

Boyacı, Tamer; Akçay, Yalçın

**Working Paper**

## Pricing when customers have limited attention

ESMT Working Paper, No. 16-01

**Provided in Cooperation with:**

ESMT European School of Management and Technology, Berlin

*Suggested Citation:* Boyacı, Tamer; Akçay, Yalçın (2016) : Pricing when customers have limited attention, ESMT Working Paper, No. 16-01, European School of Management and Technology (ESMT), Berlin,  
<https://nbn-resolving.de/urn:nbn:de:101:1-2016032215113>

This Version is available at:

<https://hdl.handle.net/10419/129123>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

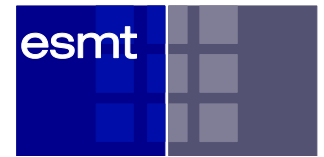
**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

ISSN 1866-3494



March 2, 2016

**ESMT Working Paper 16-01**

# Pricing when customers have limited attention

Tamer Boyacı, ESMT European School of Management and Technology

Yalçın Akçay, College of Administrative Sciences and Economics, Koç University

Copyright 2016 by ESMT European School of Management and Technology, Berlin, Germany, [www.esmt.org](http://www.esmt.org).

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, used in a spreadsheet, or transmitted in any form or by any means - electronic, mechanical, photocopying, recording, or otherwise - without the permission of ESMT.

Find more ESMT working papers at [ESMT faculty publications](#), [SSRN](#), [RePEc](#), and [EconStor](#).

Submitted to  
manuscript (Please, provide the manuscript number!)

Authors are encouraged to submit new papers to INFORMS journals by means of a style file template, which includes the journal title. However, use of a template does not certify that the paper has been accepted for publication in the named journal. INFORMS journal templates are for the exclusive purpose of submitting to an INFORMS journal and should not be used to distribute the papers in print or online or to submit the papers to another publication.

# Pricing When Customers Have Limited Attention

Tamer Boyacı

ESMT European School of Management and Technology,  
Schlossplatz 1, 10178, Berlin, Germany  
Phone: +49 30 21231-1294; Fax: +49 30 21231-1281  
tamer.boyaci@esmt.org

Yalçın Akçay

College of Administrative Sciences and Economics, Koç University,  
Rumeli Feneri Yolu, Istanbul, 34450, Turkey,  
Phone: (90) 212 338 1560; Fax: (90) 212 338 1393  
yakcay@ku.edu.tr

We study the optimal pricing problem of a firm facing customers with limited attention and capability to process information about the value (quality) of the offered products. We model customer choice based on the theory of rational inattention in the economics literature, which enables us to capture not only the impact of true qualities and prices, but also the intricate effects of customer's prior beliefs and cost of information acquisition and processing. We formulate the firm's price optimization problem and characterize the pricing and revenue implications of customer's limited attention. We test the robustness of our results under various modelling generalizations such as prices signalling quality and customer heterogeneity, and study extensions such as multiple products, competition, and joint inventory and pricing decisions. We also show that using alternative pricing policies that ignore the limited attention of customers or their ability to allocate this attention judiciously can potentially lead to significant profit losses for the firm. We discuss the managerial implications of our key findings and prescribe insights regarding information provision and product positioning.

*Key words:* pricing, choice behaviour, rational inattention, information acquisition, signalling game

---

## 1. Introduction

We are living in an era characterized by abundance of information. Firms are in a race to make use of the “big data” available on their customers and products. In a similar vein, customers have many products/services to choose from and have easy access to a wealth of information sources that can

aid in their decision processes. In theory, all the information that is available on the internet, social media, product catalogs, magazines and other published media, information broadcasted on radio, television, information obtained from friends and family are at the disposal of customers. However, almost by default, customers not only have limited time and attention, but also limited capability to process the information that is acquired. Therefore, information acquisition and processing is a “costly” endeavor. Consequently, customers have to choose how much and what type of information to pay attention to (and what to ignore), and make purchase decisions on the basis of this partial information. Understanding such limitations and how they translate into choice behaviour is of crucial concern to the selling firm (e.g., a retailer) since there are immediate implications on the pricing and information provision strategies. These constitute the primary focus of our paper.

For exemplary purposes, consider a customer who is in the market to buy a digital camera. It is relatively easy to obtain information regarding the selection of brands, models and associated prices from an on-line retailer or by walking into a physical store. Yet, cameras differ significantly in terms of features (resolution, optical/digital zoom, battery life, etc.) and performance (color reproduction, noise, distortion, etc). The customer might have some past experience with the brands and some knowledge about product features, which form her prior belief about the “quality” levels of these cameras. Naturally the customer can also learn more about the products by asking specific questions to the seller, searching facts and comparisons on-line, etc. However, she has limited time and capacity to devote to this task, hence each attempt to acquire additional information represents a cost to the customer. Furthermore, it is evidently impossible for the customer to attend to or process all relevant information that is necessary to identify the *exact, true quality* of each candidate camera. The customer has to determine how much effort to spend to acquire quality revealing information, what information to pay attention to and process while making the final decision. For the seller, the key question is how to set prices when faced with such customers.

In order to examine the optimal pricing strategies of the seller, it is essential to capture the salient features of limited attention and information processing capabilities of customers in a choice model. *Rational inattention* theory (Sims 1998, 2003, 2006) offers a compelling approach for this purpose. In

contrast to the *rational expectations* theory which assumes that customers can fully process all freely available information about the product, rational inattention theory assumes that they lack the capability to understand the available information comprehensively and translate it into decisions.<sup>1</sup> At the core of rational inattention is the understanding that attention is a scarce resource and therefore must be allocated wisely. In particular, the pioneering works of Sims (1998, 2003, 2006) propose a framework that is based on a stream of literature on information theory, which measures uncertainty by entropy and quantifies information as reduction in uncertainty. This approach does not make particular assumptions on how decision-makers acquire information and what they learn. It builds on utility-maximizing decision-makers who acquire information optimally, trading off the expected benefit of better information against the cost associated with acquiring it. Accordingly, rationally inattentive decision-makers optimally select the type and quantity of information they need, and ignore the information that is not worth obtaining and processing.<sup>2</sup> In fact, in a recent paper, Matejka and McKay (2015) show that when faced with discrete choices with stochastic (pay-off) values, a rationally inattentive decision-maker's optimal information processing strategy endogenously leads to a choice behaviour that can be characterized as generalized multinomial logit (MNL). In particular, the choice probabilities depend not only on the true realizations of the choices, but also on the prior belief of the customer and the cost of information. We utilize this characterization in our pricing framework.

We remark that rational inattention theory has been applied to a broad spectrum of economic problems and has been a powerful construct in providing explanations to some observed market and macroeconomic phenomena that extant theories fail to fully address, such as business cycles (McKowiak and Wiederholt 2015), consumption (Tutino 2013), price setting behaviour and stickiness (Matejka 2010b). The main obstacle preventing much wider applications has been the mathematical and computational complexity associated with models utilizing the rational inattention framework (Tutino 2011). As such, optimal firm-level pricing decisions in the presence of customers with limited attention, and how such behaviour impacts prices and profitability are widely unknown. We aim to bridge this gap. To this end, we seek answers to the following research questions:

<sup>1</sup> Throughout the paper we use the terms “limited attention” and “rational inattention” interchangeably.

<sup>2</sup> We discuss this theory in the context of our problem in more detail in §3.

- How should a firm price its products when customers have limited attention and cannot fully evaluate the quality of the products it offers?
- What is the impact of key market and operational factors, such as customers' prior beliefs about product quality, cost of information, on the firm's revenue and the products' prices?
- How does the optimal pricing strategy compare to those under alternative specifications of customer choice? How do such policies perform when customers have limited attention?
- What are the implications of the firm "signalling" product quality to inattentive customers through its product pricing strategy?
- What should or could the firm do in terms of its product positioning and information provision efforts to enhance its revenue when faced with rationally inattentive customers?

In order to address these questions, we first develop a *baseline model* with a single firm selling a single product to a homogenous population of rationally inattentive customers. Customers can observe the price of the product, but are not completely aware of and cannot fully assess its quality. In line with rational inattention literature discussed above, customers in our model have prior beliefs about the quality level of the product and can improve their assessment by optimally acquiring and processing costly information (which is captured in our particular discrete choice model). The firm on the other hand has more experience with the product and knows its exact quality. We formulate and comprehensively analyze the firm's price optimization problem for the baseline model, and also illustrate the profit impact of using alternative pricing policies. In addition to the baseline model, we study the case when the firm strategically uses product price to *signal* quality to inattentive customers and characterize the Perfect Bayesian Equilibria (PBE). Moreover, we incorporate customer *heterogeneity* into the baseline model by allowing a mix of attentive and inattentive customers in the market, and by allowing customers to have different prior beliefs. Finally, recognizing the potential impact of limited attention beyond the pricing strategy of a single firm, we offer insights on extensions of the baseline model by considering multiple products, competing firms, as well as joint ordering and pricing decisions.

Our contribution to the literature is three-fold: (i) From a theoretical perspective, utilizing recent advances in the theory of rational inattention, we develop a tractable price optimization framework

that captures the limited attention of customers. (ii) Utilizing our model, we characterize the pricing and revenue implications of customer's limited attention, describe how key problem features (cost of information, prior beliefs, product quality) shape these decisions, and test the robustness of our results under various generalizations (quality signalling, heterogenous customers) and extensions (multiple products, competition, inventory decisions). (iii) Our descriptive results have prescriptive value since they offer valuable insights to practicing managers. The impact of customer's cost of information can be translated into suitable information provision strategies for the firm. Hence, we can identify settings in which it is profitable for the firm to make it easier for customers to gather information about products (e.g., via salesforce, trial samples, etc) and vice versa. Likewise, we impart insights that can guide product positioning efforts of the firm.

## **2. Literature Review**

It has been long recognized that rationality of decision-makers is limited by the partial information they have, their cognitive capacities, and the finite amount of time they have to make decisions. Simon (1955) introduced "bounded rationality" as an alternative to modeling rationality as optimization, which views decision making as a fully rational process of finding an optimal choice given the information available. He proposed that decision-makers apply rationality after simplifying their available choices, and only seek a satisfactory choice rather than the optimal one (*satisficing* rather than *optimizing*). Since then, the field of behavioral economics has expanded significantly and has found applications in other disciplines, including operations management. Özer and Zheng (2012) provides a comprehensive coverage of behavioral models related to pricing and price contracting among firms.

Our paper is more closely related to the growing body of economics literature that explains the limited rationality of decision-makers as an implication of their limited capacity to process information (Shannon 1948). "Rational inattention" theory, as proposed by Sims (1998, 2003, 2006), rationalizes the choice behaviour of decision-makers who choose their preferred amount of information and allocation of attention, when all options and information pertaining to them are fully available, but attending to and processing this information is costly (i.e. information frictions). As noted in Wiederholt (2010), rational inattention has been applied to numerous areas in macroeconomics. Within this body of work,

the more relevant for our paper is the stream that applies it to price setting. Most papers in this stream have a monetary policy focus, consider sellers who are rationally inattentive to prices and show how optimal prices in the market may behave “sticky” or “rigid” (Mackowiak and Wiederholt 2009, Woodford 2009, Matejka 2010a). Matejka (2010b) extends the notion of price rigidity to the case where customers are inattentive to prices.

In the realm of pricing under rational inattention, Matejka and McKay (2012) is akin to our work since it is also based on the generalized MNL characterization of rationally inattentive customers’ response to discrete choices in Matejka and McKay (2015). Specifically, it considers multiple sellers that face random input costs and produce random quality products. Based on the realized cost and quality levels, sellers set prices (competitively), but customers are *rationally inattentive to prices*. Given the complexity of the general problem, the analysis is concentrated on very special cases, and the discussion centers around the role of prior knowledge and heterogeneity. In contrast, in our paper customers are able to perceive product prices perfectly, but are *rationally inattentive to quality*, i.e., different features and performance dimensions of the product. We are able to explicitly solve the rationally inattentive customers’ decision problem, and formulate and analyze comprehensively the seller’s revenue maximization problem under different settings.

By the very nature of the problem we study, our paper is also related to two vast streams of literature: (i) information search and (ii) quality uncertainty. Costly information search has been a rich area of investigation going back as early as Stigler (1961), with different approaches to modeling the acquisition and processing of information. On one hand, in fixed sample size search models, consumers first determine a sample of products (akin to a consideration set in consumer behavior) about which to gather information and then make a purchase decision (Roberts and Lattin 1991, Manzini and Mariotti 2014, Sahin and Wang 2014). On the other hand, in sequential search models, customers learn the value of alternatives (or gather further information about a certain product) one-by-one, and make a purchase after deciding optimally to stop collecting more information (Weitzman 1979, Branco et al. 2012). Our paper is in spirit similar to a sequential search, however it follows the rational inattention modelling approach to information frictions. It is also distinct in that it combines product quality



uncertainty and costly information acquisition induced by limited attention in the study of optimal pricing. Starting with the seminal work of Akerlof (1970), many papers considering quality uncertainty focus on the information asymmetry between customer and the firm. In particular, they show how this asymmetry impacts market structure (Bester 1998) and explore how the firm signals quality via different channels such as price endings (Stiving 2000), advertising (Erdem et al. 2008), or queue lengths (Debo et al. 2012, Debo and Veeraraghavan 2014). In this stream Bester and Ritzberger (2001) is particularly relevant since it combines information search with quality uncertainty. Specifically, it allows the customers to pay a search cost and fully reveal the quality of the product, in a price signalling context. We go a step further and endogenize the information acquisition process of the customer using rational inattention.

### 3. Customer Choice Under Limited Attention

In this section, we provide a brief overview of the application of rational inattention to discrete choice, present the resulting choice behavior, and discuss relevant key results from Matejka and McKay (2015). We also present a reduction of this framework to two commonly used choice models (multinomial logit and optimal sequential search), and draw parallels and contrasts in terms of choice behaviour. To this end, consider a customer facing  $N$  distinct choice options (which could include the no-purchase option),  $j = 1, \dots, N$ . The state of the nature is the vector  $\mathbf{v}$ . The customer has imperfect knowledge of the state of the nature so her payoff  $v^j$  from each option is uncertain. Let  $G$  denote the (joint) distribution of the customer's *prior* belief. The customer can receive signals  $\mathbf{s}$  to update her beliefs (in a Bayesian manner), and chooses the option that maximizes her expected payoff. For any posterior distribution  $B$ ,  $V(B)$  denotes the expected payoff from choosing the best option.

The novelty of the rational inattention approach is that the customer is allowed to choose the information processing strategy prior to observing any signal (i.e., information structure is completely *endogenous*). The information strategy of the customer is the selection of the joint distribution  $F(\mathbf{s}, \mathbf{v})$  of signals and states. The only requirement is that the marginal distribution over the states equals the prior distribution  $G$ , so that the customer's posterior beliefs are *consistent* with her prior:  $\int_{\mathbf{s}} F(d\mathbf{s}, \mathbf{v}) = G(\mathbf{v})$ ,  $\forall \mathbf{v}$ . Effectively, this means the customer chooses the conditional distribution  $F(\mathbf{s}|\mathbf{v})$ ; the other

conditional distribution is  $F(\mathbf{v}|\mathbf{s})$ , the posterior belief after receiving the signal  $\mathbf{s}$ . The customer determines the distribution  $F(\cdot, \cdot)$  optimally, maximizing her *ex-ante* expected payoff minus the cost of information  $\mathcal{C}(F)$  associated with generating signals of different precision levels. Formally, this problem can be stated as:

$$\max_F \int_{\mathbf{v}} \int_{\mathbf{s}} V(F(\cdot|\mathbf{s})) F(d\mathbf{s}|\mathbf{v}) dG(\mathbf{v}) - \mathcal{C}(F), \quad (1)$$

$$s.t. \int_{\mathbf{s}} F(d\mathbf{s}, \mathbf{v}) = G(\mathbf{v}) \quad \forall \mathbf{v}. \quad (2)$$

Following the works of Sims (1998, 2003, 2006), the rational inattention literature assumes that customers process information through channels with limited capacity. Accordingly, the cost of information is given by  $\mathcal{C}(F) = \lambda\kappa$ , where  $\lambda$  is the unit cost of acquiring and processing information that the customer deems useful (simply referred to as cost of information hereon), and  $\kappa$  is the amount of information processed. Information acquisition is quantified in terms of reduction in uncertainty, which is measured by (Shannon) entropy.<sup>3</sup> In our context, the customer is sharpening her beliefs on the realized values of  $\mathbf{v}$  by acquiring and processing information. The amount of information processed  $\kappa$  is then the expected difference between the entropy of prior beliefs and the posterior entropy after observing the signal. This difference is referred to as the (Shannon) mutual information between the actual state and the signal. Mathematically speaking, letting  $H(\cdot)$  denote the entropy, we have

$$\mathcal{C}(F) = \lambda(H(G) - E_{\mathbf{s}}[H(F(\cdot|\mathbf{s}))]). \quad (3)$$

According to the above specification, the customer is optimally choosing: (i) how much attention to pay, i.e., how much information to process; (ii) what to pay attention to, i.e. what information to process; and (iii) what action to select given the information. As Matejka and McKay (2015) argues, this process is akin to asking a series of “yes-or-no questions” with a cost associated with each question. The more questions the customer asks, the tighter her posterior beliefs become.<sup>4</sup> Furthermore, it is

<sup>3</sup> For any distribution  $X$  with pdf  $f$ , entropy is defined as  $-\int_x f(x)\ln(f(x))dx$ .

<sup>4</sup> From the coding theorem of information theory (Shannon 1948), the cost function (3) is proportional to the expected number of questions needed to implement the information strategy.

shown that the choice behavior that arises from the optimization problem (1)-(2) can be found as the solution to a simpler maximization problem that does not make any reference to signals or posterior beliefs. Specifically, let  $S^j$  denote the signals that lead to the choice of option  $j$ . Then the induced conditional probability  $\pi^j(\mathbf{v})$  of choosing option  $j$  is given by

$$\pi^j(\mathbf{v}) = \int_{\mathbf{s} \in S^j} F(d\mathbf{s}|\mathbf{v}). \quad (4)$$

Based on this, the *unconditional* choice probabilities  $\pi_0^j$ 's can be specified as

$$\pi_0^j = \int_{\mathbf{v}} \pi^j(\mathbf{v}) G(d\mathbf{v}). \quad (5)$$

Reformulating (1) and (2) in terms of choice probabilities, Matejka and McKay (2015) establish in Theorem 1 that for any information cost  $\lambda > 0$ , the optimal information processing strategy of the customer results in conditional choice probabilities that satisfy a *generalized multinomial logit (GMNL)* formula:

$$\text{(GMNL)} \quad \pi^j(\mathbf{v}) = \frac{\pi_0^j e^{v^j/\lambda}}{\sum_{j=1}^N \pi_0^j e^{v^j/\lambda}} \quad \text{almost surely,} \quad (6)$$

for any  $j = 1, \dots, N$ . If  $\lambda = 0$ , the customer selects the option with the highest payoff with probability 1.

