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IFN Working Paper No. 1089, 2015

## **Global Engagement, Complex Tasks, and the Distribution of Occupational Employment**

Carl Davidson, Fredrik Heyman, Steven Matusz,  
Fredrik Sjöholm and Susan Chun

# Global Engagement, Complex Tasks, and the Distribution of Occupational Employment

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**Abstract:** Building on a framework introduced by Chaney and Ossa (2013), we construct a task-based model of the firm's choice of occupational inputs to examine how that choice varies with greater global engagement. We depart from Chaney and Ossa by assuming that more complex tasks are more costly to complete. Within the structure of our model, firms skew employment toward occupations engaged in more complex tasks. Moreover, the distribution of employment is more skewed for more globalized firms, while it is less skewed for larger firms. These results are consistent with our empirical findings in Davidson, et al (2015).

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## 1. Introduction

Producing abroad or selling goods on global markets may require firms to hire workers with different skill sets than if they confined all production and marketing activities to their home market. Language and legal barriers need to be overcome, coordination across units located in different countries takes special skills, and different production processes may be more appropriate given the composition of the foreign market labor pool. Thus, one would expect firms that are globally engaged to use a somewhat different occupational mix of workers than their domestic counterparts. And, there is substantial empirical evidence that this is the case. For example, the literature on foreign direct investment (FDI) has emphasized that multinational firms (MNEs) are more skill intensive than domestic firms (see Markusen 1995 and Barba-Navaretti and Venables 2004 for surveys) while Bernard and Jensen (1997) have documented that exporters are more skill-intensive than non-exporters.<sup>1</sup>

Many of the researchers working on this topic have been forced, due to data limitations, to (a) confine their attention to manufacturing and (b) use production workers to measure unskilled employment and non-production workers to measure skilled employment. The emergence of new, richer data sets has recently allowed for a more detailed analysis of this issue. In particular, in a recent paper, using comprehensive Swedish matched employer-employee data that tracks employment in roughly 100 occupations across the entire Swedish private sector, we show that the differences in occupational mix across firms with different levels of global engagement are remarkably strong, regular and robust (see Davidson, Heyman, Matusz, Sjöholm and Zhu 2015).

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<sup>1</sup> See also Frias, Kaplan and Verhoogen (2009), Shank, Schnabel, and Wagner (2007), and Amiti and Cameron (2012) for evidence on exporters versus non-exporters. Hakkala-Nilsson, Heyman, and Sjöholm (2014) and Poole (2013) provide evidence on multinationals. Becker, Ekholm, and Muendler (2013) examine the empirics of tasks related to offshoring.

Some of these differences are displayed in Figure 1, which comes directly from our earlier paper. This figure shows the aggregate distribution of occupations, ranked by skill, for three firm types: Local firms (non-multinationals that do not export); Exporters that are not multinationals; and Multinationals. On the horizontal axis we have the percentile ranking of occupations by skill levels<sup>2</sup>, with the skill level increasing as we move left to right; and on the vertical axis we have the cumulative employment share of the labor force accounted for by each skill level. Figure 1 shows that Exporters use a distribution of occupations that tracks closely to the 45° line, indicating that their employees are roughly evenly distributed over all occupations. Local firms have a distribution that is skewed towards low-skilled occupations; while Multinationals have a distribution that is skewed towards high-skilled occupations. Moreover, the distribution for Local (Multinational) firms lies entirely to the left (right) of the Exporter distribution, indicating that the skill-ranking of the occupational mixes used by these firms holds for every sub-set of occupations that one chooses to focus on. These main findings do not change when we control for firm size, productivity, firm age, offshoring activities and/or R&D activities. They hold when we use different methods to rank occupations by skill and when we use different measures of global engagement. In addition, the results for manufacturing and non-manufacturing industries are quite similar.

The usual explanation for these empirical findings is that global engagement requires additional fixed investment and that fixed costs are more skill intensive than variable costs.<sup>3</sup> Yet, the extent to which firms make fixed investments is endogenous, depending on the nature of

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<sup>2</sup> In our earlier paper we rank occupations in two ways: by wages for all firms in 1997 and based on a Mincer wage regression ranking (see Davidson, et al 2015 for details). The two rankings yield remarkably similar figures. In this paper, Figure 1 is based on the ranking by average wages.

<sup>3</sup> Alternative explanations include quality and technology upgrading of exporters that require a workforce with higher skills. See Verhoogen (2008) for the link between quality upgrading and exports. Bustos (2011) provides evidence on the impact of exports on technology adoption.

the production process that the firm chooses to use and the manner in which the firm designs its production chain. In this paper, we present a model in which the design of the firm's production chain is endogenous and show that the result that globally engaged firms are more skill intensive follows naturally from an assumption that more complex tasks are more expensive to complete. To do so, we employ the framework of Chaney and Ossa (2013) which assumes that endogenously designed teams are trained to complete different tasks in the production process. Firms hire ex-ante identical workers to work as teams to complete a set of tasks. Each team is trained to have a core competency and then completes a set of tasks that are sufficiently close to their core competency. In Chaney and Ossa, all tasks are equally costly to complete and the teams are symmetric in terms of the range of tasks that they complete. As the firm expands in size, it adds more teams and the measure of tasks completed by each team shrinks.

Our point of departure from Chaney and Ossa is the assumption that some tasks are more complex than others, with more complex tasks being more difficult (expensive) to complete. As a result, we show that the equilibrium structure of the production chain is asymmetric in that workers specialized to less-complex tasks are given a more narrow set of tasks to complete.<sup>4</sup> We show that small firms that are not globally engaged will have production processes that are biased in two ways. First, these firms tend to make use of a relatively large number of low-skilled occupations so that the distribution of occupations is skewed in favor of those that are used to complete low-skilled tasks. Second, since high-skilled tasks must be completed for production to take place and since the firm is making use of a relatively small number of high-skilled occupations, employment is skewed in favor of high-skilled workers. As firm expand, either in size or in terms of the markets they serve (global engagement), these biases change with

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<sup>4</sup> The use of the term "production chain" suggests that tasks are sequenced, as in Chaney and Ossa (2013). While we do not explicitly assume sequencing, our framework is consistent with a notion that later tasks are more complex than earlier tasks.

the size effect and the global engagement effect working in opposite directions. Firms that become larger without becoming more globally engaged will start to make use of more high-skill occupations and will see the bias in employment shift back towards low-skilled employment. That is, increases in size will tend to reduce both biases in the production process. On the other hand, increases in global engagement, with firm size held fixed, will tend to increase both biases. In particular, globally engaged firms employ occupational mixes of workers that are more skill intensive than their domestic counterparts. These results are consistent with the stylized facts suggested by Figure 1 and the empirical results provided in our earlier paper.

We formally describe the Chaney and Ossa (2013) model with our modifications in the next section and show that the range of tasks assigned to each occupation is positively related to the complexity of the occupation's specialization. We also show that this implies that the production process is skewed in the two ways described above. In Section 3, we examine how these biases change as firm size and global engagement vary, assuming that the design of the production chain is held fixed. In Section 4, we solve for the optimal number of occupations and describe the nature of the optimal production chain, paying particular attention to how the design varies with firm size and global engagement. In section 5, we revisit the results derived in Section 3 that describe the link between globalization and occupational mix, explaining how our results need to be modified when the design of the production chain is allowed to vary. We offer some concluding remarks in section 6.

## 2. The Model

Our model has the same basic set-up as Chaney and Ossa (2013). We assume that production requires the completion of a set of tasks, and that ex-ante identical workers are hired and trained to perform a given set of tasks. Tasks differ in order of complexity, and workers

trained to complete different tasks have, ex-post, different skills sets and are employed in different occupations. We use  $\omega \in [0,1]$  to index tasks and assume that task  $\omega''$  is more complex than task  $\omega'$  for all  $\omega'' > \omega'$ .

