Baumgärtner, Stefan; Drupp, Moritz A.; Meya, Jasper N.; Munz, Jan M.; Quaas, Martin F.

Working Paper

Income inequality and willingness to pay for public environmental goods


Provided in Cooperation with:
Christian-Albrechts-University of Kiel, Department of Economics

Suggested Citation: Baumgärtner, Stefan; Drupp, Moritz A.; Meya, Jasper N.; Munz, Jan M.; Quaas, Martin F. (2016) : Income inequality and willingness to pay for public environmental goods, Economics Working Paper, No. 2016-04, Kiel University, Department of Economics, Kiel

This Version is available at:
http://hdl.handle.net/10419/129619

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
income inequality and willingness to pay for public environmental goods

by Stefan Baumgärtner, Moritz A. Drupp, Jasper N. Meya, Jan M. Munz and Martin F. Quaas

issn 2193-2476
Income inequality and willingness to pay for public environmental goods

Stefan Baumgärtner\textsuperscript{a,b,\ast}, Moritz A. Drupp\textsuperscript{c,d,e}, Jasper N. Meya\textsuperscript{f}, Jan M. Munz\textsuperscript{e} and Martin F. Quaas\textsuperscript{c}

\textsuperscript{a} Department of Environment and Natural Resources, University of Freiburg, Germany
\textsuperscript{b} Department of Sustainability Sciences and Department of Economics, Leuphana University of Lüneburg, Germany
\textsuperscript{c} Department of Economics, University of Kiel, Germany
\textsuperscript{d} Department of Geography and Environment, London School of Economics and Political Science, UK
\textsuperscript{e} University of Tübingen, Germany
\textsuperscript{f} Department of Economics, University of Oldenburg, Germany

February 29, 2016

Abstract: We study how the distribution of income among members of society, and income inequality in particular, affects social willingness to pay (WTP) for environmental public goods. We find that social WTP for environmental goods increases with mean income, and decreases (increases) with income inequality if and only if environmental goods and manufactured goods are substitutes (complements). Furthermore, social WTP for environmental normally changes more elastically with mean income than with income inequality. We derive adjustment factors for benefit transfer to control for differences in income distributions between a study site and a policy site. For illustration, we quantify how social WTP for environmental public goods depends on the respective income distribution for empirical case studies in Sweden, China and the World. We find that the effects of adjusting for income inequality can be substantial.

JEL-Classification: Q51, D63, H23, H43

Keywords: environmental goods, public goods, income distribution, inequality, willingness to pay, benefit transfer, sustainability policy

\ast Corresponding author: Chair of Environmental Economics and Resource Management, University of Freiburg, Tennenbacher Str. 4, D-79106 Freiburg, Germany, phone: +49 761 203-3753, email: stefan.baumgaertner@ere.uni-freiburg.de.
1 Introduction

Estimation of willingness to pay (WTP) for non-market-traded environmental goods has become a major subfield of environmental economics, with growing importance for environmental management (Bateman et al. 2011, Smith 2000). Lately, this development has been particularly spurred by the emergence and now wide-spread use of benefit transfer (Kaul et al. 2013), that is, the transfer of benefit estimates for an environmental good from a study site to another context where this information is to be used for environmental management (“policy site”). Such benefit transfer requires knowledge of how the benefits provided by environmental goods depend on the context’s characteristics, including environmental and socio-economic variables. With this information one can control for differences in the level of these variables when doing benefit transfer.

One important determinant of the benefits of environmental goods, as measured by the WTP for these goods, is the level of income. As individual income determines individual WTP, mean income in the society considered determines social WTP. But social WTP is also determined by the (in)equality of the distribution of income among individual members of society. While there has been substantial research on how the level of (individual or societally mean) income influences (individual or social) WTP for environmental goods, the similarly relevant question of how income inequality within society influences social WTP for environmental goods has hardly been studied.

Here, we study how the distribution of income among members of society, and income inequality in particular, affects social WTP for environmental public goods.

The question of how WTP for environmental goods depends on income has been studied, so far, mainly in terms of the income elasticity of WTP. Ebert (2003), following up on previous work by Aaron and McGuire (1970), Kovenock and Sadka (1981), Kriström and Riera (1996), Flores and Carson (1997), has scrutinized the incidence of environmental benefits and has shown that the income elasticity of WTP for an environmental public good has an inverse relationship to the elasticity of substitution between a composite consumption good and the environmental good in question, assuming a constant-elasticity-of-substitution (CES) utility function. Hence, the income elastic-
ity of WTP is smaller (greater) than unity if and only if the environmental good and consumption good are substitutes (complements).

Empirical evidence, as gathered mainly from contingent valuation studies, suggests that the income elasticity of WTP for environmental goods is generally below unity – usually between 0.1 and 0.6.\(^1\) It thus follows from Ebert’s (2003) result that the environmental goods assessed in these studies are substitutes to private consumption goods. This conclusion has been challenged by Schläpfer (2006), Schläpfer and Hanley (2006). In particular, Schläpfer (2006) argues that the incidences of income elasticities of WTP smaller than unity may be an artifact of the current design of contingent valuation studies.

In benefit transfer studies, it is current practice to adjust WTP-estimates for differences in mean income between the study site and the policy site (Ready and Navrud 2006, Czajkowski and Scasny 2010). But the effects of income inequality are unaccounted for, so far. Indeed, we are not aware of studies on how the distribution of income among members of society, and in particular income inequality, affects WTP for environmental goods.

In our analysis of this issue, we employ a specification of the model of Ebert (2003), where a continuum of individual households have identical self-regarding preferences over a market-traded private consumption good and a non-market-traded pure public environmental good, represented by a CES utility function.\(^2\) While the CES utility function is a particular functional representation of preferences, and thus of limited generality, it is an appropriate basis for our analysis for the following reasons: (1) Benefit transfer is typically based on a constant income elasticity of WTP. Our approach of deriving transfer factors from the CES utility specification thus yields results that


\(^2\)Hence, we focus on statistical effects of income inequality across a population of self-regarding individuals, and do not study the potential effect of other-regarding individual preferences or behavior.
are consistent with this practice. They are thus directly relevant for environmental management. (2) The CES specification is the simplest, yet rich enough functional form that allows studying substitutability. (3) Qualitatively, our key result on how income inequality affects the mean WTP for environmental public goods holds more generally, beyond the CES functional specification. We demonstrate this in Appendix A.12.

We extend Ebert’s (2003) model by assuming that an exogenously given amount of total income is log-normally distributed over households, and consider two alternative measures of income inequality: the coefficient of variation and the standard deviation of income. These correspond to relative and absolute notions of inequality, respectively.

We find that (i) social WTP for the environmental good increases with mean income; (ii) social WTP for the environmental good decreases (increases) with income inequality if and only if the environmental good and the manufactured good are substitutes (complements); (iii) the effect of income inequality on social WTP is the stronger, the higher the mean income; (iv) social WTP for the environmental good changes more elastically with mean household income than with income inequality, except for extreme cases of parameter values. We also derive transfer factors for benefit transfer to control for differences in income distributions between a study site and a policy site.

To illustrate our theoretical results, and to estimate the potential size of these effects, we quantify how social WTP for environmental public goods depends on the respective income distribution for three empirical case studies: (1) an environmental good of cultural importance, the existence of large predator species, in Sweden (from

---

3Also this particular functional representation of unequal distribution may be generalized to any mean preserving spread of the income distribution (see Appendix A.12).

4While the coefficient of variation satisfies all standard requirements for inequality measures (weak principle of transfers, decomposability, income scale and population size independence), the standard deviation is an absolute measure and thus increases with the level of income (Cowell 2009: 72). We sketch results for the Gini coefficient, which is another popular measure of income inequality, only briefly. For, the Gini coefficient it does not satisfy the criterion of decomposability (cf. Cowell 2009: 64) and, under a log-normal income distribution, the Gini coefficient is completely determined by the standard deviation of income (cf. Cowell 2009: 153) and therefore yields fully equivalent results.
Broberg 2010), (2) water quality improvement in rural China (from Wang et al. 2013), and (3) biodiversity conservation at a global scale (from the meta-study of Jacobsen and Hanley 2009).

As for the quantitative size of effects, a benefit transfer for biodiversity conservation from the global study with high income inequality to the case context of Sweden, a country known for its low income inequality, would entail a WTP correction for income inequality of 11 percent. We further find that in a hypothetical world of a completely equal income distribution WTP for global biodiversity conservation would be 16 percent higher than it actually is under the current unequal global income distribution.

This paper is organized as follows. We present the model in Section 2, and the results of the model analysis in Section 3. In Section 4, we illustrate these results with empirical data. In Section 5, we discuss our main assumptions. Section 6 concludes. All formal proofs are contained in the Appendix.

2 Model

We employ the model of Ebert (2003) with a specific utility function and a specific distribution of income. There is a population of households whose well-being is determined by consumption of two goods – a market-traded private consumption good, $X$, and a non-market-traded pure public (i.e. non-rival and non-excludable) environmental good, $E$. Both goods may be composites, and their amounts are continuously scalable with $X, E \geq 0$. All households have identical preferences over these two goods, represented by the utility function

$$U(X, E) = \left( \alpha X^\theta + (1 - \alpha) E^\theta \right)^{\frac{1}{\theta-1}},$$

where $\theta$ with $0 < \theta < +\infty$ is the constant elasticity of substitution between the two goods, and $0 < \alpha < 1$. The utility function (1) is strictly quasi-concave, preferences are homothetic, and both the private good and the environmental public good are normal goods. An individual household’s income is exogenously given and denoted by $Y$. The distribution of income over households is described by a continuous density function.
\( f(Y) \) over non-negative incomes. While the consumption good is traded on a market at given price \( p > 0 \), consumption of the environmental good is fixed at an exogenously given level \( E > 0 \) which is the same for all households.\(^5\) Each household maximizes its utility subject to the budget constraint and fixed level of the environmental good:\(^6\)

\[
\max_{X,E} U(X, E) \quad \text{s.t.} \quad pX = Y \text{ and } E \text{ fixed} .
\] (2)

We follow Aaron and McGuire (1970) and Ebert (2003) in defining the individual income-equivalent total WTP for the environmental good at level \( E \) as the willingness to pay \( w \) per unit (Lindahl price) times the total number \( E \) of units:

\[
\text{WTP} = wE .
\] (3)

The Lindahl price \( w \) of the environmental good is implicitly defined as the virtual price that yields the environmental good level \( E \) as the ordinary (unconditional) Marshallian demand in the hypothetical choice problem where the environmental good is considered a private market good. In this hypothetical choice problem, the environmental good can be individually chosen and must be paid for at the Lindahl price, and the household has an income of \( Y \) plus the expenditures on \( E \) (Neary and Roberts 1980, Hanemann 1991: Equation 11, Flores and Carson 1997: 289). With utility function (1), a household’s total WTP for the environmental good at level \( E \) then depends on income \( Y \) and the other model parameters as follows (see Appendix A.1):

\[
\text{WTP}(Y) = \kappa Y^\eta \quad \text{with} \quad \kappa = \frac{1 - \alpha}{\alpha} (pE)^{\frac{\eta - 1}{\theta}} , \quad \eta = \frac{1}{\theta} ,
\] (4)

where \( \eta \) is the (constant) income elasticity of WTP and \( \kappa \) is a factor that depends on all parameters of the model and on the quantity of the environmental public good.

---

\(^5\)Denoting by \( E \) both the variable ‘environmental-good-consumption’ and the fixed level at which the environmental good is provided should not cause any confusion, as in our analysis the amount of the environmental good is never variable but fixed throughout.

\(^6\)In this ‘equal-preference’-model, which is standard in public economics (Buchanan 1964), households have identical preferences and differ only in terms of income, i.e. differences in the evaluation of the environmental good between rich and poor households are caused by differences in income, not by differences in preferences.
One interesting and important implication of the underlying constant-elasticity-of-substitution utility function is that the income elasticity of WTP, $\eta$, is simply the inverse of the elasticity of substitution between the consumption good and the environmental good, $\theta$. This result, which has already been obtained by Kovenock and Sadka (1981) and Ebert (2003: 452–453), merits some attention. It means that the income elasticity of WTP is larger than one, $\eta > 1$, if and only if the consumption good and the environmental public good are complements, $\theta < 1$. The income elasticity of WTP is equal to one in the Cobb-Douglas case, $\theta = 1$; and the income elasticity of WTP is smaller than one, $\eta < 1$, if and only if the private and the public good are substitutes, $\theta > 1$. It follows that WTP for the environmental good rises progressively (proportionally, regressively) with income if and only if the consumption good and the environmental good are complements (Cobb-Douglas, substitutes).\(^7\)

While all households have identical preferences, represented by utility function (1), income $Y$ is distributed unevenly over households. In particular, we assume that $Y$ is log-normally distributed with mean $\mu_Y$ and standard deviation $\sigma_Y$. For instance, the world income distribution, as well as the income distribution in many countries, can be described by a log-normal distribution with good approximation (Pinkovskiy and Sala-i-Martin 2009). The log-normal distribution is handsome for analytical purposes too, as it is completely determined by its first two statistical moments, $\mu_Y$ and $\sigma_Y$.\(^8\)

In this society, the mean (over households) total WTP for the environmental good at level $E$, $\mu_{WTP}$, is given by

$$\mu_{WTP}(\mu_Y, \sigma_Y) = \int_0^\infty f_{\ln}(Y; \mu_Y, \sigma_Y) WTP(Y) \, dY, \quad (5)$$

where $f_{\ln}(Y; \mu_Y, \sigma_Y)$ is the density function of the log-normal distribution of $Y$ with mean $\mu_Y$ and standard deviation $\sigma_Y$, and $WTP(Y)$ is given by Equation (4). This

---

\(^7\)WTP for the environmental good is said to rise progressively (proportionally, regressively) with income $Y$ if and only if $d(WTP(Y)/Y)/dY > (=, <) 0$.

\(^8\)Strictly speaking, the first two statistical moments of the log-normal distribution are $m$ and $s$. These two biuniquely determine $\mu_Y$ and $\sigma_Y$ (see Equations A.36 and A.37 in Appendix A.2).
yields (see Appendix A.2):

\[
\mu_{\text{WTP}}(\mu_Y, \sigma_Y) = \kappa \mu_Y^{1/\theta} \left(1 + \frac{\sigma_Y^2}{\mu_Y^2}\right)^{\frac{1-\theta}{2\theta}} \text{ or, equivalently,} \tag{6}
\]

\[
\mu_{\text{WTP}}(\mu_Y, \text{CV}_Y) = \kappa \mu_Y^{1/\theta} \left(1 + \text{CV}_Y^2\right)^{\frac{1-\theta}{2\theta}}, \tag{7}
\]

where \(\text{CV}_Y := \frac{\sigma_Y}{\mu_Y}\) is the coefficient of variation of income.\(^9\) While the standard deviation \(\sigma_Y\) measures the width of income distribution in monetary units, the coefficient of variation \(\text{CV}_Y\), i.e. the relative standard deviation of income, measures the width of income distribution as a percentage of mean income. Both measures of income inequality seem plausible. First, one could simply take the standard deviation \(\sigma_Y\) as a measure of income inequality. This is in line with an idea that absolute income inequality matters, that is, inequality as a mean absolute deviation (in monetary units) from the mean. Second, one could take the coefficient of variation \(\text{CV}_Y\) as a measure of income inequality. This is in line with an idea that relative income inequality matters, that is, inequality as measured as a mean relative deviation (as a percentage) from the mean income.

