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## A pro-cyclical stock market under a countercyclical monetary policy in a model of endogenous business cycles

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# FinMaP-Working Paper No.60



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FINANCIAL DISTORTIONS AND MACROECONOMIC  
PERFORMANCE: EXPECTATIONS, CONSTRAINTS AND  
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TITLE

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*JEL classification: C03, D84, E12, E32, E52, G00*

*Keywords: pro-cyclical stock market, Tobin's average Q, endogenous cycles, heterogeneous expectations, monetary policy*

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# A Pro-cyclical Stock Market under a Counter-cyclical Monetary Policy in a Model of Endogenous Business Cycles

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## Abstract

During the last 25 years, the stock market in the US has been strongly pro-cyclical in the presence of a counter-cyclical monetary policy. In this paper, we use an endogenous business cycle model to explore the factors contributing to a pro-cyclical stock market. A dynamic expectation structure in the real sector gives rise to a strong non-linearity and is responsible for the emergence of endogenous business cycles in the model. In the context of this model, we find that a timid or ineffective monetary policy allows the stock market to be dominated by the fluctuations of profits in the real sector. We model the potential ineffectiveness of monetary policy in terms of an endogenous risk premium. The model is calibrated to fit key properties of the data. In particular, it can generate a pro-cyclical stock market in the presence of a counter-cyclical monetary policy.

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## 1. Introduction

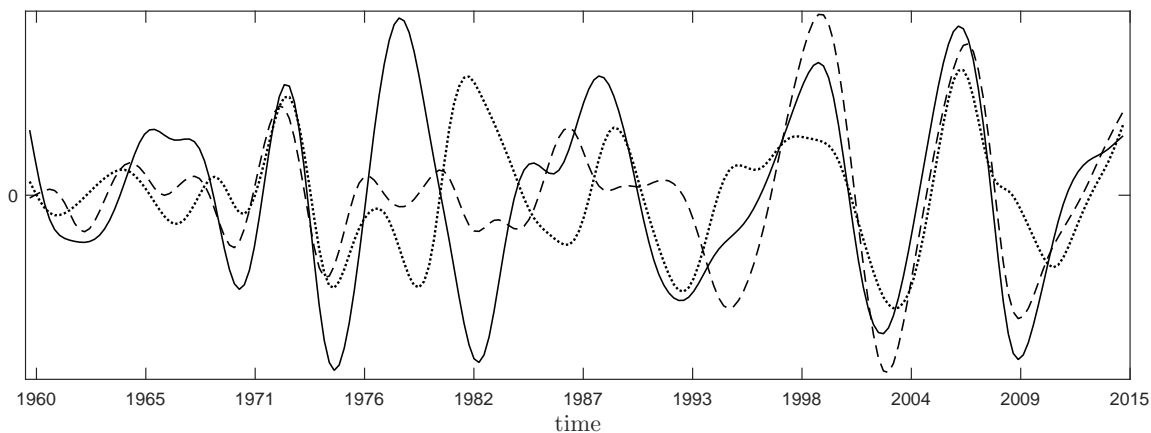
The real sector of the economy is not independent from the financial sector. On the one hand, the financial sector sets the borrowing conditions in the economy depending on the performance of the real sector and, on the other, the performance of the real sector depends on the borrowing conditions. In order to be able to better understand the evolution of the financial sector over the business cycle, it can be useful to have a dynamic model which incorporates the above-mentioned interdependence and which can reflect the inherent cyclical nature of macroeconomic data. For this purpose, an endogenous business cycle model that deals with the real sector is combined with a model of the financial sector. One distinct goal of this paper is to provide potential explanations for the co-movement (often observed in the data) of macro-variables like the output gap, the real base interest rate and stock market indices over the business cycle. In Figure 1 filtered quarterly real output, stock market and real interest rate data have been suitably re-scaled as to allow

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the reader to recognize potential patterns more easily.<sup>1</sup> Thus Figure 1 does not reflect the actual *magnitude* of the fluctuations of the respective series.<sup>2</sup> For more information on the sources of the data and on the way the cyclical component was extracted see section C of the appendix.

