Dur, Umut Mert; Ünver, M. Utku

Working Paper

Two-sided matching via balanced exchange: Tuition and worker exchanges

Koç University-TÜSİAD Economic Research Forum Working Paper Series, No. 1508

Provided in Cooperation with:
Koç University - TÜSİAD Economic Research Forum, Istanbul


This Version is available at:
http://hdl.handle.net/10419/129361

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

www.econstor.eu
TWO-SIDED MATCHING VIA BALANCED EXCHANGE: TUITION AND WORKER EXCHANGES

Umut Mert Dur
M. Utku Ünver

Working Paper 1508
May 2015

This Working Paper is issued under the supervision of the ERF Directorate. Any opinions expressed here are those of the author(s) and not those of the Koç University-TÜSİAD Economic Research Forum. It is circulated for discussion and comment purposes and has not been subject to review by referees.
Two-Sided Matching via Balanced Exchange: Tuition and Worker Exchanges*

Umut Mert Dur† M. Utku Ünver‡

March 9, 2015

Abstract

We introduce a new matching model to mimic two-sided exchange programs such as tuition and worker exchange, in which each firm has to avoid being a net-exporter of workers. These exchanges use decentralized markets, making it difficult to achieve a balance between exports and imports. We show that stable equilibria discourage net-exporting firms from exchange. We introduce the two-sided top-trading-cycles mechanism that is balanced-efficient, worker-strategy-proof, acceptable, and individually rational, and respects priority bylaws regarding worker eligibility. We prove that it is the unique mechanism fulfilling these objectives. Moreover, it encourages exchange, since full participation is the dominant strategy for firms.

JEL Classification Numbers: C71, C78, D71, D78

Keywords: Market Design, Matching Theory, Tuition Exchange, Worker Exchange, Balanced Exchange, Two-sided Matching, Two-sided Top Trading Cycles

---

*We thank John Duffy for initial discussions, and Scott Kominers, Jacob Leshno, Michael Grubb, Robert Lay, and Konrad Mierendorff for useful comments. We also acknowledge various comments we got from the audience during presentations at Texas, Rice, Zurich, Queen’s Belfast, Microsoft Research at New England, Sabanci, North Carolina State, Frontiers of Market Design’12, Spring Midwest Theory Conference’12, Stony Brook Game Theory Conference’12, Match-Up’12, Games’12, Social Choice and Welfare’12, Texas Theory Workshop, INFORMS’13, North American Econometric Society’13, Paris School of Economics and thank them. Ünver acknowledges the financial support of NSF. The earlier drafts of this paper were entitled “Tuition Exchange.”

†Affiliation: North Carolina State University, Department of Economics, 2801 Founders Drive, Raleigh, NC, 27695; e-mail: umutdur@gmail.com; web page: https://sites.google.com/site/umutdur/

‡Affiliation: Boston College, Department of Economics, 140 Commonwealth Ave., Chestnut Hill, MA, 02467; e-mail: unver@bc.edu; web page: http://www2.bc.edu/~unver
1 Introduction

The theory and design of two-sided matching markets, such as entry-level labor markets for young professionals, online dating markets, or college admissions, has been one of the cornerstones of market design for more than thirty years (cf. Gale and Shapley, 1962; Roth, 1984; Roth and Peranson, 1999; Hitsch, Hortaçsu, and Ariely, 2010). Moreover, the theory of these markets has some important applications in allocation problems such as student placement and school choice (cf. Balinski and Sönmez, 1999; Abdulkadiroğlu and Sönmez, 2003). Both sides of the market, such as firms and workers, which have preferences over each other, participate autonomously in a decentralized or centralized market to find a partner or partners. A market equilibrium concept such as stability has been the key solution concept in theory, in the practice of decentralized markets, and in the design of centralized clearinghouses.

In this paper, we introduce and model a new class of two-sided matching markets in which there is an additional fundamental constraint. Eventual market outcome is linked to an initial status-quo matching, which may give firms and workers certain rights on how future activity can play out. Eventually, a worker may end up matched with a firm different from his home institution. The most crucial property that needs to be respected in such cases is balancedness: a worker needs to be replaced with a new one at his home institution so that the market clears in a balanced manner. Throughout the world, there are several prominent examples of such exchanges, such as national and international teacher exchange programs, clinical exchange programs for medical doctors, worker exchange programs within or across firms, and student exchange programs among schools. The balancedness constraint is crucial in practice. For example, if a math teacher is matched with a school in a different country, the home school of this teacher may wish to hire a substitute math teacher for the year that he is away. This constraint induces preferences for firms not only over whom they get matched with (i.e., import), but also over whom they send out (i.e., export). The most basic kind of such preferences requires the firm to have a preference for balanced matchings, i.e., for import and export numbers to be equal. The first contribution of our paper is the introduction of this novel two-sided matching market model, the formulation of the balancedness condition, and the inspection of properties and implications of balancedness in market activity. We show that balancedness is not in general achieved through decentralized market outcomes, jeopardizing the continuation and success of such markets. Building on this observation, as our next contribution, we propose a new centralized balanced mechanism with desirable properties, and uniquely characterize it. We analyze the implications of our model in two applications of these markets: (permanent) tuition exchange and temporary worker
Some of the best-documented matching markets with a balancedness requirement are tuition exchanges among US colleges. These are semi-decentralized markets, and some have failed over the years caused by problems related to imbalanced matching activity. Therefore, these markets constitute a great natural experiment to observe and learn from. From a theoretical point of view, two-sided matching without externalities is well studied and understood within the Gale–Shapley framework (cf. Roth and Sotomayor, 1990). The formal framework for tuition exchange, on the other hand is only one-degree away from college admissions, with the additional balancedness desideratum on the side of the colleges. Therefore, by studying tuition exchange in its particular, we can also make a comparative static analysis regarding what this additional requirement both brings to and takes away from the standard Gale–Shapley model. This also helps policy makers to provide specific advice for the market applications. (The properties of the Gale–Shapley model have helped immensely in the design of real-life markets as summarized above.)

Historically, it has been difficult for small colleges and universities to compete with bigger schools in trying to hire the best and brightest faculty. Colleges located farther away from major metropolitan areas face a similar challenge. Tuition exchange programs play a prominent role for these colleges in attracting and retaining highly qualified faculty.1

In many colleges, qualified dependents of faculty are given tuition-waivers at their home institutions. Through a tuition exchange program, they can use their waivers at other colleges. This requires admittance to the other colleges. Tuition exchange has become a desirable benefit that adds value to an attractive employment package without creating additional out-of-pocket expenses for colleges.

One of the prominent programs is “The Tuition Exchange, Inc.” (TTEI),2 which is also the oldest and largest of its kind in the US. TTEI is a reciprocal scholarship program for children (and other family members) of faculty and staff employed at more than 600 participating institutions. Member colleges are spread over 47 states and the District of Columbia. Both research universities and liberal arts colleges are members. *US News and World Report* lists 38 member colleges in the best 200 research universities and 46 member colleges in the best 100 liberal arts colleges. Every year an average of 20 new institutions join the program. Through TTEI, on average, 6,000 scholarships are awarded annually, with amounts averaging about $24,000.3 Despite TTEI’s large volume, other

1“Tuition Exchange enables us to compete with the many larger institutions in our area for talented faculty and staff. The generous awards help us attract and retain employees, especially in high-demand fields like nursing and IT.” – Frank Greco, Director of Human Resources, Chatham University, from the home page of The Tuition Exchange, Inc., www.tuitionexchange.org, retrieved on 09/19/2012.


3An alternative to tuition exchange is monetary subsidization of faculty members. Any direct compen-
tuition exchange programs clear more than 50% of all exchange transactions in the US.\textsuperscript{4}

Each participating college establishes its own policies and procedures for determining the eligibility of dependents for exchange and the number of scholarships it will grant each year. Each member college has agreed to maintain a balance between the number of students sponsored by that institution ("exports") and the number of scholarships awarded to students sponsored by other member colleges ("imports"). Colleges aim to maintain a one-to-one balance between the number of exports and imports. In particular, if the number of exports exceeds the number of imports, that college may be suspended from the tuition exchange program.\textsuperscript{5} In order not to be suspended from TTEI, colleges often set the maximum number of sponsored students in a precautionary manner. Many colleges explicitly mention in their application documents that in order to guarantee their continuation in the program they need to limit the number of sponsored students.\textsuperscript{6} As a result, in many cases not all qualified dependents are sponsored. Colleges often use the length of the related employee’s tenure to prioritize eligible students.

From a market and efficiency perspective, tuition benefits at home institutions are distortionary as they cause the dependents to attend their home institutions instead of another school they may prefer. Tuition exchange programs help to correct this distortion. Making the clearinghouses employed by these programs as efficient as possible to minimize this distortion. One of our goals in this paper is to show the problems with the current exchange systems and propose a better mechanism to improve efficiency.\textsuperscript{7}

A tuition exchange program usually functions as follows: each college determines its quotas, which is the maximum number of students it will sponsor (its "eligibility quota") and the maximum number it will admit (its "import quota") through the program. Then, the eligible students apply to colleges, and colleges make scholarship decisions based on preferences and quotas. A student can get multiple offers. She declines all but one, and if possible further scholarship offers are made in a few additional rounds. Students who are not sponsored by their home colleges cannot participate in the program, and hence do not receive tuition exchange scholarship.

In the earlier two-sided matching literature, stability a la Gale and Shapley (1962) has been the central solution concept. Technically, our model is similar to a two-sided

\textsuperscript{4}In Online Appendix B, we describe the features of the other tuition exchange programs.

\textsuperscript{5}While TTEI membership requires keeping a balance in a moving three-year window, other programs have different balance requirements. In this paper we focus on keeping a yearly balance in order to adopt a minimal change from the standard Gale–Shapley model.

\textsuperscript{6}Lafayette College, Daemen College, DePaul University, and Lewis University are just a few examples.

\textsuperscript{7}See Conclusions for more discussion about why tuition exchange exists in the first place.
matching model with externalities, i.e., agents have preferences over allocations rather than their matches. However, our model has major differences from standard externality models, which generally inspect peer effects or induce different stability definitions from ours.\(^8\)

As a solution for the decentralized market, we introduce a new stability notion for the current model, which turns out to exist when colleges have plausible preferences over matchings (Proposition 1). Moreover, Proposition 2 implies that stability and balancedness are incompatible. (Because of the balancedness requirement, in practice stability is not an appropriate property for our market.)

We then inspect a decentralized quota reporting game with a stable market solution. We show that it is the best response for a college with a negative net balance\(^9\) to decrease its eligibility quota, and increasing eligibility quota is never a best response (Theorem 1). Behaving with respect to such a best response may further cause another college to have a negative net balance, as a decrease in participation never improves the negative net balances of other colleges (Theorem 2). Hence, if we take stability as a benchmark market equilibrium concept in a decentralized market, stability discourages exchange and can prevent the market from extracting the highest gains from exchange.\(^{10}\) On the other hand, in our proposed centralized balanced–efficient mechanism that we discuss below, colleges prefer to act truthfully while determining their quotas (Theorem 5).

We then restrict our attention to the set of balanced–efficient mechanisms. Unfortunately, there exists no balanced–efficient and individually rational mechanism that is non-manipulable for colleges (Proposition 5).

We propose a new two-sided matching mechanism that is balanced-efficient, student group–strategy-proof, acceptable, respecting internal priorities, individually rational,\(^{11}\) and immune to quota manipulation by colleges (Theorems 3, 4, and 5). We also show that it is the unique mechanism satisfying the first four properties (Theorem 6).\(^{12}\) The outcome

---

\(^8\)See Sasaki and Toda (1996) for externalities in two-sided matching markets and various stability definitions. Pycia (2010) explores existence in two-sided matching when agents have preferences over peers and matches. The first model is quite general; however, their stability notion, which guarantees existence, requires a very conservative definition of blocking. The second model, on the other hand, does not cover externalities regarding the balancedness requirement.

\(^9\)A college has a negative net balance if its exports are more than its imports.

\(^{10}\)That is to say, instead of considering and proposing a particular stable direct mechanism, we use any stable outcome as a best case proxy of the current market structure's equilibrium to make some comparative static exercises on incentives and try to rationalize why “withholding eligibility quota,” a documented phenomenon in real markets, could be rational.

\(^{11}\)A mechanism respects internal priorities if a college increases the number of sponsored students, then any student sponsored by that college who was matched with a college should still be matched with a college (not necessarily the same college).

\(^{12}\)Ma (1994) had previously characterized the core of a house exchange market, which can be found by Gale's TTC algorithm, when there is a single seat at each school through Pareto efficiency, individual rationality, and strategy-proofness for students. Our characterization uses a proof technique different not
of this mechanism can be computed with a variant of David Gale’s top–trading–cycles (TTC) algorithm (Shapley and Scarf, 1974) for finding core and competitive allocations for a simple “house” (or object) exchange market without money. In the school choice problem (cf. Abdulkadiroğlu and Sönmez, 2003) and house allocation problem with existing agents (cf. Abdulkadiroğlu and Sönmez, 1999), variants of mechanisms with algorithms related to Gale’s TTC have been introduced and their properties have been extensively discussed. In all these problems, one side of the market is considered to be objects to be consumed that are not included in the welfare analysis. Schools and houses have no preferences, but “priorities.” For instance, in the school choice problem, the priorities of schools over the students are determined according to test scores or proximity of their residential location to the school, whereas in the house allocation problem the priorities of houses over the students are determined by random draws or seniority. Hence, they are not strategic agents. In two-sided matching via exchange, in contrast to school choice and house allocation, both sides of the market are strategic and must be included to the welfare analysis. As we use a variant of Abdulkadiroğlu and Sönmez (2003) TTC algorithm to find its outcome, we refer to our mechanism as the two–sided top–trading–cycles (2S-TTC) mechanism, although conceptually our model, definition of the mechanisms, and the way we use TTC are not related to object allocation and exchange problems. As far as we know, this is the first time a TTC–variant algorithm has been used to find the outcome of a two-sided matching mechanism.

Although 2S-TTC is balanced–efficient, it may not match the maximum possible number of students while maintaining balancedness. We show that if the maximal balanced solution is different from the 2S-TTC outcome even for one preference profile, it can be manipulated by students (Proposition 9).

After tuition exchange, we inspect the other major application of our model, temporary worker exchange programs (such as teachers, students, academic staff, and medical doctors).13 Some of these have been running for decades,14 and thousands of participants

---

13 The Commonwealth Teacher Exchange Programme (CTEP), Fulbright Teacher Exchange Program, Erasmus Student Exchange Program and the exchange program of International Federation of Medical Students’ Associations (IFMSA) are just a few examples. There are also small bilateral staff exchange programs. See Online Appendix C for details of these programs.

14 CTEP, which allows participants to exchange teaching positions and homes with a col-
benefit from these worker exchange programs annually.\textsuperscript{15} The main difference of these programs from tuition exchange is that (1) most exchange appointments are temporary, typically lasting one year, and (2) the workers are currently employed by their associated firms, so if they cannot be exchanged, they will continue to work in their current jobs. As a result, the firm preferences are typically coarser over incoming workers than college preferences over students in tuition exchange. In that section we tweak our model slightly and assume that the quotas of the firms are fixed at the number of their current employees, and hence, firms would like to replace each agent who leaves. Moreover, firms have \textit{dichotomous} preferences over workers: a firm either wants to accept a worker or not.\textsuperscript{16}

In this model, when firm preferences over matchings are responsive to their preferences over individual workers but otherwise indifferent among matchings in which they admit the same number of acceptable workers, we prove that 2S-TTC not only carries all of its previous properties through but also is strategy-proof for the firms and stable, making it a very viable candidate (Theorem 7). Our aforementioned characterization also holds in this model.

\section{Natural Experiments: Balancedness Requirement}

Although two-sided matching via exchange induces a two-sided matching market, workers (students) cannot participate in the market activity unless their home firms (colleges) sponsor them. Hence, an import/export balance emerges as an important feature of sustainable outcomes. This is the most important feature of these markets that distinguishes them from the previously studied matching markets. We illustrate three cases in which the absence of a balanced exchange led to the failure of exchange programs in different contexts.

The Northwest Tuition Exchange program was founded in 1982 and included five members: University of Puget Sound, Reed College, Whitman College, Willamette University, and Lewis and Clark College. Unlike TTEI, this program allowed children of league from the UK, Australia, and Canada, has been running for 100 years. See http://www.cyec.org.uk/exchanges/commonwealth-teacher-exchange.

\textsuperscript{15}Every year more than 10,000 medical students and 200,000 college students around the world participate in IFMSA’s and Erasmus’ exchanges, respectively. See http://www.amsa.org/AMSA/Homepage/EducationCareerDevelopment/InternationalExchanges.aspx and http://europa.eu/rapid/press-release_IP-13-657_en.htm.

\textsuperscript{16}Dichotomous, i.e., compatibility-based, preferences are widely used in different matching models. Most notably, the kidney exchange models of Roth, Sönmez, and Ünver (2005, 2007); Sönmez and Ünver (2014) use it to model patient preferences over compatible kidneys. In a more abstract setting, Bogolomània and Moulin (2004) introduced dichotomous preferences to two-sided matching to model worker-firm interaction. Ekici (2011) also uses this modeling choice for “selling” preferences.
faculty members to attend one of the member colleges tuition free upon admission. That is, the colleges were not able to limit their exports. Because of the sizable imbalances between the imports and exports of the member colleges, members agreed to dissolve the program and it will stop accepting new applicants after Fall 2015.\textsuperscript{17}

Tuition and worker exchange markets are closely related to favor exchange markets, also known as “time banks,” where time spent doing a favor or the number of favors is used as the currency of exchange.\textsuperscript{18} The holding of the transaction currency in such markets corresponds to a positive imbalance in our model. Baby sitting co-ops are a leading example of such time banks. Negative–balance aversion as in tuition and worker exchange markets is also known to affect such banks adversely and cause the markets to shut down.\textsuperscript{19} Parallels have been drawn between such markets and complex monetary systems, where liquidity shortage is known to cause recessions (cf. Sweeney and Sweeney, 1977; Krugman, 1998).

