

Dalvai, Wilfried

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Working Paper No. 145

Agglomeration and the Product Mix

by

Wilfried Dalvai

Universität Rostock

Wirtschafts- und Sozialwissenschaftliche Fakultät
Institut für Volkswirtschaftslehre

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Wilfried Dalvai*

University of Rostock

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Worldwide trade flows are dominated by high-productivity firms, that have a large range of products. Since the product range of firms reflects partly trade flows, it is a source of economic differences in space. In this paper, I analyze the effects of the product mix of firms on agglomeration. I build a theoretical model of multi-product firms à la Mayer, Melitz, and Ottaviano (2014, AER), expand it with skilled, mobile workers and a spatial equilibrium. I show that a larger product mix of firms in a region favours dispersion. The product mix influences the indirect utility through two channels, the wage and consumer surplus. A larger product mix decreases the wage differential between the two regions through a more competitive environment and thus strengthening the dispersion force. More competition means less profits and therefore a lower wage for skilled workers. On the other hand a more competitive environment means a higher consumer surplus which diminishes agglomeration forces.

Keywords: Agglomeration, Heterogenous Firms, Product Mix, Migration.

JEL Classification: L11, F12, R11, R12

*Wilfried Dalvai, University of Rostock, Department of Economics, Ulmenstr. 69, 18051 Rostock, Germany, Email: wilfried.dalvai@uni-rostock.de, Phone: +49 381 498 4310.

1 Introduction

Worldwide trade flows are dominated by high-productivity firms, that have a large range of products. Mayer and Ottaviano (2008) show, that a part of the variations of trade flows across European countries is based on the different product-range that multi-product firms sell in different markets. Bernard et al. (2007) show the same for the U.S. and Arkolakis and Muendler (2010) for Brazil. Since the product range of firms reflects partly trade flows, it is a source of economic differences in space. It is worth, questioning what effect different levels of product mix of firms have on their location. The present paper analyzes the impact of the firm product mix on agglomeration. I build a theoretical model of multi-product firms à la Mayer, Melitz, and Ottaviano (2014), I add mobile workers and a spatial equilibrium, following Ottaviano (2012), and I highlight how product mix affects agglomeration.

In the present model the level of product mix of firms depends on their marginal costs for varieties that are not in their core competencies. When firms have lower marginal costs for their additional products, they will, everything else equal, have a more heterogeneous product mix. Migration of skilled workers is driven by regional differences in utilities. The product mix influences the indirect utility through two channels, the wage and consumer surplus. A larger product mix influences the wage differential through a more competitive environment strengthening the dispersion force. More competition means less profits and thus a lower wage for skilled workers. On the other hand a more competitive environment increases consumer surplus, which diminishes agglomeration forces. The overall effect is that a larger product mix favours dispersion.

Multi product flexibility can be interpreted as a regional technological difference. The analysis of regions with firms with a different product mix, gives several interesting insights regarding agglomeration equilibria. At low marginal costs for additional products, with increasing product mix-disparity mobile workers are driven to the region with lower marginal costs until all skilled workers will settle there. The same applies to medium levels of marginal costs. At relatively high levels of marginal costs a small shock will destabilise the agglomeration-equilibrium and all skilled workers will move to the region with a larger product mix.

In the literature there are models with multi-product firms that highlight the effect of competition on the distribution of firm product sales. Feenstra and Ma (2008) and Eckel and Neary (2010) incorporate the so called "cannibalization

effect”¹ on the supply side and analyze a single globalised world with no trade barriers. Mayer, Melitz, and Ottaviano (2014) rely on the competition effects, which are driven by variations in the number of selling firms and the average price. They analyze multiple asymmetric regions with different trade costs.

Some models of multi-product firms have nested C.E.S. preferences. Allanson and Montagna (2005) analyze the close economy. Arkolakis and Muendler (2010) and Bernard, Redding, and Schott (2011) investigate also the scenario of the open economy. Different to Mayer, Melitz, and Ottaviano (2014), all models have C.E.S. preferences and a continuum of firms produces a continuum of products, thus they have fixed markups. Moreover the “cannibalization effect” is ruled out and differences in market conditions and variation of trade costs have no effect on the choice of product mix.

To my knowledge, the present model is the first in the spatial economics literature, that analyzes the effects of the multi-product mix of firms on their location. Ottaviano (2012) studies how firm heterogeneity in terms of productivity affects the balance between agglomeration and dispersion forces including endogenous markups. The present paper differs from this model because it studies the effect of different product mix of firms.

Closely related is the model of Behrens and Robert-Nicoud (2012). They study selection in a model based on Melitz and Ottaviano (2008). Individuals have to decide to move to a rural or urban area and their productivity is revealed after that decision. In their model a larger market size increases productivity through a finer division of labor and a selection process. Higher productivity increases market size through an migration incentive to urban areas. As in the model of Behrens, Duranton, and Robert-Nicoud (2010), agents learn their ability (“luck” and “talent”) after their location decision. Talented agents sort into larger places because they find more productive jobs, which in turn leads to an increase of productivity. Places with more talented people are thus larger markets.

Other models that incorporate sorting of heterogenous firms or agents are Nocke (2006), Baldwin and Okubo (2006), Okubo, Picard, and Thisse (2010), Picard and Okubo (2012), and Davis and Dingel (2013).

2 Closed economy

Consider an economy with a mass of identical unskilled workers L and a mass of identical skilled workers H .

¹Eckel and Neary (2010) define the “cannibalization effect” as the demand linkages between the varieties multi-product firms produce.

Preferences and consumption

All individuals have identical preferences. The quasi-linear quadratic utility function includes a homogenous good q_0^c and a continuum of varieties indexed by $i \in \Omega$

$$U^c = q_0^c + \alpha \int_{i \in \Omega} q_i^c di - \frac{\gamma}{2} \int_{i \in \Omega} (q_i^c)^2 di - \frac{\eta}{2} \left(\int_{i \in \Omega} q_i^c di \right)^2,$$

where $\gamma > 0$ is measuring product differentiation and α and η are positive index of substitution between the differentiated variety and the homogenous good. The price of the homogenous good will be the numeraire so the budget constraint is

$$q_0^c + \int_{i \in \Omega} p_i q_i^c di = I^c + \bar{q}_0^c,$$

where p_i is the price of variety i , I^c is the income, and \bar{q}_0^c is an exogenous initial endowment of the homogenous good to ensure positive consumption of it.

Due to quasi-linearity of preferences, all income effects are captured by the homogenous good. The inverse demand function for a variety is

$$p_i = \alpha - \gamma q_i^c - \eta Q^c, \quad Q^c = \int_{i \in \Omega} q_i^c di, \quad (1)$$

where Q^c is total individual consumption of the differentiated varieties. If I integrate across products and solve for Q^c , I get

$$Q^c = \frac{M\alpha - P}{\gamma + \eta M}, \quad P = \int_{i \in \Omega} p_i d\omega, \quad (2)$$

where M is the measure of consumed varieties. Substituting (2) into (1) gives individual consumption q_i^c

$$q_i^c = \left(\frac{\alpha\gamma + \eta M \tilde{p}}{\gamma + \eta M} - p_i \right) / \gamma.$$

Note that the average price $\tilde{p} = P/M < \alpha$ for any positive consumption of the differentiated good $Q^c > 0$ and that, therefore, the average price \tilde{p} cannot be higher than the choke price p^{max}

$$p^{max} \equiv \frac{\alpha\gamma + \eta M \tilde{p}}{\gamma + \eta M}, \quad (3)$$

at which demand becomes zero. The individual inverse demand of a variety depends on the choke price

$$p_i = p^{max} - \gamma q_i^c.$$

Total demand and total inverse demand are

$$q_i = q_i^c(L + H) = \frac{p^{max} - p_i}{\gamma}(L + H),$$

$$p_i = p^{max} - \frac{\gamma}{L + H}q_i. \quad (4)$$

As Ottaviano (2012) shows, an increase of the number of firms and a decrease in the average price leads to a rise in the elasticity of demand and makes competition tougher for all firms, but especially for low price firms. Welfare is given by the indirect utility function

$$V^c = I^c + \frac{1}{2} \left(\eta + \frac{\gamma}{N} \right)^{-1} (\alpha - \tilde{p})^2 + \frac{1}{2} \frac{N}{\gamma} \sigma_p^2,$$

where $\sigma_p^2 = (1/N) \int_{i \in \tilde{\Theta}} (p_i - \tilde{p})^2 di$ is the variance of prices and $\tilde{\Theta}$ is the largest subset of Θ that satisfies $p_i \leq p^{max}$.