We remark that the GMNL characterization in (6) does not provide a complete explicit solution to the choice probabilities. This is because  $\pi_0^j$ 's are *not* exogenous, but are part of the of the customer's decision making process. One needs to solve (6) together with (5) and the regularity condition  $\sum_{j=1}^N \pi_0^j = 1$  to determine the exact conditional choice probabilities. Note that the unconditional choice probabilities,  $\pi_0^j$ 's, characterize the choice behaviour *prior* to processing any information. They are by definition independent of the state of the nature, but do depend on prior beliefs  $G(\mathbf{v})$  and cost of information  $\lambda$ .

The GMNL formula (6) captures the intricate effects of three factors under endogenous information acquisition and processing: *prior beliefs*, *payoffs* associated with the options, and *cost of information*. It is possible that an option with a low (high) true payoff is chosen with high (low) likelihood, if the customer's prior belief is quite favourable (unfavourable). In addition, as the cost of information increases, the customer learns less about the true nature and relies more on her prior beliefs. As such, the conditional purchase probabilities might increase, decrease, or can even be *non-monotone*.

When the customer is *a-priori* indifferent to different options, Matejka and McKay (2015) shows that the GMNL reduces to the standard MNL. Not distinguishing the options a-priori means  $G(\cdot)$  is invariant to all permutations of its entries in  $\mathbf{v}$ , under which the customer forms unconditional choice probabilities that are uniform (i.e.,  $\pi_0^j = 1/N$  for all  $j$ ). This yields conditional choice probabilities that follow the MNL formula:

$$\text{(MNL)} \quad \pi^j(\mathbf{v}) = \frac{e^{v^j/\lambda}}{\sum_{j=1}^N e^{v^j/\lambda}}. \quad (7)$$

This connection is particularly relevant in our context because a rather common approach to model bounded rationality of customers is to adopt the quantal choice model of Luce (1959), which leads to the MNL choice given in (7) (see McKelvey and Palfrey 1995). As there is no information acquisition and attention allocation in that context, the parameter  $\lambda$  is interpreted as the extent of cognitive and computational limitations, i.e., the degree of bounded rationality, of the customer (see Su 2008, Chen et al. 2012). Despite this connection, the choice behaviour under the two models can be quite different. This is mainly because rational inattention is *micro-founded* and the properties of probabilistic choice change when the environment changes (e.g., prior beliefs change, information becomes harder to obtain). In contrast, the quantal choice motivated MNL offers a *macro-view* of the choice process under information frictions, capturing the notion that customers don't always make the optimal choice, but they choose better options with higher probability. As an example of this distinction, consider what happens as  $\lambda$  becomes large. Under MNL model of bounded rationality, the limitations of the customer get tighter and rationality becomes more "bounded" with increasing  $\lambda$ , and in the limit she randomizes her choice. Under rational inattention, however, the customer acquires less information and in the limit makes her choice completely based on prior belief  $G(\cdot)$  (i.e., she chooses the option with highest expected payoff). Another major distinction can be observed when some options are a-priori very similar, and in particular identical. As shown in Matejka and McKay (2015), the GMNL choice behavior of the rationally inattentive customer does *not* follow the Independence of Irrelevant Alternatives (IIA) Axiom of Luce (1959) which generates the MNL model. Debreu (1960) criticizes IIA for creating counter-intuitive implications about duplicate choices (so called red-bus/blue-bus paradox). In sharp

contrast, a rationally inattentive customer ignores a-priori duplicate options and treats them jointly as one option.<sup>5</sup> There are other appealing features of the choice behaviour under rational inattention that sets it apart. For example, adding an option to the choice set can increase the likelihood that an existing choice is selected, which is not possible under MNL or other random utility choice models (Matejka and McKay 2015). Likewise, GMNL model is shown to be consistent with a standard theory of imperfect perception based on signal processing and choice. Hence it can be rationalized within a standard Bayesian context, while MNL and consideration set based models cannot be (Caplin and Martin 2014).

Consider now a relaxation of the information acquisition process of the inattentive customer, and assume that she can generate perfect signals with complete information. In other words, the signals reveal the true state of the nature, and therefore the expected posterior entropy is  $E_{\mathbf{s}}[H(F(\cdot|\mathbf{s}))] = 0$ . In this case, the decision problem boils down to the choice of paying the information cost  $\mathcal{C}(F) = \lambda H(G(\mathbf{v}))$  and choosing the option based on true realizations, or not paying the cost and making a decision on the basis of prior expectations. This particular decision making process of the customer is akin to a *sequential information search*. Observe that the optimal information search strategy would yield a binary choice behaviour (probability of choosing an option is either zero or one). In contrast, under rational inattention, signals are never perfect (some uncertainty always remains), and the customer displays a probabilistic choice behaviour. In this regard, the sequential search driven choice model captures the rational decision-maker's costly acquisition of information, but in doing so ignores the frictions due to limited attention and limited processing capabilities of the customers.

The differences between GMNL, MNL and sequential search have implications on the pricing decisions of the seller as well. We now turn our attention to these pricing decisions.

#### 4. Baseline Model

In this section, we study a monopolistic firm selling a single product to rationally inattentive customers. We first derive structural results regarding the optimal pricing strategy and complement these findings

<sup>5</sup> Different generalizations of the MNL model has been proposed in the literature to remedy the IIA problem, such as the nested logit, mixed MNL. See for example Aksoy-Pierson et al. (2013) for a discussion.

with numerical experiments for additional insights. Then, we examine alternative pricing policies and illustrate the impact on the optimal product price and firm profitability. In this baseline model, the market is homogeneous and the firm does *not* use the product price strategically to “signal” product quality to customers. We defer the analysis of signalling to §5, while “heterogeneity” is covered in §6.

Consider a firm selling a single product to customers with limited attention. For expositional clarity, cost of production is taken as zero, and the total market size is normalized to 1. Customers observe the price  $p$  of the product, but they are not fully aware of the true quality of the product denoted as  $q$ . This fits well with non-functional product categories such as consumer electronics and durable goods which are not purchased at very high frequencies, choices are relatively distinct and prices are posted clearly. For analytical tractability, we assume a simple canonical form with two quality levels: *high* ( $q = q_H$ ) and *low* ( $q = q_L$ ), where  $q_H > q_L \geq 0$ . Allowing more than two levels of quality level (or uniformly distributed quality) does not change our structural results. The customer’s *prior belief* is such that the product is *high*-quality ( $q = q_H$ ) with probability  $\alpha$  ( $0 < \alpha < 1$ ), and *low*-quality ( $q = q_L$ ) with probability  $(1 - \alpha)$ . These assumptions enable us to focus conspicuously on the interesting case where customers face *some* uncertainty about product quality. Customers can acquire and process information about the quality level at a unit cost  $\lambda > 0$ . The firm, on the other hand, has more experience with the product and is assumed to know the true quality nature of the product.

Following the previous section definitions, let  $\pi_0(p)$  denote the *unconditional* probability that the customer purchases the product at price  $p$ , and  $\pi_i(p)$  denote the *conditional* probability that the customer purchases the product, given that true quality level of the product is  $q_i$ ,  $i \in \{H, L\}$ . We assume that the *net value* of a product with quality  $q$  and price  $p$  is given by  $v = q - p$  (Tirole 1988). Normalizing the *net value* of the outside option (*not purchasing* the product) to 0, we obtain

$$\pi_i(p) = \frac{\pi_0(p)e^{(q_i-p)/\lambda}}{\pi_0(p)e^{(q_i-p)/\lambda} + (1 - \pi_0(p))}, \quad i \in \{H, L\}, \quad (8)$$

with

$$\pi_0(p) = \alpha\pi_H(p) + (1 - \alpha)\pi_L(p). \quad (9)$$

Substituting  $\pi_H(p)$  and  $\pi_L(p)$  from (8) into (9), we find three possible solutions to  $\pi_0(p)$  as

$$\pi_0(p) \in \{0, 1, \dot{\pi}_0(p)\}, \quad (10)$$

where

$$\dot{\pi}_0(p) = \frac{e^{p/\lambda} (e^{p/\lambda} - (1 - \alpha)e^{q_L/\lambda} - \alpha e^{q_H/\lambda})}{(e^{p/\lambda} - e^{q_H/\lambda})(e^{p/\lambda} - e^{q_L/\lambda})}. \quad (11)$$

The two solutions  $\pi_0(p) \in \{0, 1\}$  capture the cases when the customer makes the purchase decision *without* processing any information. If  $\pi_0(p) = 0$ , the customer chooses the outside option, whereas if  $\pi_0(p) = 1$ , the customer purchases the product. On the other hand,  $\pi_0(p) = \dot{\pi}_0(p)$  describes the case when the customer *processes information* to make a “random” purchase decision. As shown in Matejka and McKay (2015), when there are two non-identical options (as in our case), the solution is unique.

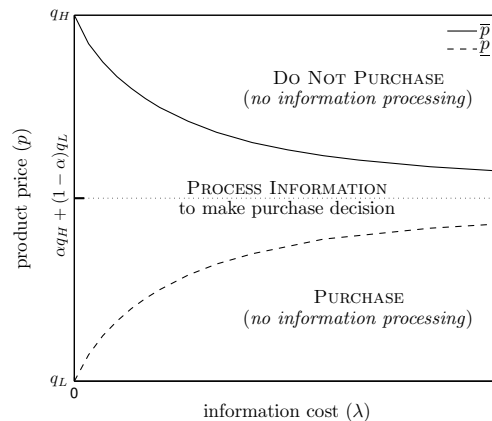


Figure 1 Product purchase and information processing strategies of an inattentive customer

Figure 1 illustrates the three possible product purchase and information processing strategies of an inattentive customer as product price and information cost varies. Clearly, if  $p \leq q_L$  or  $p \geq q_H$ , the customer does *not* need to process any information (purchase if  $p \leq q_L$ , and not purchase if  $p > q_H$ ). Note that  $\dot{\pi}_0(p) = 0$  when  $p = \bar{p} = \lambda \ln [\alpha e^{q_H/\lambda} + (1 - \alpha)e^{q_L/\lambda}]$ , and  $\dot{\pi}_0(p) = 1$  when  $p = \underline{p} = q_L + q_H - \lambda \ln [(1 - \alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}]$ . Namely,  $\dot{\pi}_0(\bar{p}) = 0$  and  $\dot{\pi}_0(\underline{p}) = 1$ . It can be easily verified that  $\underline{p} > q_L$ , while  $\bar{p} < q_H$ . Furthermore,  $\dot{\pi}_0(p) > 1$  if  $p < \underline{p}$ , and  $\dot{\pi}_0(p) < 0$  if  $p > \bar{p}$ . Accordingly, the customer processes information, only when  $\underline{p} < p < \bar{p}$ . Now, given that the customer does not process information, she

purchases the product if and only if her expected utility is at least as high as the outside option, i.e.  $p \leq \alpha q_H + (1 - \alpha)q_L$ . In Lemma 1 in Appendix, we establish that  $\underline{p} \leq \alpha q_H + (1 - \alpha)q_L \leq \bar{p}$ . This suggests that for  $q_L < p < \underline{p}$ ,  $\pi_0(p) = 1$  and for  $\bar{p} < p < q_H$ ,  $\pi_0(p) = 0$ , as seen in Figure 1. One can derive analogous thresholds for  $\lambda$ ,  $\alpha$  and  $q_H - q_L$  for which customers make purchase decision without processing information and base their decisions solely on their prior beliefs. However, we omit these for brevity.

In light of above, we can explicitly state the unconditional purchase probability as

$$\pi_0(p) = \begin{cases} 1 & \text{if } p < \underline{p} \\ \dot{\pi}_0(p) & \text{if } \underline{p} \leq p \leq \bar{p} \\ 0 & \text{if } p > \bar{p} \end{cases} \quad (12)$$

Note that the unconditional purchase probability  $\pi_0(p)$  given in (12) is continuous in  $p$ . Hence, the conditional purchase probability  $\pi_i(p)$  in (8) is also continuous in  $p$ .

Let the expected revenue of the firm as a function of price be  $\mathcal{R}_i(p) = p \times \pi_i(p)$ , for  $i \in \{H, L\}$ . We denote the optimal price that maximizes  $\mathcal{R}_i(p)$  by  $p_i^*$ , and the corresponding optimal revenue by  $\mathcal{R}_i^*$ .

**Proposition 1** *When customers have limited attention, the optimal price  $p_i^*$ ,  $i \in \{H, L\}$ , is such that*

$$q_L < \underline{p} = q_L + q_H - \lambda \ln [(1 - \alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}] \leq p_i^* \leq \bar{p} = \lambda \ln [\alpha e^{q_H/\lambda} + (1 - \alpha)e^{q_L/\lambda}] < q_H. \quad (13)$$

Proposition 1 shows that when customers are rationally inattentive, the firm will never charge the true valuation of the product. The optimal price is always sandwiched between the two true quality values. This means that a high-quality firm has to lower its price since customers can never fully assess the true quality of the product. This naturally results in a loss for the high-quality firm. In sharp contrast, a low-quality firm benefits from limited attention since costly information acquisition and resulting inattention allows the firm to charge prices higher than dictated by the true nature of the product. While intuitively appealing, this result fails to hold (as we discuss later) under the MNL choice used to model bounded rationality. Proposition 1 highlights the fact that due to customer inattention, the price range for the product is smaller than the true quality difference generating those prices, even



in the absence of any competition. Evidently, customers' limited attention restrains the ability of the firm to differentiate its product in terms of quality and charge the appropriate premium. This is an important finding that we will elaborate more on.

Based on Proposition 1, to determine the optimal price  $p_i^*$ , it is sufficient to take

$$\pi_0(p) = \dot{\pi}_0(p) = \frac{e^{p/\lambda} (e^{p/\lambda} - (1 - \alpha)e^{q_L/\lambda} - \alpha e^{q_H/\lambda})}{(e^{p/\lambda} - e^{q_H/\lambda})(e^{p/\lambda} - e^{q_L/\lambda})}. \quad (14)$$

In theory, one could substitute  $\pi_0(p)$  in (14) back into (8) and then optimize  $\mathcal{R}_i(p)$  over  $\underline{p} \leq p \leq \bar{p}$ . The extant literature suggests the revenue function is *not* concave for the standard MNL model of demand (Hanson and Martin 1996), implying the intractability of a price-based formulation. Song and Xue (2007) show that the revenue optimization problem becomes concave if market share, rather than product price, is used as the decision variable (Li and Huh 2011 propose the same approach for the nested logit model). Following this principle, from (8), we make the following variable change

$$p(\pi_i) = q_i + \lambda \ln \left[ \frac{\pi_0}{1 - \pi_0} \times \frac{1 - \pi_i}{\pi_i} \right], \quad i \in \{H, L\}, \quad (15)$$

and express the expected revenue of the firm as a function of  $\pi_i$  as  $\mathcal{R}_i(\pi_i) = p(\pi_i) \times \pi_i$ . Let  $\pi_i^*$  denote the optimal market share (conditional purchase probability) that maximizes  $\mathcal{R}_i(\pi_i)$ . Note that  $\mathcal{R}_i(\pi_i^*) = \mathcal{R}_i(p_i^*) = \mathcal{R}_i^*$ . The mapping between price and market share in (15) is well-defined when  $\pi_i \in (0, 1)$ , i.e., the customer processes information to make the purchase decision. We also know that the firm achieves  $\pi_i = 1$  for all  $p \leq \underline{p}$ . Hence, the firm sets  $p(\pi_i) = \underline{p}$  to maximize expected revenue. On the other hand, for  $\pi_i = 0$ , since it does not matter what price the firm charges as  $\mathcal{R}_i(\pi_i) = 0$  for all  $p \geq \bar{p}$ , we take  $p(\pi_i) = \bar{p}$ . As a result,  $\mathcal{R}_i(\pi_i)$  is continuous on  $\pi_i \in [0, 1]$ , and there is a one-to-one mapping between optimal market share  $\pi_i^*$  and optimal price  $p_i^*$  (obtained through the function  $\dot{\pi}_0(p)$ ).

The analysis of firm revenue through market share variables  $\pi_i$  is still quite involved and arduous since  $\pi_0$  in (15) is itself a function of  $\pi_H$  and  $\pi_L$  (as previously expressed in (9)). Nevertheless, we can prove the concavity of  $\mathcal{R}_i(\pi_i)$  in  $\pi_i$  for  $i \in \{H, L\}$ . Moreover, with the help of Proposition 1, we are able to establish that  $\mathcal{R}_H(p)$  is indeed concave in  $p$ , while  $\mathcal{R}_L(p)$  is decreasing and convex in  $p$ , over the relevant price interval  $[\underline{p}, \bar{p}]$ . It turns out that both characterizations are required for our subsequent analysis.

**Theorem 1** (i)  $\mathcal{R}_i(\pi_i)$  is concave in  $\pi_i$  for  $i \in \{H, L\}$ .

(ii)  $\mathcal{R}_H(p)$  is concave,  $\mathcal{R}_L(p)$  is decreasing convex in  $p$  for  $p \in [\underline{p}, \bar{p}]$ .

From part (ii) of Theorem 1, we can immediately see that when the true quality nature of the product is low, it is optimal for the firm to charge a (low) price such that every customer purchases the product.

**Corollary 1** *When the product is low-quality, the optimal price  $p_L^*$  and revenue  $\mathcal{R}_L^*$  are given by:*

$$\mathcal{R}_L^* = p_L^* = \underline{p} = q_L + q_H - \lambda \ln [(1 - \alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}]. \quad (16)$$

When the true quality nature of the product is high, it is not possible to obtain the optimal price  $p_H^*$  and optimal revenue  $\mathcal{R}_H^*$  in a closed-form. Despite this complication, we are able to provide additional properties of the optimal price and revenue that shed light on the role of customer's limited attention.

**Proposition 2** *When the product is low quality, the optimal revenue  $\mathcal{R}_L^*$  (and price  $p_L^*$ ) is increasing  $\lambda$ . On the other hand, when the product quality is high, the firm's optimal revenue  $\mathcal{R}_H^*$  is non-monotone  $\lambda$ . In particular, there exists a threshold  $\lambda_c$  above which the firm's optimal revenue is increasing in  $\lambda$ .*

As Proposition 2 elucidates, it is in the interest of a firm selling a low-quality product to customers with limited attention to make information acquisition more costly. The less questions asked by the customer, the less is revealed about the true (low) quality of the product, which in turn expedites the firm's ability to charge higher prices. Such firms would prefer to make product information more fuzzy and less revealing, and perhaps limit salesforce availability in order to take better advantage of limited attention of customers and costly information acquisition.