The Erasmus student exchange program among universities in Europe is another example of a market in which a lack of balancedness have caused some exchange relationships to be terminated. Member colleges that want to exchange students with each other sign bilateral contracts that set the maximum number of students to be exchanged in certain years. The renewal decision of the contract depends on whether a reasonable balance is maintained between the incoming and outgoing exchange students between these colleges. In particular, if one of the colleges has more incoming students than its outgoing students, then that college does not renew the contract.\textsuperscript{20}

3 Two–Sided Matching via Exchange: Model

In this section, we introduce our model and the desirable solution concepts.\textsuperscript{21} Let \( C = \{ c_1, \ldots, c_m \} \) and \( S = \{ s_1, s_2, \ldots, s_n \} \) be the set of colleges and students, respectively. The set of students is partitioned into \( m \) disjoint sets, i.e. \( S = \bigcup_{c \in C} S_c \) where \( S_c \) is the set of students who are applying to be sponsored by college \( c \). Let \( q = (q_c)_{c \in C} \in \mathbb{N}^m \) be the (scholarship) admission quota vector, where \( q_c \) is the maximum number of students

\textsuperscript{17}See https://www.insidehighered.com/news/2012/02/15/tuition-exchange-program-northwest-colleges-coming-end.
\textsuperscript{18}See Mobius (2001) for a dynamic analysis of a favor exchange model.
\textsuperscript{19}In the mid-1970s the Capitol Hill Baby-Sitting Coop in Washington, DC, negative–balance aversion of families resulted in imbalances between families and decreased the number of favor exchanges between families. For details see http://www.ft.com/cms/s/2/f74da156-ba70-11e1-aa8d-00144feabdc0.html.
\textsuperscript{20}See http://www.bath.ac.uk/quality/documents/QA37.pdf.
\textsuperscript{21}We will keep tuition exchange in mind in naming our concepts. The minor differences in the temporary worker exchange model will be highlighted in Section 6.
who will be admitted by $c$ with tuition exchange scholarship, and $e = (e_c)_{c \in C} \in \mathbb{N}^m$ is the (scholarship) eligibility quota vector, where $e_c$ is the number of students in $S_c$ certified eligible by $c$. Let $\triangleright_C = (\triangleright_c)_{c \in C}$ be the list of college internal priority orders, where $\triangleright_c$ is a linear order of students in $S_c$ based on some exogenous rule.\footnote{We assume that there is no tie in the internal priorities. In practice, each college breaks any ties by using lotteries.} We denote the set of eligible students that are certified eligible by $c$ by $E_c$ where $E_c = \{ s \in S_c \mid r_c(s) \leq e_c \}$ and $r_c(s)$ is the rank of student $s \in S_c$ under $\triangleright_c$. Let $E = \bigcup_{c \in C} E_c$ be the set of all eligible students. The being unassigned option, which we name the null college, is denoted by $c_0$. We set $q_c = n_c$, the number of students.

A matching is a correspondence $\mu : C \cup S \rightarrow C \cup S \cup c_0$ such that:\footnote{We will occasionally refer to singleton $\{x\}$ as $x$ with a slight abuse of notation. The only exception will be $\{0\}$.}

- $\mu(c) \subseteq S$ where $|\mu(c)| \leq q_c$ for all $c \in C$,
- $\mu(s) \subseteq C \cup c_0$ where $|\mu(s)| = 1$ for all $s \in S$,
- $s \in \mu(c)$ if and only if $\mu(s) = c$ for all $c \in C$ and $s \in S$,
- $\mu(s) = c_0$ for all $s \notin E$.\footnote{In tuition exchange, only the students who are certified eligible can be assigned to other institutions. Therefore, if $s$ is not certified eligible, i.e., if $s \in S \setminus E$, then she will be assigned to the null college.}

Let $\mathcal{M}$ be the set of matchings. Let $X^\mu_c = \{ s \in S_c \mid \mu(s) \in C \setminus c \}$ be the set of exports for $c$ under $\mu$. This is the set of certified students of $c$ who are matched with other colleges. Let $M^\mu_c = \{ s \in S \setminus S_c \mid \mu(s) = c \}$ be the set of imports for $c$ under $\mu$. This is the set of students from other colleges matched with $c$. Let $b^\mu_c \in \mathbb{R}$ be the net balance of college $c$ in matching $\mu$. We set it $b^\mu_c = |M^\mu_c| - |X^\mu_c|$. We say college $c$ has a zero (negative) [positive] net balance in matching $\mu$ if $b^\mu_c = 0$ ($b^\mu_c < 0$) [$b^\mu_c > 0$]. Less formally, a college has a zero (negative) [positive] net balance if the number of exported students is equal to (greater than) [less than] the number of imported students. In any matching $\mu \in \mathcal{M}$ the summation of the net balances of colleges is 0, i.e. $\sum_{c \in C} b^\mu_c = 0$. Therefore, if there exists a college with a positive (negative) net balance, then there exists at least one other college with a negative (positive) net balance.

Let $\succ = (\succ_S, \succ_C) = ((\succ_s)_{s \in S}), (\succ_c)_{c \in C})$ be the list of student and college preferences over matchings, where $\succ_i$ is the preference relation of agent $i \in S \cup C$ on $\mathcal{M}$. We denote the strict preference of agent $i \in S \cup C$ on $\mathcal{M}$ by $\succ_i$ and her indifference relation by $\sim_i$.

\footnote{When we say a student is matched to a college, we mean the student receives tuition exchange scholarship from that school.}
Each student \( s \in S \) cares only about her own match in a matching and has a strict preference relation \( P_s \) on \( C \cup c_0 \). Let \( R_s \) denote the at-least-as-good-as relation associated with preference relation \( P_s \) for any student \( s \in S \): \( c R_s c' \) if \( c P_s c' \) or \( c = c' \) for all \( c, c' \in C \cup c_0 \). Student \( s \)'s preferences over matchings \( \succeq_s \) is defined as follows: if \( \mu(s) R_s \mu'(s) \) then \( \mu \succeq_s \mu' \).

On the other hand, each college potentially cares not only about its admitted class of (scholarship) students but also about its net balance. We assume that colleges do not care about other types of externalities. That is, colleges rank any two matchings with the same net balance by considering the set of admitted students. Similarly, they rank any two matchings with the same admitted class by considering its net balance under both matchings. Colleges do not consider all students worth awarding a scholarship. For instance, a student who cannot satisfy regular admission requirements cannot be awarded a scholarship. To explain how colleges compare two matchings with the same balance, we need a ranking for each college over the sets of admitted students. The preference relation of a college \( c \) over matchings, \( \succeq_c \), is defined through a linear order, denoted by \( P_c \), over \( S \cup \{\emptyset\} \). Let \( P_c^* \) be the responsive (Roth, 1985) ranking of \( c \) over the subsets of students; that is, \( \forall T \subseteq S \) with \( |T| < q_s \) and \( s, s' \in S \setminus T \): (1) \( s P_c^* \emptyset \implies (T \cup s) P_c^* T \) and (2) \( s P_c^* s' \implies (T \cup s) P_c^* (T \cup s') \). Note that \( P_c^* \) is just a ranking over the sets of admitted students and is not the preference relation of \( c \) over matchings. Let \( R_c^* \) be the weak ranking over the subset of students induced by \( P_c^* \). Throughout the paper we assume that between any two matchings in which \( c \) has the same net balance, it prefers the one with the higher-ranked set of admitted students according to \( R_c^* \). Formally, for any \( c \in C \), college \( c \)'s preferences over matchings, \( \succeq_c \), satisfies the following restriction: for any \( \mu, \nu \in \mathcal{M} \), if \( b_c^\mu = b_c^\nu \) and \( \mu(c) R_c^* \nu(c) \) then \( \mu \succeq_c \nu \).\(^{26}\)

We now introduce the properties of desirable matchings. A matching \( \mu \in \mathcal{M} \) Pareto dominates another matching \( \nu \in \mathcal{M} \) if \( \mu \succeq_i \nu \) for all \( i \in C \cup S \) and \( \mu \succ_j \nu \) for some \( j \in C \cup S \). A matching \( \mu \) is Pareto efficient if it is not Pareto dominated by any other matching \( \nu \in \mathcal{M} \). A matching \( \mu \in \mathcal{M} \) is acceptable if \( s P_c^* \emptyset \) and \( c P_s^* c_0 \) for all \( c \in C \) and \( s \in \mu(c) \).

A matching \( \mu \in \mathcal{M} \) is balanced if \( b_c^\mu = 0 \) for all \( c \in C \).\(^{28}\) Balancedness is the key property in a tuition exchange market. For each tuition exchange market, the set of balanced matchings is nonempty. For instance, a matching where all students are unassigned satisfies balancedness. Moreover, there may exist multiple balanced matchings.

\(^{26}\)We say a student \( s \) is unacceptable for college \( c \) if \( c \) does not consider \( s \) worth awarding a scholarship and \( s \) is acceptable for \( c \) otherwise.

\(^{27}\)We will introduce more structure over college preferences in Sections 4 and 5.

\(^{28}\)Note that \( b_c^\mu \geq 0 \) for all \( c \in C \) and \( b_c^\mu \leq 0 \) for all \( c \in C \) imply \( b_c^\mu = 0 \) for all \( c \in C \).
for a given market. In that case, we can Pareto rank them whenever possible. We say a balanced matching \( \mu \) is **balanced-efficient** if it is not Pareto dominated by any other balanced matching.

Throughout the paper, \( C, S \), and \( \succ_C \) are fixed; each triple of a quota vector, eligibility vector, and a preference profile defines a tuition exchange market – or simply, a market – as \([q, e, \succ] \).

### 3.1 Tuition Exchange Mechanisms

The current practice of tuition exchange is implemented through indirect semi-decentralized market mechanisms. Although our new proposal can also be implemented indirectly, it will be useful to discuss it as a direct mechanism to analyze its properties. A (direct) mechanism is a systematic way of selecting a matching for each market. Let \( \varphi \) be a mechanism; then the matching selected by \( \varphi \) in market \([q, e, \succ] \) is denoted by \( \varphi[q, e, \succ] \) and the assignment of agent \( i \in S \cup C \) is denoted by \( \varphi[q, e, \succ](i) \).

In a revelation game, students and colleges report their preferences; additionally, colleges report their admission quotas and the number of students they certify eligible. We say a mechanism \( \varphi \) is **immune to preference manipulation for students** (or **colleges**) if for all \([q, e, \succ] \), there exists no \( i \in S \) (or \( i \in C \)) and \( \succ_i \) such that \( \varphi[q, e, (\succ_i, \succ_{-i})](i) \succ_i \varphi[q, e, \succ](i) \). A mechanism \( \varphi \) is **immune to preference manipulation** if it is immune to preference manipulation for both students and colleges. A mechanism \( \varphi \) is **immune to quota manipulation** if for all \([q, e, \succ] \), there exists no \( c \in C \) and \((q_c', e'_c') \in \mathbb{N}^2 \) such that \( \varphi[(q_c', q_{-c}), (e_c', e_{-c}), \succ_c](c) \succ_c \varphi[q, e, \succ](c) \). A mechanism \( \varphi \) is **strategy-proof for colleges** if for all \([q, e, \succ] \), there exists no \( c \in C \) and \((q'_c, e'_c, \succ'_c, \succ_c') \) such that \( \varphi[(q'_c, q_{-c}), (e'_c, e_{-c}), (\succ_c', \succ_{-c})](c) \succ_c \varphi[q, e, \succ](c) \). A mechanism is **strategy-proof for students** if it is immune to preference manipulation for students. Finally, a mechanism is **strategy-proof** if it is strategy-proof for both colleges and students.

One distinctive feature of the tuition exchange market is the existence of internal priorities for each \( c \in C, \succ_c \). In current practice, the internal priority order is used to determine which students will be certified eligible. This priority order is usually based on the seniority of faculty members. Based on our conservations with tuition liaison officers, when

---

29 A mechanism is balanced [balanced-efficient] (individually rational) [Pareto efficient] if it selects a balanced [balanced-efficient] (individually rational) [Pareto efficient] matching in every market. We introduce two additional properties for mechanisms.

30 Since students care only about the schools they are matched with, it will be sufficient for them to report their preferences over colleges instead of over matchings. Under an additional assumption, our proposal in Section 5 can also be implemented by colleges reporting their preferences over individual students just as “acceptable” or “unacceptable”.

31 For example, personal communication with Patricia Bussone, the director at Bradley College.
the number of scholarships is small, in general students with higher priority are awarded the scholarships. We incorporate this priority-based fairness objective into our model by introducing a new property. It is desirable that whenever a student $s$ sponsored by $c$ is assigned to a college in $\varphi[q, e, \succsim]$, she should also be assigned in $\varphi[(\tilde{q}_c, q_{-c}), (\tilde{e}_c, e_{-c}), \succsim]$ where $\tilde{e}_c > e_c$ and $\tilde{q}_c \geq q_c$. That is, the addition of students with lower internal priority should not cause $s$ to be unassigned. Formally, a mechanism $\varphi$ respects internal priorities if, whenever a student $s \in S_c$ is assigned to a college in market $[q, e, \succsim]$, then $s$ is also assigned to a college in $[(\tilde{q}_c, q_{-c}), (\tilde{e}_c, e_{-c}), \succsim]$ where $\tilde{e}_c > e_c$ and $\tilde{q}_c \geq q_c$. Given that $s$ is assigned in $[q, e, \succsim]$, then s’s internal rank should be lower than $e_c$, $r_c(s) \leq e_c$, and all the new students who are sponsored in $[(\tilde{q}_c, q_{-c}), (\tilde{e}_c, e_{-c}), \succsim]$ but not in $[q, e, \succsim]$ should have a higher internal rank (and hence a lower internal priority) than $s$. Note that respect for internal priorities is a fairness notion rather than efficiency.

4 Stability vs. Balancedness in Tuition Exchange

In this section, we analyze the current practice of tuition exchange. This market has some idiosyncratic properties different from those of previously studied two-sided matching markets. In tuition exchange — in its current implementation — an admitted class of lower-quality students can be preferable to one with higher-quality students under two different matchings, if the latter one deteriorates the net balance of the college. In particular, a higher net balance could be preferred as it keeps the college in better standing within the program in years to come. The extreme version of this preference is a college being extremely averse against negative net balance matchings, regardless of the incoming class. Maintaining a nonnegative net balance is important for the continuation of the membership to the program for a college. In particular, a college with negative net balance might be suspended from the program.

We will incorporate these features as two formal assumptions in this section. Assumption 1 states that a better admitted class is more preferable as long as the net balance does not decrease, admission of unacceptable students deteriorates the rankings of matchings regardless of their net balances, and a college deems its own students unacceptable in tuition exchange. Assumption 2 introduces negative net-balance averse preferences. In all results in this section we will use Assumption 1, while Assumption 2 will be used in only one result. We start by stating Assumption 1.

---

32 This property is used in our characterization in Section 5 where we show that this axiom does not bring additional cost to our proposed mechanism (Proposition 9).

33 As mentioned earlier, TTEI uses a variant of this penalty.
Assumption 1 For any $c \in C$ and $\mu, \nu \in \mathcal{M}$,

1. (Preference increases with better admitted class and non-deteriorating balance) if $b^c_\nu \geq b^c_\nu$ and $\mu(c)P^c_\nu \nu(c)$ then $\mu \succeq_c \nu$,

2. (Awarding unacceptable students exchange scholarships is not preferable) if there exists $s \in \nu(c) \setminus \mu(c)$, $\emptyset P_s$ and $\nu(s') = \mu(s')$ for all $s' \in S \setminus s$ then $\mu \succeq_c \nu$, and

3. (Unacceptability of the college’s own students for exchange scholarships) $\emptyset P_s$ for all $s \in S_c$.

In the current tuition exchange program, as the centralized process is loosely controlled, once each college sets its eligibility/admission quota and eligible students are determined, the market functions more like a decentralized one rather than a centralized one. Once colleges commit to the students they will sponsor, they lose their control over them. A sponsored student can sometimes get multiple offers and decide which one to accept and when to accept it. Hence, stability emerges as a relevant notion for a benchmark market equilibrium concept when there is no other friction. To adopt stability in our model, we introduce some preliminary concepts. We say a matching $\mu$ is blocked by a college $c \in C$ if there exists some $\mu' \in \mathcal{M}$ such that $\mu' \succ_c \mu$, $\mu'(s) = \mu(s)$ for all $s \in S \setminus \mu(c)$ and $\mu'(c) \subset \mu(c)$. A matching $\mu$ is blocked by a student $s \in S$ if $c_0 P_s \mu(c)$. A matching $\mu$ is individually rational if it is not blocked by any individual college or student.\(^{34}\) We say $\mu' \in \mathcal{M}$ is obtained from $\mu$ by the mutual deviation of $c$ and $s$ if $s \in \mu'(c) \subseteq \mu(c) \cup s$, and $\mu'(s') = \mu(s')$ for all $s' \in S \setminus (\mu(c) \cup s)$. A matching $\mu \in \mathcal{M}$ is blocked by college-student pair $(c, s)$ if $c P_s \mu(s)$ and $\mu' \succ_c \mu$ where $\mu' \in \mathcal{M}$ is obtained from $\mu$ by the mutual deviation of $c$ and $s$.\(^{35}\) As in any blocking condition in cooperative games with externalities, we need to take a stance on how other players act when a pair deviates. We assume that only one college or one student deviates at a time, and assume that the rest of the students and colleges do not make simultaneous decisions.

A matching $\mu$ is stable if it is individually rational and not blocked by any college-student pair. A matching $\mu$ is nonwasteful if there does not exists a blocking pair $(c, s)$ such that $|\mu(c)| < q_s$, $c P_s \mu(s)$ and $\mu' \succ_c \mu$ where $\mu'(s) = c$ and $\mu'(s') = \mu(s')$ for all $s' \in S \setminus s$.