In our analytical framework, changes in \(\mu_Y\) and \(\sigma_Y\) (or \(\text{CV}_Y\)) can be interpreted as outcomes of some stylized, not explicitly modelled policies for the growth and redistribution, respectively, of income.\(^{10}\)

Since the social WTP is the sum of individual WTPs, which is the mean WTP multiplied by the (constant) number of households, the mean WTP studied here (Equation 6 or 7) can be identified with the social WTP.

\(^9\)Denoting both functions, (6) and (7), by \(\mu_{\text{WTP}}\) saves notation but is, strictly speaking, an abuse of notation, as they depend on different variables. However, this should not cause any confusion, as we will always specify both arguments of the function. So, the reader will always know whether we speak of mean WTP as a function of mean income and standard deviation of income, or as a function of mean income and coefficient of variation of income.

\(^{10}\)We thereby abstract from restrictions to redistribution schemes that actual policy may face (e.g. Requate and Lange 2000), as a main application of our results – the theory and practice of benefit transfer – concerns cross-country comparisons and adjustments.
3 Results of model analysis

In this section, we study how the mean WTP for the environmental good, $\mu_{WTP}$ (Equation 6 and 6), changes if mean income, $\mu_Y$, and/or income inequality change. We do this for both relative and absolute measures of income inequality, coefficient of variation $CV_Y$ (Section 3.1) and standard deviation $\sigma_Y$ (Section 3.2), in turn. We start with the coefficient of variation as this yields simpler and more intuitive results.

3.1 Relative income inequality

We start by briefly stating how mean WTP for the environmental good changes if mean income changes.

Proposition 1
Mean WTP for the environmental good, $\mu_{WTP}$, increases with mean household income, $\mu_Y$:

$$\frac{\partial \mu_{WTP}(\mu_Y, CV_Y)}{\partial \mu_Y} > 0.$$  

(8)

Proof. See Appendix A.3.

The proposition states that the influence of mean household income on mean WTP is unambiguous and straightforward: mean WTP for the environmental good increases with mean income. At bottom, this reflects the fact that both the private consumption good and the environmental public good are a normal good for all households in the population, and that, therefore, individual willingness to pay for the environmental good increases with individual income. Our proposition transfers this well known result about individual WTP to the context of a society with unequally distributed income, where it becomes a statement about mean variable values.

We now come to our key result on how income inequality affects mean WTP for the environmental good.

Proposition 2

1. Mean WTP for the environmental good, $\mu_{WTP}$, decreases (increases) with relative
income inequality, \( CV_Y \), if and only if the environmental good and the consumption good are substitutes (complements):

\[
\frac{\partial \mu_{WTP}(\mu_Y, CV_Y)}{\partial CV_Y} \lesssim 0 \quad \text{if and only if} \quad \theta \gtrsim 1 .
\]  

(9)

2. \( \partial \mu_{WTP}/\partial CV_Y \) decreases (increases) with mean income, if and only if the environmental good and the consumption good are substitutes (complements):

\[
\frac{\partial^2 \mu_{WTP}(\mu_Y, CV_Y)}{\partial \mu_Y \partial CV_Y} \lesssim 0 \quad \text{if and only if} \quad \theta \gtrsim 1 .
\]  

(10)

Proof. See Appendix A.4.

Statement 1 of the proposition shows that the influence of relative income inequality on mean WTP crucially depends on whether the environmental good and the consumption good are substitutes or complements. If they are substitutes, a more equal distribution of income increases mean WTP. If they are complements, in contrast, a more equal distribution of income decreases mean WTP. The rationale behind this result is as follows. The two goods being substitutes is equivalent to an income elasticity of WTP below unity (cf. Equation 4). Then, households with lower incomes are willing to pay relatively more of their income for the environmental good than are households with higher income. This means that if a household experiences an increase (decrease) in income, their WTP increases (decreases) only by less than his income. In addition, a more equal income distribution means that some high-income households have a lower income, while some low-income household have a correspondingly higher income, and mean income remains unchanged. Taking these two effects together explains the result, as shifting income from relatively high income levels to relatively lower levels reduces the WTP of the higher income levels, and it also increases the WTP of lower incomes, and the sum of WTP increases at low-income households is larger than the sum of WTP reductions at high-income households.

The size of this effect depends on the level of mean income in the society (Statement 2 of the proposition). In particular, in the case of substitutes the negative effect of relative income inequality on mean WTP is aggravated by the mean income level: its negative effect is increased in absolute magnitude if mean income is higher.
Our key result on how income inequality affects mean WTP, is more general than stated in Proposition 2. We assumed a particular functional specification of the utility function (CES-function with substitution parameter $\theta$, Equation 1) and of the income distribution (log-normal distribution with mean $\mu_Y$ and standard deviation $\sigma_Y$), for ease of exposition and direct applicability in empirical cases (Section 4). Yet, a similar result can be casted in more general terms for any concave utility function and any regular distribution function as long as the WTP function is globally concave or globally convex (see Appendix A.12 for details).

Since both mean income and relative income inequality influence mean WTP for the environmental good, we ask: which one of the two influences is relatively stronger?

**Proposition 3**
Mean WTP for the environmental good, $\mu_{WTP}$, changes more elastically with mean household income, $\mu_Y$, than with relative income inequality, $CV_Y$, except for the extreme case where the environmental good and the consumption are strong complements, $\theta < 1/2$, and relative income inequality is larger than $\sqrt{\theta/(1-2\theta)}$. In this case, mean WTP for the environmental good changes less elastically with mean household income than with relative income inequality:

$$|\eta_{\mu_{WTP},\mu_Y}(\mu_Y, CV_Y)| := \left| \frac{\partial \mu_{WTP}(\mu_Y, CV_Y)}{\partial \mu_Y} \frac{\mu_Y}{\mu_{WTP}(\mu_Y, CV_Y)} \right| =: |\eta_{\mu_{WTP}, CV_Y}(\mu_Y, CV_Y)|$$

(11)

$$\left\{ \begin{array}{l}
> \\
= \\
<
\end{array} \right\}$$

if and only if

$$\left\{ \begin{array}{l}
\theta \geq 1/2, \text{ or } \theta < 1/2 \text{ and } CV_Y < \sqrt{\frac{\theta}{1-2\theta}} \\
\theta < \frac{1}{2} \text{ and } CV_Y = \sqrt{\frac{\theta}{1-2\theta}} \\
\theta < \frac{1}{2} \text{ and } CV_Y > \sqrt{\frac{\theta}{1-2\theta}}
\end{array} \right. .$$

(12)

**Proof.** See Appendix A.5.

This proposition means, that – except for the extreme case where the environmental good and the consumption good are strong complements and relative income inequality
is large – mean WTP for the environmental good reacts more elastically to the mean income level than to income inequality. That is, a one-percent increase in society’s mean income level will increase society’s mean WTP relatively more, i.e. by more percent, than a one-percent reduction in the coefficient of variation of income. In the extreme case, the relative effect size will be the other way round, though. For the delineation of this extreme case, what is a “large income inequality”, $CV_Y$, depends on the elasticity of substitution, $\sqrt{\theta/(1-2\theta)}$. The stronger complementary the two goods are, i.e. the smaller $\theta$, the smaller is this expression,\footnote{$\sqrt{\theta/(1-2\theta)}$ monotonically increases with $\theta$, from 0 (for $\theta \to 0$) to $+\infty$ (for $\theta \to 1/2$).} and the smaller the “large income inequality” that defines this case.

Finally, we derive adjustment factors for the effects of the income distribution on mean WTP. These can be useful for different applications, such as benefit transfer or sustainability policy. Here, we present these adjustment factors in the context of benefit transfer, while we illustrate the role of these adjustment factors for sustainability policy in Section 4.

The increasing demand for the valuation of environmental goods on the one hand and the resource-intensity of primary valuation studies on the other hand have caused a frequent application of benefit transfers, that is, the transfer of benefit measures from one site to another (Kaul et al. 2013). While benefit transfers have many limitations they are, due to budget constrains, often the only option to account for monetary values of environmental goods in the planning or policy process.

In the practice of benefit transfer, WTP-estimates are transferred from a study site, where a primary valuation study has been undertaken and which is characterized by site-specific variables ($E_{\text{study}}$, $p_{\text{study}}$, $\mu_Y^{\text{study}}$, $CV_Y^{\text{study}}$, ...), to a policy site, where benefit measures are needed and which is characterized by site-specific variables ($E_{\text{policy}}$, $p_{\text{policy}}$, $\mu_Y^{\text{policy}}$, $CV_Y^{\text{policy}}$, ...). It is widely acknowledged that a valid benefit transfer needs to correct for the difference in the amount of environmental good available at both sites, and the difference in the mean income level (e.g. Richardson et al. 2015, Ready and Navrud 2006). But so far, adjusting for the difference in income inequality has been
neglected.

**Proposition 4**

The benefit transfer function for adjusting mean WTP for the quantity of the environmental good to be valued, the market price level, the level of mean income and relative income inequality from a study site with \((E^{\text{study}}, p^{\text{study}}, \mu_Y^{\text{study}}, CV_Y^{\text{study}})\) to a policy site with \((E^{\text{policy}}, p^{\text{policy}}, \mu_Y^{\text{policy}}, CV_Y^{\text{policy}})\), assuming identical preferences \((\theta, \alpha)\) in the study and the policy sites, is given as

\[
\mu_{\text{WTP}}^{\text{policy}}(\mu_Y, CV_Y) = \Phi(E^{\text{policy}}, p^{\text{policy}}, \mu_Y^{\text{policy}}, CV_Y^{\text{policy}}, E^{\text{study}}, p^{\text{study}}, \mu_Y^{\text{study}}, CV_Y^{\text{study}}) \cdot \mu_{\text{WTP}}^{\text{study}}(\mu_Y, CV_Y)
\]

(13)

with the following disentangled transfer factor:

\[
\Phi(\ldots) = \Phi_E(E^{\text{policy}}, E^{\text{study}}) \cdot \Phi_p(p^{\text{policy}}, p^{\text{study}}) \cdot \Phi_{\mu}(\mu_Y^{\text{policy}}, \mu_Y^{\text{study}}) \cdot \Phi_{CV}(CV_Y^{\text{policy}}, CV_Y^{\text{study}})
\]

(14)

with

\[
\Phi_E(E^{\text{policy}}, E^{\text{study}}) = \left(\frac{E^{\text{policy}}}{E^{\text{study}}}\right)^{\frac{\theta-1}{\theta}}, \quad (15)
\]

\[
\Phi_p(p^{\text{policy}}, p^{\text{study}}) = \left(\frac{p^{\text{policy}}}{p^{\text{study}}}\right)^{\frac{\theta-1}{\theta}}, \quad (16)
\]

\[
\Phi_{\mu}(\mu_Y^{\text{policy}}, \mu_Y^{\text{study}}) = \left(\frac{\mu_Y^{\text{policy}}}{\mu_Y^{\text{study}}}\right)^{\frac{1}{\theta}}, \quad (17)
\]

\[
\Phi_{CV}(CV_Y^{\text{policy}}, CV_Y^{\text{study}}) = \left(\frac{1 + CV_Y^{\text{policy}}}{1 + CV_Y^{\text{study}}}\right)^{\frac{1-\theta}{2\theta^2}}. \quad (18)
\]

**Proof.** See Appendix A.6.

Our result (Proposition 4) shows that adjusting WTP-estimates for differences in income inequality is easy, as the transfer factor fully factorizes into a product of variable-specific factors. So, each of the site-specific variables can be controlled for separately. As expected after Proposition 2, the income-inequality specific transfer factor \(\Phi_{CV}\) may be greater or small than one, depending on whether the environmental good and the market good are substitutes or complements, \(\theta > 1\) or \(\theta < 1\), and whether income inequality
is greater or smaller in the policy site than in the study site, $CV_Y^{\text{policy}} > CV_Y^{\text{study}}$ or $CV_Y^{\text{policy}} < CV_Y^{\text{study}}$.

If the two goods have an elasticity of substitution of one, as for Cobb-Douglas-preferences, one has $\Phi_{CV} = 1$ so that there does not need to be any adjustment for income inequality. In this case, there is also no need to correct for differences in market prices for private goods or the endowment with the environmental public good. Yet, in the empirically more relevant case $\theta > 1$, all these correction factors will in general differ from one.

As a by-product, Proposition 4 also states how to control for the difference in market price level, $p$, that is, the purchasing power of income, between the study site and the policy site.

### 3.2 Absolute income inequality

After having discussed in the preceding Section 3.1 the effects of relative income inequality, measured by the coefficient of variation of income, $CV_Y$, on mean WTP for the environmental good, we now go through the same suite of questions and statements once more, but consider absolute income inequality, measured by the standard deviation of income, $\sigma_Y$. As some of the results will be very similar to those already obtained for relative inequality, we will be shorter in this section and focus on the qualitatively different results.

The first question is, again: how does mean WTP for the environmental good change if mean income changes?

**Proposition 5**

1. Mean WTP for the environmental good, $\mu_{WTP}$, increases with mean income, $\mu_Y$, at all levels of mean income if the environmental good and the consumption good are substitutes or weak complements:

$$\frac{\partial \mu_{WTP}(\mu_Y, \sigma_Y)}{\partial \mu_Y} > 0 \quad \text{if} \quad \theta \geq 1/2 ; \quad (19)$$

2. Mean WTP for the environmental good, $\mu_{WTP}$, decreases (increases) with mean
income if the environmental good and the consumption good are strong complements, \( \theta < 1/2 \), and mean income \( \mu_Y \) is smaller (greater) than \( \sqrt{1/\theta - 2} \sigma_Y \):

\[
\frac{\partial \mu_{WTP}(\mu_Y, \sigma_Y)}{\partial \mu_Y} \leq 0 \quad \text{for} \quad \mu_Y \leq \sqrt{1/\theta - 2} \sigma_Y \quad \text{if} \quad \theta < 1/2 . \tag{20}
\]

**Proof.** See Appendix A.7.

The proposition states that, unless the environmental good and consumption good are strong complements, the influence of mean household income on mean WTP is unambiguous and straightforward: mean WTP for the environmental good increases with mean income. A difference to the corresponding Proposition 1 for relative income inequality is that, keeping absolute income inequality fixed, mean WTP may decrease as mean income increases – namely if the two goods are strong complements, \( \theta < 1/2 \), and mean income is smaller than the threshold value of \( \sqrt{1/\theta - 2} \sigma_Y \). This threshold value increases with the degree of complementarity (it goes to \(+\infty\) for \( \theta \to 0 \)) and with absolute income inequality, \( \sigma_Y \).

In terms of absolute income inequality, our key result on how income inequality affects mean WTP for the environmental good is as follows. Again, this result can be generalized beyond the particular functional specifications used here (see Appendix A.12).

