ECONSTOR
Make Your Publications Visible.

A Service of

ZBW Leibniz-Informationszentrum Wirtschaft
Leibniz Information Centre for Economics

Karle, Heiko; Peitz, Martin

Working Paper

Competition under Consumer Loss Aversion

Working Paper Series, No. 12-8

Provided in Cooperation with:
University of Mannheim, Department of Economics

Suggested Citation: Karle, Heiko; Peitz, Martin (2012) : Competition under Consumer Loss Aversion, Working Paper Series, No. 12-8, University of Mannheim, Department of Economics, Mannheim,
https://nbn-resolving.de/urn:nbn:de:bsz:180-madoc-316425

This Version is available at:
https://hdl.handle.net/10419/129527

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

WWW.ECONSTOR.EU

Mitglied der Leibniz-Gemeinschaft

UNIVERSITY OF MANNHEIM

University of Mannheim / Department of Economics

Working Paper Series

Competition under Consumer Loss Aversion

Heiko Karle Martin Peitz

Working Paper 12-8

May 2012

Competition under Consumer Loss Aversion*

Heiko Karle†
Université Libre de Bruxelles

Martin Peitz‡
University of Mannheim

Version: May 29, 2012

Abstract

We address the effect of contextual consumer loss aversion on firm strategy in imperfect competition. Consumers are fully informed about match value and price at the moment of purchase. However, some consumers are initially uninformed about their tastes and form a reference point consisting of an expected match—value and price distribution, while others are perfectly informed all the time. We show that, in duopoly, a larger share of informed consumers leads to a less competitive outcome if the asymmetry between firms is sufficiently large and that narrowing the set of products which consumers consider leads to a more competitive outcome.

Keywords: Contextual Loss Aversion, Reference-Dependent Utility, Behavioral Industrial Organization, Imperfect Competition, Product Differentiation
JEL Classification: D83, L13, L41, M37.

---

*This paper merges and replaces the two previous papers "Consumer Loss Aversion and the Intensity of Competition" and "Pricing and Information Disclosure in Markets with Loss-Averse Consumers." We are grateful to Heski Bar–Isaac, Paul Heidhues, Justin Johnson, Emir Kamenica, Rani Spiegler, and various seminar audiences for helpful comments and suggestions. We gratefully acknowledge financial support from the National Bank of Belgium (Research Grant, "The Impact of Consumer Loss Aversion on the Price Elasticity of Demand") and the German Science Foundation (SFB TR 15).

†ECARES, Université Libre de Bruxelles (ULB), B-1050 Brussels, Belgium. E-mail: hkarle@ulb.ac.be

‡Department of Economics, University of Mannheim, 68131 Mannheim, Germany. E-mail: martin.peitz@googlemail.com. Also affiliated with CEPR, CESifo, ENCORE, and ZEW.

1 Introduction

Consumer information about price and match value of products is a key determinant of market outcomes. If consumers are loss–averse, information prior to the moment of purchase matters: Product information plays an important role at the stage at which loss–averse consumers form expectations about future transactions.

In this paper, we investigate the competitive effects of *contextual* consumer loss aversion—i.e., of consumer loss aversion when prices within a product category define the context in which consumers make their consumption decision. Our theory applies to inspection goods, with the feature that consumers readily observe prices in the market but have to inspect products before knowing the *match value*—i.e., the fit between product characteristics and consumer tastes.[1] As we argue below, this description applies to a number of product categories and, thus, is important for understanding market interaction.

Our setup is motivated by empirical and experimental evidence in the marketing and economics literatures: consumer choice behavior is influenced by reference prices according to a large body of evidence documented in the marketing literature (see Mazumdar, Raj, and Sinha (2005) for an overview). In particular, *contextual* reference prices which are based on current prices within a product category at the moment of purchase affect consumer choice—see Rajendran and Tellis (1994). According to this view consumers feel a loss if they do not buy a cheap product within a product category. In addition, loss aversion in consumer choice has been widely documented in a variety of laboratory and field settings in the economics literature, starting with Kahneman and Tversky (1979). Furthermore, loss-averse consumers may base their reference point on rational expectations, as in Kőszegi and Rabin (2006, 2007), and with respect to prices and match value, as formalized in Heidhues and Kőszegi (2008). Empirical and experimental evidence on multi-dimensional, expectation–based loss aversion is provided by Crawford and Meng (2011), Ericson and Fuster (forthcoming) and Karle, Kirchsteiger, and Peitz (2011).

Following the theory of Kőszegi and Rabin (2006, 2007), we postulate that, to make

[1] The terminology of inspection goods was introduced by Hirshleifer (1973). Inspection goods and observable prices are considered in Anderson and Renault (2009) and, more generally, in the clearing house literature.

their consumption choices, loss–averse consumers form their probabilistic reference point based on expected future transactions, which are confirmed in equilibrium. Here, a consumer's reference point is her probabilistic belief about the relevant consumption outcome (price and product match) held between the time she first focuses on the decision determining the consumption plan—i.e., when she heard about the products in a particular context, was informed about the prices for the products on offer, and formed her expectations—and the moment she actually makes the purchase.[2]

We distinguish between "informed" and "uninformed" consumers at the moment consumers form their reference point. Informed consumers know their taste ex ante and will perfectly foresee their equilibrium utility from product characteristics. Therefore, they will not face a loss or gain in product satisfaction beyond their intrinsic valuation.

Ex–ante uninformed consumers, by contrast, are uncertain about their ideal product characteristic: They form expectations about the difference between ideal and actual product characteristic which will serve as a reference point when evaluating a product along its match–value dimension. They will also face a gain or a loss relative to their expected distributions of price after learning the taste realization. Since all consumers become fully informed before making their purchasing decision, we isolate the effect of consumer loss aversion on consumption choices and abstract from the effects of differential information at the moment of purchase. Our model can be interpreted alternatively as one in which consumers know their ideal taste ex ante, but are exposed to uncertainty about product characteristics when they form their reference point.

Ex–ante uninformed consumers form expectations *before* knowing their match value attributed to a particular product, but *after* learning the prices of the products. This timing with respect to information release and reference–point formation appears to be the appropriate modeling choice when price information is more easily accessible for consumers than information about product characteristics.[3] This is true, in particular, if price infor-

[2]For evidence that expectation–based counterfactuals can affect the individual's reaction to outcomes, see Blinder, Canetti, Lebow, and Rudd (1998), Medvec, Madey, and Gilovich (1995), and Mellers, Schwartz, and Ritov (1999). Kőszegi and Rabin (2006, 2007) have developed the general theory of expectation–based reference points and the notion of personal equilibrium.

[3]In a dynamic extension in the spirit of Kőszegi and Rabin (2009) we would need that consumers' expectations about prices are updated before the consumers' expectations about match values. See the concluding section for more detail.

mation is provided at the moment consumers become aware of the existence of products, but in which match value is difficult to evaluate without closer inspection. This is arguably the case with price advertising by intermediaries such as online price comparison websites, when consumers are agnostic about prices prior to seeing the posted prices. Even if consumers did not observe product prices before entering the shop (or starting the inspection), in many cases consumers first access the information in the price dimension of each product of interest and then start learning about their product–specific match values by comparing products with each other. In particular, consumers can easily interpret price differences, but need time to digest and interpret differences of product characteristics. The inspection of product characteristics of both products is assumed to happen simultaneously. This holds trivially if consumers obtain information only from a comparison between the two products.[4]

Consider an initially uninformed consumer who decides which of two or more differentiated products to buy. She knows the prices of the products, e.g., because prices are advertised or easily available on a price search engine, but learns the match value only after spending some time to figure out how useful the products are to her. Such uninformed consumers are likely to be present in product markets in which products are bought infrequently and possess characteristics whose values take time to evaluate (e.g., electronic devices, holiday trips, tools, or furniture). This also applies to markets of products whose match value is only revealed later in time (e.g., transport services or cultural events sold only on the spot market for which some consumers do not initially know their opportunity costs for different time slots) or at the moment of inspection (e.g., perfumes for which inexperienced consumers cannot associate a brand with a particular smell).

Since consumers observe prices before forming their reference point, firms can use price as an expectation-management tool. In other words, price announcements affect not only consumer behavior given reference points, but, in addition, they endogenously change consumer preferences. If firms set different prices, uninformed consumers will face either a loss or a gain in the price dimension ex post, depending on whether they buy the more- or less-expensive product. Hence, an (ex ante) uninformed consumer's realized net utility

[4]In the same spirit, one stream of the literature on consumer search postulates that search is non-sequential (or fixed-sample)—see, e.g. Burdett and Judd (1983).

depends not only on the price of the product she buys, but also on the price of the product she does not buy. By lowering its price, a firm increases the probability the consumer expects to buy the product. Thus, the firm can manage the probabilistic reference point affecting the utility function at the moment of purchase. This analogously holds also in the match-value dimension, where a price reduction increases the probability that a consumer will buy a product that does not provide such a good fit.

Analyzing the competitive effects under consumer loss aversion, we find that consumer loss aversion in the price dimension has a pro–competitive effect whereas loss aversion in the match value dimension has an anti–competitive effect. We relate the pro-competitive effect to price comparisons by consumers. In our benchmark model (symmetric duopoly with the same degree of loss aversion in the price and match–value dimension), we find that the anti–competitive effect dominates the pro–competitive one. This implies that a larger fraction of ex–ante informed consumers makes competition more intense. In other words, more widespread product information at the ex–ante stage is pro–competitive. Thus, transparency policies for new products which increase the number of informed consumers in the market have a pro–competitive and, hence, consumer–surplus–increasing effect.

In the main part of the paper, we show that the relative weights of the two dimensions of loss aversion are altered (1) if firms are asymmetric with respect to marginal costs or (2) if the number of firms increases and analyze the competitive effects of loss aversion in these settings. In asymmetric duopoly, we find that the price difference between the less and the more efficient firm is exacerbated through the presence of uninformed loss–averse consumers. Here, the more efficient (low–cost) firm has a strong incentive to attract those consumers by increasing the price gap. In equilibrium, the pro–competitive effect in the price dimension dominates the anti–competitive effect in the match–value dimension if the cost difference (which increases the equilibrium price difference and thus the magnitude of loss aversion in the price dimension) is sufficiently large. An interpretation of this results is that under strong cost asymmetries with uninformed loss–averse consumers, a low–cost firm can be prominent by setting a low price. For instance, consider a low–cost private label competing with a high–cost national brand. Under strong cost asymmetries, we also predict that, in addition to releasing price information, the private label may dis-

close match–value relevant information and thus make consumers informed ex ante to mitigate price competition—we elaborate on this point in the discussion section.

In our second extension, we turn to symmetric oligopoly to investigate comparative statics in the number of firms. The comparative statics effects of varying the number of firms n are the same as those in an N-firm oligopoly when varying the size of the consumers' consideration set $n < N$—i.e. the number of neighboring products that consumers are aware of. For $n > 2$, there are multiple equilibria. Selecting the equilibrium that maximizes firm profits we show that a larger consideration set (a larger number of firms) relaxes competition. The explanation of this result is related to consumers' initial probability of buying the product of a single firm and, hence, the probability of facing the price of a single firm. We receive that this probability is decreasing in n. Since loss aversion in the price dimension is increasing in this probability, it follows that loss aversion in the price dimension becomes less pronounced as n increases which reduces the pro–competitive effect of loss aversion (renders price comparisons less effective as n increases). In this light, product proliferation can be interpreted as a means to make price comparisons less successful and therefore less relevant. Moreover, when we interpret targeted advertising to consist in decreasing the seize of consumers' consideration set, we find that if firms coordinate on the equilibrium that maximizes industry profits, they would be better off from jointly agreeing not to use targeted advertising since targeted advertising intensifies competition.

In our modeling effort, we follow Heidhues and Kőszegi (2008), who also consider endogenous reference points in a market setting. Our model is enriched by considering heterogeneous consumers who differ according to their knowledge of their preferences when they form their (probabilistic) reference point. Our framework has two distinguishing features: First, consumers and firms know the market environment; in particular, firms know the actual (possibly asymmetric) cost realizations.[5] Second, consumers learn posted prices *before* they form their reference points. Due to this timing, our duopoly model delivers *unique* equilibrium predictions.[6]

[5]This allows us to compare our results with those in the standard Hotelling–Salop setting. By contrast, in Heidhues and Kőszegi (2008), costs may be private information.

[6]In Heidhues and Kőszegi (2008), consumers form their reference points before knowing posted prices.

In a related paper, Zhou (2011) considers competitive effects of consumer loss aversion when consumers do not base reference points on expectations but on past observations of firms' prices and product matches. In his paper, the firm visited first (the prominent firm) may attract an excessive market share with loss–averse consumers, while in our model, the firm setting the lower price may do so. An advantage of our setup is that prominence arises endogenously due to reference pricing, whereas, in Zhou (2011), prominence is exogenous.

Heidhues and Kőszegi (2010) and Spiegler (2011b) investigate monopoly models with consumer loss aversion. In Heidhues and Kőszegi (2010)'s setting consumers are loss–averse with respect to prices and reservation utility from purchase and the monopolist initially commits to a price distribution depending on cost realizations. They show that consumer loss aversion with expectation–based reference points provides a rationale for sticky regular prices and variable sales for frequently purchased goods. Spiegler (2011b) reproduces their main result by using a simpler, sampling-based reference concept. Furthermore, he finds that loss aversion lowers expected monopoly prices and that sticky–price equilibria are more likely to arise when uncertainty stems from demand shocks rather than from costs shocks.

Herweg, Mueller, and Weinschenk (2010) and Herweg and Mierendorff (forthcoming) have applied the expectation–based loss aversion concept of Kőszegi and Rabin (2006, 2007) to agency models. More broadly, our paper contributes to the literature on behavioral industrial organization, as surveyed in Ellison (2006), DellaVigna (2009), and Spiegler (2011a). An important issue in our paper, as in Eliaz and Spiegler (2006), is the comparative statics effects in the composition of the population. In their model this composition effect is behavioral in the sense that the share of consumers with a particular behavioral pattern changes. We do not resort to this interpretation, although our analysis is compatible with it: We allow the composition effect to be informational in the sense that the arrival of information in the consumer population is changed (while the whole population is subject to the same behavioral pattern). The informational interpretation lends

Hence, firms can deviate from consumers expectations about prices. This creates a discontinuity in consumers' marginal gain–loss utility and yields to a kinked demand curve at the expected price. The kinked demand curve leads to a non-response of prices for some cost interval (focal prices) and a multiplicity of equilibria.

itself naturally to addressing questions about the effect of early information disclosure to additional consumers.

Our paper also relates to the literature on the economics of advertising—for a survey on the economics of advertising see Bagwell (2007). It belongs to the strand that consider advertising as "hard" information; in this sense we consider directly informative advertising. Our paper uncovers the role of content advertising as consumer expectation management, which provides a complementary view to Anderson and Renault (2006). Since, at the point of purchase, consumers are fully informed there is no role for content advertising at the purchasing stage. Content advertising, however, can remove the uncertainty consumers face at the ex ante stage—i.e., when forming their reference point—and can be seen as a hybrid form of informative and persuasive advertising: It changes preferences at the point of purchase which corresponds to the persuasive view of advertising albeit due to information that is received ex ante which corresponds to the informative view of advertising. It also points to the importance of the timing of advertising: For expectation management via advertising, it is important to inform consumers prior to forming their reference points.[7] Our second extension also connects to the literature on targeted advertising, as will be spelled out in Section 4.

Our paper can be seen as complementary to the work on consumer search in product markets—see, e.g., Varian (1980), Anderson and Renault (2000), Janssen and Moraga-González (2004), Armstrong and Chen (2009). Whereas that literature focuses on the effect of differential information (and consumer search) at the purchasing stage, our paper abstracts from this issue and focuses on the effect of differential information at the expectation-formation stage which is relevant if consumers are loss–averse.

The plan of the paper is as follows. In Section 2, we present the baseline model and establish the benchmark result that a larger share of informed consumers leads to lower prices. We then allow for different degrees of consumer loss aversion in the price and the

[7] Other marketing activities can also be understood as making consumers informed at the stage when they form their reference point. For instance, test drives for cars or lending out furniture, stereo equipment, and the like make consumers informed early on. Arguably, in reality, uncertainty may not be fully resolved even at the purchasing stage. However, to focus our minds, we only consider the role of marketing activities on expectation formation before purchase. In short, in our model firms may use marketing to manage expectations of loss–averse consumers at an early stage. For a complementary view, see Bar-Isaac, Caruana, and Cunat (2010).

match value dimension. In Section 3, we allow for cost asymmetries between firms in the baseline model. In Section 4, we allow for any finite number of firms in the benchmark model and link this to the size of the consideration sets of consumers. Section 5 provides some further discussion and Section 6 concludes. Some of the proofs are relegated to Appendix A. Tables of numerical illustration are contained in Appendix B. Appendix C contains relegated material on equilibrium existence and uniqueness. Appendix D contains the derivation of demand in the n-firm oligopoly. Appendix E contains an extension in which consumers receive noisy signals about their match value ex ante, as discussed in Section 5.

2 The Baseline Model

2.1 Setup

Consider a duopoly market in which firm $i = 1, 2$ incurs a constant marginal cost of production $c_i = c$. Firms are located on a circle of length 2 with maximum distance, $y_1 = 0$, $y_2 = 1$. Firms simultaneously announce prices p_i to all consumers. We consider horizontally differentiated products à la Salop (1979). A continuum of loss-averse consumers of mass 1 are uniformly distributed on the circle of length 2. A consumer's location x, $x \in [0, 2)$, represents her taste parameter. Her taste is initially—i.e., before she forms her reference point—known only to herself if she belongs to the set of informed consumers.

In our model, consumers' differential information applies to the date at which consumers determine their reference point and not to the date of purchase: A fraction $(1 - \beta)$ of loss–averse consumers, $0 \leq \beta \leq 1$, is initially uninformed about their taste. As will be detailed below, they endogenously determine their reference point and then, before making their purchasing decision, observe their taste parameter (which then becomes private information of each consumer). At the moment of purchase all consumers are perfectly informed about product characteristics, prices, and tastes. All consumers have the same reservation value v for an ideal variety and have unit demand. Their utility from not buying is $-\infty$ so that the market is fully covered.

We could alternatively consider competition on the Hotelling line. Our circle model is, in terms of market outcomes, equivalent to the Hotelling model in which consumers are uniformly distributed on the $[0, 1]$-interval and firms are located at the extreme points of the interval. However, the circle model allows for an n-firm extension and for an alternative and equivalent interpretation about the type of information some consumers initially lack: At the point in time consumers form their reference–point distribution, they all know their taste parameters, but only a fraction $(1 - \beta)$ does not know the location of the firms—these uninformed consumers only know that the two firms are located at maximal distance.

Various justifications for differential information at the ex–ante stage can be given. For instance, consumers may differ by their experience concerning the relevant product feature: Some may have previously bought the product, whereas others are new on the market. Alternatively, a share of consumers may know that they will be subject to a taste shock between forming their reference point and making their purchasing decision. These consumers then do not condition their reference point on the ex–ante taste parameter, whereas those belonging to the remaining share do.

To determine the market demand faced by the two firms, let the informed consumer type in $[0, 1]$, who is indifferent between buying good 1 and good 2, be denoted by $\hat{x}(p_1, p_2)$. Correspondingly, the indifferent uninformed consumer is denoted by $\hat{\hat{x}}(p_1, p_2)$. Since market shares on $[0, 1]$ and $[1, 2]$ are symmetric, the firms' profits are:

$$\pi_1(p_1, p_2) = (p_1 - c_1)[\beta \cdot \hat{x}(p_1, p_2) + (1 - \beta) \cdot \hat{\hat{x}}(p_1, p_2)]$$
$$\pi_2(p_1, p_2) = (p_2 - c_2)[\beta \cdot (1 - \hat{x}(p_1, p_2)) + (1 - \beta) \cdot (1 - \hat{\hat{x}}(p_1, p_2))].$$

The timing of events is as follows.

Stage 1.) *Price setting:* Firms simultaneously set prices (p_1, p_2).

Stage 2.) *Contextual reference point formation:* All consumers observe prices and

a) informed consumers observe their taste x (for them uncertainty is resolved),

b) uninformed consumers form reference–point distributions over purchase price and match value (as detailed in Subsection 2.2 below.)

Stage 3.) *Inspection:* Uninformed consumers observe their taste x—i.e., uncertainty is resolved for *all* consumers.

Purchasing: Consumers decide which product to buy:

a) informed consumers make their purchase decisions;

b) (ex–ante) uninformed consumers make their purchase decisions, based on their utility that includes realized gains and losses relative to their reference–point distribution.

We solve for subgame–perfect Nash equilibrium where firms foresee that uninformed consumers play a personal equilibrium (at stages 2b and 3b). Personal equilibrium in our context means that consumers hold rational expectation about their final purchasing decision and behave according to them in equilibrium—for the general formalization, see Kőszegi and Rabin (2006).

2.2 Consumer demand in the baseline model

2.2.1 Demand of informed consumers

Informed consumers ex ante observe prices and their taste parameter and, therefore, do not face any uncertainty when forming their reference point. Hence, their behavior ex post is only influenced by consumers' expected behavior ex ante (buy from firm 1 with probability one and face match x and price p_1 or buy from firm 2 with probability one and face match $1 - x$ and price p_2). Kőszegi and Rabin (2006) show (in their Proposition 3) that, in such an environment, it is preferable for consumers from an ex–ante point of view to hold initial plans which maximize their intrinsic utility and to follow through these plans in equilibrium (preferred personal equilibrium).[8] Therefore their behavior is

[8]The intuition behind this result is the following: In environments without uncertainty, there exists a continuum of initial plans (around the cutoff $\hat{x}$) which consumers will follow through ex post (personal equilibria). Since none of these plans will induce a net loss ex post, it is optimal for consumers to initially choose the plan which maximizes their intrinsic utility.

the same as in the standard Hotelling-Salop model. For prices p_1 and p_2, an informed consumer located at x obtains the indirect utility $u_i(x, p_i) = v - t|y_i - x| - p_i$ from buying product i, where t scales the disutility from distance between ideal and actual taste on the circle. The expression $v - t|y_i - x|$ then captures the *match value* of product i for a consumer of type x. Denote the indifferent (informed) consumer between buying from firm 1 and 2 on the first half of the circle by $\hat{x} \in [0, 1]$. The informed (interior) indifferent consumer is

$$\hat{x}(p_1, p_2) = \frac{(t + p_2 - p_1)}{2t}. \tag{1}$$

Symmetrically, a second indifferent (informed) consumer type is located at $2 - \hat{x}(p_1, p_2) \in [1, 2]$. Using symmetry and the uniform distribution of x, we receive that firm 1's demand of informed consumers is equal to $\hat{x}(p_1, p_2)$.

2.2.2 Demand of uninformed consumers

Uninformed consumers do not know their ideal taste x ex ante and, thus, are ex ante uncertain as to which product they will buy after learning their ideal taste x: They, therefore, face ex ante uncertainty in the price and match–value dimension and form reference–point distributions in these two dimensions.

Three properties of consumer behavior are worthwhile pointing out. First, consumers have gains or losses not about net utilities but about each product "characteristic", where price is then treated as a product characteristic. This is in line with much of the experimental evidence on the endowment effect—for a discussion, see, for instance, Kőszegi and Rabin (2006).[9] Second, consumers evaluate a given product by comparing it to their reference point. Due to rational expectations this reference point may depend on the rival's product characteristics. Third, we assume here that the gain–loss parameters are the same across dimensions.

[9]Gains and losses also matter in the price dimension because, even though prices are deterministic, they can be different across firms. Hence, a consumer who initially does not know her taste parameter is uncertain at this point in time about the price at which she will buy.

The uninformed consumer will buy from firm 2 if she is located close enough to firm 2—i.e., if $x \in [\hat{\hat{x}}(p_1, p_2), 2 - \hat{\hat{x}}(p_1, p_2)]$, where $\hat{\hat{x}}(p_1, p_2)$ is the location of the indifferent (uninformed) consumer we want to characterize. Hence, the uninformed consumer at x will pay p_2 in equilibrium with $Prob[p = p_2] = Prob[\hat{\hat{x}}(p_1, p_2) < x < 2 - \hat{\hat{x}}(p_1, p_2)]$ and p_1 with the complementary probability. Since x is uniformly distributed on $[0, 2]$, we obtain that from an ex ante perspective p_1 is the relevant price with probability $Prob[p = p_1] = \hat{\hat{x}}$. Correspondingly, the purchase at price p_2 occurs with probability $Prob[p = p_2] = 1 - \hat{\hat{x}}$. This defines the reference–point distribution with respect to the purchase price p. The reference–point distribution with respect to the match value refers to the reservation value v minus the distance between ideal and actual product variety, $s \in [0, 1]$, times the taste parameter t. The density of the probability distribution of the distance is denoted by $g(s) = Prob(|x - y_\sigma| = s)$, where the location of the firm is $y_\sigma \in \{0, 1\}$, and the consumer x's purchase strategy in personal equilibrium for given prices is $\sigma \in \arg\max_{j \in \{1,2\}} u_j(x, p_j, p_{-j})$.[10] The corresponding cumulative distribution function is denoted by $G(s)$.

Consider the case $\hat{\hat{x}} \geq 1/2$—i.e., $p_1 \leq p_2$ so that firm 1 has a weakly larger market share than firm 2 also for uninformed consumers. Since at $p_1 \neq p_2$ some uninformed consumers will not buy from their nearest firm, $g(s)$ is a step function with support $[0, \hat{\hat{x}}]$. The discontinuity of g on $(0, \hat{\hat{x}})$ is determined by the maximum distance that consumers are willing to accept buying the more expensive product 2, $s = 1 - \hat{\hat{x}}$, as $s \leq 1 - \hat{\hat{x}}$ holds for consumers close to either 1 or 2, while $s > 1 - \hat{\hat{x}}$ only holds for the more distant consumers of 1. Hence, the density function takes the form

$$g(s) = \begin{cases} 2 & \text{if } s \in [0, 1 - \hat{\hat{x}}] \\ 1 & \text{if } s \in (1 - \hat{\hat{x}}, \hat{\hat{x}}] \\ 0 & \text{otherwise.} \end{cases}$$

Analogously, for the case $\hat{\hat{x}} < 1/2$.