The existence of stable matchings has been widely studied in two-sided matching problems without externalities. For instance, in the college admission market, when the

\(^{34}\) Assumption 1 implies that if there exists $s \in \mu(c)$ such that $\emptyset P_s$ then $\mu$ is blocked by $c$. Moreover, if $s \in \mu(c)$ then $\mu$ is not blocked by $c$. Hence, individual rationality and acceptability are equivalent under Assumption 1.

\(^{35}\) Assumption 1 implies that if $c P_s \mu(s)$ and $|\mu(c)| < q_s$ then $(c, s)$ is a blocking pair for $\mu$. Similarly, if $s \in \mu(c)$ and $s \notin \mu(c)$ then $(c, s)$ is a blocking pair for $\mu$. 

13
college preferences are responsive, then the set of stable matching is nonempty (cf. Gale and Shapley, 1962; Roth, 1985). We prove a similar result for our environment.

**Proposition 1** Under Assumption 1, there exists at least one stable matching in any tuition exchange market.\(^{36}\)

We prove this proposition by constructing an associated Gale–Shapley college-admissions market in which the set of Gale–Shapley–stable matchings is identical to the set of stable tuition–exchange matchings.

We will illustrate how decentralized market forces moving toward stability can be at odds with the college’s objective of maintaining a zero net balance. We will also show that colleges always have incentives to decrease their quotas under stable outcomes. Hence, a decentralized market or “stable” centralized mechanisms discourage exchange.

We start with the following proposition, which shows the incompatibility between balancedness and individual rationality, and nonwastefulness.

**Proposition 2** Under Assumption 1, there may not exist an individually rational and nonwasteful matching that is also balanced.

Proposition 2 also shows that there exists no stable and balanced mechanism. One can then wonder whether there exists a stable mechanism that performs better than all other stable mechanisms in terms of balancedness. We prove otherwise.\(^{37}\)

**Proposition 3** Under Assumption 1, each college has the same net balance in all stable matchings of a given market.

We also investigate what kinds of strategic decisions a tuition exchange office in a college would face in a quota-determination game if a stable outcome emerges in the market. Here we explicitly make the aforementioned additional assumption about negative net-balance aversion on college preferences: \(^{38}\)

**Assumption 2** (Negative Net-Balance Aversion) College \(c\) prefers \(\mu \in \mathcal{M}\), such that \(b^c_\nu = 0\) and all \(s \in \mu(c)\) are acceptable, to all \(\nu \in \mathcal{M}\) with \(b^c_\nu < 0\).

\(^{36}\)All proofs are in Appendix A.

\(^{37}\)We also inspect the structure of stable matchings, as our stability concept is novel, in Online Appendix D. We show that there always exists college- and student-optimal stable matchings.

\(^{38}\)This assumption is used only in Theorem 1.
In the quota-determination game, we also fix $C$, $S$, $\succ_C$, and $\succeq$. Colleges are the players of the game and each college’s strategy is setting its admission and eligibility quotas under a simultaneous move, complete information setting. Without loss of generality, we constrain the strategy space such that a reported admission quota is not less than the reported eligibility quota. Given true quota profile, denote the action set for $c$ with $A_c$; then, it is $A_c = \{(\hat{q}_c, \hat{e}_c) \in \mathbb{N}^2 | \hat{q}_c \geq \hat{e}_c \geq 0\}$. The outcome of the game is determined by a stable mechanism, which we denote as a stable solution. In Theorem 1, by using the results of Proposition 4 below, we show that in any stable solution, if a college holds a negative net balance, then the best response is only to decrease the number of certified eligible students. Proposition 4 also gives us a comparative result regarding how the net balances of colleges change when they certify one additional student and do not decrease their admission quotas.\footnote{Weber (1997); Engelbrecht-Wiggans and Kahn (1998); Ausubel, Cramton, Pycia, Rostek, and Weretka (2014) study demand reduction in auctions.}

**Proposition 4** Under Assumption 1, for fixed preferences $\succeq$ and for any reported quota profiles $\hat{q}$ and $\hat{e}$, let $\hat{\pi}$ and $\tilde{\pi}$ be stable matchings for the induced markets $[\hat{q}, \hat{e}, \succeq]$ and $[(\hat{q}_c, \hat{q}_{-c}), (\hat{e}_c, \hat{e}_{-c}), \tilde{\succeq}]$, respectively, where $\hat{q}_c \geq \hat{e}_c$, $\tilde{q}_c \geq \tilde{e}_c$ and $\tilde{e}_c = \hat{e}_c + 1$. Then $b^\pi_c \in \{b^\hat{\pi}_c - 1, b^\hat{\pi}_c \}$ if $b^\hat{\pi}_c < 0$; and $b^\pi_c \in \{b^\hat{\pi}_c - 1, b^\hat{\pi}_c, b^\hat{\pi}_c + \hat{q}_c - \hat{e}_c \}$ if $b^\hat{\pi}_c \geq 0$.

The proposition concludes that when a college increases its eligibility quota by one without decreasing its admission quota, its overall net balance will decrease at most by one under any stable solution. Its net balance may increase only if it is a nonnegative net balance college to start with.\footnote{This is possible only if $\tilde{q}_c > \hat{q}_c$.}

**Theorem 1** Under Assumptions 1 and 2, for fixed preferences $\succeq$ and for any reported quota profiles $\hat{q}$ and $\hat{e}$, if $c$ has a negative net balance in a stable matching for market $[\hat{q}, \hat{e}, \succeq]$ where $\hat{q}_c \geq \hat{e}_c$, then its best response in any stable solution is to set only lower $\hat{e}_c$, but not higher; and in particular, there exist $\tilde{e}_c \leq \hat{e}_c$ such that college $c$ has a zero-balance in every stable matching of the market $[\hat{q}, (\tilde{e}_c, \hat{e}_{-c}), \succeq]$.

In Theorem 1, we show that if $c$ has a negative net balance then it tends to decrease the number of certified students, which will eventually increase its balance.\footnote{This result is in a similar vein as the results on college admissions where the DA mechanism is shown to be prone to admission quota manipulation of the colleges under responsive preferences, regardless of imbalance aversion (cf. Sönmez, 1997). However, Konishi and Ünver (2006) show that the DA mechanism would be immune to quota manipulation, if preferences of colleges over incoming students were responsive and monotonic in number. On the other hand, even under this restriction of preferences over the incoming class, our result would imply all stable mechanisms are manipulable with quota reports for colleges with negative net balances if colleges have negative net-balance averse preferences. (See also Kojima and Pathak, 2009.)} When college
c certifies fewer students it may cause another college $c'$ to have a negative net balance. Then college $c'$ will have a negative net balance and will certify fewer students, too. In Theorem 2 below, we show this result.

**Theorem 2** Under Assumption 1, for fixed preferences $\succeq$ and for any reported quota profiles $\hat{q}$ and $\hat{e}$, if a college $c$ is holding a negative net balance in a stable matching $\mu$ for market $[\hat{q}, \hat{e}, \succeq]$ where $\hat{q}_c \geq \hat{e}_c$ then $b^\mu_c \geq b^\mu_c$ where $\mu'$ is any stable matching of market $[(q_c, \hat{q}_{-c}), (e_c, \hat{e}_{-c}), \succeq]$, $\hat{q}_c \geq q'_c \geq e'_c$ and $\hat{e}_c > e'_c$.

Theorems 1 and 2 do not conduct an equilibrium analysis in a quota-determination game. But they do point out that in a frictionless market, the colleges that will be likely to have a negative-balance will be conservative and will decrease their eligibility quotas for exports, certifying fewer students, which will further deteriorate the balances of other colleges.

Typically, no school fully withdraws in practice, as there is often a minimum quota of participation in place. This is instituted most likely because of the reasons outlined above. Given that continued membership is an attractive benefit, often times, smaller colleges will announce that they will import and export at this minimum quota requirement (for example, it is 1 for TTEI), and will continue to be a member of the program without fully withdrawing from the system.

We conclude that under a new design for the tuition exchange market, there should be no room for quota underreporting by the colleges due to negative net-balance aversion, if possible. A fully centralized solution disregarding decentralized market stability seems to be inevitable, as it is at odds with balancedness and has various other shortcomings regarding other incentives.

Moreover, we deem such a stability concept inappropriate for our purpose as the rights of the students to participate in market activity depends on the permission of their colleges. Thus, we claim that balanced–efficiency and individual rationality are the most important features of a tuition–exchange outcome.

It turns out that we can find a plausible balanced–efficient and individually rational mechanism that is also immune to quota manipulation by the colleges (see Theorem 5 below). Full participation will lead to a thicker market and possibly more exchanges.

## 5 Two–Sided Top Trading Cycles

In this section, we propose a mechanism that is individually rational, balanced–efficient, and strategy-proof for students. Moreover, it respects colleges’ internal priorities. We
relax Assumption 1 (and also Assumption 2) on college preferences introduced in the previous section on college preferences. Assumption 3 below states that a college prefers a better scholarship class with zero net balance to an inferior scholarship class with a nonpositive net balance.

**Assumption 3** For any \( \mu, \nu \in \mathcal{M} \) and \( c \in C \), if \( b^\mu_c = 0, b^\nu_c \leq 0 \), and \( \mu(c)P^*_c \nu(c) \) then \( \mu \succ_c \nu \).

It will be useful to denote a matching as a directed graph, as we will find the outcome of our mechanism through an algorithm over directed graphs. In such graphs, colleges and students are nodes; a directed edge is between a college and a student, and it points to either the college or the student, but not both. Given a matching \( \mu \), let each \( s \in S \) point to \( \mu(s) \) and each \( c \in C \) point to all its matched sponsored students, i.e., those in \( S_c \setminus \mu(c_0) \); moreover, let \( c_0 \) point to students in \( \mu(c_0) \). In this graph, we define the following subgraph: A **trading cycle** consists of an ordered list of colleges and students \((c_1, s_1, c_2, s_2, \ldots, c_k, s_k)\) such that \( c_1 \) points to \( s_1 \), \( s_1 \) points to \( c_2 \), \ldots, \( c_k \) points to \( s_k \) and \( s_k \) points to \( c_1 \).

In the following Remark, we state that if a matching is balanced then we can decompose it into a finite number of disjoint trading cycles. We skip its proof for brevity.

**Remark 1** A matching \( \mu \) is balanced if and only if each student is in a trading cycle in the graph of the matching.

We are ready to propose a new two-sided matching mechanism. This is one of our main contributions in this paper. We will find its outcome using a **top–trading–cycles (TTC) algorithm**, similar to the ones introduced for one–sided discrete resource allocation problems such as for school choice (by Abdulkadiroğlu and Sönmez, 2003) and dormitory room allocation at college campuses (by Abdulkadiroğlu and Sönmez, 1999). These TTC algorithms were inspired by Gale’s **TTC algorithm** (Shapley and Scarf, 1974), which was used to find the core allocation of a simple discrete exchange economy, commonly referred to as the housing market. Housing market is a special case of a resource allocation problem class that we refer to as one-sided matching problems. Most common mechanisms in one-sided matching problems function through algorithms that mimic agents exchanging objects that are initially allocated to them either through individual property rights or through the mechanism’s definition to the agents (also see Pápai, 2000; Pycia and Ünver, 2009). In contrast, in our market, college slots are not objects, as the colleges are active decision makers. Therefore, our model, our definition of a mechanism, and the properties of matchings and mechanisms (except strategy-proofness for students) do not have any
analogous translations in such problems. However, since we use a TTC algorithm to find its outcome, we refer to our mechanism as \textit{two-sided (student-pointing) top-trading-cycles (2S-TTC)}. Its outcome is found for any given \([q, e, \succeq_c]\) as follows:\footnote{The converse of this process, using an algorithm originally introduced for two-sided matching markets in one-sided matching markets, has already been utilized in market design. For certain real-life one-sided problems regarding student placement and choice, Balinski and Sönmez (1999) introduced and Abdulkadiroğlu and Sönmez (2003) advocated the Pareto-dominant fair mechanism (also commonly referred to as student-optimal stable mechanism). This mechanism’s outcome can be found through the celebrated student-proposing deferred acceptance algorithm. This algorithm was originally introduced to find stable matchings in two-sided matching markets by Gale and Shapley (1962). Later on, many school districts in the US adopted this mechanism for public school admissions (cf. Abdulkadiroğlu, Pathak, Roth, and Sönmez, 2005 and Abdulkadiroğlu, Pathak, and Roth, 2005).}

The Algorithm for the Two-Sided Top-Trading-Cycles Mechanism:

\textbf{Round 1:} Assign two counters, for admission and eligibility, for each college \(c \in C\), and set them equal to \(q_c\) and \(e_c\), respectively. Each student points to her favorite college \(c' \in C \cup c_0\) that considers her acceptable (ranks her above \(\emptyset\) in \(P_c\)), and each college \(c \in C\) points to the student in \(S_c\) who has the highest internal priority in \(\succ_c\). Null college \(c_0\) points to the students pointing to it. Because of the finiteness of the sets of colleges and students, there exists at least one trading cycle. Each college and student can be part of at most one cycle. Every student in each trading cycle is assigned a seat at the college she is pointing to and removed. If the cycle does not contain \(c_0\) then the counters of each college in that cycle are reduced by one. If the cycle contains \(c_0\) then we reduce only the eligibility counter of the college whose student is in that cycle. If any counter of a college reaches zero then that college is removed and its remaining students are assigned to \(c_0\).

In general, at

\textbf{Round }k:\text{ Each remaining student points to her favorite college in }C \cup c_0\text{ that considers her acceptable among the remaining ones, and each remaining college }c \in C\text{ points to the student in }S_c\text{ who has the highest internal priority in }\succ_c\text{ among the remaining ones. Null college }c_0\text{ points to the students pointing to it. There exists at least one trading cycle. Each college and student can be part of at most one cycle. Every student in each trading cycle is assigned a seat at the college she is pointing to and removed. If the cycle does not contain }c_0\text{ then the counters of each college in that cycle are reduced by one. If the cycle contains }c_0\text{ then we reduce only the eligibility counter of the college whose student is in that cycle. If any counter of a college reaches zero then that college is removed and its remaining students are assigned to }c_0.\text{ The algorithm terminates when there are no remaining students in the market.}\footnote{As this is a two-sided matching market, we could also propose the \textit{college-pointing} version of the 2S-TTC mechanism in which in each round colleges point to their most preferred students among the ones considering them acceptable and each student points to her home college. This variant takes college preference intensity more seriously than students’. However, it can be easily shown that it gives incentives}
In the following theorem, we show that 2S-TTC is balanced–efficient, acceptable, and individually rational, and it respects internal priorities.

Theorem 3 Under Assumption 3, 2S-TTC is an individually rational, balanced–efficient, and acceptable mechanism that also respects internal priorities.

It should be noted that the balanced-efficiency of 2S-TTC is not an extension of the classical Pareto efficiency result of TTC in a one–sided market. Here, colleges are players, and they have multiple seats over which they have “responsive” preferences. Therefore, one may think that by assigning a college highly preferred students and also some unacceptable ones, an individually rational balanced matching can potentially be (weakly) improved for everyone, colleges and students alike, while violating individual rationality for colleges and yet still obeying balancedness. In this theorem, through an iterative approach, we show that it is not possible to improve over 2S-TTC’s outcome in such a fashion. Moreover, 2S-TTC satisfies an even stronger version of respecting internal priorities: each eligible student is assigned the same college seat when new students are deemed compatible.

Under a centralized mechanism, incentives for participants to truthfully reveal their preferences are desirable. Unfortunately, we show that balanced–efficiency, individual rationality, and immunity to preference manipulation for colleges (hence strategy-proofness for colleges) are incompatible properties.

Proposition 5 There does not exist an individually rational and balanced–efficient mechanism that is also immune to preference manipulation for colleges — even under Assumption 3.

We prove this theorem by constructing several small markets and showing that it is not possible to satisfy all three properties in one of these markets. The proof of Proposition 5 can be used to show incompatibility between balanced efficiency, acceptability, and immunity to preference manipulation for colleges.

Proposition 6 There does not exist a balanced–efficient and acceptable mechanism that is also immune to preference manipulation for colleges — even under Assumption 3.

From Proposition 5 and Theorem 3, it is easy to see that the 2S-TTC mechanism is not strategy-proof for colleges.

to both students and colleges for manipulation. On the other hand, the (student-pointing) 2S-TTC is group strategy-proof for students, as we state in Proposition 4.
Proposition 7 2S-TTC is not immune to preference manipulation for colleges, and hence is not strategy-proof for colleges.

Although 2S-TTC is not strategy-proof for colleges, in the following theorem we show that it is group strategy-proof for students. This result is a consequence of TTC being group strategy-proof in a house allocation/exchange market (cf. Pápai, 2000).

Theorem 4 2S-TTC is group strategy-proof for students.

As we care only whether a college finds a student acceptable or not in running 2S-TTC, it can be run as an indirect mechanism where colleges report only their acceptable incoming students. Hence, the strategy space for the colleges is very simple in using 2S-TTC in the field: their strategy is to report their admission and eligibility quotas and their sets of acceptable students based on their preferences over the matchings.

Moreover, if we focus on the game played by the tuition-exchange office of a college, when admissions preferences are fixed, truthful admission quota revelation and certifying all its own students is a (weakly) dominant strategy under 2S-TTC.

Theorem 5 Under Assumption 3 and when true eligibility quotas satisfy $e_c = |S_c|$ for all $c \in C$, 2S-TTC is immune to quota manipulation.\textsuperscript{44}

The proof uses an auxiliary lemma that we state and prove in the Appendix. This lemma shows that, as the quotas of a college weakly increase, the import and export sets of this college also weakly expand under 2S-TTC.

Theorem 5 is in stark contrast with similar results in the literature for Gale–Shapley–stable mechanisms. For example, it is well known that the student– and college–optimal Gale–Shapley–stable mechanisms are prone to admission quota manipulation by the colleges even under responsive preferences regardless of imbalance aversion (cf. Sönmez, 1997); truthful revelation does not even constitute an equilibrium, and any pure strategy equilibrium, if it exists, increases all colleges’ welfare above truthful revelation (cf. Konishi and Ünver, 2006). Thus, 2S-TTC presents a robust remedy for a common problem seen in centralized admissions that use the student–optimal Gale–Shapley–stable mechanism (such as K-12 public school choice in US school districts for one-sided matching) and also in tuition exchange in a decentralized market (cf. Theorems 1 and 2).