**Proposition 6**

1. Mean WTP for the environmental good, \( \mu_{WTP} \), decreases (increases) with absolute income inequality, \( \sigma_Y \), if and only if the environmental good and the consumption good are substitutes (complements):

\[
\frac{\partial \mu_{WTP}(\mu_Y, \sigma_Y)}{\partial \sigma_Y} \leq 0 \quad \text{if and only if} \quad \theta \geq 1 . \tag{21}
\]

2. (a) \( \partial \mu_{WTP} / \partial \sigma_Y \) does not change with mean income, \( \mu_Y \), at all if and only if \( \theta = 1/2 \) or \( \theta = 1 \):

\[
\frac{\partial \mu^2_{WTP}(\mu_Y, \sigma_Y)}{\partial \mu_Y \partial \sigma_Y} = 0 \quad \text{for all} \quad \mu_Y > 0 \quad \text{if and only if} \quad \theta = 1/2 \text{ or } \theta = 1 . \tag{22}
\]

(b) \( \partial \mu_{WTP} / \partial \sigma_Y \) decreases with mean income, \( \mu_Y \), for \( \mu_Y < \sigma_Y \sqrt{1/\theta} \) and increases with mean income, \( \mu_Y \), for \( \mu_Y > \sigma_Y \sqrt{1/\theta} \), if and only if the environmental good
and the consumption good are substitutes or strong complements:

$$\frac{\partial^2 \mu_{WTP}(\mu_Y, \sigma_Y)}{\partial \mu_Y \partial \sigma_Y} \leq 0 \quad \text{for} \quad \mu_Y \leq \sqrt{1/\theta} \sigma_Y \quad \text{if and only if} \quad \theta < 1/2 \text{ or } \theta > 1.$$  

(23)

(c) $\partial \mu_{WTP}/\partial \sigma_Y$ increases with mean income, $\mu_Y$, for $\mu_Y < \sigma_Y \sqrt{1/\theta}$ and decreases with mean income, $\mu_Y$, for $\mu_Y > \sigma_Y \sqrt{1/\theta}$, if and only if the environmental good and the consumption good are weak complements:

$$\frac{\partial^2 \mu_{WTP}(\mu_Y, \sigma_Y)}{\partial \mu_Y \partial \sigma_Y} \geq 0 \quad \text{for} \quad \mu_Y \leq \sqrt{1/\theta} \sigma_Y \quad \text{if and only if} \quad 1/2 < \theta < 1.$$  

(24)

Proof. See Appendix A.8.

Statement 1 is exactly as in the case of relative income inequality (Proposition 2, statement 1). In contrast, Statement 2 now contains an additional qualification: the effect of income inequality on social WTP not only depends on the degree of substitutability but also on the level of mean income.

Since both mean income and absolute income inequality influence mean WTP for the environmental good, we ask: which one of the two influences is relatively stronger?

**Proposition 7**

Mean WTP for the environmental good, $\mu_{WTP}$, changes more elastically with mean household income, $\mu_Y$, than with absolute income inequality, $\sigma_Y$, except for the extreme case where the environmental good and the consumption are strong complements, $\theta < 2/3$, and mean income is smaller than $\sqrt{2/\theta - 3\sigma_Y}$. In this case, mean WTP for the environmental good changes less elastically with mean household income than with
relative income inequality:

$$|\eta_{\mu_{WTP},\mu_Y}(\mu_Y, \sigma_Y)| := \left| \frac{\partial \mu_{WTP}(\mu_Y, \sigma_Y)}{\partial \mu_Y} \frac{\mu_Y}{\mu_{WTP}(\mu_Y, \sigma_Y)} \right|$$ (25)

$$\left\{\begin{array}{l}
\eta_{\mu_{WTP},\sigma_Y}(\mu_Y, \sigma_Y) := |\eta_{\mu_{WTP},\sigma_Y}(\mu_Y, \sigma_Y)| \\
\left| \frac{\partial \mu_{WTP}(\mu_Y, \sigma_Y)}{\partial \sigma_Y} \frac{\sigma_Y}{\mu_{WTP}(\mu_Y, \sigma_Y)} \right| > \\
\left| \frac{\partial \mu_{WTP}(\mu_Y, \sigma_Y)}{\partial \sigma_Y} \frac{\sigma_Y}{\mu_{WTP}(\mu_Y, \sigma_Y)} \right| =: |\eta_{\mu_{WTP},\sigma_Y}(\mu_Y, \sigma_Y)|
\end{array}\right. \quad (26)$$

if and only if

$$\left\{\begin{array}{l}
\theta \geq \frac{2}{3}, \text{ or } \theta < \frac{2}{3} \text{ and } \mu_Y > \sqrt{2/\theta - 3} \sigma_Y \\
\theta < \frac{2}{3} \text{ and } \mu_Y = \sqrt{2/\theta - 3} \sigma_Y \\
\theta < \frac{2}{3} \text{ and } \mu_Y < \sqrt{2/\theta - 3} \sigma_Y
\end{array}\right. \quad (26)$$

**Proof.** See Appendix A.9.

This result is similar as the corresponding result for relative income inequality (Proposition 3).

Finally, we derive the benefit transfer function for absolute income inequality.

**Proposition 8**

The benefit transfer function for adjusting mean WTP for the quantity of the environmental good to be valued, the market price level, the level of mean income and absolute income inequality from a study site with \((E_{\text{study}}, p_{\text{study}}, \mu_{Y_{\text{study}}}, \sigma_{Y_{\text{study}}}})\) to a policy site with \((E_{\text{policy}}, p_{\text{policy}}, \mu_{Y_{\text{policy}}}, \sigma_{Y_{\text{policy}}}})\), assuming identical preferences \((\theta, \alpha)\) in the study and the policy sites, is given as

$$\mu_{WTP}(\mu_Y, \sigma_Y) = \Phi(E_{\text{policy}}, p_{\text{policy}}, \mu_{Y_{\text{policy}}}, \sigma_{Y_{\text{policy}}}; E_{\text{study}}, p_{\text{study}}, \mu_{Y_{\text{study}}}, \sigma_{Y_{\text{study}}}) \cdot \mu_{WTP}(\mu_Y, \sigma_Y) \quad (27)$$

with the transfer factor:

$$\Phi(\ldots) = \Phi_E(E_{\text{policy}}, E_{\text{study}}) \cdot \Phi_p(p_{\text{policy}}, p_{\text{study}}) \cdot \Phi_{\mu,\sigma}(\mu_{Y_{\text{policy}}}, \sigma_{Y_{\text{policy}}}, \mu_{Y_{\text{study}}}, \sigma_{Y_{\text{study}}}) \quad (28)$$

with

$$\Phi_{\mu,\sigma}(\mu_{Y_{\text{policy}}}, \sigma_{Y_{\text{policy}}}, \mu_{Y_{\text{study}}}, \sigma_{Y_{\text{study}}}) = \left( \frac{\mu_{Y_{\text{policy}}}}{\mu_Y} \right)^{1/\theta} \cdot \left( \frac{1 + (\sigma_{Y_{\text{policy}}}/\mu_{Y_{\text{policy}}})^2}{1 + (\sigma_{Y_{\text{study}}}/\mu_{Y_{\text{study}}})^2} \right)^{1-\theta/2} \quad (29)$$

and \(\Phi_E(E_{\text{policy}}, E_{\text{study}}), \Phi_p(p_{\text{policy}}, p_{\text{study}})\) as in Equations (15) and (16).
Proof. In analogy to the proof of Proposition 4, see Appendix A.6.

The transfer factors for the level of the environmental good, \(E\), and for the market price level, \(p\), are the same as for the case of relative income inequality (cf. Proposition 4). The transfers factors for the two moments of the income distribution, \(\mu_Y\) and \(\sigma_Y\), are not algebraically decomposable and, therefore, appear as a joint factor, \(\Phi_{\mu,\sigma}(\mu_{Y}^{\text{policy}}, \sigma_{Y}^{\text{policy}}, \mu_{Y}^{\text{study}}, \sigma_{Y}^{\text{study}})\).\(^{12}\)

4 Empirical analysis

In this Section, we provide three empirical case studies to illustrate the theoretical results from Section 3, and to estimate the order of magnitude of the comparative static effects. We have chosen these cases to represent different environmental goods and different socio-economic contexts, to demonstrate a range of potential effects. We have based selection on the crucial criterion of availability of recent data on the income elasticity of WTP for the environmental goods.

The first case concerns an environmental good of high cultural importance in a developed country: the existence of large predator species in Sweden (Section 4.1.1, based on the study described in Broberg and Brännlund 2008, and in Broberg 2010). The second case features an essential environmental good in a poorly developed area: water quality improvement in Lake Puzhehei, Yunnan Province, China (Section 4.1.2, based on the study of Wang et al. 2013). The third case examines a global environmental

---

\(^{12}\)As the Gini coefficient is also often used as an inequality measure, we report here the corresponding transfer function: For the case of log-normally distributed income the Gini coefficient \((G)\) depends only on \(\sigma_Y\) and can be represented as \(G(\sigma_Y) = 2F(\sigma_Y/\sqrt{2}) - 1\) (Cowell 2009: 153), where \(F(\cdot)\) is the cumulative distribution function of the standard normal distribution \(N(0, 1)\). Hence, the transfer factor \(\Phi_{\mu,G}(\ldots)\) can be expressed in terms of the Gini coefficient as follows:

\[
\Phi_{\mu,G}(\mu_{Y}^{\text{policy}}, \sigma_{Y}^{\text{policy}}, \mu_{Y}^{\text{study}}, \sigma_{Y}^{\text{study}}) = \left(\frac{\mu_{Y}^{\text{policy}}}{\mu_{Y}^{\text{study}}}\right)^{1/\theta} \cdot \left(1 + \frac{(\sqrt{2}F^{-1}(G_{Y}^{\text{policy}}+1)/\mu_{Y}^{\text{policy}})]}{\mu_{Y}^{\text{study}})}^{2}\right)^{\frac{1-2^{1/\theta}}{2^{1/\theta}}}.
\]
good – the existence of biodiversity worldwide (Section 4.1.3, based on the meta-study of Jacobsen and Hanley 2009).

We first describe each of the three case studies separately and review how the respective data have been gathered and processed (Section 4.1). We subsequently present the results of our empirical analysis in an overview of all three cases (Section 4.2).

4.1 Data description and processing

4.1.1 Existence of large predator species in Sweden

The existence of large predator species provides a range of culturally important services to humans, including direct use and option values for hunters or wildlife tourists and in particular bequest and existence values for the broader population. In Sweden, four large predator species were threatened with extinction at the time of the study: the wolf and the wolverine are ‘critically endangerd’ and ‘endangered’, respectively, while the populations of bear and lynx are ‘vulnerable’ (Broberg and Brännlund 2008: 1066).

Broberg (2010) studied the income effects on WTP for the 2009 Swedish Predator Policy, which aims at securing the survival of these predator species. His analysis builds on survey data from 872 Swedish individuals from May 2004.\(^\text{13}\) Respondents had filled out a multiple bounded payment card matrix which was based on a polychotomous-choice question that elicited WTP according to different levels of an annual tax to be paid in the next five years with nine amounts ranging from 10 Swedish krona (SEK) to 5,000 SEK and five uncertainty levels, from “definitely yes” to “definitely no” (Broberg 2010: 7). Mean WTP of Swedish survey respondents was found to be $\mu_{WTP}^{\text{SWE}}=449.67$ SEK (Broberg and Brännlund 2008: 564).\(^\text{14}\)

\(^{13}\)The survey was sent by mail to 4,050 randomly selected Swedish individuals, who were chosen on the basis of a stratification process to ensure the selection of individuals living far from, close to and within wildlife areas. Of the 2,455 respondents, those 872 were selected to estimate the WTP function who stated a positive WTP, had non-zero income, and consistently filled out the multiple bounded payment card matrix.

\(^{14}\)Broberg and Brännlund (2008: 564) employed different estimation techniques for the WTP function, including an expansion approach where data are recoded such that “definitely yes” and “probably yes”
For each survey respondent, Broberg (2010) took household income data from the income register of Statistics Sweden, which have a very high degree of accuracy. Among other income variables, he reports annual disposable household income (net income including capital income and social benefits) in 2003. Mean annual disposable household income of all 872 survey respondents is $\mu_{\text{SWE}}^Y = 304,422$ SEK, with a standard deviation of $\sigma_{\text{SWE}}^Y = 174,879$ SEK and a corresponding coefficient of variation of $\text{CV}_{\text{SWE}}^Y = 0.57$. The constant income elasticity of WTP for the Swedish Predator Policy using annual disposable household income was estimated to be $\eta_{\text{SWE}} = 0.37$, with a standard error of $\Delta \eta_{\text{SWE}} = 0.1$ (cf. model $\text{Mod.3}_{HN}$ of Broberg 2010).

We now build on these results. To quantify the impact of changes in the income distribution on mean WTP for the Swedish Predator Policy, we need to specify the inputs to Equation (7): $\mu_{\text{SWE}}^Y$, $\sigma_{\text{SWE}}^Y$, $\text{CV}_{\text{SWE}}^Y$, $\theta_{\text{SWE}} = 1/\eta_{\text{SWE}}$ and $\kappa_{\text{SWE}}$. First, for $\mu_{\text{SWE}}^Y$, $\sigma_{\text{SWE}}^Y$ and $\text{CV}_{\text{SWE}}^Y$ we use the income data of disposable household income from Broberg (2010), which is depicted in Figure 1 in the form of a histogram as well as the curve of the best-fitting log-normal distribution, that is, the log-normal distribution with mean $\mu_{\text{SWE}}^Y$ and standard deviation $\sigma_{\text{SWE}}^Y$. We assumed this log-normal distribution to be the true income distribution. Second, the elasticity of substitution between consumption goods and the cultural environmental good, $\theta_{\text{SWE}}$, is given through the inverse income elasticity of WTP, $1/\eta_{\text{SWE}} = 1/0.37 = 2.63$ (Equation 4). Taking into account the standard error in the measurement of the income elasticity, $\Delta \eta_{\text{SWE}} = 0.1$, we obtain corresponding errors in means “yes”, and the other answers mean “no”. This approach was used by Broberg (2010) in his subsequent analysis of the income effects.

To model the relationship between WTP and income, Broberg (2010) employed a range of functional forms (linear, quadratic, linear in logarithms), different income variables and other determining factors. He found that income has a significantly positive effect on WTP, with income elasticities of WTP ranging from 0.14 to 0.4, depending on the functional form. The specification yielding a constant income elasticity of WTP “do[es] not have significantly worse overall fit” than other specifications (Broberg 2010: 15).

We assume income data in this case study to be log-normally distributed even though a Kolmogorov-Smirnov test reveals that the null-hypothesis of log-normality is rejected at common significance levels.
Figure 1: Histogram of the distribution of disposable annual household income in Sweden [in 1,000 SEK], as used by Broberg (2010), and best-fitting log-normal distribution.

\( \theta_{\text{SWE}} \) that translate into the following upper and lower bound estimates: \( \theta_{\text{SWE} \, \eta=0.47} = 2.12 \), \( \theta_{\text{SWE} \, \eta=0.27} = 3.66 \). Third, the factor \( \kappa \) is a function of all model parameters, including \( \theta \) (Equation 4). Since some of these parameters are unknown, we use \( \mu_{\text{SWE} \, \text{WTP}} = 449.67 \) as well as \( \theta_{\text{SWE}} \), \( \mu_{\text{SWE} \, Y} \) and \( \text{CV}_{\text{SWE} \, Y} \) to calculate \( \kappa_{\text{SWE}} \) indirectly via Equation (7). The residual, calibrated factor \( \kappa_{\text{SWE}} \) is 4.24. As \( \kappa \), by definition (Equation 4), depends on \( \theta = 1/\eta \), we take into account the measurement error in \( \eta \) impacting \( \kappa \). Using a standard method for estimating error propagation (Appendix A.10), we obtain: \( \kappa_{\eta=0.47} = 3.38 \) and \( \kappa_{\eta=0.27} = 5.33 \).

