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Research Report

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Search and matching frictions and business cycle fluctuations in Bulgaria: Technical Appendix

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1 Households' problem

The problem can be simplified as

$$\begin{aligned} \mathcal{L} = & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t + \phi \ln(1 - N_t) \right. \\ & - \lambda_t [C_t + K_{t+1} - (1 - \delta)K_t + b_0 S_t^\eta (1 - N_t) - (1 - \tau^l)r_t K_t - (1 - \tau^l)w_t N_t - \Pi_t - G_t^{tr}] \\ & \left. + \mu_t [(1 - \psi)N_t + p_t S_t (1 - N_t) - N_{t+1}] \right\}, \end{aligned} \quad (1.1)$$

where λ_t , μ_t are the Lagrangean multiplier in front of the households' budget constraint and the law of motion for employment.

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FOCs:

$$C_t : \frac{1}{C_t} = \lambda_t (1.2)$$

$$K_{t+1} : \lambda_t = \beta E_t \lambda_{t+1} [(1 - \tau^k) \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta)] (1.3)$$

$$S_t : \lambda_t b_0 \eta S_t^{\eta-1} (1 - N_t) = \mu_t p_t (1 - N_t) (1.4)$$

$$N_{t+1} : \beta E_t \lambda_{t+1} \left[(1 - \tau^l) w_{t+1} + b_0 S_{t+1}^\eta - \frac{\phi}{1 - N_{t+1}} \right] + \beta E_t \mu_{t+1} [(1 - \psi) - p_{t+1} S_{t+1}] = \mu_t (1.5)$$

$$\lim_{t \rightarrow \infty} \frac{1}{C_t} K_{t+1} = 0 (1.6)$$

Substitute out the Lagrangean multiplier attached to the budget constraint, as well as the probability of a match

$$\frac{1}{C_t} b_0 \eta S_t^{\eta-1} = \mu_t p_t (1.7)$$

$$\begin{aligned} -\frac{\beta \phi}{1 - N_{t+1}} + \beta E_t \frac{1}{C_{t+1}} (1 - \tau^l) w_{t+1} + \beta E_t \frac{1}{C_{t+1}} b_0 S_{t+1}^\eta \\ + \beta E_t \mu_{t+1} (1 - \psi) - \beta \mu_{t+1} p_{t+1} S_{t+1} = \mu_t \end{aligned} (1.8)$$

Note that

$$\mu_t = \frac{b_0 \eta S_t^{\eta-1}}{C_t p_t} (1.9)$$

Rearrange terms

$$\beta E_t \frac{1}{C_{t+1}} \left[(1 - \tau^l) w_{t+1} + b_0 S_{t+1}^\eta - \frac{\phi}{1 - N_{t+1}} \right] + \beta E_t \mu_{t+1} [(1 - \psi) - p_{t+1} S_{t+1}] = \mu_t (1.10)$$

or

$$\beta E_t \frac{1}{C_{t+1}} \left[(1 - \tau^l) w_{t+1} + b_0 S_{t+1}^\eta - \frac{\phi}{1 - N_{t+1}} \right] + \beta E_t \frac{b_0 \eta S_{t+1}^{\eta-1}}{C_{t+1} p_{t+1}} [(1 - \psi) - p_{t+1} S_{t+1}] = \frac{b_0 \eta S_t^{\eta-1}}{C_t p_t}$$

Multiply by p_t

$$p_t \beta E_t \frac{1}{C_{t+1}} \left[(1 - \tau^l) w_{t+1} + b_0 S_{t+1}^\eta - \frac{\phi}{1 - N_{t+1}} \right] + p_t \beta E_t \frac{b_0 \eta S_{t+1}^{\eta-1}}{C_{t+1} p_{t+1}} [(1 - \psi) - p_{t+1} S_{t+1}] = \frac{b_0 \eta S_t^{\eta-1}}{C_t}$$

or

$$\frac{b_0 \eta S_t^{\eta-1}}{C_t} = p_t \beta E_t \left\{ \frac{1}{C_{t+1}} \left[(1 - \tau^l) w_{t+1} + b_0 S_{t+1}^\eta \right] + \frac{\phi}{1 - N_{t+1}} + \frac{b_0 \eta S_{t+1}^{\eta-1}}{C_{t+1} p_{t+1}} [(1 - \psi) - p_{t+1} S_{t+1}] \right\}$$