Theorem 4, Proposition 7, and Theorem 5 point out that only colleges can benefit from manipulation, and they can manipulate the 2S-TTC mechanism by misreporting preferences.

\textsuperscript{44}In their websites, colleges are explaining the sole reason for certifying limited number of students is maintaining a balanced exchange. If colleges were not precautionary about the balance, they would like to certify all students eligible.
their preferences. Moreover, the only way to manipulate preferences is to report an acceptable student as unacceptable. Suppose we take all the admitted students in the regular admission procedure as acceptable for a tuition exchange scholarship. Then, to manipulate the 2S-TTC mechanism, a college needs to reject a student who satisfies the college admission requirements. Usually college admission decisions are made before the applicants are considered for scholarships. These decisions are made based on well-defined criteria. Violation of these criteria may lead to legal action. Proposition 8 below implies that colleges do not benefit from misreporting their ranking over incoming classes. Hence, in practice, colleges will be more likely to report their true preferences over matchings and true scholarship admission quota and to certify all their own students.

**Proposition 8** Under Assumption 3, colleges are indifferent among strategies that report preferences over matchings in which the same set of students is acceptable under the 2S-TTC mechanism.

We have shown that 2S-TTC has appealing properties. In the following theorem, we show that it is the unique mechanism satisfying respect for internal priorities, acceptability, balanced-efficiency, and strategy-proofness for students.

**Theorem 6** Under Assumption 3, 2S-TTC is the unique student-strategy-proof, acceptable, and balanced-efficient mechanism that also respects internal priorities.

In the proof of our characterization theorem, we use a different technique from what is usually employed in elegant single quota characterization proofs such as Svensson (1999) and Sönmez (1995) for the result of Ma (1994). Our proof relies on building a contradiction with the claim that another mechanism with the four properties in the theorem’s hypothesis can exist. Suppose such a mechanism exists and finds a different matching than 2S-TTC for some market. The 2S-TTC algorithm runs in rounds in which trading cycles are constructed and removed. Suppose $S(k)$ is the set of students removed in Round $k$, while running the 2S-TTC algorithm in such a way that in each round only one arbitrarily chosen cycle is removed and all other cycles are kept intact. We find a Round $k$ and construct an auxiliary market with the following three properties: (1) Eligibility quotas of home colleges of students in $S(k)$ are set such that these are the last certified students in their respective home institutions; (2) all preferences are kept intact except those of students in $S(k)$, whose preferences are truncated after their 2S-TTC assignments; and (3) all students in $S(k)$ are assigned $c_B$ under the alternative mechanism, while all students removed in the 2S-TTC algorithm before Round $k$ have the same
assignment under 2S-TTC and the alternative mechanism. This contradicts the balanced-efficiency of the alternative mechanism: we could give the students in $S(k)$ their 2S-TTC assignments while keeping all other assignments intact and obtain a Pareto-dominating balanced matching. Round $k$ and the auxiliary market are constructed in three iterative steps.

In Online Appendix F, we show the independence of the axioms mentioned in Theorem 6.

Among all the axioms, only the respect for internal priorities is based on exogenous rules. Then, one might suspect that more students will benefit from the tuition exchange program if we allow the violation of respect for internal priorities. A natural question that arises is whether there is a balanced and individually rational mechanism that never assigns fewer students than the 2S-TTC mechanism and selects a matching in which more students are assigned whenever there exists such an outcome. In Proposition 9, we show that the mechanism satisfying the above conditions is not strategy-proof for students.

**Proposition 9** Any balanced and individually rational mechanism that does not assign fewer students than the 2S-TTC, and selects a matching in which more students are assigned whenever such a balanced and individually rational outcome exists, is not strategy-proof for students.

### 5.1 Market Implementation: Tuition Remission and Exchange

Incorporating tuition remission programs of all participating colleges to tuition exchange is the best way to implement a centralized tuition clearinghouse. If parallel remission and exchange programs are run, as in current practice, a student may receive more than one scholarship offer, one from her home college and one from the tuition exchange program. If the student accepts the home college’s offer, the net balance of the college may deteriorate.\(^\text{45}\)

Although the current system is inflexible in accommodating this important detail, a clearinghouse utilizing 2S-TTC can easily combine tuition exchange with remission. Indeed, in Assumption 3 for college preferences, we allowed a college to deem its own sponsored students to be acceptable. Hence, all our results in this section go through when both programs are run together.

More specifically, in the market, we propose to run an indirect version of 2S-TTC in sequential stages in a semi-decentralized fashion: first, colleges announce their admission

\(^{45}\)Indeed, Robert Lay, the Dean of Enrollment Management at Boston College told us in personal communication that a centralized tuition exchange clearinghouse should incorporate a tuition remission program of all participating colleges for this reason.
quotas and which of their students are eligible to be sponsored for both exchange and
remission; then, eligible students apply to the colleges they find acceptable (their home
colleges or others); then colleges send out admission letters. At this stage, as students
have also learned their opportunities in the parallel running regular college admissions
market, they can form better opinions about the relative ranking of the null college, i.e.,
their options outside the tuition exchange market. Students submit rankings over the
colleges that admitted them and the relative ranking of their outside option. Finally,
2S-TTC is run centrally to determine the scholarship recipients.

6 Temporary Worker Exchanges

As noted in the Introduction, many organizations have temporary worker exchange pro-
grams that can be modeled through our balanced two–sided matching framework. 2S-
TTC can be used in such programs almost without any modification.

The first difference in such applications from tuition exchange is that these exchanges
are temporary. Hence, each firm requires a set of specific skills, e.g. a mathematics teacher
to replace their own teacher. Compatibility and ability to perform the task is the main
preference criterion rather than a strict preference ranking. E.g., finding a good teacher
with a specific degree is the first-order requirement, rather than finer details about the
rankings of all good teachers.

The second difference is that each position and each worker should be matched, unlike
the tuition exchange application. The workers are currently working for their home firms.
Thus, the firms find these workers necessarily acceptable. By contrast, in tuition exchange,
some colleges are not required to admit all the dependents of their employees without a
tuition exchange scholarship; instead, they use blind or semi-blind criteria similar to
regular admissions process. In temporary worker exchanges, a worker who does not want
to go to a different firm necessarily stays employed in his home firm.

Hence, we need to use a variant of the tuition exchange model to facilitate balanced
and efficient trade in such circumstances.

We can use the model introduced in Section 3 for the temporary worker exchange
programs, with slight changes. In particular, we need to replace colleges with firms and
students with workers. In Section 3 in the definition of a matching, students who are not
certified as eligible are taken as assigned to \( c_0 \). Since students who are not certified as
eligible are not guaranteed to be admitted by their home colleges, defining a matching
in this way is correct for tuition exchange programs; but any worker who is not certified
as eligible continues to work at her current firm. Hence, for worker exchange programs,
the workers who are not certified as eligible in a matching are assigned to their current companies. Formally, a matching is a correspondence \( \mu : C \cup S \rightarrow C \cup S \) such that, (1) \( \mu(c) \subseteq S \) where \( |\mu(c)| \leq q_c \) for all \( c \in C \), (2) \( \mu(s) \subseteq C \) where \( |\mu(s)| = 1 \) for all \( s \in S \), (3) \( s \in \mu(c) \) if and only if \( \mu(s) = c \) for all \( c \in C \) and \( s \in S \), and (4) \( \mu(s) = c \) for all \( s \in S_c \setminus E \).

Let \( \mathcal{M} \) be the set of all matchings.

To capture the features of worker exchange programs, we make assumptions on the preferences of workers and firms. Since worker \( s \in S_c \) is already working at firm \( c \), we assume that \( s \) finds \( c \) acceptable and \( c \) finds \( s \) acceptable, i.e., \( c P_s \) and \( s P_c \) for all \( s \in S_c \) and \( c \in C \). As discussed above, firms are assumed not to have a strict ranking over workers. In particular, if \( s P_c \) and \( s' P_c \) then \( s I_c s' \), where \( I_c \) denotes the indifference relation.\(^{46}\) Let \( R^*_c \) be the responsive preference relation of \( c \) over the subsets of \( S \); it is attained according to \( I_c \) and \( P_c \). The compatibility assumption and Assumption 3 together imply that each firm compares any two matchings with the same net–balance based on the numbers of acceptable workers assigned and prefers the one in which it gets more acceptable workers. We formally state these assumptions on preferences as follows.

**Assumption 4**

1. (Compatibility based college preferences) For any \( c \in C \) and \( \mu, \nu \in \mathcal{M} \), if \( b'_c = b'_c \) and \( |\{s \in \mu(c) : s P_c \}| \geq |\{s \in \nu(c) : s P_c \}| \) then \( \mu \succeq_c \nu \), and

2. (Acceptability of current match) For any \( c \in C \) and any \( s \in S_c \), \( c P_s \) and \( s P_c \).

An important feature of temporary worker exchange programs is that companies are not creating new positions for incoming workers. In particular, the number of available spots in a company is equal to the number of current staff working in that company, i.e., \( q_c = |S_c| \) for all \( c \in C \).

Based on Assumption 4, a balanced mechanism that allows employees to get better firms, which consider them acceptable, improves the total welfare without hurting anyone. In particular, 2S-TTC introduced in Section 5 can be applied to temporary worker exchange programs by replacing colleges and students with firms and workers, respectively. In this environment, 2S-TTC inherits its desired features, i.e., it is balanced-efficient, acceptable, and individually rational, and it respects internal priorities. Moreover, it is strategy–proof and stable. The uniqueness result presented in Section 5 also holds in worker exchange market.

**Theorem 7** Under Assumption 4, worker-pointing 2S-TTC is a balanced–efficient, individually rational, acceptable, strategy-proof, stable mechanism that also respects internal

\(^{46}\)Bogolomania and Moulin (2004); Roth, Sönmez, and Ünver (2005, 2007); Sönmez and Ünver (2014) also use dichotomous preferences in their matching models.
priorities, and it is the unique balanced–efficient, acceptable, student strategy-proof mechanism that respects internal priorities.

An immediate corollary of Theorem 7 is that reporting true import quota and certifying all students is a weakly dominant strategy for colleges.

**Corollary 1** Under Assumption 4, 2S-TTC is immune to quota manipulation.

## 7 Conclusions

This paper proposes a centralized market solution to overcome problems observed in decentralized exchange markets. Here, we used tuition exchange as our leading example. Our solution can also be used in temporary exchange programs, in which more than 300,000 people participate annually.

For the tuition exchange application, one concluding remark will be useful regarding the reason for the existence of such programs. Some other colleges choose to subsidize faculty directly instead of participating in tuition exchange programs; they directly pay mostly half or sometimes all of the home school’s tuition to the outgoing students. Although this may create flexibility for the students, a half-tuition benefit could be inferior to a full-tuition exchange scholarship from the faculty’s perspective (besides this direct benefit being taxable). Moreover, and more importantly, tuition exchange colleges may not want to switch to such direct compensation programs from a perspective of financial stability. For example, Boston College recently chose to remain in the Jesuit universities tuition exchange program (FACHEX) primarily to protect the school against unpredictable yearly demand shocks of direct compensation schemes, instead of paying half-tuition benefits to the faculty. Tuition exchange regimes deflate the magnitude of such demand shocks over the years and create financial stability: the marginal cost of an additional enrolled student to the college is at the order of a quarter of the full-tuition direct benefit, as fixed costs usually dominate higher education costs at the margin.\footnote{Based on private communication with the Boston College Dean of Student Enrollment, Robert Lay, in spring 2014.}

## A Appendix: Proofs

**Proof of Proposition 1.** We prove the existence by showing that for any tuition exchange market there exists an associated college admission market and the set of stable matchings are the same under both markets. Under Assumption 1, we fix a tuition
exchange market $[q, e, \succ]$. We first introduce an associated college admissions market, i.e., a Gale-Shapley (1962) two-sided many-to-one matching market, $[S, C, q, P_S, \overline{P}_C]$, where the set of students is $S$; the set of colleges is $C$; the quota vector of colleges for admissions is $q$; the preference profile of students over colleges is $P_S$; which are all the same entities imported from the tuition exchange market, and the preference profile of colleges over the set of students is $\overline{P}_C$, which we construct as follows: for all $T \subset E$ (i.e., the set of eligible students) such that $|T| < q_c$ and $i, j \in E \setminus T$, (i) \[ i P_c j \implies (T \cup i) \overline{P}_c (T \cup j), \]
(ii) \[ i P_c \emptyset \implies (T \cup i) \overline{P}_c T, \]
and (iii) $T \overline{P}_c (T \cup k)$ for all $k \in S \setminus E$. We fix $C$ and $S$ and represent such a college admission market as $[q, P_S, \overline{P}_C]$. In this college admissions market, a matching $\overline{\mu}$ is a correspondence $\overline{\mu} : C \cup S \to C \cup S \cup c_0$ such that (1) $\overline{\mu}(c) \subseteq S$ where $|\overline{\mu}(c)| \leq q_c$ for all $c \in C$, (2) $\overline{\mu}(s) \subseteq C \cup c_0$ where $|\overline{\mu}(s)| = 1$ for all $s \in S$, and (3) $s \in \overline{\mu}(c) \iff \overline{\mu}(s) = c$ for all $c \in C$ and $s \in S$. A matching $\overline{\mu}$ is individually rational if $\overline{\mu}(s) \overline{P}_c s_0$ for all $s \in S$ and for all $i \notin \overline{\mu}(c)$ we have $i \overline{P}_c \emptyset$ for all $c \in C$. A matching $\overline{\mu}$ is nonwasteful if there does not exist any $(c, s) \in C \times S$ such that \( 1 \) $cP_s \overline{\mu}(s)$, \( 2 \) $|\overline{\mu}(c)| < q_c$ and \( 3 \) $s \overline{P}_c \emptyset$. A matching $\overline{\mu}$ is blocked by a pair $(c, s) \in C \times S$ if $cP_s \mu(s)$, and there exists $s' \in \overline{\mu}(c)$ such that $s \overline{P}_c s'$. A matching $\overline{\mu}$ is stable in a college admission market if it is individually rational, nonwasteful, and not blocked by any pair.

By our construction $\overline{P}_C$ is responsive; hence there exists at least one stable matching in $[q, P_S, \overline{P}_C]$ (cf. Gale and Shapley, 1962; Roth, 1985). Let $\overline{\mu}$ be a stable matching in $[q, P_S, \overline{P}_C]$. We first show that $\overline{\mu}$ is also a matching in $[q, e, \succ]$. By the definition of a matching in a college admission market, the first three bullets of the definition of a matching in a tuition exchange market hold. Due to individual rationality, $\mu(s) = c_0$ for all $s \notin E$. Hence, $\overline{\mu}$ is a matching in $[q, e, \succ]$.

Now, we show that $\overline{\mu}$ is stable in $[q, e, \succ]$. Due to individually rationality of $\overline{\mu}$ in the college admission market, $\mu(s) \overline{P}_c s_0$ and $s \overline{P}_c \emptyset$ for all $s \in \overline{\mu}(c)$ and $c \in C$. By Assumption 1 and the definition of individual rationality in the tuition exchange market, $\overline{\mu}$ is individually rational in $[q, e, \succ]$. Whenever there exists $s \in S$ such that $cP_s \overline{\mu}(s)$, then either $s \in S \setminus E$ or $\overline{\mu} \succ_c \mu'$ for all $\mu' \in \mathcal{M}$ where $s \in \mu'(c) \cap \overline{\mu}(c) \cup s$ and $\overline{\mu}(s') = \mu'(s')$ for all $s' \in S \setminus (\overline{\mu}(c) \cup s')$. This follows from the definition of stability and construction of the college preferences in the associated college admission market and Assumption 1. Hence, $\overline{\mu}$ is stable in $[q, e, \succ]$.

Finally, we show that if $\overline{\mu}$ is not stable in $[q, P_S, \overline{P}_C]$ then it is either not a matching or unstable in $[q, e, \succ]$. Note that any matching in $[q, e, \succ]$ is also a well-defined matching in $[q, P_S, \overline{P}_C]$. Hence, it suffices to show that any matching $\mu$ in $[q, e, \succ]$ that is not stable in $[q, P_S, \overline{P}_C]$ fails to be stable in $[q, e, \succ]$. If $\mu$ is blocked by an agent in $[q, P_S, \overline{P}_C]$ then by our assumption on the preferences it is also blocked by the same agent in $[q, e, \succ]$. If $\mu$ is wasteful in $[q, P_S, \overline{P}_C]$ then there exists a college-student pair $(c, s)$ such that her addition
to the set of students admitted by \( c \) in \( \mu \) is both preferred by \( c \) and herself in \([q, e, \succeq] \).

Similarly, if \((c, s)\) is a blocking pair in \([q, P_S, P_C]\) then by our preference construction and stability definition \((c, s)\) is a blocking pair in \([q, e, \succeq] \). Thus, if \( \mu \) is a stable matching in \([q, e, \succeq] \), it is also stable in \([q, P_S, P_C]\).

Hence, the set of stable matchings under \([q, e, \succeq] \) and the set of stable matching in \([q, P_S, P_C]\) are the same.

**Proof of Proposition 2.** Consider the following market. Let \( C = \{a, b\} \) and each \( c \in C \) set \( q_c = e_c = 1 \). The set of students in each college is: \( S_a = \{1\} \) and \( S_b = \{2\} \).

The associated strict preference relations of students over colleges are given as \( P_1 : bP_1c_\emptyset \) and \( P_2 : aP_2c_\emptyset \). Student 1 is not acceptable to \( b \), i.e., \( \emptyset P_b 1 \). Student 2 is acceptable to \( a \). There is one nonwasteful matching that is not blocked by individual agents in this market: \( \mu(1) = c_\emptyset \) and \( \mu(2) = a \). This matching is not balanced, as college \( b \) has negative net balances under \( \mu \).

