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Urban Costs, Wages, and Selection

by

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Urban Costs, Wages, and Selection

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In most countries housing and commuting costs amount for one-third or more of households’ budgets. These urban costs have substantial effects on wages and income inequality. Urban costs play an important role for locational and economic decisions of individuals and firms. This paper enriches the topic on urban costs with cornerstones in much recent micro-modeling in international trade and regional and urban economics by analyzing the effects of urban costs and firm heterogeneity with endogenous markups on wages and selection. With increasing commuting technology only more productive and less firms survive. Firms have higher costs because they have to pay higher wages to compensate workers for the higher urban costs. Despite higher wages welfare decreases with larger urban costs because consumer surplus decreases and there are larger expenses for housing and commuting. Wage premia are hump-shaped with respect to urban costs.

Keywords: Urban costs, heterogenous firms, wages, selection, college wage premium, inequality.

JEL Classification: F12; R12; E24

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1. Introduction

In most countries housing and commuting costs amount for one-third or more of households’ budgets. As Cavailhès, Gaigné, and Thisse (2004) find, around 40% of income in households in the United States is spent in housing and transportation. These urban costs affect firms since they have to pay higher land rents and wages. Local firms will be less competitive because urban costs are acting as entry barriers. The striking importance of urban costs for economic activity is evident, or as Behrens, Duranton, and Robert-Nicoud (2014) phrase it: “cities result from a tradeoff between agglomeration economies and urban costs.”

Cities and urban areas exhibit substantial heterogeneity in size (Gabaix, 1999). These size differences are accompanied by differences in wages, housing prices, and productivity (Glaeser, 2008). Recent research examines the effects of city size on wages and productivity (Behrens, Duranton, and Robert-Nicoud, 2014; Behrens and Robert-Nicoud, 2012; Baum-Snow and Pavan, 2012). But how do urban costs, one of the main dispersion forces, influence wages and firm productivity? This paper is the first to analyze disparities in wages in urban areas and its consequences in a theoretical model of city structure with heterogenous firms and labor and endogenous wage disparities.

Formally, workers and firms must choose an interurban location within a secondary employment center. Three types of labor serve as factors of production. There are two sectors: a perfectly competitive sector producing a homogenous good under constant returns to scale employing manufacturers only. This good is produced in a industrial district outside the city center and is chosen as numéraire. The monopolistic competitive sector produces varieties of a horizontally differentiated good employing engineers as fixed input and clerks as variable input. Monopolistic competitive firms decide wether and where to enter the market with uncertain outcome in terms of their productivity. Once firms know their productivity they decide to produce or not. All firms that produce the differentiated varieties are situated in the central business district. The third good that individuals consume is land and it is perfectly immobile. Each region is formed by a city spread along a one-dimensional space with a given central business district in the center and two industrial districts at the edges. Land rent increases with agglomeration and commuting is costly and decreases workers effective labor supply.

Our model is able to reproduce some interesting stylized facts that are observable in the data of wages and commuting costs on city level. The emphasis on labor heterogeneity naturally yields predictions about wage inequality, which is hump-shaped with respect to urban costs. Figure 1 depicts the spatial variation in the skilled wage premium, a relative price that captures
important dimensions of wage inequality, for the 283 largest metropolitan statistical areas (MSA) in 2005. The scatterplot shows substantial cross-city variation in skill premia, measured as differences in log average weekly wages between college graduates and high school graduates, and that it is hump-shaped with respect to log mean commuting time, our measure for urban costs.

![Scatterplot of skill premium and commuting time](image)

Figure 1: Skill premium and commuting time for the 283 largest metropolitan statistical areas (MSA) in 2005. See appendix A for a detailed description of the data.

Larger urban costs increase wages, because workers must be remunerated for the higher cost of living. This is in with line what Cavailhès, Gaigné, and Thisse (2004) find. But at very high urban costs the remuneration of the high-skilled decreases because profits of firms decrease due to high variable costs.

Another stylized fact that is seen in the data and is reproduced by our model is the increase of skill premia with city size (see Figure ?? in Appendix B).

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4For additional descriptive data about urban costs, wages and socioeconomic variables see appendix B).
Additionally our model shows, that with higher urban costs only more productive firms survive. This follows the empirical findings of Combes et al. (2012). Differently than in related models of heterogenous firms (Melitz and Ottaviano, 2008; Ottaviano, 2012) there are less firms in a more competitive environment. Moreover larger urban costs increase average price and markup. The higher wage is insufficient to adjust for the higher commuting and rent costs and the lower consumer surplus, thus welfare decreases. Similar effects of higher urban costs exist in the open economy. Lower trade costs do not influence selection but affect wages.

Moreover the effects of more firm heterogeneity on wage inequality is analyzed. Inequality increases with higher firm heterogeneity. The wage of unskilled urban individuals (variable input in monopolistic competitive sector) decreases the wage of the variable input factor. The reason lies in an unproportional change in toughness of competition and the number of surviving firms.

The paper is organized as follows. Section 2 presents the related literature. Section 3 introduces the model in a closed economy and analyses the interaction of urban costs, firm heterogeneity, and wages. Section 3 looks at the effects in an open economy with two cities. Section 4 concludes.

2. Literature overview

This paper is related to three strands of the literature on that are urban economic models of monocentric city, trade models with heterogeneous firms, and wage inequality and skill premia.