For purposes of our model, we think of an occupation as a specialization in a particular task and assume that more complex tasks require more resources to master. This could be because the training required to master a more complex task utilizes more efficiency units of labor, or because more complex tasks entail more non-labor training costs, or because the firm must pay higher wages to compensate more skilled workers.<sup>5</sup> For the purpose of this paper, we assume that there is a fixed amount of labor required to master each occupation and this amount is increasing in the degree of complexity of the task.<sup>6</sup> We further assume that a worker employed in a particular occupation can perform tasks other than that to which the occupation is specialized, but the cost of doing so is increasing in distance from his specialization. Specifically, suppose that there are  $n$  occupations indexed by the  $i$ , let  $s_i \in [0,1]$  represent the specialization associated with occupation  $i$  and let  $\ell_i$  represent the number of workers in occupation  $i$  needed to produce the set of tasks  $\omega \in [\omega_l, \omega_h]$  needed to produce  $y$  units of output. We assume that

$$(1) \quad \ell_i = f + \beta s_i + y \int_{\omega_l}^{\omega_h} |s_i - \omega| d\omega, \quad f > 0, \quad \beta > 0$$

Finally, we assume that all workers are paid the same wage. It is important to note that the only difference between our model and Chaney and Ossa (2013) is the assumption that the

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<sup>5</sup> In the current version of our model, all workers are ex-ante identical. However, after training is completed some workers will have more skills than others and may be able to command higher wages.

<sup>6</sup> Alternatively, we could assume that physical inputs needed for mastery are the same for all tasks but that the wage paid to workers performing more complex tasks is higher. While the two models have similar qualitative features, an assumption that the wage paid to workers who perform more complex tasks is higher complicates the analysis since higher wages would increase the cost of deviation from a worker's specialization as that specialization increased in complexity.



fixed cost of mastering a task is increasing in  $s$ ; in their model, the fixed cost of mastering a task is the same for all tasks. In terms of the notation,  $\beta = 0$  in Chaney and Ossa (2013).

Given a fixed, common wage for all workers and given the range of tasks to be completed, the firm's objective is to minimize the employment used to produce  $y$ . Formally, this problem is:

$$(2) \quad \min_{s_i} f + \beta s_i + y \int_{\omega_l}^{\omega_h} |s_i - \omega| d\omega, \text{ subject to } s_i \in [0,1]$$

Let  $s_i^*$  represent the solution to this (2). The first-order condition for a strictly interior solution to this problem is<sup>7</sup>

$$(3) \quad \frac{\partial \ell_i}{\partial s_i} = \beta + y(s_i - \omega_l) - y(\omega_h - s_i) = 0$$

The optimally-chosen specialization is then given as follows:

$$(4) \quad s_i^* = \frac{\omega_h + \omega_l}{2} - \frac{1}{2} \frac{B}{Y}$$

where we use the normalizations  $B \equiv \beta/f$  and  $Y \equiv y/f$  in order to reduce the parameter space to two dimensions, simplifying subsequent derivations and analysis. The following lemma immediately follows from (4).

**Lemma 1:** If  $B/Y > 0$  then  $s_i^* < \frac{1}{2}(\omega_l + \omega_h)$ . Moreover, given the interval  $[\omega_l, \omega_h]$  as well as the definitions of  $B$  and  $Y$ ,  $s_i^*$  is increasing in  $y$  and decreasing in  $\beta$ .

The firm's objective is to minimize employment. If we assume  $B = 0$  so that the fixed component of employment is invariant with respect to occupational specialization, as in Chaney and Ossa (2013), the firm would choose  $s_i$  half way between the two endpoints since this would minimize the amount of variable employment needed to complete the specified range of tasks.

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<sup>7</sup> Accounting for corner solutions, the first-order condition is  $\{\beta + y(s_i - \omega_l) - y(\omega_h - s_i) = 0\}\{1 - s_i\}s_i = 0$

However, this is not the case if the fixed labor requirement increases with complexity. In this case, the firm can save on fixed employment by marginally reducing the specialization that it uses, but the cost of doing so is an increase in the variable component of employment.<sup>8</sup> A given reduction in  $s_i$  results in a larger reduction in the fixed component of employment relative to the increase in variable employment when  $B$  is large or  $Y$  is small. Much of what follows hinges on this tradeoff.

While it is true that a firm would never choose to have multiple occupations undertake the same range of tasks, it is possible that the firm could optimally choose to break a single range of tasks into mutually exclusive but collectively exhaustive regions and assign different tasks to different occupations. Consider a firm that utilizes multiple occupations with optimally-chosen specializations. In particular, consider two adjacent occupations with specializations  $s_i^* < s_{i+1}^*$ . Define  $\omega_i$  as the borderline task such that occupation  $i$  is assigned tasks  $\omega \in [s_i^*, \omega_i]$  and occupation  $i + 1$  is assigned tasks  $\omega \in [\omega_i, s_{i+1}^*]$ .<sup>9</sup> The firm's objective is to choose  $\omega_i$  to minimize  $y \int_{s_i^*}^{\omega_i} (\omega - s_i^*) d\omega + y \int_{\omega_i}^{s_{i+1}^*} (s_{i+1}^* - \omega) d\omega$ ; or,  $\frac{y}{2} \{(\omega_i - s_i^*)^2 + (s_{i+1}^* - \omega_i)^2\}$ . The first-order-condition is

$$(5) \quad y(\omega_i - s_i^*) = y(s_{i+1}^* - \omega_i)$$

We define  $\omega_i = \omega_i^*$  as the solution to (5). The interpretation of this condition is very clear. Given specializations  $s_i^*$  and  $s_{i+1}^*$ ,  $\omega_i^*$  is the task for which the amount of variable labor used by occupation  $i$  to complete tasks that are *more* complex than  $s_i^*$  just equals the amount of variable labor used by occupation  $i + 1$  to complete tasks *less* complex than  $s_{i+1}^*$ . This occurs when  $\omega_i$  is half-way between  $s_i^*$  and  $s_{i+1}^*$ . We state this as a lemma.

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<sup>8</sup> As noted above, we would obtain qualitatively similar results if we instead allowed the wage to increase with complexity.

<sup>9</sup> Occupation  $i$  will also be assigned some tasks  $\omega < s_i^*$ , while occupation  $i + 1$  will be assigned some tasks  $\omega > s_{i+1}^*$ , but this is not relevant for determining  $\omega_i$ .

Lemma 2: For any two adjacent occupations with specializations  $s_i^* < s_{i+1}^*$ , employment is minimized if tasks  $\omega \in \left[ s_i^*, \frac{s_i^* + s_{i+1}^*}{2} \right]$  are assigned to occupation  $s_i^*$ , tasks  $\omega \in \left[ \frac{s_i^* + s_{i+1}^*}{2}, s_{i+1}^* \right]$  are assigned to occupation  $s_{i+1}^*$ .

We now generalize to allow the firm to use  $n$  different occupations. The optimal choices for specializations and division of tasks can then be stated as in (6) and (7). With  $n$  occupations, the first-order conditions for the  $s_i^*$  are:

$$(6) \quad s_i^* = \frac{1}{2}(\omega_i^* + \omega_{i-1}^*) - \frac{1}{2} \frac{B}{Y}$$

$$(7) \quad \omega_i^* = \frac{s_i^* + s_{i+1}^*}{2}$$

Substituting (6) into (7), results in a recursive system of equations:

$$(8) \quad \omega_i^* = \frac{1}{2}\{\omega_{i-1}^* + \omega_{i+1}^*\} - \frac{B}{Y}, \quad i = 1, \dots, n$$

The solution to this system of equations can be represented in the following matrix form:

$$(9) \quad \begin{bmatrix} \omega_1^* \\ \vdots \\ \omega_{n-1}^* \end{bmatrix} = \frac{2}{n} \times \begin{bmatrix} b_{11} & \cdots & b_{1,n-1} \\ \vdots & \ddots & \vdots \\ b_{n-1,1} & \cdots & b_{n-1,n-1} \end{bmatrix} \times \begin{bmatrix} \frac{B}{Y} \\ \vdots \\ \frac{B}{Y} \\ \vdots \\ \frac{1}{2} - \frac{B}{Y} \end{bmatrix}$$

where

$$(10) \quad b_{ij} = \begin{cases} i \times (n - j), & i \leq j \\ j \times (n - i), & i \geq j \end{cases}$$

Therefore

$$(11) \quad \omega_i^* = \frac{b_{i,n-1}}{n} - \frac{2}{n} \frac{B}{Y} \sum_{j=1}^{n-1} b_{i,j}, \quad i = 1, \dots, n-1$$

Substituting (10) into (11) and recognizing that  $\sum_{i=1}^k i = \frac{k(k+1)}{2}$ , we arrive at the following closed-form solution:

$$(12) \quad \omega_i^* = \frac{i}{n} - i(n-i)\frac{B}{Y}, \quad i = 1, \dots, n$$

The set of tasks assigned to team  $i$  is given by  $\omega_i^* - \omega_{i-1}^* = \frac{1}{n} - [n - 2i + 1]\frac{B}{Y}$ . Since this value is increasing in  $i$ , we have our first main result.