After uncertainty is resolved, consumers experience a gain–loss utility: The reference–

[10] σ is a function of prices and consumer's location x conditional on consumer's expectation about equilibrium outcomes which are incorporated in their two-dimensional reference–point distribution. σ states a consumer's personal equilibrium. This equilibrium concept was introduced by Kőszegi and Rabin (2006) and requires that behavior-generating expectations must be self-fulfilling in equilibrium.

point distribution is split up for each dimension at the value of realization in a loss part with weight $\lambda > 1$ and a gain part with weight 1. In the loss part the realized value is compared to the lower tail of the reference–point distribution; in the gain part it is compared to the upper tail of the reference–point distribution. Following Kőszegi and Rabin (2006), we assume in this section a universal gain-loss function for all dimensions. For $p_1 \leq p_2$, the indirect utility of an uninformed consumer $x \in (1-\hat{\hat{x}}, 1]$ purchasing product 1 is then given by[11]

$$\begin{aligned} u_1(x, p_1, p_2) \quad =& (v - tx - p_1) - \lambda \cdot \text{Prob}[p = p_1](p_1 - p_1) + \text{Prob}[p = p_2](p_2 - p_1) \\ & - \lambda \cdot t \int_0^x (x - s) dG(s) + t \int_x^1 (s - x) dG(s). \end{aligned} \tag{2}$$

The first term on the right-hand side reflects the consumer's intrinsic utility from product 1. The remaining terms capture the two hedonic dimensions in which consumers experience gains and losses. The second term captures the loss in the price (or money) dimension from not facing a lower price than p_1. This term is equal to zero because p_1 is the lowest price offered in the market place. The third term is the gain from not facing a higher price than p_1, which is positive. Note that this gain is weighted by the probability of the complementary event (buying from firm 2). Intuitively, the realized gain in the price dimension is the larger the higher was the probability of facing the higher price p_2 at the initial stage. The last two terms correspond to the loss (gain) from not facing a smaller (larger) distance in the taste or match-value dimension than x. Analogously, an uninformed consumer's indirect utility from a purchase of product 2 at $p_2 \geq p_1$ is given by

$$\begin{aligned} u_2(x, p_1, p_2) \quad =& \underbrace{v - t(1-x) - p_2}_{\text{Intrinsic utility}} \underbrace{-\lambda \cdot \text{Prob}[p = p_1](p_2 - p_1)}_{\text{Loss from facing a higher } p \text{ than } p_1} \\ & \underbrace{-\lambda \cdot t \int_0^{1-x} ((1-x) - s) dG(s)}_{\text{Loss due to expecting smaller distance than } 1-x} + \underbrace{t \int_{1-x}^1 (s - (1-x)) dG(s)}_{\text{Gain due to expecting larger distance than } 1-x} \end{aligned} \tag{3}$$

[11] The indifferent uninformed consumer will be located at $x = \hat{\hat{x}}$. Therefore, $(1-\hat{\hat{x}}, 1]$ is the relevant interval for determining $\hat{\hat{x}}$ for $\Delta p \equiv p_2 - p_1 \geq 0$.

This allows us to solve a consumer's personal equilibrium by determining the location of the indifferent uninformed consumer $\hat{\hat{x}}$, which is implicitly given by $u_1(\hat{\hat{x}}, p_1, p_2) = u_2(\hat{\hat{x}}, p_1, p_2)$. Let us focus on the first half of the circle and let firm 1 be the cheaper firm—i.e., $x \in [0, 1]$ and $p_2 > p_1$.

Lemma 1. *Suppose that* $\Delta p \equiv p_2 - p_1 \geq 0$, $x \in [0, 1]$, *and* $\lambda \in (1, \lambda^c]$ *with* $\lambda^c = 3 + 2\sqrt{5} \approx 7.47$. *Then* $\hat{\hat{x}}$ *is given by*

$$\hat{\hat{x}}(\Delta p) = \begin{cases} \lambda/(\lambda - 1) - \Delta p/(4t) - S(\Delta p), & \text{if } \Delta p \in [0, \Delta\tilde{p}]; \\ 1, & \text{if } \Delta p > \Delta\tilde{p}. \end{cases} \tag{4}$$

where $\Delta\tilde{p} \equiv (\lambda + 3)t/(2(\lambda + 1)) < t$ *and*

$$S(\Delta p) = \sqrt{\frac{\Delta p^2}{16t^2} - \frac{(\lambda + 2)}{2t(\lambda - 1)}\Delta p + \frac{(\lambda + 1)^2}{4(\lambda - 1)^2}}. \tag{5}$$

We relegate the proof of this lemma to Appendix A. For $x \in [0, 1]$ and $\Delta p \geq 0$, the unique pure–strategy personal equilibrium of consumer x is described by

$$\sigma(x, \Delta p) = \begin{cases} 1 & \text{if } x \in [0, \hat{\hat{x}}(\Delta p)] \\ 2 & \text{if } x \in (\hat{\hat{x}}(\Delta p), 1]. \end{cases}$$

By symmetry and the uniform distribution of x on the circle, firm 1's demand of uninformed consumers is equal to $\hat{\hat{x}}(\Delta p)$.[12] Note that $\hat{\hat{x}}(0) = 1/2$. If $\Delta p < 0$, the location of the indifferent uninformed consumer is given by $1 - \hat{\hat{x}}(-\Delta p)$ by symmetry.

[12] The square root $S(\Delta p)$ is defined for $\Delta p \in [0, \Delta\bar{p}]$ with

$$\Delta\bar{p} \equiv \frac{2t}{(\lambda - 1)}\Big(2(\lambda + 2) - \sqrt{(2(\lambda + 2))^2 - (\lambda + 1)^2}\Big). \tag{6}$$

$\Delta\bar{p} \geq \Delta\tilde{p}$ for $\lambda \in (1, \lambda^c]$. However, for $\lambda > \lambda^c$ a critical price difference such that $\lambda/(\lambda - 1) - \Delta p/(4t) - S(\Delta p) = 1$ does not exist and we obtain a discontinuous jump up to one at $\Delta\bar{p}$. For the sake of brevity we restrict attention to the case of $\lambda \in (1, \lambda^c]$ in the following.

2.2.3 Comparison between the demand of uninformed and informed consumers

How do $\hat{\hat{x}}(\Delta p)$ and $\hat{x}(\Delta p)$ compare with one another? We can show that, for $\lambda \to 1$, the indirect utility function of uninformed consumers differs from the one of informed consumers only by a constant and we obtain $\hat{\hat{x}}(\Delta p) = \hat{x}(\Delta p)$ as a solution in this case.

Let us compare the sensitivity of uninformed consumers' demand with respect to price to the one of informed consumers. To do so, we define the critical price difference

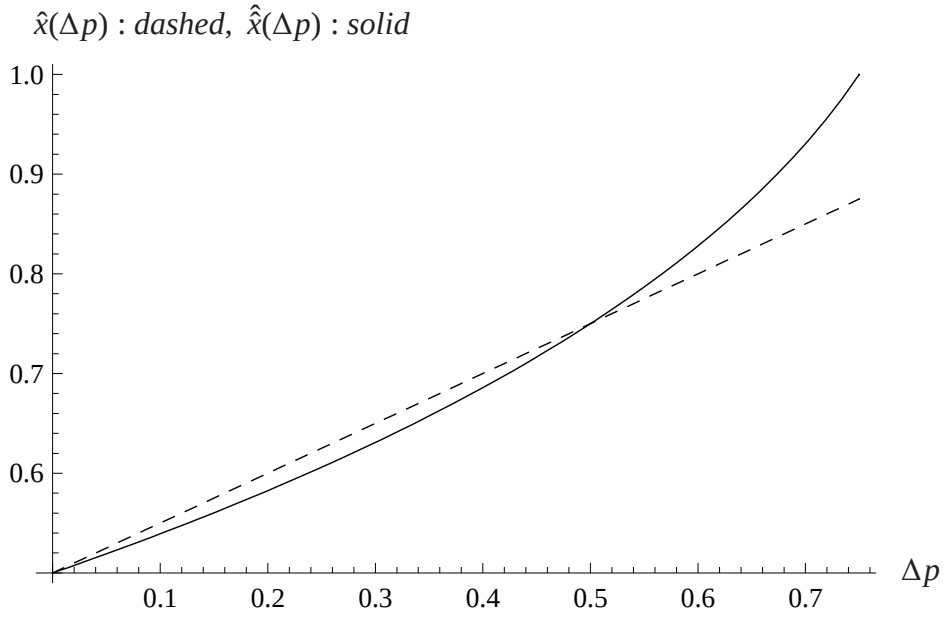
$$\Delta\hat{p} = \frac{t\left(2\sqrt{2}\cdot(2(\lambda+2)) - 3\cdot\sqrt{(2(\lambda+2))^2-(\lambda+1)^2}\right)}{\sqrt{2}(\lambda-1)}.$$

We obtain the following result:

Lemma 2. *The demand of uninformed (or loss–averse) consumers is less price sensitive than the demand of informed consumers if the price difference is sufficiently small,* $\Delta p \in [0, \Delta\hat{p})$. *The demand of uninformed (or loss–averse) consumers is more price sensitive than the demand of informed consumers if the price difference is large,* $\Delta p \in (\Delta\hat{p}, \Delta\tilde{p}]$.

The proof of the Lemma 2 is relegated to Appendix A. Demand for uninformed vs. informed consumers is illustrated in Figure 1.[13] In Section 3, we will see that this property is a driving force for our comparative static results in asymmetric markets. We note that, at a small price difference, the indifferent uninformed loss–averse consumer is harder to attract than an informed consumer by a price decrease by firm 1 because the consumer's net gain in the price dimension from buying at a lower price is outweighed by her net loss in the taste dimension if buying the more distant product 1. Thus, demand of loss–averse consumers reacts less sensitive to price in this range. The net gain and net loss in the two dimensions are equal to $(\lambda-1)\Delta p\hat{\hat{x}}(\Delta p)$ (see the difference between the second term in (20) and (21)) and $-(\lambda-1)t(1/2-2(1-\hat{\hat{x}}(\Delta p))^2)$, respectively (see the difference between the third term in (20) and (21)). Here the net gain in the price dimension is increasing and the net loss in the taste dimension is decreasing in Δp. Moreover, both

[13] We restrict attention to price differences such that the indifferent consumers are strictly interior. For larger price differences $\Delta p \in [\Delta\tilde{p}, t]$, $\hat{\hat{x}}(\Delta p) = 1$, while $\hat{x}(\Delta p) \leq 1$. Then more care is needed, as is applied in Section 3.



Location of the indifferent informed and uninformed consumer (= demand of firm 1) as a function of Δp for parameter values of $t = 1$ and $\lambda = 3$; thus, $\Delta\tilde{p} = 3/4$ and $\Delta\hat{p} = 0.2789$.

Figure 1: Demand of informed and uninformed consumers

functions are convex in Δp. For larger price differences, $\hat{\hat{x}}(\Delta p) \in [1/2, 1]$ is larger and a marginal increase in Δp increases the net gain in the price dimension more than for small price differences, while the net loss in the taste dimension decreases less than at lower price differences since $1 - \hat{\hat{x}}(\Delta p)$ is closer to zero than to $1/2$. The intuition for this finding is that, for larger price differences, the consumer is less likely to buy from the more expensive firm 2 and, thus, the avoided loss in the price dimension if not doing so ex post becomes larger. This effect dominates the reduced gain in the price dimension of buying from the cheaper firm 1 ex post which is caused by the higher probability of buying from firm 1. In addition, assigning a higher probability of buying from the more distant firm 1 leads to an expected taste difference which reduces the consumer's loss in the taste dimension if doing so ex post. Thus, the indifferent uninformed consumer as a function of the price difference is convex. For sufficiently large asymmetries, the indifferent informed consumer is closer to the center than her uninformed counterpart. Thus, the low-price firm is better able to serve uninformed than informed consumers and it becomes "prominent´´ among uninformed consumers.

2.3 Equilibrium and comparative statics

Our framework allows us to explicitly solve for equilibrium markup in symmetric duopoly, in contrast to Heidhues and Kőszegi (2008). The following lemma characterizes the symmetric equilibrium.

Lemma 3. *Any equilibrium is unique and symmetric. Equilibrium prices are given by*

$$p_i^* = c + \frac{t}{1 - \frac{(1-\beta)}{2}\frac{(\lambda-1)}{(\lambda+1)}}, i = 1, 2. \tag{7}$$

Proof. Rearranging the first-order conditions of profit maximization and using that $q_i(0;\beta) = 1/2$ for all β, we obtain

$$p_i^* - c = \frac{\frac{1}{2}}{q_i'(0;\beta)}, i = 1, 2, \tag{8}$$

where

$$q_i'(0;\beta) = -\frac{1}{4t}(1 - 3\beta) - \frac{(1-\beta)}{2(S(0))}\left(0 - \frac{(\lambda+2)}{2t(\lambda-1)}\right)$$

$$= -\frac{1}{4t}(1 - 3\beta) + \frac{(1-\beta)}{2\frac{\lambda+1}{2(\lambda-1)}}\left(\frac{(\lambda+2)}{2t(\lambda-1)}\right)$$

$$= \frac{1}{4t(\lambda+1)}\Big(2(\lambda+1) - (1-\beta)(\lambda-1)\Big).$$

Substituting into equation (8) yields the unique symmetric equilibrium price in (7). □

As shown in Appendix C.1, for all β a symmetric equilibrium exists if and only if

$$1 < \lambda \leq \lambda^c \equiv 1 + 2\sqrt{2} \approx 3.828 \tag{9}$$

In the existence proof, we have to deal with the fact that profit functions are not globally quasi-concave, since the low–price firm's profit becomes increasingly convex due to the

increasing convexity of its demand with loss-averse consumers. This violation of quasi-concavity reflects that the low–price firm may have an incentive to non-locally undercut to gain the entire demand of loss-averse consumers when the initial situation has the property that Δp is large. To deal with the non–quasi–concavity of the profit function of the low–price firm, we determine critical levels for the degree of loss aversion such that no firm has an incentive to non-locally undercut. There we use that the convexity of the low–price firm's profit function is increasing in Δp which yields that stealing the entire demand of loss-averse consumers is the only potentially optimal deviation of the low–price firm.

We define the equilibrium markup as $m^* \equiv p^* - c$. Using Lemma 3, we obtain comparative static results. In particular, as the share of informed consumers increases, each firm's markup decreases. This result follows directly from differentiating (7) with respect to β:

Proposition 1. *For $\lambda \in (1, 1 + 2\sqrt{2}]$, equilibrium markup is decreasing in the share of informed consumers β.*

In other words, uninformed loss-averse consumers exert a negative external effect on informed consumers. This contrasts the findings of a positive external effect in Gabaix and Laibson (2006) who consider a market in which only a fraction of consumers are knowledgeable about their future demand of an "add-on service", while other consumers are "naively" unaware of this.

For illustration, Table 1 reports equilibrium markups for different values of the share of informed consumers β and of the degree of loss aversion λ. At the upper bound of λ, $\lambda = \lambda^c$, the equilibrium markup reaches its maximum level of 1.414 when all consumers are loss averse. This level lies 41.4% above the level with informed (or standard) consumers.

2.4 A More Flexible Symmetric Duopoly Model

So far we imposed symmetry across the price and match value dimension. In particular, we postulated that the degrees of loss aversion are the same in the two dimensions. In this subsection, we allow for different degrees of loss aversion $\lambda_p, \lambda_m \geq 1$ in the two dimensions and verify that loss aversion in each of the two dimensions has a different

Table 1: Symmetric Equilibrium: Markups

β \ λ	1	2	3	3.8284	5	7
1	1	1	1	1	1	1
0.8	1	1.03448	1.05263	1.06222	1.07143	1.08108
0.6	1	1.07143	1.11111	1.1327	1.15385	1.17647
0.4	1	1.11111	1.17647	1.2132	1.25	1.29032
0.2	1	1.15385	1.25	1.30602	1.36364	-
0	1	1.2	1.33333	1.41421	-	-

impact on competition: in line with the insights of Lemma 2, we find that, in equilibrium, loss aversion in the price dimension is pro–competitive whereas loss aversion in the taste dimension is anti–competitive. For the sake of brevity, we only consider $\beta \in \{0, 1\}$. Extending the expression in (2), the indirect utility of buying product 1 is written as

$$\begin{aligned} u_1(x, p_1, p_2; \lambda_p, \lambda_m) \quad =& (v - tx - p_1) \\ &+ \cdot\Big(- \lambda_p \cdot \text{Prob}[p = p_1](p_1 - p_1) + \text{Prob}[p = p_2](p_2 - p_1)\Big) \\ &+ \Big(- \lambda_m \cdot t \int_0^x (x - s) dG(s) + t \int_x^1 (s - x) dG(s)\Big). \end{aligned}$$

We find that firm i's demand in duopoly is given by

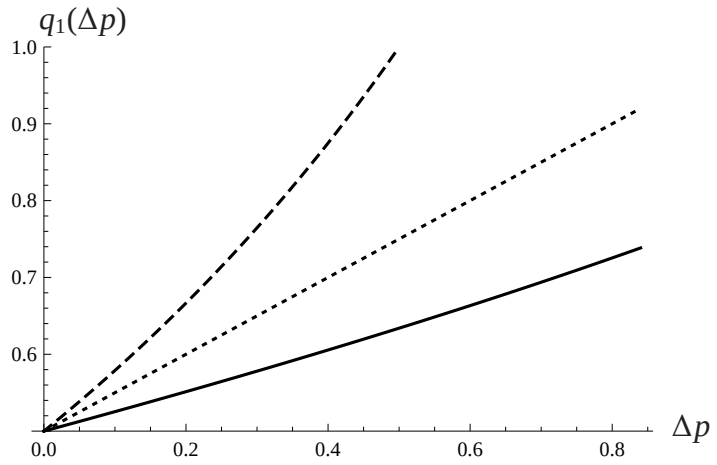
$$\hat{\hat{x}}_i(\Delta p) = \frac{\lambda_m}{(\lambda_m - 1)} - \frac{(\lambda_p - 1)}{4(\lambda_m - 1)t}\Delta p - S(\Delta p), \tag{10}$$

where

$$S(\Delta p) = \sqrt{\frac{(\lambda_p - 1)^2 \Delta p^2 - 8(\lambda_m(\lambda_m + 1) - 2)t\Delta p + 4(\lambda_m + 1)^2 t^2}{16(\lambda_m - 1)^2 t^2}} \tag{11}$$

for $\lambda_p, \lambda_m > 1$ and $\Delta p \geq 0$ (and the latter not too large). In Figure 2, we illustrate the demand of loss-averse consumers (with different degrees of loss aversion in the two dimensions).

If loss aversion in the price dimension becomes relatively more pronounced (dashed line in Figure 2), then the price sensitivity of demand increases relative to the standard case



Duopoly demand of standard and loss-averse consumers as a function of Δp for parameter values of $t = 1$: $\hat{x}_1(\Delta p) = \hat{\hat{x}}_1(\Delta p|\lambda_p = 1, \lambda_m \to 1)$: *dotted*, $\hat{\hat{x}}_1(\Delta p|\lambda_p = 1, \lambda_m = 3)$: *solid*, and $\hat{\hat{x}}_1(\Delta p|\lambda_p = 3, \lambda_m \to 1)$: *dashed*.

Figure 2: Demand with different degrees of loss aversion in the two dimensions

(dotted line). The opposite holds true if loss aversion in the taste dimension becomes relatively more pronounced (solid line).

Equilibrium prices are derived analogously to Lemma 3. For $\beta = 0$, we obtain

$$p_i^* = c + \frac{2(\lambda_m + 1)t}{\lambda_p + 3}, i = 1, 2, \tag{12}$$

provided a pure-strategy equilibrium exists. In Appendix C.2 we provide conditions for equilibrium existence for this case. Holding λ_p fixed, we observe that increasing the degree of loss aversion in the match–value dimension increases the equilibrium markup and, thus, relaxes competition. By contrast, holding λ_m fixed, we observe that increasing the degree of loss aversion in the price dimension decreases the equilibrium markup and, thus, intensifies competition.

Comparing markups in (7) (for $\beta = 1$) and (12), it follows that making loss-averse consumers informed is competitively neutral if

$$\lambda_p = 2\lambda_m - 1. \tag{13}$$

As Figure 3 illustrates, informing consumers is always pro–competitive in the baseline model where $\lambda_p = \lambda_m$. However, if the degree of loss aversion is sufficiently strong in

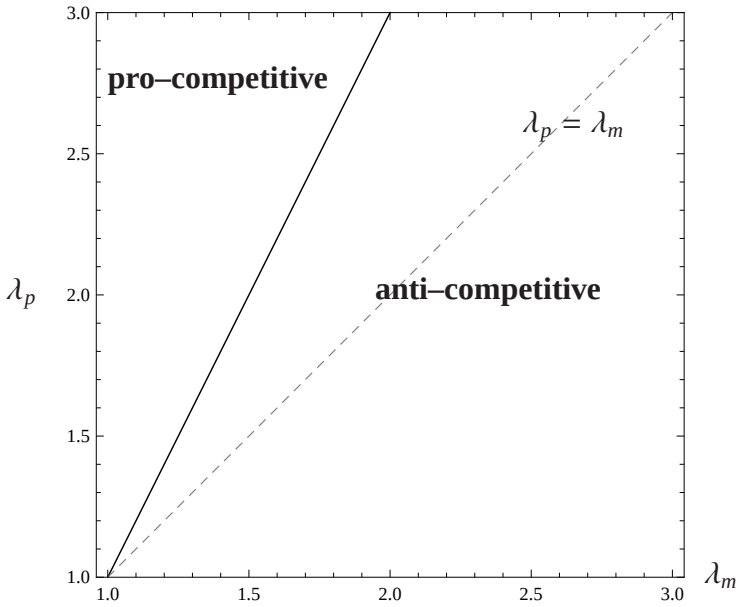


Figure 3: Pro- and anti–competitive effects of loss aversion

the price dimension relative to the match value dimension the overall implication is the reverse—i.e., competition is more intense when all consumers are ex–ante uninformed and loss-averse. We summarize our finding as follows:

Proposition 2. *Suppose that all consumers experience losses in the price dimension differently to those in the match–value dimension,* $\lambda_p \neq \lambda_m$ *and* $\lambda_p, \lambda_m \geq 1$. *If* $\lambda_p > 2\lambda_m - 1$, *the equilibrium price increases in the share of ex–ante informed consumers* $\beta \in [0, 1]$. *If the reverse inequality holds strictly, the equilibrium price decreases.*

Proof. In the main text we showed this result for the discrete change from $\beta = 0$ to $\beta = 1$. It remains to be shown that our findings in the text extend to the case with $\beta \in (0, 1)]$. The equilibrium markup in duopoly is one half divided by the first derivative of the demand function at $\Delta p = 0$; compare (8). Note that the demand function is strictly increasing in Δp. If the first derivative of the demand with uninformed consumers and flexible weights is weakly higher (resp. weakly lower) than the demand with informed consumers, then the first derivative of any convex combination of the two demand functions is as well. Thus, the result for any local increase of β with $\beta \in [0, 1)$. □

3 Cost Asymmetries

In this section, we consider asymmetric markets and provide comparative statics results with respect to β, the share of initially informed consumers. In other words, we investigate the effects of ex ante match information on market outcomes in markets with asymmetric marginal costs $c_1 \leq c_2$.

We obtain the market demand of firm 1 as the weighted sum of the demand by informed and uninformed consumers,

$$\begin{aligned} q_1(\Delta p;\beta) &= \beta \cdot \hat{x}(\Delta p) + (1-\beta) \cdot \hat{\hat{x}}(\Delta p) \\ &\equiv \phi(\Delta p;\beta) \end{aligned}$$

The demand of firm 1 is a function of the price difference Δp, which is kinked at $\Delta\tilde{p} = (\lambda+3)t/(2(\lambda+1))$ with $\Delta\tilde{p} < t$ for $\lambda > 1$. Furthermore, it approaches one as Δp approaches t.[14] Firm 2's demand is determined analogously by $q_2(\Delta p;\beta) = 1 - q_1(\Delta p;\beta)$. We focus on interior equilibria in which both products are purchased by a strictly positive share of uninformed consumers—i.e., Δp is less than $\Delta\tilde{p}$. This holds in industries in which firms are not too asymmetric.

The derivative of firm 1's demand with respect to β expresses how demand changes as the share of ex–ante informed consumers is increased. It is the difference between the demand of informed and uninformed consumers:

$$\frac{\partial\phi(\Delta p;\beta)}{\partial\beta} \equiv \phi_\beta = \hat{x}(\Delta p) - \hat{\hat{x}}(\Delta p) = \frac{3}{4t}\Delta p - \frac{\lambda+1}{2(\lambda-1)} + S(\Delta p) \gtrless 0,$$

with $\phi_\beta = 0$ at $\Delta p = 0$ and $\Delta p = t/2$. This derivative can be of positive or negative sign. As the following lemma implies, demand is decreasing in own price and increasing in the competitor's price.

[14] At $\Delta p = t$, firm 1 serves also all distant informed consumers which are harder to attract than distant uninformed consumers because the former do not perceive an overwhelming loss in the price dimension if buying from the more expensive firm 2. At $\Delta p = t$, the demand of firm 1 has another kink. We ignore the region $\Delta p > t$ since we are interested in cases in which both firms face strictly positive demand.

Lemma 4. *For $\beta < 1$, the demand of firm 1, $q_1(\Delta p;\beta) = \phi(\Delta p;\beta)$, is strictly increasing and convex in Δp for $0 \leq \Delta p \leq \Delta\tilde{p}$.*

The proof follows directly from the properties of $\hat{\hat{x}}$ and $\hat{x}$ in Section 2. In the remainder, we often refer to ϕ as a short-hand notation for $\phi(\Delta p;\beta)$. The derivative $\partial\phi/\partial(\Delta p)$ is denoted by ϕ'.

At the first stage, firms foresee consumers' purchase decisions and set prices simultaneously to maximize profits. This yields first-order conditions

$$\frac{\partial \pi_i}{\partial p_i} = \quad q_i + (p_i - c_i)\tfrac{\partial q_i}{\partial p_i} = 0 \quad , i = A, B$$

If the solution has the feature that demand of each group of consumers, informed and uninformed, is strictly positive, first-order conditions can be written as

$$\frac{\partial \pi_1}{\partial p_1} = \qquad \phi - (p_1 - c_1)\phi' = 0 \qquad (FOC_1)$$

$$\frac{\partial \pi_2}{\partial p_2} = \qquad (1 - \phi) - (p_2 - c_2)\phi' = 0. \qquad (FOC_2)$$

We refer to a solution characterized by these first-order conditions as an interior solution.

