denotes her production of effective learning hours, $h_{ij,t}^a$ and $R_{ij}^a$ are the amount of hours allocated to retraining in period $t$ and her program’s duration, respectively, and $\rho < 1$ measures the sensitivity of the learning process to this duration. All the endogenous variables are conditional on the fact that the worker retrained.

There exist two types of solutions to the optimization problem shown in (19) which can be summarized as follows (see the Appendix for a complete derivation).
If \( B_{ij}^{a^*} < T \), then \( R_{ij}^{a^*} = B_{ij}^{a^*} \);  

if \( B_{ij}^{a^*} \geq T \), then \( R_{ij}^{a^*} = T \);  

where

\[
B_{ij}^{a^*} = -\left(\frac{1 - \rho}{\rho r} \right) \ln \left( 1 - \frac{1}{\beta(t(i))} \right) \left( \frac{w_{ntj}^{*}}{w_{jt}^{*}} \right) \left( e^{-\left(\frac{1}{1 - \rho} - \frac{B_{ij}^{a^*}}{T} \right) r} - e^{-Tr} \right)
\]

Note that \( R_{ij}^{a^*} \) is decreasing in \( w_{ntj}/w_{jt}^{*} \beta(t(i)) \) (weakly and monotonically decreasing for workers in the first line and in the second line of (20), respectively). The higher \( w_{ntj}/w_{jt}^{*} \beta(t(i)) \) the higher the income increase the worker enjoys by retraining, and therefore the more willing she is to choose a short program and make her post-retraining life longer. Note also that \( R_{ij}^{a^*} \) also decreases monotonically with \( a \).

The decision on whether to retrain arises from the discounted value of the lifetime income difference between retraining and non-retraining. Using the first order condition of the problem shown in (19) this difference can be written as

\[
i_{ij}^{a,Ret} - i_{ij}^{a,NRet} = \left( w_{ntj} - w_{jt}^{*} \beta(t(i)) \right) \left( e^{-\left(\frac{R_{ij}^{a^*}}{1 - \rho} - \frac{B_{ij}^{a^*}}{T} \right) r} - e^{-Tr} \right)
\]

Note first that only workers whose retraining program is sufficiently short retrain because \( i_{ij}^{a,R} - i_{ij}^{a,NR} \) decreases monotonically with \( R_{ij}^{a^*} \). This fact implies that \( i_{ij}^{a,R} - i_{ij}^{a,NR} \) also decreases monotonically with \( a \) and \( w_{ntj}/w_{jt}^{*} \beta(t(i)) \) and, thus, allows for the derivation of two cutoffs.
For each task $ij$, there exists a cutoff level $\bar{\alpha}_{ij}$, above which workers with a higher $a$ retrain.\footnote{As noted below, $\bar{\alpha}_{ij}$ only exists for tasks with a sufficiently high $w_{ntj} / w^*_j \beta t(i)$.} This cutoff determines the extent in which employment falls in each task and is written as follows

$$R_{ij}^{\bar{\alpha}_{ij}} = T(1 - \rho).$$

(21)

The cutoff $\bar{\alpha}_{ij}$ is the $a$ value for which a worker is indifferent between retraining and keeping her old job and, thus, makes $R_{ij}^{\alpha^*}$ equal to $T(1 - \rho)$. Everything else equal, $R_{ij}^{\alpha^*}$ decreases with $w_{ntj} / w^*_j \beta t(i)$ and, therefore, $\bar{\alpha}_{ij}$ decreases with tradability and, at a given tradability level, is greater for the unskilled tasks. In other words, the extent in which employment falls is greater in tasks where service offshoring has decreased the wage by a greater amount.