Frameworks of monocentric cities

Economists have studied for decades the fundamental question how firms and workers choose their locations depending on the magnitudes of spatial frictions (Fujita, Krugman, and Venables, 1999; Fujita and Thisse, 2002). We build a monopolistic competition model with heterogeneous entrepreneurs á la Melitz and Ottaviano (2008) with a traditional monocentric urban model following Alonso (1964), Muth (1969), and Mills (1967). There exists a spatial extension that imposes commuting and land costs on the corresponding workers.

Tabuchi, and Thisse (2002). They show how preference for variety on the demand side and increasing returns on the supply side interact with urban costs to shape the space-economy.

Murata and Thisse (2005) analyze the interaction of between commodities’ transportation costs and workers’ commuting costs in framework à la Dixit and Stiglitz (1977) with homogenous firms and constant markups.

Behrens et al. (2013) analyse urban costs in their study of spatial frictions. They use the demand system of Behrens and Murata (2007) to analyse spatial equilibrium.

**Trade and spatial models with heterogeneous firms**

There are a few related papers in the trade and spatial economics literature. While most of this literature explores the effects of urban costs in a classical framework à la Krugman (1980) or Krugman (1991), this paper enriches the discussion with cornerstones in much recent micro modelling in international trade and regional and urban economics. In particular this paper analyses the effects of urban costs, firm heterogeneity with endogenous markups on selection and heterogeneous wages. This is achieved by introducing firm heterogeneity and selection à la Melitz and Ottaviano (2008). This solves two broadly known problems of the constant-elasticity-of-substitution model of Dixit and Stiglitz (1977). First, markups display heterogeneity and variability across firms and time (Loecker et al., 2012). More productive firms charge higher markups and firms offset their reductions in marginal costs by raising markups. Second, firms differ in size and the markets they serve (Bernard and Jensen, 1995). Only few firms sell to a large number of distant markets. Those firms differ along various dimensions from firms that only serve the local market.


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1 Ottaviano (2011) specifies an outlook of NEG.
Urban wage inequality and skill premia

There is wide evidence in the literature that larger agglomerations bear more inequality. Glaeser, Resseger, and Tobio (2009) and Behrens and Robert-Nicoud (2012) show that larger cities have higher gini coefficients. City-level skill inequality can explain about one-third of the variation in city-level income inequality. Baum-Snow and Pavan (2013) find that in larger cities there is a greater overall variance in nominal wages. Wheeler (2001) shows that returns on schooling rise with city size. Davis and Dingel (2013) find a strong correlation between college premia and population size.

3. Closed urban economy

Consider an economy with a mass of identical manufacturers $W$, a mass of identical clerks $L$, a mass of identical engineers $H$, and a large amount of homogenous land. Land is perfectly immobile and the city stretches along a one-dimensional space $X$. In each location $x \in X$ there is one unit land available. Two goods are produced, a homogenous good under perfect competition and a differentiated good under monopolistic competition. All firms that produce the homogenous good are situated at the edge of the city in the industrial district (ID) where also the manufacturers live and work. The monopolistically competitive firms provide services like research, banking, insurance, and business services, and are situated at the origin of $X$, which is the central business district (CBD) where $x = 0$. Clerks and engineers consume one unit of land, inelastically supply one unit of labor, and commute to the CBD. The gross labor supply (GLS) of every individual is unity and is the sum of the effective labor supply the commuting time. In equilibrium the city will have the size \[ -\frac{(L + H)}{2}, \frac{(L + H)}{2} \]. Commuting is costly and takes the form of “iceberg” costs. The effective labor supply of an individual living at a distance $|x| \leq \frac{(L + H)}{2}$ from the CBD is

\[ s(x) = 1 - 2\theta |x|, \quad x \in \left[ -(L + H)/2, (L + H)/2 \right], \quad 0 < \theta < 1, \]

with $\theta$ as the productivity loss, or as Behrens et al. (2013) call it urban frictions. If the wage of clerks and engineers is equal total effective labor supply is

\[ S = \int_{-(L + H)/2}^{(L + H)/2} s(x) \, dx = (L + H)(1 - (L + H)\theta / 2), \] (1)

and the total labor supply of clerks and engineers respectively will be

\[ S_L^w = \frac{L}{L + H} S = L(1 - (L + H)\theta / 2) \]
\[ S_H^w = \frac{H}{L + H} S = H(1 - (L + H)\theta/2) \]

On the edge of the city the land rent is zero. The wage net of commuting costs and land rent of all clerks and engineers living on the edge is then
\[ s(-(L + H)/2)w = s((L + H)/2)w = [1 - (L + H)\theta]w, \]
with \( w \) as the wage rate of clerks and engineers. The wage net of commuting costs and land rent must be equal in all areas
\[ s(x)w - R(x) = s(-(L + H)/2)w = s((L + H)/2)w = [1 - (L + H)\theta]w, \]
so that equilibrium land rent is
\[ R(x) = (1 - 2\theta|x|)w - [1 - (L + H)\theta]w. \]

If the wage of clerks is different from the wage of engineers the urban structure changes. Since the land rent does not discriminate between clerks and engineers, the factor with the higher wage rate will find it more attractive living near the CBD because its commuting costs are lower. Let us assume that \( w_H > w_L \) with \( w_L \) and \( w_H \) as the wage rate of clerks and engineers respectively. Then engineers have an advantage settling near the CBD since
\[ s(x)w_H - R(x) > s(x)w_L - R(x). \]
The effective labor supply of an engineers and clerks respectively living at a distance \(|x| \leq (L + H)/2\) from the CBD is
\[ s_H(x) = 1 - 2\theta|x| , \quad x \in [-H/2, H/2] , \quad 0 < \theta < 1, \]
\[ s_L(x) = 1 - 2\theta|x| , \quad x \in [-(L + H)/2, -H/2] \cup [H/2, (L + H)/2] , \]
The total effective labor supply is
\[ S_H = \int_{-H/2}^{H/2} s_H(x) \, dx = H(1 - H\theta/2). \] (2)
\[ S_L = \int_{-(L+H)/2}^{(L+H)/2} s_L(x) \, dx - \int_{-H/2}^{H/2} s_L(x) \, dx = L - L(2H + L)\theta/2. \] (3)
Note that to ensure positive labor supply \( H < \theta/2, L < 2/\theta \) and \( H < \frac{2 - \theta L}{2\theta} \).