It is intuitive that the market value of a firm should increase whenever the firm is able to generate high profits. It is also intuitive that the valuation should also involve the discounting of these profits by the currently prevailing opportunity cost of capital, i.e. the real interest rate. What we often observe in the data, however, is not as intuitive. In light of the counter-cyclical monetary policy being conducted in the US, the co-movement of the real sector and the stock market can appear puzzling.<sup>3</sup> Within the context of the model



**Figure 1:** The filtered fluctuations of real gross value added of the non-financial sector (solid line), real base interest rate (dotted line) and stock price index S&P 500 (dashed line) around their respective long run trends for the US over the period (1960-2014). The fluctuations have been re-scaled to make them comparable by means of visual inspection.

developed in this paper, we can identify three factors that contribute to the emergence of a pro-cyclical stock market. The first factor is related to the potential inability of the central bank to control the borrowing conditions in the economy.<sup>4</sup> If we adopt the idea that the borrowing conditions are endogenous (as suggested by Minsky (1986)) and assume that the risk premium in the model changes depending on how successful the firms are, then this interdependence can potentially offset monetary policy in a way as to allow for the stock market to be strongly pro-cyclical. This explanation is also supported by

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<sup>1</sup>The real base interest rate, as well as stock market indices seem to be pro-cyclical over most of the sample period. Around the time of the two oil price shocks in the 1970s, however, this regularity breaks down. Maybe this is due to the extreme interventions of the Fed during this period.

<sup>2</sup>The amplitude of the fluctuations of stock market indices is generally much higher than that of the fluctuations of output.

<sup>3</sup>A counter-cyclical monetary policy implies that the central bank reacts by increasing (decreasing) its base interest rate whenever the economy is in a boom (recession) and profits are high (low). If high (low) profits are being discounted by a high (low) interest rate, this should prevent the stock market from being strongly pro-cyclical (see Figure 4 at the end of the paper).

<sup>4</sup>The central bank is in control of the nominal base interest rate in the economy, while the actual borrowing conditions also depend on other factors, like, for instance, the currently prevailing risk premium.

data on the risk premium in the US, which is counter-cyclical (see Figure 5 at the end of the paper). The second factor concerns the strength of the reaction of the central bank to inflation and output gaps. Even in the absence of an endogenous risk premium, if the real base interest rate does not change much over the business cycle, the stock market would be dominated by the fluctuations in the performance of the firms.<sup>5</sup> Finally, the speed at which the financial markets reacts to changes in the fundamentals of the economy can also impact the way the stock market behaves over the business cycle.

The idea that “success breeds a disregard of the possibilities of failure” (Minsky (1986, p. 237)) can also be modeled by using the evolution of debt in the economy rather than the evolution of the risk premium (see, for example, Ryoo (2010)). In the model developed in this paper, the evolution of debt is not modeled explicitly, and as a result, the second key element of Minsky’s financial instability hypothesis is not present in the model. That is to say, unlike Ryoo (2010), this paper does not capture the fragility of economic systems with capital structures dominated by debt.

The model of the real sector is inspired by some empirical regularities concerning the evolution of the profit share over the the business cycle, as well as by the work of Franke (2008 and 2012), Bhaduri and Marglin (1990), Stockhammer (2004), Friedman (1957, 1968) and others. It is an investment-driven model in which output is determined via a multiplier stemming from an IS-relationship. Investment itself depends on the expected future demand and on profitability. Profitability is, in turn, affected by the bargaining power of labor such that high (low) levels of employment affect profitability in a negative (positive) way. The firms in the model are dynamically heterogeneous with respect to their expectations about future demand, which leads to a strong non-linearity allowing for the existence of limit cycles. This setting produces cycles characterized by the endogenous emergence of profit opportunities that are being subsequently exhausted by expansions of output. Inasmuch as the dynamics of the profit share play a central role, the model has a distinct Post-Keynesian or Goodwinian flavor.

The model of the financial sector has three components. A central bank attempts to control the borrowing conditions (or the opportunity cost of capital) in the economy via a Taylor rule. A no-arbitrage condition, equalizing the return on equity (within a given period of time) to the risk-adjusted opportunity cost of capital, yields a fundamental value of the firms in the real sector. In each period the financial markets drive equity prices towards the respective fundamental value at a certain speed. The measure of the value of a firm used in the model is Tobin’s average  $Q$  (see Tobin (1969) and Tobin and Brainard (1976)). Finally, the risk premium in the economy is made endogenous by allowing it to dependent on the performance of the real sector.

The remainder of the paper is organized as follows. Section 2 and 3 describe the model of the real sector and the financial sector, respectively. In section 4 we combine the two sectors. Section 5 discusses the factors contributing to a pro-cyclical stock market in the

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<sup>5</sup>In the context of this paper the real base interest rate refers to the nominal base rate set by the central bank less the rate of inflation.

context of the integrated model. Section 6 presents a calibration of the model that fits many properties of data from the last 25 years (1989–2014). Section 7 concludes. Some finer details and derivations are presented in sections A and B of the appendix. Issues concerning the processing of the data and its sources, as well as the parameter values used in simulations are presented in sections C and D of the appendix.