4.1.2 Water quality in Lake Puzhehei, Yunnan Province, China

Due to rapid economic expansion and lack of wastewater treatment, the water quality in many waterways and lakes in China has greatly deteriorated. This decline in water qual-
ity is accompanied by the loss of important provisioning ecosystem services. To counter this development, the World Bank supports the Government of Yunnan Province, China, to enhance the watershed environment, including an improvement of water quality in Lake Puzhehei in Qiubei County, one of the least developed regions of China (Wang et al. 2013). The specific objective is to prevent the lake from tipping towards heavy eutrophication and thus to preserve and enhance access to basic environmental infrastructure. Further projected benefits include an increase in biodiversity and property values, a reduction of water- and air-borne diseases as well as in the damage from future flooding.

We draw on the study by Wang et al. (2013), which assessed WTP for the World Bank project that aims at improving the water quality of Lake Puzhehei by one grade level. WTP [in units of Chinese yuan (CNY)] per month for five years for this project was elicited using a multiple bounded discrete choice method based on in-person interviews in 2007. Of the 507 households from Qiubei County selected using a multiple-stage stratified random sampling approach, 485 households entered the analysis as they provided complete information and stated WTP not exceeding 20% of household income. Annual household income in the previous year 2006 [in CNY] was elicited categorically, exhibiting a mean annual household income of $\mu_{\text{CHI}}=12,572$ CNY and a standard deviation of $\sigma_{\text{CHI}}=12,291$ CNY, with a coefficient of variation of $\text{CV}_{\text{CHI}}=0.98$ (Wang et al. 2013: 61).

Based on conservative assumptions, Wang et al. (2013) found that mean WTP per household for the water quality improvement project was $\mu_{\text{CHI}}=29.75$ CNY per month.

---

17 Wang et al. (2013) elicited the interviewees’ total household income in ten income classes with varying class sizes (Shi, personal communication). They transformed this categorical income data into a continuous income variable by using the respective class averages as income values. For example, if a household stated an income in the category 12,001–20,000 CNY, it was assumed that their household income is 16,000 CNY. This categorical-to-continuous-conversion generates an income distribution with 485 data points that consists of only nine different values.

18 Wang et al. (2013) assume that WTP is normally distributed within the population, in contrast to being log-normally distributed, which would yield a mean WTP of $\mu_{\text{WTP}}=33.36$ CNY.
for five years. Wang et al. (2013) further analyzed the determinants of WTP using common socio-economic characteristics, including household income and the household’s relation to the project. Their regression analysis showed that the explanatory variable ‘log(income)’ is positively significant and the estimator for the constant income elasticity of WTP is $\eta^{\text{CHI}}=0.21$, with a standard error of $\Delta \eta^{\text{CHI}}=0.05$ (Wang et al. 2013: 27, Table 6).

We now build on these results and specify the inputs to Equation (7) as follows. First, we take $\mu_Y^{\text{CHI}}=12,572$ CNY, $\sigma_Y^{\text{CHI}}=12,291$ CNY and $\text{CV}_Y^{\text{CHI}}=0.98$ as reported by Wang et al. (2013: 61). We further assume that household income in Quibai County is log-normally distributed with moments as estimated by Wang et al. (2013). Yet, we must emphasize that the quality of the income data used for this case study is very poor. Figure 2 depicts a histogram of the income data from Wang et al. (2013) as well as a curve corresponding to the assumed log-normal distribution. Second, $\theta^{\text{CHI}}$ is given through
Taking into account the standard error in $\eta^{\text{CHI}}$ of $\Delta\eta^{\text{CHI}} = 0.05$, we obtain corresponding errors in $\theta^{\text{CHI}} = 1/\eta^{\text{CHI}}$: $\theta^{\text{CHI}}_{\eta=0.26} = 3.85$, $\theta^{\text{CHI}}_{\eta=0.16} = 6.25$. Third, as above, we calculate $\kappa^{\text{CHI}}$ indirectly as 4.33. Taking into account the standard error in $\eta$ impacting $\kappa$, through a standard error propagation estimation (Appendix A.10), we obtain: $\kappa^{\text{CHI}}_{\eta=0.26} = 3.92$ and $\kappa^{\text{CHI}}_{\eta=0.16} = 4.79$.

### 4.1.3 Global biodiversity conservation

For our analysis of how mean WTP for an environmental good depends on the distribution of income on a global scale, we draw on the meta-study by Jacobsen and Hanley (2009), who gathered 145 WTP-estimates from 46 contingent valuation studies across six continents. These contingent valuation studies assessed WTP for different kinds of ecosystem service preservation projects, with a focus on existence values. Most studies included in the dataset are located in developed countries and have been conducted between 1979 and 2005. Jacobsen and Hanley (2009) estimated an income elasticity of WTP for global biodiversity conservation of $\eta^{\text{GLO}} = 0.38$, with a standard error of $\Delta\eta^{\text{GLO}} = 0.14$, through a double-log estimation with ‘WTP per year’ [in units of 2006-purchasing-power-converted-USD, “2006-PPP-USD”] as the dependent variable and ‘annual household income’ [in units of 2006-PPP-USD] as the explanatory variable (Table 3 in Jacobsen and Hanley 2009: 145) from 127 data pairs with household income. The estimated mean WTP for global biodiversity conservation is $\mu^{\text{GLO}} = 89.51$ 2006-PPP-USD.

As there is – to our knowledge – no better estimate for an income elasticity of global WTP for environmental goods, we treat it as a proxy for the global picture. The income data in the sample of Jacobsen and Hanley (2009) consist of the mean income values of the single studies. Thus, this income data is not representative of the world distribution of household income, but reflect the arbitrary study selection, with an over-proportionate representation of studies from developed countries. We therefore generated an approximation of the world household income distribution that more closely resembles the actual distribution.

We specify the inputs to Equation (7) as follows. First, for the moments of the world
distribution of household income, we draw on the study by Pinkovskiy and Sala-i-Martin (2009), who estimate log-normal income distributions for 191 countries as well as for the world, suggesting a global mean income per capita in 2006 of 9,550 USD and a standard deviation of 15,400 USD (Pinkovskiy, personal communication). To derive the world distribution of household income, we combine their per-capita income data with estimates on average national household size, which originate from the year 2002 (Dorling et al. 2010). Simple multiplication produces the moments of the global distribution of household income. We find that global mean household income is $\mu_Y^{GLO} = 37,552$ 2006-PPP-USD, with a standard deviation of $\sigma_Y^{GLO} = 60,555$ 2006-PPP-USD corresponding to a coefficient of variation of $CV_Y^{GLO} = 1.61$. The curve of the log-normal distribution with this mean and standard distribution is depicted in Figure 3. Second, $\theta^{GLO}$ is given through $1/\eta_Y^{GLO} = 1/0.38 = 2.63$. Taking into account the standard error in $\eta_Y^{GLO}$ of $\Delta \eta_Y^{GLO} = 0.14$, 

Figure 3: Best-fitting log-normal distribution of annual household income worldwide [in units of 1,000 2006-PPP-USD], based on Pinkovskiy and Sala-i-Martin (2009).
Table 1: Descriptive statistics for the three case studies.

<table>
<thead>
<tr>
<th>Existence of large predator species (Sweden)</th>
<th>Water quality improvement (China)</th>
<th>Existence of biodiversity (Global)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N 872</td>
<td>WTP mean WTP (µWTP)</td>
<td>N 127 (for WTP data)</td>
</tr>
<tr>
<td>449.67 [annual] SEK for 5 years</td>
<td>29.75 [CNY per month for 5 years]</td>
<td>89.51 [annual 2006-PPP-USD]</td>
</tr>
</tbody>
</table>
| 0.37±0.10
t - 0.10                                | 0.21±0.05
t - 0.05                  | 0.38±0.14
t - 0.14                 |
| 2.69±0.97
t - 0.57                          | 4.76±1.64
t - 0.97                  | 2.63±1.54
t - 0.71                 |
| 4.24±1.99
t - 0.86                          | 4.33±0.46
t - 0.41                  | 1.90±0.30
t - 0.26                 |
| Annual mean income (µY)                     | 304,422 [SEK]                     | 12,572 [CNY]                      | 37,522 [2006-PPP-USD]             |
| 0.57                                         | 0.98                              | 1.61                              |

we obtain corresponding errors in $\theta^\text{GLO} = 1/\eta^\text{GLO}$: $\theta^\text{GLO}_{\eta=0.52}=1.92$, $\theta^\text{GLO}_{\eta=0.24}=4.17$. Third, as above, we calculate $\kappa^\text{GLO}$ indirectly as 1.90. Taking into account the standard error in $\eta$ impacting $\kappa$, through a standard error propagation estimation (Appendix A.10), we obtain: $\kappa^\text{GLO}_{\eta=0.52}=1.64$ and $\kappa^\text{GLO}_{\eta=0.24}=2.20$.

### 4.2 Results of empirical analysis

We now quantify and illustrate how mean WTP for environmental goods depends on the distribution of income in a society (Propositions 1 through 8) using the three case studies
Table 2: Elasticities of mean WTP with respect to mean income as well as relative and absolute income inequality for the model with relative income inequality (Equations A.44, A.45) and the model with absolute income inequality (Equations A.73, A.77), with $i=$SWE,CHI,GLO.

<table>
<thead>
<tr>
<th>Existence of large predator species (Sweden)</th>
<th>Water quality improvement (China)</th>
<th>Existence of biodiversity (Global)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{\mu_{WTP},\mu_{Y}}(\mu_{Y},CV_{Y})$</td>
<td>$0.37^{+0.10}_{-0.10}$</td>
<td>$0.21^{+0.05}_{-0.05}$</td>
</tr>
<tr>
<td>$\eta_{\mu_{WTP},CV_{Y}}(\mu_{Y},CV_{Y})$</td>
<td>$0.06^{+0.00}_{-0.01}$</td>
<td>$0.08^{+0.01}_{-0.02}$</td>
</tr>
</tbody>
</table>

| Elasticities of mean WTP for the model with absolute income inequality (Proposition 7) |
|---------------------------------------------|---------------------------------|----------------------------------|
| $\eta_{\mu_{WTP},\mu_{Y}}(\mu_{Y},\sigma_{Y})$ | $0.43^{+0.10}_{-0.11}$ | $0.29^{+0.07}_{-0.07}$ | $0.55^{+0.15}_{-0.18}$ |
| $\eta_{\mu_{WTP},\sigma_{Y}}(\mu_{Y},\sigma_{Y})$ | $0.06^{+0.00}_{-0.01}$ | $0.08^{+0.01}_{-0.02}$ | $0.17^{+0.01}_{-0.04}$ |

described above. We do this in parallel for both measures of relative and absolute income inequality, the coefficient of variation and standard deviation of income, respectively. Due to the symmetry of the analysis, we only discuss in detail the results of the case study concerning the global picture (Section 4.1.3), and report the corresponding results of the two remaining case studies in Tables 2 and 3. The quantitative inputs from the three case studies to the empirical analysis are summarized in Table 1.

First, we examine how mean WTP for global biodiversity conservation changes with adjustments in mean world household income. Figure 4 illustrates this relationship for the income elasticity of WTP of $\eta_{GLO}=0.38$ ($\theta_{GLO}=2.63$), from Jacobsen and Hanley (2009), depicted as the solid black curve, with a shaded error range of one standard error in the income elasticity of WTP, while holding the coefficient of variation of income constant at the given level of $CV_{Y}^{GLO}=1.61$ (Proposition 1).

Mean WTP for global biodiversity conservation is an increasing, concave function of mean world household income. If mean world household income increased by 1%,
Figure 4: Relationship between mean WTP for global biodiversity conservation, $\mu_{WTP}^{\text{GLO}}$, and mean world household income, $\mu_Y^{\text{GLO}}$, with an error margin of one standard error of the income elasticity of WTP (shaded in grey), for a given coefficient of variation of income of $\text{CV}_Y^{\text{GLO}}=1.61$. The adjustment factor $\Phi(2\mu_Y^{\text{GLO}}, \mu_Y^{\text{GLO}})$ for a (hypothetical) doubling of mean income corresponds to an increase in mean WTP, $\Delta \mu_{WTP}^{\text{GLO}}$, of 30.13%.

and global relative income inequality stayed constant, mean WTP would rise by approximately $\eta_{\mu_{WTP},\mu_Y}^{\text{GLO}} = 0.38^{+0.14}_{-0.14}$ (Proposition 3, Equation A.44, see Table 2). The reported range corresponds to one standard error in the income elasticity of WTP.¹⁹ A hypothetical doubling of mean world household income for a constant coefficient of variation of income, corresponding to an adjustment factor $\Phi(2\mu_Y^{\text{GLO}}, \mu_Y^{\text{GLO}})$, would lead to an increase in mean WTP for environmental goods, $\Delta \mu_{WTP}^{\text{GLO}}$, of $30.13^{+13.26}_{-12.03}$ (Proposition 4, see Table 3).²⁰ For the case of a constant absolute income inequality, mean WTP would rise by $\eta_{\mu_{WTP},\mu_Y}^{\text{GLO}} (\mu_Y, \sigma_Y) = 0.55^{+0.15}_{-0.18}$ if mean household income

¹⁹This is the only source of error we report, as the quality of the data does not permit us to provide reliable standard errors for the estimation of the moments of the distribution of income.

²⁰At historical or forecasted world long-term growth rates of between 1.6% and 2% (Drupp et al. 2015), such a doubling would occur within 35 to 44 years, that is, within the lifetime of one generation.
increased by 1% (Proposition 7, Equation A.73, see Table 2), and by $42.66\%^{+15.40}_{-15.85}$ in case of a hypothetical doubling of mean income, corresponding to the adjustment factor $\Phi_{\mu,\sigma}(2\mu_Y^{GLO}, \sigma_Y^{GLO}, \mu_Y^{GLO}, \sigma_Y^{GLO})$ (Proposition 8, see Table 3).

Second, we look at how mean WTP for global biodiversity conservation changes with income inequality, as measured by either the coefficient of variation or the standard deviation of income, for a given level of mean world household income $\mu_Y^{GLO}=37,552$ 2006-PPP-USD. Figure 5 illustrates this relationship for the coefficient of variation as a measure of relative income inequality. Mean WTP for global biodiversity conservation decreases with income inequality. Increasing either relative or absolute income inequality by 1% would decrease mean WTP by $0.17^{+0.01}_{-0.04}\%$ (Propositions 3 and 7, see Table 2).

As a hypothetical scenario, reducing relative or absolute income inequality to zero and thus obtaining a perfectly equal income distribution, would yield an adjustment factor of $\Phi_{CV}(0, CV_Y^{GLO})$ or $\Phi_{\mu,\sigma}(\mu_Y^{GLO}, 0, \mu_Y^{GLO}, \sigma_Y^{GLO})$ that corresponds to an increase of mean WTP for global biodiversity conservation, $\Delta\mu_{WTP}^{GLO}$, by $16.29^{+1.05}_{-3.90}\%$. For a more realistic scenario of a benefit transfer from a global study to an application in Sweden, that is, a transfer of WTP-estimates from a study site with an income inequality of $CV_Y^{GLO} = 1.61$ to a policy site with an income inequality of $CV_Y^{SWE} = 0.57$, mean WTP would increase by $\Delta\mu_{WTP}^{GLO}$ of $11.11^{+0.62}_{-2.40}\%$, corresponding to an adjustment factor of $\Phi_{CV}(CV_Y^{SWE}, CV_Y^{GLO})$.