For each skill level, it is also possible to derive a cutoff task $\bar{T}_j$ such that employment does not fall in tasks with a lower tradability level. This cutoff is informative on the extensive margin of employment losses, i.e. the proportion of tasks in which employment falls, and is written as follows

$$R_{ij}^{\bar{T}_j} = T(1 - \rho).$$

(22)

The cutoff is $\bar{T}_j$ is the task in which the worker with the highest ability to retrain is indifferent between retraining and keeping her job and, therefore, makes $R_{ij}^{\bar{T}_j}$ equal to $T(1 - \rho)$. All else being equal, $R_{ij}^{\bar{T}_j}$ decreases with $w_{ntj} / w^*_j$ and, therefore, $\bar{T}_j$ is greater for the unskilled tasks (see Appendix for a Proof). Intuitively, since service offshoring more strongly harms the unskilled workers, the proportion of tasks in which employment falls is greater for the unskilled tasks.
5. Predictions and Matching with Current and Further Empirical Evidence

Figure 7 summarizes the results by depicting four continuums. The two continuums in the upper (lower) part depict the employment and wage results for the skilled (unskilled) workers. The comparison between the continuums labeled “Wages” shows the proportion of tasks with losers is greater for the unskilled tasks \((i_u^h > i_s^h)\). The continuums labeled “Employment” show the proportion of tasks in which employment falls is also greater for the unskilled group \((\Gamma_u > \Gamma_s)\). For a given skill level, movements along any of the continuums show that reductions in wage and employment levels increase with tradability (denoted by changes in the heaviness of the solid lines). In other words, service offshoring generates a reallocation of employment from the most to the least tradable tasks.

The model is consistent with facts documented by Crinó (2010). He finds that changes in occupational wages and employment are positively correlated in a sample of 58 white-collar occupations (and thus wage changes have been mostly driven by demand shocks). Accordingly, he conceives service offshoring as a demand shifter and estimates the elasticity of occupational demands with respect to this shifter. He finds a higher concentration of positive elasticities in the group of skilled occupations and of negative elasticities in the unskilled group. His results also show that the occupations with negative elasticities have stronger tradability characteristics than the occupations with positive elasticities. Crinó (2010) goes further and builds a continuous index of tradability to show that the probability of observing a positive elasticity decreases monotonically with tradability. This result holds even after controlling for skill-intensity.\(^{25}\) Putting his wage and employment results together suggests that wages have increased in occupations with the least pronounced tradability characteristics (although this point is suggestive and should be taken with caution).

\(^{25}\)He considers a probit where the dependent variable takes a value of 1 if the elasticity with respect to offshoring is positive. He finds that the effect of the tradability index is negative and precisely estimated, even after controlling for skill-intensity.
Several of these results are consistent with the model presented in this paper. In this model, wage changes are driven by demand shocks. As suggested by Crinó’s evidence, the negative responses of employment to service offshoring are concentrated in the group of unskilled tasks ($I_u > I_s$). The tasks with a negative response also have stronger tradability characteristics. The extent in which employment falls also increases monotonically with tradability in this model (within the set of offshored tasks). Furthermore, this result also holds at given skill level ("even after controlling for skill-intensity") because the model does not make any assumption on the correlation between tradability and skill-intensity. Finally, the results are also consistent with the fact that wages have increased in the least tradable tasks.

The setup is also consistent with some of the evidence provided by Liu and Trefler (2011), which deals with the impact of service offshoring on occupational switching. To motivate their empirical work, they embed a partial equilibrium model of trade in tasks within a Roy-type of model of occupational choice in which workers can switch occupations freely (see comments on the adjustments in employment below). The decision on whether to switch jobs in their model then depends on the wage a worker receives at her current occupation, her skill level and her unobservable characteristics. The setup presented in this paper differs from theirs because the model of tasks is a general equilibrium setup and because it justifies wage differences across occupations on the basis of task-specific knowledge and differences in tradability. Along these lines, the paper links occupational switching to retraining and, therefore, the decision on whether to switch jobs depends on the same three variables they consider but also on the wage of the non-offshored tasks (the occupation to which workers transition).27

26 Liu and Trefler (2011) also provide results on the impact of offshoring on workers’ earnings. They need assumptions on workers sorting to identify the earnings impact for occupational switchers. For occupational stayers, they find small and non-significant effects, which is a consistent with the outcomes of this paper.