## 2. The model of the real sector

The endogenous business cycle model described in this section was inspired by a particular regularity observed in the US concerning the evolution of the profit share over the business cycle. In particular, the output gap (the cyclical component of output) seems to systematically lag behind the cyclical component of the profit share of output (see Figure 2 (a)).<sup>6</sup> We now introduce the various elements of the model of the real sector.

### 2.1. The output gap

In the following, the model is derived in continuous time. A dot (  $\dot{\phantom{x}}$  ) over a variable will denote its time derivative.

The output gap in the model is determined by investment via an IS-relationship. Investment is thus predetermined in the very short run. We model a one good closed economy without a government sector. In real terms current gross investment ( $I$ ) has to equal savings. Real gross domestic income is given by  $Y = I_n + \delta K + C$ , where  $C$  stands for real consumption,  $I_n$  stands for net investment and  $\delta$  and  $K$  stand for a constant capital depreciation rate and the fixed capital stock, respectively. Real consumption depends on permanent income ( $Y^o$ ) and on actual income ( $Y$ ) in line with Milton Friedman’s permanent income hypothesis (see Friedman (1957)):

$$C(Y^o, Y) = c_1 Y^o + c_2 Y \quad (1)$$

The parameters  $c_1 > 0$  and  $c_2 > 0$  govern the extent to which consumption depends on permanent income and on actual income, respectively. Next, assume that permanent income can be proxied by potential output, that is, by output at a normal utilization rate ( $Y^o := u^o K$ ), where  $u^o$  is the “normal” output capital ratio.<sup>7</sup> We can then write:

$$C = c_1 u^o K + c_2 Y \quad (2)$$

Plugging this into the identity  $Y = I + C$ , yields:

$$Y = I + c_1 u^o K + c_2 Y \quad (3)$$

After dividing both sides by  $K$  and solving for the output capital ratio  $u = Y/K$  we get:

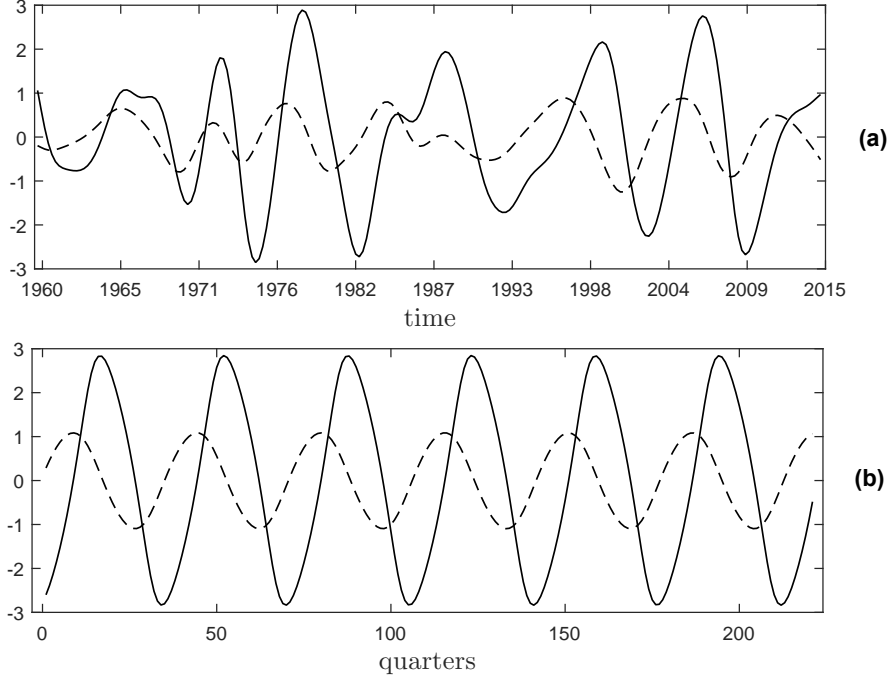
$$u = \frac{g + \delta}{1 - c_2} + \frac{c_1 u^o}{1 - c_2} \quad (4)$$

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<sup>6</sup>To a smaller extent the same is true for the profit rate. Its cyclical component is still lagging behind the output gap, but only marginally so.

<sup>7</sup>The capital stock can be considered an indicator of potential output and thus of permanent income.