On the edge of the city the land rent is zero. The wage net of commuting costs and land rent of all individuals living on the edge is
\[ s_L(-(L + H)/2)w_L = s_L((L + H)/2)w_L = [1 - (L + H)\theta]w_L. \]
The rent of individuals with the lower wage living closest to the CBD is
\[ R_L(H/2) = [(1 - (L + H)\theta) - (1 - H\theta)]w_L = L\theta w_L. \]
Individuals with the higher wage living furthest away from the CBD will also pay a land rent $L\theta w_L$. The wage net of commuting costs and land rent must be equal in all locations where clerks and engineers respectively live so that

$$s_H(H/2)w_H - L\theta w_L = s_H(x)w_H - R_H(x),$$

$$s_L((L + H)/2)w = s_L(x)w_L - R_L(x),$$

with $R_H(x)$ and $R_L(x)$ as the land rent in the area where clerks and engineers workers live respectively at a distance $|x|$ from the CBD. The equilibrium land rent will then be

$$R_H(x) = \theta(H - 2|x|)w_H + L\theta w_L,$$

$$R_L(x) = \theta((L + H) - 2|x|)w_L.$$

Figure 2 shows the urban structure and land rent. The highest rent the factor with the lower wage will pay is in $|x| = H/2$ that is $R_{Lmax} = R_L(H/2) = R_L(-H/2) = L\theta w_L$. The highest rent the factor with the higher wage will pay is near the CBD with $x = 0$ that is $R_{Hmax} = R_H(0) = H\theta w_H + L\theta w_L$.

Figure 2: The rent and urban structure of a city with $w_H > w_L$.

The aggregate land rent (ALR) is

$$ALR_H = \int_{-H/2}^{H/2} R_H(x) \, dx = (Hw_H + 2Lw_L)H\theta/2,$$

$$ALR_L = \int_{-(L+H)/2}^{(L+H)/2} R_L(x) \, dx - \int_{-H/2}^{H/2} R_L(x) \, dx = L^2w_L\theta/2.$$
Every individual owns an equal share of land and in addition to her wage she gets an income of \((ALR_L + ALR_H)/(L + H) = \frac{(H^2w_H + L(2H + L)w_L)\theta}{2(L + H)}\). The income net of urban costs is

\[
I_H = (1 - \theta H)w_H - L\theta w_L + \frac{(ALR_L + ALR_H)}{(L + H)} = w_H - \frac{(H + 2L)w_H + L^2w_L)\theta}{2(L + H)},
\]

(4)

\[
I_L = (1 - \theta(L + H))w_L + \frac{(ALR_L + ALR_H)}{(L + H)} = w_L - \frac{(H + L)w_L\theta + (H^2w_H + L(2H + L)w_L)\theta}{2(H + L)}.
\]

(5)

**Preferences**

Regarding consumption all workers have identical preferences. The quasi-linear quadratic utility function includes a homogenous good \(q^c_0\) and a continuum of varieties of a horizontally differentiated good \(q^c(\omega)\).

\[
U^c = q^c_0 + \alpha \int_0^N q^c(\omega) d\omega - \frac{\gamma}{2} \int_0^N (q^c(\omega))^2 d\omega - \frac{\eta}{2} \left( \int_0^N q^c(\omega) d\omega \right)^2,
\]

where \(N\) is the number of available individual varieties \(\omega\), \(\gamma\) is measuring product differentiation and \(\alpha\) and \(\eta\) are an index of substitution pattern between the differentiated variety and the homogenous good. All parameters are positive. The price of the homogenous good will be the numeraire. The budget constraint is

\[
q^c_0 + \int_0^N p(\omega)q^c(\omega) d\omega = I^c + q^c_0,
\]

with \(p(\omega)\) as the price of variety \(\omega\), \(I^c\) as the income, and \(q^c_0\) as an exogenous initial endowment of the homogenous good to ensure positive consumption of it.

**Technology**

The homogenous good is produced under perfect competition employing only manufacturers \(W\) as input. One unit is produced with one worker. With marginal cost pricing and by using the price of the homogenous good as numeraire gives a wage equal to unity.
The differentiated good is a continuum of varieties that are produced under monopolistic competition with increasing returns by using engineers as a fixed input and clerks as a variable input. Firms first decide whether to enter the market and pay a sunk cost investment by bidding competitively for engineers. Firms are hired to develop blueprints. Following Ottaviano (2012), firms finance their entry by borrowing from manufacturers, whose earnings on their lending are driven to zero. The engineers remuneration absorbs all average expected profits. After having bidden, every firm gets assigned its unit clerk requirement $c$ as a random draw from a common continuous differentiable distribution with cumulative distribution function (c.d.f.) $G(c)$ and support $[0, c_M]$. Based on their draw, firms then decide to produce or not. As in Melitz and Ottaviano (2008), Behrens, Duranton, and Robert-Nicoud (2014), and Behrens and Robert-Nicoud (2012), the marginal product of clerks $\varphi$ is Pareto distributed with shape parameter $k \geq 1$ and support $[1/c_M, \infty)$. The c.d.f of $c = 1/\varphi$ is $G(c) = (c/c_M)^k$, $c \in [0, c_M]$.