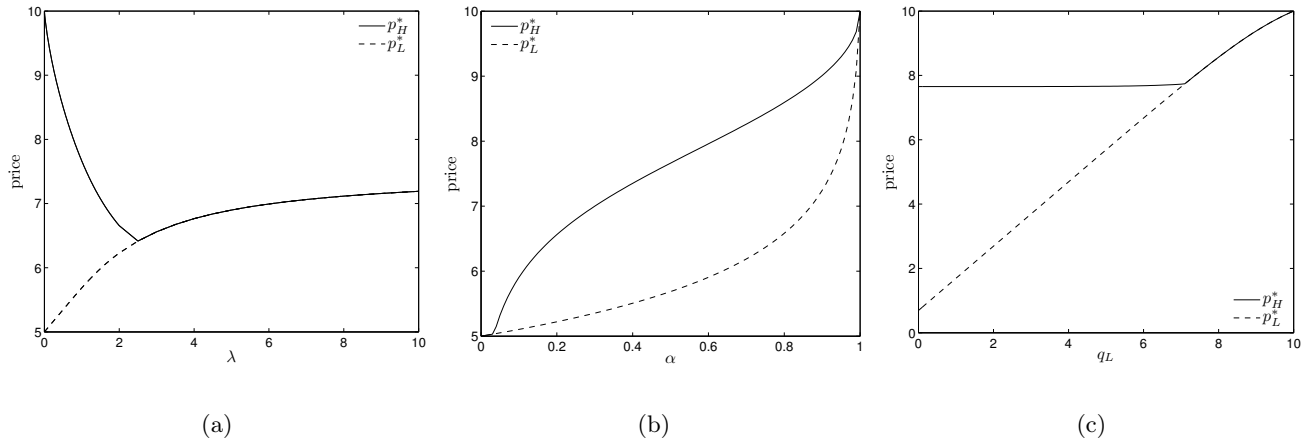
Intuitively one would expect the cost of information to play the opposite role for a high-quality product. Specifically, since a firm selling a high-quality product suffers from limited attention of the customers, it would be in the interest of the firm to make product quality information more accessible and as revealing as possible so that customers with limited attention can assess the true quality nature of the product with less effort. This intuition, however, is only *partially* correct. Increasing the cost of information  $\lambda$  initially from relatively low levels results in customers asking less questions about the

product and therefore learning less about the true (*high*) quality. In line with the intuitive thinking, this behaviour reduces the firm's profit. However, when  $\lambda$  is sufficiently large, it becomes too costly for the customers to acquire and process more information about the product so they base purchasing decisions mostly on *prior* valuations. At this point, it does not matter for the firm whether the product is high-quality or not – the pricing strategy of a high-quality product converges to that of a low-quality product, for which the optimal firm profit is increasing in  $\lambda$ . Hence, firm profit is not monotonic in  $\lambda$ . This result contrasts sharply with information search literature that finds prices to behave monotonically in the search costs (see Branco et al. 2012 for example).

The cost of information in our framework can also be viewed as a proxy for the *intensity of limitations* that customers face when attempting to obtain information about quality attributes of a product prior to purchase, which depends on the nature of the product itself. Academic literature distinguishes three types of products: *search*, *experience*, and *credence*. Search goods (Stigler 1961) are products where the customer can inspect the product and obtain relevant information prior to purchasing (relatively low information cost). Most commodities fall into this category. Experience goods (Nelson 1970) such as wine, restaurant services, have qualities that can be discovered only after full consumption (higher information costs). Credence goods (Darby and Karni 1973) like repair services, on the other hand, have qualities that cannot be evaluated, even during the time of consumption (very high information cost). Proposition 2 shows that for product/service categories where it is difficult to acquire information about the true quality, such as highly experiential or credence goods, it is better for the firm to take advantage of it (and making it even more difficult) regardless of what the true quality nature is. It also provides theoretical support for the argument that experts providing credence goods (e.g. doctors, mechanics) should charge a flat fee for all services, essentially overcharging customers for simple procedures and undercharging for elaborate procedures (Economist 2006). Dulleck and Kerschbamer (2006) reach this conclusion based on a model that features gaming opportunities for the firm, which can potentially take advantage of the customers in situations where the firm is not legally *liable* for the quality of the delivered product and the customer cannot *verify* the quality of the delivered product. Note that our result in Proposition 2 complements Dulleck and Kerschbamer (2006), as the aforementioned finding

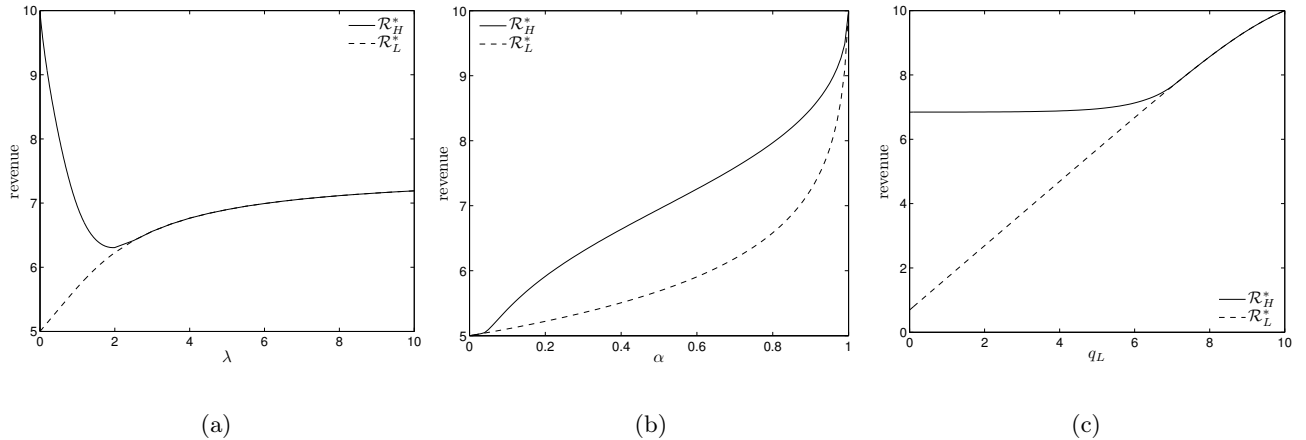
would still hold simply because of the inattentive nature of customers even when the firm does not have the potential to engage in any fraudulent behaviour.

In what follows, we augment the above insights with a numerical study, and illustrate the behaviour of the optimal price and the firm revenue for different customer and market related factors. To this end, we generate *problem settings* by systematically varying a number of key parameters of a *base scenario*. In the base scenario, we set the levels of the high-quality and low-quality as  $q_H = 10$  and  $q_L = 5$ , respectively. The customer's prior belief is such that the product is equally likely to be either high- or low-quality ( $\alpha = 0.5$ ) and the unit cost of information is 1 ( $\lambda = 1$ ). Figures 2 and 3 depict the impact of unit cost of information ( $\lambda$ ), prior belief of customers ( $\alpha$ ), and the low-quality level ( $q_L$ ) on the optimal price of the product and the firm's revenue, respectively, depending on the product's true quality level.



**Figure 2** Impact of unit information cost ( $\lambda$ ), prior belief ( $\alpha$ ), and lower quality level ( $q_L$ ) on product price

As Figure 2(a) shows, when  $\lambda = 0$ , the firm sets prices equal to the true valuation of the product, i.e.  $p_H^* = 10$  and  $p_L^* = 5$ . On the other extreme, as  $\lambda \uparrow \infty$ , prices converge to the customer's *expected* valuation for the product based solely on prior beliefs, i.e.,  $p_H^* = p_L^* = \alpha q_H + (1 - \alpha)q_L = 7.5$ . Observe that as the cost of information  $\lambda$  increases,  $p_L^*$  gradually increases (cf. Proposition 2). In contrast,  $p_H^*$  does *not* decrease monotonically (cf. Proposition 2). When  $\lambda$  is relatively small ( $\lambda \leq \lambda_c = 2.43$ ), increasing it induces inattentive customers inquiring less about the nature of the product, which forces the firm to discount the price of the high quality product. When  $\lambda = 2.43$ , the high-quality product



**Figure 3** Impact of information cost ( $\lambda$ ), prior belief ( $\alpha$ ), and lower quality level ( $q_L$ ) on firm revenue

is priced as if it is a low-quality product, and in both cases we have  $\pi_H^* = \pi_L^* = 1$ . As the cost of information  $\lambda$  increases further, it gets progressively even more expensive for the customers to learn the true quality of the product, hence the firm charges the same (increasing) price regardless of the quality. Consistent with Proposition 2, the optimal revenue  $\mathcal{R}_H^*$  is also non-monotonic with respect to  $\lambda$ , as shown in Figure 3(a).

As  $\alpha$  increases, customers' prior belief that the product is of high-quality increases. Therefore both  $p_H^*$  and  $p_L^*$  are increasing in  $\alpha$ , as seen in Figure 2(b). For a low-quality (resp., high-quality) product, one can interpret  $\alpha$  as the degree of *inaccuracy* (resp., *accuracy*) in the customers' initial assessments of the true quality nature of product. Note that the firm with a low-quality product increasingly benefits from such misjudgments (cf. Figure 3(b)). A firm with a high-quality product naturally benefits from more accurate assessment of its quality by the customers. It is also interesting to note the behavior of prices at the two extremes since these cases accentuate the effects of prior beliefs when customers have limited attention. As  $\alpha \downarrow 0$ , customers initially (very strongly) believe that the firm is selling a low-quality product, then the firm ends up pricing a high-quality product as if its quality is low to be able to sell the product. In contrast, as  $\alpha \uparrow 1$ , customers initially (very strongly) believe that the firm is selling a high-quality product, then the firm can price a low-quality product as if the product is of high-quality since they are absolutely wrong in their initial beliefs about the product's true quality nature. Clearly, customer's prior beliefs about the quality of the product has a profound impact on the optimal prices and the profit of the firm.

Figure 2(c) depicts the impact of  $q_L$  on the price of the product. As  $q_L$  increases, the true quality level of the low-quality product approaches to that of a high-quality product, essentially reducing the perceived degree of product differentiation. Note that when  $q_L$  is sufficiently lower than  $q_H$  ( $q_L \leq 7.1$ ), the relative difference between the quality levels is significant. Hence, inattentive customers face a potentially considerable *quality risk*. Therefore, they are willing to gather and process information about the true quality at cost. In this range,  $p_L^*$  steadily increases with  $q_L$ . The price  $p_H^*$  of the high-quality product also increases, though the impact is, as expected, more subdued. On the other hand, when  $q_L$  is sufficiently close to  $q_H$  ( $q_L > 7.1$ ), then the quality levels are so close that it becomes prohibitively expensive for customers to ask enough number of questions to differentiate between a high- or a low-quality product. Consequently, customers purchase without processing any information, the product is priced the same regardless of its true quality level, and this price continues to increase with the quality level  $q_L$ . Put differently, lack of significant quality distinction curbs the ability for price differentiation when customers are rationally inattentive. Finally, note that the fact that optimal prices increase in the perceived level of quality implies that a firm will benefit from any kind of false advertising or customer misperceptions that would inflate the true quality nature of its product in a monopolistic market. This is simply because the firm can charge higher prices when customers value the product at a higher level.

#### 4.1. Alternative Pricing Schemes

In the previous section, we have established the existence and some key properties of the optimal pricing strategy of a firm selling to rationally inattentive customers. Computing this optimal price still requires considerable effort on behalf of the firm, involving the estimation of the cost of information as well as computing the optimal prices numerically (for a high-quality product at least). There are natural alternative pricing schemes that do not depend on the cost of information and are easier to compute, as well as those that are derived from alternative specifications of the consumer choice process discussed earlier. We now take for a fact that customers have limited attention, and investigate the pricing and profit impacts of using these pricing strategies, which essentially overlook this fundamental characteristic of the customers.

**EX-POST PRICING:** When  $\lambda = 0$ , customers can process all relevant information *freely* and choose the utility-maximizing option with probability 1. Accordingly, the firm charges the true quality valuation of the product, i.e.  $p_i^{\text{EP}} = q_i$ ,  $i \in \{H, L\}$ . We refer to this policy as EX-POST pricing.

**EX-ANTE PRICING:** When  $\lambda \rightarrow \infty$ , customers do *not* process any information and purchase the product solely based on the observed price and *prior* beliefs (i.e., expected quality level). Since  $\lim_{\lambda \rightarrow \infty} \underline{p} = \lim_{\lambda \rightarrow \infty} \bar{p} = \alpha q_H + (1 - \alpha)q_L$ , the firm sets the price as  $p_i^{\text{EA}} = \alpha q_H + (1 - \alpha)q_L$ ,  $i \in \{H, L\}$ . We refer to this pricing strategy as EX-ANTE pricing. The profit gap occurring from using ex-ante pricing reflects the *impact of information processing* of customers with limited attention.

**MNL-BASED PRICING:** Matejka and McKay (2015) show that when all options are a-priori homogeneous, i.e.,  $G(\mathbf{v})$  is invariant to all permutations of  $\mathbf{v}$ , the unconditional probabilities are uniform. Consequently, the conditional purchase probability reduces to

$$\pi_i^{\text{MNL}}(p) = \frac{e^{(q_i - p)/\lambda}}{e^{(q_i - p)/\lambda} + 1}. \quad (17)$$

This is the standard MNL equation we provided in (7) with  $v = q_i - p$ . It can be shown that the optimal price under MNL is  $p_i^{\text{MNL}} = \lambda [1 + \mathcal{W}(e^{q_i/\lambda - 1})]$  for  $i \in \{H, L\}$ , where  $\mathcal{W}$  is the Lambert function (see Akcay et al. 2010). Note that customers being invariant to  $q_i - p$  means they do not distinguish their choices before acquiring information, regardless of quality or price. Hence, the gap between the optimal revenue  $\mathcal{R}_i^*$  and  $\mathcal{R}_i(p_i^{\text{MNL}})$  identifies the impact of *prior knowledge* of rationally inattentive customers.

Another reason for exploring MNL-based pricing is its wide spread use, especially in the context of modeling boundedly rational choice. As noted in §3, there are notable differences in the purchasing behaviors of a boundedly rational customer described by MNL and a rationally inattentive customer described by GMNL. These behavioral discrepancies naturally translate into differences in the optimal prices. On one extreme, as  $\lambda \downarrow 0$ , the optimal prices converge to EX-POST prices under both MNL and GMNL. For any  $\lambda > 0$ , and especially at higher levels of  $\lambda$ , optimal prices differ significantly. In fact, under MNL the optimal prices can well exceed true quality levels, while they remain between  $q_L$  and  $q_H$  under GMNL (Proposition 1). As  $\lambda \uparrow \infty$ , under MNL, customers increasingly randomize choice, causing prices to increase further, while under GMNL they converge to *ex-ante* prices.

**SEARCH-BASED PRICING:** Consider now the *sequential search* model of consumer choice introduced in §3, and suppose that a customer can learn the true quality nature of the product at cost  $\Lambda$ . This is equivalent to assuming that the inattentive customer receives “perfect information” signals, at cost  $\mathcal{C}(F) = \lambda H(G(\mathbf{v}))$ . Hence,  $\Lambda = \mathcal{C}(F) = -\lambda[\alpha \ln \alpha + (1 - \alpha) \ln(1 - \alpha)]$  for a meaningful comparison. The profit gap as a result of using SEARCH-BASED pricing strategy  $p_i^{\text{SPB}}$  signifies the impact of *information frictions* due to cognitive/processing limitations of the customers, which enables them to generate only imperfect signals.

Note that under SEARCH-BASED pricing, there is no probabilistic choice. However, the firm can affect the customer’s search and purchase behaviour by selecting  $p$ . In fact, customer’s information acquisition and purchasing decisions can be represented by a simple decision tree, and based on this, the optimal search-based price  $p_i^{\text{SPB}}$  maximizing the firm’s expected revenue can be determined as<sup>6</sup>

$$p_i^{\text{SPB}} = \begin{cases} \max\{q_H - \frac{\Lambda}{\alpha}, \alpha q_H + (1 - \alpha)q_L\}, & \text{if } i = H \\ \min\{q_L + \frac{\Lambda}{1 - \alpha}, \alpha q_H + (1 - \alpha)q_L\}, & \text{if } i = L \end{cases}.$$

It can be verified that the optimal price under SEARCH-BASED pricing is asymptotically equivalent to that under pricing for inattentive customers when  $\lambda \rightarrow 0$  (EX-POST prices) and  $\lambda \rightarrow \infty$  (EX-ANTE pricing), but it is quite different in general.

Next, we carry out a numerical study to compare the profit impact of the aforementioned strategies (EX-POS, EX-ANTE, MNL-BASED, SEARCH-BASED). In this study, we characterize a problem setting by a particular combination of parameter values for  $\lambda$ ,  $\alpha$  and  $q_L$ . For each problem setting,  $\lambda$  assumes a value from the set  $\{0, 0.5, 1, \dots, 9.5, 10\}$ ,  $\alpha$  assumes a value from the set  $\{0, 0.1, 0.2, \dots, 0.9, 1\}$ , and  $q_L$  assumes a value from the set  $\{0, 1, 2, \dots, 9, 10\}$  (we fix  $q_H = 10$  in all problems). In total, we generate 2079 problem scenarios. Table 1 provides an overall summary of all results; i.e. the price, revenue, and revenue gap (measured as the lost revenue as a percentage of the optimal revenue) associated with each scheme, averaged over all instances.

As seen from the results, the average profit loss of using a suboptimal pricing strategy is quite high (24% to 100% depending on the strategy). For a firm selling a high-quality product, the profit loss is

<sup>6</sup> We omit details for space considerations. Details are available upon request.



		OPTIMAL	EX-ANTE	EX-POST	SEARCH-BASED	MNL-BASED
$q = q_H$	Average Price	7.45	7.50	10	7.84	10.78
	Average Revenue	7.28	4.85	0	5.40	2.51
	Revenue Gap	–	30.08%	100%	24.45%	70.54%
$q = q_L$	Average Price	6.65	7.50	5.00	7.16	8.35
	Average Revenue	6.65	2.59	5.00	3.36	2.50
	Revenue Gap	–	62.67%	30.76%	50.62%	62.71%

**Table 1 Comparison of Alternative Pricing Strategies**

the least when the firm adopts SEARCH-BASED pricing, while for a firm selling a low-quality product, EX-POST pricing is best. Recall from Corollary 1 that when the product quality is low, it is optimal for the firm to maintain a low price and aim to sell to all customers. This price is naturally close the EX-POST price which equals the valuation of the product and does not take advantage of customers' inattention. Using the same pricing strategy for a high-quality product, however, leads to a very high price that renders customers to not consider the product at all (cf. Corollary 1). In this case, the price has to be lowered down, and SEARCH-BASED as well as EX-ANTE pricing do precisely that. We should also mention that compared to the optimal price, SEARCH-BASED pricing undercharges in 86.6% of the cases for a high-quality product, whereas it overcharges in 95.2% of the cases for a low-quality product. Further, the impact of overcharging customers on firm profitability is more severe compared to that of undercharging. Also, MNL-BASED pricing performs significantly worse than SEARCH-BASED pricing.