Since the profit function of the low-cost firm is not quasi-concave, we cannot use standard results to establish equilibrium existence.[15] We rule out non-interior solutions and show equilibrium existence in Appendix C.3. In asymmetric markets, existence requires an adjustment of the upper bound of the degree of loss aversion to cost differences.

We now turn to the characterization of interior equilibria (p_1^*, p_2^*).

Lemma 5. *In an interior asymmetric equilibrium with equilibrium prices (p_1^*, p_2^*), the*

[15] Anderson and Renault (2009) face an, at first glance, similar fixed point problem. They consider a general differentiated product Bertrand duopoly with covered markets in which asymmetries arise due to quality differences between firms. The authors show uniqueness and existence of a pure–strategy price equilibrium under the assumption of strict log-concavity of firms' demand. Although strict log-concavity allows for some convexity of demand, in our setup this property is not met since for large price differences and a high degree of loss aversion the convexity of the low-price firm's demand rises above any bound—i.e., $\phi''(\Delta p) \to \infty$ for $\Delta p \to \Delta\tilde{p}$ and $\lambda \to \lambda^c$.

price difference $\Delta p^* = p_2^* - p_1^*$ *satisfies*

$$\Delta p^* \quad = \quad \Delta c + f(\Delta p^*; \beta), \tag{14}$$

where $\Delta c = c_2 - c_1$ *and* $f(\Delta p; \beta) = (1 - 2\phi)/\phi'$.

Proof. Combining (FOC_1) and (FOC_2) yields the required equilibrium condition as a function of price differences. □

Thus, (14) implicitly defines the equilibrium price difference Δp^* as a function of the parameters Δc, β, λ, and t, where the latter two parameters affect the functional form of f via ϕ.

For any $\Delta c > 0$, it is not possible to obtain explicit analytical solutions to equilibrium prices—see Appendix B for numerical solutions at particular parameter values. Nevertheless, we obtain the following analytical comparative statics results.

Proposition 3. *Suppose that firm 1 is the more efficient firm,* $c_1 < c_2$.

a) *The equilibrium price difference* $\Delta p^*(\beta)$ *is decreasing in the share of informed consumers* β.

b) *The equilibrium price of the low-cost firm* $p_1^*(\beta)$ *is monotone or inversely U-shaped in the share of informed consumers* β. *In particular,* $p_1^*(\beta)$ *may be globally increasing in* β.

c) *The equilibrium price of the high-cost firm* $p_2^*(\beta)$ *is monotone or inversely U-shaped in the share of informed consumers* β. *In particular,* $p_2*(\beta)$ *may be globally increasing in* β.

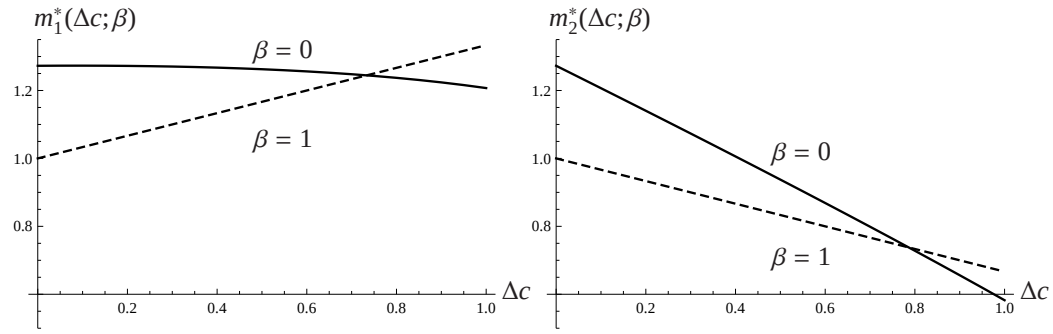
The proof is relegated to the Appendix A. In the following, we discuss the implications of this proposition. Price tends to be decreasing in β for a small cost difference (since the markup is higher with uninformed, loss–averse consumers in this case) and increasing for a large cost difference (since the markup is lower with uninformed, loss–averse consumers in this case). In addition, a larger share of uninformed, loss–averse consumers leads to

a larger price difference.[16] We observe that, in strongly asymmetric markets, lack of ex–ante information amplifies the asymmetry in market share and price between firms. In other words, relative price and market share of the "prominent" firm is larger than in a setting in which consumers are fully informed ex ante. This implies that, with loss–averse consumers, setting a low price (and making consumers aware of this) provides a means for a low–cost firm to become prominent in a market—compare the literature on prominence in search markets, in particular, Armstrong, Vickers, and Zhou (2009). Possible examples are the presence of private labels in the food or non–food grocery industry in the U.S. (Steiner 2004, p. 115) or of low–cost holiday trip providers such as Neckermann in Europe.

Firms have to trade–off the business–stealing effect with the effect on the profit margin. This trade-off is affected by the share of informed consumers which, in many markets, can be considered to be increasing over time. We find that the price of the high–margin (i.e., low–cost) firm is monotone (i.e., globally increasing or decreasing) or inverse U-shaped in β, depending on the parameter constellation:[17] in strongly asymmetric markets (when the effect of loss aversion is pro–competitive) the price of the low–cost firm may be increasing over time if more consumers become informed as the market matures. In these markets, we predict that a low–cost firm prefers to use low introductory prices. This describes a novel rationale for low introductory prices in the absence of quality differences. This is distinct from the classical result in Nelson (1970) where low introductory prices signal high product quality and other explanations on dynamic consumer behavior. By contrast, the price of the low–cost firm decreases over time in moderately asymmetric markets

[16] This is in contrast to one of the main findings in Heidhues and Kőszegi (2008) who show that, in their setting, consumer loss aversion is a rationale for focal prices. In Heidhues and Kőszegi (2008) consumers do not observe prices before forming their two-dimensional reference–point distribution. Firms therefore can deviate from consumers expectations about prices. This creates a discontinuity in consumers' marginal gain–loss utility and yields to a kinked demand curve at the expected price. The kinked demand curve leads to price rigidities for some cost interval and a multiplicity of equilibria. This discontinuity does not arise in our duopoly model since prices are observed ex ante and consumers hold correct expectations also off equilibrium. In absence of consumer loss aversion, firms would condition prices on their marginal costs. Using our terminology, Heidhues and Kőszegi compare a setting with mass 1 of uninformed consumers—i.e., $\beta = 0$, to a setting with mass 0 of uninformed consumers, which corresponds to a world without consumer loss aversion. The message by Heidhues and Kőszegi (2008) is that consumer loss aversion tends to lead to (more) equal prices; our finding, by contrast, says that consumer loss aversion leads to a larger price difference in a market with asymmetric firms.

[17] For instance, Krishnan, Bass, and Jain (1999) report that price of color TVs and clothes dryers are either monotonically declining or show an increase–decrease pricing pattern.



Equilibrium markups of firm 1 and 2 for markets in which either all consumers are uninformed ($\beta = 0$) or informed (=benchmark case, $\beta = 1$) as a function of cost differences Δc for parameter values of $t = 1$ and $\lambda = 2.5$: $\Delta c^{nd}(\beta = 0) = 1.27681$.

Figure 4: Equilibrium markup of both firms

(when the effect of loss aversion is anti–competitive). Here, low introductory prices are not chosen by the low-cost firm.

The findings for the extreme cases $\beta = 0$ and $\beta = 1$ can be inferred from Figure 4—compare firm 1's markups at low, intermediate, and high cost differences. The critical price difference (that implies the critical cost difference) at which price locally does not respond to β (c.p. Δp—i.e., the partial effect) can be solved for analytically. The critical Δp is a function of λ and t and is independent of β:

$$\Delta p|_{\frac{\partial p_1}{\partial \beta}=0}(\lambda, t) \quad = \quad \frac{t}{4(3+5\lambda)}\left((9-(26-15\lambda)\lambda) + \sqrt{3}\cdot|-1+5\lambda|\sqrt{(2(\lambda+2))^2-(\lambda-1)^2}\right)$$

For example, for parameters $\lambda = 3$ and $t = 1$ the critical price difference, at which the price of the low-cost firm reaches its maximum, satisfies $\Delta p|_{\frac{\partial p_1}{\partial \beta}=0}(3,1) = 0.2534$. It is also insightful to evaluate the derivative in the limit as β turns to 1. In this case, we can also solve analytically for a critical Δp at which the total derivative of p_1 is zero—i.e., $\frac{dp_1^*(\Delta p^*(\beta);\beta)}{d\beta} = 0$:

$$\Delta p|_{\frac{dp_1}{d\beta}=0}(\lambda, t) \quad = \quad t\frac{3(\lambda(31\lambda+42)-41) - \sqrt{21}\cdot|7-11\lambda|\sqrt{(\lambda+3)(3\lambda+5)}}{2(\lambda-3)(9\lambda-1)} \quad \text{at } \beta = 1.$$

In the example, $\Delta p|_{\frac{dp_1}{d\beta}=0}(3,1) = 7/26 = 0.2692$ at $\beta = 1$. This means that, given parameters $\lambda = 3$ and $t = 1$, if the equilibrium price difference satisfies $\Delta p^*(1) < 0.2692$ a small decrease in the share of informed consumers leads to a higher price of the more efficient

firm, $dp_1^*/d\beta < 0$. By contrast, for $\Delta p^*(1) > 0.2692$, the reverse inequality holds—i.e., $dp_1^*/d\beta > 0$.

For the high-cost firm, our result is qualitatively similar: The price tends to be decreasing in β for small cost differences and increasing for large cost differences.[18]

We briefly discuss the firms' incentives to disclose information—i.e., we investigate the effect of β on profits. Here, private information disclosure can be seen as the firms' management of consumer expectations (i.e., reference points). Note that in our simple setting information disclosure by one firm fully reveals the information of both firms since consumers make the correct inferences from observing the match value for one of the two products. We confine attention to a numerical example. The critical value of Δp such that $d\pi_1/d\beta = 0$ at $\beta = 1$ and $\lambda = 3$ and $t = 1$, $c_1 = 0.25$, and $c_2 = 1$ is $\Delta p = 0.2581$. The critical value of Δp such that $d\pi_2/d\beta = 0$ at the same parameter values as above is $\Delta p = 0.2870$. The critical value at $\beta = 1$ is $\Delta p^*(1) = 0.25$ (see Table 3 in Appendix B). Hence, the critical values of Δp at $\beta < 1$ are larger than $\Delta p^*(1)$. Moreover, $\Delta p|_{\frac{dp_2}{d\beta}=0} > \Delta p|_{\frac{dp_1}{d\beta}=0}$.

Our numerical example shows that there are cases where increasing the initial share of ex–ante informed consumers, first none, then one and then both of the firms gain from information disclosure. Since disclosing information about match value to a positive number of consumers is profitable, such a strategy will be chosen by profit–maximizing firms (if disclosure is not too costly). This also implies that, for sufficiently large cost asymmetries, a "prominent" firm might disclose product match information to consumers at an

[18] With respect to firm 1, we also solve for critical values at which the marginal effect of firm 2's price changes sign:

$$\Delta p|_{\frac{dp_2}{d\beta}=0}(\lambda, t) = \frac{t}{2(\lambda+1)(\lambda+7)}\left((-23+(\lambda-10)\lambda)+|5-\lambda|\sqrt{(2(\lambda+2))^2-(\lambda-1)^2}\right)$$

For instance, $\Delta p|_{\frac{dp_2}{d\beta}=0}(3,1) = 0.3201$. At $\beta = 1$ we can solve analytically for a critical Δp at which the total derivative of p_2 is zero—i.e., $(dp_2^*(\Delta p^*(\beta);\beta))/d\beta = 0$:

$$\Delta p|_{\frac{dp_2}{d\beta}=0}(\lambda, t) = \frac{t\left(3(\lambda(17\lambda+6)-55)-\sqrt{15}\cdot|11-7\lambda|\sqrt{(\lambda+3)(3\lambda+5)}\right)}{4\lambda(3\lambda-11)}$$

We have $\Delta p|_{\frac{dp_2}{d\beta}=0}(3,1) = 1/2\cdot(5\sqrt{35}-29) = 0.2902$ at $\beta = 1$. Thus, for $\Delta p^*(1) < 0.2902$, we obtain $dp_2/d\beta < 0$ at $\beta = 1-\epsilon$, while, for $\Delta p^*(1) > 0.2902$, we obtain $dp_2/d\beta > 0$ at $\beta = 1$. Thus, the overall effect of a marginal increase in β can indeed become positive if cost asymmetries are sufficiently large.

early stage. In this case, the "prominent" firm prefers to give up a higher market share with uninformed consumers in favor of higher markups with informed consumers. This result shows that if competition becomes too intense it can become profitable for private labels (low–cost firms) to disclose product information. Our finding provides a rationale for truthfully advertising product characteristics at an early stage, although all consumers would learn them prior to purchase even in the absence of advertising. Without consumer loss aversion it would be irrelevant for market demand and market outcomes whether or not a firm advertises product characteristics ex ante.

While the focus of our analysis has been on the effect of a change of the share of ex–ante informed consumers, we may also want to compare markets with different asymmetries between firms. For this purpose, we state a comparative statics result with respect to the degree of cost asymmetry—i.e., the level of $\Delta c = c_2 - c_1$.

Proposition 4. *The equilibrium price difference* $\Delta p^*(\Delta c, \beta)$ *is an increasing function of the cost asymmetry between firms,* Δc*. It reacts more sensitive to* Δc *than in a market in which all consumers are informed ex ante,* $d(\Delta p^*)/d(\Delta c) > 1/3$ *for* $\beta > 0$.

The proof is relegated to Appendix A.2. Proposition 4 says that the more pronounced the cost asymmetry the larger the price difference between high-cost and low-cost firm. The marginal effect of an increase in cost differences on price variation is stronger if some loss-averse consumers are uninformed. We thus predict exacerbated price variation in markets with uninformed loss–averse consumers in response to a larger asymmetry. Intuitively, the more efficient firm (firm 1) is tempted to use consumer expectation management to increase its market share or prominence: announcing a very low price ex ante makes loss–averse consumers more reluctant than standard consumers to buy from the less efficient firm (firm 2) later on.

Finally, we would like to comment on the equilibrium markup of the low-cost firm $m_1^*(\Delta c) \equiv p_1^*(\Delta c, c_1) - c_1$. While in the standard Hotelling world with only informed consumers ($\beta = 1$) the markup of the more efficient firm is increasing in the cost difference, a local increase of the cost difference may have the reverse effect under consumer loss aversion ($\beta < 1$, $\lambda > 1$). This holds true in strongly asymmetric markets: The price sensitivity of demand is larger than in the standard Hotelling world due to the dominating loss in

the price dimension. We note that, under very large cost differences, firm 1's markup might fall below its level in the standard Hotelling world, as has been illustrated in Figure 4—see, also, Tables 3 and 4 in Appendix B.

4 Consideration Sets

In this section, we introduce consideration sets and focus on comparative statics with respect to the size of this set. To this effect we analyze a symmetric oligopoly. For expositional reasons, we only consider the situations in which none or all consumers are ex–ante informed, i. e. $\beta \in \{0, 1\}$. We consider comparative statics in the number of firms in an n-firm oligopoly and argue that this is equivalent to varying the size of the consideration set for a given number of firms $N > n$ in the industry. Suppose that the length of the circle is $L = n$ (while the consumer mass is equal to 1); this implies that the equilibrium markup in the model with standard consumers (as in Salop (1979)) is independent of the number of firms, as additional firms do not affect the degree of product differentiation between any direct neighbors. Hence, we are able to isolate the role played by consumer loss aversion as any change in the equilibrium markup is due to the presence of consumer loss aversion. Furthermore, we observe that, under the alternative timing proposed by Heidhues and Kőszegi (2008) that consumers form reference points before observing prices, the set of symmetric equilibrium prices is independent of the number of firms. The reason is that consumers expect a particular reference point distribution which is independent of the number of firms as a local price change given these reference points has the same effect on a firm's demand independent of the number of firms in the market. Thus, any comparative statics results in the number of firms are due to the fact that price changes are observed initially and, thus, affect the reference–point distribution.

As shown in Appendix D, firm i's demand $n\hat{\hat{x}}_i(\Delta p, p')$ for a small price decrease that does not steal any adjacent market ($\hat{\hat{x}}_i \in [1/n, 2/n]$, $p_i \leq p'$, and $p_j = p'$ for all $j \neq i$; $\Delta p \equiv p' - p_i$) satisfies

$$\hat{\hat{x}}_i(\Delta p, p') = \left(\frac{4}{(\lambda - 1)(n + 2)} + \frac{3n + 2}{n(n + 2)}\right) - \frac{2\Delta p}{n(n + 2)t} - 2S(\Delta p), \tag{15}$$

where

$$S(\Delta p) = \sqrt{\frac{\Delta p^2(\lambda-1)^2 - (\lambda-1)(\lambda(3n+2)+n(2n+5)-2)t\Delta p + (1+\lambda)^2 n^2 t^2}{(\lambda-1)^2\left(2n+n^2\right)^2 t^2}} \quad (16)$$

for $\lambda > 1$ and $\Delta p \geq 0$ and sufficiently small. Firm i's demand for a sufficiently small price increase can be derived analogously. Moreover, its demand for a larger price decrease is reported in the n–firm existence proof provided in Appendix C.4.

We observe that, at symmetric prices of the firm $j \neq i$, firm i's demand is kinked for $n > 2$. This means that demand in oligopoly with more than two firms behaves qualitatively differently than duopoly demand because setting a slightly lower price than the competitor leads to a different marginal effect in absolute value than setting a slightly higher price if there is more than one competitor.[19] The kinked demand is illustrated in Figure 5 for $n = 100$ (keeping the competitors' prices fixed at the duopoly equilibrium price $p^*(2)$).

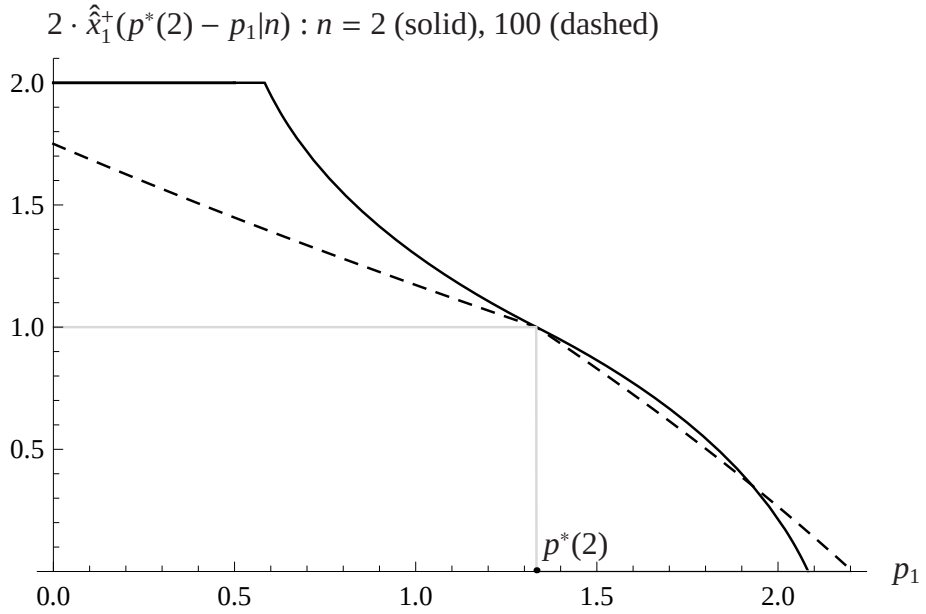
Due to kinked demand, there is a continuum of equilibria for $n > 2$. Suppose that firms coordinate on the symmetric equilibrium that maximizes industry profits.[20] The maximal equilibrium markup is derived below.

Establishing equilibrium existence in n-firm oligopoly is rather involved, since there might arise profitable non-local deviations by stealing consumers in distant sub-markets. Although, for large n, conditions for maximal equilibrium existence carry over from the duopoly case, stricter conditions are required in markets with a small number of firms. The next lemma reports sufficient conditions, which are derived in detail in Appendix C.4.

Lemma 6. *A symmetric maximal equilibrium with n firms and prices,* $p^*(n) = m^*(n) + c = ((1+\lambda)nt)/(\lambda - 1 + 2n) + c$, *exists*

[19] For $\Delta p \leq 0$, firm i's demand (resp. location of the indifferent loss-averse consumer) can be derived considering the indirect utility functions for a price *increase* of firm i. For $n = 2$, firm i deviating from symmetric prices (p, p) by a price increase to p' is equivalent to firm $-i$ deviating from symmetric prices (p', p') by a price decrease to p. Therefore demand is symmetric around $\Delta p = 0$ in this case and no kink arises.

[20] Cf. literature on cheap–talk games. Note that there is also a continuum of equilibria under the alternative timing proposed by Heidhues and Kőszegi (2008). However, since consumers do not observe prices under this alternative timing, firms do not only have to solve the coordination problem, but consumers must be included as well in the coordination of beliefs, which equilibrium will be played.



Loss-averse consumers covered by firm 1 (= twice location of the indifferent, loss-averse consumer or n-times demand of firm 1) for varying n as a function of p_1 for parameter values of $t = 1$ and $\lambda = 3$: $p^*(2) = 4/3$

Figure 5: Location of the indifferent, loss-averse consumer ($n = 2, 100$)

1. *for all* $\lambda \in (1, \lambda^c]$ *with* $\lambda^c = 1 + 2\sqrt{2} \approx 3.828$ *if* $n = 2$ *or* $n > 6$,

2. *for all* $\lambda \in (1, \lambda^{cc}]$ *with* $\lambda^{cc} = 1/4\left(1 + \sqrt{57}\right) \approx 2.137$ *if* $n \in \{3, 4, 5, 6\}$.

The maximal equilibrium markup can be derived from the first–order conditions of firm i's maximization problem using (15) and symmetry—i.e., $\hat{\hat{x}}_i(0) = 1/n$. The maximal equilibrium markup equals

$$m^*(n) = \frac{(\lambda + 1)nt}{(\lambda - 1) + 2n}, \tag{17}$$

which is illustrated in Figure 6—the upper line shows the maximal equilibrium markup. The grey line shows the minimal equilibrium markup $m^{*,-}(n) = \frac{(\lambda+1)nt}{\lambda(n-1)+n+1}$, which is decreasing in n.

Our main result in this section is that the maximal equilibrium markup positively depends on the number of firms in the consideration set, in contrast to the model with standard consumers where it is independent of the number of firms. This holds because the reference price distribution reacts less sensitive to a price change after an increase of the number

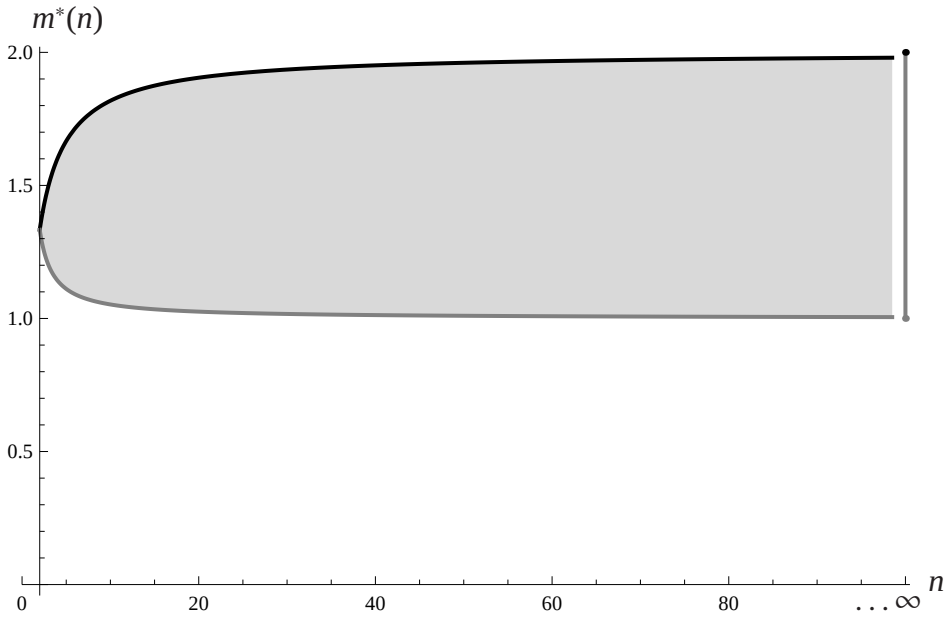


Figure 6: Markups with loss aversion in both dimensions

of firms: in particular, in the duopoly model, consumers expect that they are likely to be affected by a price deviation and thus adjust their reference–point distribution accordingly, while, given a larger number of firms, the reference–point distribution reacts less sensitive to a firm's deviation from the maximal equilibrium strategy. Hence, since consumers' probability of buying the product of a particular firm—and thus their probability of facing the price of that firm—is decreasing in n, loss aversion in the price dimension becomes less pronounced as n increases. This is due to the fact that this probability is a multiplicative term in the consumers' gain–loss utility in the price dimension (e.g. see $Prob[p = p_1]$ in (3)). Using the insights of Section 2.4 and focussing on the maximal equilibrium, it follows that an increase in n has an anti–competitive effect.

We summarize our result as follows:

Proposition 5. *In the Salop model with $L = n$ and informed consumers, the number of firms does not affect competition. By contrast, with uninformed loss-averse consumers the maximal equilibrium price is increasing in the number of firms.*

For $n \to \infty$, the maximal equilibrium price $p^*(\infty) \equiv \lim_{n\to\infty} p^*(n)$ is the upper bound of the equilibrium set that results in the model in which consumers do not observe price before forming their reference–point distribution. Thus, treating n as a continuous variable,

as n turns to infinity, the correspondence of symmetric equilibria converges to the set of symmetric equilibria under the alternative timing (which is invariant in n).

The comparison of markets with a different number of firms and an adjusted circumference of the circle was introduced as an intermediate step.[21] Let us compare the duopoly market to a market with $n > 2$ firms (n-firm oligopoly). The latter captures a situation in which all n firms belong to the consideration set of consumers because, at the ex–ante stage, they do not know their location. We may think of each firm advertising the existence and price of its product to *all* consumers—i.e., firms engage in non–targeted advertising.