**Proof of Proposition 3.** Under Assumption 1, we fix a market \([q, e, \succeq] \). The case in which we have a unique stable matching in \([q, e, \succeq] \) is trivial. Hence, we consider the case in which there are at least two stable matchings. Let \( \nu \) and \( \mu \) be any two stable matchings of \([q, e, \succeq] \). By the proof of Proposition 1, \( \nu \) and \( \mu \) are also stable under the associated college admission market \([q, P_S, P_C]\).

Let \( S^\nu \) and \( S^\mu \) be the set of students assigned to a college in \( \nu \) and \( \mu \), respectively. Due to Assumption 1 Part 3 and individual rationality, \( M^\mu_c = \mu(c) \), \( M^\nu_c = \nu(c) \) for all \( c \in C \). In the rural hospital theorem (Roth, 1986) it is shown that the number of students assigned to a college is the same in all stable matchings, \( |\nu(c)| = |\mu(c)| \) for each \( c \in C \). Moreover, the set of students assigned to a real college is the same in all stable matchings, i.e., \( S^\nu = S^\mu \). Since \( X^\mu_c = S^\mu \cap S_c \), \( X^\nu_c = S^\nu \cap S_c \), and \( S^\nu = S^\mu \), we have \( X^\mu_c = X^\nu_c \). Then, \( b^\mu_c = |\mu(c)| - |S^\mu \cap S_c| = |\nu(c)| - |S^\nu \cap S_c| = b^\nu_c \) for all \( c \in C \).

We first state and prove the following Lemma, which is used in proving Proposition 4 and Theorem 2:

**Lemma 1** Under Assumption 1, let \( \tilde{\pi} \) be a stable matching of \([\tilde{q}, \tilde{e}, \succeq] \) and \( \tilde{\pi} \) be a stable matching of \([(\tilde{q}_c, \tilde{q}_-c), (\tilde{e}_c, \tilde{e}_-c), \succeq] \) where \( \tilde{e}_c = e_c + 1 \), and \( \tilde{q}_c = \tilde{q}_c \) if \( |\pi(c)| = \tilde{q}_c \) and \( \tilde{q}_c \geq \tilde{q}_c \) otherwise. Then we have \( b^\tilde{\pi}_c \in \{b^\tilde{\pi}_c - 1, b^\tilde{\pi}_c \} \) and \( b^\tilde{\pi}_c \in \{b^\tilde{\pi}_c, b^\tilde{\pi}_c + 1\} \) for all \( c' \in C \setminus c \).

**Proof.** Denote the newly certified student of \( c \) by \( i \) in \([(\tilde{q}_c, \tilde{q}_-c), (\tilde{e}_c, \tilde{e}_-c), \succeq] \). The number of positions filled by each college is the same at every stable matching by Proposition 3. Moreover, \( \tilde{\pi} \) is stable in the associated college admissions market \([\tilde{q}, P_S, P_C]\) by the proof of Proposition 1. Thus, without loss of generality, we assume \( \tilde{\pi} \) to be the outcome of the student-proposing DA algorithm in \([\tilde{q}, P_S, P_C]\).
First, consider the market \( [(\hat{q}_c, \hat{q}_{-c}), \hat{e}_c, \hat{z}] \). Let \( [(\hat{q}_c, \hat{q}_{-c}), P_S, \overline{P}_C] \) be the associated college admissions market. If \( |\hat{\pi}(c)| < \hat{q}_c \), then adding new seats to an underdemanded college will not change the set of students assigned to \( c \), and the DA selects the same outcome in \( [\hat{q}, P_S, \overline{P}_C] \) and \( [(\hat{q}_c, \hat{q}_{-c}), P_S, \overline{P}_C] \). If \( |\hat{\pi}(c)| = \hat{q}_c \), \( \hat{q}_c = \hat{q}_c \) by assumption. Hence, the DA selects the same outcome in \( [(\hat{q}_c, \hat{q}_{-c}), P_S, \overline{P}_C] \) and \( [\hat{q}, P_S, \overline{P}_C] \).

Denote the associated college admissions market of \( [(\hat{q}_c, \hat{q}_{-c}), (\hat{e}_c, \hat{e}_{-c}), \hat{z}] \) by \( [(\hat{q}_c, \hat{q}_{-c}), P_S, \overline{P}_C'] \). We will apply the sequential DA algorithm introduced by McVitie and Wilson (1971) in \( [(\hat{q}_c, \hat{q}_{-c}), P_S, \overline{P}_C'] \), where the new agent \( i \) will be considered at the end. Let \( \hat{\pi} \) be the outcome of the DA for \( [(\hat{q}_c, \hat{q}_{-c}), P_S, \overline{P}_C] \).

Let \( C_< \) be the set of colleges that could not fill all their seats, and \( C_= \) be the set of colleges that did, in \( \hat{\pi} \). Formally, \( C_< = \{ c \in C : |\hat{\pi}(c)| < \hat{q}_c \} \) and \( C_= = \{ c \in C : |\hat{\pi}(c)| = \hat{q}_c \} \). Now, when it is the turn of \( i \) to apply in the sequential version of the student-proposing DA, the current tentative matching is \( \hat{\pi} \). After \( i \) starts making offers in the algorithm, let \( \tilde{c} \) be the first college that does not reject \( i \). Since \( \emptyset P_c i, c \neq \tilde{c} \).

In the rest of the proof, as we run the sequential DA, we run the following cases iteratively, starting with student \( i \).

1. If \( \tilde{c} = c_0 \), then the algorithm terminates; \( b^\tilde{c} = b^\tilde{c} \).

2. If \( \tilde{c} \in C_< \), then \( i \) will be assigned to \( \tilde{c} \) and the algorithm terminates; \( b^\tilde{c}_i = b^\tilde{c} - 1 \), \( b^\tilde{c}_c = b^\tilde{c} + 1 \), and \( b^\tilde{c}_{-\{c,\tilde{c}\}} = b^\tilde{c}_{-\{c,\tilde{c}\}} \).

3. If \( \tilde{c} \in C_= \), then student \( i \) who has the lowest priority among the students in \( \hat{\pi}(\tilde{c}) \) is rejected in favor of \( i \). We consider two cases:

   3.a. Case \( \tilde{i} \in S_\cdot \): The net balance of no college will change since the beginning, and we continue from the beginning above again using student \( \tilde{i} \) instead of \( i \).

   3.b. Case \( \tilde{i} \notin S_\cdot \): The instantaneous balance of \( c \) will deteriorate by 1 as \( i \) is tentatively accepted. Now, it is \( \tilde{i} \)'s turn in the sequential DA to make offers. In this series of offers, suppose the first college that does not reject student \( \tilde{i} \) is \( \tilde{c} \). Denote the home college of \( \tilde{i} \) by \( c' \) (note that \( c' \neq c \)).

   3.b.i. If \( \tilde{c} \in c_0 \cup (C_<) \), then the algorithm will terminate, and \( b^\tilde{c}_i \in \{b^\tilde{c} - 1, b^\tilde{c}\}, b^\tilde{c}_c \in \{b^\tilde{c}_c, b^\tilde{c}_c + 1\} \) for all \( \tilde{c} \in C \setminus c \).

   3.b.ii. If \( \tilde{c} \in C_= \), then the lowest-priority student held by \( \tilde{c} \) will be rejected in favor of \( \tilde{i} \). Let this student be \( \tilde{i} \). There are two further cases:

   3.b.ii.a. Case \( \tilde{i} \in S_\cdot \): Then, \( \tilde{c} \neq c \). The instantaneous balance of \( c \) will increase by 1, and we will start from the beginning again above with \( \tilde{i} \) instead of \( i \). The total change in \( c \)'s balance since the beginning will be 0. Also, no other college's balance has changed since the beginning.

   3.b.ii.b. Case \( \tilde{i} \notin S_\cdot \): We start from Step 3.b above with student \( \tilde{i} \) instead of \( i \).
Thus, whenever we continue from the beginning, the instantaneous balance of \( c \) is \( b^*_c \), and whenever we continue from Step 3.b, the instantaneous balance of \( c \) is \( b^*_c - 1 \) or \( b^*_c \) and the instantaneous balances of all other colleges either increase by one or stay the same. Due to finiteness, the algorithm will terminate at some point at Steps 1 or 2, or Steps 3.b.i or 3.b.ii; and the net balance of \( c \) at the new DA outcome will be \( b^*_c \) or \( b^*_c - 1 \). Moreover, whenever the algorithm terminates, the net balance of any other college has gone up by one or stayed the same.

We are ready to prove the results stated in the main text:

**Proof of Proposition 4.** Let \( \bar{q}, P_S, \bar{P}_C \) and \( [\bar{q}_c, \bar{q}_{-c}), P_S, \bar{P}'_C \) be the associated college admissions problems of \( [\bar{q}, \bar{e}, \bar{x}], [(\bar{q}_c, \bar{q}_{-c}), (\bar{e}_c, \bar{e}_{-c}), \bar{z}] \), respectively. Let \( \hat{\pi} \) and \( \hat{\pi} \) be the outcome of the DA in \( \bar{q}, P_S, \bar{P}_C \) and \( [(\bar{q}_c, \bar{q}_{-c}), P_S, \bar{P}'_C] \), respectively. By Propositions 1 and 3, it is sufficient to prove the proposition for \( \hat{\pi} \) and \( \hat{\pi} \). Note that \( M^*_c = \hat{\pi}(c) \), by Assumption 1 Part 3, and \( \hat{\pi} \) is stable.

Two cases are possible:

**Case 1:** \( b^*_c < 0 \): We have \( |\hat{\pi}(c)| = |M^*_c| < |X^*_c| \leq \hat{e}_c \leq \bar{q}_c \). Then, by Lemma 1, \( b^*_c \in \{b^*_c - 1, b^*_c \} \).

**Case 2:** \( b^*_c \geq 0 \): We have two cases again:

2.a. \( |\hat{\pi}(c)| < \bar{q}_c \) or \( \bar{q}_c = \hat{q}_c \): By Lemma 1, \( b^*_c \in \{b^*_c - 1, b^*_c \} \).

2.b. \( |\hat{\pi}(c)| = \bar{q}_c \) and \( \bar{q}_c = \hat{q}_c + k \) for \( k > 0 \): Denote the newly certified student of \( c \) by \( i \) in market \( [(\bar{q}_c, \bar{q}_{-c}), (\bar{e}_c, \bar{e}_{-c}), \bar{z}] \). We first consider the outcome of the DA in the associated college admissions market of \( [(\bar{q}_c, \bar{q}_{-c}), (\bar{e}, \bar{z})] \), which we denote by \( \pi'' \). We first show that the number of students imported by \( c \) in \( \pi'' \) cannot be less than the one in \( \hat{\pi} \). Let \( C_< = \{c \in C : |\hat{\pi}(c)| < \bar{q}_c \} \). By our construction, in any stable matching of an associated college admissions market all students in \( S \setminus E \) are assigned to \( c_0 \). Due to the nonwastefulness of \( \hat{\pi} \), \( \hat{\pi}(s) P_s \tilde{c} \) for all \( s \in E \setminus \hat{\pi}(\tilde{c}) \) and \( \tilde{c} \in C_< \). We know that the DA is resource monotonic: when the number of seats increases, then every student will be weakly better off (cf. Kesten, 2006). That is, \( \pi''(s) R_s \tilde{c} \) for all \( s \in E \). By combining the resource monotonicity and individual rationality of the DA, we can say if a student is assigned to a college in \( \hat{\pi} \) then he will also be assigned to a college in \( \pi'' \). Hence, we can write:

\[
\sum_{c' \in C} |\pi''(c')| \geq \sum_{c' \in C} |\hat{\pi}(c')|.
\]

(1)

Note that the difference between the left-hand side and the right-hand side of the equation can be at most \( k \). This follows from the fact that in \( \pi'' \) no new student will be assigned to a college in \( C_< \), the number of students assigned to other colleges can increase only for \( c \), and the maximum increment is \( k \).
By combining nonwastefulness and resource monotonicity we can write:

$$\sum_{c \in C_<} |\pi''(c)| \leq \sum_{c \in C_<} |\hat{\pi}(c)|. \tag{2}$$

Then, if we subtract the left–hand side of Equation 2 from the left–hand side of Equation 1 and the right–hand side of Equation 2 from the right–hand side of Equation 1, we get the following inequality:

$$\sum_{c' \in C \setminus C_<} |\pi''(c')| \geq \sum_{c' \in C \setminus C_<} |\hat{\pi}(c')|. \tag{3}$$

Given that each college in \(C \setminus C_<\) fills its seats in \(\hat{\pi}\), when we subtract \(\sum_{c' \in C \setminus (C_< \cup c)} \hat{q}_c\) from both sides of Equation 3 we get the following inequality:

$$|\pi''(c)| + \sum_{c' \in C \setminus (C_< \cup c)} (|\pi''(c')| - \hat{q}_c) \geq |\hat{\pi}(c)|. \tag{4}$$

The term \(\sum_{c' \in C \setminus (C_< \cup c)} (|\pi''(c')| - \hat{q}_c)\) is nonnegative since \(\pi''(c')| \leq \hat{q}_c\) for all \(c' \in C \setminus (C_< \cup c)\). Therefore, \(|\pi''(c)| \geq |\hat{\pi}(c)|\).

If \(|\pi''(c)| = |\hat{\pi}(c)|\) then \(|\pi''(c')| = |\hat{\pi}(c')|\) for all \(c' \in C\). This follows from Equation 4, Equation 2, and the fact that \(|\pi''(c')| \leq |\hat{\pi}(c')|\) for all \(c' \in C \setminus \{c\}\). Therefore, \(c\) cannot export and import more students and \(b_c'' = b_c\). If \(|\pi''(c)| > |\hat{\pi}(c)|\), then at most \(k\) more students can be assigned to a college in \(\pi''\) among the eligible students who were not assigned to a college in \(\hat{\pi}\). It is possible that some of the students belong to \(S_c\). Thus, \(b_c'' \in \{b_c, \ldots, b_c + k\}\).

Thus, by Lemma 1, as we increase the eligibility quota of college \(c\) by 1 and keep the admission quota at \(\hat{q}_c\), we have \(b_c \in \{b_c'' - 1, b_c''\}\), and hence, \(b_c \in \{b_c - 1, b_c, \ldots, b_c + k\}\).

**Proof of Theorem 1.** Given Proposition 4, when \(c\) decreases its certification quota by one keeping its admission quota the same, its balance in any stable outcome of the new market will either be the same or increase by one. Since \(c\) will have a nonnegative balance in any stable outcome of the market \([\hat{q}, (\hat{e}_c = 0, \hat{e}_{-c}), \bar{\pi}]\), there exists \(0 \leq \hat{e}_c \leq \hat{e}_c\) such that \(c\) has a zero–balance in every stable matching of the market \([\hat{q}, (\hat{e}_c, \hat{e}_{-c}), \bar{\pi}]\). ■

**Proof of Theorem 2.** We consider two problems: \([\hat{q}, \hat{e}, \bar{\pi}]\) and \([(\hat{q}'_c, \hat{q}_{-c}), (\hat{e}_{c} - 1, \hat{e}_{-c}), \bar{\pi}]\) with \(\hat{q}_c \geq \hat{e}_c\) and \(\hat{q}_c \geq \hat{q}'_c \geq \hat{e}_c - 1\) such that for \(c\), \(b_c' < 0\) for a stable matching \(\mu\) of the first market. Let \(\mu'\) be an arbitrary stable matching of the second market. We want to show that \(b_c' > 0\). From Proposition 4, we know that \(b_c' < 0\) or \(b_c' = 0\).
By Proposition 3, without loss of generality we assume that \( \mu \) and \( \mu' \) are the outcome of the sequential DA algorithm for the associated college admissions market of \([\hat{q}, \hat{e}, \hat{\nu}]\) and \([(q', \hat{q}_{c-1}, \hat{e}_{c-1}), \hat{\nu}]\), respectively. We have two cases:

**Case 1:** \( b^e_{c'} < 0 \). We have \(|\mu'(c)| = |M^e_c| < |X^e_c| \leq e_c - 1 \leq \min\{\hat{q}_c, q'_c\} \). Hence, as \( c \) did not fill its admission quota at \( \mu' \) under both \( \hat{q}_c \) and \( q'_c \), in market \([\hat{q}, (\hat{e}_{c-1}, \hat{e}_{c-1}), \hat{\nu}]\) \( \mu' \) will still be the outcome of DA for the associated college admissions market. When we add a new student \( i \) from \( c \) to the set of eligible students, we obtain \([\hat{q}, \hat{e}, \hat{\nu}]\). By Lemma 1, we have \( b^e_{c'} \in \{b^e_{c'}, b^e_{c'} + 1\} \) for all \( c' \in C \setminus c \).

**Case 2:** \( b^e_{c'} = 0 \). There are two possibilities: (a) \(|\mu'(c)| < q'_c \) and (b) \(|\mu'(c)| = q'_c \).

2.a. If \(|\mu'(c)| < q'_c \), then by Lemma 1, we have \( b^e_{c'} \in \{b^e_{c'}, b^e_{c'} + 1\} \) for all \( c' \in C \setminus c \).