Proposition 1: If the cost of mastering tasks is increasing in the complexity of the task, low-skilled occupations will be assigned a more narrow set of tasks to complete than high-skilled occupations.

Having solved for the optimal division of tasks, we can now substitute (12) into (6) to solve for optimal specializations.

$$(13) \quad s_i^* = \frac{i}{n} - \frac{1}{2n} - \left\{ \frac{n}{2} + (n-i)(i-1) \right\} \frac{B}{Y}$$

If we look at the distance between specializations,  $s_i^* - s_{i-1}^* = \frac{1}{n} - [n - 2i + 2]\frac{B}{Y}$ , we see that this value is increasing in  $i$ , providing us with the following Corollary to Proposition 1.

Corollary 1: The distance between specializations is increasing in the complexity of the task to be completed. Thus, the design of the production chain is skewed towards low-skilled occupations.

In Section 3 we explore this issue of skewness further, investigating the manner in which the bias in the production chain is tied to firm size and global engagement. To do so, we need a formal measure of skewness. There are many such measures, and they all behave in similar ways with respect to our model, so we choose the Groeneveld and Meeden coefficient due to its tractability in our framework. This measure is defined as

$$(14) \quad SK \equiv \frac{\mu - \nu}{E|s_i^* - \nu|}$$

Where  $\mu$  and  $\nu$  are the mean and median of the distribution of specializations.

Summing over  $s_i^*$  in (13) and dividing by the number of specializations ( $n$ ), we obtain  $\mu$  for any  $n$ . Specifically, we obtain  $\mu = \frac{1}{2} - \frac{1}{6}(n^2 + 2)\frac{B}{Y}$ . The median specialization is given by  $s_{(n+1)/2}^*$ ; or, from (13),  $\nu = \frac{1}{2} - \frac{1}{4}(n^2 + 1)\frac{B}{Y}$ . Note that  $\mu - \nu = \frac{1}{12}(n^2 - 1)\frac{B}{Y} > 0$  for all  $\frac{B}{Y} > 0$ , indicating that the production process is biased towards low-skilled occupations regardless of the number of specializations. In the Appendix (Result R.1) we show that the denominator of (14) is equal to  $\frac{1}{4}$  if  $n$  is even and  $\frac{1}{4}\frac{n^2-1}{n^2}$  if  $n$  is odd, yielding the following

$$(15) \quad SK = \begin{cases} \frac{1}{3}(n^2 - 1)\frac{B}{Y} & \text{if } n \text{ is even} \\ \frac{1}{3}n^2\frac{B}{Y} & \text{if } n \text{ is odd} \end{cases}$$

It is important to note that the fact that the distribution of specializations is skewed in favor of low-skilled occupations is *not* directly related to Figure 1, since that figure shows the cumulative distribution of *employment* across occupations for different types of firms. However, given our framework, this result does have indirect implications for employment patterns across skill levels. In particular, since the firm is creating relatively more low-skill occupations and assigning them fewer tasks, Proposition 1 and Corollary 1 imply that the firm must employ a relatively large number of high-skilled workers to complete a relatively wide range of complex tasks. In other words, *employment will be skewed towards high-skilled occupations*.

To show this, note that, given the number of distinct occupations employed by the firm, (13) specifies the identity of those occupations while (12) specifies the tasks assigned to each occupation. We can use these two equations to solve for the number of workers the firm hires into each occupation. We can then sum over all occupations to obtain total employment. Having

calculated these variables, we can then use them to compute the distribution of employment by occupation as well as various moments of that distribution. We begin by deriving  $E_i(n)$ , the total employment of workers in occupation  $i$ . That is, the total number with specialization  $s_i^*$ .

By definition,

$$(16) \quad E_i(n) = f + \beta s_i^* + y \int_{\omega_{i-1}^*}^{\omega_i^*} |s_i^* - \omega| d\omega$$

Substituting (12), (13) into (16), we obtain:<sup>10</sup>

$$(17) \quad E_i(n) = f \left\{ 1 + \frac{Y}{4} \left( \frac{1}{n} \right)^2 + \frac{B^2}{Y} \left\{ \frac{n^2 + 4n + 2}{4} \right\} - \frac{B(n+2)}{2n} + \left\{ \frac{2B}{n} - \frac{2B^2(1+n)}{Y} \right\} i + \frac{2B^2}{Y} i^2 \right\}$$

Occupation-specific employment depends on parameters  $B$  and  $Y$  as well as the total number of occupations used by the firm. This latter measure is a choice variable that will be optimally chosen by the firm to minimize cost. For now, we take  $n$  as exogenous, deferring until Section 4 a discussion of its determinants.

We next use (17) to solve for total employment of workers in the first  $k$  occupational categories, keeping in mind that categories are sequenced such that  $s_i^* < s_{i+1}^*$  for all  $i$ .

$$(18) \quad E^k(n) = \sum_{i=1}^k E_i(n) = f \left[ \left\{ 1 + \frac{Y}{4} \left( \frac{1}{n} \right)^2 + \frac{B^2}{Y} \left\{ \frac{3n^2 - 2}{12} \right\} - \frac{B}{2} \right\} k + \left\{ \frac{B}{n} - \frac{B^2 n}{Y} \right\} k^2 + \frac{2B^2}{3Y} k^3 \right]$$

Substituting  $k = n$  into (18) and simplifying, we derive  $E^n(n)$ , which is total employment:

$$(19) \quad E^n(n) = nf \left[ 1 + \frac{Y}{4} \left( \frac{1}{n} \right)^2 - \frac{B^2}{12Y} \{n^2 + 2\} + \frac{B}{2} \right]$$

Equation (19) expresses total employment as a function of the number of occupations. In deriving (19), we used optimal choices for specializations and task assignments for each of the

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<sup>10</sup> We use the additional fact that  $\sum_{i=1}^k i^2 = \frac{2k+1}{3} \sum_{i=1}^k i$ .

$n$  occupation. We are now in position to formally define the share of the firm's workforce that is employed in each occupation,  $\lambda_i(n)$ , which follows from (17) and (19)

$$(20) \quad \lambda_i(n) = \frac{E_i(n)}{E^n(n)}$$

First suppose that  $B = 0$ , as in Chaney and Ossa (2013). From (17) and (19), it is evident that employment is uniformly distributed across occupations:  $\lambda_i(n) = 1/n$  for all  $i$ . In contrast, when  $B > 0$ , the share of employment devoted to occupations specialized to less-complex tasks will be less than  $1/n$  while those specialized to more-complex tasks will be larger than  $1/n$ . We state this result as follows (the proof is provided as Result R.2 in the Appendix).

Proposition 2: When  $B > 0$ , there exists an occupation  $m$  such that  $\lambda_i(n) \leq 1/n \leq \lambda_j(n)$  for all  $i \leq m \leq j$ . Moreover,  $\lambda_i(n)$  is convex in  $i$ .

Since the individual employment shares are convex in  $i$ , the cumulative distribution function, which is just a sum of these terms, must be convex in  $i$  as well. For convenience, we state this as a corollary to Proposition 2.

Corollary 2: The cumulative distribution function  $F(k) \equiv \frac{E^k(n)}{E^n(n)}$  is convex in  $k$ .

The cumulative distribution function is depicted as the solid curve in Figure 2. As is evident from Figure 2, employment is weighted in favor of high-skill occupations. We now turn to the issue of how this relationship is tied to the firm's size and its level of global engagement.