The two parameters $c_M$ and $k$ regulate the cost draws, hence they decide the degree of heterogeneity of firms. The scale parameter $c_M$ regulates the “richness” or “number” of different unit labor requirements that can be drawn. A larger $c_M$ means that entering firms can draw larger $c$. The shape parameter $k$ measures the similarity of probability of cost draws, that we shall call “evenness”. With $k = 1$ the distribution is uniform and evenness is maximal. With increasing $k$ the distribution becomes more concentrated at higher cost draws near $c_M$. With $k$ going to infinity all draws will be $c_M$. Smaller $k$ leads to more heterogeneity by making low-cost draws more likely.

**Consumption**

Due to quasi-linearity of preferences, all income all income effects are captured by the homogenous good. The inverse demand function for a variety is

$$p(\omega) = \alpha - \gamma q^c(\omega) - \eta Q^c, \quad Q^c = \int_0^N q^c(\omega) \, d\omega, \quad (6)$$

where $Q^c$ is total individual consumption of the differentiated varieties. If we integrate across products and solve for $Q^c$ we get

$$Q^c = \frac{N\alpha - P}{\gamma + \eta N}, \quad P = \int_0^N p(\omega) \, d\omega. \quad (7)$$

Substituting (7) into (6) gives individual consumption $q^c(\omega)$

$$q^c(\omega) = \left( \frac{\alpha \gamma + \eta N \bar{p}}{\gamma + \eta N} - p(\omega) \right) / \gamma.$$
Note that the average price $\bar{p} = P/N < \alpha$ for any positive consumption of the differentiated good $Q^c > 0$ and that, therefore, the average price $\bar{p}$ cannot be higher than the choke price $p_C$

$$p_C \equiv \frac{\alpha \gamma + \eta N \bar{p}}{\gamma + \eta N}, \quad (8)$$

at which demand becomes zero. The individual inverse demand of any variety can be rewritten as

$$p(\omega) = p_C - \gamma q^c(\omega).$$

Total demand and total inverse demand are

$$q(\omega) = q^c(\omega)(W + L + H) = \frac{p_C - p(\omega)}{W + L + H},$$

$$p(\omega) = p_C - \frac{\gamma}{W + L + H} q(\omega). \quad (9)$$

As Ottaviano (2012) shows, an increase of the number of firms and a decrease in the average price leads to a rise in the elasticity of demand and makes competition tougher for all firms but especially for low price firms. Welfare is given by indirect utility function

$$V^c = I^c + \frac{1}{2} \left( \eta + \frac{\gamma}{N} \right)^{-2} (\alpha - \bar{p})^2 + \frac{1}{2} \sigma_p^2,$$

where $\sigma_p^2 = (1/N) \int_0^{c_C} (p(\omega) - \bar{p})^2 d\omega$ is the variance of prices and $c_C$ is the cut-off unit requirement of the variable input.

**Production**

Since marginal revenue must equal marginal cost, the profit-maximizing output of a monopolistically competitive firm with unit clerk requirement $c$ is

$$q(c) = \frac{W + L + H}{2 \gamma} (p_C - cw_L),$$

with $w_L$ as the wage of the clerks. The cut-off unit clerk requirement $c_C$, such that $q(c_C) = 0$, will be $c_C = p_C/w_L$. Only firms that have $c \leq c_C$ will be producing. If the unit clerk requirement distribution of producers is $G^*(c) = G(c)/G(c_C) = (c/c_C)^k$,

$$\rho = \left( \frac{c_C}{c_M} \right)^k, \quad \sigma_p^2 = \frac{k}{(k+1)^2(k+2)} c_C^2 w_L, \quad \bar{c} = \frac{k}{k+1} c_C,$$
with $\rho$ as the success rate of entrance and $\tilde{c}$ as the average unit clerk requirement. With $p_C = c_C w_L$, firm output and price with the profit maximisation condition will be

$$q(c) = \frac{W + L + H}{2\gamma} w_L (c_C - c) \quad , \quad p(c) = \frac{w_L}{2} (c_C + c).$$

Markup, revenue and profit functions are

$$\mu(c) = \frac{w_L}{2} (c_C - c) \quad , \quad r(c) = \frac{W + L + H}{4\gamma} w_L^2 (c_C^2 + c^2) \quad , \quad \pi(c) = \frac{W + L + H}{4\gamma} w_L^2 (c_C - c)^2.$$

Firms with higher productivity (lower $c$) have a higher output and a higher revenue and charge lower prices but higher markups than firms with higher productivity. The higher the wage of clerks the higher is the price a firm charges. Now average price, average markup and average output can be evaluated

$$\bar{p} = p(\bar{c}) = \frac{2k + 1}{2(k + 1)} c_C w_L, \quad \bar{\mu} = \mu(\bar{c}) = \frac{c_C w_L}{2(k + 1)}, \quad \bar{q} = q(\bar{c}) = \frac{W + L + H}{2\gamma(k + 1)} c_C w_L.$$