2.b. If \(|\mu'(c)| = q'_c \), then \(|\mu'(c)| = \hat{e}_c - 1 = q'_c \). We first increase the admission quota of \( c \) from \( q'_c \) to \( \hat{q}_c \) and keep its eligibility quota at \( \hat{e}_c - 1 \). Suppose the number of students assigned to \( c \) increases at the outcome of the DA under the associated college admissions market of \([\hat{q}, (\hat{e}_{c-1}, \hat{e}_{c-1}), \hat{\nu}]\), which we denote by \( \mu'' \), i.e., \(|\mu''(c)| > |\mu'(c)| = \hat{e}_c - 1 \). Thus, \( b^e_{c''} > 0 \). When we also increase the eligibility quota of \( c \) from \( \hat{e}_c - 1 \) to \( \hat{e}_c \), then by Lemma 1, \( b^e_{c} \in \{b^e_{c''} - 1, b^e_{c''}\} \), and hence, \( b^e_{c} \geq 0 \). However, this contradicts the fact that \( b^e_{c'} < 0 \). Therefore, \(|\mu'(c)| = |\mu'(c)| = q'_c \leq \hat{q}_c \). Hence, under both associated college admissions problems of \([(q', \hat{q}_{c-1}), (\hat{e}_{c-1}, \hat{e}_{c-1}), \hat{\nu}]\) and \([\hat{q}, (\hat{e}_{c-1}, \hat{e}_{c-1}), \hat{\nu}]\), the DA chooses the same matching, i.e., \( \mu'' = \mu' \). When we increase the eligibility quota of \( c \) from \( \hat{e}_c - 1 \) to \( \hat{e}_c \) and keep the admission quota at \( \hat{q}_c \), the DA outcome changes from \( \mu'' = \mu' \) to \( \mu \) under the associated college admissions market. By Lemma 1, we have \( b^e_{c'} \in \{b^e_{c'}, b^e_{c'} + 1\} \) for all \( c' \in C \setminus c \).

In either case, \( b^e_{c'-c} \leq b^e_{c''-c} \).

**Proof of Theorem 3. Individual Rationality:** Let \( \pi \) be the matching selected by 2S-TTC. Since each \( s \in S \) is assigned to a college better than \( c_0 \), \( s \) does not block \( \pi \). Since all students in \( \pi(c) \) are ranked above \( \emptyset \) in \( P_e \) for each \( c \in C \), \( \pi(c) \subseteq R^*_{c} \hat{S} \) for any \( \hat{S} \subseteq \pi(c) \). In any matching \( \mu \in \mathcal{M} \) such that \( \mu(s) = \pi(s) \) for all \( s \in S \setminus \pi(c) \) and \( \mu(c) \subseteq \pi(c) \), \( c \in C \) has a nonpositive net balance. By Assumption 3, \( \pi \) is not blocked by \( c \).

**Acceptability:** Students will be assigned to the null college, \( c_0 \), whenever they point to it, and hence, they will never need to point to an unacceptable college. Moreover, a student cannot point to a college that considers her unacceptable. Therefore, the students ranked below \( \emptyset \) in \( P_e \) cannot be assigned to \( c \). Thus, the 2S-TTC is acceptable.

**Respect for Internal Priorities:** Suppose, contrary to the claim, that 2S-TTC does not respect internal priorities. Then there exists a student \( s \in S_c \) who is assigned to a college in \([q, e, \nu]\), but not assigned to a college in \([(\hat{q}_c, q_{c-1}), (\hat{e}_c, e_{c-1}), \nu]\) where \( \hat{q}_c > q_c \).
and $\tilde{e}_c > e_c$. We use a variation of the 2S-TTC in which the students with the highest priority point to a college $c \in C$ in each round. Since only the top priority students and students pointing to $c_0$ can form a cycle in each round under both versions 2S-TTC, they will select the same outcome. Let $S(k)$ and $\tilde{S}(k)$ be the set of students assigned in Round $k$ of 2S-TTC applied to the problems $[q, e, \succ]$ and $[(\tilde{q}_c, q_{-c}), (\tilde{e}_c, e_{-c}), \succ]$, respectively. In both problems, the same set of agents will be active in the first round. Since we consider the same preference profile, $S(1) = \tilde{S}(1)$. Then, if $s \in S(1)$, we are done. If not, consider the second round. Since the same set of students is removed with their assignments, the set of active students and the remaining colleges in the second step of 2S-TTC applied to the problems will be the same. Moreover, students will be pointing to the same colleges in both problems. Hence, $S(2) = \tilde{S}(2)$. Then, if $s \in S(2)$, we are done. If not, we can repeat the same steps and show that $s$ will be assigned in the matching selected by the 2S-TTC in market $[(\tilde{q}_c, q_{-c}), (\tilde{e}_c, e_{-c}), \succ]$.

**Balanced-efficiency:** Since the matching selected by the 2S-TTC consists of trading cycles in which students and their assignments form unique cycles, its outcome is balanced by Remark 1. Let $\pi$ be the matching selected by 2S-TTC. Let $S(k)$ be the set of students assigned in Round $k$ of 2S-TTC. We will prove that $\pi$ cannot be Pareto dominated by another balanced matching in two parts.

**Part I:** We first prove that $\pi$ cannot be Pareto dominated by another acceptable balanced matching. If $s \in S(1)$, then $\pi(s)$ is the highest ranked college in her preference list that considers her acceptable. That is, no agent $s \in S(1)$ can be assigned to a better college considering her acceptable. If there exists $\nu \in \mathcal{M}$ such that $\nu \succ_s \pi$ then $\nu(s)$ considers $s$ unacceptable. That is, $\pi$ cannot be Pareto dominated by another $\nu$ in which at least one student in $S(1)$ is better off under $\nu$, and all students are assigned to a college that considers them acceptable.

If a student $s \in S(2)$ is not assigned to a more preferred $c \in C$ that considers her acceptable, then $c$ should either fill its quota in Round 1 by another $s' \in S$, or $e_c = 1$ and the only eligible student $\tilde{s} \in E_c$ is assigned to $c_0$. For the first case, since $s'$ is assigned in Round 1, $\pi(s') = c$ is her favorite college among the ones considering her acceptable. That is, in any $\nu \in \mathcal{M}$ in which $s$ is assigned to $\pi(s')$, $s'$ will be made worse off. Hence, $\pi$ cannot be Pareto dominated by another balanced $\nu$ in which at least one student in $S(2)$ is better off under $\nu$. For the second case, $\tilde{s}$ needs to be assigned to a college in order for $s$ to be assigned to $c$. Otherwise, balancedness will be violated. This will violate acceptability, since $\tilde{s}$ considers all colleges considering her acceptable as unacceptable.

We similarly show the same for all other rounds of the 2S-TTC. Thus, no student can be assigned to a better college without harming any other student among the colleges.
that consider her acceptable. Hence, no college can be made better off without harming another college either, if we focus on matchings that are acceptable.

Part II: Next we show that there does not exist a unacceptable balanced matching that Pareto dominates \( \pi \). To the contrary of the claim, suppose there exists an individually irrational balanced matching \( \nu \) that Pareto dominates \( \pi \). Then each \( i \in C \cup S \) weakly prefers \( \nu \) to \( \pi \), and at least one agent \( j \in C \cup S \) strictly prefers \( \nu \) to \( \pi \). Due to the individual rationality of the 2S-TTC, every student weakly prefers her assignment in \( \pi \) to \( c_0 \) or to being assigned to an unacceptable college. Therefore, every assigned student in \( \pi \) is also assigned to an acceptable college under \( \nu \). Thus, due to the balancedness of both \( \pi \) and \( \nu \), \( |\nu(c)| \geq |\pi(c)| \) for all \( c \in C \). As \( \nu \) is acceptable, there exists some \( c_0 \in C \) such that \( s_0 \in \nu(c_0) \) is unacceptable for \( c_0 \). As \( \nu \geq_{c_0} \pi \), there should be at least one student \( s_1 \in \nu(c_0) \setminus \pi(c_0) \) such that \( s_1 \) is acceptable for \( c_0 \) by Assumption 3 and \( \nu(s_1)P_{s_1}\pi(s_1) \).

We need to consider two cases regarding \( \pi(s_1) \):

1. First, suppose \( \pi(s_1) = c_0 \). Denote the home college of \( s_1 \) by \( c_1 \). Hence, \( |\nu(c_1)| > |\pi(c_1)| \) by balancedness of \( \nu \) and \( \pi \). By Assumption 3, \( \nu(c_1)P_{c_1}\pi(c_1) \) and there exists a student \( s_2 \in \nu(c_1) \setminus \pi(c_1) \) such that \( s_2 \) is acceptable for \( c_1 \) and \( \nu(s_2)P_{s_2}\pi(s_2). \)

2. Next, suppose \( \pi(s_1) \in C \). Denote \( \pi(s_1) \) by \( c_1 \). As \( |\nu(c_1)| \geq |\pi(c_1)| \), there exists \( s_2 \in \nu(c_1) \setminus \pi(c_1) \) and \( s_2 \) is acceptable for \( c_1 \) by Assumption 3. We also have \( \nu(s_2)P_{s_2}\pi(s_2). \)

We continue with \( s_2 \) and her assignment \( \pi(s_2) \), similarly construct \( c_2 \) and then \( s_3 \). As we continue, by finiteness, we should encounter the same student \( s_k = s_\ell \) for some \( k > \ell \geq 1 \), that is, we’ve encountered her before in the construction. Consider the students \( s_{\ell+1}, s_{\ell+2}, \ldots, s_k \). Let \( s_{k'} \) be the student who is assigned in the earliest round of the 2S-TTC in this list. By definition, she points to \( \pi(s_{k'}) \). However, she prefers college \( c_{k'-1} \) to her assignment, and she is acceptable at \( c_{k'-1} \). Moreover, we know that college \( c_{k'-1} \) has not been removed yet from the algorithm, because if \( c_{k'-1} \) was constructed in Case 1 above then \( q_{c_{k'-1}} > |\pi(c_{k'-1})| \) and \( s_{k'-1} \in S_{c_{k'-1}} \) is still not removed, and if \( c_{k'-1} \) was constructed in Case 2 above then \( s_{k'-1} \in \pi(c_{k'-1}) \) is still not removed. Therefore, \( s_{k'} \) should have pointed to \( c_{k'-1} \) in 2S-TTC in that round. This is a contradiction to \( \nu \) Pareto dominating \( \pi \).

Proof of Proposition 5. Suppose that there does exist such a mechanism. Denote it by \( \psi \). In this proof we will use different examples to show our result.

Case 1: Let \( C = \{a,b,c\} \) and \( S_a = \{1,2\}, S_b = \{3\}, S_c = \{4\} \). Let \( q = e = (2,1,1) \). Let \( \succeq_S \) be the student preference profile with associated rankings over \( C \cup c_\emptyset \): \( P_1 : bP_1cP_1c_\emptyset, P_2 : cP_2c_\emptyset, P_3 : aP_3c_\emptyset \) and \( P_4 : aP_4c_\emptyset \). Let \( \succeq_C \) be the college preference profile with associated rankings over students \( P_a : 3P_4a4P_a\emptyset, P_b : 1P_b\emptyset \) and \( P_c : 1P_22P_c\emptyset \). We assume that \( \succeq_C \) satisfies Assumption 3. There are two balanced–efficient and individually rational
matchings: $\mu_1 = \left( \begin{array}{ccc} a & b & c \\ 4 & \emptyset & 1 \end{array} \right)$ and $\mu_2 = \left( \begin{array}{ccc} a & b & c \\ \{3,4\} & 1 & 2 \end{array} \right)$.

If $\psi$ selects $\mu_1$, then $a$ can manipulate $\psi$ by submitting $\succsim_a^1$ where $P_a^1 : 3P_a^1 \emptyset$ and any matching $\pi$ such that $4 \notin \pi(a)$ is preferred to $\mu_1$. Then the only individually rational and balanced–efficient matching is $\mu_3 = \left( \begin{array}{ccc} a & b & c \\ 3 & 1 & \emptyset \end{array} \right)$. Therefore, $\psi[q,e,\succsim_a] = \mu_2$.

**Case 2:** We consider the same example with a slight change in $a$’s preferences. Let $\succsim_a^2$ be $a$’s preferences over the matchings with associated ranking $P_a^2 : 4P_a^2 3P_a^2 \emptyset$. In this case, $\mu_1$ and $\mu_2$ are the only two balanced–efficient and individually rational matchings.

If $\psi$ selects $\mu_1$, then $a$ can manipulate $\psi$ by submitting $\succsim_a^1$. Then we will be in Case 1 and $\mu_2$ will be selected, which makes $a$ better off. Therefore, $\psi[q,e,(\succsim_a^2, \succsim_a^-)] = \mu_2$.

**Case 3:** Now consider the case where colleges report the following preferences $\succsim_a^3$ where $\succsim_a = \succsim_a^1$, $\succsim_b = \succsim_b$, $P_c^3 : 1P_c^3 \emptyset$ is the associated ranking with $\succsim_c^3$ and any matching $\pi$ such that $2 \notin \pi(c)$ is preferred to $\mu_2$ under $\succsim_c^3$. Then there are two individually rational and balanced–efficient matchings: $\mu_4 = \left( \begin{array}{ccc} a & b & c \\ 4 & \emptyset & 1 \end{array} \right)$ and $\mu_5 = \left( \begin{array}{ccc} a & b & c \\ 3 & 1 & \emptyset \end{array} \right)$.

If $\psi$ selects $\mu_4$, then in Case 2 $c$ can manipulate $\psi$ by reporting $\succsim_c^3$. Therefore, $\psi[q,e,\succsim_c^3] = \mu_5$.

**Case 4:** Now consider the case where colleges report the following preferences $\succsim_a^4$ where $\succsim_a = \succsim_a^1$, $\succsim_a = \succsim_a^1$, $P_a^4 : 4P_a^4 \emptyset$ is the associated ranking with $\succsim_a^4$ and any matching $\pi$ such that $3 \notin \pi(a)$ is preferred to $\mu_5$ under $\succsim_a^4$. There is a unique balanced-efficient and individually rational matching: $\mu_4$. In Case 3, $a$ can manipulate $\psi$ by reporting $\succsim_a^4$; then we will be in Case 4 and $a$ will be better off with respect to Case 3 preferences.

Therefore, there does not exist a balanced–efficient, individually rational mechanism that is immune to preference manipulation by colleges. ■

**Proof of Theorem 4.** Consider the preference relations of each student who ranks as acceptable only those colleges that find her acceptable. If we consider only these preferences as possible preferences to choose from for each student, we see that the 2S-TTC is group strategy-proof for students, as Pápai (cf. 2000) showed that the TTC is group strategy-proof. In the 2S-TTC, observe that students are indifferent among reporting preference relations that rank the colleges finding themselves as acceptable in the same relative order. Thus, the 2S-TTC is group strategy-proof for students. ■

**Proof of Proposition 8.** The 2S-TTC mechanism takes into account only the set of acceptable students based on the submitted preferences of colleges over the matchings. Hence, for any two different preference profiles with the same set of acceptable students, the 2S-TTC selects the same outcome. ■

The following Lemma is used in proving Theorem 5:
Lemma 2. Let \( \pi \) and \( \tilde{\pi} \) be the outcome of 2S-TTC in \([q, e, \succsim]\) and \([(\tilde{q}_c, q_{-c}), (\tilde{e}_c, e_{-c}), \succsim]\) where \( \tilde{q}_c \leq q_c \) and \( \tilde{e}_c \leq e_c \) for some \( c \in C \). Then, \( M^\pi_{c} \subseteq M_c^\pi \) and \( X^\pi_{c} \subseteq X_c^\pi \).

Proof. We have three cases to consider:

Case 1: \( \tilde{q}_c = q_c \) and \( \tilde{e}_c < e_c \). We consider the case in which one more student is certified by \( c \), i.e., \( e_c + 1 = e_c \). Denote the student added to the eligible set by \( s' \). Let \( s' \in S_c \) and \( r_c(s') = r_c(s) - 1 \). Consider the execution of the 2S-TTC for this new market.

If \( c \) imports \( \tilde{q}_c \) students before \( s' \)'s turn, then \( c \) will be removed, and certifying one more student will not affect the set of students exported and imported by \( c \). Now consider the case in which \( c \) imports less than \( \tilde{q}_c \) before \( s' \)'s turn. Denote the intermediate matching that we have just after \( s' \) is processed by \( \nu \). Since \( c \) is removed just after \( s' \) is processed in \([(\tilde{q}_c, q_{-c}), (\tilde{e}_c, e_{-c}), \succsim]\), \( M^\pi_{c} = M_c^\nu \) and \( X^\pi_{c} = X_c^\nu \). If \( s \) is assigned to a college \( c' \in C \setminus c \) at market, \( c \) will import one more acceptable student. Denote that matching by \( \mu \). Given that one more student is exported and imported compared to the ones in \( \nu \) we have \( M^\pi_{c} = M_c^\nu \subseteq M^\pi_{c} \) and \( X^\pi_{c} = X_c^\nu \subseteq X_c^\pi \). If \( s \) is assigned to the \( c_0 \) or \( c \), then \( c \) will have the same import and export set. If we keep certifying all \( e_c - \tilde{e}_c \) students one at a time, similarly we will have \( M^\pi_{c} \subseteq M_c^\pi \) and \( X^\pi_{c} \subseteq X_c^\pi \) where \( \pi \) is the outcome of the 2S-TTC in \([q, e, \succsim]\).

Case 2: \( \tilde{q}_c < q_c \) and \( \tilde{e}_c = e_c \). Let \( \pi \) and \( \nu \) be the outcomes of the 2S-TTC in \([q, e, \succsim]\) and \([(\tilde{q}_c, q_{-c}), e, \succsim]\), respectively. If \( |M_c^\nu| < \tilde{q}_c \) then the 2S-TTC will select \( \nu \) when \( c \) reports either \( \tilde{q}_c \) or \( q_c \). Therefore, \( M^\pi_{c} = M_c^\nu \) and \( X^\pi_{c} = X_c^\nu \). If \( |M_c^\nu| = \tilde{q}_c \) and if all eligible students of \( c \) are considered in \([(\tilde{q}_c, q_{-c}), e, \succsim]\) when it is removed, then it will not make a difference if \( c \) reports either \( \tilde{q}_c \) or \( q_c \). If \( |M_c^\nu| = \tilde{q}_c \) and if \( c \) is removed before all its eligible students are considered, then one more student \( s \in S_c \) might be considered when \( c \) reports \( q_c \). As in the previous case, \( c \) may import and export at least one more student. At the end, we get \( M^\pi_{c} \subseteq M_c^\nu \subseteq M_c^\pi \) and \( X^\pi_{c} \subseteq X_c^\nu \subseteq X_c^\pi \) if some of the students who are considered only when \( c \) reports \( q_c \) are assigned. Otherwise, \( M^\pi_{c} = M_c^\nu \) and \( X^\pi_{c} = X_c^\nu \).