Where  $g$  is the net investment rate under the assumption of a constant capital depreciation rate ( $\delta$ ). In other words, the capital stock grows at the rate  $g = \dot{K}/K$ , where  $\dot{K}$  stands for the change in capital stock. We assume, at this point, that there exists a preference to smooth consumption and impose the following restriction:  $0 < c_2 < 1$ . If we assume that



**Figure 2:** (a) – The fluctuations of the profit share (dashed line) and of real gross value added of the non-financial sector (solid line) around their respective long-run trends for the US; (b) – The fluctuations of the profit share (dashed line) and of real output (solid line) generated by a simulation of the model of the real sector. The dashed line corresponds to the evolution of  $h - h^o$ , because of this it fluctuates around 0.

on average the capital stocks in the economy increases by  $g^o$  because of labor augmenting technical progress, then the “normal” rate of utilization is given by  $u^o = (g^o + \delta)/(1 - c_2) + (c_1 u^o)(1 - c_2)$ . Finally, by defining the output gap as  $y := (Y - Y^o)/Y^o$ , so that  $y = (u - u^o)/u^o$ , we can solve for the investment driven output gap (OG):<sup>8</sup>

$$y = \frac{g - g^o}{(1 - c_2) u^o} \quad (\text{OG})$$

It is assumed that due to the presence of excess capacity (not explicitly modeled here) output is able to immediately adjust to aggregate demand. In other words, the “normal” output-capital ratio  $u^o$  represents a situation in which not all of the capital is being used to produce the respective amount of output. We assume, for simplicity, that there is an upper limit of utilization  $\bar{u}$  after which the output cannot be accommodated by the existing capital stock and that the model economy never reaches this limit. We also assume that labor is perfectly flexible. Labor demand has an effect on the real wage in the economy, but the necessary amount of labor is always supplied to accommodate the

<sup>8</sup>Recall that the potential output was defined as  $Y^o := u^o K$ .



aggregate demand for the real sector good  $Y$  (see Section 2.3).

It is worth noting, at this point, that the overall savings rate in the economy ( $\sigma = I/Y$ ) is endogenous in this setting. We can solve eq. (3) for  $\sigma$ . After some algebra we arrive at:

$$\sigma = 1 - c_2 - \frac{c_1}{1 + y} \quad (5)$$

We see that the savings rate is pro-cyclical ( $\partial\sigma/\partial y > 0$ ), which is in line with the idea of consumption smoothing.<sup>9</sup>

## 2.2. The investment function

The aggregate investment function (7) in the model has two main components. The aggregate net investment rate ( $g$ ) depends, on the one hand, on the current expectation structure ( $x$ ) across firms (regarding future demand) and, on the other, on the currently prevailing normal rate of profit ( $r^n$ ):

$$g = f(x, r^n) \quad (6)$$

Where  $\partial g/\partial x > 0$  and  $\partial g/\partial r^n > 0$ .

Next, we look at the two components determining investment in more detail.

### 2.2.1. The normal rate of profit

Define  $\Pi$  and  $\Pi_a$  as gross profit in real terms and gross profit adjusted for depreciation in real terms ( $\Pi_a := \Pi - \delta K$ ), respectively. The actual gross profit rate in the economy is defined as  $r := \Pi_a/K$ . We can express the real gross profit adjusted for depreciation as  $\Pi_a = Y - \delta K - (1 - h)Y$ , where  $h := \Pi/Y$  is the profit share and  $(1 - h)$  is the labor share. After dividing by  $K$  and recalling that ( $u = Y/K = (1 + y)u^o$ ) we arrive at:

$$r = h(1 + y)u^o - \delta \quad (\text{APR})$$

The normal rate of profit is the profit rate associated with normal utilization ( $u^o$ ) or, in other words, with a zero output gap ( $y = 0$ ):

$$r^n = h u^o - \delta \quad (\text{NPR})$$

This profit rate concept is often associated with the Bhaduri-Marglin investment function (see Bhaduri and Marglin (1990)). In the investment function (see eq. (6)), the firms first consider how much to invest in order to avoid over/under-utilization given their expectations about future demand (captured by the expectation structure ( $x$ )). In the second step, they consider the profitability of that investment provided they were able to avoid over/under-utilization. The corresponding measure of profitability is thus  $r^n$ .

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<sup>9</sup>The endogenous savings rate is able to mitigate the unrealistically strong multiplier effect of a constant savings rate in IS-relationships. For a discussion on the strength of the multiplier under a constant savings rate see, for example, Franke (2015a).

Finally, aggregate demand ( $Y$ ) is assumed to be accommodated by the firms according to their capital stock, such that the output capital ratio ( $u$ ) and the output gap ( $y$ ) are the same across all firms. This implies that the actual rate of profit ( $r$ ) also does not vary across firms.<sup>10</sup>

### 2.2.2. *The structure of the expectations*

The firms in the model are heterogeneous with respect to their expectations about the future demand (and the associated future utilization). There are two types of expectations in the model. The individual firms can either expect a positive output gap or a negative one (meaning under- or over-utilization).<sup>11</sup> The key point in this framework is that the firms influence each other when forming their expectations. The currently observed output gap ( $y$ ) is seen as evidence in favor of the expectation of the firms that predicted demand correctly (these are the firms with an expectation matching the sign of  $y$ ). Accordingly, the firms that got it wrong adjust their expectations towards the expectation of the firms that got it right. In other words, the key assumption here is that information (in this case the observed output gap) is not independent of the agents in the economy. Information rather flows through the firms as they influence each other when forming their expectations. This assumption is in the spirit of the idea of ecological rationality (see, for example, Simon (1955) or Lavoie (2014, Section 2.2)).