Third, we study whether the negative effect of income inequality on mean WTP for environmental goods depends on the level of mean income. As shown in Proposition 4, the adjustment factor $\Phi_{CV}$ that concerns the relationship between mean WTP for environmental goods and relative income inequality is independent of the level of mean income. This means, reducing relative income inequality to zero from its original value $CV_Y^{GLO}$ raises mean WTP by $\Delta\mu_{WTP}^{GLO} = 16.29\%$ irrespective of the initial level of mean income. This finding does not hold for the relationship between mean WTP

---

21 The respective figure for the standard deviation as a measure of absolute income inequality shows exactly the same curve and error margin.

22 Sweden is a country known for its relatively low income inequality.
Figure 5: Relationship between mean WTP for global biodiversity conservation, $\mu_{WTP}^{GLO}$, and the coefficient of variation of world household income, $CV_Y^{GLO}$, for a given level of mean world household income $\mu_Y^{GLO} = 37,552$ in 2006-PPP-USD, with an error margin of one standard error (shaded in grey). The adjustment factors $\Phi_{CV}(0, CV_Y^{GLO})$ and $\Phi_{CV}(CV_Y^{SWE}, CV_Y^{GLO})$ correspond to an (hypothetical) increase in mean WTP, $\Delta \mu_{WTP}^{GLO}$, of 16.29% and 11.11%, respectively. While the former is the WTP-adjustment for the extreme case of complete income equality, the latter is the WTP-adjustment for income inequality between the global situation to the setting in Sweden.

For environmental goods and absolute income inequality as measured by the standard deviation, $\sigma_Y$ (cf. Proposition 8). We find that for the world mean household income level of $\mu_Y^{GLO}$, reducing absolute income inequality to zero, corresponding to an adjustment factor $\Phi_{\mu,\sigma}(\mu_Y^{GLO}, 0, \mu_Y^{GLO}, \sigma_Y^{GLO})$, leads to an increase of mean WTP for global biodiversity conservation by $\Delta \mu_{WTP}^{GLO} = 16.29^{+1.05}_{-3.90} \%$. In comparison, for a 30% lower world mean household income level, the same reduction of absolute income inequality, corresponding to an adjustment factor $\Phi_{\mu,\sigma}(0.7\mu_Y^{GLO}, 0, 0.7\mu_Y^{GLO}, \sigma_Y^{GLO})$, leads to an increase of mean WTP for global biodiversity conservation by $\Delta \mu_{WTP}^{GLO} = 24.23^{+1.61}_{-5.91} \%$; and for a 30% higher world household income level, corresponding to an adjustment factor
Table 3: Changes in mean WTP for environmental goods $\Delta \mu_i^{WTP}$ in percent corresponding to the adjustment factors $\Phi$, with $\Delta \mu_i^{WTP} = \Phi - 1$, and $i=$SWE,GLO,CHI.

<table>
<thead>
<tr>
<th>Adjustment factor</th>
<th>Existence of large predator species</th>
<th>Water quality improvement</th>
<th>Existence of biodiversity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi(2\mu_Y^i, \mu_Y^i)$</td>
<td>29.41±9.20 –8.58</td>
<td>15.67±4.39 –4.23</td>
<td>30.13±13.26 –12.03</td>
</tr>
<tr>
<td>$\Phi_{CV}(0, CV_Y^i)$</td>
<td>3.39±0.23 –0.52</td>
<td>5.72±1.03 –1.20</td>
<td>16.29±1.05 –3.90</td>
</tr>
<tr>
<td>$\Phi_{CV}(CV_Y^{SWE}, CV_Y^i)$</td>
<td>0</td>
<td>3.28±0.57 –0.75</td>
<td>11.11±0.62 –2.49</td>
</tr>
</tbody>
</table>

Changes in mean WTP for the model with relative income inequality (Proposition 4)

<table>
<thead>
<tr>
<th>Adjustment factor</th>
<th>Existence of large predator species</th>
<th>Water quality improvement</th>
<th>Existence of biodiversity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi(2\mu_Y^i, \mu_Y^i)$</td>
<td>32.56±9.65 –9.23</td>
<td>20.13±5.37 –5.28</td>
<td>42.66±15.49 –15.85</td>
</tr>
<tr>
<td>$\Phi_{\mu,\sigma}(\mu_Y^i, 0, \mu_Y^i, \sigma_Y)$</td>
<td>3.39±0.23 –0.52</td>
<td>5.72±1.03 –1.20</td>
<td>16.29±1.05 –3.90</td>
</tr>
<tr>
<td>$\Phi_{\mu,\sigma}(0.7\mu_Y^i, 0, 0.7\mu_Y^i, \sigma_Y)$</td>
<td>6.20±0.42 –0.96</td>
<td>9.39±1.69 –2.00</td>
<td>24.23±1.61 –5.94</td>
</tr>
<tr>
<td>$\Phi_{\mu,\sigma}(1.3\mu_Y^i, 0, 1.3\mu_Y^i, \sigma_Y)$</td>
<td>2.11±0.14 –0.32</td>
<td>3.79±0.66 –0.79</td>
<td>11.60±0.73 –0.73</td>
</tr>
</tbody>
</table>

$\Phi_{\mu,\sigma}(1.3\mu_Y^{GLO}, 0, 1.3\mu_Y^{GLO}, \sigma_Y^{GLO})$, the increase is $\Delta \mu_{WTP}^{GLO} = 11.60^{+0.73\%}_{-2.73\%}$ (Table 3). So, the negative effect of income inequality on mean WTP is more than twice as strong, in percent, when the income level doubles.

Fourth, since both mean income and income inequality influence global mean WTP for biodiversity conservation, we study which one of the two influences is relatively stronger (Propositions 3 and 7, see Table 2). The elasticity of mean WTP with respect to mean income for $CV_Y$ as the measure of relative income inequality is simply the inverse of the elasticity of substitution between the composite environmental good and consumption good: $\eta_{\mu_Y^{WTP},\mu_Y}^{GLO} = 0.38^{+0.14\%}_{-0.14\%}$ (Equation A.44). The elasticity of mean WTP with respect to mean income for $\sigma_Y$ as the measure of absolute income inequality is $\eta_{\mu_Y^{WTP},\sigma_Y}^{GLO} = 0.55^{+0.15\%}_{-0.18\%}$. (cf. Equation A.73). The elasticity of mean WTP with respect to both absolute and relative income inequality is
\[ \eta_{\text{GLO}^{\text{CV}_Y}}^{\text{GLO}^{\text{CV}_Y}}(\text{CV}_Y) = \eta_{\text{GLO}^{\mu_{Y}, \sigma_{Y}}}(\mu_{Y}, \sigma_{Y}) = 0.17^{+0.01}_{-0.04}\% \text{ (Equations A.45 and A.77).} \]

It thus follows that the influence of a change in mean income on mean WTP is relatively stronger than a change in either relative or absolute income inequality, while this relative effect is greater for the case of absolute income inequality.

5 Discussion

In this section, we discuss to what extent assumptions made in this analysis limit the generality of our results. First, our model applies to pure environmental public goods only. The meta-study of Jacobsen and Hanley (2009), employed in our empirical illustration, draws on contingent valuation studies that elicit WTP for biodiversity conservation with a particular focus on existence values. Although these habitat and species preservation projects will not benefit all households equally on a global scale, existence values may be regarded as a prime example of pure-public-good-type benefits. However, there are many environmental goods with only a limited spatial range of benefits, or with at least some degree of rivalry in consumption. Our analysis does not cover cases of such impure public environmental goods.

Second, the CES-utility specification implies that both the private consumption good and the environmental good are normal goods, and not Giffen or luxury goods. It further implies that the income elasticity of WTP is constant, an assumption that is supported by some empirical evidence (e.g. Jacobsen and Hanley 2009, Broberg 2010) and adopted in many benefit-transfer applications. There is, however, also empirical evidence that the income elasticity of WTP may vary with mean income (Barbier et al. 2015, Ready et al. 2002). Again, our model does not capture this effect. An extension of our analysis that could capture a non-constant income elasticity is to assume non-homothetic preferences, for example by taking into account a minimum (subsistence) consumption level (Baumgärtner et al. 2015).

Third, we assume that households have identical preferences and differ only with respect to income. Our results continue to hold, however, if households have different utility functions, as long as for each household the elasticity of substitution between
environmental goods and market consumption goods is constant, and these elasticities as well as the other utility parameters (e.g. the relative weight of market consumption goods to environmental goods in utility) and utility-determining variables (e.g. education, social norms and relations) are not systematically correlated with the distribution of income. In particular, it is easy to show that the basic structure of the model is unaltered if the income elasticity of WTP is normally distributed over households, which is a common assumption in empirical studies (see Appendix A.11). Thus, our results generalize to a setting where preferences of all households are described by a constant elasticity of substitution between the private and environmental goods, but this elasticity may be different across households.

Fourth, our analysis rests on the assumption that income is log-normally distributed among members of society. While there is sound evidence that this is the case at the global level and in many countries, there are also suggestions (e.g. by Bandourian et al. 2003, Giesen et al. 2010) that actual income distributions may have a 'fatter tail' than the log-normal distribution. As our calculation of mean WTP rests on the assumption of log-normal distribution of income, these results would quantitatively, but not qualitatively, change if one assumed a different kind of distribution (see last point of discussion).

Fifth, we have only examined the statistical effect of how the income distribution, and income inequality in particular, affects mean WTP for environmental public goods. A further channel of influence may be through behavioral responses to income inequality affecting mean WTP due to social preferences. The experimental findings provide hitherto inconclusive and contradictory results: while some studies find that heterogeneously endowed players in public good games contribute the same percentage of their income (e.g. Hofmeyr et al. 2007, Rapoport and Suleiman 1993), others find that players contribute the same absolute amount, meaning that low-income players contribute a higher relative share of their income (e.g. Buckley and Croson 2006). Furthermore, a recent study by Broberg (2014) suggests that relative income effects may play a role in determining WTP for environmental goods. Yet, we are not aware of any study that relates (income inequality dependent) contributions to a public good to the substitutability
between market-traded consumption goods and public goods. We therefore cannot con-
clude whether taking into account behavioral reactions would amplify or dampen our
results, which crucially depend on the elasticity of substitution. Scrutinizing the in-
teraction of social preferences and the income inequality effects described in this paper
represents a fruitful area for further research.

Finally, while we have derived all our results from a particular functional specification
of the model to allow for an empirical quantification, one could qualitatively derive our
main result more generally, using more general concepts of utility, substitutability, and
income inequality. We show this in Appendix A.12.

6 Conclusion

We have studied how the distribution of income among members of society, in particular
income inequality, affects the social willingness to pay for environmental public goods.
We found that if income is unevenly distributed among otherwise identical households
(i) social WTP for environmental goods increases with mean income; (ii) social WTP
for environmental goods decreases (increases) with income inequality if and only if envi-
ronmental goods and manufactured goods are substitutes (complements); (iii) the effect
of income inequality on social WTP is the stronger, the higher the mean income; and
(iv) social WTP for environmental goods changes more elastically with mean household
income than with income inequality, except for extreme cases of parameter values.

Our results are relevant in several respects. First, for benefit transfer, one should
correct WTP-estimates for differences in both mean income and income inequality. We
provide ready-to-use adjustment factors for this purpose. With data from empirical
case studies we have demonstrated that the size of this adjustment is considerable: a
WTP-transfer for biodiversity conservation from a global case study with high income
inequality to a society with relatively low income inequality, such as Sweden, would
entail a WTP correction for income inequality of more than ten percent.

Second, as the income-inequality effect on social WTP is the stronger, the higher
the level of mean income, it is more important to take it into account in studies and
management applications in rich countries than in poor countries.

Third, when giving policy recommendations aimed at both allocative efficiency and distributive justice ("sustainability policy", Baumgärtner and Quaas 2010), the effect of the income distribution on WTP has to be known. Assessment of allocative efficiency may require monetary valuation of non-market goods, while the distribution of income influences this monetary valuation in turn. The two aspects are thus mutually interlinked and need to be studied and addressed simultaneously. For instance, one may correct WTP-estimates for unjust income inequality, and use inequality-corrected WTP-estimates for efficiency (e.g. cost-benefit)-analysis. In the case of global WTP for biodiversity conservation this adjustment might lead to an increase in WTP of up to 16 percent, depending on the (in-)equality preferences of society.

Overall, our analysis demonstrates the importance of taking into account economic inequality and equity considerations when doing benefit transfer and economic policy in general.

Acknowledgments

We are especially grateful to Maxim Pinkovskiy, Thomas Broberg, Jette Jacobsen and Yuyan Shi for making their data sets and analyses accessible and understandable to us. We also thank Rodrigo Arriagada, Johannes Bröcker, John-Oliver Engler, Maik Heinemann, Till Requate, Felix Schläper, Lilly Schön and Armin Schmutzler, as well as conference participants at the AURÖ 2012, BIOECON 2011, EAERE 2012, EnvEcon 2013, ESEE 2013, ISEE 2014 and TEEB 2012 for helpful comments. Last, not least, we are grateful to the German National Academic Foundation, and to Marcus Lippe in particular, for providing a most stimulating academic environment for our working group on Sustainability Economics in the Social Sciences College 2009–2011. MD further gratefully acknowledges financial support from the DAAD and the BMBF under grant 01LA1104C.
Appendix

A.1 Derivation of WTP(Y) (Equation 4)

Total WTP for the environmental good at level $E$ is given as the marginal WTP $w$ times the number of units of $E$:

$$WTP = wE.$$  \hspace{1cm} (A.30)

The marginal WTP $w$ can be derived from the agent’s indirect utility function $V(p, E, Y)$ by an extension of Roy’s identity (Ebert 2003: 440).

$$w = \frac{\partial V(p, E, Y)/\partial E}{\partial V(p, E, Y)/\partial Y}.$$  \hspace{1cm} (A.31)

With the CES-utility function (Equation 1) the indirect utility function is

$$V(p, E, Y) = \left( \alpha \left( \frac{Y}{p} \right)^{\frac{\theta + 1}{\theta}} + (1 - \alpha) E^{\frac{\theta + 1}{\theta}} \right)^{\frac{\theta}{\theta - 1}}$$  \hspace{1cm} (A.32)

and, employing (A.31), the marginal WTP is then

$$w = p \frac{1 - \alpha}{\alpha} \left( \frac{Y}{pE} \right)^{\frac{1}{\theta}}.$$  \hspace{1cm} (A.33)

Plugging this into Equation (A.30) yields

$$WTP(Y) = \frac{1 - \alpha}{\alpha} (pE)^{\frac{\theta + 1}{\theta}} Y^{1/\theta}.$$  \hspace{1cm} (A.34)

A.2 Derivation of $\mu_{WTP}$ (Equation 6)

The density function of the log-normal distribution of income $Y$ with mean $\mu_Y$ and standard deviation $\sigma_Y$ is given by

$$f_{\ln}(Y; \mu_Y, \sigma_Y) = \frac{1}{Y \sqrt{2\pi s^2}} \exp \left( -\frac{(\ln Y - m)^2}{2s^2} \right)$$  \hspace{1cm} (A.35)

with

$$m = \ln \mu_Y - \frac{1}{2} \ln \left( 1 + \frac{\sigma^2_Y}{\mu^2_Y} \right),$$  \hspace{1cm} (A.36)

$$s^2 = \ln \left( 1 + \frac{\sigma^2_Y}{\mu^2_Y} \right).$$  \hspace{1cm} (A.37)
Equation (5) then becomes

\[ \mu_{\text{WTP}} = \int_0^\infty \frac{\kappa Y}{\sqrt{2\pi s^2}} \exp \left( -\frac{(\ln Y - m)^2}{2s^2} \right) dY \]

(A.35), (4)

\[ \ln Y = Z \]

\[ \frac{\kappa}{\sqrt{2\pi s^2}} \int_{-\infty}^\infty \exp(\eta Z) \exp \left( -\frac{(Z - m)^2}{2s^2} \right) dZ \]

(A.36), (A.37)

\[ \eta = 1/\theta \]

\[ \kappa \mu_Y^{1/\theta} \left( 1 + \frac{\sigma_Y^2}{\mu_Y^2} \right)^{1-\theta/2} \].