By contrast, if each firm can identify whether a consumers is located somewhere between the firm's location and the location of an adjacent firm, they may inform only consumers in their vicinity about existence and price. We may call such a practice *targeted advertising*. Effectively, the consideration set of consumer $x \in [i, i+1]$ is $\{i, i+1\}$ at the ex–ante stage. The n-firm oligopoly model with such targeted advertising is outcome-equivalent to the duopoly model in the paper. In particular, using symmetry, the re-scaled first-order condition of profit maximization in the n-firm model with targeted advertising is the same as the one of the duopoly model. Therefore, equilibrium prices are the same. This implies that our comparative statics results with respect to the number of firms (going from n to 2 firms) can be interpreted as resulting from a switch in advertising technology from non-targeted to targeted advertising in n-firm oligopoly. If firms coordinate on the equilibrium that maximizes industry profits, they would be better off from jointly agreeing not to use targeted advertising since this intensifies competition. However, since impressions on distant consumers are wasted, we conjecture that firms would opt to target ads if they were given the possibility, in order to avoid the associated costs.

We can extend our n–firm setup to one in which the mass of consumers is increasing in the number of products n such that a firm's profit with standard consumers will be constant in n (for instance due to market integration). We receive that, under contextual consumer loss aversion, firms' profits are higher in larger markets than in smaller ones because consumers are less price sensitive in the former markets. To the extent that firms offer

[21] The direct interpretation would be in the spirit of non–address models of imperfect competition of the Dixit–Stiglitz–type, where an additional variant may not affect the pricing of firms.

multiple products that do not directly cannibalize market share from each other, product proliferation in our model can be seen as a means to make price comparisons less relevant.

5 Discussion

5.1 Empirical support and implications

Empirical support for consumer loss aversion. According to the marketing literature on reference prices (for an overview see Mazumdar, Raj, and Sinha (2005)), contextual reference prices are important to explain consumer choices, in particular, as Rajendran and Tellis (1994, p.33) point out, "when brand preference is weak, brand sampling is wide, and shopping is infrequent."[22] While that literature typically proceeds by making some ad hoc assumptions on the contextual reference point, our work can be seen as providing a particular choice–based foundation how posted prices affect reference points. In particular, we show that contextual reference prices are a market–share–weighted sum of product prices (in equilibrium, market shares reflect purchase probabilities which are inversely related to price levels). The fact that we consider loss aversion in the match–value dimension in addition to loss aversion in the price dimension does not qualitatively change our results concerning relative price levels (compare Section 2.4). Loss aversion in the match–value dimension does, however, increase the overall price level.

Recent experimental work from the lab and the field provides evidence that economic outcomes are well explained by expectation–based loss aversion. Such evidence comes from exchange experiments (see Ericson and Fuster (forthcoming)) and lab experiments in which participants are compensated for exerting effort in a boring task (see Abeler, Falk, Goette, and Huffman (2011)). Similarly, there is evidence that expectation–based reference dependence affects golf players' performance (see Pope and Schweitzer (2011)) and cab drivers' labor supply decision (see Crawford and Meng (2011)). Karle, Kirchsteiger, and Peitz

[22] Our model focuses on markets with infrequent purchases. By contrast, for environments with frequent purchase decisions, e.g. Kalyanaram and Winer (1995) highlight the relevance of *temporal* reference prices which depend on past prices. Temporal reference prices and occasional sales are not the focus of our paper. See Zhou (2011) and Heidhues and Kőszegi (2010) for formal investigations on this topic.

(2011) find evidence for expectation–based reference dependence in a controlled consumer choice setting with real consumption. In the lab, they isolate contextual reference dependence and show that loss–averse consumers are more price–sensitive than standard consumers when price difference is sufficiently large relative to taste difference.

Implications and predictions. Our comparative statics results with respect to the fraction of ex–ante informed consumers lend themselves for empirical predictions, in particular with respect to the equilibrium prices. We may relate the fraction of informed consumers to the frequency with which consumers buy in a particular market or region. If purchases are on average frequent this corresponds to a market or region with a large fraction of informed consumers. Our baseline model then predicts that prices are lower in markets or regions with more frequent purchases.

We focus our discussion on firm asymmetries and the size of the consumers' consideration set. We have shown that, for product categories in which brand preferences are weak and shopping is infrequent (i.e., contextual reference pricing matters), price–cost margins are low when producers are sufficiently asymmetric and/or the consumers' consideration set of products is small. This is due to the fact that, in those environments, loss aversion in the price dimension, which has a pro–competitive effect, is less pronounced.

As markets evolve over time, initially, only few consumers of those active at that point in time possess match–value information when forming their reference point. The fraction of informed consumers supposedly increases over time as the market matures. We consider a dynamic interpretation of our comparative statics results useful even though such an interpretation may be criticized on the ground that our model ignores possibly relevant dynamic effects of consumer and firm behavior. Comparative statics results about prices translate into price paths in an evolving market (provided that the products' lifetime is exogenous). In particular, in markets for complex products of a particular generation, we would expect that, as the market matures, more and more consumers become informed about their valuation of product features well ahead of the inspection stage. Thus, a more mature market refers to a market in which a larger fraction of consumers is ex–ante informed about their match value (resp. to a market in which all consumers have initially more precise information about their match value; see below). Our baseline model then

predicts that prices are decreasing over time as opposed to models that highlight the importance of low introductory prices with uninformed consumers as in Nelson (1970).

A particular application is the study of asymmetric duopoly markets for consumer goods. In strongly asymmetric markets the price of the high-margin firm may be increasing over time if more consumers become informed as the market matures. This is likely to be the case if a national brand competes against a private label. Here, with a large fraction of ex–ante uninformed consumers, relative price and market share of the low–cost private label is larger than in the case in which consumers are fully informed ex ante. Setting low prices therefore provides a means for the private label to be prominent in the market. By contrast, our theory predicts that the price of the high–margin firm decreases over time in moderately asymmetric markets. This is likely to be the case in a market in which two national brands compete with each other.

Our theory provides a new perspective on information disclosure and advertising. Since all consumers are fully informed at the purchasing stage, standard theory would predict that it is irrelevant how far in advance of the purchasing stage information is revealed. Our theory predicts that consumer behavior and market outcomes depend on whether and to what extent match–value relevant information is revealed at an early stage. In particular, our model predicts that advertising and other marketing instruments (as, e.g., the offer of test drives or trial products) that allow for voluntary early information disclosure about match value are more likely to be used in markets characterized by large asymmetries between firms because one or both firms gain from information disclosure. Applied to private labels, it may be profitable for private labels to disclose information on product characteristics ex ante. A case in point may be Walmart's announcement concerning food labeling and its modified composition of nutrients and ingredients to make the private–label products more healthy (see e.g. New York Times, January 20, 2011, "Wal–Mart Shifts Strategy to Promote Healthy Foods" by Sheryl Gay Stolberg).

Targeting is becoming common for online retailers. For instance, Amazon recommends particular products within a particular product category based on the inferred characteristics of consumers. The literature has, in particular, looked at targeted advertising as a means to increase profits, see e.g. Esteban, Gil, and Hernandez (2001). It has also pointed out that targeted advertising can sometimes be used to fragment the market, see

e.g. Iyer, Soberman, and Villas-Boas (2005). Our theory suggests a potential downside of targeted advertising from the firms' perspective: as the advertising technology allows for better targeting, the consumers' consideration sets become smaller, which makes firms set prices more aggressively, and, in equilibrium, this leads to lower prices. Thus, as the technological possibilities of targeting, e.g. of search portals, are improved, prices may decrease.

Our symmetric n–firm extension also suggests that loss aversion in the price dimension plays a less important role if the number of firms increases. This is due to the fact that the probability of buying from a certain firm decreases in n. Focussing on the maximal equilibrium, this implies that, in symmetric markets, firms compete less if the number of products increases. Thus, as pointed out above, product proliferation can be seen as a means to make price comparisons less relevant. Allowing for asymmetries across firms we conjecture that an increase in n reduces low–cost firms' incentives to gain extra market share or prominence by setting low prices. Thus, price competition should be more pronounced in asymmetric markets in which consumers make few comparisons.

5.2 Robustness

Consumers with noisy signals. In the previous sections, we have considered consumers which are either fully informed or fully uninformed and have allowed for variations of the fraction of the two consumer groups. We can modify our setup, however, to a situation in which consumer information becomes a continuous variable. We do so by introducing noisy signals about consumers' match value which are fully revealing with a certain probability (precision) and pure noise with the complementary probability. Under full precision, consumers are ex–ante informed, while they are ex–ante uninformed if the precision of the signal is zero. In Appendix E, we show that, in symmetric settings, our results with a mixed population of fully informed and fully uninformed consumers translate to a single group of continuously informed consumers. In particular, increasing the fraction of informed consumers is qualitatively similar to increasing the precision of the noisy signals consumers receive. In asymmetric duopoly, our comparative statics results in β may not find a counterpart in the modified model in which consumers receive noisy

signals because demand is not monotone in the signal precision.

Further issues. In line with most of theory on imperfect competition, but in contrast to Heidhues and Kőszegi (2008), we considered markets in which information on the firms' costs is public (at least among firms). This has allowed us to compare our results to those in the standard Hotelling–Salop model of imperfect competition. Suppose now that each firm privately observes its marginal cost. Under the timing that consumers form reference points before observing prices, Heidhues and Kőszegi (2008) have shown that firms may decide not to condition their prices on their private information. By contrast, under our assumptions on the timing of events, namely that consumers initially observe prices and form their reference point distribution when already knowing the prices of the products, one can show that, in any interior equilibrium of the duopoly model, firms will always condition their price on their private information—i.e., the equilibrium is non–focal. This is seen as follows: For any prices set by the competitors, firm i has a unique best response. This best response is strictly increasing in c_i. Hence, there cannot exist an equilibrium in which each firm ignores its private information. The equilibrium price of firm i as a function of this firm's marginal cost must be strictly increasing.

Another feature of our setting is that consumers obtain information about the two products simultaneously. What happens if the information about products has to be acquired sequentially by uninformed, loss–averse consumers? We can show that our main qualitative insights are confirmed in a duopoly setting in which uninformed consumers have to incur a positive shopping cost to visit the second product. In such a sequential search extension consumers first investigate the offering of the more efficient firm, as price information is always available.

6 Conclusion

This paper aims at a better understanding of the competitive effects of expectation–based consumer loss aversion. For this purpose we embed consumers with expectation–based loss aversion into a Hotelling–Salop model, which is a standard workhorse in the modern industrial–organization literature (see, e.g., Tirole (1988) or Belleflamme and Peitz

(2010)). We distinguish between ex–ante informed consumers who behave identically to consumers with standard preferences and ex–ante uninformed consumers for whom loss aversion affects behavior. In the baseline model, a larger fraction of ex–ante informed consumers increases competition and thus leads to lower prices. Two forces are at play: Loss aversion in the match–value dimension has an anti–competitive effect, whereas loss aversion in the price dimension has a pro–competitive effect. The latter tends to dominate the former with strongly asymmetric efficiency levels. An increase in the share of ex–ante informed consumers is anti–competitive, in contrast to our findings in the baseline model. Furthermore, increasing the size of the consumers' consideration set is anti–competitive when selecting the payoff–dominant equilibrium.

Our model makes a novel conceptual point when embedding Kőszegi and Rabin (2006)'s framework of rational expectation–based loss aversion into a standard imperfect competition model: When price information is immediately accessed by consumers, while information about match values is not—this is most likely the case for complex or less frequently bought products—firms can use price to manage the reference–point distribution of consumers in the match–value and the price dimensions and, thus, affect their preferences at the purchase stage.[23] We have shown in this paper that this possibility admits price variation in equilibrium. This is in contrast to Heidhues and Kőszegi (2008) who predict that firms in equilibrium "insure" loss–averse consumers against price fluctuations by setting identical focal prices across products or sticky prices over time. Their finding is due the fact that loss–averse agents dislike (initially unobservable) variation in monetary outcomes.[24] In our setup, however, all uncertainty about match values and purchase prices at the reference–point–formation stage stems only from product characteristics. In other words, only match–value uncertainty generates consumer loss aversion in our setup.

[23] Also, firms' information–disclosure policy about *product characteristics* can be seen as an expectation management tool. Such information disclosure can be achieved through advertising campaigns and promotional activities, which do not generate additional information at the moment of purchase (at this point, consumers would be informed in any case), but inform consumers before they form their reference point distribution.

[24] This reasoning has been applied to a contracting problem by Herweg, Mueller, and Weinschenk (2010) to show the optimality of binary wage schemes in employment relationships with moral hazard and non–binary states.

A feature of our setting is that there are no dynamics in the formation of reference points. One may wonder whether the results are robust if price information becomes available only at an intermediate stage—i.e., consumers initially have price expectation, which may have to be corrected at an intermediate stage. Consumer utility then also includes a term that depends on the deviation of observed prices from equilibrium price expectations, as in Heidhues and Kőszegi (2008). We conjecture that the qualitative features of our comparative statics results hold true for an appropriate equilibrium selection and a sufficiently small weight (i.e., a sufficiently large discount factor) on gains and losses at the intermediate stage. More generally, one could extend our model to allow for informative signals over time, as in Kőszegi and Rabin (2009). We leave this issue for future research.

Another feature of our setting is that, if a firm releases information on the match value of its product, consumers fully infer the match value of the other product, as well. Future work may want to look at alternative settings in which information is not perfectly correlated across products, giving rise to a richer set of information-disclosure policies.

Appendix

A Relegated Proofs

A.1 Relegated proof of Section 2

Proof of Lemma 1. Using the properties of the reference–point distributions, we rewrite the utility function as

$$\begin{aligned} u_1(x, p_1, p_2) &= (v - tx - p_1) + (1 - \hat{\hat{x}})(p_2 - p_1) \\ &\quad - \lambda \cdot t\Big(\int_0^{1-\hat{\hat{x}}} 2(x - s)\, ds + \int_{1-\hat{\hat{x}}}^{x} (x - s)\, ds \Big) + t\Big(\int_x^{\hat{\hat{x}}} (s - x)\, ds \Big) \\ &= (v - tx - p_1) + (1 - \hat{\hat{x}})(p_2 - p_1) \\ &\quad - \lambda \cdot \frac{t}{2}\Big(x^2 + 2x(1 - \hat{\hat{x}}) - (1 - \hat{\hat{x}})^2 \Big) + \frac{t}{2}(\hat{\hat{x}} - x)^2 \end{aligned} \tag{18}$$

$$\begin{aligned} u_2(x, p_1, p_2) &= (v - t(1 - x) - p_2) - \lambda \cdot \hat{\hat{x}}(p_2 - p_1) - \lambda \cdot t \int_0^{1-x} 2((1 - x) - s)\, ds \\ &\quad + t\Big(\int_{1-x}^{1-\hat{\hat{x}}} 2(s - (1 - x))\, ds + \int_{1-\hat{\hat{x}}}^{\hat{\hat{x}}} (s - (1 - x))\, ds \Big) \\ &= (v - t(1 - x) - p_2) - \lambda \cdot \hat{\hat{x}}(p_2 - p_1) - \lambda \cdot t(1 - x)^2 \\ &\quad + t\Big((x - \hat{\hat{x}})^2 + (\frac{1}{2} - x - \hat{\hat{x}} + 2x\hat{\hat{x}}) \Big). \end{aligned} \tag{19}$$

To determine the location of the indifferent uninformed consumer $x = \hat{\hat{x}}$, we set $u_1 = u_2$, where

$$u_1(\hat{\hat{x}}, p_1, p_2) = (v - t\hat{\hat{x}} - p_1) + (1 - \hat{\hat{x}})(p_2 - p_1) - \lambda \cdot \frac{t}{2}\Big(1 - 2(1 - \hat{\hat{x}})^2 \Big) \tag{20}$$

$$u_2(\hat{\hat{x}}, p_1, p_2) = (v - t(1 - \hat{\hat{x}}) - p_2) - \lambda \cdot \hat{\hat{x}}(p_2 - p_1) - \Big(\lambda \cdot t(1 - \hat{\hat{x}})^2 - 2t(\frac{1}{2} - \hat{\hat{x}})^2 \Big) \tag{21}$$

If she buys product 1, the indifferent uninformed consumer will experience no gain but the

maximum loss in the match–value dimension. If she buys product 2, she will experience a gain and a loss because distance could have been smaller or larger than $1-\hat{\hat{x}}$. With respect to the price dimension the indifferent uninformed consumer (like all other consumers) faces only a loss when paying price p_2 and only a gain when paying price p_1.

$u_1(\hat{\hat{x}}, p_1, p_2) = u_2(\hat{\hat{x}}, p_1, p_2)$ can be transformed to the following quadratic equation in $\hat{\hat{x}}$,

$$0 = \quad 2t(\lambda-1)\cdot\hat{\hat{x}}^2 + \Big((\lambda-1)(p_2-p_1) - 4t\lambda\Big)\cdot\hat{\hat{x}} + \Big(2(p_2-p_1) + \frac{t}{2}(3\lambda+1)\Big). \qquad (22)$$

Solving this quadratic equation in $\hat{\hat{x}}$ leads to the expression given in the lemma. If $\lambda \in (1, \lambda^c]$, the second solution to the quadratic equation can be ruled out as it violates the restriction that it is contained in $[0,1]$ for any feasible price difference $\Delta p \in [0, \Delta\overline{p}]$. □

Proof of Lemma 2. The first derivative of $\hat{x}(\Delta p)$ with respect to Δp is equal to $1/(2t)$ for all $\Delta p \leq t$. Evaluated at $\Delta p = 0$, demand of ex–ante uninformed consumers reacts less price sensitive than demand of ex–ante informed consumers. This can be seen as follows: The derivative of $\hat{\hat{x}}(\Delta p)$ with respect to Δp for $\Delta p \in [0, \Delta\tilde{p}]$,

$$\hat{\hat{x}}'(\Delta p) = -\frac{1}{4t} - \frac{1}{2\cdot S(\Delta p)}\cdot\Big(\frac{\Delta p}{8t^2} - \frac{(\lambda+2)}{2t(\lambda-1)}\Big),$$

is strictly positive. Evaluated at $\Delta p = 0$, we obtain

$$\hat{\hat{x}}'(0) = -\frac{1}{4t} + \frac{(\lambda+2)}{2t(\lambda+1)}.$$

For $\lambda \to 1$, $\hat{\hat{x}}'(0)$ is approaching $1/(2t)$ from below.

Moreover, $\hat{\hat{x}}(\Delta p)$ is strictly convex for all $\Delta p \in [0, \Delta\tilde{p}]$, as illustrated in Figure 1.

$$\hat{\hat{x}}''(\Delta p) = \frac{(3+\lambda)(5+3\lambda)}{64t^2\cdot(S(\Delta p))^3} > 0$$

We note that the degree of convexity of $\hat{\hat{x}}(\Delta p)$ is strictly increasing in λ.

Evaluated at large price differences, the property concerning the price sensitivity, is re-

versed due to convexity of $\hat{\hat{x}}$:[25]

$$\hat{\hat{x}}'(\Delta\tilde{p}) = \frac{\lambda(7\lambda+22)+19}{4t|\lambda(\lambda-6)-11|} - \frac{1}{4t} > \frac{1}{2t} \quad for \quad \lambda \in (1, \lambda^c].$$

Thus, $\hat{\hat{x}}'(\Delta\tilde{p}) > \hat{x}'(\Delta\tilde{p})$.[26] Interior demand of uninformed consumers, evaluated at large price differences, reacts more sensitive to an increase in the price difference than the demand of informed consumers.

Since $\hat{x}'(\Delta p)$ is constant and $\hat{\hat{x}}'(\Delta p)$ continuous and monotone (with the required boundary properties), applying the mean value theorem, there exists a unique intermediate price difference $\Delta\hat{p} \in [0, \Delta\tilde{p}]$ such that $\hat{\hat{x}}'(\Delta\hat{p}) = \hat{x}'(\Delta\hat{p}) = 1/(2t)$. This critical price difference can be explicitly calculated as

$$\Delta\hat{p} = \frac{t\left(2\sqrt{2}\cdot(2(\lambda+2)) - 3\cdot\sqrt{(2(\lambda+2))^2-(\lambda+1)^2}\right)}{\sqrt{2}(\lambda-1)},$$

which is strictly positive for all $\lambda > 1$ since numerator and denominator of $\Delta\hat{p}(\lambda)$ are strictly positive in this range. □

A.2 Relegated proof of Section 3

Proof of Proposition 4.

$$\begin{aligned} \frac{d(\Delta p^*)}{d(\Delta c)} &= -\frac{(\phi')^2}{3(\phi')^2+\phi''(1-2\phi)}\cdot(-1) \\ &= \frac{(\phi')^2}{3(\phi')^2+\phi''(1-2\phi)} \end{aligned} \tag{23}$$

Denote the denominator of $d(\Delta p^*(\Delta c))/d(\Delta c)$ by $D(\Delta p^*;\beta)$. We show that, on the relevant domain of price differences, $D(\Delta p^*;\beta)$ is strictly positive. We have that

$$D(0;\beta) = 3(\phi'(0;\beta))^2 + \phi''(0;\beta)\cdot 0$$

[25] Note that $\Delta\tilde{p} = (\lambda+3)t/(2(\lambda+1)) < t$ for $\lambda > 1$.

[26] For $\lambda \to \lambda^c$, $\hat{\hat{x}}'(\Delta\tilde{p}) \to \infty$.

$$= \quad 3(\phi'(0;\beta))^2 > 0$$

The sign of the derivative is of ambiguous sign:

$$\begin{aligned}\frac{\partial D(\Delta p;\beta)}{\partial \Delta p} &= 6\phi'\phi'' + \phi'''(1-2\phi) - 2\phi''\phi' \\ &= 4\phi'\phi'' + \phi'''(1-2\phi)\end{aligned}$$

Thus $D(\Delta p^*;\beta)$ is not necessarily non-negative. However, since $D(\Delta p^*;\beta)$ is equivalent to the tangent condition (36) which approaches zero at $\Delta p = \Delta p^{ta}(\lambda, t)$ we conclude that

$$\frac{d(\Delta p^*)}{d(\Delta c)} > 0 \tag{24}$$

for $\Delta p < \Delta p^{ta}(\lambda, t)$, which is the relevant domain for equilibrium existence. Moreover, since $\phi''(1-2\phi) = 0$ for $\Delta c = 0$ (i.e., $\Delta p = 0$) and $\phi''(1-2\phi) \leq 0$ for $\Delta c > 0$, it holds true that $d(\Delta p^*(\Delta c))/d(\Delta c) \geq 1/3$. □

B Tables

Table 2: Small Cost Differences:

The table shows the analytical solution of the market equilibria for parameter values of $t = 1$, $\lambda = 3$, $c_1 = 0.25$, $c_2 = 0.5$:

β	$p_1^*(\beta)$	$p_2^*(\beta)$	$\Delta p^*(\beta)$	$q_1(\Delta p^*)$	$\hat{x}(\Delta p^*)$	$\hat{\hat{x}}(\Delta p^*)$	π_1^*	π_2^*	CS^*	CS^*_{in}	CS^*_{un}
1.0	1.33333	1.41667	0.0833333	0.541667	0.541667	0.532453	0.586806	0.420139	1.37674	1.37674	1.16648
0.8	1.37274	1.45643	0.0836887	0.539995	0.541844	0.532597	0.606272	0.439961	1.29508	1.33717	1.12672
0.6	1.41524	1.49932	0.0840806	0.538326	0.54204	0.532755	0.627281	0.461361	1.21022	1.29448	1.08382
0.4	1.46121	1.54572	0.0845149	0.536662	0.542257	0.532931	0.650008	0.484522	1.12178	1.24832	1.03742
0.2	1.51103	1.59603	0.0849986	0.535002	0.542499	0.533127	0.674653	0.509652	1.02934	1.19828	0.987112
0.0	1.56518	1.65072	0.0855405	0.533347	0.54277	0.533347	0.701446	0.536986	0.932421	1.14388	0.932421

Table 3: Intermediate Cost Differences

The table shows the analytical solution of the market equilibria for parameter values of $t = 1$, $\lambda = 3$, $c_1 = 0.25$, $c_2 = 1$:
Prices of both firms are first increasing and then decreasing in β.

β	$p_1^*(\beta)$	$p_2^*(\beta)$	$\Delta p^*(\beta)$	$q_1(\Delta p^*)$	$\hat{x}(\Delta p^*)$	$\hat{\hat{x}}(\Delta p^*)$	π_1^*	π_2^*	CS^*	CS^*_{in}	CS^*_{un}
1.0	1.5	1.75	0.25	0.625	0.625	0.605992	0.78125	0.28125	1.14063	1.14063	0.834921
0.8	1.5039	1.758	0.254109	0.62324	0.627054	0.60798	0.781477	0.285586	1.07357	1.13519	0.827071
0.6	1.50553	1.76414	0.25861	0.621651	0.629305	0.61017	0.780502	0.289112	1.00758	1.13188	0.821115
0.4	1.50448	1.76803	0.263546	0.62026	0.631773	0.612585	0.778104	0.29165	0.942908	1.13111	0.81744
0.2	1.50029	1.76925	0.26896	0.619097	0.63448	0.615251	0.774048	0.293008	0.879835	1.13332	0.816464
0.0	1.49248	1.76737	0.274896	0.618194	0.637448	0.618194	0.768092	0.292988	0.818625	1.13897	0.818625

Table 4: Large Cost Differences:

The table shows the analytical solution of the market equilibria for parameter values of $t = 1$, $\lambda = 3$, $c_1 = 0.25$, $c_2 = 1.25$: Non-existence for $\beta = 0$ (see Figure 8).

β	$p_1^*(\beta)$	$p_2^*(\beta)$	$\Delta p^*(\beta)$	$q_1(\Delta p^*)$	$\hat{x}(\Delta p^*)$	$\hat{\hat{x}}(\Delta p^*)$	π_1^*	π_2^*	CS^*	CS_{in}^*	CS_{un}^*
1.0	1.58333	1.91667	0.333333	0.666667	0.666667	0.648371	0.888889	0.222222	1.02778	1.02778	0.673468
0.8	1.5623	1.90417	0.341863	0.66734	0.670931	0.652973	0.875753	0.217615	0.974147	1.04598	0.686806
0.6	1.5361	1.88738	0.351282	0.668631	0.675641	0.658117	0.859926	0.211208	0.923306	1.06911	0.7046
0.4	1.5043	1.86596	0.361666	0.670654	0.680833	0.663868	0.841199	0.202865	0.87537	1.09757	0.727236
0.2	1.46663	1.83971	0.373075	0.673535	0.686538	0.670284	0.819444	0.192519	0.830299	1.13163	0.754968
0.0	-	-	-	-	-	-	-	-	-	-	-

C Appendix: Equilibrium Existence and Uniqueness

C.1 Symmetric Duopoly

Here, we investigate equilibrium existence. Equilibrium uniqueness in symmetric duopoly follows from Lemma 7 below. For any interior solution, quasi-concavity of the profit functions would assure that the solution to the first-order conditions characterizes an equilibrium. However, profit functions are not quasi-concave. If firm i sets a much lower price than firm j, firm i's profit becomes increasingly convex due to the increasing convexity of its demand with loss-averse consumers.