Technology and production

The homogenous good is produced under perfect competition employing only workers L as input. One unit is produced with one worker so that the wage of workers is equal to unity.

The differentiated good is a continuum of varieties that are produced under monopolistic competition with increasing returns by using skilled as a fixed input and workers as a variable input. Firms first decide whether to enter the market and pay a sunk cost investment by bidding competitively for skilled workers. E skilled workers are hired to develop blueprints. Production of each variety exhibits constant returns to scale. As in Ottaviano (ibid.) firms finance their entry by borrowing from unskilled workers, whose earnings on their lending are driven to zero. The remuneration of skilled workers absorbs all average expected profits. Every firm may produce more than one variety but has one key variety corresponding to its “key competency”. After having bidden for skilled workers, every firm gets assigned the marginal costs for its key product c , equal to unit unskilled worker requirement, as a random draw from a common known continuous differentiable distribution with cumulative distribution function (c.d.f.) $G(c)$ and support $[0, c_M]$. Based on their draw, firms then decide to produce or not.

Firms can introduce new varieties, but each new variety comes with a customization cost because it pulls resources away from the firms core competency. The additional varieties have thus higher marginal costs of production.

Mayer, Melitz, and Ottaviano (2014) refer to this incremental production cost as a customization cost.

Let m be the index of varieties produced by a firm in increasing order of distance from the core variety $m = 0$. $v(m, c) = \omega^{-m}c$ is the marginal cost of variety m produced by a firm with core marginal cost c , where $\omega \in (0, 1)$. If ω goes to zero, firms will only be able to produce their core variety and the model is like the one of Melitz and Ottaviano (2008).

Since marginal revenue must equal marginal cost, the profit-maximizing output of a monopolistically competitive firm with cost v is

$$q(v) = \frac{L+H}{\gamma}[p(v) - v].$$

The cut-off cost v_D must be equal the profit maximizing rise $p(v_D) = v_D = p^{max}$, such that $q(v_D) = 0$. Only firms that have $v \leq v_D$ will be producing. Let $r(v) = p(v)q(v)$, $\pi(v) = r(v) - q(v)v$, $\mu(v) = p(v) - v$ be the revenue, profit, and markup of a variety with cost v , that can be rewritten as a function of v and v_D :

$$p(v) = (v_D + v)/2 \quad , \quad \mu(v) = (v_D - v)/2 \quad , \quad q(v) = \frac{L+H}{2\gamma}(v_D - v)$$

$$r(v) = \frac{L+H}{4\gamma}[(v_D)^2 - v^2] \quad , \quad \pi(v) = \frac{L+H}{4\gamma}(v_D - v)^2.$$

Firms that have too high costs for its core competency, that is $v > v_D$, will exit the market because they cannot produce profitably their core variety. This implies that $c_D = v_D$, which in turn defines the threshold for firm survival. All firms with core costs $c < c_D$ have positive profits taking also into account the entry cost. Some of the surviving firms will produce more than only the core variety, such that $v(m, c) \leq v_D \iff c \leq \omega^m c_D$. The number of varieties produced by a firm with core cost c is

$$M(c) = \begin{cases} 0 & \text{if } c > c_D, \\ \max\{m \mid c \leq \omega^m c_D\} + 1 & \text{if } c \leq c_D. \end{cases}$$

The number of varieties produced by a firm is an integer number and is decreasing for all $c \in [0, c_M]$ and I assume, that $c_M > c_D$ so exit rates are always positive. Fig. 1 shows the number of varieties $M(c)$ as an increasing step-function of the firm's productivity $1/c$. Firms with higher core productivity produce more varieties.

Once the mass of entrants $N_E = H/E$ is determined, the distribution of costs is set by the optimal product range of a firm $M(c)$ and the distribution of

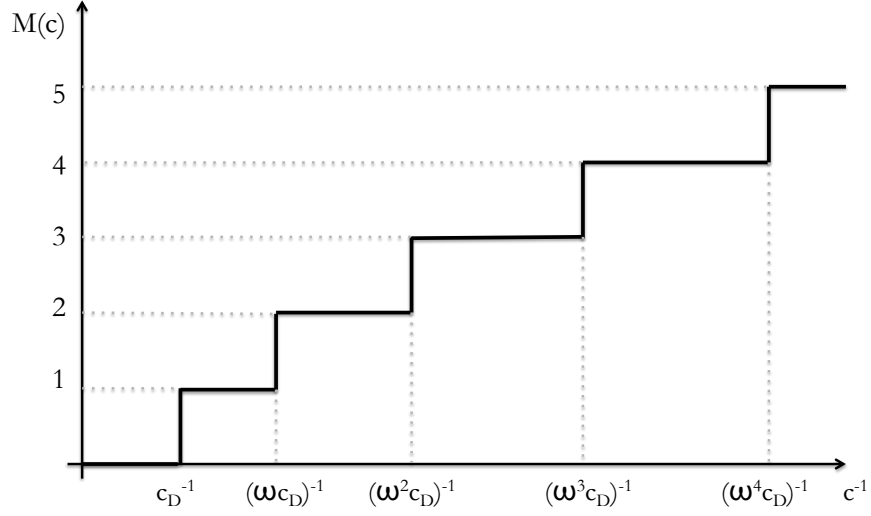


Figure 1: Number of varieties produced depending on firm productivity.

the core variety $G(c)$. The normalized measure of varieties per unit mass entrants $J(v)$ is defined exogenously by $G(\cdot)$ and ω such as $J(v) \equiv M_v(v)/N_E = \sum_{m=0}^{\infty} G(\omega^m v)$, with $M_v(v)$ as the measure of varieties produced at cost v or lower, given N_E entrants. If the mass of entrants is given, then a mass of $G(v)$ of core varieties will be produced at cost v or less, a mass of $G(\omega v)$ first additional varieties will be produced at cost v or less, a mass of $G(\omega^2 v)$ second additional varieties will be produced at cost v or less, and so on.