Table 1 also shows that information processing strategy of the customers has an important impact on the optimal prices and resulting profits. Looking at the revenue gap of EX-ANTE pricing strategy, it is clear that a firm can garner significantly higher profits by considering customer's information processing, especially for a low-quality product. Likewise, the revenue gap of MNL-BASED pricing signifies the importance of customer's prior knowledge. Roughly 60-70% revenue loss for the firm originates from ignoring that customers might have some knowledge and (non-uniform) preferences prior to acquiring information. Even accounting for costs associated with information acquisition does not diminish the gap significantly, as seen from the performance of the SEARCH-BASED pricing.

Two remarks are in order. First, we conducted further analysis elaborating on scenarios where the revenue gaps are relatively low (below 5%), and yet again observed substantial variability in the performance of each strategy. This reinforces that it is not possible to unequivocally ascertain the relative

performance of any alternative pricing scheme. Second, we considered alternative specifications of the MNL-BASED pricing policy (e.g. using a deterministic valuation for the outside option, or based on a valuation using the expected instead of the true quality level of the product), and confirmed that the revenue gaps still remain rather significant. We omit the associated details for brevity.

## 5. When Prices Signal Quality

In our analysis so far, customers do not infer quality from the observed prices. However, it is plausible that prices can be strategically used by the (informed) firm to “signal” the quality of the product to the (uninformed) customers. In this section, we generalize our pricing framework by incorporating the signalling game between the firm and customers with limited inattention.

Suppose that as before, customers are homogenous with belief  $\alpha$  that the product is of high-quality and incur information costs at rate  $\lambda$ . The actual quality of the product is realized and revealed to the selling firm. Accordingly, there are two firm types: high-quality and low-quality. First the firm decides on the product price  $p$  which is freely observed by the customers. After observing the price of the product, customers are allowed to revise their beliefs about the quality of the product. Then, they optimally acquire and process information about the quality, and make their optimal purchase decisions. Firm profit is realized based on the optimal purchasing behavior of the customers.

We search for Perfect Bayesian Equilibria (PBE) of this sequential-move, strategic pricing signalling game. Firm strategy is the pricing decision. Let  $\sigma_H(p)$  (resp.,  $\sigma_L(p)$ ) denote the probability that a high-quality (resp., low-quality) firm charges the price  $p$ . Customer strategy is the purchasing decision, i.e., the probability that a customer buys the product when the price charged is  $p$ . This probability naturally depends on the quality nature of the product. The customer’s decision process is as follows. After observing price  $p$ , the customer forms an *interim* belief about the quality of the product, which is denoted as  $\mu(p)$ . More specifically,  $\mu(p)$  is the probability the customers believe that the observed price  $p$  is associated with a *high-quality* product. Under PBE, this belief has to be consistent with the Baye’s rule whenever possible. The customer chooses the information strategy optimally and with no restrictions on the information structure, except that the posterior is consistent with the prior belief

(which in this case is  $\mu(p)$ ). This leads to the GMNL choice behavior as before. To highlight the dependency of the choice behavior on beliefs, we use a slightly different notation, and represent the GMNL conditional choice probabilities for high- and low-quality products as  $\pi_H^\mu(p)$  and  $\pi_L^\mu(p)$  respectively. The resulting expected profits for the high- and low-quality firms are given as  $\mathcal{R}_H(p, \mu(p)) = p\pi_H^\mu(p)$  and  $\mathcal{R}_L(p, \mu(p)) = p\pi_L^\mu(p)$  respectively.

The PBE equilibrium  $\{\hat{\sigma}, \hat{\pi}, \hat{\mu}\}$  of the game is characterized as follows:

1. The firm assigns positive probability only to prices that maximize its profit:

$$\mathcal{R}_H(p, \hat{\mu}(p)) \geq \mathcal{R}_H(p', \hat{\mu}(p')) \quad \forall p' \in [q_L, q_H] \quad \text{whenever } \hat{\sigma}_H(p) > 0 \quad (18)$$

$$\mathcal{R}_L(p, \hat{\mu}(p)) \geq \mathcal{R}_L(p', \hat{\mu}(p')) \quad \forall p' \in [q_L, q_H] \quad \text{whenever } \hat{\sigma}_L(p) > 0 \quad (19)$$

2. Customers make purchase decisions optimally:

$$\hat{\pi}_H(p) = \pi_H^{\hat{\mu}}(p) \quad (20)$$

$$\hat{\pi}_L(p) = \pi_L^{\hat{\mu}}(p) \quad (21)$$

3. Customer beliefs are consistent with the Baye's rule whenever possible:

$$\hat{\mu}(p) = \frac{\alpha \hat{\sigma}_H(p)}{\alpha \hat{\sigma}_H(p) + (1 - \alpha) \hat{\sigma}_L(p)} \quad \text{whenever } \hat{\sigma}_H(p) + \hat{\sigma}_L(p) > 0. \quad (22)$$

Most signalling games have multiple equilibria since the PBE concept does not put restrictions on the belief structure of customers off the equilibrium path. Nevertheless, utilizing the results of the previous section, we are able to provide a fine characterization of the equilibria.

**Theorem 2** *The following are true for pure strategy PBE:*

1. *There exists a unique separating equilibrium, which is the perfect information equilibrium: The high-quality firm charges  $\hat{p}_H = q_H$  with probability 1 (i.e.,  $\hat{\sigma}_H(p) = 1$  if  $p = q_H$ ,  $\hat{\sigma}_H(p) = 0$  othw.), while the low-quality firm charges  $\hat{p}_L = q_L$  with probability 1 (i.e.,  $\hat{\sigma}_L(p) = 1$  if  $p = q_L$ ,  $\hat{\sigma}_L(p) = 0$  othw.).*

2. *For any  $\lambda \geq \lambda_c$ , there exists a unique pooling equilibrium: Both type of firms charge same price  $\hat{p} = q_L + q_H - \lambda \ln [(1 - \alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}]$  with probability 1 (i.e.,  $\hat{\sigma}_H(p) = \hat{\sigma}_L(p) = 1$  if  $p = \hat{p}$  and  $\hat{\sigma}_H(p) = \hat{\sigma}_L(p) = 0$  othw.).*

3. *For any  $\lambda < \lambda_c$ , there does not exist a pooling equilibrium.*

Theorem 2 supports the validity of key insights obtained under our baseline model. In particular, when the cost of information is sufficiently high, there exists an equilibrium in which high- and low-quality firms charge the same price. Consequently, firms selling highly experiential or credence goods should charge a flat price regardless of the quality of the services rendered, even if customers interpret prices as signals of quality. On the other hand, when the cost of information is low, this structure is no longer supported in equilibrium. In such environments, the only pure strategy equilibrium that exists is one where firms charge differentiated prices equal to actual quality levels, perfectly signalling to customers the true product quality. In addition, note that the (pooling) equilibrium price at sufficiently high information costs is the same as in our baseline model. From Proposition 2, this price is increasing in the cost of information. Hence, the firm with the high-quality product can experience equilibrium prices that are non-monotonic in information costs.<sup>7</sup>

In order to address the multiplicity of equilibria in signalling games, refinement concepts such as the Intuitive Criterion (IC) by Cho and Kreps (1987) or Divinity Criterion (D1) by Banks and Sobel (1987) are usually applied. Such refinement concepts employ restrictions on out-of-equilibrium beliefs to rule out counter-intuitive equilibrium outcomes. As a result, the validity of equilibria satisfying (surviving) the criterion is strengthened, as well as the predictive power of the model. Next we show that both the separating and the pooling equilibria characterized in Theorem 2 belong to this class.

**Proposition 3** *The separating and pooling equilibria defined in Theorem 2 satisfy the IC and D1.*

Proposition 3 substantiates the predictive value of the pure strategy equilibria of the strategic pricing signalling game. It does not ascertain however the non-existence of other equilibria. An important candidate is a highly informative equilibrium that partially reveals quality levels (see Bester and Ritzberger 2001). In this semi-separating/pooling equilibrium, the high-quality firm plays a pure strategy and charges a high price, while the low-quality firm adopts a mixed strategy by randomizing between imitating the high-quality firm's price and revealing its low-quality by charging  $q_L$ . Letting  $f$  denote the

<sup>7</sup> It is possible that for  $\lambda < \lambda_c$   $\hat{p}_H = q_H$ , but a switch occurs at  $\lambda \geq \lambda_c$  and  $\hat{p}_H = \hat{p} < q_H$ , after which  $\hat{p}$  increases in  $\lambda$ .

probability that the low-quality firm imitates the high-quality firm price, this semi-separating/pooling PBE can be specified in terms of  $\{\hat{\sigma}, \hat{\pi}, \hat{\mu}, \hat{f}\}$  :

$$\hat{\sigma}_H(p) = 1 \text{ if } p = \hat{p}_H = \arg \max_{q_L \leq p \leq q_H} \mathcal{R}_H(p, \hat{\mu}) \quad \hat{\sigma}_L(p) = \begin{cases} \hat{f} & \text{if } p = \hat{p}_H \\ 1 - \hat{f} & \text{if } p = q_L \end{cases} \quad (23)$$

$$\hat{\pi}_H(p) = \pi_H^{\hat{\mu}}(p) \quad \hat{\pi}_L(p) = \pi_L^{\hat{\mu}}(p) \quad (24)$$

$$\hat{\mu}(p) = \begin{cases} \frac{\alpha}{\alpha + (1-\alpha)\hat{f}} & \text{if } p = \hat{p}_H \\ 0 & \text{otherwise} \end{cases} \quad \hat{f} = \arg \max_{0 \leq f \leq 1} \{f\mathcal{R}_L(\hat{p}_H, \hat{\mu}) + (1-f)q_L\} \quad (25)$$

Under this equilibrium, when customers observe the low price  $q_L$ , they believe (perfectly) that the product is low-quality. On the other hand, when they observe the high price  $\hat{p}_H > q_L$ , they form interim beliefs that the product is high-quality according to the Baye's Rule ( $\hat{\mu} = \alpha/(\alpha + (1-\alpha)\hat{f})$ ). Hence, they need to acquire and process additional information to distinguish the true quality of the product. As before, customers optimally acquire and process quality information, and make choices according to GMNL, consistent with their updated beliefs. The (high) price chosen by the high-quality firm maximizes his profit under the equilibrium belief structure. In a similar vein, at the equilibrium  $\hat{f}$ , the low-quality firm profit is maximized.

Notice that as  $\lambda \downarrow 0$ , customers can determine actual quality very easily regardless of their prior beliefs, implying  $\hat{p}_H$  will approach  $q_H$ . Since imitating this price would be very costly to the low-quality firm, we would expect  $\hat{f}$  to approach zero, and the equilibrium to converge to the perfect information separating equilibrium. In contrast, when  $\lambda$  is sufficiently large (and in particular  $\lambda \geq \lambda_c$ ),  $\hat{p}_H$  will be very close to low-quality firm's optimal price  $\arg \max_{q_L \leq p \leq q_H} \mathcal{R}_L(p, \hat{\mu})$ . Hence, we would expect the low-quality firm to prefer imitating the high-quality firm price ( $\hat{f} = 1$ ), and making the equilibrium identical to the pooling equilibrium.

It can be checked easily that the low-quality firm's profit (see RHS of 25) is not necessarily unimodal in  $f$  and  $\hat{f}$  can take extreme values  $\{0, 1\}$  or an intermediate value. Hence, although an equilibrium always exists, it is not possible to characterize its properties analytically. Nevertheless, our numerical experiments show that the above intuition holds. Specifically, as seen in Figure 4(a), for the base scenario, when  $\lambda$  is low,  $\hat{f} = 0$ . For intermediate  $\lambda$  levels, the strict semi-separating/pooling equilibrium

(denoted as S/P E on the figures) emerges ( $0 < \hat{f} < 1$ ), while for high  $\lambda$  levels,<sup>8</sup>  $\hat{f} = 1$ . Overall, the imitation rate of the high price of the high-quality firm increases as information becomes more costly. We also illustrate the equilibrium prices and profits in Figures 4(b) and 4(c) respectively. For the low-quality firm, for any  $\lambda$  we take an average of the prices and revenue (over the equilibrium  $f$ ). The central observation is the close resemblance of the behaviour of the equilibrium prices and revenues to those under our baseline model. This further testifies to the robustness of results and insights obtained under our baseline model in §4.

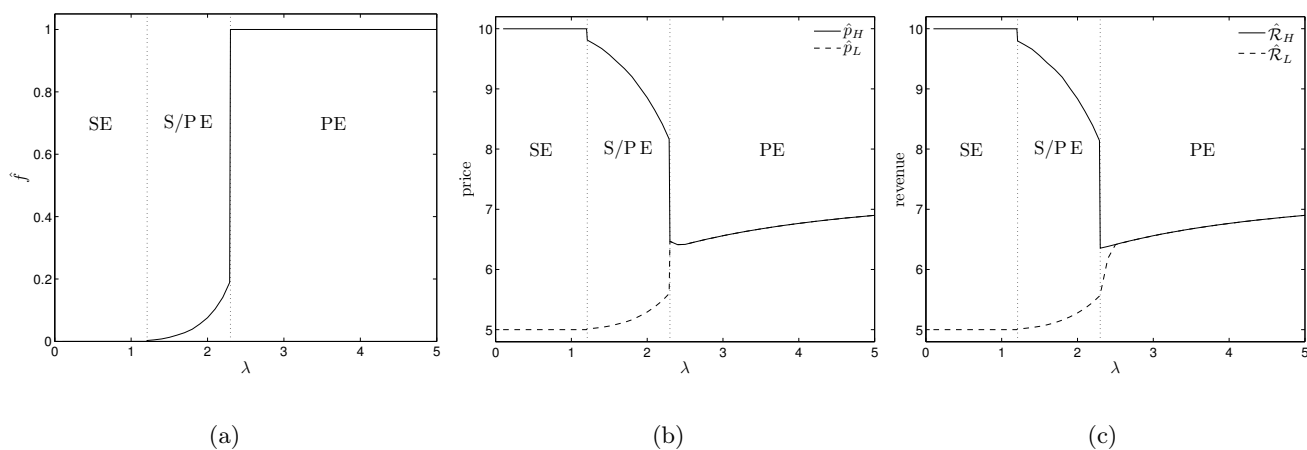


Figure 4 Separating (SE), Semi Separating/Pooling (S/P E) and Pooling (PE) Equilibria as  $\lambda$  varies

## 6. Heterogeneous Customers

Thus far we have assumed that customers are homogenous; (i) they are equally informed about product quality and have identical information costs, and (ii) they share the same prior belief about product quality. We now incorporate customer heterogeneity by relaxing these assumptions.

### 6.1. Heterogeneity in Information Costs

Suppose that, as in Debo et al. (2012), customers are heterogeneously informed about the quality, such that only a fraction of them are *uninformed* about quality as in our baseline model, while others are fully *informed* (and hence know the true quality of the product). Equivalently, we can consider the market to be heterogenous in information costs, consisting of both *inattentive* customers with cost of

<sup>8</sup> In this example,  $\hat{f} = 1$  for any  $\lambda \geq 2.3$ . The critical  $\lambda_c = 2.43$ .

information of  $\lambda > 0$  and perfectly *attentive* customers with zero information cost. Let the fraction of inattentive customers be  $\beta$ . Customer heterogeneity brings forth new trade-offs for the firm in choosing the optimal product price. In this case, the expected profit can be written as

$$\mathcal{R}_i(p) = \beta p \pi_i(p) + (1 - \beta) p 1_{p \leq q_i} \quad \text{for } i \in \{H, L\}, \quad (26)$$

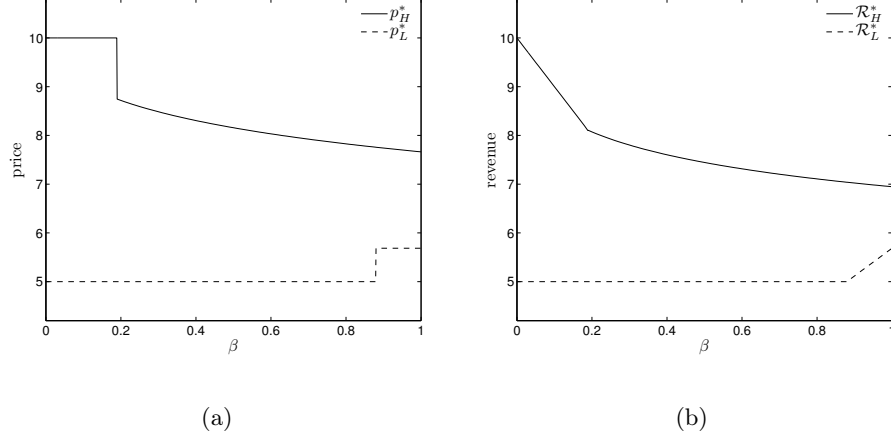
where  $1_x$  is the indicator function. Clearly, the firm has to decide which segments of the market to serve in setting its price when customers are heterogenous. Next, we characterize the optimal strategy.

**Proposition 4** *The optimal pricing strategy under information cost heterogeneity is as follows:*

1. *When the product quality is high, there exists a threshold  $\underline{\beta} = \frac{q_H - \tilde{p}_H}{q_H - (1 - \pi(\tilde{p}_H))\tilde{p}_H}$  such that for any  $\beta \geq \underline{\beta}$ , the firm serves both market segments with price  $\tilde{p}_H$ , which uniquely maximizes  $\mathcal{R}_H(p) = \beta p \pi_H(p) + (1 - \beta)p$ . If  $\beta < \underline{\beta}$ , the firm serves attentive customers only with price  $q_H$ .*
2. *When the product quality is low, there exists a threshold  $\bar{\beta} = \frac{q_L}{q_L + q_H - \lambda \ln[(1 - \alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}]}$  such that for any  $\beta \leq \bar{\beta}$ , the firm serves both market segments with price  $q_L$ . If  $\beta > \bar{\beta}$ , the firm serves inattentive customers only with price  $p_L^* = q_L + q_H - \lambda[(1 - \alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}]$ .*

Evidently, the optimal pricing strategy is more complicated when customers are heterogenous in quality information or information costs. Nevertheless, the effects of heterogeneity is rather reasonable. First, the higher the fraction of informed/attentive customers (lower  $\beta$ ), the closer the prices to the true quality levels of the products. This can also be seen in Figure 5, which illustrates the firm's optimal pricing strategy and optimal profit for the base scenario. Naturally, a high-quality firm benefits from an increased proportion of informed/attentive customers while a low-quality firm loses from it.