27 In my model, the unobserved characteristics are workers’ abilities to retrain.
In their empirical approach, Liu and Trefler (2011) find that the impact of service offshoring on occupational switching is gradual so that adjustment in employment takes some time. My model also predicts a gradual adjustment in employment in which the full effects of service offshoring are observed sometime after the shock. In line with their results, my model predicts that occupational switching is stronger among unskilled workers and among workers employed in routine tasks (a proxy for tradability based on Autor et al., 2003). Whereas their evidence suggests that some of the adjustment process may concern transitions to unemployment (the model motivating their results does not specify the mechanism governing the adjustment process), my setup emphasizes the relevance of task-occupation-specific (Kambourov and Manovskii, 2009a and 2009b and Ritter, 2008) and considers retraining as the mechanism governing the adjustment. It is important to note, on this regard, that transitions to unemployment and retraining may be complementary mechanisms (see Hummels et al., 2012, for retraining and transitions to unemployment).

![Figure 7. Wage and Employment Predictions](image-url)
6. Conclusions

Tradability is relevant to understanding the wage and employment effects of service offshoring because of the particular characteristics of the ICT revolution. This revolution has eased the delivery of service tasks, reducing the offshoring costs of labor tasks. However, the cost reduction has not been homogeneous across tasks. In particular, the extent of the costs reduction has varied (and will) across tasks because they are of a varying nature: Tasks differ in their degree of complexity, their requirements for personal interaction and their level of routineness. Since the reduction in offshoring costs has not been homogeneous and the tradability characteristics are varying, one would expect the effect of service offshoring to vary across tasks. An emerging empirical literature has started to show that tradability is relevant in shaping wage and employment effects. It has also been argued that the tradability and skill-intensity of a task are not correlated. The implication is crucial because the standard classification of labor into skill groups may no longer be useful to classify winners and losers from trade.

In this paper, I have shown the tradability of a task determines the impact of service offshoring on its wage and employment level. The higher the tradability degree of a task, the more service offshoring increases its exposure to foreign competition, and therefore the greater the wage and employment decreases are. I have also shown that service offshoring increases the wage and employment level of the least tradable tasks, and that some workers gain from offshoring even though their task types are offshored. On the other hand, this paper does neglect the standard comparative advantage argument, according to which unskilled-labor abundant countries should export a relatively higher proportion of unskilled tasks. The paper shows that both tradability and skill-intensity are relevant in shaping wage and employment effects and does not rely on a potential correlation between tradability and skill-intensity to obtain this result. By proving the relevance of both labor dimensions, the paper extends the standard classification of labor into skill groups to account for tradability.
7. References


Head, Keith, Thierry Mayer, and John Ries (2009) “How Remote is the Offshoring Threat?”


8. Appendix Section

DIFFERENT $I_u$ AND $I_s$ WITH PRODUCTION FUNCTION SHOWN IN (1).

The proof proceeds in two steps. First, consider the following expressions for $w_{nt}(J_j = I_u)/ (w_j^* \beta t(J_j = I_u))$

$$\frac{w_{ntu}(I_u)}{w_u^* \beta t(I_u)} = \exp \left\{ \frac{\ln(A\tau) - \ln(\beta) - \int_0^{I_u} \ln(t(i)) di}{1-I_u} \right\} = 1,$$

$$\frac{w_{nts}(I_u)}{w_s^* \beta t(I_u)} = \exp \left\{ \frac{\ln(A/\tau) - \ln(\beta) - \int_0^{I_u} \ln(t(i)) di}{1-I_u} \right\} < 1.$$

Since $A\tau = (w_u/w_u^*) > (w_s/w_s^*) = A/\tau$, then $w_{nts}(J_s)/(w_s^* \beta t(J_s)) < 1$ at $J_s = I_u$. In equilibrium, $w_{nts}(J_s)/(w_s^* \beta t(J_s)) = 1$ and, thus, $w_{nts}(J_s)/(w_s^* \beta t(J_s))$ must increase. In the second step, note that

$$\frac{d}{dJ_s} \left( \frac{w_{nts}(J_s)}{w_s^* \beta t(J_s)} \right) = \frac{w_{nts}(J_s)}{w_s^* \beta t(J_s)} \left[ \frac{\ln(w_{nts}(J_s)/w_s^* \beta t(J_s))}{1-J_s} - \frac{t'(J_s)}{t(J_s)} \right] < 0 \text{ at } J_s = I_u.$$

$-t'(J_s)/(t(J_s))$ is negative for every $J_s$ and $\ln(w_{nts}(J_s)/(w_s^* \beta t(J_s)))/1-J_s$ is negative at $J_s = I_u$. This implies that $w_{nts}(J_s)/(w_s^* \beta t(J_s)) < 1$ for any $J_s > I_u$. Hence, $I_u > I_s$.