The average profit of producing firms is

$$\bar{\pi} = \int_0^{c_C} \pi(c) \, dG^*(c) = \frac{W + L + H}{2\gamma(k + 2)(k + 1)} c_C^2 w_L^2.$$

The free entry condition states that average expected profits from entering the market must be zero $\rho \bar{\pi} - f_E w_H = 0$ that is

$$f_E w_H = \rho \frac{W + L + H}{2\gamma(k + 2)(k + 1)} c_C^2 w_L^2, \quad (10)$$

where $w_H$ is the wage of engineers. The zero cut-off profit condition ($c_C w_L = p_C$) gives the number of firms that survive and produce

$$N = \frac{2\gamma(k + 1) \alpha - c_C w_L}{\eta c_C w_L}. \quad (11)$$

Since $N_E = N/\rho$ entrants employ $f_E$ engineers and $N$ firms need $\bar{c} = \frac{k}{k+1} c_C$ clerks to produce, labor demand for clerks and engineers respectively is

$$D_L = \frac{k}{k + 1} c_C N, \quad (12)$$

$$D_H = \frac{f_E}{\rho} N. \quad (13)$$
Equilibrium

Substituting (11) in (12) and (13) and using (2) and (3) yields labor market equilibrium. Combined with (10) we obtain an equation system with three unknown variables $w_L$, $w_H$, and $c_C$

$$L - L(2H + L)\theta/2 = \frac{k}{k+1} \frac{c_C 2\gamma(k+1) \alpha - c_C w_L}{c_C w_L},$$  

(14)

$$H(1 - H\theta/2) = \frac{f_E}{\left(\frac{c_C}{c_M}\right)} \frac{2\gamma(k+1) \alpha - c_C w_L}{c_C w_L},$$  

(15)

$$w_H = \left(\frac{c_C}{c_M}\right)^k \frac{W + L + H}{2\gamma(k+2)(k+1)f_E} c_C^2 w_L^2.$$  

(16)

The equation system can be solved analytically only for given integer values of $k$ and we use numerical simulation. Welfare can be evaluated by the following indirect utility function

$$V^c = I^c + \frac{1}{2\eta} (\alpha - c_C w_L) \left( \alpha - \frac{k+1}{k+2} c_C w_L \right),$$

where the second term on the right hand side is consumer surplus. Using (4) and (5), the indirect utility of engineers and clerks respectively is

$$V_H = w_H - \frac{(H(H + 2L)w_H + L^2w_L)\theta}{2(L + H)} + \frac{1}{2\eta} (\alpha - c_C w_L) \left( \alpha - \frac{k+1}{k+2} c_C w_L \right),$$

$$V_L = w_L - (H + L)w_L \theta + \frac{(H^2 w_H + L(2H + L)w_L)\theta}{2(H + L)} + \frac{1}{2\eta} (\alpha - c_C w_L) \left( \alpha - \frac{k+1}{k+2} c_C w_L \right).$$

Firm heterogeneity

Before we analyse the effects of urban costs on wages and selection let us briefly check what influence firm heterogeneity has on wage disparity and average productivity. More cost-increasing richness (larger $c_M$) decreases the survival probability $\rho$ and therefore the number of producing firms will decreases (smaller $N$). Less producing firms means less competition and the cut-off unit clerk requirement increases (larger $c_C$). Fig. 3 and Fig. 4 show, that

\[\text{Parameters are taken from Ottaviano (2012).}\]
if firm heterogeneity grows because there are additional bad draws, selection decreases.

The opposite applies for more cost-decreasing richness. A smaller $k$ increases survival probability and thus more (larger $N$) and only more efficient firms (smaller $c_C$) will produce at a lower average input requirement $\bar{c}$. If heterogeneity grows because probability of already existing good draws increases, selection gets tougher and more firms produce. This effect is known from related models (Melitz and Ottaviano, 2008; Ottaviano, 2012).

$$\alpha=10; \gamma=2; \delta=1; \eta=10; H=20; L=20; W=30$$

Figure 3: The effect of more firm heterogeneity (higher $c_M$, lower $k$) on the cost cutoff.

The effect of more firm heterogeneity on wages is ambiguous. The wage of clerks decreases with more cost-increasing richness. With a larger $c_M$ average variable input requirement $\bar{c}$ increase and more variable input factor is needed. But with a larger $c_M$ there are also less firms that produce. The number of producing firms decreases proportionally stronger than average clerk input requirement increases and following equation (12) therefore $w_L$ will decrease. Also more cost-decreasing evenness decreases the wage of clerks. A lower $k$ decreases average variable input requirement $\bar{c}$ but increases the number of producing firms. Since a smaller $k$ decreases the average clerk input requirement proportionally stronger than the number of producing firms, the wage of clerks will decrease.
Figure 4: The effect of more firm heterogeneity (higher $c_M$, lower $k$) on the number of surviving firms (selection).

The wage of engineers increases with more cost-increasing richness. A larger $c_M$ increases average profits $\bar{\pi}$ proportionally stronger than survival probability so that expected average profits increase. Since engineers get all expected average profits and labor demand stays constant, $w_H$ increases. But as Fig. 5 shows, at higher levels of $c_M$ and lower levels of $k$ this effect decreases and the wage of engineers stagnates. The reason is, that at lower $k$ competition is tougher so average expected profits are lower. The effect of more cost-decreasing evenness (smaller $k$) on the wage of engineers depends on the level of cost-increasing richness. At low levels of $c_M$ more cost-decreasing evenness (smaller $k$) decreases $w_H$. At low $c_M$ a smaller $k$ lowers average expected profits more than at high $c_M$ because competition is tougher.