(A.38)

A.3 Proof of Proposition 1

Taking the derivative of \( \mu_{\text{WTP}} \) (Equation 7) with respect to \( \mu_Y \) yields

\[ \frac{\partial \mu_{\text{WTP}}(\mu_Y, CV_Y)}{\partial \mu_Y} = \kappa \frac{1}{\theta^2} \mu_Y^{1-\theta} (1 + CV_Y^2) \frac{1-\theta-2\theta^2}{2\theta^2} \],

which is strictly greater than zero because \( \kappa, \theta, \mu_Y, CV_Y > 0 \).

A.4 Proof of Proposition 2

Ad 1. Taking the derivative of \( \mu_{\text{WTP}} \) (Equation 7) with respect to \( CV_Y \) yields

\[ \frac{\partial \mu_{\text{WTP}}(\mu_Y, CV_Y)}{\partial CV_Y} = \kappa \frac{1 - \theta}{\theta} \frac{1}{\mu_Y^2} CV_Y (1 + CV_Y^2) \frac{1-\theta-2\theta^2}{2\theta^2} \]

(A.40)

Because \( \theta, \kappa, \mu_Y, CV_Y > 0 \), the sign of \( \partial \mu_{\text{WTP}}/\partial CV_Y \) is determined by the sign of \( 1 - \theta \):

\[ \frac{\partial \mu_{\text{WTP}}(\mu_Y, CV_Y)}{\partial CV_Y} \leq 0 \quad \text{if and only if} \quad \theta \leq 1 \].

(A.41)

Ad 2. The cross derivative of mean WTP (Equation 7) is obtained by taking the derivative of (A.40) with respect to \( \mu_Y \):

\[ \frac{\partial^2 \mu_{\text{WTP}}(\mu_Y, CV_Y)}{\partial \mu_Y \partial CV_Y} = \kappa \frac{1 - \theta}{\theta^3} \mu_Y^{1-\theta} CV_Y (1 + CV_Y^2) \frac{1-\theta-2\theta^2}{2\theta^2} \].

(A.42)
Again, because $\theta, \kappa, \mu_Y, CV_Y > 0$, the sign of $\partial^2 \mu_{WTP}/\partial \mu_Y \partial CV_Y$ is determined by the sign of $1 - \theta$:

$$\frac{\partial^2 \mu_{WTP}(\mu_Y, CV_Y)}{\partial \mu_Y \partial CV_Y} \lesssim 0 \text{ if and only if } \theta \gtrsim 1.$$  \hfill (A.43)

### A.5 Proof of Proposition 3

The elasticity of mean WTP with respect to mean income can be calculated from Equations (7) and (A.39) as

$$\eta_{\mu_{WTP},\mu_Y}(\mu_Y, CV_Y) := \frac{\partial \mu_{WTP}(\mu_Y, CV_Y)}{\partial \mu_Y} \frac{\mu_Y}{\mu_{WTP}(\mu_Y, CV_Y)} = \frac{1}{\theta} > 0.$$ \hfill (A.44)

Hence, $|\eta_{\mu_{WTP},\mu_Y}(\mu_Y, CV_Y)| = \eta_{\mu_{WTP},\mu_Y}(\mu_Y, CV_Y)$.

The elasticity of mean WTP with respect to relative income inequality can be calculated from Equations (7) and (A.40) as

$$\eta_{\mu_{WTP},CV_Y}(\mu_Y, CV_Y) := \frac{\partial \mu_{WTP}(\mu_Y, CV_Y)}{\partial CV_Y} \frac{\mu_Y}{\mu_{WTP}(\mu_Y, CV_Y)} = \frac{1 - \theta}{\theta^2} \frac{1}{1 + 1/CV_Y^2} \lesssim 0 \text{ for } \theta \gtrsim 1.$$ \hfill (A.45)

Hence

$$|\eta_{\mu_{WTP},CV_Y}(\mu_Y, CV_Y)| = \begin{cases} 
\eta_{\mu_{WTP},CV_Y}(\mu_Y, CV_Y) & < 1 \\
0 & = 1 \\
-\eta_{\mu_{WTP},CV_Y}(\mu_Y, CV_Y) & > 1. 
\end{cases}$$ \hfill (A.46)

To determine which of the two elasticities is greater in absolute terms, we have to distinguish three cases.

#### Case 1: $\theta > 1$

$$|\eta_{\mu_{WTP},\mu_Y}(\mu_Y, CV_Y)| < 1 \quad -\eta_{\mu_{WTP},CV_Y}(\mu_Y, CV_Y) = |\eta_{\mu_{WTP},CV_Y}(\mu_Y, CV_Y)|$$ \hfill (A.47)

$$\frac{1}{\theta} < 1 \quad \frac{\theta - 1}{\theta^2} \frac{1}{1 + 1/CV_Y^2} \lesssim 0 \text{ for } \theta \gtrsim 1.$$ \hfill (A.48)

$$\Leftrightarrow \quad 1/CV_Y^2 < -1/\theta.$$ \hfill (A.49)

As $CV_Y > 0$ and $\theta > 0$, the LHS (which is positive) is always greater than the RHS (which is negative). Hence, we have

$$|\eta_{\mu_{WTP},\mu_Y}(\mu_Y, CV_Y)| > |\eta_{\mu_{WTP}(\mu_Y, CV_Y),CV_Y}| \text{ for } \theta > 1.$$ \hfill (A.50)
Case 2: $\theta = 1$

$$|\eta_{\mu_{\text{WTP}, Y}}(\mu_Y, CV_Y)| = 1 > 0 = |\eta_{\mu_{\text{WTP}, CV_Y}}(\mu_Y, CV_Y)| \text{ for } \theta = 1.$$ \hfill (A.51)

Case 3: $\theta < 1$

$$|\eta_{\mu_{\text{WTP}, Y}}(\mu_Y, CV_Y)| < ? > \eta_{\mu_{\text{WTP}, CV_Y}}(\mu_Y, CV_Y) = |\eta_{\mu_{\text{WTP}, CV_Y}}(\mu_Y, CV_Y)| \text{ (A.52)}$$

$$\frac{1}{\theta} < ? > \frac{1 - \theta}{\theta^2} > \frac{1}{1 + 1/CV_Y^2} \quad (A.53)$$

$$\Leftrightarrow \frac{1}{CV_Y^2} < ? > \frac{1}{\theta - 2} \quad (A.54)$$

As $CV_Y > 0$, if $\theta \geq 1/2$ the LHS (which is positive) is always greater than the RHS (which is non-positive), and we have

$$|\eta_{\mu_{\text{WTP}, Y}}(\mu_Y, CV_Y)| > |\eta_{\mu_{\text{WTP}, CV_Y}}(\mu_Y, CV_Y)| \text{ for } 1/2 \leq \theta < 1.$$ \hfill (A.55)

If $\theta < 1/2$, the RHS is positive and

$$\frac{1}{CV_Y^2} < ? > \frac{1}{\theta - 2} \quad (A.56)$$

$$\Leftrightarrow \sqrt{\frac{\theta}{1 - 2\theta}} < ? > CV_Y \quad (A.57)$$

Hence, we have

$$|\eta_{\mu_{\text{WTP}, Y}}(\mu_Y, CV_Y)| > |\eta_{\mu_{\text{WTP}, CV_Y}}(\mu_Y, CV_Y)| \text{ for } \theta < 1/2 \text{ and } CV_Y > \sqrt{\frac{\theta}{1 - 2\theta}}.$$ \hfill (A.58)

Considering all three cases together, we thus have

$$|\eta_{\mu_{\text{WTP}, Y}}(\mu_Y, CV_Y)| \begin{cases} < \quad &|\eta_{\mu_{\text{WTP}, CV_Y}}(\mu_Y, CV_Y)| \text{ if and only if } \begin{cases} \theta < \frac{1}{2} \text{ and } CV_Y > \sqrt{\frac{\theta}{1 - 2\theta}} \\ \text{else} \end{cases} \\ > \end{cases} \quad (A.59)$$
A.6 Proof of Proposition 4

The transfer function is defined as the quotient of the mean WTPs at the policy and the study sites, and is given as:

$$
\Phi(E_{\text{policy}}, p_{\text{policy}}, \mu_{Y}^{\text{policy}}, CV_{Y}^{\text{policy}}; E_{\text{study}}, p_{\text{study}}, \mu_{Y}^{\text{study}}, CV_{Y}^{\text{study}}) \equiv \frac{\mu_{WTP}^{\text{policy}}(\mu_{Y}, CV_{Y})}{\mu_{WTP}^{\text{study}}(\mu_{Y}, CV_{Y})} = \frac{1 - \frac{\alpha}{\theta}(pE_{\text{policy}})^{\frac{\alpha - 1}{\sigma}}(\mu_{Y}^{\text{policy}})^{\frac{\alpha - 1}{\sigma}}(1 + CV_{Y}^{\text{policy}})^{\frac{1 - \theta}{2\sigma^2}}}{1 - \frac{\alpha}{\theta}(pE_{\text{study}})^{\frac{\alpha - 1}{\sigma}}(\mu_{Y}^{\text{study}})^{\frac{\alpha - 1}{\sigma}}(1 + CV_{Y}^{\text{study}})^{\frac{1 - \theta}{2\sigma^2}}} = \left(\frac{E_{\text{policy}}}{E_{\text{study}}}\right)^{\frac{\alpha - 1}{\sigma}} \cdot \left(\frac{p_{\text{policy}}}{p_{\text{study}}}\right)^{\frac{\alpha - 1}{\sigma}} \cdot \left(\frac{\mu_{Y}^{\text{policy}}}{\mu_{Y}^{\text{study}}}\right)^{\frac{1}{\theta}} \cdot \left(1 + CV_{Y}^{\text{policy}}\right)^{\frac{1 - \theta}{2\sigma^2}} \cdot \left(1 + CV_{Y}^{\text{study}}\right)^{\frac{1 - \theta}{2\sigma^2}}
$$

A.7 Proof of Proposition 5

Taking the derivative of $\mu_{WTP}$ (Equation 6) with respect to $\mu_{Y}$ yields

$$
\frac{\partial \mu_{WTP}(\mu_{Y}, \sigma_{Y})}{\partial \mu_{Y}} = \kappa \frac{1}{\theta} \mu_{Y}^{\frac{1}{\theta} - 1} \left[1 + \frac{\sigma_{Y}^{2}}{\mu_{Y}^{2}}\right]^{\frac{1 - \theta}{2\sigma^2}} \left[1 - \left(1 - \frac{1}{\theta} - 1\right) \frac{1}{\mu_{Y}^{2} + 1}\right]. \quad (A.60)
$$

Because $\kappa, \theta, \mu_{Y}, \sigma_{Y} > 0$,

$$
\frac{\partial \mu_{WTP}(\mu_{Y}, \sigma_{Y})}{\partial \mu_{Y}} \gg 0 \quad \text{for} \quad \left[1 - \left(1 - \frac{1}{\theta} - 1\right) \frac{1}{\mu_{Y}^{2} + 1}\right] \gg 0 \quad (A.61)
$$

$$
\Leftrightarrow \quad \frac{\mu_{Y}^{2}}{\sigma_{Y}^{2}} \gg \frac{1}{\theta} - 2. \quad (A.62)
$$

For $\theta \geq 1/2$, the RHS is non-positive while the LHS is strictly positive, so that the inequality holds with $>$. Thus, $\partial \mu_{WTP}/\partial \mu_{Y}$ is strictly positive for all levels of mean income $\mu_{Y}$.

In contrast, for strong complementarity, $\theta < 1/2$, $\partial \mu_{WTP}/\partial \mu_{Y}$ (Equation A.60) can have either sign. $\mu_{WTP}$ attains a unique minimum at the mean income level $\mu_{Y}^{\text{min}} = \sqrt{1/\theta - 2} \sigma_{Y}$, where $\partial \mu_{WTP}/\partial \mu_{Y}$ (Equation A.60) equals zero: $\mu_{WTP}$ falls with mean income for mean income levels below $\mu_{Y}^{\text{min}}$ and increases with mean income above $\mu_{Y}^{\text{min}}$. 40
A.8 Proof of Proposition 6

Taking the derivative of $\mu_{WTP}$ (Equation 6) with respect to $\sigma_Y$ yields

$$\frac{\partial \mu_{WTP}(\mu_Y, \sigma_Y)}{\partial \sigma_Y} = \kappa \frac{1}{\theta} \left( \frac{1}{\theta} - 1 \right) \mu_Y \left( 1 + \frac{\sigma_Y^2}{\mu_Y^2} \right)^{\frac{1-\theta-2\theta^2}{2\theta^2}} \sigma_Y \frac{1}{\mu_Y^2}. \quad (A.63)$$

Because $\kappa, \theta, \mu_Y, \sigma_Y > 0$ it follows directly that

$$\frac{\partial \mu_{WTP}(\mu_Y, \sigma_Y)}{\partial \sigma_Y} \leq 0 \quad \text{if and only if} \quad \theta \geq 1. \quad (A.64)$$

This establishes the first part of Proposition 6. To prove the second part of the proposition, we take the cross derivative of $\mu_{WTP}$ (Equation 6) as the derivative of (A.63) with respect to $\mu_Y$ and find:

$$\frac{\partial^2 \mu_{WTP}(\mu_Y, \sigma_Y)}{\partial \mu_Y \partial \sigma_Y} = F1 \times F2 \times F3 \quad \text{with}$$

$$F1 := \kappa \frac{1}{\theta} \mu_Y^{\frac{1}{\theta}-3} \sigma_Y \left( 1 + \frac{\sigma_Y^2}{\mu_Y^2} \right)^{\frac{1-\theta-2\theta^2}{2\theta^2}}, \quad (A.66)$$

$$F2 := \frac{1}{\theta} - 1, \quad (A.67)$$

$$F3 := \frac{1}{\mu_Y^2} + 1 \left( \frac{1}{\theta} - 2 \right) \left( \frac{\mu_Y^2}{\sigma_Y^2} - \frac{1}{\theta} \right). \quad (A.68)$$

As $F1 > 0$ for all parameter values, the sign of (A.65) depends on the signs of the factors $F2$ and $F3$. As for $F2$ (Equation A.67), we have

$$F2 \geq 0 \quad \text{if and only if} \quad \theta \leq 1. \quad (A.69)$$

As for $F3$ (Equation A.68), we have

$$F3 = 0 \quad \text{if and only if} \quad \theta = \frac{1}{2} \text{ or } \mu_Y = \sqrt{1/\theta} \sigma_Y. \quad (A.70)$$

As $\mu_Y > 0$ and $\sigma_Y > 0$, we also have that for $\theta < 1/2$:

$$F3 \leq 0 \iff \frac{\mu_Y^2}{\sigma_Y^2} - \frac{1}{\theta} \leq 0 \iff \mu_Y \leq \sqrt{1/\theta} \sigma_Y, \quad (A.71)$$

and for $\theta > 1/2$:

$$F3 \geq 0 \iff \frac{\mu_Y^2}{\sigma_Y^2} - \frac{1}{\theta} \geq 0 \iff \mu_Y \geq \sqrt{1/\theta} \sigma_Y. \quad (A.72)$$
Hence, the signs of $F_1$, $F_2$ and $F_3$ and, consequently, the sign of $d^2\mu_{WTP}/d\mu_Y d\sigma_Y$ (Equation A.65) are as follows:

<table>
<thead>
<tr>
<th></th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$F_3$</th>
<th>$d^2\mu_{WTP}/d\mu_Y d\sigma_Y (\mu_Y, \sigma_Y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta &lt; 1/2$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>$\geq 0$ for $\mu_Y \geq \sqrt{1/\theta_\sigma Y}$</td>
<td>$\leq 0$ for $\mu_Y \leq \sqrt{1/\theta_\sigma Y}$</td>
</tr>
<tr>
<td>$\theta = 1/2$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>$= 0$</td>
<td>$= 0$</td>
</tr>
<tr>
<td>$1/2 &lt; \theta &lt; 1$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>$\geq 0$ for $\mu_Y \leq \sqrt{1/\theta_\sigma Y}$</td>
<td>$\geq 0$ for $\mu_Y \leq \sqrt{1/\theta_\sigma Y}$</td>
</tr>
<tr>
<td>$\theta = 1$</td>
<td>$&gt; 0$</td>
<td>$= 0$</td>
<td>$= 0$</td>
<td>$= 0$</td>
</tr>
<tr>
<td>$\theta &gt; 1$</td>
<td>$&gt; 0$</td>
<td>$&lt; 0$</td>
<td>$\geq 0$ for $\mu_Y \leq \sqrt{1/\theta_\sigma Y}$</td>
<td>$\leq 0$ for $\mu_Y \leq \sqrt{1/\theta_\sigma Y}$</td>
</tr>
</tbody>
</table>

This establishes the second part of Proposition 6.