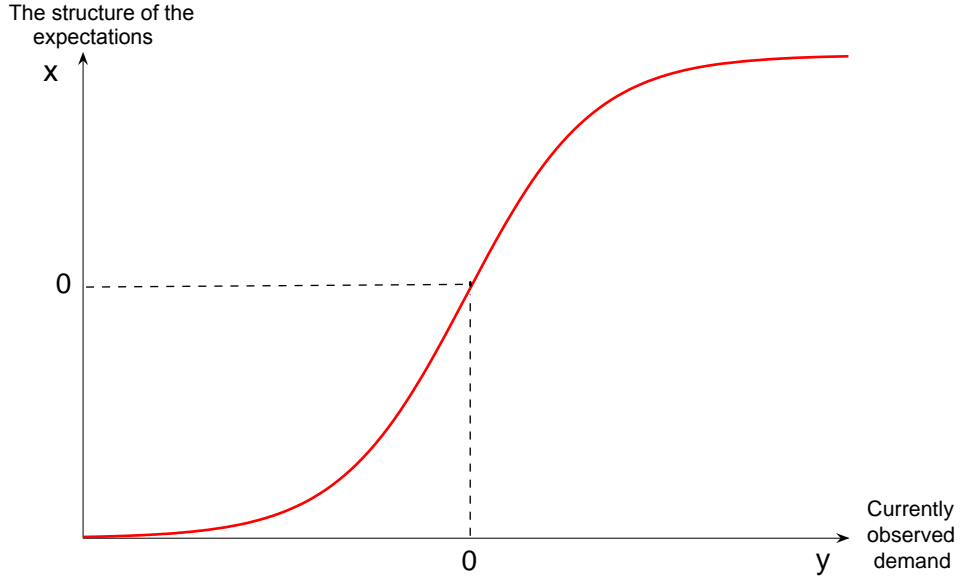
This makes the heterogeneity of the population of firms with respect to their expectations dynamic. There are two extremes to consider. In the first extreme all the firms have the right expectation (e.g. the observed output gap is positive ( $y > 0$ ) and all the firms expect it to be positive). In this case the observed output gap ( $y$ ) would have no impact on the structure of the expectations ( $x$ ). In the other extreme half the firms expect a positive output gap, while the other half expects a negative one. In this case the observed output gap ( $y$ ) would have a very strong impact on the structure of the expectations ( $x$ ) because there is a large pool of firms that can switch to the expectation that matches the sign of  $y$ .

Figure 3 is a stylized representation of the resulting relationship between the structure of the expectations ( $x$ ) and the observed output gap ( $y$ ). The variable  $x$  is standardized in such a way as to take on values between  $-1$  and  $1$ , where the former value is associated with all firms expecting a negative output gap and the latter with all firms expecting a positive output gap. Figure 3 combined with the investment function (eq.6) show us that in this model the relationship between the observed output gap ( $y$ ) and the ensuing investment is strongly non-linear.

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<sup>10</sup>Even though the profit rate is assumed to be identical across firms, the firms still have an incentive to correctly guess future demand, since the profit of each firm in absolute terms depends on this (see section 2.2.2). The same profit rate is applied to capital stocks that can differ across firms.

<sup>11</sup>Introducing fixed magnitudes as demand expectations or a continuous distribution of expectations (as opposed to the current specification in which there are two types of expectations that represent intervals:  $y > 0$  or  $y < 0$ ) would not change the setting conceptually. Because of this, we stick to the simplest possible specification.



**Figure 3:** A stylized representation of the relationship (resulting from eq. **SE**) between the structure of the demand expectations in the economy and the observed demand.

A mathematical translation of this idea can be made by using a framework developed by Weidlich and Haag (1983). Formally, the relationship between the structure of the expectations ( $x$ ) and the observed output gap ( $y$ ) can be represented by the following differential equation:<sup>12</sup>

$$\dot{x} = \nu [(1 - x) \exp(\vartheta_y y) - (1 + x) \exp(-\vartheta_y y)] , \quad -1 \leq x \leq 1 \quad (\mathbf{SE})$$

This equation is derived in section A of the appendix. The positive parameter  $\vartheta_y$  governs the strength with which the signal ( $y$ ), as opposed to the idiosyncratic characteristics of the firms, affects their decisions to switch from one expectation to another. The positive parameter  $\nu$ , on the other hand, determines how quickly the switching takes place.