### 3. Firms Size, Global Engagement and Occupational Mix: Initial Findings

Motivated by Melitz (2003) and Helpman, Melitz, and Yeaple (2004), we think that globalization tends to be associated with firms that are larger and that have greater fixed costs of production compared with firms that are not globalized. We also think of the additional fixed costs as being skewed toward the more complex tasks. For example, additional fixed costs

associated with exporting might be concentrated in tasks dealing with writing contracts and managing exchange risk rather than product assembly. In the context of our model,  $B$  and  $Y$  are both increasing in the degree of globalization, but for different reasons. In particular, we think of changes in  $B$  as capturing the direct effect of greater global engagement, holding firm size fixed. One of the advantages of our theoretical framework is that it allows us to separate the size effect (operating through  $Y$ ) from this pure global-engagement effect (operating through  $B$ ).<sup>11</sup> This separation of size from global engagement closely matches our earlier empirical work where we found that the skill-intensity of occupational mix was negatively related to firm size while positively related to the degree of global engagement (see Table 3 of Davidson et al 2015).

Our interest in this paper is in analyzing the impact of globalization on the firm's distribution of occupations. We begin by examining the impact of changes in  $Y$  and  $B$  on the distribution of specializations and the distribution of employment when the number of occupations is held fixed at  $n$ . In subsequent sections we solve for the optimal number of occupations (and hence, the design of the optimal production chain) and see how the results derived in this section need to be modified.

As stated in Corollary 1, the design of the production chain is skewed towards low-skilled occupations. The magnitude of skewness depends on our two parameters,  $B$  and  $Y$ . More precisely, from (15) and the discussion preceding it the mean and median of the optimal distribution of occupations along with our measure of skewness are all tied directly to  $\frac{B}{Y}$ . The following propositions tell us how these measures vary with firm size and global engagement.

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<sup>11</sup> Suppose we were to merge our framework with a Melitz (2003)-style model. All firms with a given degree of globalization (for example, all exporters) would have the same value of  $B$ , which would be larger than the value of  $B$  for less-globalized firms. However, more productive exporters would be larger than less productive exporters.



Proposition 3: An increase in firm size, brought about via globalization, increases the skill level of the mean and median occupations employed by the firm. In addition, the distribution of specializations becomes less skewed towards low-skilled occupations as  $Y$  increases.

Proposition 4: An increase in global engagement decreases the skill level of the mean and median occupations employed by the firm. In addition, the distribution of occupations becomes more skewed towards low-skilled occupations as  $B$  increases.

The intuition underlying Propositions 3 and 4 is straight forward. The firm balances the fixed costs entailed with the establishment of an occupation with the variable costs involved in determining the range of tasks assigned to each occupation. If all occupations were equally costly to establish, the firm would evenly space all occupations and assign each an equal range of tasks. In our framework, the cost of establishing occupations in more complex tasks varies directly with  $B$ , therefore inducing firms to establish less complex occupations in response to an increase in  $B$ . On the other hand, variable cost increases in output. As output increases, the firm is more concerned with reducing variable cost at the expense of increased fixed cost, resulting in a more evenly-spaced distribution of occupations.

Turn next to the distribution of employment. The cumulative distribution function is convex in the occupational index (Corollary 2). A standard measure of convexity is the Arrow-Pratt measure, which, in this case, is the second derivative of the function with respect to the occupational index divided by the first derivative with respect to the index. If we use  $A(\lambda_i)$  to denote the Arrow-Pratt measure, then from (17), (19) and (20) we have

$$(21) \quad A(\lambda_i) = \frac{2B}{\frac{Y}{n} - B(1+n) + 2Bi}.$$

Straightforward differentiation of (21) with respect to  $B$  and  $Y$  with  $n$  fixed reveals that  $A(\lambda_i)$  is increasing in  $B$  and decreasing in  $Y$ . Holding output constant, greater global engagement, represented by an increase in  $B$ , shifts the cumulative distribution function to the broken line in Figure 2. This change in the cumulative distribution function is consistent with empirical evidence as summarized in Figure 1 -- that is, the cumulative distribution of employment is more convex for more globally-engaged firms. Moreover, holding global engagement constant, an increase in firm size ( $Y$ ) reduces the convexity of the cumulative distribution of employment. This result is also consistent with our earlier empirical work in which firm size consistently enters in with a significant negative sign in our regressions designed to explain the skill-distribution of employment (in particular, see Table 3 of Davidson et al 2015).

The relationship between firm size, global engagement and the distribution of employment with a fixed number of occupations can be summarized by the following propositions.

Proposition 5: An increase in firm size, brought about via globalization, results in a distribution of employment that is *less* skewed towards high-skilled occupations.

Proposition 6: An increase in global engagement results in a distribution of employment that is more heavily skewed towards high-skilled occupations.

It is important to remember that Propositions 3 - 6 all depend upon our assumption that the number of occupations is fixed at  $n$ . However, in designing its production chain the firm will choose  $n$  to minimize its production costs; and the optimal  $n$  will be influenced by firm size and the degree of global engagement. In the next section, we derive the optimal number of occupations and describe how the design of the cost-minimizing production chain varies with  $B$

and  $Y$ . Then, in Section 5, we revisit Propositions 3 - 6 and examine whether they need to be modified when  $n$  is allowed to vary with  $B$  and  $Y$ .

#### 4. The Optimal Production Chain and Occupational Mix

We assume that all workers are paid the same wage, determined in a perfectly competitive labor market. Combined with the assumption that labor is the only input, cost minimization is then equivalent to employment minimization. Total employment for any  $n$  is given by (19). Differentiating (19) with respect to  $n$ , and rearranging terms, we have

$$(22) \quad \frac{dE^n(n)}{dn} \leq 0 \leftrightarrow \left(4Y - \frac{2}{3}B^2\right) \leq \left(\frac{Y}{n} - Bn\right)^2$$

Inspection of (22) reveals that this derivative is negative for all  $n$  if  $6Y \leq B^2$ , implying that the firm would like to use as many occupations as possible. This result stems from our assumption that fixed costs are linearly increasing in the complexity of the occupation's specialty so that  $s_i < 0$  would reduce fixed costs. From (13) we see that for  $s_1^* \geq 0$  we need  $n \leq \sqrt{Y/B}$ , placing an upper limit on the firm's choice of  $n$ . That is, when  $6Y \leq B^2$ , the firm's optimization problem results in the corner solution  $n^* = \sqrt{Y/B}$ , where we again use an asterisk to denote the optimal solution.<sup>12</sup>

When  $6Y \geq B^2$ , the optimal solution satisfies (23).

$$(23) \quad \left(\frac{Y}{n^*} - Bn^*\right)^2 = \left(4Y - \frac{2}{3}B^2\right)$$

The right-hand side of (23) is positive given our restrictions on  $B$  and  $Y$ . Taking square roots of both sides and rearranging terms allows us to construct a quadratic in terms of  $n^*$ . This equation has two roots, but only one of the roots satisfies the second-order condition for a minimum. Specifically, the employment-minimizing number of occupations can be written as:

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<sup>12</sup> It is worth noting that  $n \leq \sqrt{Y/B}$  is also the second order condition for the employment minimization problem.

$$(24) \quad n^* = \begin{cases} \sqrt{Y/B} & \text{if } 6Y \leq B^2 \\ \sqrt{Y/B + Y/B^2 - 1/6} - \sqrt{Y/B^2 - 1/6} & \text{otherwise} \end{cases}$$

The nature of this solution can be more clearly seen in the two panels of Figure 3 where we graph total employment as a function of  $n$ , first assuming that  $6Y < B^2$  (panel a), then assuming that  $6Y > B^2$  (panel b). For the remainder of the paper, we focus on the case in which the firm's employment minimization problem has an interior solution.