With $\Pi(c) = \sum_{m=0}^{M(c)-1} \pi(v(m, c))$ as the profit of a firm with cost c , the equilibrium free entry condition, where all expected profits are driven to zero, is

$$\int_0^{c_D} \Pi(c) dG(c) = \sum_{m=0}^{\infty} \left[\int_0^{\omega^m c_D} \pi(\omega^{-m} c) dG(c) \right] = wE. \quad (5)$$

Average expected profits must be equal to the entry cost, that is the remuneration of the skilled factor. (5) determines the cost cutoff $c_D = v_D$ which determines the aggregate mass of varieties

$$M = \frac{2\gamma}{\eta} \frac{\alpha - v_D}{v_D - \bar{v}}, \quad (6)$$

where \bar{v} is the average cost of all varieties

$$\bar{v} = \frac{1}{M} \int_0^{v_D} v dM_v(v) = \frac{1}{N_E J(v_D)} \int_0^{v_D} v N_E dJ(v) = \frac{1}{J(v_D)} \int_0^{v_D} v dJ(v),$$

which depends only from the cutoff v_D similar to the average price

$$\bar{p} = \frac{1}{M} \int_0^{v_D} p(v) dM_v(v) = \frac{1}{J(v_D)} \int_0^{v_D} p(v) dJ(v).$$

Parametrization of technology

So far the model applies for any distribution of the core cost draws $G(c)$. I assume that the marginal product of workers φ is Pareto distributed with shape parameter $k \geq 1$ and support $[1/c_M, \infty]$. The c.d.f of $c = 1/\varphi$ is $G(c) = (c/c_M)^k$, $c \in [0, c_M]$. If $k = 1$ the cost distribution is uniform on $[0, c_M]$ and if k goes to infinity, all firms have $c = c_M$. The truncated cost distribution of surviving firms will be given by $G_D(c) = (c/c_D)^k$, $c \in [0, c_D]$. All product varieties will have the same Pareto distribution as core competencies

$$J(c) = \sum_{m=0}^{\infty} G(\omega^m c) = \Omega G(c) \quad , \quad \Omega = (1 - \omega^k)^{-1}.$$

Ω is an index of multi-product flexibility, that must be equal to the average number of products produced

$$\frac{M}{N} = \frac{J(v_D) N_E}{G(c_D) N_E} = \Omega, \quad (7)$$

with N as the number of producer.

Free entry is possible if the cost of entry wE are equal average expected profits $\rho\tilde{\pi}$, with w as the wage of skilled workers. Solving for w , gives the free entry condition

$$w = \left(\frac{c_D}{c_M} \right)^k \frac{(L + H)c_D^2}{2\gamma(k+2)(k+1)E(1 - \omega^k)}. \quad (8)$$

If customisation cost for non-core varieties become very large ($\omega \rightarrow 0$), then multi-product flexibility goes to unity and (8) looks as in Ottaviano (2012).

The free entry condition (8) summarises how technology, market size, product differentiation, multi-product flexibility and market competition influence wages. A bigger market (larger L), less competition (smaller c_D), and a larger multi-product flexibility (lower ω) lead to a larger wage for the skilled factor. Technology improvements (smaller E or c_M) and an increase in product substitutability (larger γ) lead to smaller wages.

Given the Pareto parametrisation, the “zero-cutoff profit” condition, average price and welfare of unskilled and skilled, respectively, are given by

$$M = \frac{2\gamma(k+1)}{\eta} \frac{\alpha - c_D}{c_D}, \quad (9)$$

$$\bar{p} = \frac{2k+1}{2(k+1)} c_D,$$

$$V_L = 1 + \frac{1}{2\eta} (\alpha - c_D) \left(\alpha - \frac{k+1}{k+2} c_D \right),$$

$$V = w + \frac{1}{2\eta} (\alpha - c_D) \left(\alpha - \frac{k+1}{k+2} c_D \right).$$

An increase of skilled workers or toughness of competition does not change the average number of products produces by a firm $M/N = \Omega$ because the mass of surviving firms N rises at the same amount as the mass of varieties M .

Substituting $N = \rho N_E = \left(\frac{c_D}{c_M}\right)^k H/E$ into (7) gives

$$M = \left(\frac{c_D}{c_M}\right)^k \Omega H/E. \quad (10)$$

Combining Eq. (9) and Eq. (10) yields

$$\frac{2\gamma(k+1)}{\eta} \frac{\alpha - c_D}{c_D} = \left(\frac{c_D}{c_M}\right)^k (1 - \omega^k)^{-1} H/E. \quad (11)$$

(11) and (8) give a system of two equations with two unknown variables c_D and w . There is a unique value of c_D solving the “zero-cutoff profit” equilibrium condition (11). Its left-hand side is increasing whereas its right-hand side is decreasing in c_D .

Comparative statics of Eq. (11) are easily assessed because of their monotonicity of the left- and right-hand-sides. More skilled workers (larger H), lower entry costs (smaller E), weaker product differentiation (smaller γ), weaker preference for differentiated varieties with respect to the homogenous good (smaller α or larger η), all lead to a smaller c_D and therefore to tougher competition. If firm heterogeneity c_M increases, c_D increases but less than proportionate. On the left-hand side lower k decreases $(k+1)$ and on the right-hand side lower k raises the probability of entrance $(c_D/c_M)^k$ given that $c_D/c_M < 1$, and increases the product mix $\Omega = (1 - \omega^k)^{-1}$, so that c_D has to fall to keep Eq. (11). This means that in the present model if heterogeneity grows because the probability of already existing good draws increases, the increase in selection is larger compared to Ottaviano (2012). An increased product mix (larger Ω

i.e. larger ω) leads to a smaller c_D and therefore to tougher competition, or formally:

$$\frac{dc_D}{d\omega} < 0.$$

Once the equilibrium cutoff has been defined, the “free entry condition” Eq. (8) defines the equilibrium wage of skilled workers as an increasing function of c_D :

$$w = \left(\frac{c_D}{c_M}\right)^k \frac{(L+H)c_D^2}{2\gamma(k+2)(k+1)E(1-\omega^k)}.$$

For any given equilibrium cutoff a larger number of consumers (larger L or H , weaker differentiation (smaller γ), lower entry costs (smaller E), and larger product mix (larger ω) increase the wage of skilled workers. Some parameters have an ambiguous overall effect because they also influence Eq. (11) and therefore the equilibrium cutoff. More skilled workers lead to tougher competition and therefore lower expected profit but also to a larger market and larger expected profit. Lower entry costs lead to tougher competition and thus lower expected profit but also to fewer skilled workers sharing expected profits. Weaker product differentiation leads to tougher competition but also to larger firm size and therefore larger expected profits. As firms have a larger product mix it leads on the one hand to more competition but on the other hand to larger expected profits.

After the cutoff, the wage of skilled workers, and the number of firms are set, indirect utility of skilled workers can be evaluated:

$$V = w + \frac{1}{2\eta}(\alpha - c_D) \left(\alpha - \frac{k+1}{k+2}c_D \right), \quad (12)$$

where $S = V - w$ is consumer surplus. Tougher competition (lower c_D) increases consumer surplus because it leads to lower prices and more product variety. More competition lowers the wage of skilled workers because it reduces the probability of survival and average expected profits are lower.

3 Open Economy

To apply a spatial equilibrium I must extend the model to the open economy. While Mayer, Melitz, and Ottaviano (2014) allow for an arbitrary number of regions, in this case the two-regions scenario is sufficient, because I use a core-periphery spatial equilibrium. Let the two regions denote Home and Foreign.

All terms of Foreign are identical to the expressions of Home but marked with an asterisk. There are $L/2$ unskilled workers in each region and they are immobile. Skilled workers are geographically mobile. λ is the share of H that resides in the Home region while $1 - \lambda$ is the share that is living in the Foreign region with $\lambda \in [0, 1]$. Any variety can be exported from one region to the other subject to iceberg trade costs $\tau > 1$. The delivery cost for a variety m by a firm with core competency c is $\tau v(m, c) = \tau \omega^{-m} c$.