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -2q_i' + (p_i - c)q_i'', \tag{25}$$

where $q_i'' = \partial^2 q_i(\Delta p)/\partial \Delta p^2$ which is positive for $\Delta p > 0$ but negative for $\Delta p \leq 0$ due to symmetry (since $q_i(-|\Delta p|) = 1 - q_i(|\Delta p|)$). Using that $(p_i - c) = q_i/q_i'$ by FOC_i, the second-order condition of firm i can be expressed as

$$-2(q_i')^2 + q_i q_i'' < 0. \tag{26}$$

For $\beta < 1$, equation (33) is satisfied for Δp sufficiently small, while it is violated for $\Delta p \to \Delta\overline{p}$, as q_i'' goes faster to infinity in Δp than $(q_i')^2$.

The next proposition clarifies the issue of equilibrium existence. It deals with the non-quasi-concavity of firm i's profit function by determining critical levels for the degree of loss aversion such that no firm i has an incentive to non-locally undercut prices. We use that the convexity of firm i's profit function is increasing in Δp which yields that stealing the entire demand of loss-averse consumers is the only possibly optimal deviation of firm i. We focus on the most critical case for equilibrium existence, the case in which all consumers are loss-averse.[27]

[27] Adding more standard consumers always reduces the problem of non-quasi-concavity of firm i's profit function since the demand of standard consumers is linear. Thus, the upper bound on the degree of loss aversion with only loss-averse consumers is sufficient for existence with a positive share of standard consumers. Cf. an older working paper version of this paper (SFB/TR 15 Discussion Paper, 319).

Proposition 6. *Suppose that all consumers are loss averse ($\beta = 0$) and there are two firms in the market. A symmetric equilibrium with prices p_i^* for all $i \in \{1, 2\}$ exists if and only if*

$$1 < \lambda \leq \lambda^c \text{ with } \lambda^c = 1 + 2\sqrt{2} \approx 3.828. \tag{27}$$

Proof of Proposition 6. In this proof we rule out non-local deviations from symmetric price equilibrium in the duopoly case—i.e., when firms only compete in their neighboring sub-markets. Let firm i be the deviating firm. It has been shown that firm i's profit is concave if the price difference Δp is sufficiently small—i.e., Δp is negative or not too positive. Therefore, non-local price increases are never profitable. Since the degree of "convexity" of firm i's profit increases in Δp, firm i's most profitable price deviation is a price reduction stealing the entire demand of loss-averse consumers.[28]

We next derive the critical upper bound of the degree of loss aversion for which stealing the entire demand of loss-averse consumers is not profitable. To steal the entire market, firm i sets a deviation price $p_i^d = p^* - \Delta\tilde{p}$. For $\beta = 0$, the firm i's deviation profit, π_i^d, can be expressed as follows,

$$\pi_i^d = (p_i^d - c) \cdot 1 = (p^* - c) - \Delta\tilde{p}. \tag{28}$$

Firm i's profit in symmetric equilibrium is equal to

$$\pi_i^* = (p_i^* - c) \cdot q_i(0) = \frac{(p^* - c)}{2}. \tag{29}$$

Thus, a deviation from symmetric equilibrium is not profitable if and only if

$$\begin{aligned}
& \pi_i^*(\lambda) \geq \pi_i^d(\lambda) \\
\Leftrightarrow \quad & \Delta\tilde{p}(\lambda) \geq \frac{p^*(\lambda) - c}{2} && \text{by (28) and (29)} \\
\Leftrightarrow \quad & \frac{(\lambda + 3)t}{2(\lambda + 1)} \geq \frac{t}{2 - \frac{(\lambda - 1)}{(\lambda + 1)}} && \text{by (4) and (7)}
\end{aligned}$$

[28] The intuition behind this result is that for sufficiently large price differences loss-averse consumers try to avoid buying the more expensive product. Furthermore, this avoidance is the more attractive the higher the degree of loss aversion. This holds true because the degree of convexity of firm i's demand increases in the degree of loss aversion—i.e., $\partial q_i'' / \partial \lambda > 0$.

$$\Leftrightarrow \qquad (\lambda + 3)^2 \geq 2(\lambda + 1)^2$$

Since $\lambda > 1$, we receive the unique solution $\lambda \leq \lambda^c \equiv 1 + 2\sqrt{2}$. □

C.2 A More Flexible Symmetric Duopoly Model

Proposition 7 (Existence in duopoly with different degrees of loss aversion)**.** *Suppose that there are two firms in the market and all consumers are loss averse ($\beta = 0$) with different degrees of the two dimensions of loss aversion ($\lambda_p, \lambda_m > 1$). A symmetric equilibrium with prices*

$$p_i^* = c + \frac{2(\lambda_m + 1)t}{\lambda_p + 3}, i = 1, 2,$$

exists if and only if $\lambda_m > 1$ and $\lambda_p \in (1, \lambda_p^c(\lambda_m)]$ with

$$\lambda_p^c(\lambda_m) = \frac{\lambda_m + 7}{\lambda_m - 1}. \tag{30}$$

Proof of Proposition 7. Analogously to the proof of Proposition 6, a deviation from the symmetric candidate equilibrium is not profitable if and only if

$$\begin{aligned}
& \pi_i^*(\lambda_p, \lambda_m) \geq \pi_i^d(\lambda_p, \lambda_m) \\
\Leftrightarrow \qquad & \Delta\tilde{p}(\lambda_p, \lambda_m) \geq \frac{p_i^*(\lambda_p, \lambda_m) - c}{2} && \text{by (28) and (29)} \\
\Leftrightarrow \qquad & \frac{(\lambda_m + 3)t}{2(\lambda_p + 1)} \geq \frac{t}{2 - \frac{2(\lambda_m+1)t}{\lambda_p+3}} \\
\Leftrightarrow \qquad & \lambda_p \leq \frac{\lambda_m + 7}{\lambda_m - 1} \equiv \lambda_p^c(\lambda_m).
\end{aligned}$$

□

C.3 Asymmetric Duopoly

C.3.1 Equilibrium Uniqueness in Asymmetric Duopoly

In Lemma 7 we provide sufficient conditions under which an interior equilibrium is unique. Given parameters λ and t, the condition states that the cost asymmetry between firms is not too large.[29]

Lemma 7. *An equilibrium is the unique interior equilibrium if*

$$\Delta c < \Delta c^u(\lambda) \equiv \Delta\bar{p} = \frac{2t}{(\lambda-1)}\left(2(\lambda+2) - \sqrt{(2(\lambda+2))^2 - (\lambda+1)^2}\right). \tag{31}$$

Proof of Lemma 7. We have to derive a number of useful properties of $f(\Delta p;\beta) = (1-2\phi)/\phi'$: First, $f(0;\beta) = 0/\phi'(0) = 0\,\forall\beta$. Second, at $\lambda = \lambda^c$: $\lim_{\Delta p\uparrow\Delta\tilde{p}} f(\Delta p;\beta) = 0$ since $\lim_{\Delta p\uparrow\Delta\tilde{p}} \phi'(\Delta p;\beta) = \infty\,\forall\beta < 1$ as $S(\Delta\tilde{p}) = 0$, and $f(\Delta\tilde{p};1) = -2\Delta\tilde{p} < 0$. Third,

$$f'(\Delta p;\beta) \;=\; \frac{-2(\phi')^2 - \phi''(1-2\phi)}{(\phi')^2} = -\left(2 + \frac{\phi''(1-2\phi)}{(\phi')^2}\right) \lesseqgtr 0 \quad \forall\beta < 1,$$

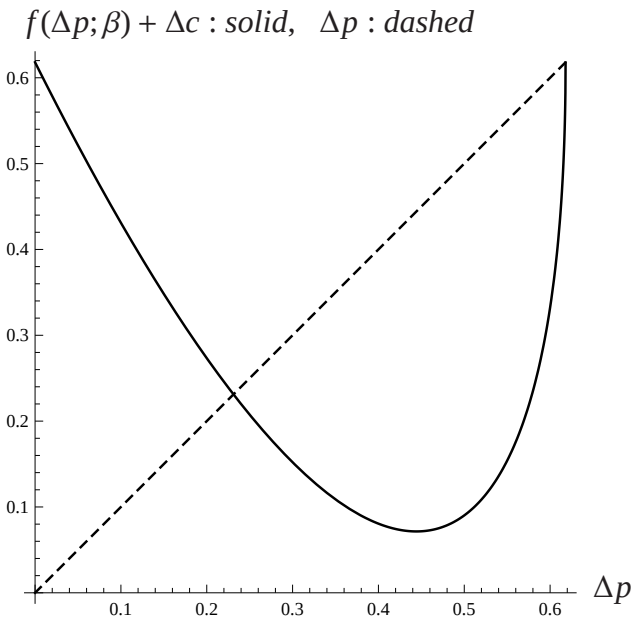
since $f'(0;\beta) = -2 < 0 \quad \forall\beta$ and $f'(\Delta\tilde{p};\beta) > 0 \quad \forall\beta < 1$. Moreover, $f'(\Delta p;1) = -2 \quad \forall\Delta p$.

It has to be shown that $f(\Delta p;\beta)$ is strictly convex in $\Delta p \in [0,\Delta\tilde{p}]$ for $\beta < 1$. We find that

$$f''(\Delta p;\beta) \;=\; -\frac{(\phi'\phi''' - 2(\phi'')^2)(1-2\phi) - 2(\phi')^2}{(\phi')^3} > 0.$$

We first consider the case of $\lambda = \lambda^c \approx 7.47$, for which $\Delta\tilde{p} = \Delta\bar{p}$. Figure 7 illustrates the equilibrium condition (14) at $\Delta c = \Delta\bar{p}$. Now, if $\beta < 1$ by continuity of $f(\Delta p)$ for $\Delta p \in [0;\Delta\bar{p}]$, $f(0;\beta) = 0$, $\lim_{\Delta p\uparrow\Delta\bar{p}} f(\Delta p;\beta) \to 0$, $f'(0;\beta) < 0$, $\lim_{\Delta p\uparrow\Delta\bar{p}} f'(\Delta p;\beta) = \infty > 1$, and strict convexity of $f(\Delta p)$ for $\beta < 1$, we know that, for $\Delta c \geq \Delta\bar{p}$, there are two candidate interior equilibria since the $(f(\Delta p) + \Delta c)$-curve shifts up and intersects the Δp-line twice. For values of Δc lower than $\Delta\bar{p}$, $(f(\Delta\bar{p};\beta < 1) + \Delta c)$ is always smaller than $\Delta\bar{p}$ and no other equilibrium can exist.

[29] Since t turns out to simply scale equilibrium markups, $m_i^* = p_i^* - c_i$, we set $t = 1$ here.



Equilibrium condition (14) at $\Delta c = \Delta\bar{p}$ for parameter values of $\beta = 0$, $t = 1$, and $\lambda = \lambda^c$: $\Delta\tilde{p} = \Delta\bar{p} = 0.6180$.

Figure 7: Two potential interior equilibria

If $\beta = 1$, $f(\Delta p;\beta)$ is strictly decreasing for all Δp and at most one intersection between $f(\Delta p;1)+\Delta c$ and Δp exists (standard Hotelling case).[30]

Secondly, in the case of $1 < \lambda < \lambda^c$ all uninformed consumers buy from firm 1 at $\Delta p = \Delta\tilde{p}$, which is smaller than $\Delta\bar{p}$. Since f is continuous, $f(\Delta\tilde{p};\beta) < 0$, and $f(\Delta p;\beta) = (1-2(\beta\hat{x}(\Delta p)+(1-\beta)))\cdot 2t/\beta$ is strictly decreasing for $\Delta p > \Delta\tilde{p}$, $\Delta c < \Delta\bar{p}$ is sufficient to rule out other equilibria in this case. □

In the lemma $\Delta\bar{p}$ depicts the upper bound of Δp such that $S(\Delta p)$ in $\hat{\hat{x}}(\Delta p)$ is equal to zero (Cf. equation (6)). For $\lambda = \lambda^c$, $\Delta\tilde{p} = \Delta\bar{p}$. It is easy to check that $\Delta c^u(\lambda)$ is strictly decreasing in λ.

We also can provide conditions that non-interior equilibria do not exist. For the sake of brevity, here we restrict attention to the case $\beta = 0$.

Lemma 8. *Suppose that all consumers are uninformed ($\beta = 0$) and the degree of loss*

[30] In this case an analytical solution for (14) can be determined: $\Delta p^* = \Delta c/3$.

aversion, $\lambda \in (1, 1 + 2\sqrt{2}]$. *Non-interior equilibria do not exist if*

$$\Delta c \leq \Delta c^{ni}(\lambda) \equiv 2\Delta\tilde{p} = \frac{(\lambda + 3)t}{\lambda + 1}. \tag{32}$$

Proof of Lemma 8. The candidate non-interior equilibrium is $p_1^{**} = c_2 - \Delta\tilde{p}$, $p_2^{**} = c_2$. The associated profits are $\pi_1^{**} = (c_2 - \Delta\tilde{p} - c_1) \cdot 1 = \Delta c - \Delta\tilde{p}$ and $\pi_2^{**} = 0$. Note that for $\Delta p^{**} = \Delta\tilde{p}$ setting $p_2 = c_2$ is the local best response of firm 2.

We consider a non-local deviation by firm 1 to $p_1 = c_2$. The associated profit is $(c_2 - c_1)\phi(0)$ at $p_1 = c_2$. Hence, a sufficient condition for the non-existence of non-interior equilibria is

$$\Delta c - \Delta\tilde{p} \leq (c_2 - c_1)\phi(0) = \frac{\Delta c}{2}.$$

This is equivalent to

$$\Delta c \leq 2\Delta\tilde{p}.$$

For $\lambda \in (1, 1 + 2 \cdot \sqrt{2}]$, $\Delta\tilde{p}$ is equal to $\Delta\tilde{p}(\lambda) = (\lambda + 3)t/(2(\lambda + 1))$, which completes the proof. □

Combining Lemmata 7 and 8 we obtain the following proposition:

Proposition 8. *For* $\Delta c < \min\{\Delta c^u, \Delta c^{ni}\}$, *any equilibrium is unique and interior.*

C.3.2 Equilibrium Existence in Asymmetric Duopoly

For any interior solution, concavity of the profit functions would assure that the solution characterizes an equilibrium.

$$\frac{\partial^2\pi_1}{\partial p_1^2} = -2\phi' + (p_1 - c_1)\phi'' < 0 \qquad (SOC_1)$$

$$\frac{\partial^2\pi_2}{\partial p_2^2} = -2\phi' - (p_2 - c_2)\phi'' < 0. \qquad (SOC_2)$$

Given the properties of ϕ (in particular, that ϕ is strictly increasing and convex in Δp for $\beta < 1$) SOC_2 holds globally, whereas SOC_1 is not necessarily satisfied. Using that $(p_1 - c_1) = \phi/\phi'$ by FOC_1, SOC_1 can be expressed as

$$-2(\phi')^2 + \phi\phi'' < 0. \tag{33}$$

It can be shown that (33) is satisfied for small Δp, while it is violated for $\Delta p \to \Delta\tilde{p}$ and $\lambda \to \lambda^c$, as ϕ'' goes faster to infinity in Δp than $(\phi')^2$.[31] This violation of SOC_1 reflects that firm 1 may have an incentive to non-locally undercut prices to gain the entire demand of uninformed consumers when Δp is large. The driving force behind this is that loss aversion in the price dimension increasingly dominates loss aversion in the match–value dimension if price differences become large. Moreover, excessive losses in the price dimension if buying the expensive product 2 make also nearby consumers of 2 more willing to opt for product 1.

The next proposition clarifies the issue of equilibrium existence. It deals with the non-quasiconcavity of firm 1's profit function by determining critical levels of market asymmetries and the degree of loss aversion such that firm 1 has no incentive to non-locally undercut prices. Here, we make use of the increasing convexity of firm 1's profit function in $-p_1$ which yields that stealing the entire demand of uninformed consumers is the unique optimal deviation of firm 1. For notational convenience, we focus on the most demanding setting for equilibrium existence. This is the one in which all consumers are uninformed.[32]

Proposition 9. *Suppose that all consumers are uninformed ($\beta = 0$) and the degree of loss aversion, λ, lies within the interval* $(1, 1 + 2\sqrt{2}]$. *An interior equilibrium with prices* (p_1^*, p_2^*) *exists if and only if*

$$\Delta c \leq \Delta c^{nd}(\lambda) \equiv \Delta p^{nd}(\lambda) - f(\Delta p^{nd}(\lambda); 0), \tag{34}$$

[31] This implies that π_1 is not globally concave. It is easy to check that it is neither globally quasi-concave. This is illustrated in Figure 8 in Appendix C. Moreover, the non-concavity of π_1 is increasing in Δp (resp. $-p_1$) for $\Delta p \leq \Delta\tilde{p}$ (resp. $p_1 \geq p_2 - \Delta\tilde{p}$).

[32] Adding more informed consumers always makes the non-quasiconcavity problem less severe as the demand of informed consumers is linear. Thus, the derived upper bound on cost asymmetries with only uninformed consumers is sufficient for existence with a positive share of informed consumers.

with $\Delta p^{nd}(\lambda)$ being implicitly defined as the solution $\Delta p^{nd}(\lambda) \neq \Delta\tilde{p}$ of the equation

$$\Delta p = \Delta\tilde{p} - \frac{\phi(\Delta p;\beta)\cdot(1-\phi(\Delta p;\beta))}{\phi'(\Delta p;\beta)}. \tag{35}$$

Before presenting its proof, let us comment on this proposition. The result shows that an equilibrium exists if firm 1 has no incentive to non-locally undercut prices. In fact, the incentive to undercut prices increases in more asymmetric industries or for more loss–averse consumers. For a low degree of loss aversion ($1 < \lambda < 1 + 2\sqrt{2} \approx 3.828$), an equilibrium exists if the cost difference between firms is not too large (see (34)).[33] In this case, an equilibrium exists for all values of β. However, if the degree of loss aversion rises further, equilibria only exist if there is a sufficiently large share of informed consumers which reduces the undercutting incentive of firm 1. This tradeoff is illustrated in Table 5 below.

Table 5: Non-deviation condition

Variation of Δp^{nd} and Δc^{nd} in β and λ. ($t = 1$)

	$\lambda = 3$		$\lambda = 6$	
β	$\Delta p^{nd}(\lambda,\beta)$	$\Delta c^{nd}(\lambda,\beta)$	$\Delta p^{nd}(\lambda,\beta)$	$\Delta c^{nd}(\lambda,\beta)$
0.8	0.648337	1.75869	0.372669	1.07069
0.6	0.543254	1.45317	0.23824	0.686206
0.4	0.459237	1.22329	0.107415	0.314749
0.2	0.377489	1.00993	-	-
0.0	0.278889	0.75963	-	-

In the proof of Proposition 9 we first provide the critical level of Δc for which the equilibrium condition in (14) is satisfied for candidate interior equilibria. We next identify the set of candidate interior equilibria which are robust to local and non-local price deviations of firm 1.

Proof of Proposition 9. 1. To find an upper bound on Δc for which the equilibrium

[33] For instance, experimental work by Tversky and Kahneman (1992) suggests that λ takes the value 2.25, which is within this range.

condition (14) is satisfied, we determine the point at which $f(\Delta p;\beta)$ is a tangent on the Δp-line. In Figure 7 this corresponds to an upward shift of the $f(\Delta p;\beta)$-curve. The tangent condition is

$$f'(\Delta p;\beta) = 1 \qquad \Leftrightarrow \qquad 3(\phi')^2 + \phi''(1-2\phi) = 0 \tag{36}$$

It is sufficient to consider $\beta = 0$ as the most problematic case with respect to existence. The reason is that for $\beta > 0$ there is a positive weight on the demand of informed consumers which is linear. Denote the critical price difference that satisfies (36) at $\beta = 0$ as $\Delta p^{ta}(\lambda)$. We note that it is decreasing in λ.

Then, the equilibrium condition in (14) is fulfilled if and only if Δc satisfies the following condition

$$\Delta c \leq \Delta c^{ta}(\lambda) \equiv \Delta p^{ta}(\lambda) - f(\Delta p^{ta}(\lambda); 0). \tag{37}$$

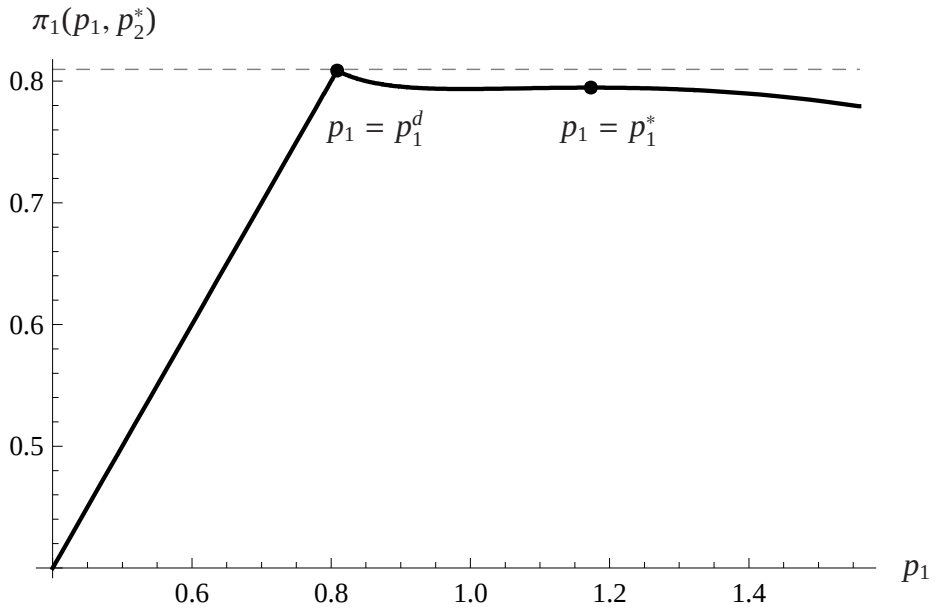
$\Delta c^{ta}(\lambda)$ is uniquely determined by $\Delta p^{ta}(\lambda)$, using equilibrium condition (14) because at the tangent point there is a one-to-one relationship between the two variables.[34]

2. At this step, we show that a solution to the first-order condition is a local maximizer. Suppose, by contrast, that, at $\Delta p = \Delta p'$, SOC_1 is not satisfied. Then, at $\Delta p'$, firm 1's profit takes a minimum and $\Delta p'$ cannot be an equilibrium. Now, define $\Delta p^s(\lambda)$ as the critical price difference which satisfies the second-order condition of firm 1 (33) with equality. There is a unique $\Delta p^s(\lambda)$ for any given λ because the convexity of π_1 is strictly decreasing in p_1. Thus, SOC_1 holds for $\Delta p \leq \Delta p^s(\lambda)$. We next show that SOC_1 implies the tangent condition—i.e., $\Delta p^s(\lambda) < \Delta p^{ta}(\lambda)$. Rearranging (33) and (36) leads to

$$\frac{\phi}{2} \leq \frac{(\phi')^2}{\phi''}, \tag{33'}$$

$$\frac{(2\phi-1)}{3} \leq \frac{(\phi')^2}{\phi''}. \tag{36'}$$

[34] For $\Delta c^u(\lambda) \leq \Delta c < \Delta c^{ta}(\lambda)$ there might arise two candidate interior equilibria. However as we see next, the second one does not survive the local SOC_1 criterion.



Profit of firm 1, $\pi_1(p_1, p_2^*)$, as a function of its own price p_1 given $p_2 = p_2^*$ for $\Delta c = 1$ ($c_1 = 0, c_2 = 1$) and parameter values of $\beta = 0$, $t = 1$, and $\lambda = 3$: $p_1^* = 1.17309$, $p_1^d = p_2^* - \Delta\tilde{p} = 0.80863$, $p_2^* = 1.55863$, $\Delta p^* = 0.385537$, and $\Delta\tilde{p} = 3/4$.

Figure 8: Non-existence

Hence, $\Delta p^s(\lambda) < \Delta p^{ta}(\lambda)$ holds if and only if $\phi/2 > (2\phi - 1)/3$. This inequality is satisfied for all $\phi \in [1/2, 1]$.

3. Due to the increasing convexity of π_1 in $-p_1$ a candidate interior equilibria which locally satisfy SOC_1 might be ruled out as an equilibrium because a non-local deviation may be profitable. This is the case when the convexity is sufficiently large: A non-local price decrease becomes a profitable deviation for firm 1—an example of this kind is presented in Figure 8. Given the increasing convexity of π_1, the unique optimal deviation of firm 1 (if it exists) is characterized by firm 1 serving the entire market of uninformed consumers—i.e., p_1^d such that $\Delta p^d = \Delta\tilde{p}$. Decreasing p_1^d further is not profitable since firm 1 does not attract more consumers, while its profit margin goes down for all consumers. Hence, in the following we will restrict our attention to price deviations by firm 1 that steal the entire demand of uninformed consumers. If deviating is profitable, firm 1 sets $p_1^d = p_2^* - \Delta\tilde{p}$. For $\beta = 0$, firm 1's deviation profit π_1^d is equal to $(p_1^d - c_1)\cdot 1$ since $\phi(\Delta\tilde{p}; 0) = 1$. Using that $p_1^d = p_2^* - \Delta\tilde{p}$

we receive

$$\begin{aligned}\pi_1^d &= \left(p_2^* - \Delta\tilde{p} - c_1\right)\cdot 1 \\ &= \left(\frac{1-\phi}{\phi'} + \Delta c - \Delta\tilde{p}\right)\cdot 1 && \text{by } FOC_2 \\ &= \left(\Delta p^* + \frac{\phi}{\phi'} - \Delta\tilde{p}\right)\cdot 1 && \text{by (14)} \end{aligned} \tag{38}$$

For the candidate interior equilibrium, firm 1's profit is equal to $\pi_1(\Delta p^*) = (p_1^* - c_1)\phi$, which in turn is equal to ϕ^2/ϕ' by FOC_1.