DIFFERENT $I_u$ AND $I_s$ WITH LEONTIEF PRODUCTION FUNCTION

The proof proceeds in the same two steps. Let’s assume:

$$Y_j = AM \ln \{z_{ij}, z_{ij}, z_{ij}, \ldots, \ldots\}, \ i \in [0,1], \ j = s, u,$$

Thus, we have:

$$\frac{w_{ntu}(I_u)}{w_u^* \beta t(I_u)} = \frac{(Ar) - \beta \int_0^{I_u} t(i) di}{(1-I_u)\beta t(I_u)} = 1.$$

$$\frac{w_{nts}(I_u)}{w_s^* \beta t(I_u)} = \frac{(A/\tau) - \beta \int_0^{I_u} t(i) di}{(1-I_u)\beta t(I_u)} < 1.$$

Since $w_u/w_u^* > w_s/w_s^*$, then $w_{nts}(J_s)/(w_s^* \beta t(J_s)) < 1$ at $J_s = I_u$. We also have:
\[
\frac{d}{dJ_s} \left( \frac{w_{nts}(J_s)}{w_s^* \beta t(J_s)} \right) = \frac{w_s/w_s^* - \beta \int_0^{J_s} t(i) di - (1 - J_s) \beta t(J_s) - t'(J_s)(1 - J_s) \beta}{((1 - J_s) \beta t(J_s))^2} < 0 \text{ at } J_s = I_u.
\]

\( w_s/w_s^* - \beta \int_0^{J_s} t(i) di - (1 - J_s) \beta t(J_s) < 0 \) because \( w_{nts}(J_s)/(w_s^* \beta t(J_s)) < 1 \) at \( J_s = I_u \), then

\[
dw_{nts}(J_s)/d(w_s^* \beta t(J_s)) < 0. \text{ This proves the result.}
\]

**COMPARATIVE STATICS FOR BOTH TYPES OF PRODUCTION FUNCTIONS**

Employing the expressions for \( w_{ntf}(J_j = I_u)/(w_j^* \beta t(J_j = I_u)) \) derived above for both types of production functions, I write:

\[
\frac{dl_j}{d\tau} = \frac{\frac{d}{d\tau} \left( \frac{w_{ntf}(J_j)}{w_j^* \beta t(J_j)} \right)_{J_j = I_j}}{\left( \frac{d}{d\tau} \left( \frac{w_{ntf}(J_j)}{w_j^* \beta t(J_j)} \right) \right)_{J_j = I_j}}.
\]

Whereas the numerator is negative for the case of skilled tasks, it is positive for the case of unskilled tasks. This proves that \( dl_s/d\tau < 0 \) and \( dl_u/d\tau > 0 \) given that denominator is negative for both cases.

The following expression studies the effect of changes in the Hicks parameter

\[
\frac{dl_j}{d\tau} = -\frac{dl_j}{dA} = -\frac{\frac{d}{dA} \left( \frac{w_{ntf}(J_j)}{w_j^* \beta t(J_j)} \right)_{J_j = I_j}}{\left( \frac{d}{dA} \left( \frac{w_{ntf}(J_j)}{w_j^* \beta t(J_j)} \right) \right)_{J_j = I_j}}
\]

The numerator is negative and the denominator is negative for both cases. Hence, \( (dl_s)/(dA) > 0 \) and \( (dl_u)/(dA) > 0 \).