### A.9 Proof of Proposition 7

The elasticity of mean WTP with respect to mean income can be calculated from Equations (6) and (A.60) as

$$\eta_{\mu_{WTP},\mu_Y}(\mu_Y, \sigma_Y) := \frac{\partial \mu_{WTP}(\mu_Y, \sigma_Y)}{\partial \mu_Y} \frac{\mu_Y}{\mu_{WTP}(\mu_Y, \sigma_Y)} = \frac{1}{\theta} \left[1 - \left(\frac{1}{\theta} - 1\right) \frac{\mu_Y}{\frac{\mu_Y^2}{\sigma_Y} + 1}\right].$$

This is strictly positive for all levels of mean income $\mu_Y$ for an elasticity of substitution $\theta \geq 1/2$. In contrast, for $\theta < 1/2$, it can have either sign:

$$\frac{1}{\theta} \left[1 - \left(\frac{1}{\theta} - 1\right) \frac{\mu_Y}{\frac{\mu_Y^2}{\sigma_Y} + 1}\right] \begin{cases} 
\geq 0 & \text{for } \mu_Y \geq \sqrt{1/\theta - 2 \sigma_Y} \\
< 0 & \text{for } \mu_Y < \sqrt{1/\theta - 2 \sigma_Y}.
\end{cases} \quad (A.74)$$

Hence,

$$|\eta_{\mu_{WTP},\mu_Y}(\mu_Y, \sigma_Y)| = \begin{cases} 
\eta_{\mu_{WTP},\mu_Y}(\mu_Y, \sigma_Y) & \text{for } \theta \geq 1/2, \text{ or } \theta < 1/2 \text{ and } \mu_Y > \sqrt{1/\theta - 2 \sigma_Y} \\
0 & \text{for } \theta < 1/2 \text{ and } \mu_Y = \sqrt{1/\theta - 2 \sigma_Y} \\
-\eta_{\mu_{WTP},\mu_Y}(\mu_Y, \sigma_Y) & \text{for } \theta < 1/2 \text{ and } \mu_Y < \sqrt{1/\theta - 2 \sigma_Y} 
\end{cases} \quad (A.75)$$

for

$$\begin{cases} 
\theta \geq 1/2, \text{ or } \theta < 1/2 \text{ and } \mu_Y > \sqrt{1/\theta - 2 \sigma_Y} \\
\theta < 1/2 \text{ and } \mu_Y = \sqrt{1/\theta - 2 \sigma_Y} \quad \text{(A.76)} \\
\theta < 1/2 \text{ and } \mu_Y < \sqrt{1/\theta - 2 \sigma_Y}
\end{cases}$$
The elasticity of mean WTP with respect to absolute income inequality can be calculated from Equations (6) and (A.63) as

$$
\eta_{\mu_{\text{WTP}},\sigma_Y}(\mu_Y, \sigma_Y) := \frac{\partial \mu_{\text{WTP}}(\mu_Y, \sigma_Y)}{\partial \sigma_Y} \frac{\sigma_Y}{\mu_{\text{WTP}}(\mu_Y, \sigma_Y)} = \frac{1}{\theta} \left( \frac{1}{\theta} - 1 \right) \frac{1}{\sigma_Y^2} + 1 \geq 0 \quad \text{for } \theta \leq 1. \quad \text{(A.77)}
$$

Hence,

$$
|\eta_{\mu_{\text{WTP}},\sigma_Y}(\mu_Y, \sigma_Y)| = |\eta_{\mu_{\text{WTP}},\sigma_Y}(\mu_Y, \sigma_Y)| = \begin{cases} 
\eta_{\mu_{\text{WTP}},\sigma_Y}(\mu_Y, \sigma_Y) & \text{for } \theta < \frac{1}{2} \leq 1 \\
0 & \text{for } \theta = 1 \\
-\eta_{\mu_{\text{WTP}},\sigma_Y}(\mu_Y, \sigma_Y) & \text{for } \theta > \frac{1}{2}.
\end{cases} \quad \text{(A.78)}
$$

To determine which of the two elasticities is greater in absolute terms, we have to distinguish the following cases:

| Case | Condition | $|\eta_{\mu_{\text{WTP}},\mu_Y}(\mu_Y, \sigma_Y)| - |\eta_{\mu_{\text{WTP}},\sigma_Y}(\mu_Y, \sigma_Y)|$ |
|------|-----------|-------------------------------------------------|
| 1    | $\theta > 1$ | $\eta_{\mu_{\text{WTP}},\mu_Y} - \eta_{\mu_{\text{WTP}},\sigma_Y}$ |
| 2    | $\theta = 1$ | $\eta_{\mu_{\text{WTP}},\mu_Y}$ |
| 3    | $1/2 \leq \theta < 1$ | $\eta_{\mu_{\text{WTP}},\mu_Y}$ |
| 4    | $\theta < 1/2$ and $\mu_Y > \sqrt{1/\theta - 2} \sigma_Y$ | $\eta_{\mu_{\text{WTP}},\mu_Y}$ |
| 5    | $\theta < 1/2$ and $\mu_Y = \sqrt{1/\theta - 2} \sigma_Y$ | $0$ |
| 6    | $\theta < 1/2$ and $\mu_Y < \sqrt{1/\theta - 2} \sigma_Y$ | $-\eta_{\mu_{\text{WTP}},\mu_Y}$ |

Case 1: $\theta > 1$

$$
|\eta_{\mu_{\text{WTP}},\mu_Y}(\mu_Y, \sigma_Y)| - |\eta_{\mu_{\text{WTP}},\sigma_Y}(\mu_Y, \sigma_Y)| = \eta_{\mu_{\text{WTP}},\mu_Y}(\mu_Y, \sigma_Y) + \eta_{\mu_{\text{WTP}},\sigma_Y}(\mu_Y, \sigma_Y) - \frac{1}{\theta} > 0. \quad \text{(A.79)}
$$

Hence

$$
|\eta_{\mu_{\text{WTP}},\mu_Y}(\mu_Y, \sigma_Y)| > |\eta_{\mu_{\text{WTP}},\sigma_Y}(\mu_Y, \sigma_Y)|. \quad \text{(A.80)}
$$

Case 2: $\theta = 1$

$$
|\eta_{\mu_{\text{WTP}},\mu_Y}(\mu_Y, \sigma_Y)| - |\eta_{\mu_{\text{WTP}},\sigma_Y}(\mu_Y, \sigma_Y)| = \eta_{\mu_{\text{WTP}},\mu_Y}(\mu_Y, \sigma_Y) - 0 = 1 > 0. \quad \text{(A.81)}
$$
Hence
\[ |\eta_{\text{WTP}, \mu_Y}(\mu_Y, \sigma_Y)| > |\eta_{\text{WTP}, \sigma_Y}(\mu_Y, \sigma_Y)|. \quad (A.82) \]

Case 3: \( \frac{1}{2} \leq \theta < 1 \)

\[ |\eta_{\text{WTP}, \mu_Y}(\mu_Y, \sigma_Y)| - |\eta_{\text{WTP}, \sigma_Y}(\mu_Y, \sigma_Y)| = \eta_{\text{WTP}, \mu_Y}(\mu_Y, \sigma_Y) - \eta_{\text{WTP}, \sigma_Y}(\mu_Y, \sigma_Y) = \frac{1}{\theta} \left[ 1 - 2 \left( \frac{1}{\theta} - 1 \right) \frac{1}{\frac{\mu_Y^2}{\sigma_Y^2} + 1} \right] \quad (A.83) \]

Hence
\[ |\eta_{\text{WTP}, \mu_Y}(\mu_Y, \sigma_Y)| \gtrless |\eta_{\text{WTP}, \sigma_Y}(\mu_Y, \sigma_Y)| \iff \left[ 1 - 2 \left( \frac{1}{\theta} - 1 \right) \frac{1}{\frac{\mu_Y^2}{\sigma_Y^2} + 1} \right] \gtrless 0 \quad (A.85) \]
\[ \iff \frac{\mu_Y^2}{\sigma_Y^2} \gtrless \frac{2}{\theta} - 3 \quad (A.86) \]

For \( \theta \geq 2/3 \) the RHS is non-positive. As \( \mu_Y, \sigma_Y > 0 \), the LHS is strictly positive. It follows that \( |\eta_{\text{WTP}, \mu_Y}| > |\eta_{\text{WTP}, \sigma_Y}| \) for \( \theta \geq 2/3 \). For \( \theta < 2/3 \), the RHS is strictly positive and we have
\[ |\eta_{\text{WTP}, \mu_Y}(\mu_Y, \sigma_Y)| \gtrless |\eta_{\text{WTP}, \mu_Y}(\mu_Y, \sigma_Y)| \iff \mu_Y \gtrless \sqrt{\frac{2}{\theta} - 3 \sigma_Y}. \quad (A.87) \]

Case 4: \( \theta < 1/2 \) and \( \mu_Y > \sqrt{1/\theta - 2 \sigma_Y} \)

\[ |\eta_{\text{WTP}, \mu_Y}(\mu_Y, \sigma_Y)| = \eta_{\text{WTP}, \mu_Y}(\mu_Y, \sigma_Y) \gtrless \eta_{\text{WTP}, \sigma_Y}(\mu_Y, \sigma_Y) = |\eta_{\text{WTP}, \sigma_Y}(\mu_Y, \sigma_Y)| \quad (A.88) \]

\[ \iff \frac{1}{\theta} \left[ 1 - \left( \frac{1}{\theta} - 1 \right) \frac{1}{\frac{\mu_Y^2}{\sigma_Y^2} + 1} \right] \gtrless \frac{1}{\theta} \left( \frac{1}{\theta} - 1 \right) \frac{1}{\frac{\mu_Y^2}{\sigma_Y^2} + 1} \quad (A.89) \]
\[ \iff \frac{1}{\theta} \left[ 1 - 2 \left( \frac{1}{\theta} - 1 \right) \frac{1}{\frac{\mu_Y^2}{\sigma_Y^2} + 1} \right] \gtrless 0 \quad (A.90) \]
\[ \iff \frac{\mu_Y^2}{\sigma_Y^2} \gtrless \frac{2}{\theta} - 3 \quad (A.91) \]
\[ \iff \mu_Y \gtrless \sqrt{\frac{2}{\theta} - 3 \sigma_Y}. \quad (A.92) \]
As $\sqrt{2/\theta - 3} > \sqrt{1/\theta - 2}$ for $\theta < 1/2$, all three potential relations between the LHS and the RHS are feasible.

Case 5: $\theta < 1/2$ and $\mu_Y = \sqrt{1/\theta - 2}\sigma_Y$

$$|\eta_{\muWTP,\muY}(\mu_Y, \sigma_Y)| \geq \frac{1}{\theta} \left( \frac{1}{\theta} - 1 \right) \frac{1}{\frac{\mu_Y^2}{\sigma_Y^2} + 1} \iff 0 \geq \frac{1}{\theta} \left( \frac{1}{\theta} - 1 \right) \frac{1}{\frac{\mu_Y^2}{\sigma_Y^2} + 1}.$$  \hspace{1cm} (A.93)

As the RHS is strictly positive for all $\mu_Y, \sigma_Y$ and $\theta < 1/2$, we have that

$$|\eta_{\muWTP,\muY}(\mu_Y, \sigma_Y)| < |\eta_{\muWTP,\sigmaY}(\mu_Y, \sigma_Y)|.$$  \hspace{1cm} (A.94)

Case 6: $\theta < 1/2$ and $\mu_Y < \sqrt{1/\theta - 2}\sigma_Y$

$$|\eta_{\muWTP,\muY}(\mu_Y, \sigma_Y)| = -\eta_{\muWTP,\muY}(\mu_Y, \sigma_Y) \iff \eta_{\muWTP,\sigmaY}(\mu_Y, \sigma_Y) = |\eta_{\muWTP,\sigmaY}(\mu_Y, \sigma_Y)|.$$  \hspace{1cm} (A.95)

As the LHS is strictly negative for all $\theta < 1/2$, we have that

$$|\eta_{\muWTP,\muY}(\mu_Y, \sigma_Y)| < |\eta_{\muWTP,\sigmaY}(\mu_Y, \sigma_Y)|.$$  \hspace{1cm} (A.96)

Putting the different cases and sub-cases together we have the following results.