Following Ottaviano (2012) the timing of the events is as follows. Firms decide whether and where to enter the market taking the share of skilled workers in a region as given. If they enter they competitively bid for the local stock of skilled workers. After that each entrant is assigned its unit low-skilled worker requirement c as a random draw from a common known distribution $G(c)$ with support $[0, c_M]$ that is the same in both regions. Based on their draw entrants decide to produce or not in the location they entered. Firms finance their entry by borrowing from local workers whose earnings on their lending are driven to zero. In each location the local remuneration of the skilled workers absorbs all expected profits from entry. The wage of skilled is therefore $w = \rho \bar{\pi} / E$.

p^{max} is the choke price, that is the price threshold for positive demand

$$p^{max} = \frac{1}{\eta M + \gamma} (\gamma \alpha + \eta M \bar{p}),$$

where M is the total number of varieties sold in the Home region, that is domestically produced and imported varieties, and \bar{p} is the average price of varieties. The cost cutoffs for profitable domestic production and for profitable exports must satisfy

$$v_D = p^{max} \quad , \quad v_X = (p^{max})^* / \tau.$$

This means that $v_X = v_D / \tau$. The cutoff v_D summarises all effects of market conditions. Firm profit π is a function of the cutoff

$$\begin{aligned} \pi_D(v) &= \frac{L/2 + \lambda H}{4\gamma} (v_D - v)^2, \\ \pi_X(v) &= \frac{L/2 + (1 - \lambda)H}{4\gamma} \tau^2 (v_X - v)^2 = \frac{L/2 + (1 - \lambda)H}{4\gamma} \tau^2 (v_D^* - \tau v)^2. \end{aligned}$$

The cutoff for firm survival in the Home region will be $c_D = v_D$ and $c_X = v_X$ will be the firm export cutoff. No firm with $c > c_X$ can profitably export any varieties. A firm with core competency c will produce all varieties m such that $\pi_D(v(m, c)) \geq 0$ and will export a subset of varieties m such that $\pi_X(v(m, c)) \geq 0$. The total number of varieties produced and exporter by a firm with cost c are

$$M_D = \begin{cases} 0 & \text{if } c > c_D, \\ \max\{m | c \leq \omega^m c_D\} + 1 & \text{if } c \leq c_D, \end{cases}$$

$$M_X = \begin{cases} 0 & \text{if } c > c_X, \\ \max\{m | c \leq \omega^m c_X\} + 1 & \text{if } c \leq c_X. \end{cases}$$

With total number of sold products I can calculate total profits from domestic sales and exports

$$\Pi_D = \sum_{m=0}^{M_D(c)-1} \pi_D(v(m, c)) \quad , \quad \Pi_X = \sum_{m=0}^{M_X(c)-1} \pi_X(v(m, c)).$$

Before bidding for skilled workers, firms chose the location where to enter the market and produce. Setting the expected profit equal the entry cost yields

$$w = \frac{(L/2 + \lambda H)c_D^{k+2} + (L/2 + (1 - \lambda)H)(c_D^*)^{k+2}\tau^{-k}}{2\gamma(k+1)(k+2)c_M^k(1 - \omega^k)E}, \quad (13)$$

$$w^* = \frac{(L/2 + (1 - \lambda)H)(c_D^*)^{k+2} + (L/2 + \lambda H)c_D^{k+2}\tau^{-k}}{2\gamma(k+1)(k+2)c_M^k(1 - \omega^k)E}, \quad (14)$$

with $\lambda \equiv H$ and $1 - \lambda \equiv H^*$ as the fraction of workers residing in the Home and Foreign region, respectively. The indirect utilities for unskilled workers are

$$V_L = 1 + \frac{1}{2\eta}(\alpha - c_D) \left(\alpha - \frac{k+1}{k+2}c_D \right),$$

$$V_L^* = 1 + \frac{1}{2\eta}(\alpha - c_D^*) \left(\alpha - \frac{k+1}{k+2}c_D^* \right),$$

and the indirect utilities for skilled workers are

$$V = w + \frac{1}{2\eta}(\alpha - c_D) \left(\alpha - \frac{k+1}{k+2}c_D \right),$$

$$V^* = w^* + \frac{1}{2\eta}(\alpha - c_D^*) \left(\alpha - \frac{k+1}{k+2}c_D^* \right).$$

Indirect utilities are determined by the two cutoffs c_D and c_D^* and the geographical distribution of skilled workers λ . The two cutoffs are themselves a function of λ . To evaluate equilibrium I first need to establish the zero-cutoff profit condition.

The threshold price condition yields, together with the Pareto distribution of all prices for varieties sold in the Home location, the zero-cutoff profit condition linking the variety cutoff $v_D = c_D$ to the mass of varieties sold in the Home and the Foreign location, respectively

$$M = \frac{2(k+1)\gamma}{\eta} \frac{\alpha - c_D}{c_D},$$

$$M^* = \frac{2(k+1)\gamma}{\eta} \frac{\alpha - c_D^*}{c_D^*}.$$

Given the mass of entrants in the Foreign region $N_E^* = (1 - \lambda)H/E$, there will be $G(c_X^*)N_E^*$ firms exporting $\Omega\tau^{-1}G(c_X^*)N_E^*$ varieties to the Home location. The number of varieties sold in the Home and Foreign location, that is the locally produced and imported, are then

$$M = \left(\frac{c_D}{c_M}\right)^k (1 - \omega^k)^{-1} \lambda H/E + \left(\frac{c_D}{c_M}\right)^k \tau^{-k} (1 - \omega^k)^{-1} (1 - \lambda) H/E,$$

$$M^* = \left(\frac{c_D^*}{c_M}\right)^k (1 - \omega^k)^{-1} (1 - \lambda) H/E + \left(\frac{c_D^*}{c_M}\right)^k \tau^{-k} (1 - \omega^k)^{-1} \lambda H/E,$$

The spatial equilibrium

Recall $\lambda \equiv H$ and $1 - \lambda \equiv H^*$ are the fraction of skilled workers residing in the Home and Foreign region respectively. They are attracted by the region that gives them a higher indirect utility.

The equation system is solvable only by numerical solution and gives us the indirect utilities as a function of the spatial allocation of workers $V(\lambda)$ and $V^*(\lambda)$. The two equilibrium marginal cost cutoffs c_D and c_D^* are uniquely associated with any given λ . The two cutoffs define also consumer surplus and yield the indirect utilities.

As Ottaviano (2012) shows, a distribution $\lambda[0, 1]$ is a spatial equilibrium if and only if no skilled worker can get a higher indirect utility by changing location. If $\Delta V(\lambda) \equiv V(\lambda) - V^*(\lambda)$ there is a spatial equilibrium at $\lambda \in (0, 1)$ if $V(\lambda) = 0$ or at $\lambda = 0$ if $\Delta V(0) \leq 0$ or at $\lambda = 1$ if $\Delta V(1) \geq 0$.

Local markets adjust instantly when the mobile factor moves. The driving force is the current indirect utility differential:

$$\dot{\lambda} \equiv d\lambda/dt = \begin{cases} \Delta V(\lambda) & \text{if } 0 < \lambda < 1, \\ \min\{0, \Delta V(\lambda)\} & \text{if } \lambda = 1, \\ \max\{0, \Delta V(\lambda)\} & \text{if } \lambda = 0, \end{cases}$$

with t as time. I assume also, that every period firms have to invest the fix cost E and then draw their unit worker requirements, which are not brought to the next period. There is only a spatial equilibrium if $\lambda = 0$ because if $\Delta V(\lambda) > 0$, some skilled workers will move from the Home to the Foreign region. Conversely if $\Delta V(\lambda) < 0$ some will move to the Home region. A spatial equilibrium is stable if a marginal deviation brings the spatial allocation of skilled workers back to the original one. A dispersed equilibrium, that is $\lambda \in (0, 1)$, is an equilibrium if the slope of $\Delta V(\lambda)$ is negative in $\lambda = 1/2$. If $\lambda = 1$ or $\lambda = 0$ is an equilibrium it is always stable.

Agglomeration, dispersion and the product mix

In this section I examine what impact a different product mix will have on agglomeration. From the analysis of the isolated economy from Mayer, Melitz, and Ottaviano (2014) it is known that higher competition brings high cost firms to exit the market and remaining firms drop their high cost products. This leads to higher productivity on the firm and aggregate level. In the open economy firms respond to tougher competition in export markets in very similar ways by skewing their export product mix towards their better performing products, which is confirmed in their data. If firms have a different product mix, it must have an effect on the indirect utility of skilled workers, thus on the level of agglomeration in a region.