We also observe that heterogeneity does not lead to an unexpected price ordering between high- and low-quality products (this contrasts with Debo et al. (2012) which shows that a high-quality firm can choose a slower rate of service due to heterogeneity). It is indeed never optimal for the firm to set the price of the low-quality product higher than that of the high-quality product. Likewise, a low-quality product never garners higher profits for the firm than a high-quality product.



**Figure 5** Impact of information cost heterogeneity on product price and firm revenue

## 6.2. Heterogeneity in Prior Beliefs

Suppose that customers have heterogeneous priors about product quality – a  $\gamma$  fraction of customers believe that the product is high-quality with probability  $\alpha_1$  (or expect the quality to be  $\alpha_1 q_H + (1 - \alpha_1)q_L$ ), whereas the remaining  $1 - \gamma$  fraction believe this said probability is  $\alpha_2$  (or expect the quality to be  $\alpha_2 q_H + (1 - \alpha_2)q_L$ ). Here we assume  $\alpha_1 > \alpha_2$  without loss of generality. Each customer (segment) selects her information strategy optimally, forming posterior beliefs that are consistent with her *own* prior. Then, we can express the firm's profit as follows

$$\mathcal{R}_i(p) = \gamma p \pi_{i,1}(p) + (1 - \gamma) p \pi_{i,2}(p) \quad \text{for } i \in \{H, L\}, \quad (27)$$

where  $\pi_{i,k}(p)$  is the conditional purchase probability of the product with quality  $q_i$ ,  $i \in \{H, L\}$  by customers with belief  $\alpha_k$ ,  $k \in \{1, 2\}$ . The following proposition characterizes the firm's optimal strategy.

**Proposition 5** *The optimal pricing strategy under heterogeneity in prior beliefs is as follows:*

1. When the product quality is high, the optimal price  $p_H^*$  can be uniquely determined since  $\mathcal{R}_H(p)$  is piecewise concave in  $p$ .
2. When the product quality is low, there exists a threshold  $\underline{\gamma} = \frac{q_L + q_H - \lambda \ln[(1 - \alpha_2)e^{q_H/\lambda} + \alpha_2 e^{q_L/\lambda}]}{q_L + q_H - \lambda \ln[(1 - \alpha_1)e^{q_H/\lambda} + \alpha_1 e^{q_L/\lambda}]}$  such that for any  $\gamma \geq \underline{\gamma}$ , the firm sets the price as  $p_L^* = q_L + q_H - \lambda[(1 - \alpha_1)e^{q_H/\lambda} + \alpha_1 e^{q_L/\lambda}]$ ; otherwise the firm sets the price lower as  $p_L^* = q_L + q_H - \lambda[(1 - \alpha_2)e^{q_H/\lambda} + \alpha_2 e^{q_L/\lambda}]$ .

Next, we illustrate the impact of customer heterogeneity in prior beliefs on the firm's revenue using our bases scenario. First, recall that when customers are homogenous ( $\alpha_1 = \alpha_2$ ), both  $\mathcal{R}_H^*$  and  $\mathcal{R}_L^*$



increase as the prior belief that the product is high-quality gets stronger (see Figure 3(b)). Let  $\gamma = 0.5$  and  $\alpha_1 + \alpha_2 = 1$ , so that the *average* belief about product quality ( $\gamma\alpha_1 + (1 - \gamma)\alpha_2 = 0.5$ ) is the same with the base scenario ( $\alpha = 0.5$ ). In Figure 6(a), we observe that heterogeneity hurts revenues – the firm enjoys its highest revenue when customers are homogenous, and as the degree of heterogeneity (captured by  $\alpha_1 - \alpha_2$ ) increases,  $\mathcal{R}_H^*$  and  $\mathcal{R}_L^*$  decrease. Practically speaking, this suggests that the firm should aim for “consistency” in the quality perceptions of its customers. Moreover, the firm benefits from having a larger fraction of customers who have higher quality perceptions, as seen in Figure 6(b) with  $\alpha_1 = 0.75$  and  $\alpha_2 = 0.25$  –  $\mathcal{R}_H^*$  increases monotonically with  $\gamma$ , while  $\mathcal{R}_L^*$  remains constant when  $\gamma < \underline{\gamma}$  (the firm serves all customers) and increases when  $\gamma \geq \underline{\gamma}$  (the firm serves all customers with higher perceptions, but only *some* of the customers with low perceptions, see Proposition 5). Finally, Figure 6(c) shows that our key observations from the baseline model about the impact of information cost on the firm’s revenue are still valid when customers have heterogeneous priors ( $\alpha_1 = 0.75$ ,  $\alpha_2 = 0.25$  and  $\gamma = 0.5$ ).

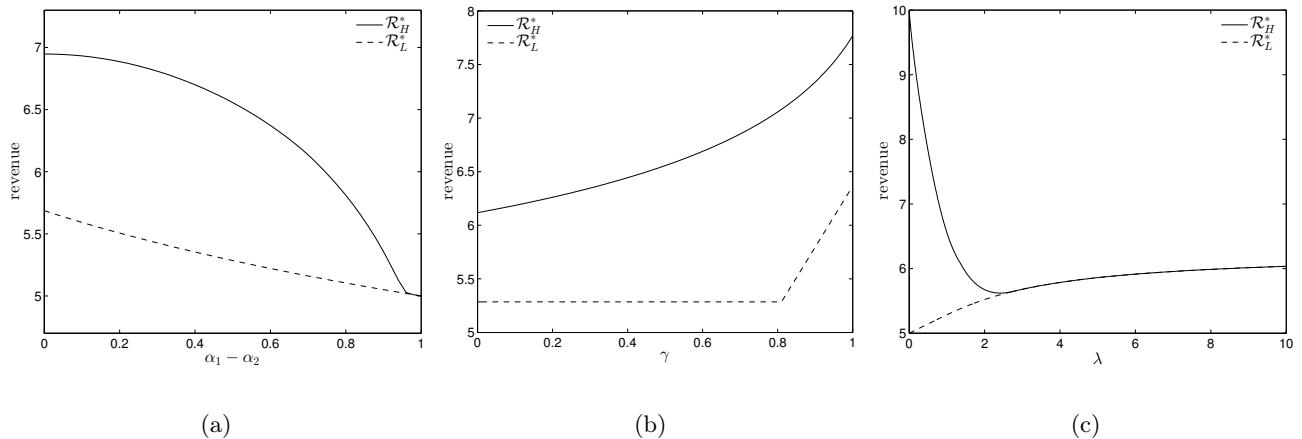


Figure 6 Impact of heterogeneity in customer beliefs on firm revenue

## 7. Extensions and Applications

Limited attention of customers and resulting choice behavior have implications beyond the pricing strategy of a single firm, and these extend well into more complicated market structures (e.g., multiple products, competition) as well as operational domains (e.g. product assortment optimization, inventory levels). A comprehensive analysis of each of these applications is a major undertaking in its own regard

and is beyond the scope of the paper. Nevertheless, there are immediate extensions of our framework that would form the initial building blocks of these applications. In this section, we introduce and discuss three such extensions.

### 7.1. Multiple Products

Rational inattention to choice can have a profound impact on a firm's optimal product assortment decisions. These include the selection of product varieties to offer (and to exclude) and the prices associated with each variety in the assortment. Clearly, there are also costs associated with product assortment decisions (e.g., due to shelf-space allocation, managing inventories). A first step of analysis of assortment decisions would be to put these costs aside and elaborate on the revenue side. To this end, we consider an extension of our baseline model with two products, each of which can either be high- or low- quality. Let  $q_j$  and  $p_j$  be the quality and the price of product  $j$ , for  $j \in \{1, 2\}$ , respectively. Note that when there are multiple distinct products in the market, there could be some correlation in their quality levels. Consequently, as the rational customer is acquiring information about one product, she might do so for the other as well. We capture such correlation by assuming that the customers' priors are such that either product is high-quality with probability 0.5, but the quality of the two products have a correlation of  $\rho$ . This leads to the following joint distribution of prior beliefs:

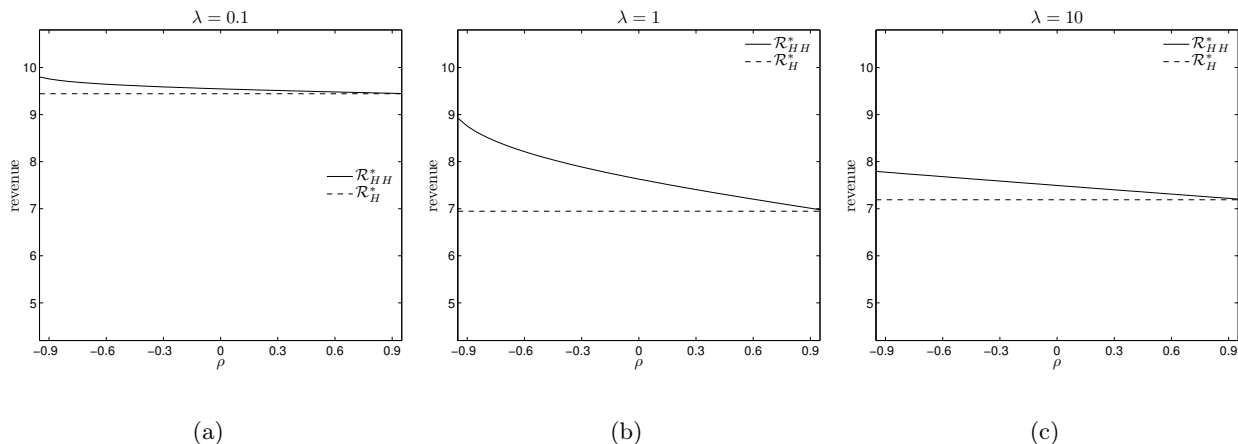
$$\begin{aligned} P(q_1 = q_L, q_2 = q_L) &= \frac{1}{4}(1 + \rho) & P(q_1 = q_H, q_2 = q_L) &= \frac{1}{4}(1 - \rho) \\ P(q_1 = q_L, q_2 = q_H) &= \frac{1}{4}(1 - \rho) & P(q_1 = q_H, q_2 = q_H) &= \frac{1}{4}(1 + \rho) \end{aligned}$$

Before making the purchase decision, the customer has the opportunity to study both products at a cost  $\lambda$ . As before,  $\pi_0^j$  is the unconditional purchase probability for product  $j$ ,  $j \in \{1, 2\}$ . Let  $\pi_{q_1, q_2}^j(\mathbf{p})$  be the conditional purchase probability for product  $j$  for given price vector  $\mathbf{p} = (p_1, p_2)$  when the quality levels are  $q_1$  and  $q_2$ . Normalizing the net value of the outside option of not purchasing to zero, we have

$$\pi_{q_1, q_2}^j(\mathbf{p}) = \frac{\pi_0^j(\mathbf{p})e^{(q_j - p_j)/\lambda}}{\pi_0^1(\mathbf{p})e^{(q_1 - p_1)/\lambda} + \pi_0^2(\mathbf{p})e^{(q_2 - p_2)/\lambda} + (1 - \pi_0^1(\mathbf{p}) - \pi_0^2(\mathbf{p}))}, \quad j \in \{1, 2\}. \quad (28)$$

Consequently, the total revenue of the firm becomes  $\mathcal{R}_{q_1, q_2}(\mathbf{p}) = p_1\pi_{q_1, q_2}^1(p_1, p_2) + p_2\pi_{q_1, q_2}^2(p_1, p_2)$ . Let  $\mathcal{R}_{q_1, q_2}^*$  denote the optimal profit of the firm.

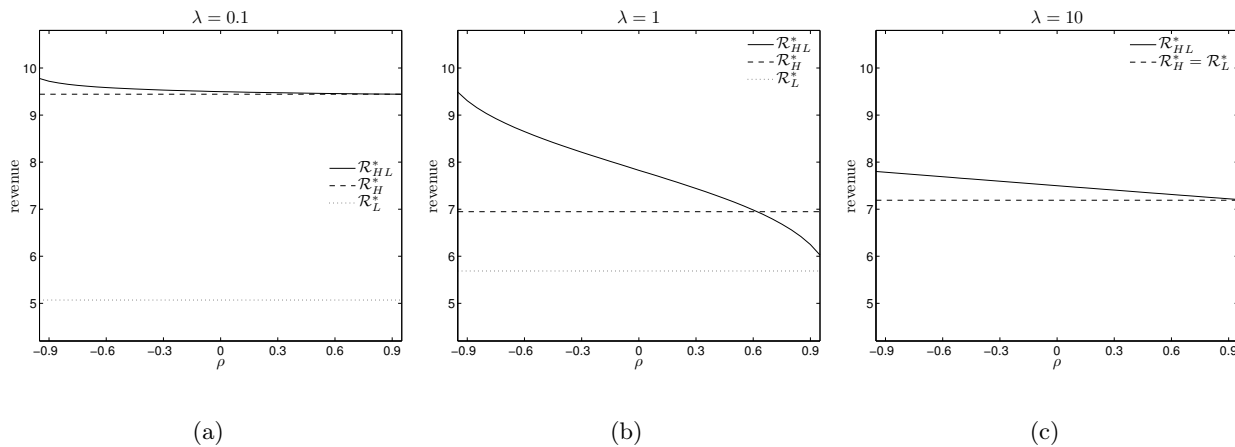
We first explore the case when the true quality of the two products are the same. Figure 7 depicts the firm’s total revenue when two products are high-quality for the base scenario (with  $\rho = 0$ ). The case when both products are low-quality produces essentially same results and is omitted.



**Figure 7** Impact of correlation ( $\rho$ ) on revenue when  $q_1 = q_2 = q_H$

There are several interesting observations. First, the firm is always better-off by offering two products with the same quality as opposed to one, provided that customers assign some positive probability that their qualities could be different (i.e.,  $\rho \neq 1$ ). In this case, the customer has to spend additional effort to distinguish relative quality levels (beyond the efforts to determine absolute levels of quality), and the firm can take advantage of limited attention and command higher prices. As expected the gains are higher for moderate levels of information costs  $\lambda$ , where the customer processes most information. Furthermore, the gains improve as  $\rho$  decrease since when customers increasingly believe that product qualities differ, they face higher “quality risks” and have to spend more effort to learn and distinguish product qualities. Putting it differently, as  $\rho$  increases, customer prior beliefs are increasingly more aligned with the actual quality levels, and therefore the incremental gain from maintaining both products in the assortment decreases for the firm. The limiting case deserves mentioning as well. When  $\rho = 1$ , customers treat the two products identical in *quality*. The firm could well price the products differently, create asymmetry in the net valuations, and thereby attempt to take advantage of limited attention. It turns out that this delusion tactic is suboptimal for the firm. The firm chooses to price

the two products the same, and as a result, they become duplicates in the eyes of the customer.<sup>9</sup> Consequently, there is no added value of having both products in the assortment.



**Figure 8** Impact of correlation ( $\rho$ ) on revenue when  $q_1 = q_H$  and  $q_2 = q_L$

Figure 8 illustrates the case when the two product qualities actually differ. We see that for low and high information costs, the key insights remain the same. When the information costs are moderate however, there could be interesting outcomes. Observe that when customer beliefs on products being the same quality gets stronger ( $\rho$  increases), it becomes harder for the firm to reflect the quality differences into the prices and hence profits decrease. The effects can be detrimental to the firm when  $\rho$  is sufficiently high. In this case, beliefs are highly misaligned with the actual quality levels, and hence significant effort is required to glean true quality. The firm is pressurized to price the high-quality product similar to the low-quality one so much that it would actually prefer to only sell the high-quality product. Clearly, this exemplifies the possibility that adding a new product to an assortment can actually reduce the profits; i.e., “less” can indeed be “more” when customers are rationally inattentive.

## 7.2. Competing Firms

The optimal pricing strategy of a firm facing rationally inattentive customers can be significantly different when there are other firms with competing products. In order to demonstrate the effects of competition, we consider two firms (FIRM 1 and FIRM 2) each selling a single product (using the same setup for prior beliefs in §7.1). One expected effect of competition is the reduction in prices

<sup>9</sup> We remark that when prices signal quality, there would even be less incentive for deceitful pricing.

and revenues. We find that this is indeed true and that the impact is stronger when the competing products have the same (or similar) quality or customer beliefs are such. For crisper insights and in the interest of brevity, we concentrate on the case when actual product qualities are different. The impact of information cost  $\lambda$ , correlation factor  $\rho$ , and quality level  $q_L$  are depicted in Figure 9 respectively. Here, each FIRM  $j$ ,  $j \in \{1, 2\}$ , optimizes its own revenue  $p_j \pi_{q_1, q_2}^j(p_1, p_2)$ , where  $\pi_{q_1, q_2}^j(\mathbf{p})$  is given by (28). Let  $\mathcal{R}_i^j$  denote the Nash-equilibrium revenue of FIRM  $j$  selling product with quality  $q_i$ .

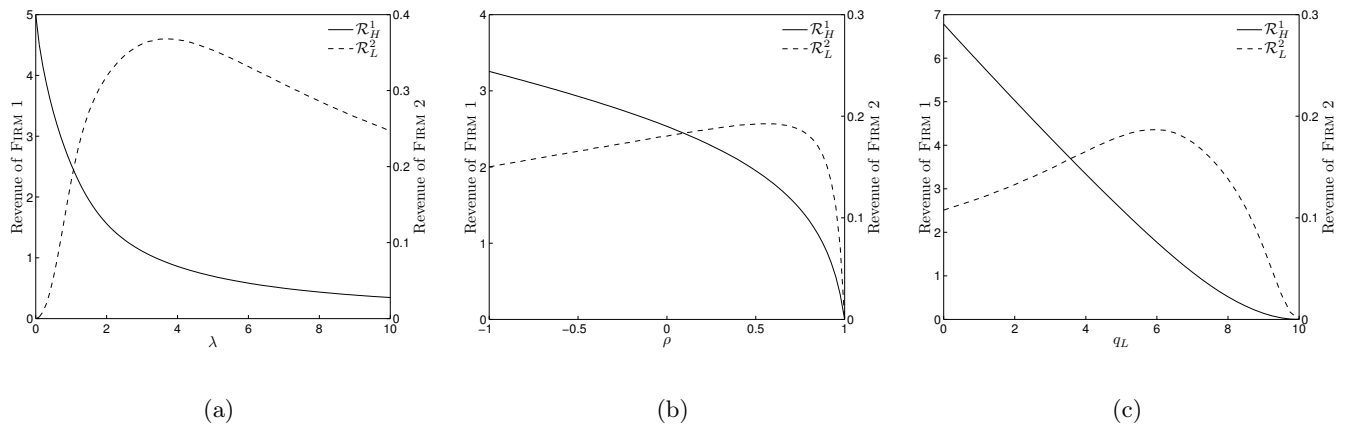


Figure 9 Revenues for two competing firms (Firm 1 with  $q_H$  and Firm 2 with  $q_L$ )

Observe from Figure 9(a) that just as in our single-firm baseline model, revenues do not behave monotonically with respect to information cost. The precise behaviour however is starkly different. When  $\lambda = 0$ , only FIRM 1 sells its high-quality product at price  $q_H - q_L = 5$ , as a consequence of the resulting Bertrand's duopoly game. As  $\lambda$  increases, FIRM 2 selling the low-quality product can charge higher prices due to customers' inattention and starts gaining market share, and increasing profits. In contrast, FIRM 1 is forced to reduce its price and faces losses. This continues until the cost of information starts to get prohibitively expensive and customers increasingly base their decisions on their prior beliefs, hence the profits of FIRM 2 also start declining. This suggests that, contrary to the single-firm case, it is in the interest of the firms to make quality-revealing information more accessible to the customers for highly experiential or credence goods.