Wage inequality is examined through the skill premium, that is the difference in average wages of employees whose highest educational attainment is a bachelor’s degree and those whose is a high school degree. In our model the skill premium is the difference between the wage rate of engineers and clerks. More cost-increasing richness increases the skill premium (Fig. 6). Income inequality occurs from the selection effect. Everything else equal, in markets with weaker competition (higher $c_C$) the skill premium will be larger. With a larger $c_M$ the cut-off will be larger. First, average expected profits will increase, and second, the wage of clerks will decrease. More cost-decreasing evenness
also increases inequality. A smaller $k$ decreases the wage of clerks. This effect dominates the impact of a smaller $k$ on the wage of engineers. The literature finds a positive relationship between skill premium and city size (Wheeler, 2001; Baum-Snow and Pavan, 2012; Davis and Dingel, 2013). Larger cities are affiliated to more selection (smaller $c_C$), that is more cost-decreasing evenness. Interestingly in our model also more cost-increasing richness creates more inequality.

After assessing the effect of firm heterogeneity on the cut-off and the wage of clerks, we can analyse the effects of firm heterogeneity on variable costs of the least profitable firm, that is $c_C \times w_L$. This term is important since it determines average price, average markup, average cost, and average output. Fig. 7 shows that the variable costs of the least productive firm increase with more cost-increasing richness (larger $c_M$).

This increases average price, average markup, average cost, and average output. More cost-decreasing evenness (lower $k$) has the opposite effect and decrease the variable costs of the least productive firm.

**Urban costs**

From the urban structure model we known that urban costs reduce effective labor supply and indirect utility. Let us first look how urban costs influence
Figure 6: More firm heterogeneity (smaller $k$, larger $c_M$) increases wage inequality.

Figure 7: The effect of firm heterogeneity on the variable input costs of the least productive firm.
selection. Fig. 8 and Fig. 9 show that an increase in urban costs increases selection (lower $c_C$) and lowers the number of firms. Differently than in related models (Melitz and Ottaviano, 2008; Ottaviano, 2012), where a lower cut-off is related to a higher number of producing firms, more urban costs reduce both, the cut-off and the number of surviving firms.

$$\alpha = 10; \gamma = 2; \kappa = 1; \kappa = 10; \eta = 20; \kappa = 20; W = 30$$

**Figure 8:** Larger urban costs (larger $\theta$) bears more selection.

Urban costs decrease the supply of engineers and therefore reduce the number of entrants $N_E = S_H / f_E$. Since $N_E = N(c_M / c_C)^k$, the number of producers has to decline or the cut-off has to increase. Eq. (12) implies that a smaller supply of clerks through higher urban costs reduces $N$ or $c_C$ or both. Less clerk supply increases the $w_L$ so Eq. (11) implies the cut-off has to decrease which means probability of survival $\rho$ will be smaller. $N_E = N(c_M / c_C)^k$ implies that a smaller $\rho$ lets $N$ decrease.

The remuneration of engineers is hump-shaped with respect to urban costs. At low levels of $\theta$ the wage increases. Less supply of engineers means less entrants that have to share average expected profits. But as shown in Fig 9, at very high levels of $\theta$ the wage of engineers decreases. Firms have to pay high wages for clerks and average expected profits decrease. This is in line with what Cavailhès, Gaigné, and Thisse (2004) find. Firms must pay higher wages to offset urban costs. With higher urban costs only more productive
firms (with lower \(c\)) survive. This follows the empirical findings of Combes et al. (2012).

Figure ?? shows, that wage inequality is hump-shaped with respect to urban costs. At lower levels of \(\theta\) the remuneration of engineers increases proportionally stronger than the wage for clerks. At very high levels of \(\theta\) average expected profits of firms, and thus the engineers remuneration, decrease and the skill premium declines.

Fig. ?? shows, that larger urban costs decrease welfare for both factors. The magnitude of increased expenditures for the land rent and a decreased consumer surplus is larger than the increased wages. Fig. ?? shows, that the variable costs of the least productive firm increase with larger \(\theta\). This means that higher urban costs imply a higher average price, higher average markup and higher average output.

**City size**

The effect of a larger city, and thus a larger market, depends heavily upon which factor will increase. If more manufacturers live in the city the structure of the city will not be affected but the market gets larger. Since more manufacturers influence only average profits, the wage of engineers will increase
Figure 10: Urban costs and wages.

Figure 11: Urban costs and inequality.
\( \alpha = 10; \gamma = 2; \fe = 1; \cm = 15; \eta = 10; \H = 20; \L = 20; \W = 30; k = 2; \)

Figure 12: Higher urban costs reduce welfare.

\( \alpha = 10; \gamma = 2; \fe = 1; \cm = 15; \eta = 10; \H = 20; \L = 20; \W = 30; k = 2; \)

Figure 13: Higher urban costs increase average prices.
leaving the cut-off and the wage of clerks unchanged. This follows the empirical findings of Wheeler (2001), Baum-Snow and Pavan (2012) and Davis and Dingel (2013), who show, that the skill premium increases with population size.

An larger number of clerks in the city has two main effects: first it increases labor supply of clerks and second it changes the urban structure. A larger market means a higher average profit which increases the wage of engineers. The larger labor supply reduces the wage of clerks. Welfare of clerks is smaller since they earn less and on average have to commute more. More clerks in a city increase the cut-off, which means selection gets weaker. This stands in contrast with the result of Melitz and Ottaviano (2008), where more consumers (larger markets) reduce the cut-off. Since in the present model more clerks decrease the wage of the variable input factor and increase average profit, competition will be less tough.