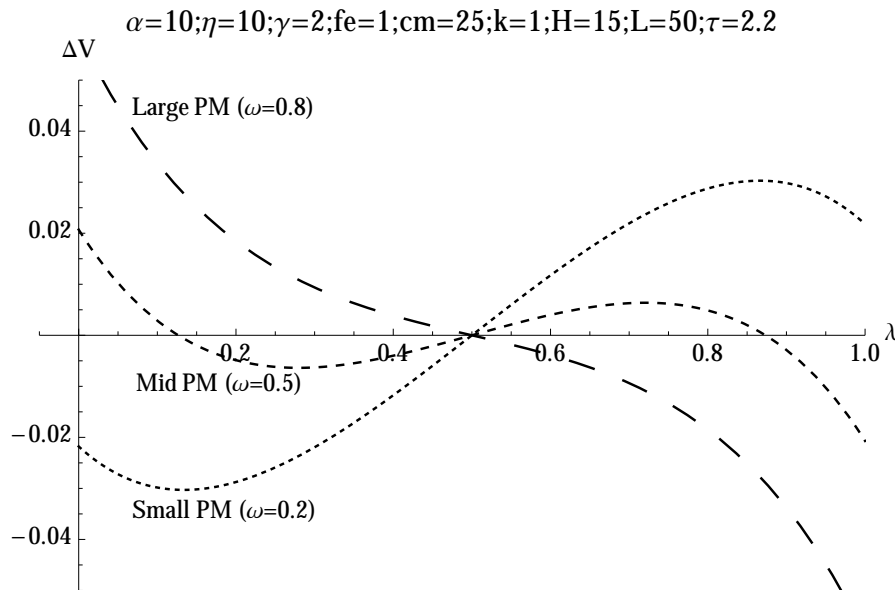


Figure 2: A larger product mix (PM) fosters dispersion.

Fig. 2 shows the effect of a larger product mix (larger ω) on agglomeration. A larger ω means firms have lower marginal costs for varieties in increasing order of distance from a firm's core competency. Recall, if ω goes to zero, firms produce only one product. When firms have lower marginal costs for their additional products, they will, everything else equal, have a more heterogeneous product mix. Fig. 2 shows, that a larger product mix (larger ω) shifts the balance in favour of dispersion forces.

Let us see how exactly the larger product mix is affected by the smaller marginal cost for additional products. Recall from Eq. (12) that the indirect utility differential is determined by two components: (A) the income differential and (B) the consumer surplus differential. Fig. 3 shows the wage and consumer surplus of skilled workers as a function of ω at the symmetric equilibrium $\lambda = 1/2$. The wage is monotonically increasing in ω while consumer surplus (and the cutoff) is a monotonically decreasing function of ω . The net effect of these two functions gives the hump-shaped indirect utility of skilled workers as a function of ω that is shown in Fig. 4.

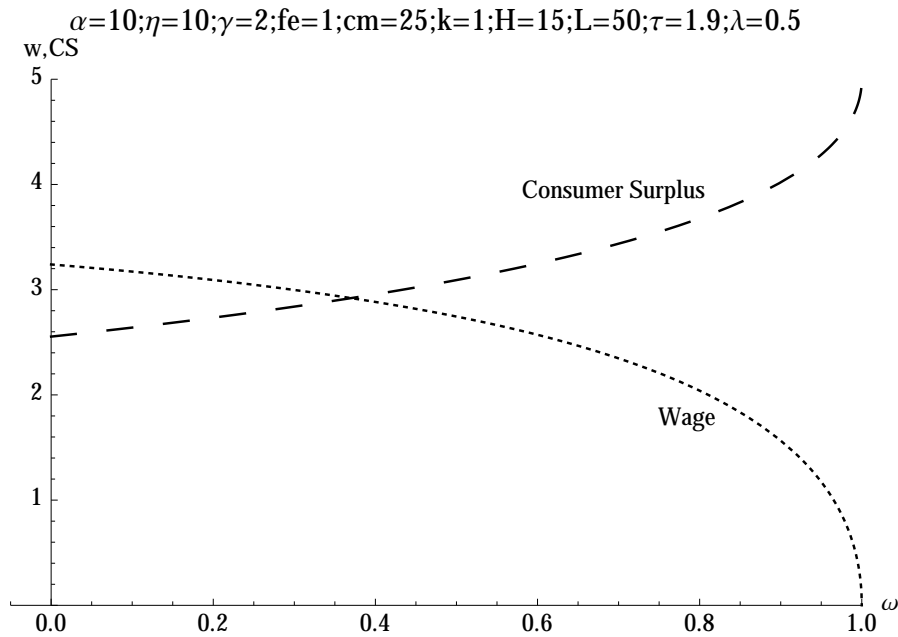


Figure 3: At the symmetric equilibrium a larger product mix increases competition (larger consumer surplus) but decreases expected profits (smaller wage).

The model does not allow for the derivation of a closed form “break point” because the value of the cutoff in the symmetric equilibrium $\lambda = 1/2$ is itself an implicit function of trade costs, so $c_D = c_D^* = c_S$, where $c_S = c_D(\lambda =$

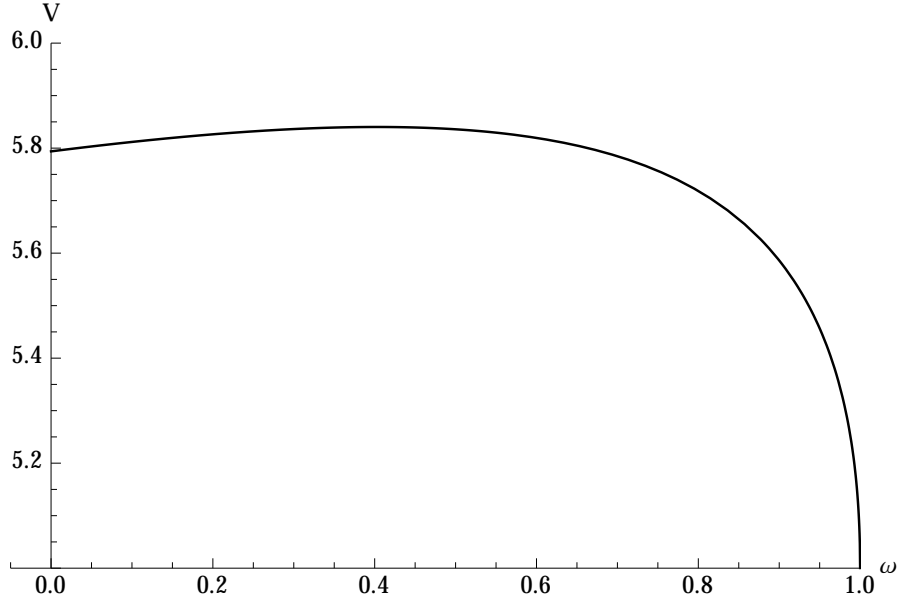


Figure 4: At the symmetric equilibrium a larger product mix has an hump-shaped effect on indirect utility of skilled workers.

$1/2) = c_D^*(\lambda = 1/2)$. This is shown in the “zero-cutoff profit condition” at the symmetric equilibrium

$$\frac{1 + \tau^{-k}}{2} \left(\frac{c_S}{c_M} \right)^k \frac{H}{E(1 - \omega^k)} = \frac{2\gamma(k+1)\alpha - c_S}{\eta c_S}, \quad (15)$$

where the sub letter S stands for the variables in the symmetric equilibrium $\lambda = 1/2$. If there is no trade ($\tau \rightarrow \infty$) and the endowment of skilled and unskilled workers is the same in each region, then Eq. (15) is the same as Eq. (9) in the closed economy. The comparative statics of the product mix with respect to the cutoff in the open economy are therefore the same as in the closed economy

$$\frac{\partial c_S}{\partial \omega} < 0. \quad (16)$$

To illustrate how a different product mix acts on agglomeration and dispersion, I use a numerical simulation and analyze the variation of wage and consumer surplus if there are small deviations from the symmetric equilibrium.