2 Firm's problem

The problem of the firm is also one of constrained optimization nature:

$$\max_{\{K_t, V_t, N_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_t \left\{ A_t K_t^\alpha N_t^{1-\alpha} - w_t N_t - r_t K_t - a V_t - \nu_t [(1-\psi)N_t + q_t V_t - N_{t+1}] \right\}, \quad (2.1)$$

where

$$\beta_t = \beta E_{t-1} \left(\frac{C_t}{C_{t-1}} \right) \quad (2.2)$$

is the stochastic discount factor.

FOCs:

$$K_t : \alpha \frac{Y_t}{K_t} = r_t \quad (2.3)$$

$$V_t : \nu_t q_t = -a \quad (2.4)$$

$$N_t : \beta_t \left[(1-\alpha) \frac{Y_{t+1}}{N_{t+1}} - w_{t+1} - \nu_{t+1}(1-\psi) \right] = -\nu_t \quad (2.5)$$

From the optimality condition for vacancies, express $\nu_t = -a/q_t$ and plug it into the FOC for employment to obtain

$$\beta_t \left[(1-\alpha) \frac{Y_{t+1}}{N_{t+1}} - w_{t+1} + \frac{a(1-\psi)}{q_{t+1}} \right] = \frac{a}{q_t} \quad (2.6)$$

or

$$\beta E_{t-1} \left(\frac{C_t}{C_{t-1}} \right) \left[(1-\alpha) \frac{Y_{t+1}}{N_{t+1}} - w_{t+1} + \frac{a(1-\psi)}{q_{t+1}} \right] = \frac{a}{q_t} \quad (2.7)$$

In the literature, this is also referred to as the job creation condition (JCC). In other words, this equation characterizes the firm's labor demand.

3 Steady-State

$$1 = \beta[(1 - \tau^k)\alpha\frac{y}{k} + 1 - \delta] \quad (3.1)$$

$$\frac{a}{c} = \mu(1 - \gamma)\frac{m}{v} \quad (3.2)$$

$$\frac{b_0\eta s^{\eta-1}(1-n)}{c} = \mu\gamma\frac{m}{s} \quad (3.3)$$

$$m = \psi n \quad (3.4)$$

$$u = 1 - n \quad (3.5)$$

$$m = v^{1-\gamma}[su]^\gamma \quad (3.6)$$

$$c + \delta k + b_0 s^\eta(1-n) + av = y \quad (3.7)$$

$$y = Ak^\alpha n^{1-\alpha} \quad (3.8)$$

$$\frac{\beta}{c}[(1 - \tau^l)(1 - \alpha)\frac{y}{n} + b_0 s^\eta] + \frac{\beta\phi}{1-n} = \mu[1 - \beta(1 - \psi - \gamma\frac{m}{1-n})] \quad (3.9)$$

$$p = \frac{m}{s(1-n)} = \left(\frac{\theta}{s}\right)^{1-\gamma} \quad (3.10)$$

$$\theta = \frac{m}{1-n} \quad (3.11)$$

$$q = \frac{m}{v} = \left(\frac{s}{\theta}\right)^\gamma \quad (3.12)$$

4 Log-linearization

4.1 Capital accumulation equation

$$k_{t+1} = i_t + (1 - \delta)k_t \quad (4.1)$$

Take natural logarithms from both sides to obtain

$$\ln k_{t+1} = \ln[i_t + (1 - \delta)k_t] \quad (4.2)$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln k_{t+1}}{dt} = \frac{d \ln[i_t + (1 - \delta)k_t]}{dt} \quad (4.3)$$

$$\hat{k}_{t+1} = \frac{d[i_t + (1 - \delta)k_t]}{dt} \frac{1}{i + (1 - \delta)k} \quad (4.4)$$