The effect of costly information is accentuated in Figure 9(b). When customers believe that products are of similar quality, costly information acquisition intensifies the price competition between the two

firms, and beats down revenues. Firm revenues improve when customers believe that qualities are different, and the effects are much more pronounced for the high-quality firm (the low-quality firm might even face a slight decrease). These results highlight again the critical role prior beliefs play when customers have limited attention.

Finally, notice that as  $q_L$  increases, the competition between the two products intensifies. This naturally reduces the prices and profits of the firm selling the high-quality product. For the firm selling the low-quality product the impact is more convoluted. As seen in Figure 9(c), an increase in  $q_L$  initially benefits the firm since it makes this a better alternative to the no-purchase option. However, as  $q_L$  further increases, competition intensifies with the high-quality product, and the profit starts declining. Hence there is only a limited benefit of increasing the quality level of the product. Notice that these results contrast sharply with the baseline model with a single firm, where an increase in the low-quality level always improves the profitability of firm (refer to Figures 2(c) and 3(c)). The practical implication is that false advertising or any other initiative that is aimed to improve the quality perception of a product may not be profitable for a firm in a competitive market. Instead, the firm might be better off by pitching to the customers that it is offering a different, *superior* product.

### 7.3. Joint Pricing and Ordering

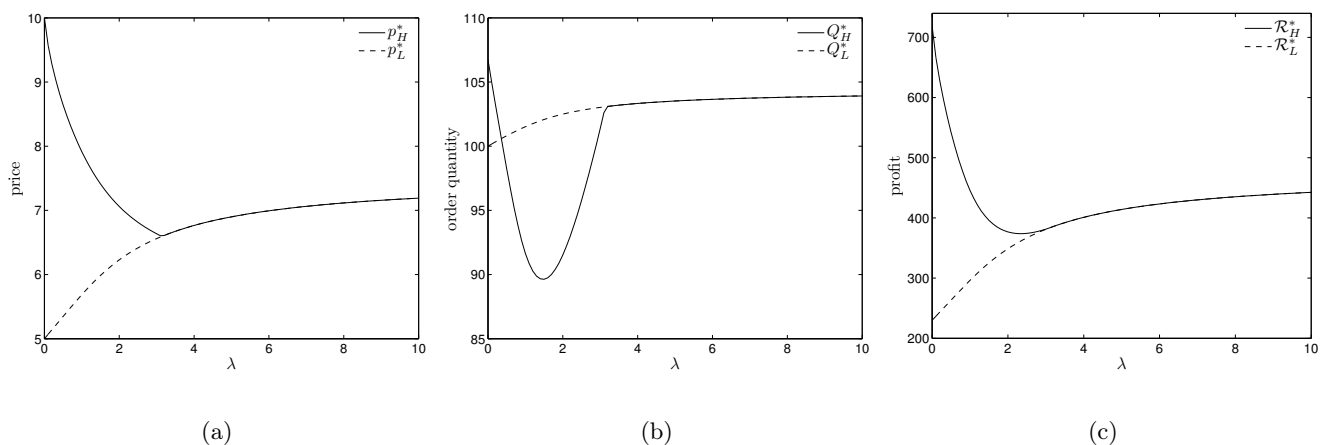
As a final extension, we study the impact of limited attention on inventory decisions of the firm. To this end, consider the case in which the firm sells a finite initial inventory of the product with quality  $q_i$ ,  $i \in \{H, L\}$ , over a selling season. Accordingly, the firm decides on the optimal order quantity  $Q_i^*$  at the start of the selling season, in addition to setting the optimal product price  $p_i^*$ . Let  $c_i$  be the unit purchasing cost of the product, and without loss of generality, assume zero salvage value and zero holding cost. Further, suppose that customers arrive according to a Poisson process with rate  $\mu$ , and demand for the product is realized from customers arriving to the firm during the finite selling period. For any given price  $p$ , we approximate the random demand with a normal distribution with mean  $\mu\pi_i(p)$  and standard deviation  $\sqrt{\mu\pi_i(p)}$ . Note that both the mean and the standard deviation of demand are functions of the price. This is essentially a newsvendor model with normal demand, and for a given price  $p$ , the optimal order quantity and resulting profit can be written as

$$Q_i(p) = \mu\pi_i(p) + \Phi^{-1}(1 - c_i/p)\sqrt{\mu\pi_i(p)} \quad (29)$$

$$\mathcal{R}_i(p) = \mu\pi_i(p)(p - c_i) - \phi\left(\Phi^{-1}(1 - c_i/p)\right) \sqrt{\mu\pi_i(p)} \quad (30)$$

where  $\phi(\cdot)$  and  $\Phi(\cdot)$  are the density and cumulative distribution functions for the standard normal, respectively. The optimal price  $p_i^*$  that maximizes  $\mathcal{R}_i(p)$  can be obtained using numerical search (Madhah and Tarhini 2014). Based on  $p_i^*$ , we define  $Q_i^* = Q_i(p_i^*)$  and  $\mathcal{R}_i^* = \mathcal{R}(p_i^*)$  for  $i \in \{H, L\}$ .

Figure 10 illustrates the impact of information cost on the optimal price, order quantity and profit in this particular price-setting newsvendor context. In our computations, we use the base scenario and assume  $\mu = 100$  and  $c_i = 2.5$ , for  $i \in \{H, L\}$ . As evident from Figures 10(a) and 10(c), the behavior of the optimal prices and profits are identical to those under the baseline model. Hence, inclusion of purchasing costs and inventory related decisions and costs do not alter the main results.



**Figure 10** Impact of unit information cost ( $\lambda$ ) on product price, order quantity and profit

Next, we elaborate on the inventory ordering decisions in Figure 10(b). Note that the two key components of the order quantity in (29) are the *demand* and the *critical fractile*. When product quality is low, the critical fractile,  $1 - \frac{c_L}{p_L^*}$ , increases with  $\lambda$ , as  $p_L^*$  is an increasing function of  $\lambda$  (see Figure 10(a)). Since  $\pi(p_L^*) = 1$  for all  $\lambda$ ,  $Q_L^*$  is solely driven by the critical fractile, and hence increases in  $\lambda$ . On the other hand, when product quality is high,  $p_H^*$  and subsequently the critical fractile  $1 - \frac{c_H}{p_H^*}$  are both non-monotone in  $\lambda$  – decreasing until  $\lambda_c (= 3.14)$  and increasing afterwards. When  $\lambda \leq \lambda_c$ ,  $\pi_H(p_H^*)$  is also non-monotone in  $\lambda$ , which is a result of the interplay between the effects of information cost and product price on  $\pi_H(p_H^*)$ . When  $\lambda = 0$ ,  $\pi_H(p_H^*) = 1$  because customers have perfect information about

the product. As  $\lambda$  increases, customers start inquiring less about the product, and they no longer have perfect information, which drives  $\pi_H(p_H^*)$  down, i.e.,  $\pi_H(p_H^*) < 1$ . As  $\lambda$  increases further, customers rely more on their priors,  $p_H^*$  decreases and the net value from purchasing increases, which eventually drive  $\pi_H(p_H^*)$  up to 1 as  $\lambda = \lambda_c$ . The non-monotonic behavior of  $Q_H^*$  on  $\lambda \leq \lambda_c$  in Figure 10(b) is the net outcome of the decreasing critical fractile and the non-monotone demand. When  $\lambda > \lambda_c$ , we have  $Q_H^* = Q_L^*$ , and hence  $Q_H^*$  is increasing in  $\lambda$ .

Next, we formalize the impact of limited attention on the firm's optimal order quantity.

**Proposition 6** *The firm orders less of the high-quality product and more of the low-quality product when facing customers with limited attention compared to those cases with perfectly attentive customers.*

From Proposition 1, we know that when customers are rationally inattentive, the firm will *not* charge “perfectly attentive prices” (i.e. true quality levels) – overprices a low-quality product, and underprices a high-quality product. In turn, the underage risk – the risk associated with unsatisfied demand – increases for the low-quality product and decreases for the high-quality product, compared to the case with attentive customers. For a low-quality firm, the market is served fully in either case. In contrast, when product quality is high, the demand of inattentive customers is less than (or equal to) that of attentive customers albeit the price break given to inattentive customers. Accordingly, to maximize profit, the firm orders more of the low-quality product and less of the high-quality product when information processing is costly.

## 8. Discussion and Concluding Remarks

In this paper, we develop a framework to analyze pricing decisions in the presence of customers with limited attention. We consider a scenario where customers are ex-ante unable to assess the exact “quality” aspects of the products, but can observe the prices. In accordance with the theory of rational inattention, the customers in our model can attend to information regarding the quality of the products at some cost, and make the attention allocation decisions optimally. The firms, on the other hand, have more experience with the product, and know the exact quality of the products. Anticipating rationally inattentive customer behaviour (which is shown in the literature to generate conditional



purchase probabilities that follow a generalized multinomial logit form) and the resulting demand, the firms select prices to maximize revenue.

Our model provides a realistic representation of limited customer attention and fits most non-functional product (goods or services) markets where customers do not purchase the product at very high frequencies, and hence pay attention to the posted prices of the distinct choices. It is also general in that it allows signalling of quality via prices and customer heterogeneity, offers unifying interpretations across multiple product types (search, experience and credence goods) via its stylized costly information acquisition and processing aspect, and has connections to the quantal choice models of bounded rationality as well as costly information search models.

Through analytical characterizations and complementary numerical experiments, we provide a full account of optimal prices and firm revenues in the presence of customers with limited attention, and show how they are shaped by (i) ease of obtaining information, (ii) prior beliefs about product qualities, and (iii) relative quality levels of the products, in different settings including multiple products, competition, among others. In order to avoid repetition, instead of providing a summary of these results, we discuss here the prescriptive results and managerial insights generated therefrom, together with some practical examples. We can group these insights in two categories: *information provision* (how easy/difficult should the firm make the access of customers to quality revealing information) and *product positioning* (how should the firm try to shape the quality perceptions of the customers).<sup>10</sup>

We show that when the cost of information is relatively low (search goods such as sports equipment, cameras, LED TVs), it is always in the interest of a firm selling a low-quality product to make it difficult for customers to gather information about its product. We see examples of this in practice too. For example, many knock-off smartphone and tablet producers do not even maintain a proper website to inform customers about their (cheap) imitation products. A firm with a high quality product, on the other hand, should do just the opposite. This can be done by proactive targeted advertising, salesforce,

<sup>10</sup> Needless to say, these actions might bear some costs on behalf of the firm, which are not considered here. In this regard, we are only talking about the benefits to the firm, which should be traded-off against potential costs (if any).

providing demo/trial models etc. Samsung for example employs own sales associates within Best Buy stores to inform customers about its high-end TVs.

When the cost of information is sufficiently high (credence or highly experiential good such as medical services, wine), the information provision preferences depend on market competition. Specifically, if the firm is enjoying a rather dominating position in the market, then regardless of the true quality of its product, it should make it more difficult for the customers to assess it. In sharp contrast, when there are distinct competing products in the market, it should do the opposite even if it is selling the lower quality good. The benefit of increasing the customer's ability to distinguish the products is the softening of price competition, which is already amplified due to difficulties in learning the true qualities of the products. Hence, while a restaurant offering only one choice of wine should deliberately not reveal much information about it (and ambiguously refer to it as the "house wine"), it is in the interest of a winery to offer tours (as they often do), enabling the customers to sample the lower quality wines along with high-quality ones. We also remark that as the cost of information increases, the optimal price of high- and low-quality products converge in monopolistic as well as competitive markets (in the latter case prices are further reduced). When it is difficult for customers to attend to quality revealing information, they start to ignore it by making choice decisions mostly based on prior beliefs. Knowing this, firms with high- and low-quality products charge the same price. This result provides alternative support as to why in unregulated markets prices of credence goods should converge. It is also robust as it is likely to hold even when prices signal quality to customers.

Our model and results offer insights as to how firms should try to influence quality perceptions and position its products accordingly. There are two fundamental ways the firm can do this in our framework: It can try to improve the perceived quality level of its product or it can try to improve the belief that it is offering the superior ("higher" quality) product. We show that the latter strategy is a safe one since it improves firm profits even for a firm with a low-quality product. It is effective even when the quality levels of the products are similar. Take for example, Samsung's Galaxy Note and Apple's iPhone. If the entire customer base were to believe that these products are identical, we know extreme price competition would reduce revenues of the firm significantly. The existence of the

so-called “fans” (Apple’s *Fanboys* and Samsung’s *Fandroids*), who believe one product to be better than the other one, softens the price competition in this fiercely competitive market and allows them to charge high prices and retain revenues. It is therefore no surprise that both firms regularly take highly publicized jabs at each other to promote their superiority (e.g., *Apple has a green dig at Samsung in new ad* (cnet.com 2014), *Samsung uses Steve Jobs quote against Apple: “No one is going to buy a big phone”* (idownloadblog.com 2014)). Improving the perceived quality of the product can also be effective, if it is increasing the relative quality distinction. Hence, there are risks involved with this strategy, in particular for the firm with lower quality product. We show that if the cost of information is high, when facing competition from a higher end product, it is better for the low-quality firm to reduce its perceived quality level. This would improve the profitability of both firms. A recent example of a company which has gone through this experience is Mulberry, a producer of (luxury) ladies leather handbags. After failing to push its brand upmarket with increased prices, it repositioned itself on the lower luxury market by promoting “affordable luxury” (Financial Times 2014).

We find that market heterogeneity can have a compounding effect in both information provision and product positioning strategies. Since a firm selling a high-quality product suffers from the inattention of customers, it should proactively try to enhance awareness of its high-quality (increase the size of more informed customers), by allowing pre-launch screenings (as in movies), advertising, social marketing, or through other means. Naturally, the reverse is true for a firm selling a low-quality product. Heterogeneity in customer’s prior belief about quality, however, is detrimental to both low- and high-quality firms. Therefore, achieving a consistent brand quality perception should be a universal goal when customers have limited attention.

## References

- Akcay, Y., H. P. Natarajan, S. H. Xu. 2010. Joint dynamic pricing of multiple perishable products under consumer choice. *Management Science* **56**(8) 1345–1361.
- Akerlof, G. A. 1970. The market for "lemons": Quality uncertainty and the market mechanism. *The quarterly journal of economics* 488–500.

- Aksoy-Pierson, M., G. Allon, A. Federgruen. 2013. Price competition under mixed multinomial logit demand functions. *Management Science* **59**(8) 1817–1835.
- Banks, J. S., J. Sobel. 1987. Equilibrium selection in signaling games. *Econometrica* **55**(3) pp. 647–661.
- Bester, H. 1998. Quality uncertainty mitigates product differentiation. *The RAND Journal of Economics* 828–844.
- Bester, H., K. Ritzberger. 2001. Strategic pricing, signalling, and costly information acquisition. *International Journal of Industrial Organization* **19**(9) 1347–1361.
- Branco, F., M. Sun, J. M. Villas-Boas. 2012. Optimal search for product information. *Management Science* **58**(11) 2037–2056.
- Caplin, A., D. Martin. 2014. A testable theory of imperfect perception. *The Economic Journal* **125**(February) 184–202.
- Chen, Y., X. Su, Xi. Zhao. 2012. Modeling bounded rationality in capacity allocation games with the quantal response equilibrium. *Management Science* **58**(10) 1952–1962.
- Cho, I., D. Kreps. 1987. Signaling games and stable equilibria. *The Quarterly Journal of Economics* **102**(2) 179–221.
- cnet.com. 2014. Apple has a green dig at samsung in new ad. <http://www.cnet.com/news/apple-has-another-dig-at-samsung-in-green-ad/>. Accessed: 2014-11-20.
- Darby, M. R., E. Karni. 1973. Free competition and the optimal amount of fraud. *Journal of Law & Economics* **16** 67–88.
- Debo, L.G., C. Parlour, U. Rajan. 2012. Signaling quality via queues. *Management Science* **58**(5) 876–891.
- Debo, L.G., S. Veeraraghavan. 2014. Equilibrium in queues under unknown service times and service value. *Operations Research* **62**(1) 38–57.
- Debreu, G. 1960. Review of individual choice behavior by r.d. luce. *The American Economic Review* **50**(1) pp. 186–188.
- Dulleck, U., R. Kerschbamer. 2006. On doctors, mechanics, and computer specialists: The economics of credence goods. *Journal of Economic Literature* **44**(1) 5–42.

- Economist. 2006. Economics focus: Sawbones, cowboys and cheats. *The Economist* **April 15, 2006**.
- Erdem, T., M. P. Keane, B. Sun. 2008. A dynamic model of brand choice when price and advertising signal product quality. *Marketing Science* **27(6)** 1111–1125.
- Financial Times. 2014. Mulberry aims for “affordable luxury” after failed move upmarket. *Financial Times* **April 17, 2014**.
- Hanson, W., K. Martin. 1996. Optimizing multinomial logit profit functions. *Management Science* **42(7)** 992–1003.
- idownloadblog.com. 2014. Samsung uses steve jobs quote against apple: No one is going to buy a big phone. <http://www.idownloadblog.com/2014/09/10/samsung-jobs-quote-big-phone/>. Accessed: 2011-11-20.
- Li, H., W. T. Huh. 2011. Pricing multiple products with the multinomial logit and nested logit models: concavity and implications. *Manufacturing & Service Operations Management* **13(4)** 549–563.
- Luce, R.D. 1959. *Individual Choice Behavior: A Theoretical Analysis*. Wiley, New York.
- Mackowiak, B., M. Wiederholt. 2009. Optimal sticky prices under rational inattention. *American Economic Review* **99(3)** 769–803.
- Maddah, E. K., B. and Bish, H. Tarhini. 2014. Newsvendor pricing and assortment under poisson decomposition. *IIE Transactions* **46(6)** 567–584.
- Manzini, P., M. Mariotti. 2014. Stochastic choice and consideration sets. *Econometrica* **82(3)** 1153–1176.
- Matejka, F. 2010a. Rationally inattentive seller: Sales and discrete pricing. Tech. Rep. wp408, The Center for Economic Research and Graduate Education - Economic Institute, Prague.
- Matejka, F. 2010b. Rigid pricing and rationally inattentive consumer. Tech. Rep. wp409, The Center for Economic Research and Graduate Education - Economic Institute, Prague.
- Matejka, F., A. McKay. 2012. Simple market equilibria with rationally inattentive consumers. *American Economic Review (Papers and Proceedings)* **102(3)** 24–29.
- Matejka, F., A. McKay. 2015. Rational inattention to discrete choices: A new foundation for the multinomial logit model. *American Economic Review* **105(1)** 272–298.
- McKelvey, R. D., T. R. Palfrey. 1995. Quantal response equilibria for normal form games. *Games and economic behavior* **10(1)** 6–38.