Fig. (5) shows the effect of a larger product mix on the wage differential of the two regions at different levels of geographic distribution of skilled workers. The more skilled workers in a region (larger λ) the larger is the negative wage differential, i.e. skilled workers will earn less. More skilled workers in a region have to share profits and have thus a lower income.

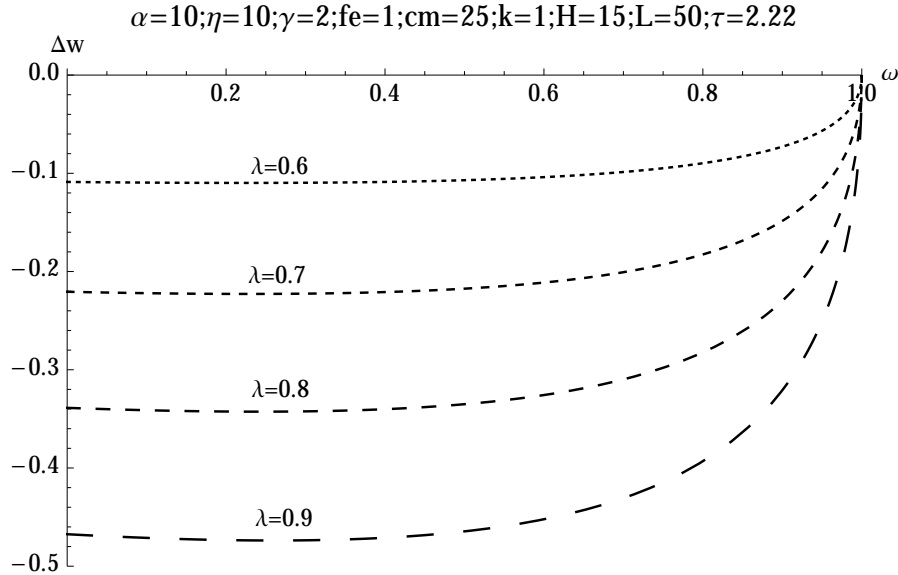


Figure 5: A larger product mix (larger ω) first slightly increases the wage gap and then decreases it sharply until it vanishes at $\omega = 1$.

A larger product mix (larger ω) first slightly increases the wage gap and then it decreases it sharply until it vanishes at $\omega = 1$. The increase is shown in Fig. 6 where the axis of the wage differential is on a smaller scale. This can be explained through the multiple effects ω has on the wage. On one hand product mix directly influences the wage through $(1 - \omega^k)$ and increases it. On the other hand there is an indirect effect through the cutoff that decreases the wage. At low levels of ω the wage differential gets larger because the effect on the cutoff is proportionally stronger. For larger values of ω the direct effect on the wage is proportionally stronger in the large region.

To highlight the different channels the product mix influences agglomeration and dispersion, I use the linearisation that is applied by Behrens, Duranton, and Robert-Nicoud (2010) and Ottaviano (2012). The wage differential of skilled workers is

$$\frac{w - w^*}{w_S} = 4 \left(\frac{1 - \tau^{-k}}{1 + \tau^{-k}} \right) \frac{H}{L + H} \frac{\lambda - 1/2}{1/2} - 2 \left(\frac{1 - \tau^{-k}}{1 + \tau^{-k}} \right)^2 \frac{k + 2}{k + \frac{\alpha}{\alpha - c_S}} \frac{\lambda - 1/2}{1/2}, \quad (17)$$

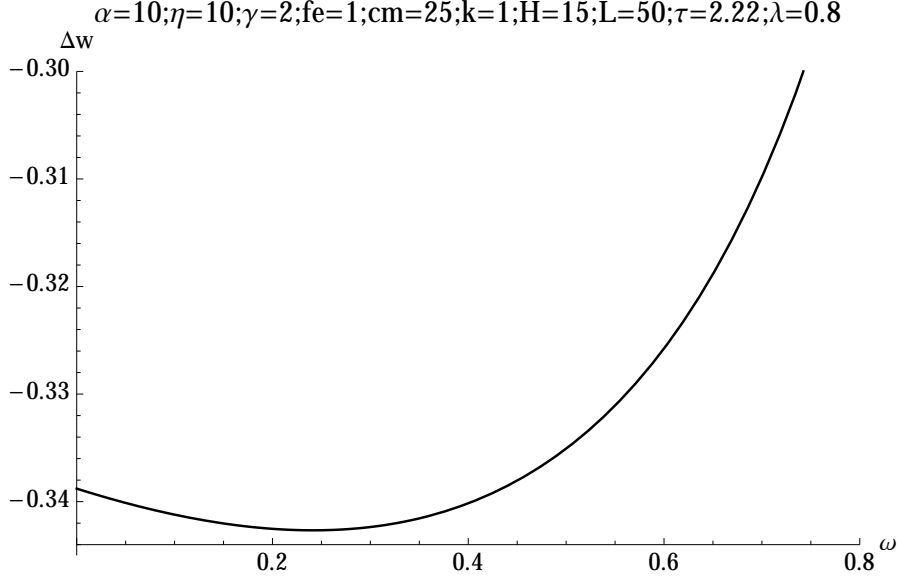


Figure 6: The u-shaped effect of a larger product mix (larger ω) at $\lambda = 0.8$.

where the first term on the right hand side captures agglomeration forces and the second term dispersion forces. Recall from Eq. (16) that ω influences the symmetric-equilibrium cutoff c_S . Since a larger ω decreases c_S , ceteris paribus, the second term on the right hand side will be smaller, strengthening the dispersion force.

Fig. 7 shows the effect of a larger product mix on the consumer surplus differential between the two regions at different levels of geographic distribution of skilled workers. The consumer surplus differential is positive and a monotonically decreasing function of the product mix. The larger a region the larger is the consumer surplus.

Differential surplus as a linearisation of equilibrium equations around symmetry gives:

$$\frac{S - S^*}{S_S} = 2 \frac{1 - \tau^{-k}}{1 + \tau^{-k}} \frac{1}{k + \frac{\alpha}{\alpha - c_S}} \left(\frac{c_S}{\alpha - c_S} + \frac{\frac{k+1}{k+2} c_S}{\alpha - \frac{k+1}{k+2} c_S} \right) \frac{\lambda - 1/2}{1/2} \quad (18)$$

Eq. (18) shows that the larger location enjoys higher consumer surplus. A larger product mix (larger ω) has an implicit impact on $\frac{S-S^*}{S_S}$ through c_S . Recall from Eq. (16), that a larger product mix decreases c_S , which means that agglomeration forces are weaker.