Equations (12), (13), (17), and (24) completely characterize the choices of the firm and describe the optimal design of the production chain:  $n^*$ ,  $\omega_i^*(n^*)$ ,  $s_i^*(n^*)$ , and  $E_i(n^*)$ . Our next concern is how changes in  $Y$  and  $B$  alter the design of this chain. That is, we are interested in how changes in these variables affect  $n^*$ , the number of occupations that the firm chooses to use. Intuitively, we would expect larger firms to employ a greater number of occupations since an increase in output makes minimization of variable employment a greater priority. Similarly, an increase in  $B$  raises the cost of adding additional occupations, leading the firm to use fewer occupations. This intuition is largely correct, but there is a small portion of parameter space for which the optimal number of occupations is actually increasing in  $B$ . Moreover, the optimal number of occupations is decreasing in  $Y$  for a subset of that range of parameters. Specifically<sup>13</sup>,

$$(25) \quad n_B^* \equiv \frac{\partial n^*}{\partial B} \leq 0 \leftrightarrow Y \geq \frac{1}{6}B^2 + \frac{2}{3}B$$

$$(26) \quad n_Y^* \equiv \frac{\partial n^*}{\partial Y} \geq 0 \leftrightarrow Y \geq \left(\frac{1}{6}B^2 + \frac{1}{3}B\right) \left(\frac{B}{1+B}\right)$$

To understand these restrictions, we refer to (13). When  $B = 0$ , firms choose to evenly space occupations, thereby minimizing variable cost. As  $B$  increases, firms tradeoff higher variable cost in return for saving fixed cost, skewing the production chain toward less complex tasks (Corollary 1). Additional increases in  $B$  (or reductions in  $Y$  when  $B > 0$ ) further skew the

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<sup>13</sup> See Result R.3 in the Appendix for derivations.

production chain and increase variable costs. For relatively small levels of output, the resulting percentage increase in cost is large compared to the percentage increase in fixed costs. To offset this, firms choose to utilize more occupations. We should underscore that these perverse results hold only for a very narrow range in the parameter space. In general, increases in  $Y$  and  $B$  have the expected influence over the number of occupations employed by the firm. We summarize this result as Proposition 7 and graph  $n^*$  as a function of  $B$  and  $Y$  in Figure 4 (the proof of Proposition 7 is given as Result R.3 in the Appendix).

Proposition 7: If  $Y \geq \frac{1}{6}B^2 + \frac{2}{3}B$ , then  $n^*$  is increasing in  $Y$  and decreasing in  $B$ .

## 5. Firms Size, Global Engagement and Occupational Mix Revisited

We began to investigate the impact of globalization on the firm's distribution of occupations in Section 3 where we held  $n$  fixed while varying  $B$  and  $Y$ . We found that greater global engagement would make the distribution of occupations more skewed towards occupations specialized in less complex tasks while making the distribution of employment more skewed towards occupations specialized in more complex tasks. Increases in firm size had the opposite effects. In this section, we seek to understand how those results must be modified when we allow the feedback effects that occur when changes in size or global engagement affect the number of occupations that the firms chooses to create.

We start with the skewness of the optimal production chain. From the derivation of (15) the mean and median of the distribution of occupations are both inversely related to  $\frac{B}{Y}n^2$ . Moreover, for large values of  $n$ , our measure of skewness is directly related to this measure. From Proposition 7 we know that an increase (decrease) in global engagement (firm size) tends

to reduce  $n$ , implying that  $n^2$  and  $\frac{B}{Y}$  move in opposite directions. Nevertheless, the following lemma tells us that the key variable,  $\frac{B}{Y}n^2$ , always moves in the same direction as  $\frac{B}{Y}$ .

Lemma 3:  $\frac{B}{Y}n^2$  is decreasing in  $Y$  and increasing in  $B$ .

Proof: See Result R. 4 in the Appendix.

The immediate implication of Lemma 3 is that Propositions 3 and 4 extend to the setting in which  $n$  is chosen optimally. It is worth noting that the first part of Propositions 3 and 4, which include statements about the mean and the median of the distribution of occupations, hold for all parameter values; while the statements about the skewness measure hold when parameters are such that the optimal number of specializations is sufficiently large.

Finally, we consider the effect of global engagement and firm size on employment when the firm optimally chooses the number of occupations. With an exogenous number of occupations, these effects, spelled out in Propositions 5 and 6, are illustrated in Figure 2. Before proceeding, we note that we chose to mimic Figure 1 as closely as possible by putting occupations (a category variable) on the horizontal axis of Figure 2. A change in the number of occupations will change the shape of the cumulative distribution function, but there will be no economic content to that change. However, we can reintroduce economic content by recognizing that equation (18), upon which Figure 2 is based, does double duty. This equation represents cumulative employment up to occupation  $k$ , but each occupation is associated with a specialization, therefore (18) also shows cumulative employment of workers with specialization less than or equal to  $s_k$ . This relationship is graphed as the solid line in Figure 5. This cumulative distribution matches the solid-line cumulative distribution in Figure 2, translated to the left by matching each occupation with its specialization.

Substitute  $i = n$  into (13) and differentiate with respect to  $B$  and  $Y$  to obtain:

$$(27) \quad \frac{\partial s_n^*}{\partial B} = - \left\{ \frac{1}{2Y} + \left( \frac{B}{Y} - \frac{1}{n^2} \right) n_B \right\} \leq 0$$

$$(28) \quad \frac{\partial s_n^*}{\partial Y} = \frac{1}{2} \frac{n_B}{Y^2} - \left( \frac{B}{Y} - \frac{1}{n^2} \right) n_Y \geq 0$$

where the inequalities follow from using the second-order condition for employment minimization along with Proposition 7.

To examine the effect of changes in  $B$  and  $Y$  on the convexity of the employment distribution, we next differentiate (21) allowing  $n$  to vary. We obtain

$$(29) \quad \text{sign} \frac{\partial A(\lambda_i)}{\partial B} = \text{sign} \left\{ \frac{Y}{n} + \left[ B + \frac{Y}{n^2} \right] n_B \right\}$$

$$(30) \quad \text{sign} \frac{\partial A(\lambda_i)}{\partial Y} = \text{sign} \left\{ -\frac{1}{n} + \left[ B + \frac{Y}{n^2} \right] n_Y \right\}$$

In (29) and (30), the first term on the right hand side represents the direct effect of the change in  $B$  or  $Y$  as described in Propositions 5 and 6, while the second term represents the indirect effect as the firm optimally adjusts the design of its production chain. From Proposition 7, increases in  $B$  ( $Y$ ) usually lead to a reduction (increase) in  $n$ , in which case the direct and indirect effects tend to offset each other, suggesting that Propositions 5 and 6 may need to be modified when feedback effects are taken into account. However, we show in Result R.5 of the Appendix that, given the structure of our model, the direct effect is always dominant so that Propositions 5 and 6 generalize as stated. The broken line in Figure 5 then represents a new the distribution of employment by specialization for a situation where  $B$  is larger ( $Y$  smaller).

Thus, the results derived in Section 3 in which the design of the production chain is held fixed, largely generalize when feedback effects are taken into account. The one exception is in the impact on our measure of skewness for the distribution of specializations. In that case, our results generalize when our underlying parameters lead to an equilibrium with a large number of occupations.

## 6. Conclusion

Any production process requires the completion of a series of tasks to generate output and bring the resulting goods to the market. These tasks differ in complexity. Some may be quite simple (performing straightforward tasks along an assembly line) while others may require considerable skill and knowledge (designing performance contracts or computer software). It is up to the firm to decide how to train their workforce and assign the various tasks to their employees in order complete production. In this paper, we have built on the insights of Chaney and Ossa (2013) to develop a model in which a cost minimizing firm does just that. In the Chaney and Ossa framework, firms create occupations by training workers to master a specific task and then assigning them a range of tasks to complete that are sufficiently close to the worker's core competency. Our contribution is two-fold. First, unlike Chaney and Ossa, we assume that tasks differ in complexity and that more complex tasks require more resources to master. We show that as a result, cost-minimizing firms will assign a more narrow set of tasks to low-skill occupations. We also show that the cost-minimizing production chain is biased in two dimensions – the distribution of occupations is biased in favor of low-skilled occupations but the distribution of employment is biased in favor of high-skilled workers.

Our second set of contributions stem from our assumption that greater global engagement requires the completion of relatively more complex tasks. With this assumption in place, we explore the effects of increased firm size and increased global engagement on the nature of the production process. We show that these two effects tend to work in opposite directions with increased global engagement exacerbating both biases while increases in firm size tend to moderate both biases. In other words, increased global engagement (firm size) leads the firm to



increase (decrease) the skill-intensity of the workforce that they employ. These results are consistent with recent empirical evidence on the link between globalization and the skill intensity of the production process.