Thus, a deviation of firm 1 is not profitable if and only if $\pi_1(\Delta p^*) \geq \pi_1^d$. Rearranging yields

$$\Delta p^* \leq \Delta\tilde{p} - \frac{\phi\cdot(1-\phi)}{\phi'}. \tag{39}$$

This is the required non-deviation condition. We define $\Delta p^{nd}(\lambda)$ as the non-trivial solution different from $\Delta\tilde{p}$ to (39) holding with equality. We have

$$\Delta p^{nd}(\lambda) = \Delta\tilde{p} - \frac{\phi(\Delta p^{nd}(\lambda);0)\cdot\left(1-\phi(\Delta p^{nd}(\lambda);0)\right)}{\phi'(\Delta p^{nd}(\lambda);0)}.$$

Lemma 9 below shows that $\Delta p^{nd}(\lambda)$ is uniquely determined by this non-deviation condition if the trivial solution, $\Delta\tilde{p}$, is excluded. Furthermore, the set of non-negative $\Delta p^{nd}(\lambda)$ is non-empty for $\lambda \in (1, 1+2\sqrt{2}]$.

Again by using the equilibrium condition (14), an interior equilibria exists if and only if $\Delta c \leq \Delta c^{nd}(\lambda) \equiv \Delta p^{nd}(\lambda) - f(\Delta p^{nd}(\lambda))$.[35]

4. Taken together, due to increasing convexity of π_1, the non-deviation condition implies local concavity of the firms' profit function and therefore, as shown above, the tangent condition. Thus any price difference resulting from a candidate interior

[35] Since Δc^{nd} is a function of β, while Δc^u is not, we have to be more careful to have a uniqueness statement for $\beta > 0$. Fix some $\beta > 0$. For $\Delta c^u(\lambda) \leq \Delta c < \Delta c^{nd}(\lambda)$ (where, in an abuse of notation, the latter critical value is adjusted for β), the equilibrium condition (14) may not make a unique selection—i.e., there might arise a second solution to (14), Δp^{**}. This solution can be ruled out, however, because, by construction, Δp^{**} is larger than $\Delta p^{ta}(\lambda)$ and, hence, larger than $\Delta p^{nd}(\lambda)$.

equilibrium which satisfies the non-deviation condition can be supported in equilibrium, as $\Delta p^{nd}(\lambda) < \Delta p^{s}(\lambda) < \Delta p^{ta}(\lambda) < \Delta\tilde{p}(\lambda)$. This provides a bound on the admissible cost asymmetry that is given in the proposition.

□

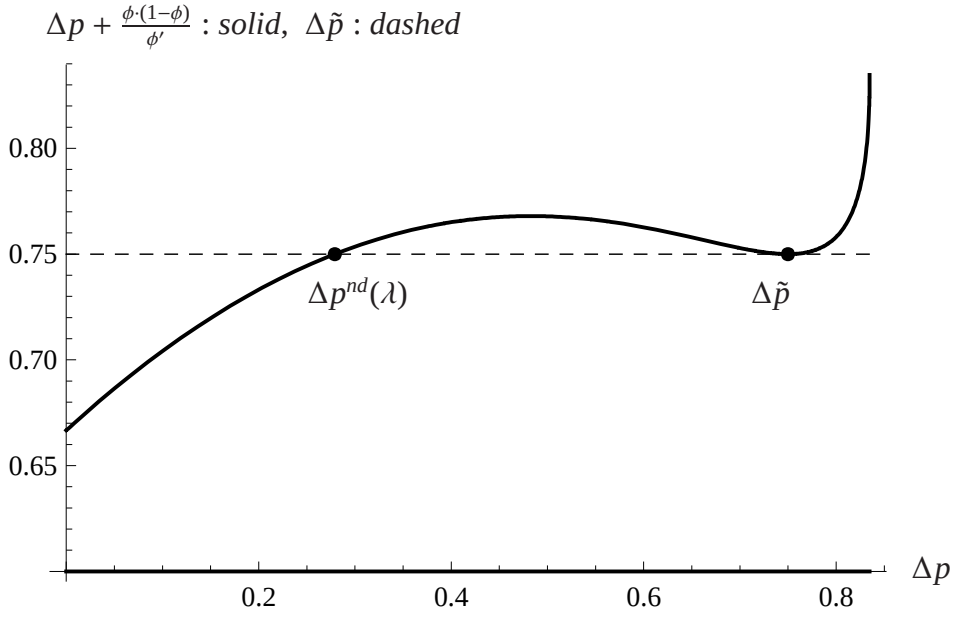
Lemma 9. *For $\beta = 0$ and $\lambda \in (1, 1+2\sqrt{2}]$, $\Delta p^{nd}(\lambda)$ is the unique non-trivial solution (i.e., $\Delta p^{nd}(\lambda) \neq \Delta\tilde{p}$) to equation* (35)*. Moreover, $\Delta p^{nd}(\lambda)$ is non-negative.*

Proof of Lemma 9. Note that the non-deviation condition is trivially satisfied at $\Delta p = \Delta\tilde{p}$ since $\phi(\Delta\tilde{p};\beta) = 1$ for $\beta = 0$ (see Figure 9 below for a graphical illustration of the non-deviation condition). It can be shown that $A(\Delta p) \equiv \Delta p + \phi(1-\phi)/\phi'$ approaches $\Delta\tilde{p}$ from above for $\Delta p < \Delta\tilde{p}$. For $\Delta p \geq 0$ but Δp being small, $A(\Delta p)$ is strictly increasing and strictly concave. Moreover, $A(\Delta p)$ is continuous and exhibits at most one maximum for $\Delta p \in [0, \Delta\tilde{p})$. Taken together, there exists a unique $\Delta p \in [0, \Delta\tilde{p})$ at which the non-deviation condition is satisfied if and only if, at $\Delta p = 0$, $A(\Delta p)$ is smaller or equal than $\Delta\tilde{p}$. For $\beta = 0$, $A(0) = (\lambda+3)/(4t(\lambda+1))$ and $\Delta\tilde{p} = (\lambda+3)t/(2(\lambda+1))$. It is easy to check that $A(0) \leq \Delta\tilde{p}$ if and only if $\lambda \in (1, 1+2\sqrt{2}]$. Denoting the solution to the non-deviation condition by $\Delta p^{nd}(\lambda)$ completes the proof.[36] □

If the degree of loss aversion becomes sufficiently high ($\lambda > 1+2\sqrt{2} \approx 3.828$), the set of non-negative $\Delta p^{nd}(\lambda)$ becomes empty. Here, deviating is profitable even in symmetric industries ($\Delta c = 0$). However, restricting the share of uninformed consumers can establish existence of symmetric equilibria also in this case.

For illustration, we provide a numerical example on equilibrium existence and uniqueness. For $\lambda = 3$, $t = 1$ and $\beta = 0$, the following price differences arise: $\Delta p^{nd}(3) = 0.27889$, $\Delta p^{u}(3) = 0.31072$, $\Delta p^{s}(3) = 0.48259$, $\Delta p^{ta}(3) = 0.69532$, $\Delta\tilde{p} = 0.75$, and $\Delta\bar{p} = 0.83485$. Moreover, $\Delta c^{nd}(3)$ is equal to $(\Delta p^{nd}(3) - f(\Delta p^{nd}(3);0)) = 0.75963$—i.e., an equilibrium exists for $\Delta c < 0.75963$. The other critical values are $\Delta c^{u}(3) = 0.83485$, $\Delta c^{ta}(3) = 1.40396$, and $\Delta c^{ni}(3) = 1.5$. Since $\Delta c^{nd}(3) < \Delta c^{u}(3)$, the equilibrium is the unique interior equilibrium. Since $\Delta c^{nd}(3) < \Delta c^{ni}(3)$, there does not exist a non-interior

[36] We receive $\Delta p^{nd}(1+2\sqrt{2}) = 0$ and, for $\lambda \to 1$, $\Delta p^{nd}(\lambda) \to \Delta\tilde{p}$.



Non-deviation condition of firm 1, as a function of the price difference Δp for $\Delta c = 0.25$ ($c_1 = 0.25, c_2 = 0.5$) and parameter values of $\beta = 0$, $t = 1$, and $\lambda = 3$: $\Delta p^{nd}(3) = 0.27889$, $\Delta c^{nd}(3) = (\Delta p^{nd}(3) - f(\Delta p^{nd}(3); 0)) = 0.75963$, and $\Delta\tilde{p} = 3/4$. Non-deviation for $\Delta p \leq \Delta p^{nd}(3) = 0.27889$.

Figure 9: Non-deviation in asymmetric industries

equilibrium. For equilibrium values at $\Delta c = 0.25$ and 0.75, see Tables 2 and 3 in Appendix B. An example for non-existence at $\beta = 0$ is provided in Figure 8 with $\Delta c = 1$.

C.4 Symmetric *n*-Firm Oligopoly

In *n*-firm oligopoly there might arise profitable non-local deviations by stealing consumers in distant sub-markets. We next establish existence of symmetric maximal equilibria in this setup.[37] Although conditions for existence carry over from the duopoly case for *n* sufficiently large, there might arise additional existence problems in markets with a small number of firms when consumers are loss averse up to the level that constitutes the upper bound of the duopoly case ($\lambda^c \approx 3.828$). As mentioned before, in contrast to

[37] Note that symmetric maximal equilibria are the symmetric equilibria, for which deviations by a price *decrease* are most profitable. Moreover, deviations from symmetric equilibrium candidates by a price *increase* are never optimal due to demand concavity in this price range. Thus, maximal equilibrium candidates are the most critical for symmetric equilibrium existence.

Heidhues and Kőszegi (2008), in our setup consumers observe prices ex ante and adjust their reference–point distributions to price deviations.

We restrict the analysis to the most demanding case: All consumers are loss-averse ($\beta = 0$). Divide the circle of length $L = n$ into $2n$ sub-markets of length $1/2$. Thus, there are n sub-markets on each half of a circle and 2 between each pair of neighboring firms. In a symmetric maximal equilibrium, a firm located at y_i serves all consumers on the left and the right neighboring sub-market—i.e., all consumers x within $[y_i - 1/2; y_i + 1/2]$.[38]

Due to symmetry, it suffices to consider deviations on one half of the circle only and to locate the deviating firm at $y_1 = 0$. This firm (=firm 1) is supposed to deviate from the symmetric maximal equilibrium by lowering its price. If it attracts consumers up to the mth sub-market (on the first half of the circle), firm 1's (right) indifferent consumer is located at $\hat{\hat{x}}_1^+ \in [\frac{(m-1)}{2}, \frac{m}{2}]$ with $2 \leq m \leq n$. Its total demand equals $2\hat{\hat{x}}_1^+/n$ due to the uniform distribution of x. Loss-averse consumers who expect $\hat{\hat{x}}_1^+$ to be located in the mth sub-market for given prices, form the following reference–point distribution with respect to the match-value dimension,

- for even m:

$$G_m(s|n) = \begin{cases} \frac{2}{n}(n-(m-2))s, & s \in [0, 1-(\hat{\hat{x}}_1^+ - \frac{m-2}{2})]; \\ \frac{2}{n}(n-(m-1))s + a(\hat{\hat{x}}_1^+, m, n), & s \in (1-(\hat{\hat{x}}_1^+ - \frac{m-2}{2}), \frac{1}{2}]; \\ \frac{2}{n}s + b(\hat{\hat{x}}_1^+, m, n), & s \in (\frac{1}{2}, \hat{\hat{x}}_1^+]. \end{cases}$$

  with $a(\hat{\hat{x}}_1^+, m, n) = (m-1)/n - 2\hat{\hat{x}}_1^+/n$ and $b(\hat{\hat{x}}_1^+, m, n) = 1 - 2\hat{\hat{x}}_1^+/n$ being the required constants for the kinked cdf.

- for odd m:

$$G_m(s|n) = \begin{cases} \frac{2}{n}(n-(m-1))s, & s \in [0, \hat{\hat{x}}_1^+ - \frac{m-1}{2}]; \\ \frac{2}{n}(n-(m-2))s + \tilde{a}(\hat{\hat{x}}_1^+, m, n), & s \in (\hat{\hat{x}}_1^+ - \frac{m-1}{2}, \frac{1}{2}]; \\ \frac{2}{n}s + \tilde{b}(\hat{\hat{x}}_1^+, m, n), & s \in (\frac{1}{2}, \hat{\hat{x}}_1^+]. \end{cases}$$

[38] Since the set of consumers is restricted to mass one and x is uniformly distributed on $[0; n]$, the demand of firm i on $[y_i - 1/2; y_i + 1/2]$ is equal to $1/n$.

It can be easily seen that both distributions coincide for $\hat{\hat{x}}_1^+$ reaching the boundaries between two neighboring sub-markets: e.g., for $\hat{\hat{x}}_1^+ = 1$ $G_2(s|n) = G_3(s|n)$ and for $\hat{\hat{x}}_1^+ = 3/2$ $G_3(s|n) = G_4(s|n)$ and so on. For $n = m = 2$, we are back in the duopoly case.

To see how the reference–point distributions can be derived, consider the case of $m = 3$ and $n \geq 3$: $\hat{\hat{x}}_1^+ \in [1; 3/2]$ means that the deviating firm 1 steals all consumers up to the location of its right neighbor (firm 2 located at $y_2 = 1$) and some even in the neighbor's backyard market. Therefore, an equilibrium taste difference s within $[0; \hat{\hat{x}}_1^+ - 1] \subseteq [0; 1/2]$ can be expected by consumers on each of the n sub-markets on the first half of the circle, except for the two sub-markets neighboring firm 2 ($m = 2, 3$). This holds true since consumers who turn out to be located in these two sub-markets, will be attracted by the deviating firm 1 which is located further apart, while consumers on all other sub-markets will buy from the firm closest by. The resulting probability of facing a taste difference in this interval equals $(2/n)(n-2)s$. An equilibrium taste difference $s \in (\hat{\hat{x}}_1^+ - 1; 1/2]$ can be expected on $n-1$ sub-markets (on the first half of the circle) since also consumers on sub-market $m = 3$ with $x \in (\hat{\hat{x}}_1^+; 3/2]$ will be buying from their closest firm, which is firm 2 located at $y(\hat{\hat{x}}_1^+)^2 = 1$. Thus, $G_3(s|n)$ is equal to $2/n(n-1)s$ plus a constant in this interval. Facing an equilibrium taste difference $s \in (1/2; \hat{\hat{x}}_1^+ - 1] = (1/2; 1] \cup (1; \hat{\hat{x}}_1^+ - 1]$, there is each time one particular sub-market consumers expect to be located in: $m = 2$ for $s \in (1/2; 1]$ and $m = 3$ for $s \in (1; \hat{\hat{x}}_1^+ - 1]$. Hence, the probability of $s \in (1/2; \hat{\hat{x}}_1^+ - 1]$ is equal to $2/n \cdot s$ plus a constant.

From the functional form of $G_m(s|n)$ it follows directly that, for given n, a distribution with a higher m first-order stochastically dominates the ones with lower m. This is because consumers expect to be attracted by the deviating firm with a higher probability when it steals a large market share. Therefore, buying from the closest firm becomes less likely: Consumers put less weight on taste differences less than $1/2$ and positive weight on taste differences greater than $1/2$.[39] An increase in the number of firms has exactly the opposite effect to an increase in the number of stolen sub-markets by the deviating firm: For a given m, the reference–point distribution puts more mass on small taste differences if the

[39]For this updating behavior the observability of prices is crucial. In contrast to this, consumers in Heidhues and Kőszegi (2008) cannot adjust their reference point to price deviations because prices become observable only after forming their reference point.

number of firms n increases. Here, the chance of being affected by a price cut of a single firm simply washes out if the total number of firms increases without bound.

The probability of buying from the deviating firm 1 (=probability of facing purchase price p_1) is $\hat{\hat{x}}_1^+$ in the duopoly and generalizes to $2\hat{\hat{x}}_1^+/n$ in the n-firm case. The intuition for this mirrors the one just given above: If the number of firms rises, consumers are less likely to be affected by a price cut of a single firm. Using the generalized reference–point distribution in both dimensions, we can derive a generalized demand function for symmetric markets with n firms. Consider, for instance, the indirect utility functions of a consumer x who has learned to be located in sub-market m (with m even) which is the sub-market consumers ex ante expected the indifferent loss-averse consumer to be located in,[40] given prices ($p_1 < p^*$). Moreover, suppose this consumer is the indifferent loss-averse consumer on this side of the circle, $x = \hat{\hat{x}}_1^+ \in [(m-1)/2; m/2]$. Then, her indirect utility if buying from the deviating firm 1 can be expressed as follows,[41]

$$\begin{aligned} u_1(\hat{\hat{x}}_1^+, p_1, p^*, ..., p^*) \quad &= v - t\hat{\hat{x}}_1^+ - p_1 + \left(1 - \frac{2\hat{\hat{x}}_1^+}{n}\right)(p^* - p_1) \\ &\quad - \lambda t\Bigg(\int_0^{1-(\hat{\hat{x}}_1^+ - \frac{(m-2)}{2})} (\hat{\hat{x}}_1^+ - s)\frac{2}{n}(n-(m-2))ds \\ &\quad + \int_{1-(\hat{\hat{x}}_1^+ - \frac{(m-2)}{2})}^{1/2} (\hat{\hat{x}}_1^+ - s)\frac{2}{n}(n-(m-1))ds + \int_{1/2}^{\hat{\hat{x}}_1^+} (\hat{\hat{x}}_1^+ - s)\frac{2}{n}ds\Bigg) \\ &= v - t\hat{\hat{x}}_1^+ - p_1 + \left(1 - \frac{2\hat{\hat{x}}_1^+}{n}\right) - \frac{\lambda t}{4n}\left(-8(\hat{\hat{x}}_1^+)^2 + 4(m+n)\hat{\hat{x}}_1^+ - ((m-1)m+n)\right). \end{aligned}$$

It can be seen that the indifferent loss-averse consumer faces only a gain in the price dimension (last term in the first line) when purchasing the product of the deviating firm. In the match–value dimension she faces the maximum loss (second and third line). If buying from firm $i + m/2$ instead, her indirect utility equals

$$u_{1+m/2}(\hat{\hat{x}}_1^+, p_1, p^*, ..., p^*) \quad = v - t(1 - (\hat{\hat{x}}_1^+ - \frac{(m-2)}{2})) - p^* - \lambda\left(\frac{2\hat{\hat{x}}_1^+}{n}\right)(p^* - p_1)$$

[40] We use this latter condition here, since, as we show later, the mapping from $\Delta p = p^* - p_1 \in \mathbb{R}_0^+$ into $m \in [2, 3, ..., n-1, n]$ is not a function but a correspondence—i.e., for given price difference Δp, there may exist several personal equilibria $\hat{\hat{x}}_1^+$ within different sub-markets.

[41] Compare the indirect utility function for $m = 2$ in the proof of Lemma 11 and consult Section 2 for a detailed exposition of the utility function with reference dependence.

$$
\begin{aligned}
&- \lambda t \int_0^{1-(\hat{\hat{x}}_1^+ - \frac{(m-2)}{2})} (1 - (\hat{\hat{x}}_1^+ - \frac{(m-2)}{2}) - s)\frac{2}{n}(n-(m-2))ds \\
&+ t\Bigg(\int_{1-(\hat{\hat{x}}_1^+ - \frac{(m-2)}{2})}^{1/2} (s - (1 - (\hat{\hat{x}}_1^+ - \frac{(m-2)}{2})))\frac{2}{n}(n-(m-1))ds \\
&+ \int_{1/2}^{\hat{\hat{x}}_1^+} (s - (1 - (\hat{\hat{x}}_1^+ - \frac{(m-2)}{2})))\frac{2}{n}ds\Bigg) \\
=& v - t(1 - (\hat{\hat{x}}_1^+ - \frac{(m-2)}{2})) - p^* - \lambda\left(\frac{2\hat{\hat{x}}_1^+}{n}\right)(p^* - p_1) \\
&+ \frac{t}{4n}\Big(4(2-(\lambda-1)\overline{n})(\hat{\hat{x}}_1^+)^2 + 4(((\lambda-1)\overline{n}-1)m+n)\hat{\hat{x}}_1^+ \\
&+ ((1-(\lambda-1)\overline{n})m - 2n - 1)m + n\Big),
\end{aligned}
$$

with $\overline{n} \equiv ((n-m)+2)$. Here, the indifferent loss-averse consumer only faces a loss in the price dimension but losses and gains in the taste dimension—cf. the proof of Lemma 11 where $m = 2$. By setting $u_1 = u_{1+m/2}$, we can solve the consumers' personal equilibrium and determine $\hat{\hat{x}}_1^+$ for given n and given that ex ante consumers expect $\hat{\hat{x}}_1^+ \in [(m-1)/2; m/2]$ for given prices.[42] Firm 1's demand from loss-averse consumers in even sub-market m, $q_1(\Delta p|m,n,\beta = 0)$, is then characterized by $2\hat{\hat{x}}_1^+/n$. Firm 1's demand for odd sub-markets m can be derived analogously.

To analyze whether deviations to sub-markets m, $m \geq 3$, are profitable, we first consider consumers located on the boundaries of the sub-markets, $\hat{\hat{x}}_1^+ = 1, 3/2, ..., (n-1)/2, n/2$. For $\hat{\hat{x}}_1^+$ being an integer, firm 1 attracts consumers up to the location of a competing firm, while for $\hat{\hat{x}}_1^+ = j + 1/2$, $j \in \mathbb{N}$, it also attracts the entire backyard market of competitor j. As is known from the standard Salop oligopoly, the price differences for $\hat{\hat{x}}_1^+ = j$ and $\hat{\hat{x}}_1^+ = j + 1/2$ coincide. This means that firm 1's demand has a discontinuous jump of size $1/2 \cdot 2/n = 1/n$ at this price difference. It can be shown, however, that despite this

[42] $0 = u_1 - u_{1+m/2}$ is equivalent to

$$
\begin{aligned}
0 =& \Big((n-m)+4\Big)(\lambda-1)t \cdot (\hat{\hat{x}}_1^+)^2 - \Big(\left(((\lambda-1)m + \lambda + 3)n - (\lambda-1)(m-3)m\right)t - 2(\lambda-1)\Delta p\Big) \cdot \hat{\hat{x}}_1^+ \\
&+ \frac{1}{4}\Big(8n\Delta p + nt\left((\lambda-1)m^2 + \lambda + 4m - 1\right) - (\lambda-1)m((m-3)m+1)t\Big).
\end{aligned}
$$

We do not present the functional form of $\hat{\hat{x}}_1^+(\Delta p|m,n)$ here for two reasons. First of all, it is lengthy and tedious to derive, as $u_i - u_{i+m/2} = 0$ describes a quadratic equation in $\hat{\hat{x}}_1^+$. Secondly, since we are mainly interested in deviations to the boundaries of a sub-market m, we can fix $\hat{\hat{x}}_1^+$ at $(m-1/2)$ or $m/2$ and solve for the corresponding price difference Δp.

feature non-local deviation are never profitable in the standard Salop model. To check this in a world with loss-averse consumers, we next derive the deviation price differences for $\hat{\hat{x}}_1^+ = 1, 3/2, ..., (n-1)/2, n/2$. For a deviation covering an even number of sub-markets m (resp. an odd number of sub-markets $m' = m-1$), replace $\hat{\hat{x}}_1^+$ in $u_1 - u_{1+m/2} = 0$ by $m/2$ (resp. $(m-1)/2$) and solve for Δp.

$$\Delta p^{even}(m,n) = \frac{\Big(2(\lambda+1)m - (\lambda-1)\Big)n + (\lambda-1)(m-1)m}{4(\lambda-1)m + 8n}t, \quad m \text{ even and } n \geq m \geq 2,$$

$$\Delta p^{odd}(m',n) = \frac{\Big(2(\lambda+1)n - (\lambda-1)(m'-1)\Big)(m'-1)}{4(\lambda-1)m' + 8n}t, \qquad m' \text{ odd and } n \geq m' \geq 3.$$

It can be shown that both deviation price differences are increasing in m and n. The first implication of this is very intuitive: For a given number of firms n, attracting consumers on more sub-markets m requires a larger price difference—i.e., a larger price cut by the deviating firm. Secondly and more interestingly, if the number of firms n increases, a larger price cut is necessary to steal a given number of sub-markets m. The intuition for this is that, for a larger number of firms, consumers expect to be less often affected by a certain price cut of a single firm and, therefore, expect their equilibrium taste difference to be low. This increases the loss in the taste dimension for those consumers who ex post happen to buy from the more distant deviating firm, and this makes it more difficult for the deviating firm to steal a large share of the market. Consider for example two markets with $n = 3$ and 5, $(\lambda = 3, t = 1)$: $\Delta p^{even}(2,3) = 19/20 < \Delta p^{even}(2,5) = 33/28 < \Delta p^{even}(4,5) = 7/4$. Similarly, $\Delta p^{odd}(3,3) = 5/6 < \Delta p^{odd}(3,5) = 9/8 < \Delta p^{odd}(5,5) = 8/5$. It can also be seen here that the price difference necessary to steal the entire backyard sub-market of a competitor is lower than the one necessary to steal consumers up to the location of this competitor—i.e., $\Delta p^{odd}(m+1,n) < \Delta p^{even}(m,n)$. This demonstrates a violation of the *law of demand* which is caused by the fact that consumer's indirect utility functions if buying the cheap or the most-liked product are decreasing in consumer's location x on odd sub-markets. Hence, to describe a personal equilibrium, $\hat{\hat{x}}_1^+$ must be decreasing in Δp on odd sub-markets. This makes deviations under which the deviating firm steals an odd number of sub-markets particularly profitable, as will be shown in the next paragraph. In the example, the demand of the deviating firm is given by $m/2 \cdot$

$2/n = m/n$ and the corresponding markup in symmetric maximal equilibrium equals $m^*(3) = 3/2$ and $m^*(5) = 5/3$. This illustrates that the deviation price difference might become larger than the maximal equilibrium markup if the number of firms n and the number of deviations m become sufficiently large: In the example we find $m^*(5) = 5/3 < \Delta p^{even}(4,5) = 7/4$. Therefore, those kind of deviations generate losses for the deviating firm and are, therefore, never optimal.