- $|\eta_{\muWTP,\muY}(\mu_Y, \sigma_Y)| > |\eta_{\muWTP,\sigmaY}(\mu_Y, \sigma_Y)|$ in the following cases:

  - $\square\theta > 1$
  - $\square\theta = 1$
  - $\square 2/3 \leq \theta < 1$
  - $\square 1/2 \leq \theta < 2/3$ and $\mu_Y > \sqrt{2/\theta - 3}\sigma_Y$
  - $\square \theta < 1/2$ and $\mu_Y > \sqrt{2/\theta - 3}\sigma_Y$ (which implies $\mu_Y > \sqrt{1/\theta - 2}\sigma_Y$)
\[ |\eta_{\text{WTP, } Y}(\mu_Y, \sigma_Y)| = |\eta_{\text{WTP, } Y}(\mu_Y, \sigma_Y)| \text{ in the following cases:} \]

- \( \Box \frac{1}{2} \leq \theta < \frac{2}{3} \) and \( \mu_Y = \sqrt{2/\theta - 3\sigma_Y} \)
- \( \Box \theta < \frac{1}{2} \) and \( \mu_Y = \sqrt{2/\theta - 3\sigma_Y} \) (which implies \( \mu_Y > \sqrt{1/\theta - 2\sigma_Y} \))

\[ |\eta_{\text{WTP, } Y}(\mu_Y, \sigma_Y)| < |\eta_{\text{WTP, } Y}(\mu_Y, \sigma_Y)| \text{ in the following cases:} \]

- \( \Box \frac{1}{2} \leq \theta < \frac{2}{3} \) and \( \mu_Y < \sqrt{2/\theta - 3\sigma_Y} \)
- \( \Box \theta < \frac{1}{2} \) and \( \sqrt{1/\theta - 2\sigma_Y} < \mu_Y < \sqrt{2/\theta - 3\sigma_Y} \)
- \( \Box \theta < \frac{1}{2} \) and \( \mu_Y = \sqrt{1/\theta - 2\sigma_Y} \)
- \( \Box \theta < \frac{1}{2} \) and \( \mu_Y < \sqrt{1/\theta - 2\sigma_Y} \)

### A.10 Error propagation

Equation (4) shows how \( \kappa \) depends on \( \eta = 1/\theta \):

\[
\kappa = \left( 1 - \frac{\alpha}{\theta} \right)^{\frac{\theta}{\alpha}} = G^{1-\eta}. \tag{A.99}
\]

Taking the natural logarithm, we obtain

\[
\ln \kappa = (1 - \eta) \cdot \ln G. \tag{A.100}
\]

Taking \( G \) as exactly measured, and denoting by \( \Delta z \) the absolute standard error of some variable \( z \), standard error propagation from \( x \) to \( y \) yields (Bronstein and Semendjajew 1987: 99–100)

\[
\frac{\Delta y}{y} = \frac{\Delta x}{x} \iff \frac{\Delta \ln \kappa}{\ln \kappa} = -\frac{\Delta \eta}{1 - \eta} \iff \Delta \ln \kappa = -\frac{\Delta \eta}{1 - \eta} \cdot \ln \kappa. \tag{A.101}
\]

As \( \kappa = \exp (\ln \kappa) \), the standard error in \( \eta \) gives rise to an interval around \( \kappa_\eta \) – the value of \( \kappa \) obtained from \( \eta \) according to Equation (A.99) – which is bounded by the following values:

\[
\kappa_{\eta \pm \Delta \eta} = \exp (\ln \kappa_\eta \pm \Delta \ln \kappa) = \exp \left( 1 \mp \frac{\Delta \eta}{1 - \eta} \cdot \ln \kappa_\eta \right). \tag{A.102}
\]
A.11 Heterogenous preferences

Assuming that $\eta$ is normally distributed and uncorrelated with income, the mean WTP is

$$
\mu_{\text{WTP}}(\mu_Y, \sigma_Y) = \int_0^\infty f_{\ln}(Y; \mu_Y, \sigma_Y) \left[ \int_{-\infty}^\infty f_{\text{norm}}(\eta; \mu_\eta, \sigma_\eta) \kappa Y^\eta d\eta \right] dY
$$

$$
= \int_0^\infty f_{\ln}(Y; \mu_Y, \sigma_Y) \kappa Y^{\mu_\eta + \sigma_\eta^2} dY
$$

$$
= \kappa \mu_Y^{\mu_\eta + \sigma_\eta^2} \left( 1 + \frac{\sigma_Y^2}{\mu_Y^2} \right) \left( \frac{1}{\mu_\eta + \sigma_\eta^2} - \frac{1}{\mu_\eta} \right), \quad (A.103)
$$

which has the same structural form as Equation (6), except that $\eta = 1/\theta$ is replaced by $\mu_\eta + \sigma_\eta^2/2$.

Similarly, if $\kappa$ follows some statistical distribution, for example because the parameter $\alpha$ of the utility function (1) has different values for different households, one has to replace $\kappa$ in (6) by its mean value $\mu_\kappa$.

A.12 Generalization of main result

Our main result in this paper – that the mean willingness to pay for the environmental good decreases (increases) with the inequality of the income distribution, for constant mean income, if and only if the environmental good and the consumption good are substitutes (complements) (Proposition 2 and Proposition 6, Statement 1) – can be be shown to hold more generally, that is, beyond the particular functional specifications of the utility function (CES) and the income distribution function (log-normal) used there. We sketch the line of argument of such a more general proof in the following.

Consider the following setting. There are $m$ private goods $X_j$ that are market-traded at prices $p_j$ ($j = 1, ..., m$) and a non-market-traded public environmental good $E$. There are $n$ individuals with identical preferences over the $j + 1$ goods that are represented by a utility function $U(X_1, ..., X_m, E)$ that is strictly increasing in all arguments and strictly quasi-concave. Income $Y$ is distributed over individuals according to some regular distribution over non-negative incomes.
In order to classify the environmental good as a “substitute” or a “complement” for the market-traded goods we build on the partial elasticities of substitution introduced by Allen and Uzawa: the partial elasticity of substitution \( \theta_{AU}(\hat{X}_j, \hat{E}) \) between the environmental good and the private good \( j \) is defined as the percentage change of the ratio of the quantity \( E \) to \( X_j \) arising from a percentage change in the price ratio \( p_j/w \) keeping utility \( U \) unchanged (cf. Uzawa 1962), where \( w \) is the Lindahl price (“virtual price”), i.e. the willingness-to-pay per unit, for the environmental good and \( \hat{X} \) and \( \hat{E} \) denote the ordinary (unconditional) Marshallian demand in a hypothetical setting where the environmental good was market-traded at price \( w \) and income was adjusted to \( \hat{Y} = Y + w\hat{E} \), so that the consumer would choose \((X, E)\).

For the following, consider the following aggregate Allen-Uzawa elasticity of substitution between the environmental good and the market-traded goods,

\[
\theta_{AU_{aggr}}(\hat{X}, \hat{E}) := \sum_{j=1}^{m} \frac{p_j X_j}{Y} \theta_{AU}(\hat{X}_j, \hat{E}),
\]

which is a weighted sum of the partial elasticities of substitution where the weights are the budget shares of market-traded good \( X_j \) with respect to original income \( Y \). This definition of the aggregate elasticity of substitution allows us, for any utility function \( U(X, E) \) and depending on the consumption levels \( X = (X_1, ..., X_m) \) and \( E \), a classification of the environmental good as being a substitute (\( \theta_{AU_{aggr}} > 1 \)) or a complement (\( 0 \leq \theta_{AU_{aggr}} < 1 \)) for the market-traded consumption goods, in an aggregate manner. If \( \theta_{AU}(\hat{X}_j, \hat{E}) = \theta \) for all \( j = 1, ..., m \), as it is the case for the CES-utility function, \( \theta_{AU_{aggr}}(\hat{X}, \hat{E}) = \theta \).

Ebert (2003: Result 9) has shown that

\[
\eta_{WTP,Y} = \frac{\eta_{E,Y}}{\theta_{AU_{aggr}}(\hat{X}, \hat{E})}.
\]

That is, in a hypothetical setting where the environmental good was market-traded at the price \( w \), the income elasticity of WTP depends positively on the income elasticity of demand for the environmental good, and inversely on the aggregate Allen-Uzawa elasticity of substitution between the environmental and the market-traded goods. All three quantities in Equation (A.105) depend on the level of income \( Y \), so that this
equation generally holds for any level of income $Y$. Result (A.105) is a generalization of $\eta = 1/\theta$ (Equation 4) for any utility function, and for a CES-utility function reduces to the latter, as $\eta_{E,Y} \equiv 1$ and $\theta^{AUaggr}(\hat{X}, \hat{E}) \equiv \theta$ for CES-utility functions.

Result (A.105) implies that the income elasticity of WTP, $\eta_{WTP,Y}$, increases with the income elasticity of demand for the environmental good, $\eta_{E,Y}$, and decreases with the aggregate Allen-Uzawa elasticity of substitution, $\theta^{AUaggr}(\hat{X}, \hat{E})$. It thus depends on both, the value of $\eta_{E,Y}$ and the value of $\theta^{AUaggr}(\hat{X}, \hat{E})$, whether $\eta_{WTP,Y}$ is smaller or larger than one, that is, whether WTP increases with income in a progressive or regressive way. Ebert (2003: Section 4.3) illustrates this result with a number of examples of different kinds of utility functions.

Define

$$\tilde{\theta} := \eta_{E,Y},$$

(A.106)
such that for all $\theta^{AUaggr}(\hat{X}, \hat{E}) > (<) \tilde{\theta}$ one has $\eta_{WTP,Y} < (> 1$ and WTP increases with income in a regressive (progressive) way. In other words, WTP increases with income in a regressive (progressive) way if and only if the aggregate Allen-Uzawa elasticity of substitution is larger (smaller) than some threshold value $\tilde{\theta}$ which is given by the income elasticity of demand for the environmental good, $\eta_{E,Y}$ (Definition A.106). WTP is then a concave (convex) function of income $Y$ over that range(s) of income for which $\theta^{AUaggr}(\hat{X}, \hat{E}) > (<) \tilde{\theta}$ holds. In general, $WTP(Y)$ may not be a globally concave (convex) function of income $Y$.

To define what it means to say that an income distribution is “more (un)equal” than another one we employ a fundamental axiom of inequality measurement: an income distribution that emerges from another one through a Pigou-Dalton-transfer is “more

---

23“Globally” here means that the WTP($Y$)-function is concave (convex) over the entire support of the income distribution.

24Consider a society of $n$ individuals with income distribution $Y = (Y_1, ..., Y_n)$, i.e. $Y_i \geq 0$ is the income of individual $i$ with $i = 1, ..., n$. Now take two individuals $j$ and $k$ with $j, k \in \{1, ..., n\}, j \neq k$ and $Y_j < Y_k$. An income transfer $\delta > 0$ from individual $k$ to individual $j$ with $Y_j + \delta \leq Y_k - \delta$ which leaves the incomes of all other members of society unaltered is called a Pigou-Dalton-transfer (Dalton 1920, following Pigou 1912).
equal” than the original one. In a more general sense, an income distribution that emerges from another one through a sequence of Pigou-Dalton-transfers is “more equal” than the original one.

For a globally concave (convex) WTP(Y)-function a more equal distribution of income Y implies a higher (lower) mean value of WTP, µ_{WTP}. This can be seen as follows. Consider a society of n individuals with income distribution \( Y = (Y_1, ..., Y_n) \), i.e. \( Y_i \geq 0 \) is the income of individual \( i \) with \( i = 1, ..., n \). Consider in particular two individuals \( j \) and \( k \) with \( j, k \in \{1, ..., n\}, j \neq k \) and \( Y_j < Y_k \) and an income transfer \( \delta > 0 \) from individual \( k \) to individual \( j \) with \( Y_j + \delta \leq Y_k - \delta \) which leaves the incomes of all other members of society unaltered. This Pigou-Dalton transfer generates an income distribution \( Y' = (Y_1, ..., Y_j + \delta, ..., Y_k - \delta, ..., Y_n) \) which is, by definition, more equal than the distribution \( Y \). Also, the following inequality holds:

\[
Y_j < Y_j + \delta \leq Y_k - \delta < Y_k . \tag{A.107}
\]

Define

\[
\lambda := 1 - \frac{\delta}{Y_k - Y_j} . \tag{A.108}
\]

This can be rearranged into \( \delta = (1 - \lambda)(Y_k - Y_j) \), so that

\[
Y_j + \delta = \lambda Y_j + (1 - \lambda)Y_k , \tag{A.109}
\]

\[
Y_k - \delta = (1 - \lambda)Y_j + \lambda Y_k . \tag{A.110}
\]

As \( \lambda \in [0, 1] \), this shows that for all possible Pigou-Dalton-transfers of \( \delta \), the after-transfer incomes of individuals \( j \) and \( k \) can be expressed as convex combinations of their before-transfer incomes.

Under the original income distribution \( Y \), mean income and mean WTP for the environmental good are

\[
\mu_Y = \frac{Y_1 + ... + Y_j + ... + Y_k + ... + Y_n}{n} , \tag{A.111}
\]

\[
\mu_{WTP} = \frac{WTP(Y_1) + ... + WTP(Y_j) + ... + WTP(Y_k) + ... + WTP(Y_n)}{n} . \tag{A.112}
\]

Under the more equal income distribution \( Y' \), mean income and mean WTP for the
environmental good are

$$
\mu'_{Y'} = (Y_1 + ... + Y_j + \delta + ... + Y_k - \delta + ... + Y_n)/n
$$

(A.113)

$$
\mu'_{WTP} = [WTP(Y_1) + ... + WTP(Y_j + \delta) + ... + WTP(Y_k - \delta) + ... + WTP(Y_n)]/n
$$

(A.114)

Obviously, \( \mu_{Y'} = \mu_Y \), that is, the Pigou-Dalton-transfer income leaves mean income unaltered, and

$$
\mu'_{WTP} - \mu_{WTP} = \{WTP(Y_j + \delta) + WTP(Y_k - \delta) - [WTP(Y_j) + WTP(Y_k)]\} / n
$$

(A.115)

Suppose the \( WTP(Y) \)-function is globally concave (convex). Then, as there exists a \( \lambda \in [0, 1] \) such that \( Y_j + \delta = \lambda Y_j + (1-\lambda)Y_k \) (Equation A.109) and \( Y_k - \delta = (1-\lambda)Y_j + \lambda Y_k \) (Equation A.110), one has – by definition of concavity (convexity)\(^{25}\) – that

\[
WTP(Y_j + \delta) = WTP(\lambda Y_j + (1-\lambda)Y_k) \geq (\leq) \lambda WTP(Y_j) + (1-\lambda)WTP(Y_k) \tag{A.116}
\]

\[
WTP(Y_k - \delta) = WTP((1-\lambda)Y_j + \lambda Y_k) \geq (\leq) (1-\lambda)WTP(Y_j) + \lambda WTP(Y_k) \tag{A.117}
\]

This implies that

\[
WTP(Y_j + \delta) + WTP(Y_k - \delta) \\
\geq (\leq) \lambda WTP(Y_j) + (1-\lambda)WTP(Y_k) + (1-\lambda)WTP(Y_j) + \lambda WTP(Y_k) \\
= WTP(Y_j) + WTP(Y_k) . \tag{A.119}
\]

With this and Equation (A.115), it becomes obvious that

$$
\mu'_{WTP} \geq (\leq) \mu_{WTP} . \tag{A.120}
$$

That is, for a globally concave (convex) \( WTP(Y) \)-function a more equal distribution of income \( Y \) implies a higher (lower) mean value of WTP, \( \mu_{WTP} \).

\(^{25}\)A real function \( \phi(x) \) is called \( \text{convex} \) if for all \( x_1, x_2 \) from its domain and all \( \lambda \in [0, 1] \), it holds that

$$
\phi(\lambda x_1 + (1-\lambda)x_2) \leq \lambda \phi(x_1) + (1-\lambda)\phi(x_2) . \tag{A.116}
$$

If the inequality is reversed, i.e. it holds with \( \geq \), the function is called \( \text{concave} \).
Putting the pieces together, we can make the following general proposition. Assume that the utility function $U(X_1, \ldots, X_m, E)$ is such that the associated WTP($Y$) function is globally concave (globally convex). Then, there exists a threshold value of the aggregate Allen-Uzawa elasticity of substitution between the environmental and the market-traded goods, $\tilde{\theta}$, such that the following holds: the mean willingness to pay for the environmental good, $\mu_{WTP}$, decreases (increases) with the inequality of the income distribution – for given mean income – if and only if the aggregate Allen-Uzawa elasticity of substitution is larger (smaller) than $\tilde{\theta}$.

References