Fig. 8 shows the net effect of wage differential and consumer surplus differential depending on the product mix and at different levels of agglomera-

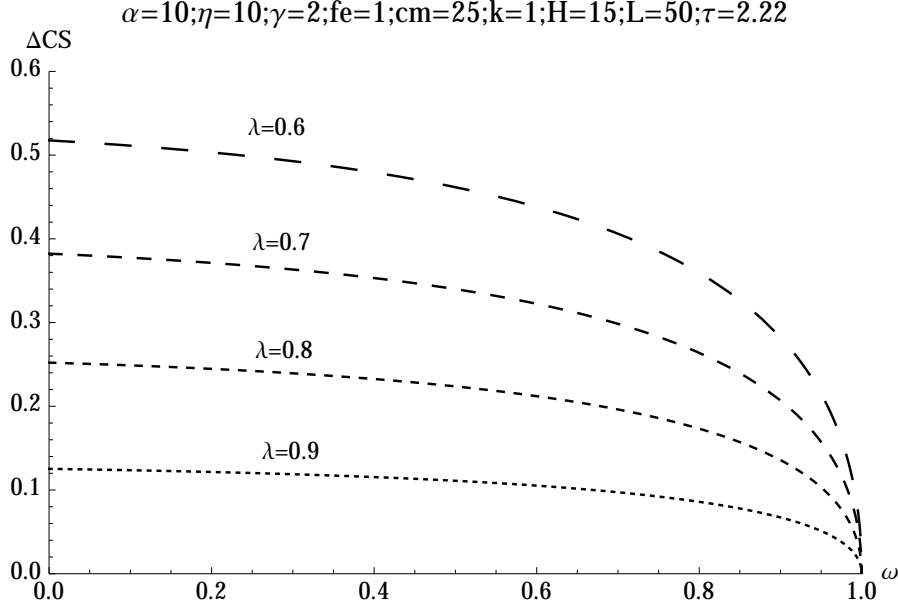


Figure 7: A larger product mix (larger ω) decreases the consumer surplus gap.

tion. With increasing ω the indirect utility differential decreases and the larger region has a larger indirect utility differential. Both findings are plausible following the results of the previous analysis of the linearised differentials. A larger product mix strengthens the dispersion force of wage differentials and weakens the agglomeration forces of consumer surplus differential. On both, the supply and demand side, there dispersion is fostered.

Agglomeration and asymmetric multi-product flexibility

Rather than interpreting Ω as the average number of products produced across all surviving firms, it can be seen as an exogenous change of multi product flexibility. Recall that $v(m, c) = \omega^{-m}c$ are the marginal costs for variety m produced by a firm with core marginal cost c . Mayer, Melitz, and Ottaviano (2014) show that as multi-product flexibility increases, firms respond by introducing more products. This additional production is skewed towards the better performing firms and leads to tougher competition (smaller cutoff). The values for aggregate productivity as industry output per worker, $\bar{\Phi}$, and industry deflated sales per worker, $\bar{\Phi}_R$, are:

$$\bar{\Phi} = \bar{\Phi}_R = \frac{k+2}{c_D k} \quad (19)$$

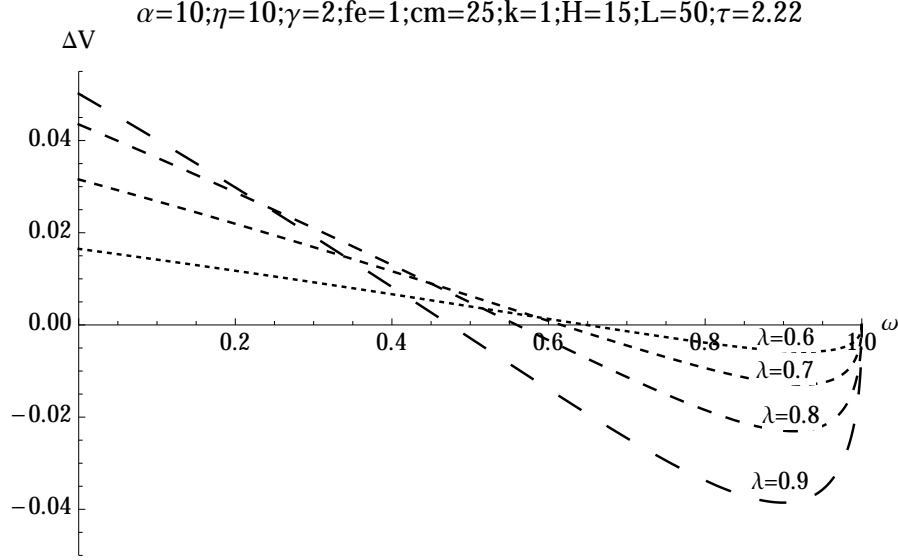


Figure 8: Indirect utility differential at different levels of agglomeration.

Since a larger ω decreases the cutoff, Eq. (19) shows, that aggregate productivity increases with more product flexibility. In other words, a larger ω can be seen as a exogenous technological progress. This has been largely ignored by the new economic geography literature because a symmetric increase in each region's productivity exerts no effect on the long-run location pattern in traditional models. Ehrlich and Seidel (2013) introduce firm heterogeneity and selection effects and show that this fosters agglomeration.

If multi product flexibility is interpreted as regional technological difference it might be worth looking at asymmetric regions. With $\Omega = (1 - \omega^k)$ and $\Omega^* = (1 - (\omega^*)^k)$ as the level of multi product flexibility in the Home and Foreign region respectively, the equilibrium conditions are:

$$w = \frac{(L/2 + \lambda H)c_D^{k+2} + (L/2 + (1 - \lambda)H)(c_D^*)^{k+2}\tau^{-k}}{2\gamma(k+1)(k+2)c_M^k(1 - \omega^k)E},$$

$$w^* = \frac{(L/2 + (1 - \lambda)H)(c_D^*)^{k+2} + (L/2 + \lambda H)c_D^{k+2}\tau^{-k}}{2\gamma(k+1)(k+2)c_M^k(1 - (\omega^*)^k)E},$$

$$\frac{2(k+1)\gamma}{\eta} \frac{\alpha - c_D}{c_D} = \left[\left(\frac{c_D}{c_M} \right)^k H/E \right] \left[(1 - \omega^k)^{-1}\lambda + \tau^{-k}(1 - (\omega^*)^k)^{-1}(1 - \lambda) \right],$$

$$\frac{2(k+1)\gamma}{\eta} \frac{\alpha - c_D^*}{c_D^*} = \left[\left(\frac{c_D^*}{c_M} \right)^k H/E \right] \left[(1 - (\omega^*)^k)^{-1}(1 - \lambda) + \tau^{-k}(1 - \omega^k)^{-1}\lambda \right].$$

Several interesting insights come up with regard to agglomeration equilibria. Fig. 9 illustrates three combinations of product mix at low levels of ω . Recall that ω regulates the marginal costs of additional varieties. A symmetric product mix $\omega = \omega^* = 0.2$ implies a stable equilibrium at $\lambda = 1/2$. Once the Foreign region has lower marginal costs for additional varieties (larger ω), the indirect utility differential shifts downward. Let us first look at small deviations, that is the medium-dashed line with $\omega = 0.2$ and $\omega^* = 0.201$. The difference of welfare between the Home and Foreign region is smaller for any given geographically distribution of skilled workers than at $\omega = \omega^* = 1/2$. The indirect utility differential is zero only at $\lambda \approx 0.55$. Increasing ω^* to 0.205 (large-dashed line) results in an unstable equilibrium at $\lambda \approx 0.65$ and a stable at $\lambda \approx 0.98$. Increasing the product mix even further gives a strictly negative welfare difference and only full agglomeration in the Foreign region with the larger ω is a stable equilibrium. With increasing ω -disparity mobile workers are driven more and more to the region with a larger ω . Once utility differential is strictly negative all skilled workers are agglomerated in the Foreign region.