Next, let us aggregate investment so as to arrive at the aggregate net investment rate  $g$  (see eq. (6)).  $E^j(y)$  denotes firm  $j$ 's expectation about future demand. On average each firm invests at a positive rate  $g^o$  due to labor augmenting technical progress. Firm  $j$ 's net investment rate ( $g_j$ ) depends on the normal profit rate ( $r^n$ ). Whenever profitability ( $r^n$ ) exceeds a certain threshold  $r^o$ , this results in additional investment. We keep  $r^o$  constant for now.<sup>13</sup> We define this threshold as  $r^o = h^o u^o - \delta$ , where  $h^o$  stands for a "normal" profit rate around which the economy fluctuates ( $0 < h^o < 1$ ). In addition, an expectation about a positive (negative) output gap is associated with a higher (lower) investment rate. The parameters  $g^o$ ,  $\phi_x$  and  $\phi_r$  are all positive and uniform across firms.

<sup>12</sup>Figure 3 shows the relationship between the structure of the expectations ( $x$ ) and the observed output gap (according to eq. **SE**) after the adjustments of  $x$  have finished. In other words, Figure 3 shows the relationship between  $x$  and  $y$  if we impose  $\dot{x} = 0$  and solve **SE** for  $x$ .

<sup>13</sup>This threshold will later be made endogenous (in the presence of a financial sector) and will reflect the currently prevailing borrowing conditions in the economy.

The investment function of firm  $j$  is specified as:

$$g_j = g^o + \phi_r (r^n - r^o) + \begin{cases} \phi_x & \text{if } E^j(y) > 0 \\ -\phi_x & \text{if } E^j(y) < 0 \end{cases}$$

Across the  $N$  firms in the economy the aggregate net investment rate is approximately  $g = (\sum_{j=1}^N g_j)/N$ . The approximation is good if the inequality in the distribution of the capital stock across firms is not too extreme. We use this approximation in order to avoid having to track the capital stock of all firms over time. The same aggregation approach is used in Franke (2012). Define the fraction of firms expecting a positive output gap as  $n^+$ . Then the aggregate net investment rate becomes:

$$g = g^o + \phi_r (r^n - r^o) + n^+ \phi_x - (1 - n^+) \phi_x \quad (7)$$

The variable describing the structure of the demand expectations is defined as  $x := n^+ - n^-$ , where  $n^-$  stands for the fraction of firms expecting a negative output gap ( $n^- = 1 - n^+$ ). The aggregate investment function can thus be written as:

$$g = g^o + \phi_r (r^n - r^o) + \phi_x x \quad (\mathbf{IF}')$$

### 2.3. The evolution of profitability

As discussed in the previous section the normal rate of profit ( $r^n$ ) is one of the two driving forces of investment in the model. In section 2.2.1 we saw that profitability itself depends on the profit share (see eq. **NPR**). Next, we derive the equation governing the evolution of the profit share from the dynamics of the real wage. The price and wage setting in the economy are modeled independently and later combined to arrive at the dynamics of the real wage. A similar approach has been taken, for example, in Chiarella et al. (2005, pp. 27f).

#### 2.3.1. The price Phillips curve

We start by looking at the price ( $p$ ) setting for the real sector good. The firms follow a mark-up pricing rule of the type  $p = \Upsilon^f w^e/z$ , where  $\Upsilon^f$  stands for the mark-up factor that the firms apply, while  $w^e$  and  $z$  are the expected nominal wage and labor productivity, respectively. Labor is homogeneous, all firms pay the same nominal wage ( $w$ ), apply the same mark-up factor ( $\Upsilon^f$ ) and form the same expectation about the nominal wage ( $w^e$ ). In addition, assume that the observed output gap  $y$  (as a measure of aggregate demand) triggers a reaction on the part of the firms with respect to the mark-up factor  $\Upsilon^f$ . The mark-up factor increases (decreases) if the observed output gap is positive (negative). Finally, we assume that the level of the mark-up factor also depends on the duration of the observed output gap. Hence we use a dynamic specification:

$$\hat{\Upsilon}^f = \kappa y \quad (8)$$

Where  $\kappa$  is a non-negative parameter and  $\hat{\Upsilon}^f$  is the growth rate of  $\Upsilon^f$ . Note that for  $\kappa = 0$  the firms use a constant mark-up rule. The mark-up factor  $\Upsilon^f$  is an ex ante