## Appendix

Result R.1: We want to derive the denominator of (14),  $E|s_i^* - \nu|$ , where  $\nu$  is the median of the specialization distribution. Consider the case in which  $n$  is even. Then  $\nu$  is the mid-point between  $s_{n/2}^*$  and  $s_{(n/2)+1}^*$ , with an equal number of specializations  $\left(\frac{n}{2}\right)$  above and below  $\nu$ . From (13), we can write  $s_i^*$  in the form  $s_i^* = \alpha_i - \theta_i \frac{B}{Y}$ , with  $\alpha_i = \frac{i}{n} - \frac{1}{2n}$  and  $\theta_i = \frac{n}{2} + (n-i)(i-1)$ . There is some symmetry that we can now exploit. It is straight-forward to show that from (13) we have  $\theta_1 = \theta_n, \theta_2 = \theta_{n-1}, \theta_3 = \theta_{n-2}$ , and so on (in general,  $\theta_i = \theta_{n+1-i}$ ) and that  $\frac{1}{2} - \alpha_1 = \alpha_n - \frac{1}{2}, \frac{1}{2} - \alpha_2 = \alpha_{n-1} - \frac{1}{2}$  and so on (in general,  $\frac{1}{2} - \alpha_i = \alpha_{n+1-i} - \frac{1}{2}$ ). Thus, given that  $\nu = \frac{1}{2} - \frac{1}{4}(n^2 + 1) \frac{B}{Y}$ , when we sum the terms  $\nu - s_i^*$  for  $s_i^* < \nu$  and  $s_i^* - \nu$  for  $s_i^* > \nu$ , two things are immediately apparent: all of the  $\frac{B}{Y}$  terms cancel out and the sum of the  $\frac{1}{2} - \alpha_i$  terms for the specializations below the median is equal to the sum of the  $\alpha_i - \frac{1}{2}$  terms for the specializations above the median. It follows that

$$(A.1) \quad E|s_i^* - \nu| = \frac{2}{n} \sum_{i=1}^{n/2} \left( \frac{1}{2} - \alpha_i \right)$$

Substituting for the  $\alpha_i$  terms yields

$$(A.2) \quad E|s_i^* - \nu| = \frac{2}{n} \sum_{i=1}^{n/2} \left[ \frac{1}{2} - \frac{i}{n} + \frac{1}{2n} \right] = \frac{2}{n} \left[ \frac{n+1}{4} - \frac{1}{n} \sum_{i=1}^{n/2} i \right]$$

Making use of the fact that  $\sum_{i=1}^k i = \frac{k(k+1)}{2}$ , we obtain

$$(A.3) \quad E|s_i^* - \nu| = \frac{2}{n} \left[ \frac{n+1}{4} - \frac{n+2}{8} \right] = \frac{1}{4} \text{ which is our desired result.}$$

Now, consider the case in which  $n$  is odd. In this case,  $v = s_{(n+1)/2}^*$  with an equal number of specializations  $\binom{n-1}{2}$  above and below  $v$ . Applying the same logic as above, we have

$$(A.4) \quad E|s_i^* - v| = \frac{2}{n} \sum_{i=1}^{(n-1)/2} \left( \frac{1}{2} - \alpha_i \right)$$

Substituting for the  $\alpha_i$  terms yields

$$(A.5) \quad E|s_i^* - v| = \frac{2}{n} \sum_{i=1}^{(n-1)/2} \left[ \frac{1}{2} - \frac{i}{n} + \frac{1}{2n} \right] = \frac{2}{n} \left[ \frac{n^2-1}{4n} - \frac{1}{n} \sum_{i=1}^{(n-1)/2} i \right]$$

Again, using the fact that  $\sum_{i=1}^k i = \frac{k(k+1)}{2}$

$$(A.6) \quad E|s_i^* - v| = \frac{2}{n} \left[ \frac{n^2-1}{4n} - \frac{n^2-1}{8n} \right] = \frac{1}{4} \frac{n^2-1}{n^2},$$

Which is the desired result. #

Result R.2: The proof of Proposition 2 follows. We first show that  $\lambda_1(n) < 1/n$ . Next we show that  $\lambda_i(n)$  is increasing and convex in  $i$ . Combining these results with the fact that all shares have to add up to 1, it must be the case that the employment shares for the occupations specialized to the most complex tasks must exceed the average employment share.

To show  $\lambda_1(n) < 1/n$ , use (17) and (19):

$$(A.7) \quad \lambda_1(n) = \frac{1}{n} \left\{ \frac{1 + \frac{Y}{4} \left( \frac{1}{n} \right)^2 + \frac{B}{n} \frac{B}{2} + \frac{B^2}{4Y} (n^2 - 4n + 2)}{1 + \frac{Y}{4} \left( \frac{1}{n} \right)^2 + \frac{B}{2} - \frac{B^2}{12Y} (n^2 + 2)} \right\}$$

Therefore,  $\lambda_1(n) < 1/n$  if  $\frac{B}{n} - \frac{B}{2} + \frac{B^2}{4Y} (n^2 - 4n + 2) < \frac{B}{2} - \frac{B^2}{12Y} (n^2 + 2)$ . After rearranging and cancelling terms, this inequality can be written as (A.8):

$$(A.8) \quad \frac{Y}{B} > \frac{1}{3} n^2 - \frac{2}{3} n.$$

By the second order condition,  $n \leq \sqrt{Y/B}$ , so

$$(A.9) \quad \frac{Y}{B} \geq n^2 > \frac{1}{3} n^2 - \frac{2}{3} n.$$

From (17)

$$(A10) \quad \frac{\partial \lambda_i(n)}{\partial i} \geq 0 \leftrightarrow \frac{2B}{n} - \frac{2B^2(1+n)}{Y} + \frac{4B^2}{Y}i \geq 0$$

The left-hand-side of (A.10) is smallest when  $i = 1$ . Thus, after rearranging and cancelling terms, the inequality in (A.10) is equivalent to  $Y/B \geq n^2 - n$ , which, again referring to the second-order condition, will hold as a strict inequality. Since the left-hand side of the inequality is increasing in  $i$ , we infer that this derivative is positive for all  $i$ .

Convexity follows from (A.10), since the second derivative with respect to  $i$  is positive.

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Result R.3: The proof of Proposition 7 follows. First, differentiate (23) with respect to  $B$ :

$$(A.11) \quad \frac{dn^*}{dB} = -\left(\frac{Y}{B} + \frac{Y}{B^2} - \frac{1}{6}\right)^{-\frac{1}{2}} \left(\frac{Y}{2B^2} + \frac{Y}{B^3}\right) + \left(\frac{Y}{B^2} - \frac{1}{6}\right)^{-\frac{1}{2}} \frac{Y}{B^3}$$

Multiply through by  $B^3/Y$  and rearrange terms. We then have

$$(A.12) \quad \frac{dn^*}{dB} \leq 0 \leftrightarrow \left(1 + \frac{6BY}{6Y - B^2}\right)^{\frac{1}{2}} \leq 1 + \frac{B}{2}$$

The left-hand side of (A.12) becomes infinite as  $6Y - B^2$  goes to zero, in which case  $\frac{dn^*}{dB}$  is infinitely positive. Square both sides of (A.12), hold  $B$  constant, and let  $Y$  become infinitely large. The inequality becomes  $1 + B \leq 1 + B + B^2/4$ , which is satisfied for all  $B \geq 0$ , proving that the derivative is negative for large  $Y$ . To find the minimum value of  $Y$  for which the derivative is negative, treat (A.12) as an equality, square both sides, and apply the quadratic formula to solve for  $B$  as a function of  $Y$ . Invert the result.