Using that in steady state $k = i + (1 - \delta)k$, or $i = \delta k$

$$\hat{k}_{t+1} = \frac{di_t}{k dt} + (1 - \delta) \frac{dk_t}{k dt} \quad (4.5)$$

$$\hat{k}_{t+1} = \delta \hat{i}_t + (1 - \delta) \hat{k}_t \quad (4.6)$$

4.2 Employment dynamics equation

$$n_{t+1} = (1 - \psi)n_t + m_t \quad (4.7)$$

Take natural logarithms from both sides to obtain

$$\ln n_{t+1} = \ln[(1 - \psi)n_t + m_t] \quad (4.8)$$

Totally differentiate w.r.t. time to obtain

$$\frac{d \ln n_{t+1}}{dt} = \frac{d \ln[(1 - \psi)n_t + m_t]}{dt} \quad (4.9)$$

$$\frac{dn_{t+1}}{dt} \frac{1}{n} = \frac{d[(1 - \psi)n_t + m_t]}{dt} \frac{1}{(1 - \psi)n + m} \quad (4.10)$$

Using that in steady state $n = (1 - \psi)n + m$,

$$\frac{dn_{t+1}}{dt} \frac{1}{n} = (1 - \psi) \frac{dn_t}{dt} \frac{1}{n} + \frac{dm_t}{dt} \frac{1}{n} \quad (4.11)$$

or

$$\frac{dn_{t+1}}{dt} \frac{1}{n} = (1 - \psi) \frac{dn_t}{dt} \frac{1}{n} + \frac{dm_t}{dt} \frac{1}{n} \frac{m}{m} \quad (4.12)$$

Pass to log-deviations to obtain

$$n\hat{n}_{t+1} = (1 - \psi)n\hat{n}_t + m\hat{m}_t \quad (4.13)$$

4.3 Matching function

$$m_t = v_t^{1-\gamma} [s_t(1 - n_t)]^\gamma \quad (4.14)$$

Take natural logarithms from both sides to obtain

$$\ln m_t = (1 - \gamma) \ln v_t + \gamma \ln s_t + \gamma \ln(1 - n_t) \quad (4.15)$$

Totally differentiate w.r.t. time to obtain

$$\frac{d \ln m_t}{dt} = (1 - \gamma) \frac{d \ln v_t}{dt} + \gamma \frac{d \ln s_t}{dt} + \gamma \frac{d \ln(1 - n_t)}{dt} \quad (4.16)$$

Pass to log-deviations

$$\hat{m}_t = (1 - \gamma)\hat{v}_t + \gamma\hat{s}_t - \frac{\gamma n}{1 - n}\hat{n}_t \quad (4.17)$$