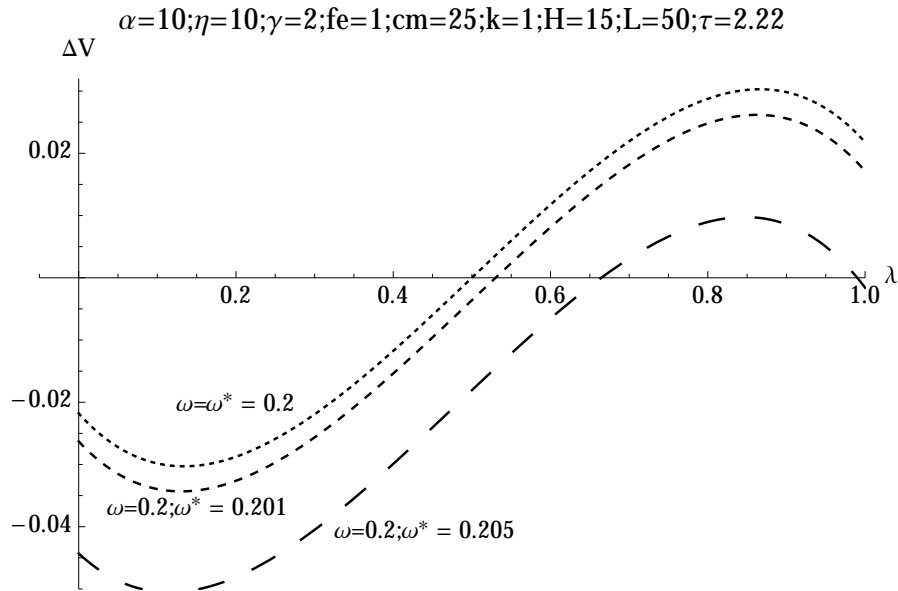


Figure 9: Equilibria for low levels of asymmetric product mix.

In the medium product mix scenario the stable interior stable equilibrium increases first from $\lambda = 1/2$ to $\lambda \approx 0.65$ and the unstable equilibria move from $\lambda \approx 0.075$ to $\lambda \approx 0.125$ and from $\lambda \approx 0.0875$ to $\lambda \approx 0.8$. With an even larger product mix there is only full agglomeration in the region with a larger ω . Also in this case with increasing ω -disparity mobile workers are driven more and

more to the region with a larger ω . Once utility differential is strictly negative all skilled workers are agglomerated in the Foreign region.

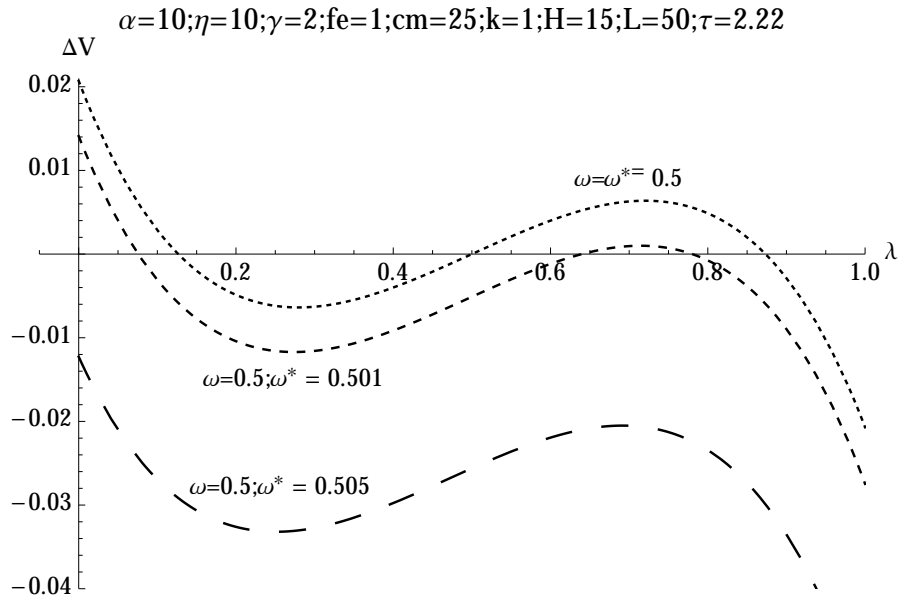


Figure 10: Equilibria for medium levels of asymmetric product mix.

The third scenario is with relatively large values of ω around 0.8. In Fig. 11 the symmetric equilibrium is unstable even in the symmetric product mix scenario $\omega = \omega^* = 0.8$. If the asymmetry regarding the marginal cost for additional varieties remains moderate ($\omega = 0.8, \omega^* = 0.801$) agglomeration in the Home region is stable as well as agglomeration in the Foreign region. This is quite unlikely because a relatively small shock in ω ($\Delta \approx 0.004$) will destabilize the agglomeration-equilibrium in the Home region and all skilled workers will settle in the Foreign region.

4 Conclusion

I have investigated how the product mix of firms effects agglomeration and dispersion in a region. By using state of the art trade theory models, I develop a two-region framework with heterogenous firms with a variable product mix, endogenous markups, and mobile skilled workers. Since a different product mix of firms effects the remuneration of skilled workers and their consumer surplus, there arise incentives to migrate.

Finally the findings qualify in two respects. First, my analysis points out that a larger product mix of firms favours dispersion. A larger product mix

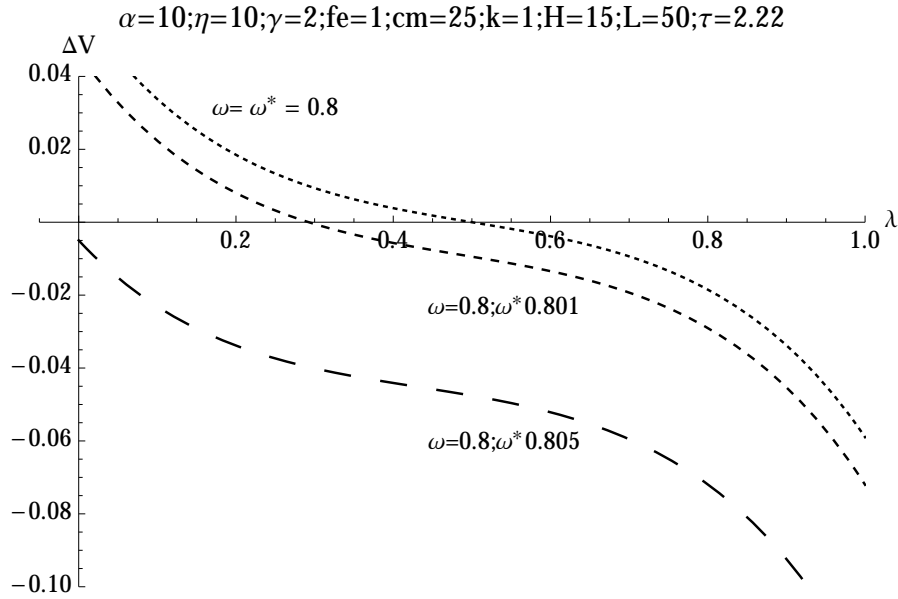


Figure 11: Equilibria for high levels of asymmetric product mix.

means firms have lower marginal costs for varieties in increasing order of distance from a firm's core competency. When firms have lower marginal costs for their additional products, they will, everything else equal, have a more heterogeneous product mix. Migration of skilled workers is driven by regional differences in utilities. The product mix influences the indirect utility through two channels, the wage and consumer surplus. A larger product mix influences the wage differential through creating a more competitive environment and thus strengthening the dispersion force. More competition means less profits and thus a lower wage for skilled workers. On the other hand a more competitive environment means a higher consumer surplus which diminishes agglomeration forces. The overall effect is that a larger product mix favours dispersion.

Second, product flexibility can be interpreted as regional technological difference. This analysis of asymmetric regions gives several interesting insights with regard to agglomeration equilibria. In the scenario with low marginal cost for additional products, with increasing product mix-disparity mobile workers are driven more and more to the region with lower marginal cost until all skilled workers are agglomerated in the Foreign region. The same applies at medium levels of marginal cost. At relatively high levels of marginal costs a small shock will destabilise the agglomeration-equilibrium in the Home region and all skilled workers will settle in the Foreign region with the lower marginal cost.

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