More engineers in a city increase their labor supply and thus decrease their wage because expected average profits have to be divided among more engineers. Since more entrants are trying to survive, selection gets tougher and the cut-off decreases. More engineers change the urban structure of a city. Clerks benefit from the fact, that their own effective labor supply decreases if more engineers live in the city. The smaller labor supply of clerks and the larger market increase the wage of clerks.

4. Open urban economy

The closed-economy model of the previous sector can be extended with a second city. All terms of the foreign city are identical to the expressions of the home city but marked with an asterisk. Each region is endowed with $W$ identical workers, $L$ identical clerks, and $H$ identical engineers. All workers supply inelastically one unit of their type of labor and are not mobile. Clerks and engineers commute to the CBD and consume one unit of land. Preferences and technology are the same as in the closed economy, with the difference that the homogenous good is freely tradable between the cities and varieties are subject to “iceberg” transportation costs of $\tau \geq 1$. Since trade of varieties is costly, the number of firms producing and selling in a location may be different. Let $N_P$ be the mass of firms surviving and producing in a location and $N_S$ be the firms selling to a location, that is firms selling locally and foreign firms exporting to the region. The choke price then is

$$p_O \equiv \frac{\alpha \gamma + \eta N_S \hat{p}}{\gamma + \eta N_S}. \quad (17)$$
With $c_D = p_O / w_L$ and $c_X = p_O^* / \tau w_L$, firms located in the home region have a profit maximising price and quantity for sales to domestic market and sales to the foreign respectively

$$q_D(c) = \frac{W + L + H}{2\gamma} w_L (c_D - c) , \quad p_D(c) = \frac{w_L}{2} (c_D + c),$$

$$q_X(c) = \frac{W + L + H}{2\gamma} \tau w_L (c_X - c) , \quad p_X(c) = \frac{\tau w_L}{2} (c_X + c).$$

Markups for domestic sales and exports are

$$\mu_D(c) = \frac{w_L}{2} (c_D - c) , \quad \mu_X(c) = \frac{\tau w_L}{2} (c_X - c),$$

respectively. The corresponding maximised profits as a function of the firm’s own marginal costs are

$$\pi_D(c) = \frac{W + L + H}{4\gamma} w_L^2 (c_D - c)^2 , \quad \pi_X(c) = \frac{W + L + H}{4\gamma} \tau^2 w_L^2 (c_X - c)^2.$$

With $c_X = c_D^* w_L^* / w_L \tau$ and $c_X^* = c_D w_L / \tau w_L^*$ profits from exporting gives

$$\pi_X(c) = \frac{W + L + H}{4\gamma} (c_D^* w_L^* - \tau w_L c)^2.$$

If $\rho_D = G(c_D)$ and $\rho_X = G(c_X)$, expected average profits for a firm in the home location are

$$\rho_D \bar{\pi}_D + \rho_X \bar{\pi}_X = \frac{(W + L + H) c_D^{k+2} w_L^2 + (W* + L* + H*)(c_D^*)^2 (w_L^*)^2 \left( \frac{c_D^* w_L^*}{w_L \tau} \right)^k}{2\gamma (k+1)(k+2)c_M^k}. \quad (18)$$

The free entry condition states that average expected profits must be equal fixed entry costs

$$f_E w_H = \frac{(W + L + H) c_D^{k+2} w_L^2 + (W* + L* + H*)(c_D^*)^2 (w_L^*)^2 \left( \frac{c_D^* w_L^*}{w_L \tau} \right)^k}{2\gamma (k+1)(k+2)c_M^k},$$

$$f_E w_H^* = \frac{(W + L + H)(c_D^*)^{k+2}(w_L^*)^2 + (W* + L* + H*)c_D^2 w_L^2 \left( \frac{c_D w_L}{w_L \tau} \right)^k}{2\gamma (k+1)(k+2)c_M^k}.$$
By using (17) we can compute the zero cut-off profit condition and get the number of firms selling to a region, that is domestic producers and foreign exporters

\[ N_S = \frac{2\gamma(k+1)\alpha - c_D w_L}{\eta c_D w_L}, \]  
(19)

\[ N_S^* = \frac{2\gamma(k+1)\alpha - c_D^* w_L^*}{\eta c_D^* w_L^*}. \]  
(20)

Since \( N_S = N_P + N_P^*(c_X^*/c_D^*)^k \) and \( N_S^* = N_P^* + N_P(c_X/c_D)^k \) and \( c_X = \frac{c_D w_L}{w_L^*}, \) the labor market for engineers is in equilibrium if

\[ N_S = \frac{S_H}{f_E} \left( \frac{c_D}{c_M} \right)^k + \frac{S_H^*}{f_E} \left( \frac{c_D w_L}{w_L^* \tau} \right)^k, \]  
(21)

\[ N_S^* = \frac{S_H^*}{f_E} \left( \frac{c_D^*}{c_M^*} \right)^k + \frac{S_H}{f_E} \left( \frac{c_D^* w_L^*}{w_L^* \tau} \right)^k, \]  
(22)

where \( S_H \) and \( S_H^* \) is the effective labor supply of engineers in the home and foreign region respectively. The labor market for clerks will be in equilibrium if \( N_S = \tilde{c} N_P + \tilde{c}^* N_P^* \left( \frac{c_X}{c_D} \right)^k \) that is