4.4 Log-linearizing transition probability from unemployment to employment

$$p_t = \frac{M_t}{S_t(1 - N_t)} \quad (4.18)$$

$$\hat{p}_t = \hat{m}_t - \hat{s}_t + \frac{n}{1 - n}\hat{n}_t \quad (4.19)$$

Alternatively

$$p_t = \left(\frac{\theta_t}{S_t} \right)^{1-\gamma} \quad (4.20)$$

or

$$\hat{p}_t = (1 - \gamma)\hat{\theta}_t - (1 - \gamma)\hat{s}_t \quad (4.21)$$

4.5 Log-linearizing market tightness

$$\theta_t = \frac{M_t}{1 - N_t} \quad (4.22)$$

$$\hat{\theta}_t = \hat{m}_t + \frac{n}{1 - n} \hat{n}_t \quad (4.23)$$

4.6 Log-linearizing transition probability from an unfilled vacancy to a filled one

$$q_t = \frac{m_t}{v_t} \quad (4.24)$$

$$\hat{q}_t = \hat{m}_t - \hat{v}_t \quad (4.25)$$

Alternatively,

$$q_t = \left(\frac{S_t}{\theta_t} \right)^\gamma \quad (4.26)$$

or

$$\hat{q}_t = \gamma \hat{s}_t - \gamma \hat{\theta}_t \quad (4.27)$$

4.7 Production function

$$y_t = A_t k_t^\alpha n_t^{1-\alpha} \quad (4.28)$$

Take natural logarithms from both sides to obtain

$$\ln y_t = \ln A_t + \alpha \ln k_t + (1 - \alpha) \ln n_t \quad (4.29)$$

Totally differentiate w.r.t. time to obtain

$$\frac{d \ln y_t}{dt} = \frac{d \ln A_t}{dt} + \alpha \frac{d \ln k_t}{dt} + (1 - \alpha) \frac{d \ln n_t}{dt} \quad (4.30)$$

Pass to log-deviations to obtain

$$\hat{y}_t = \hat{A}_t + \alpha \hat{k}_t + (1 - \alpha) \hat{n}_t \quad (4.31)$$

4.8 Euler equation

$$\frac{1}{c_t} = \beta E_t \frac{1}{c_{t+1}} [(1 - \tau^k) \alpha \frac{y_{t+1}}{k_{t+1}} + (1 - \delta)] \quad (4.32)$$

Take natural logarithms from both sides to obtain

$$\ln \frac{1}{c_t} = \ln \beta E_t \frac{1}{c_{t+1}} [\alpha \frac{y_{t+1}}{k_{t+1}} + (1 - \delta)] \quad (4.33)$$

$$-\ln c_t = \ln \beta - \ln E_t c_{t+1} + \ln E_t [(1 - \tau^k) \alpha \frac{y_{t+1}}{k_{t+1}} + (1 - \delta)] \quad (4.34)$$

Totally differentiate w.r.t. time and simplify to obtain

$$-\frac{d \ln c_t}{dt} = -\frac{d \ln E_t c_{t+1}}{dt} + \frac{d \ln E_t [(1 - \tau^k) \alpha \frac{y_{t+1}}{k_{t+1}} + (1 - \delta)]}{dt} \quad (4.35)$$

or

$$-\hat{c}_t = -\hat{c}_{t+1} + \frac{d E_t [(1 - \tau^k) \alpha \frac{y_{t+1}}{k_{t+1}} + (1 - \delta)]}{dt} \frac{1}{\alpha \frac{y}{k} + (1 - \delta)} \quad (4.36)$$

Noting that in equilibrium $\beta = (1 - \tau^k) \alpha \frac{y}{k} + (1 - \delta)$

$$-\hat{c}_t = -\hat{c}_{t+1} + \beta (1 - \tau^k) \alpha \frac{d E_t \frac{y_{t+1}}{k_{t+1}}}{dt} \quad (4.37)$$

$$-\hat{c}_t = -\hat{c}_{t+1} + \beta (1 - \tau^k) \alpha [\frac{d y_{t+1} k - d k_{t+1} y}{d t k^2}] \quad (4.38)$$

$$-\hat{c}_t = -\hat{c}_{t+1} + \beta (1 - \tau^k) \alpha \frac{y}{k} \hat{y}_{t+1} - \beta (1 - \tau^k) \alpha \frac{y}{k} \hat{k}_{t+1} \quad (4.39)$$

4.9 FOC vacancy rate

$$\frac{a}{c_t} = \mu_t (1 - \gamma) \frac{m_t}{v_t} \quad (4.40)$$

Take natural logarithms from both sides to obtain

$$\ln \frac{a}{c_t} = \ln \mu_t (1 - \gamma) \frac{m_t}{v_t} \quad (4.41)$$

$$\ln a - \ln c_t = \ln \mu_t + \ln(1 - \gamma) + \ln m_t - \ln v_t \quad (4.42)$$