- McKowiak, B., M. Wiederholt. 2015. Business cycle dynamics under rational inattention. *Review of Economic Studies*. Forthcoming.
- Munoz-Garcia, F., A. Espinola-Arredondo. 2011. The intuitive and divinity criterion: Interpretation and step-by-step examples. *Journal of Industrial Organization Education* **5**(1) 1–20.
- Nelson, P. 1970. Information and consumer behavior. *Journal of Political Economy* **78**(2) 311–29.
- Özer, Ö., Y. Zheng. 2012. Behavioral issues in pricing management. Ö. Özer, R. Phillips, eds., *The Oxford Handbook of Pricing Management*. Oxford University Press, Oxford, 415–465.
- Roberts, J.H., J. M. Lattin. 1991. Development and testing of a model of consideration set composition. *Journal of Marketing Research* 429–440.
- Sahin, O., R. Wang. 2014. The impact of consumer search cost on assortment planning and pricing. *Working Paper*. Available at SSRN 2503490.
- Shannon, C. E. 1948. The mathematics theory of communication. *Bell Syst. Tech. J* **27** 379–423.
- Simon, H. A. 1955. A behavioral model of rational choice. *The Quarterly Journal of Economics* **69**(1) 99–118.
- Sims, C. A. 1998. Stickiness. *Carnegie-Rochester Conference Series on Public Policy* **49**(1) 317–356.
- Sims, C. A. 2003. Implications of rational inattention. *Journal of Monetary Economics* **50**(3) 665–690.
- Sims, C. A. 2006. Rational inattention: Beyond the linear-quadratic case. *The American Economic Review* **96**(2) 158–163.
- Song, J. S., Z. Xue. 2007. Demand management and inventory control for substitutable products. *Working paper* .
- Stigler, G. J. 1961. The economics of information. *Journal of Political Economy* **69**(3) 213–225.
- Stiving, M. 2000. Price-endings when prices signal quality. *Management Science* **46**(12) 1617–1629.
- Su, X. 2008. Bounded rationality in newsvendor models. *Manufacturing & Service Operations Management* **10**(4) 566–589.
- Tirole, J. 1988. *The theory of industrial organization*. MIT press.
- Tutino, A. 2011. 'Rational Inattention' Guides Overloaded Brains, Helps Economists Understand Market Behavior. *Economic Letter* **6**(mar) 1–4.

- Tutino, A. 2013. Rationally inattentive consumption choices. *Review of Economic Dynamics* **16**(3) 421–439.
- Weitzman, M. L. 1979. Optimal search for the best alternative. *Econometrica: Journal of the Econometric Society* 641–654.
- Wiederholt, M. 2010. Rational inattention. Steven N. Durlauf, Lawrence E. Blume, eds., *The New Palgrave Dictionary of Economics*. Palgrave Macmillan, Basingstoke.
- Woodford, M. 2009. Information-constrained state-dependent pricing. *Journal of Monetary Economics* **56**(S) 100–124.

## Appendix. Technical Proofs

**Lemma 1** (i)  $\bar{p} \geq \alpha q_H + (1 - \alpha)q_L$ , and (ii)  $\underline{p} \leq \alpha q_H + (1 - \alpha)q_L$ .

**Proof.** We can re-write the inequality in (i)  $\ln [\alpha e^{q_H/\lambda} + (1 - \alpha)e^{q_L/\lambda}] \geq \alpha(q_H/\lambda) + (1 - \alpha)(q_L/\lambda)$  substituting for  $\bar{p}$ . Taking exponents of both sides, we obtain  $\alpha e^{q_H/\lambda} + (1 - \alpha)e^{q_L/\lambda} \geq e^{\alpha(q_H/\lambda) + (1 - \alpha)(q_L/\lambda)}$ , which is true due to the convexity of the function  $f(x) = e^x$ . Similarly, we can simplify the inequality in (ii) as  $\ln [(1 - \alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}] \geq (1 - \alpha)(q_H/\lambda) + \alpha(q_L/\lambda)$  after substituting for  $\underline{p}$ . This inequality holds true based on our argument for part (i).  $\square$

**Lemma 2**  $\coth x \geq \frac{1}{x}$  for  $x \geq 0$ .

**Proof.** We re-write the relation in the lemma as  $x \geq \frac{1}{\coth x}$ . When  $x = 0$ , the inequality is satisfied as the LHS and RHS are equal to zero. When  $x > 0$ , both  $x$  and  $\frac{1}{\coth x}$  are increasing in  $x$  ( $\frac{\partial}{\partial x} \frac{1}{\coth x} = (\operatorname{sech} x)^2 \geq 0$ ). Further, note that the rate of increase of  $x$  is larger than that of  $\frac{1}{\coth x}$ , i.e.,  $(\operatorname{sech} x)^2 \leq 1$  for all  $x \geq 0$ . Hence, the inequality given in the lemma is true.  $\square$

**Proof of Proposition 1.** First, note that  $\dot{\pi}_0(p)$  is non-increasing in  $p$  since

$$\frac{\partial \dot{\pi}_0(p)}{\partial p} = - \frac{\alpha \left[ \operatorname{csch} \left( \frac{p - q_L}{2\lambda} \right) \right]^2 + (1 - \alpha) \left[ \operatorname{csch} \left( \frac{p - q_H}{2\lambda} \right) \right]^2}{4\lambda} \leq 0.$$

When  $p \leq \underline{p}$ , we have  $\dot{\pi}_0(p) \geq 1$ . Subsequently  $\pi_0(p) = 1$  and  $\pi_L(p) = \pi_H(p) = 1$ . Since  $\mathcal{R}_i(p) = p \times \pi_i(p)$  decreases in  $p$  in this interval, we have  $p_i^* \geq \underline{p}$ . Moreover,  $\underline{p}$  is increasing in  $\alpha$  and hence approaches  $q_L$  as  $\alpha \downarrow 0$ , i.e.,  $\underline{p} > q_L$ . On the other hand, when  $p \geq \bar{p}$ , we have  $\dot{\pi}_0(p) \leq 0$ . Hence  $\pi_0(p) = 0$  and also  $\pi_L(p) = \pi_H(p) = 0$ . Since  $\mathcal{R}_i(p) = 0$  in this interval,  $p_i^* \leq \bar{p}$ . Furthermore,  $\bar{p}$  is increasing in  $\alpha$  and approaches  $q_H$  as  $\alpha \uparrow 1$ , i.e.,  $\bar{p} < q_H$ .  $\square$

**Proof of Theorem 1.** (i) Letting  $\xi(\pi_i) = \frac{\pi_0}{1 - \pi_0} \times \frac{1 - \pi_i}{\pi_i}$ , we re-write the firm's revenue function as  $\mathcal{R}_i(\pi_i) = q\pi_i + \lambda\pi_i \cdot \ln [\xi(\pi_i)]$ . Taking derivatives, we obtain  $\frac{\partial^2 \mathcal{R}_i(\pi_i)}{\partial \pi_i^2}$  as

$$\frac{\partial^2 \mathcal{R}_i(\pi_i)}{\partial \pi_i^2} = \frac{2\lambda \xi^{(1)}(\pi_i)}{\xi(\pi_i)} + \lambda\pi_i \times \left\{ - \left( \frac{\xi^{(1)}(\pi_i)}{\xi(\pi_i)} \right)^2 + \frac{\xi^{(2)}(\pi_i)}{\xi(\pi_i)} \right\} \quad (31)$$

where  $\xi^{(1)}(\cdot)$  and  $\xi^{(2)}(\cdot)$  denote the first and second derivatives of  $\xi(\pi_i)$  with respect to  $\pi_i$ , respectively. From (31),  $\mathcal{R}_i(\pi_i)$  is concave in  $\pi_i$  if: (1)  $\xi(\pi_i)$  is non-negative, i.e.,  $\xi(\pi_i) \geq 0$ , (2)  $\xi(\pi_i)$  is non-increasing in  $\pi_i$ , i.e.,  $\xi^{(1)}(\pi_i) \leq 0$ , and (3)  $\xi(\pi_i)$  is concave in  $\pi_i$ , i.e.,  $\xi^{(2)}(\pi_i) \leq 0$ .

Clearly,  $\xi(\pi_i)$  is always non-negative. We next verify that  $\xi^{(1)}(\pi_i) \leq 0$  and  $\xi^{(2)}(\pi_i) \leq 0$  for  $i \in \{H, L\}$ . Note that the conditional purchase probabilities  $\pi_H$  and  $\pi_L$ , using (8), are given by

$$\pi_H = \frac{e^{(q_H-p)/\lambda}}{e^{(q_H-p)/\lambda} + \frac{(1-\pi_0)}{\pi_0}} \quad \text{and} \quad \pi_L = \frac{e^{(q_L-p)/\lambda}}{e^{(q_L-p)/\lambda} + \frac{(1-\pi_0)}{\pi_0}}$$

respectively. Then, we express  $\pi_L$  in terms of  $\pi_H$  as  $\pi_L = \frac{\pi_H e^{q_L/\lambda}}{e^{q_H/\lambda} - \pi_H(e^{q_H/\lambda} - e^{q_L/\lambda})}$ , plug into (9) to obtain

$$\pi_0 = \alpha\pi_H + \frac{(1-\alpha)\pi_H e^{q_L/\lambda}}{e^{q_H/\lambda} - \pi_H(e^{q_H/\lambda} - e^{q_L/\lambda})}$$

and consequently  $\xi(\pi_H)$  as follows

$$\xi(\pi_H) = \frac{\alpha(1-\pi_H)e^{q_H/\lambda} + (1-\alpha(1-\pi_H))e^{q_L/\lambda}}{(1-\alpha\pi_H)e^{q_H/\lambda} + \alpha\pi_H e^{q_L/\lambda}}$$

As a result,  $\xi^{(1)}(\pi_H)$  and  $\xi^{(2)}(\pi_H)$  are calculated as

$$\xi^{(1)}(\pi_H) = -\frac{\alpha(1-\alpha)(e^{q_H/\lambda} - e^{q_L/\lambda})^2}{((1-\alpha\pi_H)e^{q_H/\lambda} + \alpha\pi_H e^{q_L/\lambda})^2} \quad \text{and} \quad \xi^{(2)}(\pi_H) = -\frac{2\alpha^2(1-\alpha)(e^{q_H/\lambda} - e^{q_L/\lambda})^3}{((1-\alpha\pi_H)e^{q_H/\lambda} + \alpha\pi_H e^{q_L/\lambda})^3}$$

respectively. We observe that  $\xi^{(1)}(\pi_H) \leq 0$  and  $\xi^{(2)}(\pi_H) \leq 0$ , since  $0 < \alpha < 1$ ,  $0 \leq \pi_H \leq 1$  and  $q_H > q_L$ .

Similarly, we express  $\pi_H$  in terms of  $\pi_L$  as  $\pi_H = \frac{\pi_L e^{q_H/\lambda}}{e^{q_L/\lambda} + \pi_L(e^{q_H/\lambda} - e^{q_L/\lambda})}$ , plug into (9) to get

$$\pi_0 = \frac{\alpha\pi_L e^{q_H/\lambda}}{e^{q_L/\lambda} + \pi_L(e^{q_H/\lambda} - e^{q_L/\lambda})} + (1-\alpha)\pi_L$$

Then  $\xi(\pi_L)$  is calculated as

$$\xi(\pi_L) = \frac{(\pi_L + \alpha(1-\pi_L))e^{q_H/\lambda} + (1-\alpha)(1-\pi_L)e^{q_L/\lambda}}{(1-\alpha)\pi_L e^{q_H/\lambda} + (1-(1-\alpha)\pi_L)e^{q_L/\lambda}}$$

Accordingly,  $\xi^{(1)}(\pi_L)$  and  $\xi^{(2)}(\pi_L)$  follow as

$$\xi^{(1)}(\pi_L) = -\frac{\alpha(1-\alpha)(e^{q_H/\lambda} - e^{q_L/\lambda})^2}{((1-\alpha)\pi_L e^{q_H/\lambda} + (1-(1-\alpha)\pi_L)e^{q_L/\lambda})^2} \quad \text{and} \quad \xi^{(2)}(\pi_L) = -\frac{2\alpha(1-\alpha)^2(e^{q_H/\lambda} - e^{q_L/\lambda})^3}{((1-\alpha)\pi_L e^{q_H/\lambda} + (1-(1-\alpha)\pi_L)e^{q_L/\lambda})^3}$$

respectively. Once again, we find that  $\xi^{(1)}(\pi_L) \leq 0$  and  $\xi^{(2)}(\pi_L) \leq 0$ , as  $0 < \alpha < 1$ ,  $0 \leq \pi_L \leq 1$  and  $q_H > q_L$ .

As a result, we conclude that  $\xi^{(1)}(\pi_i) \leq 0$  and  $\xi^{(2)}(\pi_i) \leq 0$  for  $i \in \{H, L\}$ . Hence,  $\mathcal{R}_i(\pi_i)$  is concave in  $\pi_i$ .

(ii) First, suppose that product quality is high. Since  $\pi_H(p)$  is given by

$$\pi_H(p) = \frac{\pi_0(p)e^{(q_H-p)/\lambda}}{\pi_0(p)e^{(q_H-p)/\lambda} + (1-\pi_0(p))},$$

using (14), we can express  $\pi_H(p)$  in closed-form for  $\underline{p} \leq p \leq \bar{p}$  as follows:

$$\pi_H(p) = \frac{e^{q_H/\lambda}}{\alpha} \left[ \frac{1}{e^{q_H/\lambda} - e^{q_L/\lambda}} - \frac{1-\alpha}{e^{q_H/\lambda} - e^{p/\lambda}} \right]$$

Therefore,  $\mathcal{R}_H(p) = p \times \pi_H(p) = \frac{pe^{q_H/\lambda}}{\alpha} \left[ \frac{1}{e^{q_H/\lambda} - e^{q_L/\lambda}} - \frac{1-\alpha}{e^{q_H/\lambda} - e^{p/\lambda}} \right]$ . The second derivative of  $\mathcal{R}_H(p)$  w.r.t.  $p$  is

$$\frac{\partial^2 \mathcal{R}_H(p)}{\partial p^2} = -\frac{(1-\alpha) \left[ \text{csch}\left(\frac{p-q_H}{2\lambda}\right) \right]^2}{4\lambda^2\alpha} \times \left[ 2\lambda - p \cdot \text{coth}\left(\frac{p-q_H}{2\lambda}\right) \right].$$



Since  $-\frac{(1-\alpha)\left[\operatorname{csch}\left(\frac{p-q_H}{2\lambda}\right)\right]^2}{4\lambda^2\alpha} \leq 0$  for all  $0 < \alpha < 1$ ,  $\lambda > 0$ , and  $\operatorname{coth}\left(\frac{p-q_H}{2\lambda}\right) \leq 0$  for  $p \leq q_H$ , we have  $\frac{\partial^2 \mathcal{R}_H(p)}{\partial p^2} \leq 0$ . As a result,  $\mathcal{R}_H(p)$  is concave in  $p$  over  $\underline{p} \leq p \leq \bar{p}$ .

Next, suppose that product quality is low. As we have

$$\pi_L(p) = \frac{\pi_0(p)e^{(q_L-p)/\lambda}}{\pi_0(p)e^{(q_L-p)/\lambda} + (1-\pi_0(p))},$$

we can express  $\pi_L(p)$  in closed-form for  $\underline{p} \leq p \leq \bar{p}$  as follows:

$$\pi_L(p) = \frac{e^{q_L/\lambda}}{1-\alpha} \left[ \frac{\alpha}{e^{p/\lambda} - e^{q_L/\lambda}} - \frac{1}{e^{q_H/\lambda} - e^{q_L/\lambda}} \right]$$

Hence,  $\mathcal{R}_L(p) = p \times \pi_L(p) = \frac{pe^{q_L/\lambda}}{1-\alpha} \left[ \frac{\alpha}{e^{p/\lambda} - e^{q_L/\lambda}} - \frac{1}{e^{q_H/\lambda} - e^{q_L/\lambda}} \right]$ . The first derivative of  $\mathcal{R}_L(p)$  w.r.t.  $p$  is

$$\frac{\partial \mathcal{R}_L(p)}{\partial p} = -\frac{e^{q_L/\lambda}}{1-\lambda} \left[ \frac{1}{e^{q_H/\lambda} - e^{q_L/\lambda}} + \frac{e^{p/\lambda}\alpha(p-\lambda) + e^{q_L/\lambda}\alpha\lambda}{(e^{p/\lambda} - e^{q_L/\lambda})^2\lambda} \right]$$

Clearly,  $e^{p/\lambda}\alpha(p-\lambda) + e^{q_L/\lambda}\alpha\lambda > 0$  if  $p \geq \lambda$ . Now assume  $p < \lambda$ . In order to have  $e^{p/\lambda}\alpha(p-\lambda) + e^{q_L/\lambda}\alpha\lambda \geq 0$ , we need show that  $\lambda e^{q_L/\lambda} \geq (\lambda-p)e^{p/\lambda}$  which implies  $\frac{\lambda}{\lambda-p} \geq e^{(p-q_L)/\lambda}$ . As  $p \downarrow q_L$  (note  $q_L < p < \lambda$ ), the latter inequality is satisfied (strictly). Also observe that LHS and RHS increase in  $p$ , but there exists no real  $p$  that solves the said inequality as an equality. Since both LHS and RHS are continuous, the inequality is always satisfied, hence  $e^{p/\lambda}\alpha(p-\lambda) + e^{q_L/\lambda}\alpha\lambda > 0$ . As a result, for all  $p \in [\underline{p}, \bar{p}]$ , we have  $\frac{\partial \mathcal{R}_L(p)}{\partial p} < 0$ , and  $\mathcal{R}_L(p)$  is decreasing in  $p$ . To check for convexity, we calculate second derivative of  $\mathcal{R}_L(p)$  w.r.t.  $p$  as

$$\frac{\partial^2 \mathcal{R}_L(p)}{\partial p^2} = -\frac{(1-\alpha)\left[\operatorname{csch}\left(\frac{p-q_L}{2\lambda}\right)\right]^2}{4\lambda^2\alpha} \times \left[ 2\lambda - p \cdot \operatorname{coth}\left(\frac{p-q_L}{2\lambda}\right) \right].$$

First note that  $-\frac{(1-\alpha)\left[\operatorname{csch}\left(\frac{p-q_L}{2\lambda}\right)\right]^2}{4\lambda^2\alpha} \leq 0$  for all  $0 < \alpha < 1$  and  $\lambda > 0$ . Moreover, based on Lemma 2, we have  $\operatorname{coth}\left(\frac{p-q_L}{2\lambda}\right) \geq \frac{2\lambda}{p-q_L}$ . If we re-write this inequality, we obtain  $2\lambda - (p-q_L)\operatorname{coth}\left(\frac{p-q_L}{2\lambda}\right) \leq 0$ . Hence, we also have  $2\lambda - p \operatorname{coth}\left(\frac{p-q_L}{2\lambda}\right) \leq 0$  for  $p \geq q_L$ . As a result,  $\frac{\partial^2 \mathcal{R}_L(p)}{\partial p^2} \geq 0$  and  $\mathcal{R}_L(p)$  is convex in  $p$ .  $\square$

**Proof of Corollary 1.** Theorem 1 and Proposition 1 together directly imply the result.

**Proof of Proposition 2.** We first show that  $p_L^*$  and  $\mathcal{R}_L^*$  are increasing in  $\lambda$ . We know that  $p_L^* = q_L + q_H - \lambda \ln[(1-\alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}]$ . Taking the derivative of  $p_L^*$  w.r.t.  $\lambda$  we check if

$$\frac{\partial p_L^*}{\partial \lambda} = \frac{(1-\alpha)q_H e^{q_H/\lambda} + \alpha q_L e^{q_L/\lambda}}{\lambda[(1-\alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}]} - \ln[(1-\alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}] \geq 0.$$

We re-write the above condition as follows:

$$(1-\alpha)\frac{q_H}{\lambda}e^{q_H/\lambda} + \alpha\frac{q_L}{\lambda}e^{q_L/\lambda} \geq [(1-\alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}] \ln[(1-\alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}]$$

Note that the RHS of the condition is of the form  $x \ln(x)$  which is increasing convex for  $x > 0$ . Hence, the above condition directly follows from the definition of convexity.