Next, differentiate (23) with respect to  $Y$ ; we obtain

$$(A.13) \quad \frac{dn^*}{dY} = \frac{1}{2} \left(\frac{Y}{B} + \frac{Y}{B^2} - \frac{1}{6}\right)^{-\frac{1}{2}} \left(\frac{1}{B^2} + \frac{1}{B}\right) - \frac{1}{2} \left(\frac{Y}{B^2} - \frac{1}{6}\right)^{-\frac{1}{2}} \frac{1}{B^2}$$

Rearranging terms:

$$(A.14) \quad \frac{dn^*}{dY} \geq 0 \leftrightarrow \left(1 + \frac{6BY}{6Y-B^2}\right)^{\frac{1}{2}} \leq 1 + B$$

The remainder of the proof follows as above. Note that  $\frac{dn^*}{dY} \geq 0$  if  $\frac{dn^*}{dB} \leq 0$ . #

Result R.4: The proof of Lemma 3 follows. Though we are interested in  $\frac{B}{Y}n^2$ , the proof is

simplified if we work with  $x(B, Y) = n\sqrt{\frac{B}{Y}}$ . From (23):

$$(A.15) \quad x(B, Y) = n\sqrt{1 + \frac{1}{B} - \frac{1B}{6Y}} - \sqrt{1 - \frac{1B}{6Y}}$$

Then

$$(A.16) \quad \frac{\partial x}{\partial Y} = \frac{B}{6Y^2} \left( \frac{1}{\sqrt{1 + \frac{1}{B} - \frac{1B}{6Y}}} - \frac{1}{\sqrt{1 - \frac{1B}{6Y}}} \right) < 0$$

$$(A.17) \quad \frac{\partial x}{\partial B} = -\left(\frac{1}{B^2} + \frac{1}{6Y}\right) \left( \frac{1}{\sqrt{1 + \frac{1}{B} - \frac{1B}{6Y}}} - \frac{1}{\sqrt{1 - \frac{1B}{6Y}}} \right) > 0 \quad \#$$

Result R.5: In order to determine the signs of the derivatives in (29) and (30), we need to solve for  $n_B$  and  $n_Y$ . Though derived explicitly in (A.11) and (A.13), the proof is simpler if we apply the implicit function theorem to (22), the first-order condition for employment minimization.

We can write that first-order condition as  $Y - Bn^2 = nz$ , where  $z = 4Y - \frac{2}{3}B^2$ . Totally

differentiate this function with respect to  $B$  and  $Y$ . Using the implicit function theorem:

$$(A.18) \quad n_B = -\frac{n^2 + nz_B}{2Bn + z}$$

$$(A.19) \quad n_Y = -\frac{1 - nz_Y}{2Bn + z}$$

where we note that the denominator of (A.18) and (A.19) is the second derivative of employment with respect to the number of occupations, and the second-order condition for employment minimization therefore ensures that this is positive.

Substitute (A.18) and  $Bn^2 = Y - nz$  into (29). After rearranging terms, we find a sufficient condition for  $\frac{\partial A}{\partial B} > 0$  is  $\frac{2Bn+z_B}{2Bn+z} < 1$ . This inequality is satisfied since  $z_B < 0 < z$ .

Similarly, substitute (A.19) and  $z = \frac{Y}{n} - Bn$  into (30). We find a sufficient condition for  $\frac{\partial A}{\partial Y} < 0$  is  $\frac{-nz_Y+z}{2Bn+z} < 1$ . This inequality is satisfied since  $-nz_Y < 0 < 2Bn$ .

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## References

- Amiti, M and L Cameron (2012). Trade Liberalization and the Wage Skill Premium: Evidence from Indonesia. *Journal of International Economics* 87: 277 – 287.
- Barba-Navaretti, G. and A. Venables (2004), *Multinational Firms in the World Economy*, Princeton: Princeton University Press.
- Becker, S., K. Ekholm, and M. Muendler (2013). Offshoring and the Onshore Composition of Occupations, Tasks, and Skills. *Journal of International Economics*. 90: 91 – 106.
- Bernard, A. and J. Jensen (1997). Exporters, skill upgrading, and the wage gap. *Journal of International Economics*, 42: 3-31.
- Bustos, P. (2011). Trade liberalization, exports and technology upgrading: Evidence on the impact of MERCOSUR on Argentinian firms. *American Economic Review* 101(1): 304-40.
- Chaney, T. and R. Ossa (2013). Market size, division of labor, and firm productivity. *Journal of International Economics* 90 (1): 177-80.
- Davidson, C.; F. Heyman; S. Matusz; F. Sjöholm; and S. Zhu (2015). Globalization and the occupational structure of firms. Michigan State University.
- Frias, J., D. Kaplan, and E. Verhoogen (2009). Exports and Wage Premia: Evidence from Mexican Employer-Employee Data. Working Paper, Columbia University.
- Hakkala-Nilsson, K.; F. Heyman and F. Sjöholm (2014). Multinational firms, acquisitions and job tasks. *European Economic Review* 66(C): 248-65.
- Helpman, E; M. Melitz, and S. Yeaple (2004). Export versus FDI with heterogeneous firms. *American Economic Review* 94(1): 300-16.
- Markusen, J. (1995). The boundaries of multinational enterprises and the theory of international trade. *Journal of Economic Perspective* 9: 169-189.
- Melitz, M. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica* 71: 1695-1725.

Poole, J. (2013). Knowledge Transfers from Multinational to Domestic Firms: Evidence from Worker Mobility. *The Review of Economics and Statistics* 95: 393 – 406.

Schank, T, C. Schnabel, and J. Wagner (2007). Do Exporters Really Pay Higher Wages? First Evidence from German Linked Employer-Employee Data. *Journal of International Economics* 72: 52 – 74.

Verhoogen, E. (2008). Trade, quality upgrading, and wage inequality in the Mexican manufacturing sector. *Quarterly Journal of Economics* 123 (2): 489-530.

Cumulative  
Frequency

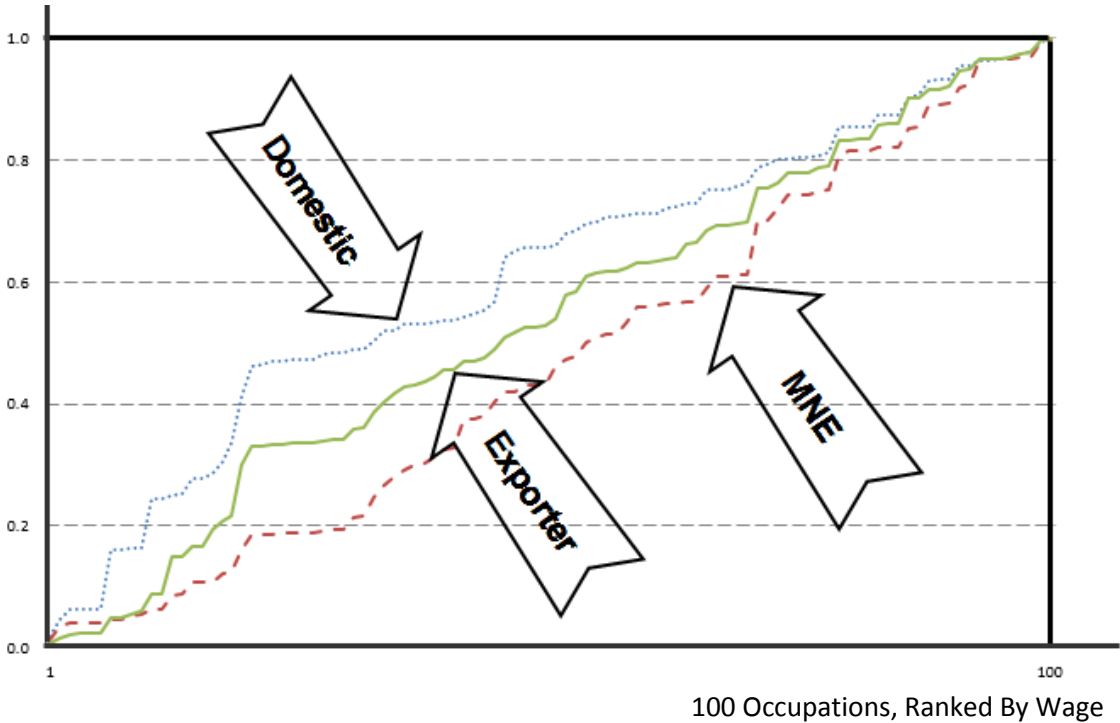


Figure 1: Empirical Cumulative Distribution of Employment by Occupation.