Totally differentiate w.r.t. time to obtain

$$\frac{d \ln a}{dt} - \frac{d \ln c_t}{dt} = \frac{d \ln \mu_t}{dt} + \frac{d \ln(1 - \gamma)}{dt} + \frac{d \ln m_t}{dt} - \frac{d \ln v_t}{dt} \quad (4.43)$$

Simplify to obtain

$$-\frac{d \ln c_t}{dt} = \frac{d \ln \mu_t}{dt} + \frac{d \ln m_t}{dt} - \frac{d \ln v_t}{dt} \quad (4.44)$$

Pass to log-deviations

$$-\hat{c}_t = \hat{\mu}_t + \hat{m}_t - \hat{v}_t \quad (4.45)$$

4.10 FOC search effort

$$\frac{b_0 \eta s_t^{\eta-1} (1 - n_t)}{c_t} = \mu_t \gamma \frac{m_t}{s_t} \quad (4.46)$$

Take natural logarithms from both sides to obtain

$$\ln \frac{b_0 \eta s_t^{\eta-1} (1 - n_t)}{c_t} = \ln \mu_t \gamma \frac{m_t}{s_t} \quad (4.47)$$

$$\ln b_0 + \ln \eta + (\eta - 1) \ln s_t + \ln(1 - n_t) - \ln c_t = \ln \mu_t + \ln \gamma + \ln m_t - \ln s_t \quad (4.48)$$

Totally differentiate w.r.t time and simplify to obtain

$$(\eta - 1) \frac{d \ln s_t}{dt} + \frac{d \ln(1 - n_t)}{dt} - \frac{d \ln c_t}{dt} = \frac{d \ln \mu_t}{dt} + \frac{d \ln m_t}{dt} - \frac{d \ln s_t}{dt} \quad (4.49)$$

$$(\eta - 1) \hat{s}_t - \frac{n}{1 - n} \hat{n}_t - \hat{c}_t = \hat{\mu}_t + \hat{m}_t - \hat{s}_t \quad (4.50)$$

or

$$\eta \hat{s}_t - \frac{n}{1 - n} \hat{n}_t - \hat{c}_t = \hat{\mu}_t + \hat{m}_t \quad (4.51)$$

4.11 Dynamic FOC employment

$$[E_t \frac{\beta}{c_{t+1}} [(1 - \alpha) \frac{y_{t+1}}{n_{t+1}} + b_0 s_{t+1}^\eta] - \frac{\beta \phi}{1 - n_{t+1}}] = \mu_t - \beta E_t \mu_{t+1} [1 - \psi - \gamma \frac{m_{t+1}}{1 - n_{t+1}}] \quad (4.52)$$

Take natural logarithms from both sides and totally differentiate w.r.t time to obtain

$$\frac{d[E_t \frac{\beta}{c_{t+1}} [(1 - \alpha) \frac{y_{t+1}}{n_{t+1}} + b_0 s_{t+1}^\eta] - \frac{\beta \phi}{1 - n_{t+1}}]}{dt} = \frac{d \mu_t}{dt} - \frac{d \beta E_t \mu_{t+1} [1 - \psi - \gamma \frac{m_{t+1}}{1 - n_{t+1}}]}{dt} \quad (4.53)$$

$$\begin{aligned} & \frac{\beta}{c} (1 - \alpha) \frac{y}{n} E_t \hat{c}_{t+1} + \frac{\beta}{c} (1 - \alpha) \frac{y}{n} E_t \hat{y}_{t+1} - \frac{\beta}{c} (1 - \alpha) \frac{y}{n} E_t \hat{n}_{t+1} \\ &= \mu \hat{\mu}_t - \beta \mu E_t \hat{\mu}_{t+1} [1 - \psi - \gamma \frac{m}{1 - n}] + \beta \gamma \mu \frac{m}{1 - n} E_t \hat{m}_{t+1} + \beta \gamma \mu \frac{mn}{(1 - n)^2} E_t \hat{n}_{t+1} \end{aligned} \quad (4.54)$$