In order to prove the result regarding the high-quality product, first recall that as  $\lim_{\lambda \downarrow 0} \mathcal{R}_H^* = q_H > \alpha q_H + (1-\alpha)q_L = \lim_{\lambda \uparrow \infty} \mathcal{R}_H^*$ . To show that optimal profit is non-monotone in this region, we will establish the existence of a finite  $\lambda_c$  such that  $\mathcal{R}_H^*$  is increasing in  $\lambda$  on  $\lambda \geq \lambda_c$ . This will follow when we prove that on  $\lambda \geq \lambda_c$  for some finite  $\lambda_c$ , it is optimal for the firm serve the entire market ( $\pi_H^* = 1$ ), implying that the optimal price and revenue will be the same as the low-quality product (which is increasing in  $\lambda$  as proved above). We will utilize the concavity of  $\mathcal{R}(\pi_H)$  shown in Theorem 1 for this result. Specifically, we are going to investigate the first

derivative of  $\mathcal{R}(\pi_H)$  w.r.t.  $\pi_H$  as  $\pi_H \uparrow 1$  and show that it is increasing in  $\lambda$  and changes sign once (at  $\lambda_c$ ) from negative to positive. From concavity, this implies that  $\pi_H^* = 1$  for  $\lambda \geq \lambda_c$ .

Observe that the first of derivative of  $\mathcal{R}(\pi_H)$  w.r.t.  $\pi_H$  is given as

$$q_H + \lambda \ln[\xi(\pi_H)] + \lambda \pi_H \frac{\xi^{(1)}(\pi_H)}{\xi(\pi_H)},$$

which in the limit as  $\pi_H \uparrow 1$  becomes (after some manipulation)

$$q_H + \left\{ \lambda \ln \left[ \frac{e^{q_L/\lambda}}{(1-\alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}} \right] - \lambda \alpha (1-\alpha) \frac{(e^{q_H/\lambda} - e^{q_L/\lambda})^2}{e^{q_L/\lambda}((1-\alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda})} \right\}.$$

Let's denote the terms within the bracket  $\{\}$  as  $B$ . Since  $\frac{e^{q_L/\lambda}}{(1-\alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}} \leq 1$  and  $\frac{(e^{q_H/\lambda} - e^{q_L/\lambda})^2}{e^{q_L/\lambda}((1-\alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda})} \geq 0$ , we have  $B \leq 0$ . Moreover,  $B$  is also increasing in  $\lambda$ . To show this, we examine the first derivative of  $B$  with respect to  $\lambda$  which is given as

$$\begin{aligned} \frac{\partial B}{\partial \lambda} &= \underbrace{\frac{(1-\alpha)(q_H - q_L)e^{q_H/\lambda}}{\lambda((1-\alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda})} + \ln \left[ \frac{e^{q_L/\lambda}}{(1-\alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}} \right]}_C \\ &\quad + \underbrace{\frac{e^{-q_L/\lambda}(e^{q_H/\lambda} - e^{q_L/\lambda})(1-\alpha)\alpha}{\lambda((1-\alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda})^2}}_D \\ &\quad \times \underbrace{\left( (1-\alpha)(q_H - q_L - \lambda)e^{2q_H/\lambda} + \alpha \lambda e^{2q_L/\lambda} + ((q_H - q_L)(1+\alpha) + \lambda(1-2\alpha))e^{(q_H+q_L)/\lambda} \right)}_E \end{aligned}$$

Firstly,  $C$  is decreasing in  $\lambda$  since

$$\frac{\partial C}{\partial \lambda} = -\frac{\alpha(1-\alpha)(q_H - q_L)^2 e^{(q_H+q_L)/\lambda}}{\lambda^3((1-\alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda})^2} \leq 0,$$

and  $\lim_{\lambda \rightarrow \infty} C = 0$ . Hence,  $C \geq 0$  for all  $\lambda \geq 0$ . Secondly,  $D \geq 0$ . Thirdly,  $E$  is convex in  $q_H$  for any given  $\lambda$ ,  $\alpha$  and  $q_L$  since

$$\frac{\partial^2 E}{\partial q_H^2} = \frac{e^{q_H/\lambda} (4e^{q_H/\lambda} (q_H - q_L)(1-\alpha) + e^{q_L/\lambda} ((q_H - q_L)(1+\alpha) + 3\lambda))}{\lambda^2} \geq 0,$$

$E$  is minimized at  $q_H = q_L$ , assuming the value 0. Hence,  $E \geq 0$ .

Having established that  $B$  is increasing in  $\lambda$ , we further note that  $\lim_{\lambda \rightarrow 0} B = -\infty$  and  $\lim_{\lambda \rightarrow \infty} B = -(1-\alpha)(q_H - q_L)$ . This means that in the limit as  $\pi_H \uparrow 1$ ,  $\partial \mathcal{R}(\pi_H)/\partial \pi_H = q_H + B$  is increasing in  $\lambda$ , approaches  $-\infty$  as  $\lambda \downarrow 0$  and approaches  $q_L + \alpha(q_H - q_L) > 0$  when  $\lambda \uparrow \infty$ . Hence, there exists a finite threshold  $\lambda = \lambda_c$  above which  $\pi_H^* = 1$ .  $\square$

**Proof of Theorem 2.** Note that under any separating equilibrium the high- and low-quality firms charge different prices  $p_H > p_L$  and these prices reveal the quality of the product to the customers. Hence  $\hat{\mu}(p) = 1$  for  $p \geq p_H$  and  $\hat{\mu}(p) = 0$  for  $p < p_H$ . When customers infer the true quality, there is no need to incur more information cost to further test the quality. The customers make decisions by comparing the observed prices to true quality levels  $q_H$  and  $q_L$ , as in the perfect information case. Firm optimality conditions (18) and (19) imply that the firm charges the highest possible price under each quality level,  $\hat{p}_H = q_H$  and  $\hat{p}_L = q_L$ . These prices are unique and hence the separating equilibrium.

Under any pooling equilibrium, the high- and low-quality firms charge the same price  $p_H = p_L = p$ . As such, the observed prices do not reveal any additional information to the customers, meaning  $\hat{\mu}(p) = \alpha$  for any  $p$ . Then,

we know from Proposition 1 and Corollary 1 (see also Figure 2(a)) that when  $\lambda \geq \lambda_c$ ,  $\arg \max_{q_L \leq p \leq q_H} \mathcal{R}_H(p, \alpha) = \arg \max_{q_L \leq p \leq q_H} \mathcal{R}_L(p, \alpha) = q_L + q_H - \lambda \ln [(1 - \alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}] = \hat{p}$ . As this price is unique, so is the pooling equilibrium. By the same token, for any  $\lambda < \lambda_c$ , firm optimality cannot be ascertained at the same price for high- and low-quality types,  $\arg \max_{q_L \leq p \leq q_H} \mathcal{R}_H(p, \alpha) \neq \arg \max_{q_L \leq p \leq q_H} \mathcal{R}_L(p, \alpha)$ . Hence there is no pooling equilibrium for low information costs. In this range, the higher quality firm would like to charge a price different (higher) than  $q_L + q_H - \lambda \ln [(1 - \alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}]$ .  $\square$

**Proof of Proposition 3.** In our pricing context, Intuitive Criterion requires that for any out-of-equilibrium price  $p$ ,  $\hat{\mu}(p) = 1$  if  $\mathcal{R}_H(p, 1) > \hat{\mathcal{R}}_H \doteq \mathcal{R}_H(\hat{p}_H, \hat{\mu}(\hat{p}_H))$  and  $\mathcal{R}_L(p, 1) < \hat{\mathcal{R}}_L \doteq \mathcal{R}_L(\hat{p}_L, \hat{\mu}(\hat{p}_L))$  (see Bester and Ritzberger 2001). This suggest that price  $p$  signals to the customer a high quality product if, given this belief, only the high-quality firm would like to deviate to  $p$ . With two quality levels, this is also the D1 Criterion (see Munoz-Garcia and Espinola-Arredondo 2011), so it suffices to show that the equilibria survives the IC.

First, note that  $\mathcal{R}_H(p, 1) = \mathcal{R}_L(p, 1) = p$  if  $p \leq q_H$  and  $\mathcal{R}_H(p, 1) = \mathcal{R}_L(p, 1) = 0$  otherwise. Consider now the separating equilibrium in Theorem 2.  $\mathcal{R}_L(p, 1) < \hat{\mathcal{R}}_L$  requires  $p < q_L$ , which also means  $\mathcal{R}_H(p, 1) < q_L < \hat{\mathcal{R}}_H = q_H$ . As there is no  $p$  that satisfies the two requirements, the equilibrium survives the IC. Consider now the pooling equilibrium in Theorem 2. Recall that in this range of information costs, every customer is buying the product. Hence  $\hat{\mathcal{R}}_H = \hat{\mathcal{R}}_L = \hat{p}_H = \hat{p}_L = \hat{p}$ . Now  $\mathcal{R}_L(p, 1) < \hat{\mathcal{R}}$  requires  $p < \hat{p}$ , which also means  $\mathcal{R}_H(p, 1) = p < \hat{\mathcal{R}}_H = \hat{p}$ . Again, as there is no  $p$  that satisfies the two requirements, the equilibrium survives the IC.  $\square$

**Proof of Proposition 4.** Attentive customers purchase the product with quality  $q$  and price  $p$  with probability 1 as long as  $p \leq q$ . First, suppose that  $q = q_H$ . If the firm chooses to serve both attentive and inattentive customers, based on Proposition 1, it would have to set price  $p$  such that  $p \leq \lambda[\alpha e^{q_H/\lambda} + (1 - \alpha)e^{q_L/\lambda}] < q_H$ . Otherwise, inattentive customers will choose the outside option without processing any information. In this case, the firm earns  $\beta p \pi_H(p) + (1 - \beta)p$ . Let  $\tilde{p}_H$  denote the price which maximizes this particular revenue function. Based on Theorem 1,  $\tilde{p}_H$  is uniquely defined. If the firm chooses to serve attentive customers only, it would set the price at  $q_H$  and earn  $(1 - \beta)q_H$ . Accordingly, the firm's optimal strategy is to serve both segments only if the proportion of attentive customers in the market is higher than a certain threshold, i.e.,  $\beta \geq \frac{q_H - \tilde{p}_H}{q_H - (1 - \pi(\tilde{p}_H))\tilde{p}_H}$ . If this is the case, the firm sets the price as  $\tilde{p}_H$ . Otherwise, it is optimal for the firm to serve attentive customers only and set the price as  $q_H$ .

Next assume  $q = q_L$ . If the firm chooses to serve both attentive and inattentive customers, it would have to set the product price as  $q_L$ , since attentive customers would not buy the product at a higher price. At this particular price, we know that all inattentive customers would also purchase the product, i.e.,  $\pi(q_L) = 1$ ; hence, the firm earns  $q_L$ . If the firm chooses to serve inattentive customers only, it would set the price as  $p_L^* = q_H + q_L - \lambda[(1 - \alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}]$  and earn  $\beta(q_H + q_L - \lambda[(1 - \alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}])$ , as shown in Corollary 1. As a result, the optimal strategy of the firm is to serve both customer segments by setting the price as  $q_L$  only if the proportion of inattentive customers is below a certain threshold, i.e.,  $\beta \leq \frac{q_L}{(q_L + q_H - \lambda \ln[(1 - \alpha)e^{q_H/\lambda} + \alpha e^{q_L/\lambda}])}$ . Otherwise, the firm should only sell to inattentive customers and set the price as  $p_L^*$ .  $\square$

**Proof of Proposition 5.** First we define  $\underline{p}_k$  and  $\bar{p}_k$  as the lower and upper bounds on the optimal price that maximizes the firm's revenue from customers with belief  $\alpha_k$ ,  $k \in \{1, 2\}$ , as described in Proposition 1. If  $\alpha_2 \geq \frac{\alpha_1 e^{q_L/\lambda}}{(1 - \alpha_1)e^{q_H/\lambda} + \alpha_1 e^{q_L/\lambda}}$ , we have  $\bar{p}_1 \geq \bar{p}_2 \geq \underline{p}_1 \geq \underline{p}_2$ ; otherwise  $\bar{p}_1 \geq \underline{p}_1 > \bar{p}_2 \geq \underline{p}_2$ . For brevity, we assume that

the first condition is satisfied (a similar proof can easily be extended to the second case). Then, we express the firm's revenue function as follows

$$\mathcal{R}_i(p) = \begin{cases} p & \text{if } p < \underline{p}_2 \\ \gamma p + (1 - \gamma)p\pi_{i,2}(p) & \text{if } \underline{p}_2 \leq p < \underline{p}_1 \\ \gamma p\pi_{i,1}(p) + (1 - \gamma)p\pi_{i,2}(p) & \text{if } \underline{p}_1 \leq p < \bar{p}_2 \\ \gamma p\pi_{i,1}(p) & \text{if } \bar{p}_2 \leq p \leq \bar{p}_1 \\ 0 & \text{if } p > \bar{p}_1 \end{cases} \quad \text{for } i \in \{H, L\} \quad (32)$$

Since  $\pi_{H,1}(p)$  and  $\pi_{H,2}(p)$  are concave in  $p$  based on Theorem 1,  $\mathcal{R}_H(p)$  is piecewise concave in  $p$ , and a unique solution to  $p_H^*$  exists. On the other hand,  $\mathcal{R}_L(p)$  is increasing if  $p < \underline{p}_2$ , convex if  $\underline{p}_2 \leq p < \underline{p}_1$ , and convex decreasing if  $\underline{p}_1 \leq p < \bar{p}_1$ . Hence,  $p_L^* \in \{\underline{p}_1, \underline{p}_2\}$ . Following from (32), the seller would set  $p_L^* = \underline{p}_2$  if  $\underline{p}_2 > \gamma \underline{p}_1 + (1 - \gamma)\underline{p}_1 \pi_{L,2}(\underline{p}_1)$ , or equivalently if  $\gamma < \underline{\gamma} = \frac{\underline{p}_2 - \underline{p}_1 \pi_{L,2}(\underline{p}_1)}{\underline{p}_1(1 - \pi_{L,2}(\underline{p}_1))}$ . If  $\gamma \geq \underline{\gamma}$ , then  $p_L^* = \underline{p}_1$ .  $\square$

**Proof of Proposition 6.** Let  $p_i^a$  and  $Q_i^a$  denote the optimal price and order quantity, respectively, when selling to perfectly attentive customers. Clearly,  $p_H^a = q_H$  and  $p_L^a = q_L$ . Based on Proposition 1, we know that  $p_H^a > p_H^*$  and  $p_L^a < p_L^*$ . Therefore, regarding the critical ratios, we have  $1 - c_H/p_H^a > 1 - c_H/p_H^*$  and  $1 - c_L/p_L^a < 1 - c_L/p_L^*$ . Moreover,  $\pi_H(p_H^a) = \pi_L(p_L^a) = \pi(p_L^*) = 1$  and  $\pi(p_H^*) \leq 1$ . As a result,  $Q_H^a \geq Q_H^*$  and  $Q_L^a \leq Q_L^*$  directly follows from (29).  $\square$

## Recent ESMT Working Paper

	ESMT No.
<b>Regional state aid control in Europe: A legal and economic assessment</b>	15-05
Hans W. Friederiszick, ESMT European School of Management and Technology Massimo Merola, Bonelli Erede, College of Europe of Bruges	
<b>Fund flows inducing mispricing of risk in competitive financial markets</b>	15-04
Axel Stahmer, ESMT European School of Management and Technology	
<b>LeChatelier-Samuelson principle in games and pass-through of shocks</b>	15-03
Alexei Alexandrov, Consumer Financial Protection Bureau Özlem Bedre-Defolie, ESMT European School of Management and Technology	
<b>Contracts as a barrier to entry when buyers are non-pivotal</b>	15-02
Özlem Bedre-Defolie, ESMT European School of Management and Technology Gary Biglaiser, University of North Carolina, Chapel Hill	
<b>What do patent-based measures tell us about product commercialization? Evidence from the pharmaceutical industry</b>	14-01 (R1)
Stefan Wagner, ESMT European School of Management and Technology Simon Wakeman, New Zealand Productivity Commission	
<b>Hedge fund flows and performance streaks: How investors weigh information</b>	15-01
Guillermo Baquero, ESMT European School of Management and Technology Marno Verbeek, Rotterdam School of Management, Erasmus University	