**WINNERS AND LOSERS FOR BOTH TYPES OF PRODUCTION FUNCTIONS**

The indifferent task, which has been indicated by \( h \), is written as follows

\[
\frac{w_j}{w_j^*} = t(i_j^h).
\]

If \( w_u/w_u^* > w_s/w_s^* \) and \( t(.) \) is increasing in \( i \), then \( i_s^h < i_u^h \).
RETRAINING

To solve (19) I will first ignore the time constraint, $R_{ij}^a \leq T$ and then impose it on the solution arising from the one-constraint problem. I solve this problem in two steps. First, I take the program’s duration as given and maximize the worker’s lifetime income with respect to $h_{ij,t}^a$. Second, I substitute the solution into $I_{ij}^{a,R}$ and maximize with respect to $R_{ij}^a$. The first step is summarized as follows

$$
\lambda_{ij}^a = \frac{w_j^* \beta t(i)}{a} \left( \left( 1 - \frac{\rho}{r} \right) \left( \exp \left( \frac{r \rho R_{ij}^a}{1 - \rho} \right) - 1 \right) \right)^{-\frac{1}{\rho}} ,
$$

$$
h_{ij,t}^a = \frac{\theta}{a} \left( \left( 1 - \frac{\rho}{r} \right) \left( \exp \left( \frac{r \rho R_{ij}^a}{1 - \rho} \right) - 1 \right) \right)^{-\frac{1}{\rho}} \exp \left\{ \frac{rt}{1 - \rho} \right\} ,
$$

where $\lambda_{ij}^a$ is the marginal cost of an effective learning hour. Plugging this result into $I_{ij}^{a,R}$ and maximizing with respect to $R_{ij}^a$. yields the following condition

$$
(w_{nt} - w_j^* \beta t(i)) \exp \{-R_{ij}^a r\} = \left( \frac{\theta w_j^* \beta t(i)}{a} \right)^{1 - \rho} \left( \frac{1 - \rho}{\rho} \right)^{(1 - \rho)} \exp \left\{ \frac{r \rho R_{ij}^a}{1 - \rho} \right\} \left( \exp \left\{ \frac{r \rho R_{ij}^a}{1 - \rho} \right\} - 1 \right)^{-\frac{1}{\rho}}
$$

Finally, note that replacing this equation into $I_{ij}^{a,R} - I_{ij}^{a,NR}$ yields Equation (20) The RHS of the Equation displayed above shows the impact of decreasing $R_{ij}^a$ on retraining costs: As can be seen, in Equation (19), $\lambda_{ij}^a$ decreases with $R_{ij}^a$ because of the C.E.S. specification (see footnote at the beginning of Section 4 on this regard). The LHS shows the marginal benefit from decreasing $R_{ij}^a$: The sooner the retraining process ends, the sooner the worker starts to benefit from the wage increase.

When the wage increase is sufficiently small no real value of $R_{ij}^a$ solves for the equation. The marginal net benefit from increasing $R_{ij}^a$ is $\left( \frac{\theta w_j^* \beta t(i)}{a} \right)^{1 - \rho} \left( 1 - \frac{\rho}{r} \right)^{(1 - \rho)} \exp \left\{ r \rho R_{ij}^a / (1 - \rho) \right\} - 1 \right)^{-\frac{1}{\rho}} - \exp \{-r/(1 - \rho)R_{ij}^a\} \left( w_{nt} - w_j^* \beta t(i) \right)$. The two terms are monotonically decreasing in $R_{ij}^a$ and therefore the marginal benefit is positive for $R_{ij}^a < R_{ij}^a$. The worker increases the program’s duration as much as she can and, therefore, her best strategy is to set the duration to $T$ periods. When
the wage increase is not sufficiently small, $R_{ij}^{\alpha^*}$ arises from balancing the marginal benefit and marginal cost. The solution is given in Equation (19).

**CUTOFF RETRAINING TASKS**

Replacing the optimal plan’s duration in equation (22) yields the following

$$I_j = t' \left( \frac{w_{ntj}}{w_j^* \beta} \left( \frac{\bar{a}}{\bar{b}} \left( 1 + e^{\tau r \rho} \right)^{1-\rho} \right) \right).$$

This retraining cutoff increases in the zero-profit to RW wage ratio and therefore is greater for the unskilled tasks.