4.12 Market clearing

$$y_t = c_t + k_{t+1} - (1 - \delta)k_t + g_t + b_0 s_t^\eta (1 - n_t) + av_t \quad (4.55)$$

Take natural logarithms from both sides and totally differentiate w.r.t to time to obtain

$$\frac{d}{dt} \ln y_t = \frac{d}{dt} \left[c_t + k_{t+1} - (1 - \delta)k_t + g_t + b_0 s_t^\eta (1 - n_t) + av_t \right] \quad (4.56)$$

$$y\hat{y}_t = c\hat{c}_t + \hat{k}k_{t+1} - (1 - \delta)\hat{k}k_t + g\hat{g}_t + \eta b_0 s^\eta (1 - n)\hat{s}_t - b_0 s^\eta n\hat{n}_t + av\hat{v}_t \quad (4.57)$$

4.13 Nash bargaining rule

$$w_t = \gamma \left((1 - \alpha) \frac{y_t}{n_t} + \frac{av_t}{1 - n_t} \right) + (1 - \gamma) \left(- \frac{1 - n_t}{\phi c_t} - b_0 s_t^\eta (1 - n_t) \right) \quad (4.58)$$

Take natural logarithms from both sides and totally differentiate w.r.t to time to obtain

$$\frac{d}{dt} \ln w_t = \frac{d}{dt} \ln \left\{ \gamma \left((1 - \alpha) \frac{y_t}{n_t} + \frac{av_t}{1 - n_t} \right) + (1 - \gamma) \left(- \frac{1 - n_t}{\phi c_t} - b_0 s_t^\eta (1 - n_t) \right) \right\} \quad (4.59)$$

$$w\hat{w}_t = \frac{d}{dt} \left\{ \gamma \left((1 - \alpha) \frac{y_t}{n_t} + \frac{av_t}{1 - n_t} \right) + (1 - \gamma) \left(- \frac{1 - n_t}{\phi c_t} - b_0 s_t^\eta (1 - n_t) \right) \right\} \quad (4.60)$$

$$w\hat{w}_t = \gamma(1 - \alpha) \frac{y}{n} \hat{y}_t - \gamma(1 - \alpha) \frac{y}{n} \hat{n}_t + \frac{d}{dt} \left\{ \left(\frac{av_t}{1 - n_t} \right) + (1 - \gamma) \left(- \frac{1 - n_t}{\phi c_t} - b_0 s_t^\eta (1 - n_t) \right) \right\}$$

$$w\hat{w}_t = \gamma(1 - \alpha) \frac{y}{n} \hat{y}_t - \gamma(1 - \alpha) \frac{y}{n} \hat{n}_t + \frac{av}{1 - n} \hat{v}_t - \frac{avn}{(1 - n)^2} \hat{n}_t + \frac{d}{dt} \left\{ (1 - \gamma) \left(- \frac{1 - n_t}{\phi c_t} - b_0 s_t^\eta (1 - n_t) \right) \right\}$$

$$w\hat{w}_t = \gamma(1 - \alpha) \frac{y}{n} \hat{y}_t - \gamma(1 - \alpha) \frac{y}{n} \hat{n}_t + \frac{av}{1 - n} \hat{v}_t - \frac{avn}{(1 - n)^2} \hat{n}_t + (1 - \gamma) \frac{1 - n}{\phi c} \hat{c}_t + (1 - \gamma) \frac{n}{\phi c} \hat{n}_t - \eta b_0 s^\eta (1 - n) \hat{s}_t + b_0 s^\eta n \hat{n}_t$$