We next evaluate whether there exist profitable deviations from the symmetric maximal equilibrium with $n > 2$ firms and $\lambda \leq \lambda^c = 1 + 2\sqrt{2} \approx 3.828$ (compare Prop. 6). The maximal equilibrium profit, $\pi^*(n)$, can be expressed by

$$\pi^*(n) = m^*(n) \cdot \frac{1}{n} = \frac{(1+\lambda)t}{(\lambda - 1 + 2n)},$$

with the maximal equilibrium markup, $m^*(n)$, being derived in Section 4 (cf. equation (17)). The deviation profits for even and odd deviations are equal to

$$\pi^{even}(m,n) = \left(m^*(n) - \Delta p^{even}(m,n)\right) \cdot \frac{m}{n}$$
$$\pi^{odd}(m',n) = \left(m^*(n) - \Delta p^{odd}(m',n)\right) \cdot \frac{m'}{n}.$$

Deviation profits change monotonously in n and m: $\pi^{odd}(m,n)$ and $\pi^{even}(m,n)$ are monotonously decreasing in n and m. This is shown in Table 6, where we restrict attention to $\lambda = \lambda^c$, the highest level of loss aversion at which a symmetric maximal equilibrium exists for $n = 2$. For smaller levels of loss aversion with $\lambda > 1$ deviating is less profitable, but the monotonicity in n and m is preserved. The table illustrates that deviating becomes less profitable if the number of firms n in the market increases[43] and that within the class of odd (resp. even) deviations stealing a small number of sub-markets m is preferable to stealing a larger number of sub-markets. Moreover, it is depicted that for a given number of firms n stealing an odd number of sub-markets $m' = m + 1$ is more profitable than stealing an even number of sub-markets m. Thus, the deviation profit is highest in a three-firm oligopoly when the deviating firm steals the entire market ($m = 3$). Note that $m = 1$ can be excluded since $\Delta p^{odd}(1,n)$ coincides with $\Delta p^*(n) = 0$, the symmetric maximal

[43] This also implies that non-local deviations in the home market ($m = 2$), as considered in the duopoly case, are less profitable if n raises.

Table 6: Deviation profits with n firms

The table shows the variation of $\pi^{odd}(m,n)/t$ and $\pi^{even}(m,n)/t$ in n and m for $\lambda = \lambda^c = 1 + 2\sqrt{2}$ (and $\beta = 0$).

n \ m	2	3	4	5	6	7	8
2	0.7071						
3	0.4676	0.8360					
4	0.3400	0.5877	0.3694				
5	0.2624	0.4373	0.1444	0.3506			
6	0.2111	0.3388	0.0096	0.1418	-0.1676		
7	0.1751	0.2705	-0.0750	0.0060	-0.3607	-0.2384	
8	0.1486	0.2211	-0.1296	-0.0851	-0.4861	-0.4151	-0.7769

Table 7: Extra profit from deviating

The table shows the variation of $(\pi^{odd}(m,n) - \pi^*(n))/t$ and $(\pi^{even}(m,n) - \pi^*(n))/t$ in n and m for $\lambda = \lambda^c = 1 + 2\sqrt{2}$ (and $\beta = 0$).

n \ m	2	3	4	5	6	7	8
2	0						
3	-0.0793	0.2891					
4	-0.1059	0.1418	-0.0765				
5	-0.1139	0.0610	-0.2320	-0.0258			
6	-0.1145	0.0131	-0.3160	-0.1839	-0.4932		
7	-0.1118	-0.0165	-0.3619	-0.2809	-0.6476	-0.5253	
8	-0.1078	-0.0354	-0.3860	-0.3415	-0.7426	-0.6715	-1.033

equilibrium.

To identify the deviations that are the most critical for existence, the difference between deviation and maximal equilibrium profit are presented in Table 7. By construction, $\pi^{even}(2,2) = \pi^*(2)$ at $\lambda = \lambda^c$ (cf. Prop. 6). It can be seen that there exist profitable deviations from symmetric maximal equilibrium for $\lambda = \lambda^c$. However, only deviations stealing $m = 3$ sub-markets are profitable if the number of firms is not too large—i.e., $n \in \{3,4,5,6\}$. More generally, this can be shown by solving for the critical number of

firms $n^{odd}(m,\lambda)$ in $\pi^{odd}(m,n) - \pi^*(n) = 0$ with $n^{odd}(m,\lambda)$ being the only positive solution.

$$n^{odd}(m,\lambda) = (\lambda-1)\frac{(\lambda+m)m + \sqrt{(m\lambda^2 + 2(3(m-2)m+4)\lambda + (m-2)(m+6)m + 8)\,m}}{4(\lambda+1)(m-2)}$$

Deviating is profitable for given λ, m, and n if $n < n^{odd}(m,\lambda)$ and $m \leq n$. Moreover, $n^{odd}(m,\lambda)$ is strictly decreasing in m for $n^{odd}(m,\lambda) > m$ and strictly increasing in λ. Therefore, $m = 3$ is the most critical deviation and profitable for $n < n^{odd}(3,\lambda^c) \approx 6.3890$. A critical n can be derived for even deviations analogously. We skip this step here since even deviations are dominated by odd ones. To rule out deviations from symmetric maximal equilibrium for all $n \geq 2$, the maximum degree of loss aversion λ has to be below $\lambda^c = 1 + 2\sqrt{2} \approx 3.828$.

Before stating the conditions for symmetric maximal equilibrium to exist, we return to the issue of multiple personal equilibria for given prices. Since $\Delta p^{odd}(3,n) < \Delta p^{even}(2,n)$, consumers facing a price difference $\Delta p = \Delta p^{odd}(3,n)$ between the deviating firm and non-deviating firms could expect $\hat{\hat{x}}_1^+$ to be located either on the second or the third sub-market (on the first half of the circle). Expecting $m = 3$ rather than $m = 2$ given $\Delta p = \Delta p^{odd}(3,n)$ is preferable for the deviating firm because it receives a strictly larger market share, but it is not necessarily preferable for consumers. For instance, consumers who do not buy the lower-priced product will ex post experience a higher loss in the price dimension since the probability of low purchase price increases in $\hat{\hat{x}}_1^+$.[44] Therefore, the deviations considered above use the most conservative personal equilibrium and deliver the strictest conditions for a maximal equilibrium to exist.

Lemma 10. *A symmetric maximal equilibrium with n firms and prices,* $p^*(n) = m^*(n)+c = ((1+\lambda)nt)/(\lambda - 1 + 2n) + c$, *exists if* $n \geq n^{odd}(3,\lambda)$ *with* $\lambda > 1$.

The derivation of $n^{odd}(m,\lambda)$ and the relevance of $n^{odd}(3,\lambda)$ is provided in the text. We finally provide a proof of Proposition 6.

Proof of Proposition 6. $n^{odd}(3,\lambda^c) \approx 6.3890$. Thus, $n = 2$ or $n > 6$ suffice for existence at $\lambda = \lambda^c$. Maximal equilibrium existence holds for $1 < \lambda < \lambda^c$ since $n^{odd}(3,\lambda)$ is increasing

[44] Cf. the concept of (consumer's) preferred personal equilibrium of Kőszegi and Rabin (2006) and Kőszegi and Rabin (2007).

in λ. Existence for $n \in \{3,4,5,6\}$ follows from the same property: $n^{odd}(3,\lambda) = 3$ for $\lambda = \lambda^{cc} = 1/4\left(1 + \sqrt{57}\right) \approx 2.137$. □

Hence, existence in the duopoly case carries over to the n-firm oligopoly case for sufficiently large n. For symmetric markets with a small number of firms, however, maximal equilibrium might fail to exist for intermediate values of λ ($\lambda < \lambda^c$).

D Appendix: Demand in n-Firm Oligopoly

In this appendix we first specify utility functions for any vector of prices. We then derive demand for price vectors such that one firm deviates from symmetric prices. To obtain the two–dimensional reference–point distribution of loss–averse consumers, suppose that the price vector $\boldsymbol{p} = (p_1, \ldots, p_n)$ is such that any sub-market between two neighboring firms is served by only these two firms—i.e., the maximum price difference between any two neighboring firms is not too large in absolute terms.[45] The rank order of the price difference, $\Delta p_i^+ = p_{i+1} - p_i$, and distance between firm i and her indifferent loss-averse consumer on the right, $\hat{\hat{x}}_i^+ - y_i = \hat{\hat{x}}_i^+ - (i-1) \in [0,1]$, are identical.[46] This holds true since the reference comparison induced by reference-dependent utility is, by construction, rank-order maintaining. For example, if $p_i = p_{i+1}$ ($\Delta p_i = 0$), then $\hat{\hat{x}}_i^+ - (i-1) = 1/2$ (by symmetry), while $\hat{\hat{x}}_j^+ - (j-1) > 1/2$ if $p_j < p_{j+1}$ ($\Delta p_j > 0$). The reference–point distribution in the price dimension, $F(p)$, is the probability that the equilibrium purchase price p^* is not larger than p. Recall that due to consumers' initial taste uncertainty, the equilibrium purchase price is not known when consumers form their reference point, even though firms' prices are already disclosed. Under the uniform distribution of x, we obtain

$$F(p) = \sum_{i \in \{i | p_i \leq p\}} \frac{(\hat{\hat{x}}_i^+ - \hat{\hat{x}}_i^-)}{n}. \tag{40}$$

[45] The case in which a single firm serves several sub-markets is considered in Section C in the Appendix.

[46] Note that the index i for Δp_i^+ is modulo n—i.e., $\Delta p_n^+ = p_1 - p_n$.

We next define the distances z_j between an indifferent consumer's location and the locations of her two neighboring firms,[47]

$$\forall j \in \{1, ..., 2n\}: \qquad z_j = \begin{cases} \hat{\hat{x}}_i^+ - (i-1), & \text{if } j = 2i-1; \\ 1 - (\hat{\hat{x}}_i^+ - (i-1)), & \text{if } j = 2i. \end{cases} \tag{41}$$

Distances z_j can be ordered by rank. Let $z[k]$ describe the kth smallest distance in $\{z_j\}_{j=1}^{2n}$ and $\#(z[k])$ the number of distances of size $z[k]$.[48] $\sigma(x)$ describes consumer x's purchase decision (pure-strategy personal equilibrium), which requires that, for given prices $\boldsymbol{p}$, consumers correctly anticipate the locations of the indifferent consumers $\{\hat{\hat{x}}_i^+\}_{i=1}^n$. The reference–point distribution in the taste dimension, $G(s)$, is the probability that the equilibrium taste difference between the consumer's ideal taste x and the taste of the purchased product $y_{\sigma(x)}$ is smaller than s—i.e., $G(s) = Prob(|x - y_{\sigma(x)}| \leq s)$. We obtain,

$$G(s) = \begin{cases} 2s, & s \in [0, z[1]]; \\ 2s\frac{2n-\#(z[1])}{2n} + a_1, & s \in (z[1], z[2]]; \\ \vdots & \vdots \\ 2s\frac{2n-\sum_{j=1}^{k}\#(z[j])}{2n} + a_k, & s \in (z[k], z[k+1]]; \\ \vdots & \vdots \\ 2s\frac{2n-\sum_{j=1}^{K-1}\#(z[j])}{2n} + a_{K-1}, & s \in (z[K-1], z[K]]; \\ a_K = 1, & s \in (z[K], 1]. \end{cases} \tag{42}$$

with $\{a_k\}_{k=1}^K$ being the required constants for the $(K-1)$-times kinked, piecewise linear cdf. If all prices are the same, then consumers expect to buy from their closest firm ex post with probability one. The distribution of the expected taste difference, $G(s)$, is not kinked in this case and approaches the uniform distribution: $K = 1$ and $G(s) = 2s$ for $s \in (0, 1/2]$. If there are two or more different prices p_i in the market, then there are at least two different distances z_j. For small realized taste differences, $s \in [0, z[1]]$, consumers expect

[47] Note that $\max\{z_{2i-1}, z_{2i}\}$ represents the maximum taste difference consumers located between firm i and $i+1$ are willing to accept for given prices. Also note that $\max_{j\in\{1,...,2n\}}\{z_j\}$ reflects consumers' maximum acceptable taste difference in the entire market and corresponds to the largest price difference between two neighboring firms.

[48] Obviously, if there are no ties between price differences and between distances, then $\#(z[k]) = 1$ for all $k \in \{1, ..., K\}$ and $K = 2n$.

to buy from their closest firm ex post and, thus, $G(s) = 2s$. For a larger taste difference, however, consumers anticipate that they will be attracted with positive probability to the more distant, cheaper firm ex post. For this to happen, given $s \in (z[1], z[2]]$, the realization of x must be sufficiently close to the more expensive firm in the sub-market with the largest price difference. Let, for instance, $\Delta p_i^+ = p_{i+1} - p_i$ be the (unique) maximum price difference for given $\boldsymbol{p}$. Then, the indifferent consumer, $\hat{\hat{x}}_i^+$, in this sub-market is more closely located to the high-price firm $i+1$ ($y_{i+1} = i$). Moreover, the distance between firm $i+1$ and the indifferent consumer $\hat{\hat{x}}_i^+$ is the smallest distance in the entire market—i.e., $y_{i+1} - \hat{\hat{x}}_i^+ = i - \hat{\hat{x}}_i^+ = 1 - (\hat{\hat{x}}_i^+ - (i-1)) = z[1]$. Thus, if the realization of x lies in the interval $[y_{i+1} - z[2], \hat{\hat{x}}_i^+]$, the consumer will be attracted by the low-price firm i. Therefore, the consumer will not buy from her closest firm in equilibrium. This means that for $s \in (z[1], z[2]]$, only $2n-1$ sub-markets are relevant for the probability of facing s and $G(s)$, therefore, equals $2s(2n-1)/2n$ plus a constant. This argument carries over to all $s \in (z[k], z[k+1]]$ with $1 \leq k \leq K \leq 2n$. $G(s)$ shows up to $2n-1$ kinks if there n distinct price differences in the market.

Consider the indirect utility functions of a consumer who has learned, after forming her reference–point distribution given prices, that her ideal taste x lies in the sub-market between firm i and firm $i+1$. Suppose further that this consumer is the indifferent loss-averse consumer on this sub-market—i.e., $x = \hat{\hat{x}}_i^+ \in [i-1, i]$. The consumer faces a distance of $\hat{\hat{x}}_i^+ - (i-1) = z_{2i-1}$ to firm i and $1 - z_{2i-1}$ to firm $i+1$. Her indirect utility if buying from firm i can be expressed as

$$\begin{aligned} u_i(x = \hat{\hat{x}}_i^+, \boldsymbol{p}) \quad =& v - t z_{2i-1} - p_i \\ &+ \left(- \lambda \sum_{j \in \{j | p_j \leq p_i\}} \frac{(\hat{\hat{x}}_j^+ - \hat{\hat{x}}_j^-)}{n} (p_i - p_j) + \sum_{j \in \{j | p_j > p_i\}} \frac{(\hat{\hat{x}}_j^+ - \hat{\hat{x}}_j^-)}{n} (p_j - p_i) \right) \\ &+ \left(- \lambda t \int_0^{z_{2i-1}} (z_{2i-1} - s) dG(s) + t \int_{z_{2i-1}}^1 (s - z_{2i-1}) dG(s) \right), \end{aligned}$$

where the first line describes the consumer's intrinsic utility from product i. As in the duopoly model, v represents the common reservation value for one unit of any product, and t scales the disutility from distance between ideal and actual taste on the circle. The first term in the second line shows the loss in the price dimension from not facing a lower

price than p_i, while the second term in this line shows the gain from not facing a higher price than p_i. The two terms in the third line correspond to the loss (gain) from not facing a smaller (larger) distance in the taste dimension than $\hat{\hat{x}}_i^+ - (i-1) = z_{2i-1}$. If buying from firm $i+1$ instead, the indifferent consumer's indirect utility is

$$\begin{aligned} u_{i+1}(x = \hat{\hat{x}}_i^+, \boldsymbol{p}) \quad &= v - t(1 - z_{2i-1}) - p_{i+1} \\ &+ \Bigg(- \lambda \sum_{j \in \{k | p_k \leq p_{i+1}\}} \frac{(\hat{\hat{x}}_j^+ - \hat{\hat{x}}_j^-)}{n} (p_{i+1} - p_j) + \sum_{j \in \{k | p_k > p_{i+1}\}} \frac{(\hat{\hat{x}}_j^+ - \hat{\hat{x}}_j^-)}{n} (p_j - p_{i+1}) \Bigg) \\ &+ \Bigg(- \lambda t \int_0^{(1-z_{2i-1})} ((1 - z_{2i-1}) - s) dG(s) + t \int_{(1-z_{2i-1})}^1 (s - (1 - z_{2i-1})) dG(s) \Bigg). \end{aligned}$$

By setting $u_i - u_{i+1} = 0$ for all i and solving for $\{\hat{\hat{x}}_i^+\}_{i=1}^n$, we determine the locations of indifferent loss-averse consumers (consumers' personal equilibria) for any given $\boldsymbol{p}$ (provided that a solution exists).

Since the focus of this paper is on symmetric firms and symmetric price equilibria, we can restrict our attention to prices that are the same for all firms but one. The variation in the price of one firm is required to determine the symmetric equilibrium prices in stage 1 of the game. Let $p_i \neq p'$ be the price set by firm i and $p_j = p'$, $j \neq i$, the price of other firms in the market. By symmetry, the location of indifferent consumers in any sub-market with zero price difference lies exactly in the middle between the two firms on this sub-market—i.e., $\hat{\hat{x}}_j^+ - (j-1) = 1/2$. The location of indifferent consumers in the two sub-markets around firm i is further apart from firm i than $1/2$, if firm i has set a lower price than any neighboring firm—i.e., $\hat{\hat{x}}_i^+ - (i-1) = (i-1) - \hat{\hat{x}}_i^- > 1/2$ for $p_i < p'$—and vice versa if firm i has set a higher price than any neighboring firm. In the following lemma, we solve for the location of the indifferent consumer $\hat{\hat{x}}_i^+$ as a function of the price difference $\Delta p = p' - p_i \geq 0$, conditional on the number of firms n in the market. In an abuse of notation, let $\hat{\hat{x}}_i^+$ depend on the size of the price deviation Δp and the price p' that is set by all other firms, i.e. $\boldsymbol{p} = (p', ..., p', p_i, p', ...p')$.

Lemma 11. *Suppose that* $\hat{\hat{x}}_i^+ \in [(i-1) + 1/2, i]$, $p_i \leq p'$, *and* $p_j = p'$ *for all* $j \neq i$. *Moreover,* $\lambda > 1$. *Then* $\hat{\hat{x}}_i^+$, *as a function of the price difference* $\Delta p \equiv p' - p_i \in [0, \overline{\Delta p}]$

given the price vector $\boldsymbol{p} = (p', ..., p', p_i, p', ..., p')$*, is*

$$\hat{\hat{x}}_i^+(\Delta p, p') = (i-1) + \left(\frac{4}{(\lambda-1)(n+2)} + \frac{3n+2}{n(n+2)}\right) - \frac{2\Delta p}{n(n+2)t} - 2S(\Delta p), \tag{43}$$

where

$$S(\Delta p) = \sqrt{\frac{(\lambda-1)^2 \cdot \Delta p^2 - (\lambda-1)\Lambda t \cdot \Delta p + (1+\lambda)^2 n^2 t^2}{((\lambda-1)n(n+2)t)^2}}$$

with $\Lambda = (2(\lambda-1) + n(3\lambda + 2n + 5))$ *and* $\Delta\bar{p}$ *being the upper bound of* Δp *for which the square root* $S(\Delta p)$ *is defined.*[49]

From the general form of $\hat{\hat{x}}_i^+(\Delta p, p')$ in Lemma 11, we can derive the demand of loss-averse consumers for a price decrease of firm i, $\hat{\hat{x}}_i(\Delta p, p')$: Using the uniform distribution of x and symmetry we obtain

$$\hat{\hat{x}}_i(\Delta p, p') = \frac{\hat{\hat{x}}_i^+(\Delta p, p') - \hat{\hat{x}}_i^-(\Delta p, p')}{n} = \frac{2}{n}\left(\hat{\hat{x}}_i^+(\Delta p, p') - (i-1)\right) = \frac{2}{n} z_{2i-1}. \tag{44}$$

In the proof of Lemma 11, we make use of the fact that there exist only two indifferent consumers whose locations are different from $1/2$, the indifferent consumers to the right and the left of firm i. Since their locations are symmetric, it suffices to solve a system of one (quadratic) equation and one unknown—i.e., to solve $u_i - u_{i+1} = 0$ for $\hat{\hat{x}}_i^+$. For $\lambda \to 1$, $u_i - u_{i+1} = 0$ collapses to a linear equation and $\hat{\hat{x}}_i^+(\Delta p, p')$ shows a much simpler form.

Proof of Lemma 11. We rewrite the indirect utility functions for the indifferent consumer to the right of firm i and solve for her location (personal equilibrium).

Since the price differences in the sub-market between firm $i-1$ and firm i and in the one between firm i and firm $i+1$ are the same—i.e., the taste differences which the two

[49] For $x \in [i-1, i]$, consumer x's personal equilibrium (determining her product choice—i.e., $\sigma(x, \Delta p) \in \arg\max_{j \in \{i, i+1\}} u_j(x, p_j, p_{-j})$) is described by

$$\sigma(x, \Delta p) = \begin{cases} i & \text{if } x \in [y_i, \hat{\hat{x}}_i^+(\Delta p, p')], \\ i+1 & \text{if } x \in (\hat{\hat{x}}_i^+(\Delta p, p'), y_{i+1}]. \end{cases}$$

indifferent consumers of firm i face are the same—we have $\hat{\hat{x}}_i^+ - (i-1) = (i-1) - \hat{\hat{x}}_i^-$. We, therefore, can simplify $\hat{\hat{x}}_i^+ - \hat{\hat{x}}_i^-$ in $F(p)$ to $2(\hat{\hat{x}}_i^+ - (i-1))$ or, equivalently, to $2z_{2j-1}$. Furthermore, using that $p_j = p'$ for all $j \neq i$, we obtain that

$$F(p) = \begin{cases} \frac{2z_{2j-1}}{n} & \text{if } p \in [p_i, p') \\ 1 & \text{if } p \geq p'. \end{cases}$$

A price deviation $p_i < p'$ implies that $\hat{\hat{x}}_i^+ - (i-1) = z_{2j-1} > 1/2$. Thus, the smallest critical taste distance in the market exists between $\hat{\hat{x}}_i^+$ and firm $i+1$ (and between $\hat{\hat{x}}_i^-$ and firm $i-1$). This distance is equal to $1 - z_{2j-1}$. The next larger critical taste distance is the one in sub-markets with symmetric prices. It is equal to $1/2$. Finally, only the consumers that will be attracted by firm i ex post face up to the maximum critical taste distance which is z_{2j-1}. Hence, $G(s)$ can be rewritten as

$$G(s) = \begin{cases} 2s & \text{if } s \in [0, 1 - z_{2j-1}] \\ 2s\frac{n-1}{n} + a_1 & \text{if } s \in (1 - z_{2j-1}, \frac{1}{2}] \\ 2s\frac{1}{n} + a_2 & \text{if } s \in (\frac{1}{2}, z_{2j-1}], \end{cases}$$

where $a_1 = \frac{1-2z_{2j-1}}{n}$ and $a_2 = (1 - \frac{2z_{2j-1}}{n})$. Using the properties of the reference–point distributions, we rewrite the indirect utility functions of consumers buying from firm i or $i+1$,

$$\begin{aligned} u_i(\hat{\hat{x}}_i^+, p') &= v - tz_{2i-1} - p_i + (1 - \frac{2z_{2i-1}}{n})(p' - p_i) + \frac{\lambda t}{4n}\left(8z_{2i-1}^2 - 4(2+n)z_{2i-1} + 2 + n\right) \\ u_{i+1}(\hat{\hat{x}}_i^+, p') &= v - t(1 - z_{2i-1}) - p' - \lambda\frac{2z_{2i-1}}{n}(p' - p_i) \\ &\quad + \frac{t}{4n}\left(n\left((2z_{2i-1} - 1)^2 - 4\lambda(z_{2i-1} - 1)^2\right) + 2(2z_{2i-1} - 1)^2\right). \end{aligned}$$

Next, we determine the location of the indifferent loss-averse consumer by setting $u_i = u_{i+1}$. Rearranging leads to the following quadratic equation in z_{2j-1},

$$\begin{aligned} 4(\lambda-1)(n+2)t \cdot z_{2i-1}^2 + \Big(8(\lambda-1)\Delta p - 4(2n + (2(\lambda-1) + (3\lambda-1)n))t\Big) \cdot z_{2i-1} \\ + \Big(4(2)n\Delta p + 2(\lambda-1)t + (4 + (5\lambda-1))nt\Big) = 0 \end{aligned}$$

Solving this quadratic equation w.r.t. z_{2j-1} and adding $(i-1)$ leads to $\hat{\hat{x}}_i^+(\Delta p)$, the expression given in the lemma. The second solution to the quadratic equation can be ruled out because it does not lie in the interval $[1/2, 1]$. □

E Appendix: Noisy Signals

In this appendix we modify our benchmark model and postulate that ex ante all consumers are identical. Suppose that consumers receive a signal r about their location ex ante. Let r be equal to the true ideal taste x with probability a and be noise with probability $1-a$ which is uniformly distributed on the circle.

E.1 Certainty Case

If the precision of the signal r is equal to 1 ($a = 1$)—i.e., if consumer know their location ex ante for sure, we are back to the situation in which consumers are perfectly informed ex ante. Here, a consumer's initial plan is the only determinant of her reference point. This raises the issue of multiple personal equilibria (PE) (see below) which can be resolved by focussing on consumer's preferred personal equilibria (PPE) (from an ex ante point of view). In Proposition 3 of Kőszegi and Rabin (2006) it is shown that, in the PPE of an environment without uncertainty, consumers expect to choose the alternative which maximizes their intrinsic utility with probability one. In our model, they simply behave like standard Hotelling consumers: they buy from firm 1 if $x \in [0, \hat{x}(\Delta p)]$ and from firm 2 otherwise.