Case 3: \( \tilde{q}_c < q_c \) and \( \tilde{e}_c < e_c \). Let \( \mu \) be the outcome of the 2S-TTC in \([q, (\tilde{e}_c, e_{-c}), \succsim]\). Then, we have \( M^\pi_{c} \subseteq M_c^\mu \subseteq M_c^\pi \) and \( X^\pi_{c} \subseteq X_c^\mu \subseteq X_c^\pi \), where the first and second subset relations come from invoking Case 1 and Case 2, respectively.

Proof of Theorem 5. We prove a stronger version of Theorem 5: Under the 2S-TTC, suppose that preference profiles are fixed for colleges such that no college reports an unacceptable student as acceptable in its preference report. In the induced quota-reporting game, under Assumption 3, it is a weakly dominant strategy for any \( c \in C \) to certify all its students and to reveal its true admission quota.

Take a market \([q, e, \succsim]\) and a college \( c \). Suppose that preference reports are fixed such that \( c \) does not report any unacceptable students as acceptable in these reports. We have
two cases to consider for possible quota manipulations by $c$:

**Case 1:** $c$ reports $\tilde{q}_c \leq q_c$ and $\tilde{e}_c \leq |S_c|$: In Lemma 2 we have shown that when $c$ reports its admission and eligibility quotas higher, the set of students imported by $c$ (weakly) expands. By Assumption 3, reporting $\tilde{q}_c \leq q_c$ and $\tilde{e}_c \leq |S_c|$ is weakly dominated by reporting the true admission quota and certifying all students.

**Case 2:** $c$ reports $\tilde{q}_c > q_c$: This strategy is weakly dominated by reporting its true admission quota $q_c$. We prove this as follows: Let $\nu$ and $\mu$ be the matchings that the 2S-TTC mechanism selects when $c$ reports $\tilde{q}_c$ and $q_c$, respectively. If $|M_c^\nu| \leq q_c$, then $M_c^\nu = M_c^\mu$ and $X_c^\nu = X_c^\mu$; thus, it is indifferent between the two matchings. However, if $|M_c^\nu| > q_c$, among the balanced matchings by Assumption 3, colleges preferences depend on the preferences on admitted students which are only responsive up to true admission quota, and $\mu$ is individually rational, then it prefers $\mu$ to $\nu$.

**Proof of Theorem 6.** Here, we use a variant of the 2S-TTC. In this variant, we select only one cycle in one round. If there is more than one cycle, then the selection is done randomly. Let $S(k)$ be the set of students removed in Round $k$. Suppose the theorem does not hold. Let $\psi$ be the mechanism satisfying all four axioms, and select a different matching for $[q,e,\succ]$. Denote the outcome of 2S-TTC for $[q,e,\succ]$ by $\mu$. In the rest of the proof, we work on students’ preferences over colleges, $P_S$, instead of matchings, $\succ_S$.

We first prove the following claim:

**Claim:** If there exists a student in $S(1)$ who prefers her assignment in $\psi[q,e,\succ]$ to the one in $\mu$, then there exists another student in $\bigcup_{k=1}^{k-1} S(k)$ who prefers her assignment in $\mu$ to the one in $\psi[q,e,\succ]$.

**Proof of Claim:** We use induction in our proof. Consider the students in $S(1)$. If $S(1)$ is a singleton, then the student in $S(1)$ is assigned to $c_0$. Any college that she prefers to $c_0$ considers her unacceptable. If she prefers her assignment under $\psi$ to $c_0$, then she is assigned to a college considering her unacceptable by $\psi$. Therefore, $\psi$ is not acceptable. If she prefers $c_0$ to her assignment under $\psi$, then $\psi$ is not acceptable. Then any acceptable mechanism will assign her to $c_0$. If $S(1)$ is not a singleton, then all students in $S(1)$ are assigned to the best colleges considering themselves as acceptable, and they prefer their assignment in $\mu$ to $c_0$. If $s \in S(1)$ prefers her assignment in $\psi[q,e,\succ]$ to $\mu(s)$ then $\psi$ is not acceptable. Hence, all students in $S(1)$ weakly prefer their assignment in $\mu$.

In the inductive step, assume that for all Rounds 1, ..., $k - 1$, for some $k > 1$, the claim is correct. Consider Round $k$. If there exists a student $s \in S(k)$ such that $c = \psi[q,e,\succ](s) P_s \mu(s)$ then either $c$ considers $s$ acceptable and its seats are filled in Rounds $k^* = \bigcup_{k'=1}^{k-1} S(k') = \emptyset$.\footnote{We take $\bigcup_{k'=1}^{k-1} S(k') = \emptyset$.}
the same college in 1, ..., k - 1 of 2S-TTC, or s is unacceptable for c. In the latter case, ψ is not acceptable. Consider the former case. Let s’ be assigned to c under μ in Round k’ ≤ k - 1 but not under ψ[q, e, ˜c], as s is assigned instead of her. If she prefers c to ψ[q, e, ˜c](s’) then we are done. If she does not, k’ > 1, and by the inductive step, there exists a student s'' ∈ S(k'') for some k'' < k' ≤ k - 1 who prefers μ(s'') to ψ[q, e, ˜c](s’’).

One can observe that, if there does not exist a student strictly preferring ψ[q, e, ˜c] to μ, then there exists at least one student strictly preferring μ to ψ[q, e, ˜c]. Otherwise, μ = ψ[q, e, ˜c].

Now we are ready to prove the theorem. By the Claim and the observation above, as μ ̸= ψ[q, e ˜c], there exists a student s and some round k such that s ∈ S(k), she prefers μ(s) to ψ[q, e, ˜c](s), and μ(s’) = ψ[q, e, ˜c](s’) for all s’ ∈ ∪_{k’=1}^{k-1} S(k’).

We will construct our proof in three steps. Assign to each round of the 2S-TTC mechanism a counter and set it as Counter(k') = |S(k')| - 1 for all rounds k'.

**Step 1:** Construct a preference profile ˜c with associated ranking ˜P as follows: Let student s ∈ Sc rank only μ(s) as acceptable in ˜P and ˜s_s = ˜c_s. 2S-TTC will select μ for [q, e, ˜c]. Since ψ is student strategy-proof and acceptable, ψ[q, e, ˜c](s) = c_0.

Then, we check whether the assignments of students in ∪_{k’=1}^{k-1} S(k’) are the same in ψ[q, e, ˜c] and μ. If not, then for some ˜k < k, there exists a student ˜s ∈ S(˜k) preferring μ(˜s) to ψ[q, e, ˜c](˜s) and each student in ∪_{k’=1}^{k-1} S(k’) gets the same college in μ and ψ[q, e, ˜c]. Then we repeat Step 1 by taking ˜c := ˜c, s := ˜s, and k := ˜k.

This repetition will end by the finiteness of rounds and the fact that ∪_{k’=1}^{k-1} S(k’) = ∅. When all students in ∪_{k’=1}^{k-1} S(k’) get the same college in μ and ψ[q, e, ˜c], then we proceed to Step 2.

**Step 2:** In Step 1, we have shown that s prefers μ(s) to ψ[q, e, ˜c](s) = c_0. Suppose c is the home college of s. Set a new eligibility quota ˜e_c equal to the rank of student s in c’s internal priority order, that is, ˜e_c = r_c(s), and let ˜e_e = e_e. In [q, ˜e, ˜c], the 2S-TTC assigns all students in ∪_{k’=1}^{k} S(k’) to the same college as in μ. ψ[q, ˜e, ˜c](s) = c_0 since ψ respects internal priorities and we weakly decreased c’s eligibility quota. We check whether the assignments of students in ∪_{k’=1}^{k-1} S(k’) are the same in both ψ[q, ˜e, ˜c] and μ. If not, then by the Claim, there should exist ˜s ∈ S(˜k) preferring μ(˜s) to ψ[q, ˜e, ˜c](˜s) and each student in ∪_{k’=1}^{k-1} S(k’) gets the same college in μ and ψ[q, ˜e, ˜c] where ˜k < k; then we restart from Step 1 by taking ˜c := ˜c, s := ˜s, k := ˜k, and e := ˜e.

Eventually, by the finiteness of the rounds of the 2S-TTC and as we reduce the round k in each iteration, we reach the point in our proof such that students in ∪_{k’=1}^{k-1} S(k’) get the same college in μ and ψ[q, ˜e, ˜c].

Observe that s is the last remaining eligible student of c in Round k of 2S-TTC for
\[ [q, \tilde{e}, \tilde{s}] \text{ by the choice of } \tilde{e}_c = r_c(s). \text{ Since for all } s'' \in \bigcup_{k' = 1}^{k-1} S(k'), \mu(s'') = \psi[q, \tilde{e}, \tilde{s}](s') \text{ and } \mu(s) \tilde{P}_s \psi[q, \tilde{e}, \tilde{s}](s) = c_0, \text{ if } s' \in S(k) \cap \mu(c) \text{ will be assigned to a different college in } \psi[q, \tilde{e}, \tilde{s}] \text{ than } c. \text{ Otherwise, } \psi \text{ is not balanced. As for all } s'' \in \bigcup_{k' = 1}^{k-1} S(k'), \mu(s'') = \psi[q, \tilde{e}, \tilde{s}](s'') \text{ and } s' \text{ points to the best available college that finds her acceptable in Round } k, \ c = \mu(s') \tilde{P}_s \psi[q, \tilde{e}, \tilde{s}](s'). \text{ We decrease Counter}(k) \text{ by } 1. \text{ If Counter}(k) > 0 \text{ then we turn back to Step 1 by taking } \tilde{s} := \tilde{s} \text{ and } s := s'; \text{ otherwise we continue with Step 3. Note that eventually we will find a Step } K \text{ such that } \text{Counter}(K) = 0, \text{ because we weakly decrease all counters and decrease one counter by } 1 \text{ in each iteration of Step 2.}

\textbf{Step 3:} \text{ By construction above, each } \tilde{s} \in S(k) \text{ ranks only } \mu(\tilde{s}) \text{ as acceptable in } \tilde{P}_s \text{ and is the last certified student by her home college in } [q, \tilde{e}, \tilde{s}]. \text{ In Step 2, we showed that there exist at least } 2 \text{ students } s_1(=s \text{ in Step 2}) \text{, } s_2(=s \text{ in Step 2}) \text{ in } S(k) \text{ who are not assigned to } \mu(s_1) \text{ and } \mu(s_2) = c_1 \text{ (=} c \text{ in Step 2), respectively, in } \psi[q, \tilde{e}, \tilde{s}], \text{ where } c_1 \text{ is the home college of } s_1. \text{ Then, they are assigned to } c_0 \text{ in } \psi[q, \tilde{e}, \tilde{s}], \text{ by the individual rationality of } \psi. \text{ Recall that in the } 2S-\text{TTC for } [q, \tilde{e}, \tilde{s}], \text{ each student certified by the home colleges of } s_1 \text{ and } s_2 \text{ – colleges } c_1 \text{ and } c_2, \text{ respectively – other than } s_1 \text{ and } s_2 \text{ is removed in a round earlier than } k. \text{ Suppose for } s_3 \in S(k), \mu(s_3) = c_2. \text{ Since } \psi[q, \tilde{e}, \tilde{s}](s_2) = c_0, \text{ for all } \tilde{s} \in \bigcup_{k' = 1}^{k-1} S(k'), \psi[q, \tilde{e}, \tilde{s}](\tilde{s}) = \mu(\tilde{s}) \text{ (by Step 2), and } \psi \text{ is balanced, } s_3 \text{ cannot be assigned to } c_2 \text{ in } \psi[q, \tilde{e}, \tilde{s}], \text{ and hence, } \psi[q, \tilde{e}, \tilde{s}](s_2) = c_0. \text{ We continue similarly with } s_3 \text{ and home college of } s_3, \text{ say college } c_3, \text{ eventually showing that for all } \tilde{s} \in S(k), \psi[q, \tilde{e}, \tilde{s}](\tilde{s}) = c_0. \text{ Recall that students in } S(k) \text{ had formed a trading cycle in which each agent in the cycle was assigned in } \mu \text{ the home college of the next student in the cycle. Thus, } \psi[q, \tilde{e}, \tilde{s}] \text{ is Pareto dominated by the balanced matching } \nu \text{ obtained as } \nu(\tilde{s}) = \psi[q, \tilde{e}, \tilde{s}](\tilde{s}) \text{ for all } \tilde{s} \in S \setminus S(k) \text{ and } \nu(\tilde{s}) = \mu(\tilde{s}) \text{ for all } \tilde{s} \in S(k); \text{ that is, } \nu \text{ is obtained from } \psi[q, \tilde{e}, \tilde{s}] \text{ by students in } S(k) \text{ trading their assignments with each other to get their assignments in } \mu. \text{ This is because each college in the cycle of Round } k \text{ gets one acceptable student more and each student weakly prefers } \mu \text{ to } \psi[q, \tilde{e}, \tilde{s}]. \text{ This contradicts the balanced–efficiency of } \psi. \text{ Hence, } \psi[q, e, \tilde{s}] = \mu, \text{ i.e., } \psi \text{ is equivalent to } 2S-\text{TTC.} \]
and student preference profiles are given as:

\[
\begin{array}{ccc}
\succ_a & \succ_b & \succ_c \\
1 & 3 & 4 \\
\end{array}
\quad
\begin{array}{cccc}
P_1 & P_2 & P_3 & P_4 \\
b & c & a & b \\
c_\emptyset & c_\emptyset & c_\emptyset & a \\
c_\emptyset & \\
\end{array}
\]

The 2S-TTC selects \( \mu = (\begin{array}{ccc}
a & b & c \\
\{3, 4\} & 1 & 2 \\
\end{array}) \). \( \psi \) will also select \( \mu \), since any other matching in which all students are assigned is individually irrational (and unacceptable).

If student 4 reports \( \succ'_4 \) with associated ranking \( P'_4 : bP'_4c_\emptyset P'_4a \) then 2S-TTC will select \( \mu' = (\begin{array}{ccc}
a & b & c \\
3 & 1 & \emptyset \\
\end{array}) \). The only balanced and individually rational (acceptable) matching in which more than two students are assigned is \( \mu'' = (\begin{array}{ccc}
a & b & c \\
3 & 4 & 2 \\
\end{array}) \). Therefore, the outcome of \( \psi \) when 4 reports \( \succ'_4 \) is \( \mu'' \). Hence, 4 can manipulate \( \psi \).

**Proof of Theorem 7.** Recall that, the student-pointing 2S-TTC mechanism can be run by using the set of acceptable students by colleges, and that while proving the properties of 2S-TTC we consider only these sets of acceptable students. Hence, we can use the same proofs for the worker-pointing 2S-TTC. We refer to the proof of Theorem 3 and Theorem 4 for balance–efficiency, acceptability, individual rationality, strategy-proofness for students, and respecting internal priorities. We refer to the proof of Theorem 6 for uniqueness.

**Immunity to Preference Manipulation by Colleges:** Given our assumption on the firm preferences, firms are indifferent between any balanced and individually rational matching that fills their quota. Since the 2S-TTC mechanism selects a balanced and individually rational matching that fills all firms’ quotas, firms cannot be better off by manipulating their preferences over the matchings and reporting quotas different from their true quotas.

**Stability:** Denote the outcome of the 2S-TTC by \( \mu \). Recall that \( q_c = |S_c| \) for all \( c \in C \), all workers consider their current jobs acceptable, all firms consider their current workers acceptable, and workers who are not certified remain at their current jobs. Hence, \( |\mu(c)| = q_c \) for all \( c \in C \). Since \( \mu \) fills all colleges’ quotas, \( \mu \) is nonwasteful. Since all employees in \( \mu(c) \) are acceptable, replacing one of the employee in \( \mu(c) \) with another one in \( S \setminus \mu(c) \) cannot make \( c \) better off. Hence, \( \mu \) cannot be blocked by a worker-firm pair. Moreover, \( \mu \) is individually rational.
References


B  Online Appendix: Tuition Exchange Programs

The Tuition Exchange Inc (TTEI): In addition to information provided in the Introduction, here we give more detail. In TTEI, every participating institution determines the number of outgoing students it can certify, as well as how many TTEI awards it will grant to incoming students each year. Each college determines its export and import quotas. Then each faculty member submits the TTEI application to the registration office of their college. If the number of applicants is greater than the number of outgoing students that the college is willing to certify, then the college decides whom to certify based on years of service or some other criterion (internal priority order).

Each student who is certified eligible submits a list of colleges to the liaison office of her home institution. Each liaison office sends a copy of the TTEI “Certificate of Eligibility” to the TTEI liaison officer at the participating colleges and universities listed by the eligible dependents. Certification only means that the student is eligible for a TTEI award; it is not a guarantee of an award. The eligible student must apply for admission to the college(s) in which she is interested, following each institution’s application procedures and deadlines. After admission decisions have been made, the admissions offices or TTEI liaisons at her listed institutions inform her whether she will be offered a TTEI award. TTEI scholarships are competitive and some eligible applicants may not receive them. That is, the sponsoring institution cannot guarantee that an “export” candidate, regardless of qualifications, will receive a TTEI scholarship. Institutions choose their scholarship recipients (“imports”) based on the applicants’ academic profiles. Some export candidates may receive more than one scholarship offer. It is also possible that an export candidate may not be admitted to her listed colleges during the admission process, and as a result, she will not be considered for a scholarship.

The Council of Independent Colleges Tuition Exchange Program (CIC-TEP): CIC-TEP is an extensive exchange program composed of approximately 400 small to mid-sized colleges across the nation, although most colleges are based in the Midwest or on the East Coast. All full-time employees of a CIC-TEP participating institution are eligible for the benefit, along with their spouses and dependents. Colleges determine their own policies and guidelines for the eligibility of their own employees. Each participating college is required to accept a minimum of three new students per year. There is no limitation on the number of exported students, and all eligible students can apply for the scholarship. Students must be admissible at the importing institution to be considered for the scholarship.