Note: This figure shows the cumulative distribution of occupations by skill levels for three types of firms: multinationals (MNEs), exporters, and local firms (i.e., non-MNEs that do not export). The horizontal axis is the skill percentile ranking of occupations based on average wages for 1997.



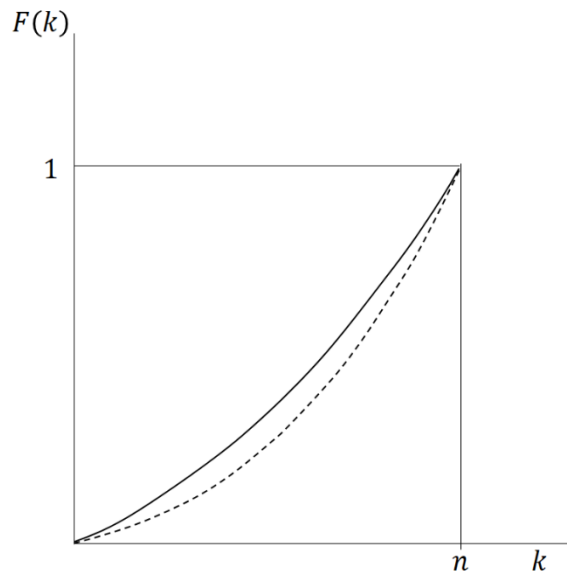


Figure 2: Predicted Cumulative Distribution of Employment by Occupation

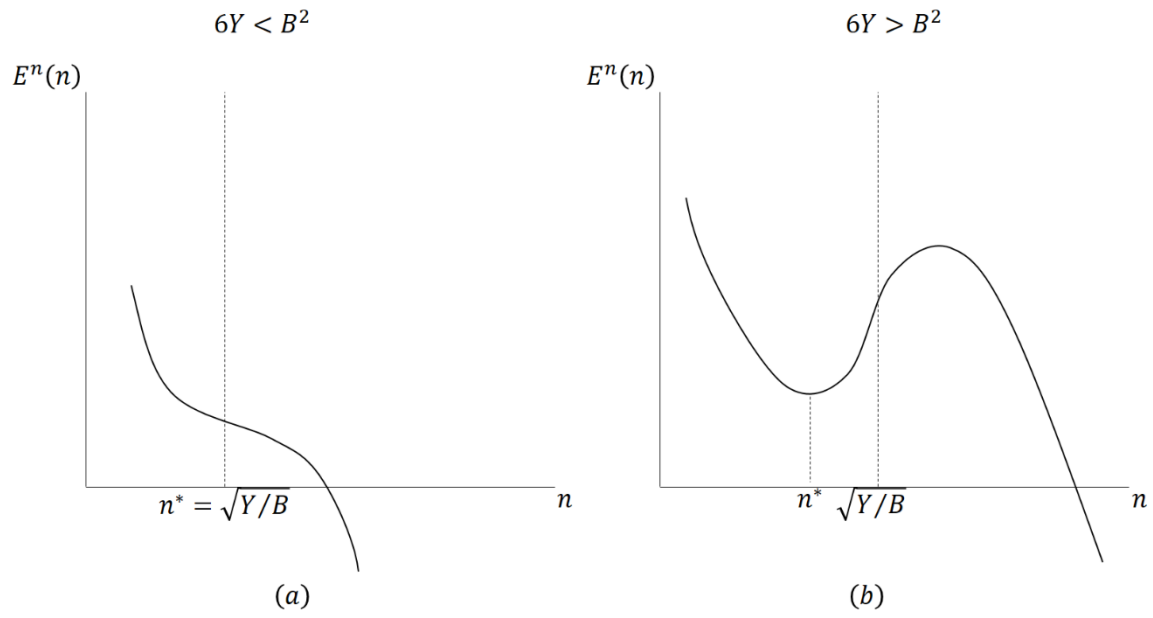


Figure 3: Employment as a Function of the Number of Occupations

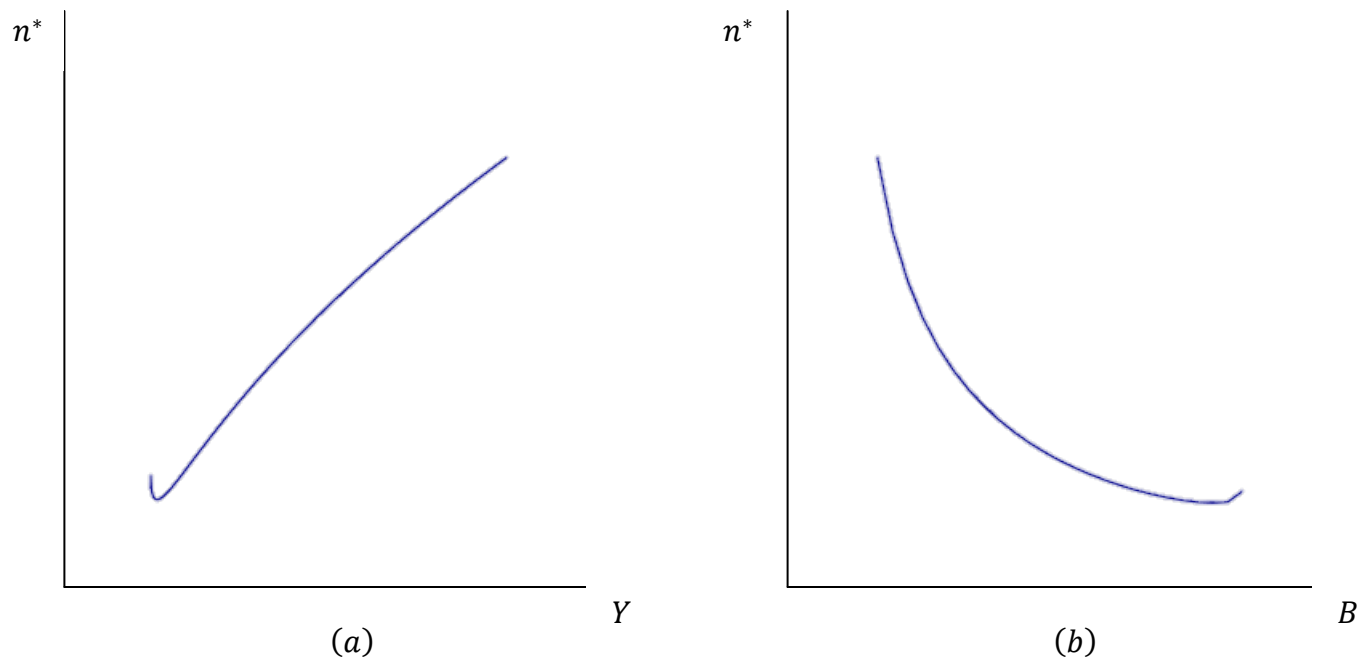


Figure 4: Employment-minimizing number of occupations as a function of  $Y$  and  $B$

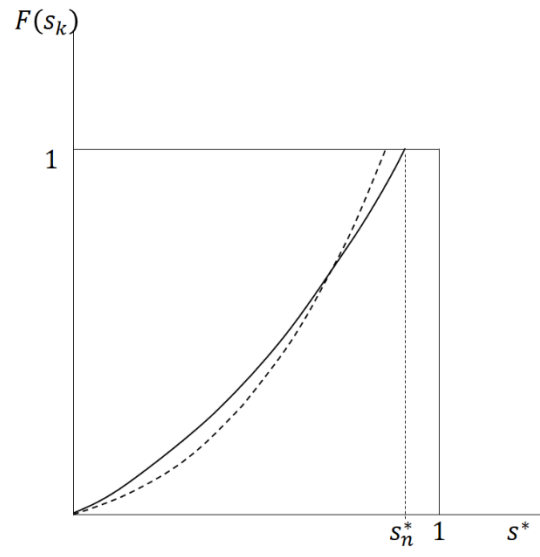


Figure 5: Predicted Cumulative Distribution of Employment by Specialization