\[ N_S = \frac{S_L k c_D}{(k+1)} + \left( \frac{c_D w_L}{c_D^* w_L^*} \right)^k \frac{S_L^* k c_D}{(k+1)}, \]  
(23)

\[ N_S^* = \frac{S_L^* k c_D^*}{(k+1)} + \left( \frac{c_D^* w_L^*}{c_D w_L} \right)^k \frac{S_L^* k c_D}{(k+1)}, \]  
(24)

with \( S_L \) and \( S_L^* \) as the effective labor supply of clerks in the home and foreign city respectively. Substituting (19) in (21) and (23) and substituting (20) in (22) and (24) gives together with (19) and (20) an equation system that can be solved numerically for the unknown variables \( w_L, w_L^*, w_H, w_H^*, c_D, c_D^* \).

If there is no trade between the cities, that is if \( \tau \to \infty \), the expressions look like the one for the closed urban economy. If trade of varieties is costless (\( \tau = 1 \)) and cities are symmetric in population, all firms that survive, sell in the home and foreign region (\( c_X = c_D = c_D^* = c_X^* \)).

The numerical simulation for symmetric cities shows, that higher trade costs do not influence the cut-off, hence selection. All effects of increased trade costs are transmitted to wages. Since higher trade costs means less varieties in a city, because there will be less imports, labor market equilibrium implies that
$w_L$ increases. Since the cut-off does not change and $w_L$ increases, there will be higher average profits and therefore higher $w_H$, because the survival probability stays constant. A higher $w_L$ and invariant $c_D$ means also a higher average price and a higher markup. The wage of engineers increases more than the wage of clerks bearing higher inequality. As in the closed economy, higher urban costs create more selection (lower $c_D$) and increase wages, thus high urban and trade costs reduce overall welfare.

Let us assume cities are not symmetric in size. If the home city is larger than the foreign city, that is $W > W^*, L > L^*, H > H^*$, then the cut-off is lower in the larger city. Larger markets create more selection (Melitz and Ottaviano, 2008). Tougher (weaker) competition decreases (increase) profits and therefore the wage of engineers in the larger (smaller) city. Since the number of producers increases (decreases) proportionally stronger in the larger (smaller) city than the cut-off decreases (increases), the wage of clerks will be higher (lower). Lower trade costs have no influence on the cut-offs but decrease wages in both regions. The effect of higher urban costs is the same as in the scenario with symmetric regions. More congestion (higher $\theta$) means more selection (lower $c_D$) and increase wages.

5. Conclusion

In this paper we analysed the effect of urban costs in a model of monopolistic competition with endogenous markups. Moreover the impact of firm heterogeneity on selection and wages is investigated.

Costly commuting and land rent shape the urban structure, so that individuals with higher wages will live near the central business district, while individuals with lower wages will settle farther away.

More firm heterogeneity decreases the wage of lower skilled urban citizens. The wage of entrepreneurs is ambiguously effected by firm heterogeneity. Wage inequality increases if there are more different firms in a city. More cost-increasing richness decreases selection and more cost-decreasing evenness increases selection.

Larger urban costs increase selection. Only more productive and less firms survive. In a more congested city firms have to pay higher wages. Although individuals earn more, costs for commuting, housing and a lower consumer surplus predominate and welfare decreases. Inequality first increases with larger urban costs but then decreases at very high levels. Average price, average markups and average output increase with larger urban costs.
These results show, that urban costs matter for policy makers. A different urban structure, better commuting technology or increases population density have a considerable impact on the decision of individuals and firms to settle in a city or metropolitan region (see also Ahlfeldt et al. (2012) and Behrens et al. (2013)).
References


A. Data

We use data from different sources to construct aggregate variables and indices at the Metropolitan Statistical Area (MSA), our unit of observation, in the year 2005. Average labor productivity is measured as hourly wage, which is calculated as yearly salary by workers from age 16 to 65 divided by weeks worked per year and divided by hours worked during the week. We exclude observations missing the age, education, or wage. Following Acemoglu and Autor (2011), we exclude observations reporting an hourly wage below USD 1.675 per hour in 1982 USD, using the GDP PCE deflator. We use the Integrated Census Public Use Microdata Sample (IPUMS) provided by Ruggles et al. (2010) to calculate average wages for individuals with a highschool degree and a Bachelor’s degree. We weight observations by the “person weight” variable provided by IPUMS.

Average commuting time on the MSA-level is calculated from the IPUMS-data. It is the total amount of time, in minutes, that an individual usually takes to get from home to work in a week.

Data about rents are taken from the IPUMS. We calculate average rent per room per year for each MSA using the “household weight” variable provided by IPUMS.

Data for MSA population are taken from the U.S. Bureau of Economic Analysis (BEA). We use data from the U.S. Census Bureau for the MSA land area in square miles.

B. Additional data plots
Figure 14: Log average wages of individuals with highschool degree and log average rents in U.S. MSA’s in 2005.

Figure 15: Log average wages of individuals with Bachelor’s degree and log average rents in U.S. MSA’s in 2005.
Figure 16: Log average wages of individuals with highschool degree and log mean commuting time in U.S. MSA’s in 2005.

Figure 17: Log average wages of individuals with Bachelor’s degree and log mean commuting time in U.S. MSA’s in 2005.
Figure 18: Skill premia and log population for U.S. MSA’s in 2005.

Figure 19: Skill premia and log population density for U.S. MSA’s in 2005.