After the determination of the sponsored students, the exporting institution completes the tuition exchange participation form and directs the form to the institutions to which
their sponsored students are applying. Each student also applies for admission directly to the institutions of her choice and submits all required financial aid information.

**Catholic College Cooperative Tuition Exchange (CCCTE):** There are 70 members comprising schools throughout the country, some large and some small. Each member defines its own benefit levels for tuition exchange. Each institution determines who among its employees is eligible to apply for tuition exchange at another member college. All dependents must be accepted to the college or university of their choice before applying for the benefit. Acceptance at the institution does not guarantee the availability of the benefit. Each participating college is not allowed to import more than five students per academic year over the number of students it exports to other participating institutions. There is no limitation on the number of students it exports in a given academic year. Hence, each employee who is determined to be eligible can apply for the scholarship.

**Great Lakes Colleges’ Association (GLCA):** GLCA is a consortium of thirteen private liberal arts colleges in Pennsylvania, Michigan, Ohio, and Indiana. The eligibility of the student’s parent is determined by the college where the parent is employed. All other policies affecting the student are determined by the college the student attends. Under this program, each participant’s family will pay a fee equal to 15% of the GLCA mean tuition. The remaining 85% is paid by the exporting institution. Thus there are no concerns about maintaining a balance. Hence, it is no longer an exact exchange program; previously, the association was using an exchange program. After problems related to maintaining balances surfaced, they converted to the current model.

**Associated Colleges of the Midwest (ACM):** This program is composed of fourteen liberal arts colleges in Wisconsin, Minnesota, Iowa, Illinois, and Colorado. Eligibility is set by the exporting college. Anyone eligible for the importing college’s tuition remission program is considered eligible for the program, and she is placed in the applicant pool.

Each participating ACM college compensates 50% tuition to all imported students from participating colleges. Each exporting college determines the level of benefit it offers its employees, though it must be at least 80% of the importing college’s tuition. If the exporting college benefit is 80%, the family is responsible for the remaining 20% of tuition. The exporting college might choose a benefit of 90% or 100% or any other level, as long as it is at least 80% of the tuition of the college to be attended.

The current ACM exchange replaces a previous exchange that resulted in imbalances between the member colleges. Since the current system does not have a balancing feature, popular colleges with few qualified employees with dependent students could find themselves with a greater number of incoming students.

**Faculty and Staff Children Exchange Program (FACHEX):** There are 28 partici-
pating Jesuit colleges in this program. The student first must submit a regular application for admission to the FACHEX-participating college involved. This must be done in accordance with that college's regular admission requirements and procedures. Eligibility to participate in the FACHEX program does not qualify a student for admission, nor does admission qualify a student for FACHEX tuition remission. No institution is obligated to enroll more than three FACHEX students over the number it exports.

Council for Christian Colleges and Universities Tuition-Waiver Exchange Program (CCCU-TWEP): There are 100 participating colleges in this program. Each participating college agrees to accept at least one (with a recommendation of three) new student from other participating institutions. Applicant students apply directly to the institutions of their choice and must meet normal admissions requirements in order to be considered for the exchange program, due to limited space.

C Online Appendix: Temporary Worker Exchange Programs

C.1 Teacher Exchange

The Fulbright Teacher Exchange Program, established by an act of the US Congress in 1946, provides opportunities to US primary and secondary school teachers to participate in direct exchange of positions with teachers from other countries, including the Czech Republic, France, Hungary, India, Mexico, and the United Kingdom. Fulbright program staff match US and overseas candidates, and each candidate and each school involved in the application process must be approved before final selection to the program takes place.

The Commonwealth Teacher Exchange Programme (CTEP) has been running for 100 years, initiated by the League for the Exchange of Commonwealth Teachers. Participating teachers exchange jobs and homes with their exchange partner usually for a year, while remaining employed by their own school. Countries that have participated have included Australia, Canada, and the UK since the earliest times, but also India, Jamaica, Malawi, New Zealand and South Africa, as well as non-Commonwealth countries in Europe and Asia, and the United States. More than 40,000 teachers have benefited from the CTEP. If applicants are found a match they are notified between February and July. Detailed forms of exchange partners are posted to each principal for consideration. Principals have the right to veto any proposed exchange they think will not be appropriate for their school.
The Educator Exchange Program is run by the Canadian Education Exchange Foundation (CEEF). CEEF offers both interprovincial and international exchanges. Destinations for international teacher exchanges are Australia, Denmark, France, Germany, Switzerland, the UK and Colorado, US. Exchanges are done reciprocally. Exchange coordinators determine the possible matches by looking at preferences, family needs, accommodations offered, and accommodations needed.

The Manitoba Teacher Exchange enables teachers in Manitoba to exchange their positions with teachers in Australia, the UK, the US, Germany, and other Canadian provinces. Once a complete application is received, applications are forwarded for consideration to the relevant country. When a prospective match is made, the incoming teacher’s file is forwarded to the Manitoba applicant, the principal, and the employing authority for consideration. For the exchange to be completed, all these agents must accept the incoming applicant.

In the Saskatchewan Teacher Exchange, a teacher who is employed by a public school division in Saskatchewan and has at least five years of teaching experience is eligible to apply for exchange positions with teachers in the UK, the US, and Germany. The authorities in the UK, US, and Germany suggest a suitable exchange partner based on similar teaching assignments and personal and professional interests. The proposal is forwarded to applicant’s director of education for consideration. If the match is acceptable to the applicant’s director and principals she will be given an opportunity to consider it. The exchange is not considered completed until the applicant, her partner, both employers, and both of the exchange authorities concerned have agreed that it is acceptable.

The Northern Territory Teacher Exchange Program gives Northern Territory teachers the opportunity to apply for a reciprocal exchange with teachers from the UK, Canada (British Columbia, Alberta and Ontario), the US, New Zealand, and the Australian States of New South Wales, Queensland, South Australia, and Western Australia. An application summary is sent to the interstate/overseas education authorities in the preferred locations of each applicant. If a potential match is identified for an applicant, the application is sent to the other exchange authority. When an exchange is proposed, the applicant, in consultation with his or her school principal, makes the decision to accept or reject the proposal. When all parties on each side accept and sign their undertakings, an exchange match is confirmed.

The Western Australian Teacher Exchange Program provides public school teachers the opportunity to apply for a reciprocal exchange with teachers from school systems in different states, territories, or countries. When a suitable exchange match is identified, the principal will have the right to accept or refuse a proposal. An exchange proposal
must be accepted in both applicant jurisdictions.

The Rural Teacher Exchange Program (TRTEP) enables teachers in rural and remote schools in New South Wales, Australia, to exchange their positions with teachers at other schools for one year. TRTEP’s objective is to allow rural and remote teachers to experience a different teaching environment. There are more than 800 rural schools in New South Wales. If a teacher can find a possible exchange counterpart, then they swap their positions before entering the central exchange mechanism. Otherwise, exchanges are determined according to the submitted preference list.

C.2 Clinical Exchange

The International Clinical Exchange Program allows individual medical students to do a clinical rotation in a foreign country. This international exchange program operates through the International Federation of Medical Students’ Association (IFMSA). Annually, approximately 10,000 medical students around the world participate in IFMSA's exchanges. The Standing Committee on Professional Exchange (SCOPE) organizes one-month clinical experiences, which all IFMSA medical members are eligible to participate in. The exchanges are done bilaterally: for every student who goes to a certain country, one student of that country can come to the student’s home country for a clerkship. The exact number of available clerkships in a country is determined by the number of contracts signed between the country of origin and the host country.

The MICEFA Medical Program has promoted the exchange of medical students studying in France and the US for thirty years. Students are allowed to participate in a medical exchange abroad for one to two months in one of the partner university hospitals in almost all specializations (in both clinical and research electives). Students are exchanged on a one-to-one basis. Tuitions are paid to the home institute. Reciprocity is considered as the key element of the success of the exchange program. This reciprocal exchange program is strictly limited to medical students enrolled in partnering medical schools in the United States and France. Faculty members can also benefit from this exchange program. American faculty members will receive a corresponding French salary, whereas the French faculty member will receive the corresponding salary in the US.

C.3 Student Exchange

The National Student Exchange (NSE) is a consortium of nearly 200 colleges in the United States, Canada, Guam, Puerto Rico, and the US Virgin Islands. Through NSE, these member institutions provide exchange opportunities for undergraduate students.
NSE was established in 1968 and has provided exchange opportunities to more than 105,000 students. The program features a tuition reciprocity system that allows students to attend their host institution by paying either the in-state tuition/fees of their host institution or the normal tuition/fees of their home campus.

The University of California Reciprocal Exchange program (UCREP) is the system-wide study-abroad program for the University of California. UCREP is partnered with more than 120 universities in 33 countries. Around 4,000 students benefit from this program annually. Reciprocity students are recruited and selected by the home university. Reciprocal exchanges seek to balance the costs and benefits of import and export students for each university participating in the exchange program.

The University Mobility in Asia and the Pacific Exchange Program (UMAEP) is a student exchange program between 500 universities in 34 Asia-Pacific countries. It was established in 1993. UMAEP involves two programs: a bilateral exchange program and a multilateral exchange program. In the bilateral exchange program, home colleges select the exchange students and exchanges are done through bilateral agreements signed between colleges. In the multilateral exchange program, the selection of exchange students is done by each host university. The maximum number of exchange students will not exceed two (2) per academic year in each university.

The International Student Exchange Program (ISEP) was founded in 1979. Around 40,000 from 45 countries have benefited from this exchange program. It is a reciprocal exchange program: for each student sent to another college, the college receives one in return. Each exchange student pays tuition to her home college. Exchanges are based on the balance of students exchanging places rather than a monetary exchange, so a university abroad can only accept as many students as it sends out.

The Erasmus Student Exchange Program is a leading exchange program between the universities in Europe. Close to 3 million students have participated since it started in 1987. The number of students benefiting from the program is increasing each year; in 2011, more than 231,408 students attended a college in another member country as an exchange student. The number of member colleges is more than 4,000. Each college needs to sign bilateral agreements with the other member institutions. In particular, the student exchanges are done between the member universities that have signed a bilateral contract with each other. The bilateral agreement includes information about the number of students who will be exchanged between the two universities in a given period. The selection process of the exchange students is mostly done as follows. The maximum number of students that can be exported to a partner university is determined based on the bilateral agreement with that partner and the number of students who
have been exported since the agreement was signed. The students submit their list of preferences over the partner universities to their home university. Each university ranks its own students based on predetermined criteria, e.g., GPA and seniority. Based on the ranking, a serial dictatorship mechanism is applied to place students at the available slots. Finally, the list of the students who received the slots of the partner universities is sent to the partners. The partner universities typically accept all the students on the list. An exchange student pays her tuition to her own college, not the one importing her.

There are huge imbalances between the number of students exported and imported by each country. Moreover, countries with high positive balances are not often willing to match the quota requests of the net exporter countries. This precautionary behavior may lead to inefficiencies as in tuition exchange markets.

### C.4 Scientific Exchange

The Mevlana Exchange Program aims to exchange academic staff between Turkish universities and universities in other other countries. Turkish public universities are governed by Turkish Higher Education Council and professors are public servants. Therefore, the part of the exchange that is among public universities can be seen as a staff exchange program, while the exchange among public and private Turkish universities and foreign universities can be seen as a worker exchange program. Any country can benefit from this program. Academic staff may lecture abroad from one week (minimum) to three months (maximum). In 2013, around 1,000 faculty members benefited from this program.

### D Online Appendix: Structure of Stable Matchings

In this Online Appendix, we inspect the structure of stable matchings, as our stability concept is novel. In the college admissions market, there always exist student–optimal and college–optimal Gale–Shapley–stable matchings (cf. Gale and Shapley, 1962; Roth, 1985).\(^\text{49}\) Under Assumption 1, we can guarantee the existence of college– and student–optimal stable tuition–exchange matchings. This result’s proof also uses the associated Gale–Shapley college admissions market for each tuition–exchange market and the properties of Gale–Shapley stable matchings in these markets.\(^\text{50}\)

---

\(^{49}\)A matching is student–(or college–)optimal stable if it is preferred to all the other stable matchings by all students (or colleges).

\(^{50}\)The lattice property of Gale–Shapley–stable college–admissions matchings can also be used to prove an analogous lattice property for stable matchings in tuition–exchange markets under Assumption 1. We skip it for brevity.
**Proposition 10** Under Assumption 1, there exist college- and student-optimal matchings in any tuition exchange market.

**Proof of Proposition 10.** By the proof of Proposition 1, Gale and Shapley (1962), and Roth (1985), there exists a student-optimal stable matching for each tuition-exchange market. By Assumption 1 Part 1 and Proposition 3, colleges compare only the stable matchings through the admitted set of students. By Gale and Shapley (1962) and Roth (1985), there exists a college-optimal stable matching for each tuition-exchange market.

E  **Online Appendix: An Example of the 2S-TTC Mechanism**

We illustrate the dynamics of the 2S-TTC mechanism with an example below:

**Example 1 (2S-TTC)** Let \( C = \{a, b, c, d, e\} \), \( S_a = \{1, 2\} \), \( S_b = \{3, 4\} \), \( S_c = \{5, 6\} \), \( S_d = \{7, 8\} \), and \( S_e = \{9\} \). Let each college certify all its students as eligible and \( q = (2, 2, 2, 1, 1) \). The internal priorities and the rankings of agents associated with their preferences over matchings are given as:

<table>
<thead>
<tr>
<th>( P_a )</th>
<th>( P_b )</th>
<th>( P_c )</th>
<th>( P_d )</th>
<th>( P_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>9</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>7</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>( o_e )</td>
<td>( o_a )</td>
<td>( o_e )</td>
<td>( o_a )</td>
<td>( o_e )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( P_4 )</th>
<th>( P_5 )</th>
<th>( P_6 )</th>
<th>( P_7 )</th>
<th>( P_8 )</th>
<th>( P_9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>e</td>
<td>c</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>c</td>
<td>a</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>d</td>
</tr>
<tr>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
</tr>
</tbody>
</table>

Let \( o_e \) and \( o_a \) be the vectors representing the eligibility and admission counters of colleges, respectively. Then we set \( o_e = (2, 2, 2, 1) \) and \( o_a = (2, 2, 2, 1, 1) \).
Round 1: The only cycle formed is \((b, 3, a, 1)\). Therefore, 1 is assigned to \(b\) and 3 is assigned to \(a\). Observe that although college \(a\) is the most preferred college of student 6, she is not acceptable to \(a\), and hence, she points to college \(b\) instead. The updated counters are \(o_e = (1, 1, 2, 2, 1)\) and \(o_a = (1, 1, 2, 1, 1)\).

Round 2: The only cycle formed in Round 2 is \((c, 6, b, 4)\). Therefore, 6 is assigned to \(b\) and 4 is assigned to \(c\). The updated counters are \(o_e = (1, 0, 1, 2, 1)\) and \(o_a = (1, 0, 1, 1, 1)\). College \(b\) is removed.

Round 3: The only cycle formed in Round 3 is \((a, 2, c, 5)\). Therefore, 5 is assigned to \(a\) and 2 is assigned to \(c\). The updated counters are \(o_e = (0, 0, 0, 2, 1)\) and \(o_a = (0, 0, 0, 1, 1)\). Colleges \(a\) and \(c\) are removed.

Round 4: The only cycle formed in Round 4 is \((c_\emptyset, 7)\). Therefore, 7 is assigned to \(c_\emptyset\). Given that we have a trivial cycle, we only update \(o_e\). The updated counters are \(o_e = (0, 0, 0, 1, 1)\) and \(o_a = (0, 0, 0, 1, 1)\).

Round 5: The only cycle formed at this round is \((e, 9, d, 8)\). Therefore, 8 is assigned to \(e\) and 9 is assigned to \(d\). The updated counters are \(o_e = (0, 0, 0, 0, 0)\) and \(o_a = (0, 0, 0, 0)\).

All agents are assigned, so the algorithm terminates and its outcome is given by matching

\[
\mu = \begin{pmatrix}
a & b & c & d \\
\{3, 5\} & \{1, 6\} & \{2, 4\} & 9
\end{pmatrix}.
\]

\[\diamond\]
Appendix: Independence of Axioms

- A student–strategy-proof, acceptable but not balanced–efficient mechanism that also respects internal priorities: A mechanism that always selects the null matching for any market.

- A student–strategy-proof, balanced–efficient, acceptable mechanism that does not respect internal priorities: Consider a variant of the 2S-TTC mechanism in which each college points to the certified student who has the lowest priority among the certified ones. This mechanism is strategy-proof for students, balanced–efficient, and individually rational, but it fails to respect internal priorities.

- A balanced–efficient, acceptable, but not student–strategy-proof mechanism that respects internal priorities: Consider the following market. There are three colleges $C = \{a, b, c\}$ and four students $S_a = \{1, 2\}$, $S_b = \{3\}$, and $S_c = \{4\}$. The ranking $P$ associated with preference preference profile $\succeq_S$ is given as

<table>
<thead>
<tr>
<th></th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$b$</td>
<td>$a$</td>
<td>$a$</td>
</tr>
<tr>
<td></td>
<td>$c$</td>
<td>$c$</td>
<td>$c_\emptyset$</td>
<td>$c_\emptyset$</td>
</tr>
<tr>
<td></td>
<td>$c_\emptyset$</td>
<td>$c_\emptyset$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Let mechanism $\psi$ select the same matching as the 2S-TTC for each market except the market $[q = (2, 1, 1), e = (2, 1, 1), \succeq]$, and for this market it assigns 1 to $c$, 2 to $b$, 3 to $a$, and 4 to $a$. This mechanism is balanced-efficient, acceptable, and respecting internal priorities. However, it is not student–strategy-proof, because when 1 excludes $c$, $\psi$ and 2S-TTC will assign 1 to $b$.

- A balanced–efficient, student–strategy-proof, but not individually rational mechanism that respects internal priorities: Consider a variant of the 2S-TTC in which students are not restricted to point to those colleges consider them acceptable. This mechanism is balanced–efficient, strategy-proof, and respecting internal priorities, but it is not acceptable since an unacceptable student can be assigned to a college.