We next derive a set of personal equilibria that will be useful for analyzing the uncertainty case in the next subsection. There exists an interval $[\underline{x}(\Delta p), \overline{x}(\Delta p)]$ including $\hat{x}(\Delta p)$ in which any element can be supported as a PE given corresponding expectations of consumers ex ante. For instance, consider a consumer x_1 who ex ante expects to buy the cheaper product 1 ($\Delta p \geq 0$) with probability one, $\sigma = 1$. This means that the probability of paying price p_2 and buying product 1 is zero. Her indirect utility when buying product

1 ex post equals,

$$u_1(x_1, p_1, p_2|\sigma = 1) = v - tx_1 - p_1. \tag{45}$$

Ex post for $x_1 > 1/2$, her indirect utility when buying product 2 instead equals,

$$u_2(x_1, p_1, p_2|\sigma = 1) = v - t(1 - x_1) - p_2 - \lambda\Delta p + t(x_1 - (1 - x_1)). \tag{46}$$

Note that, at $x_1 = \hat{x}(\Delta p)$, the consumer would buy product 1, since $\Delta u = u_2 - u_1 < 0$. Buying product 1 for sure reflects her PPE. The maximal location at which a consumer x_1 would buy from firm 1, however, is given by $\Delta u = u_2 - u_1 = 0$ and is equal to

$$\hat{x}(\Delta p|\sigma = 1) = \frac{\lambda + 1}{2}\frac{\Delta p}{2t} + \frac{1}{2}. \tag{47}$$

Note that the maximal location for buying from firm 1 is larger than the location of the indifferent Hotelling consumer, i.e., $\hat{x}(\Delta p|\sigma = 1) > \Delta p/(2t) + 1/2 = \hat{x}(\Delta p)$. We define $\hat{x}(\Delta p|\sigma = 1) \equiv \overline{x}(\Delta p)$. Thus, buying from firm 1 with probability one is a PE for all consumers $x_1 \in [0, \overline{x}(\Delta p)]$.

We turn to the more general case in which a consumer x_σ holds the initial plan to purchase from firm 1 with probability σ, $\sigma \in [0, 1]$. Her indirect utility when buying product 1 ex post equals,

$$u_1(x_\sigma, p_1, p_2|\sigma) = v - tx_\sigma - p_1 + \Delta p(1 - \sigma) - \lambda t(x_\sigma - (1 - x_\sigma))(1 - \sigma) \tag{48}$$

Her indirect utility when buying product 2 ex post equals,

$$u_2(x_\sigma, p_1, p_2|\sigma) = v - t(1 - x_\sigma) - p_2 - \lambda\Delta p\sigma + t(x_\sigma - (1 - x_\sigma))\sigma. \tag{49}$$

$\Delta u = 0$ yields the following location of the indifferent consumer,

$$\hat{x}(\Delta p|\sigma) = \frac{(\lambda - 1)\sigma + 2}{(\lambda - 1)(1 - \sigma) + 2}\frac{\Delta p}{2t} - \frac{1}{2}. \tag{50}$$

For $\sigma = 0$, the minimal location for buying from firm 2 is lower than the location of the indifferent Hotelling consumer—i.e., $\hat{x}(\Delta p|\sigma = 0) < \Delta p/(2t) + 1/2 = \hat{x}(\Delta p)$. Defining

$\underline{x}(\Delta p)$ as $\hat{x}(\Delta p|\sigma = 0)$, determines the interval in which multiple PE can exist. For instance, buying from firm 2 with probability one is a PE for all consumers $x_2 \in [\underline{x}(\Delta p), 1]$. Thus, there exists an overlap in $[\underline{x}(\Delta p), \overline{x}(\Delta p)]$ with the set of pure-strategy PE of buying from firm 1. This means that in $[\underline{x}(\Delta p), \overline{x}(\Delta p)]$ any pure–strategy PE (buying from firm i, $i \in \{1, 2\}$) can be supported by corresponding initial expectations.

For any $\sigma \in [0, 1]$, there exists a mixed-strategy PE for a consumer located at $x_\sigma = \hat{x}(\Delta p|\sigma)$. It follows from (50) that firm 1's demand is equal to that in the standard Hotelling case for $\sigma = 1/2$ (or $\lambda = 1$). Buying from both firms with equal probability is a mixed-strategy PE for a consumer located at $x_{1/2} = \hat{x}(\Delta p)$ for all $\Delta p \geq 0$ such that $\hat{x}(\Delta p) \in [0, 1]$. We will return to this PE in the next subsection when we introduce uncertainty.

E.2 Uncertainty Case

If $a < 1$, consumers face some exogenous uncertainty when receiving a signal r: with probability $1 - a$ their location is uniformly distributed on the circle. Consumers form rational expectations about their ideal taste x after receiving their signal r. The probability of buying from firm 1 ex post is equal to $a\sigma + (1 - a)\hat{\hat{x}}(\Delta p|a, \sigma)$, where $\hat{\hat{x}}(\Delta p|a, \sigma)$ is the location of the indifferent consumer given precision a of the signal and consumer's strategy σ to buy from firm 1 with $a, \sigma \in [0, 1]$. For a consumer whose signal is close to zero (the location of firm 1), $\sigma = 1$ is optimal $\forall a$, while the opposite holds for a consumer whose signal is close to one (the location of firm 2). To determine the cutoff, we will consider the consumer located at $\hat{\hat{x}}(\Delta p|a, \sigma = 1/2)$ who is indifferent between choosing product 1 and 2.

For $p_1 \leq p_2$, we receive the following indirect utility for the general indifferent consumer, $x = \hat{\hat{x}}(\Delta p|a, \sigma)$, who received signal, $r = x$, when buying from firm 1 ex post:

$$\begin{aligned} u_1(\hat{\hat{x}}, \hat{\hat{x}}, p_1, p_2|a, \sigma) \quad =& (v - t\hat{\hat{x}} - p_1) + \Delta p(a(1 - \sigma) + (1 - a)(1 - \hat{\hat{x}})) \\ & - \lambda \cdot t\Bigg(\int_0^{1-\hat{\hat{x}}} (\hat{\hat{x}} - s)2(1 - a)ds + \int_{1-\hat{\hat{x}}}^{\hat{\hat{x}}} (\hat{\hat{x}} - s)(1 - a)ds \\ & + a(1 - \sigma)(\hat{\hat{x}} - (1 - \hat{\hat{x}}))\Bigg). \end{aligned} \tag{51}$$

Analogously, the general indifferent consumer's indirect utility from a purchase of product 2 is given by

$$\begin{aligned} u_2(\hat{\hat{x}}, \hat{\hat{x}}, p_1, p_2|a, \sigma) \quad =& (v - t(1 - \hat{\hat{x}}) - p_2) - \lambda\Delta p(a\sigma + (1 - a)\hat{\hat{x}}) \\ & - \lambda \cdot t\Big(\int_0^{1-\hat{\hat{x}}} ((1 - \hat{\hat{x}}) - s)2(1 - a)ds \Big) + t \int_{1-\hat{\hat{x}}}^{\hat{\hat{x}}} (s - (1 - \hat{\hat{x}}))(1 - a)ds \\ & + ta\sigma(\hat{\hat{x}} - (1 - \hat{\hat{x}})). \end{aligned} \tag{52}$$

Solving for the location of the general indifferent uninformed consumer $\hat{\hat{x}}$, which is implicitly given by $\Delta u = 0$, yields

$$\hat{\hat{x}}(\Delta p|a, \sigma) = \frac{2\lambda - a(\lambda - 1)(\sigma + 1)}{2(1 - a)(\lambda - 1)} - \frac{\Delta p}{4t} - S(\Delta p|a, \sigma), \tag{53}$$

where

$$S(\Delta p|a, \sigma) = \sqrt{\frac{\Delta p^2}{16t^2} - \frac{(2(\lambda + 2) - a(\lambda - 1)(1 - \sigma))\Delta p}{4(1 - a)(\lambda - 1)t} + \frac{((\lambda + 1) + a\sigma(\lambda - 1)}{4(1 - a)^2(\lambda - 1)^2}} \tag{54}$$

and $\Delta p \geq 0$ but sufficiently small. It follows from (53) that, $\forall\sigma$, $\hat{\hat{x}}(\Delta p|a, \sigma)$ converges to $\hat{\hat{x}}(\Delta p)$ if $a \to 0$—i.e., if there is pure noise. Note that $\hat{\hat{x}}(\Delta p|a, \sigma)$ is increasing in σ and thus, $\hat{\hat{x}}(\Delta p|a, \sigma = 0) < \hat{\hat{x}}(\Delta p|a, \sigma = 1/2) < \hat{\hat{x}}(\Delta p|a, \sigma = 1)$. This means that consumers who expect to buy from firm 1 are more likely to buy from firm 1 ex post and vice versa.

We next consider the PE in which consumers use the cutoff $\hat{\hat{x}}(\Delta p|a, \sigma = 1/2)$ to evaluate their initial signal. Note that this cutoff converges to $\hat{x}(\Delta p)$ for $a \to 1$.[50] In this PE all consumers who received a signal $r \in [0, \hat{\hat{x}}(\Delta p|a, \sigma = 1/2)]$ expect to buy from firm 1 with probability $\sigma = 1$ if the signal is correct, while consumers who received a signal $r \in (\hat{\hat{x}}(\Delta p|a, \sigma = 1/2), 1]$ expect to buy from firm 1 with probability $\sigma = 0$ if the signal is correct. In case their signal turns out to be incorrect, consumers who expected to buy from firm 1 with probability one (resp. zero) buy from firm 1 with probability $\hat{\hat{x}}(\Delta p|a, \sigma = 1)$

[50] Cf. equation (50) in the previous subsection. By continuity in a, we receive that convergence to $\hat{x}(\Delta p)$ is a property which is also satisfied by the cutoff of the PPE. In the following, we will use the cutoff $\hat{\hat{x}}(\Delta p|a, \sigma = 1/2)$ for $a < 1$ instead of the cutoff of the PPE since the derivation of the latter is analytically intractable as it requires the maximization of consumer's expected utility at the initial stage over this cutoff for all possible signals and all possible realized match values.

(resp. $\hat{\hat{x}}(\Delta p|a,\sigma=0)$) depending on the realization of their true match value.

It can easily be shown that the cutoff $\hat{\hat{x}}(\Delta p|a,\sigma=1/2)$ is monotonous in the precision a. This means that $\hat{\hat{x}}(\Delta p|a,\sigma=1/2)$ is either monotonously increasing in a if $\Delta p \geq 0$ is small such that $\hat{\hat{x}}(\Delta p) < \hat{x}(\Delta p)$ or monotonously decreasing in a if $\Delta p \geq 0$ is sufficiently large such that $\hat{\hat{x}}(\Delta p) > \hat{x}(\Delta p)$—see Figure 1.

The corresponding demand of firm 1, $q_1(\Delta p, a)$ equals,

$$q_1(\Delta p,a) = \int_0^{\hat{\hat{x}}(\Delta p|a,1/2)} \left(a+(1-a)\int_0^{\hat{\hat{x}}(\Delta p|a,1)} ds\right)dr + \int_{\hat{\hat{x}}(\Delta p|a,1/2)}^1 \left((1-a)\int_0^{\hat{\hat{x}}(\Delta p|a,0)} ds\right)dr$$

$$= a\hat{\hat{x}}(\Delta p|a,1/2) + (1-a)\left(\hat{\hat{x}}(\Delta p|a,1/2)\hat{\hat{x}}(\Delta p|a,1) + [1-\hat{\hat{x}}(\Delta p|a,1/2)]\hat{\hat{x}}(\Delta p|a,0)\right) \tag{55}$$

Since by construction $\hat{\hat{x}}(\Delta p|a,1/2)$ converges to $\hat{x}(\Delta p,1/2) = \hat{x}(\Delta p)$ for $a \to 1$,[51] so does $q_1(\Delta p,a)$. The symmetric equilibrium markup for a demand of $\hat{\hat{x}}(\Delta p|a,\sigma)$ can be calculated as

$$p_i^*(a,\sigma) - c = \frac{2((\lambda+1)-(\lambda-1)a\sigma)t}{(\lambda+3)+(\lambda-1)a(2\sigma-1)}, i = 1,2. \tag{56}$$

It follows that the symmetric equilibrium markup for a demand of $\hat{\hat{x}}(\Delta p|a,1/2)$,

$$p^*(a,1/2) - c = (2(\lambda+1)-(\lambda-1)a)t/(\lambda+3)$$

which is decreasing in precision of the signal a. This symmetric equilibrium markup is an upper bound on the corresponding markup for demand $q_1(\Delta p,a)$; see Lemma 12 below. (Note that $p^*(0,1/2)-c$ is equal to the markup formula in (7) and $p^*(1,1/2)-c=t$.) This yields monotonicity of symmetric markups in a and thus robustness of our results in symmetric settings to the introduction of noisy signals which decrease the uncertainty of uninformed, loss–averse consumers before they form their reference point distributions.

Lemma 12. *In symmetric duopoly, the markup for a demand of* $\hat{\hat{x}}(\Delta p|a,1/2)$ *is weakly*

[51] More precisely, the solution of $\Delta u(\sigma=1/2)=0$ converges to $\hat{x}(\Delta p)$ for $a \to 1$. In the limit $\Delta u(\sigma=1/2)=0$ becomes a linear equation of $\hat{x}$.

larger than the equilibrium markup for the demand with noisy signals $q_1(\Delta p, a)$ *for all* $a \in [0,1]$.

Proof of Lemma 12. Applying the markup formula for symmetric duopoly in (8), $m_{\hat{\hat{x}}(\sigma=1/2)}(a) \geq m_{q_1}(a)$, for all $a \in [0,1]$, is equivalent to $\hat{\hat{x}}'(\Delta p = 0|a, 1/2) \leq q_1'(\Delta p = 0|a)$, for all $a \in [0,1]$. It follows from (53) that $\hat{\hat{x}}(\Delta p = 0|a, \sigma) = 1/2$ for all $\sigma \in [0,1]$. Using this fact, the first derivative of $q_1(\Delta p, a)$ w.r.t. Δp at $\Delta p = 0$ can be expressed as

$$q_1'(\Delta p = 0|a) = a\hat{\hat{x}}'(\Delta p = 0|a, 1/2) + (1-a)\left(\frac{1}{2}(\hat{\hat{x}}'(\Delta p = 0|a, 1) + \hat{\hat{x}}'(\Delta p = 0|a, 0))\right) \quad \forall a \in [0,1].$$

Then, $q_1'(\Delta p = 0|a)$ is weakly larger than $\hat{\hat{x}}'(\Delta p = 0|a, 1/2)$ for all $a \in [0,1]$ if and only if

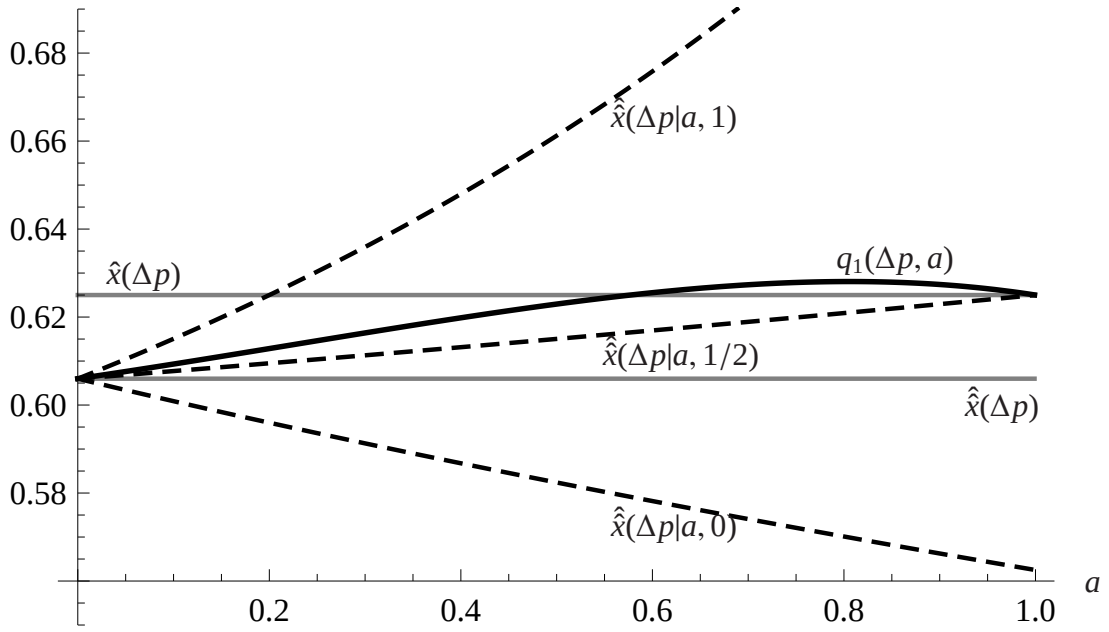
$$\frac{1}{2}\left(\hat{\hat{x}}'(\Delta p = 0|a, 1) + \hat{\hat{x}}'(\Delta p = 0|a, 0)\right) \geq \hat{\hat{x}}'(\Delta p = 0|a, 1/2).$$

Using (53), the previous inequality can be transformed to

$$\frac{a^2(\lambda-1)^2((3-a)\lambda + 5 + a)}{8(\lambda+1)((1-a)\lambda + 1 + a)(\lambda + 2 + a)t} \geq 0 \quad \forall a \in [0,1),$$

This inequality is always satisfied which completes the proof. □

However, even though the initial cutoff $\hat{\hat{x}}(\Delta p|a, 1/2)$ is monotonous in a, we cannot rule out that the resulting demand $q_1(\Delta p, a)$ is non–monotonous in a. An example is illustrated in Figure 10. The reason for the non–monotonicity in a is that consumers who receive a signal in favor of buying from the more efficient firm with a very high precision a are locked in into buying from that firm due to a high expected gain in the price dimension. Hence, they will not "switch" to the less efficient firm even though the initial signal turns out to be incorrect and consumers' true match value strongly favors the latter firm. This boosts the demand of the more efficient firm for very precise signals. While the non-monotonicity does not affect qualitatively our results in a symmetric setting, it may well affect them in asymmetric duopoly.



The cutoffs $\hat{\hat{x}}(\Delta p|a,1/2)$ (which defines whether the initial signal is in favor of buying from firm 1 or 2), $\hat{\hat{x}}(\Delta p|a,1)$ (if the initial signal in favor of buying from firm 1 was incorrect), and $\hat{\hat{x}}(\Delta p|a,0)$ (if the initial signal if the initial signal in favor of buying from firm 2 was incorrect) as a function of the signal precision a for given price difference $\Delta p = 0.2$ (which is sufficiently small such that $\hat{\hat{x}}(\Delta p) < \hat{x}(\Delta p)$) and $\lambda = 3$; $\hat{\hat{x}}(\Delta p|a = 0,1/2) = \hat{\hat{x}}(\Delta p) = 0.6060$ and $\hat{\hat{x}}(\Delta p|a = 1,1/2) = \hat{x}(\Delta p) = 0.625$.

Figure 10: Noisy Signals: Cutoffs for Small Price Difference

References

ABELER, J., A. FALK, L. GOETTE, AND D. HUFFMAN (2011): "Reference Points and Effort Provision," *American Economic Review*, 101(2), 470–492.

ANDERSON, S. P., AND R. RENAULT (2000): "Consumer Information and Firm Pricing: Negative Externalities from Improved Information," *International Economic Review*, 31(3), 721–741.

——— (2006): "Advertising Content," *American Economic Review*, 96(1), 93–113.

——— (2009): "Comparative Advertising: Disclosing Horizontal Match Information," *RAND Journal of Economics*, 40(3), 558–581.

ARMSTRONG, M., AND Y. CHEN (2009): "Inattentive Consumers and Product Quality," *Journal of the European Economic Association*, 7(2-3), 411–422.

ARMSTRONG, M., J. VICKERS, AND J. ZHOU (2009): "Prominence and Consumer Search,"

The RAND Journal of Economics, 40(2), 209–233.

BAGWELL, K. (2007): *The Economic Analysis of Advertising*,vol. 3 of *Handbook of Industrial Organization*, chap. 28, pp. 1701–1844. Elsevier.

BAR-ISAAC, H., G. CARUANA, AND V. CUNAT (2010): "Information Gathering and Marketing," *Journal of Economics and Management Strategy*, 19(2), 375–401.

BELLEFLAMME, P., AND M. PEITZ (2010): *Industrial Organization: Markets and Strategies*. Cambridge University Press.

BLINDER, A. S., E. R. D. CANETTI, D. E. LEBOW, AND J. B. RUDD (1998): "Asking About Prices: A New Approach to Understanding Price Stickiness," *Russell Sage Foundation*.

BURDETT, K., AND K. L. JUDD (1983): "Equilibrium Price Dispersion," *Econometrica*, 51(4), 955–969.

CRAWFORD, V. P., AND J. MENG (2011): "New York City Cabdrivers' Labor Supply Revisited: Reference-Dependent Preferences with Rational-Expectations Targets for Hours and Income," *American Economic Review*, 101(5), 1912–32.

DELLAVIGNA, S. (2009): "Psychology and Economics: Evidence from the Field," *Journal of Economic Literature*, 47(2), 315–372.

ELIAZ, K., AND R. SPIEGLER (2006): "Contracting with Diversely Naive Agents," *Review of Economic Studies*, 73(3), 689–714.

ELLISON, G. (2006): *Bounded Rationality in Industrial Organization*,vol. 2 of *Advances in economics and econometrics: theory and applications, Ninth World Congress*, chap. 5, pp. 142–219. Cambridge University Press.

ERICSON, K. M. M., AND A. FUSTER (forthcoming): "Expectations as Endowments: Evidence on Reference-Dependent Preferences from Exchange and Valuation Experiments," *Quarterly Journal of Economics*.

ESTEBAN, L., A. GIL, AND J. M. HERNANDEZ (2001): "Informative Advertising and Optimal Targeting in Monopoly," *Journal of Industrial Economics*, 49, 161–180.

GABAIX, X., AND D. LAIBSON (2006): "Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets," *Quarterly Journal of Economics*, 121(2), 505–540.

HEIDHUES, P., AND B. KŐSZEGI (2008): "Competition and Price Variation when Consumers are Loss Averse," *American Economic Review*, 98(4), 1245–1268.

——— (2010): "Regular Prices and Sales," *mimeo*.

HERWEG, F., AND K. MIERENDORFF (forthcoming): "Uncertain Demand, Consumer Loss Aversion, and Flat–Rate Tariffs," *Journal of the European Economic Association*.

HERWEG, F., D. MUELLER, AND P. WEINSCHENK (2010): "Binary Payment Schemes: Moral Hazard and Loss Aversion," *American Economic Review*, forthcoming.

HIRSHLEIFER, J. (1973): "Where Are We in the Theory of Information?," *American Economic Review*, 63(2), 31–39.

IYER, G., D. SOBERMAN, AND J. VILLAS-BOAS (2005): "The Targeting of Advertising," *Marketing Science*, 24, 461–476.

JANSSEN, M. C. W., AND J. L. MORAGA-GONZÁLEZ (2004): "Strategic Pricing, Consumer Search and the Number of Firms," *Review of Economic Studies*, 71(4), 1089–1118.

KAHNEMAN, D., AND A. TVERSKY (1979): "Prospect Theory: An Analysis of Decision under Risk," *Econometrica*, 47(2), 263–291.

KALYANARAM, G., AND R. S. WINER (1995): "Empirical Generalizations from Reference Price Research," *Marketing Science*, 14(3), 161–169.

KARLE, H., G. KIRCHSTEIGER, AND M. PEITZ (2011): "The Impact of Contextual Reference Dependence on Purchase Decisions: An Experimental Study," *mimeo*.

KŐSZEGI, B., AND M. RABIN (2006): "A Model of Reference-Dependent Preferences," *Quarterly Journal of Economics*, 121(4), 1133–1165.

——— (2007): "Reference-Dependent Risk Attitudes," *American Economic Review*, 97(4), 1047–1073.

——— (2009): "Reference-Dependent Consumption Plans," *American Economic Review*, 99(3), 909–936.

KRISHNAN, T. V., F. M. BASS, AND D. C. JAIN (1999): "Optimal Pricing Strategy for New Products," *Management Science*, 45(12), 1650–1663.

MAZUMDAR, T., S. RAJ, AND I. SINHA (2005): "Reference Price Research: Review and Propositions," *Journal of Marketing*, 69(4), 84–102.

MEDVEC, V. H., S. F. MADEY, AND T. GILOVICH (1995): "When Less Is More: Counterfactual Thinking and Satisfaction Among Olympic Medalists," *Journal of Personality and Social Psychology*, 69(4), 603–610.

MELLERS, B., A. SCHWARTZ, AND I. RITOV (1999): "Emotion-Based Choice," *Journal of Experimental Psychology*, 128(3), 332–345.

NELSON, P. (1970): "Information and Consumer Behavior," *Journal of Political Economy*,

78(2), 311–329.

POPE, D. G., AND M. E. SCHWEITZER (2011): "Is Tiger Woods Loss Averse? Persistent Bias in the Face of Experience, Competition, and High Stakes," *American Economic Review*, 101, 129–157.

RAJENDRAN, K. N., AND G. J. TELLIS (1994): "Contextual and Temporal Components of Reference Price," *Journal of Marketing*, 58(1), 22–34.

SALOP, S. C. (1979): "Monopolistic Competition with Outside Goods," *The Bell Journal of Economics*, 10(1), 141–156.

SPIEGLER, R. (2011a): *Bounded Rationality and Industrial Organization*. Oxford University Press.

——— (2011b): "Monopoly Pricing when Consumers are Antagonized by Unexpected Price Increases: A "Cover Version" of the Heidhues-Kőszegi-Rabin Model," *Economic Theory*, April, 1–17.

STEINER, R. (2004): "The Nature and Benefits of National Brand/Private Label Competition," *Review of Industrial Organization*, 24, 105–127, 10.1023/B:REIO.0000033351.66025.05.

TIROLE, J. (1988): *The Theory of Industrial Organization*. The MIT Press, Cambridge, Massachusetts.

TVERSKY, A., AND D. KAHNEMAN (1992): "Advances in prospect theory: Cumulative representation of uncertainty," *Journal of Risk and Uncertainty*, 5, 297–323, 10.1007/BF00122574.

VARIAN, H. R. (1980): "A Model of Sales," *American Economic Review*, 70(4), 651–659.

ZHOU, J. (2011): "Reference Dependence and Market Competition," *Journal of Economics and Management Strategy*, 20(4), 1073–1097.