mark-up factor that the firms use to set  $p$ . Ex post, the actual mark-up factor would also depend on the actual evolution of the nominal wage ( $w$ ). When setting  $p$  the firms need to form an expectation about the evolution of the nominal wage. All firms form the same expectation about its evolution:  $\hat{w}^e = \pi^c + \hat{z}$ , where  $\pi^c$  stand for the current inflation climate in the economy, while  $\hat{z}$  denotes the growth rate of labor productivity. In other words, the firms expect the nominal wage to grow by expected inflation ( $\pi^c$ ) plus the growth rate of labor productivity ( $\hat{z}$ ). After inserting eq. (8) and the nominal wage expectations ( $\hat{w}^e = \pi^c + \hat{z}$ ) into the price setting rule expressed in terms of growth rates ( $\pi = \hat{Y}^f + \hat{w}^e - \hat{z}$ ) we get:

$$\pi = \pi^c + \kappa y \tag{PC}$$

This is a standard representation of a Phillips curve.

The inflation climate ( $\pi^c$ ) constitutes adaptive expectations that adjust to the observed inflation and are influenced by the central bank's target inflation rate  $\pi^*$ . The specification is taken from Franke (2012, p.12) and allows for a very flexible inflation process:

$$\dot{\pi}^c = \alpha [\gamma \pi^* + (1 - \gamma) \pi - \pi^c] \tag{IC}$$

The parameter  $\alpha$  governs how quickly the climate adjustment to the observed inflation ( $\pi$ ), while the parameter  $\gamma$  can be interpreted as the central bank's credibility with respect to its target inflation rate ( $\pi^*$ ) and governs the general variability of the inflation climate. Both parameter take on values between 0 and 1. For  $\gamma$  close to 1 the climate is dominated by the constant target rate  $\pi^*$  and thus barely fluctuates, while for  $\gamma = 0$  we have standard adaptive expectations.

### 2.3.2. The nominal wage Phillips curve

Define the employment gap as  $e = (L - L^o)/L^o$ , where  $L$  stands for the amount of labor employed to produce  $Y$ , while  $L^o$  is the amount of labor needed to produce the potential output  $Y^o$ . If we assume that labor productivity ( $z$ ) is independent of output, we can rewrite the output gap as  $y = (Lz - L^o z)/(L^o z)$ . We see that in this case  $y = e$  and thus the output gap can be considered a proxy of the labor market conditions.<sup>14</sup>

Next, we assume that the presence of an employment gap represents a difference in bargaining power between the workers and the firms in the economy. Whenever the workers have more bargaining power ( $e > 0$ ) they are able to drive up the nominal wage. Accordingly, for  $e < 0$  there would be downward pressure on the nominal wage. In addition, we assume that in the absence of bargaining power (i.e.  $e = y = 0$ ) factor labor is able to negotiate nominal wage growth reflecting the current inflation climate and the

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<sup>14</sup>The author is aware of the empirical regularity known as Okun's law. It states that in reality employment does not move one-to-one with output. This effect can be achieved in the model by relaxing the assumption that productivity is independent of output. However, since this adds complexity to the model and since (for the purposes of this paper) we are rather interested in the sign of the relationship (between employment and output), we simply take the output gap as a proxy of the labor market conditions.

growth in labor productivity. As in eq. (8) we assume that the effect of bargaining power depends on its duration. A standard nominal wage Phillips curve satisfies these assumptions:

$$\hat{w} = \pi^c + \kappa_{nw} y + \hat{z} \quad (9)$$

Where  $\kappa_{nw}$  is a non-negative parameter.

### 2.3.3. The real wage Phillips curve and profitability

We can arrive at the real wage Phillips curve by combining the nominal wage with the price Phillips curve (eq. **PC** with eq. (9)). The growth rate of the real wage ( $\hat{\omega} = \hat{w} - \pi$ ) is thus given by:

$$\hat{\omega} = (\kappa_{nw} - \kappa) y + \hat{z} \quad (10)$$

Next, we assume that the nominal wage (see eq. (9)) reacts more strongly to an output gap than the price of the real sector good does (see eq. **PC**), i.e. we assume  $\kappa_{nw} > \kappa$ . We can justify this assumption by arguing that the existence of bargaining power (i.e.  $y \neq 0$ ) naturally implies that one side is likely to gain an advantage in the process of negotiations. For the firms this advantage is expressed in terms of the ability to employ labor at a lower real wage, while if factor labor has bargaining power, it can enjoy a higher real wage. Hence, for  $y > 0$  ( $y < 0$ ) the real wage increases (decreases).<sup>15</sup> Note that under a more strict assumption – that the firms use a constant mark-up factor  $\Upsilon^f$  when setting the price of the real sector good (i.e.  $\kappa = 0$ ) – the inequality  $\kappa_{nw} > \kappa$  holds automatically. After inserting the composite parameter  $\kappa_{rw} := (\kappa_{nw} - \kappa) > 0$  the real wage Phillips curve becomes:

$$\hat{\omega} = \kappa_{rw} y + \hat{z} \quad (11)$$

The evolution of the real wage has direct implications for the evolution of the profit share ( $h$ ) and thus (via eq. **NPR**) for the normal rate of profit ( $r^n$ ). To see this we can start from the definition of the labor share:  $l := (wL)/(pY) = w/(zp)$ , where  $w$ ,  $p$  and  $z$  stand for the nominal wage, the goods market price level and labor productivity, respectively. In terms of growth rates the labor share can be expressed as:

$$\hat{l} = \hat{w} - \pi - \hat{z} \quad (12)$$

After inserting eq. (11) in the above equation we get:

$$\hat{l} = \hat{w} - \pi - \hat{z} = \hat{\omega} - \hat{z} = \kappa_{rw} y \quad (13)$$

Finally, by recalling that the profit share can be expressed as  $h = 1 - l$  and after applying some algebra we arrive at the equation governing the evolution of the profit share:

$$\dot{h} = -(1 - h) \kappa_{rw} y, \quad 0 < h < 1 \quad (\text{PS})$$

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<sup>15</sup>For a more detailed discussion of this assumption see Chiarella et al. (2005, p. 228).

Equation **PS** describes the result of a conflict between factor labor and the firms in an economy. In this sense, it can be considered in the Post-Keynesian tradition. Stockhammer (2004, ch. 2), for example, also considers a relationship between the profit share and the conditions on the labor market. Note that since  $p$ ,  $w$  and  $\Upsilon^f$  are not firm-specific,  $h$  is not firm-specific and thus the normal rate of profit ( $r^n$ ) also does not differ across firms.

Equation **PS** also implies certain dynamics for the actual ex post mark-up factor ( $\Upsilon^a$ ). The ex post mark-up factor is simply a residual of the actual price, wage and productivity dynamics. The price of the real sector good can be expressed as:  $p = \Upsilon^a w/z$ . In terms of growth rates this reads:  $\pi = \hat{\Upsilon}^a + \hat{w} - \hat{z}$ . By combining this expression with eq. (11) and after rearranging we arrive at the implied dynamics of the actual ex post mark-up factor:

$$\hat{\Upsilon}^a = -\kappa_{rw} y \quad (14)$$

Franke (2015b, p. 22) proposes a similar relationship for the mark-up dynamics in a different context.

Equation **PS** together with equation **NPR** determine how the normal rate of profit ( $r^n$ ) evolves in the model.

#### 2.4. Key features of the model of the real sector

The model of the real sector is driven by the dynamic equations **SE** and **PS** together with the contemporaneous equations **IF'**, **OG** and **NPR**:

$$\dot{x} = \nu [(1-x) \exp(\vartheta_y y) - (1+x) \exp(-\vartheta_y y)] , \quad -1 \leq x \leq 1 \quad (\text{SE})$$

$$\dot{h} = -(1-h) \kappa_{rw} y , \quad 0 < h < 1 \quad (\text{PS})$$

$$g = g^o + \phi_r (r^n - r^o) + \phi_x x \quad (\text{IF}')$$

$$y = \frac{g - g^o}{(1 - c_2) u^o} \quad (\text{OG})$$

$$r^n = h u^o - \delta \quad (\text{NPR})$$

$$r^o = h^o u^o - \delta$$

The above system greatly resembles the model developed in a similar context by Franke (2008). Since this system of differential equations is two dimensional, we can analytically explore many of its properties. This is done in section B of the appendix. Here we present the results of the analysis, simulate the system and provide intuition.

First, note that the system has a point of rest at  $(x, h) = (0, h^o)$ . The system can also have points of rest associated with corner solution involving  $h = 1$  and  $h = 0$ . However, since such points of rest are hardly economically meaningful, we exclude the potentially attractive ones by applying a restriction on the parameters (see sections B.1 and B.2 in the appendix):

$$\phi_x / \phi_r < (0.5 - |h^o - 0.5|) u^o \quad (\text{R1})$$

The intuition behind this restriction is that the stabilizing forces coming from the changes in profitability via the parameter  $\phi_r$  must be sufficiently stronger than the destabilizing forces coming from the changes in the structure of the expectations via the parameter  $\phi_x$ . If this is the case, there are no attractive points of rest associated with the corner solutions  $h = 1$  and  $h = 0$ .

In the following, we only consider the parameter space satisfying **R1**. In this case, since the system is globally stable, sustained cyclical behavior occurs if the point of rest  $(x, h) = (0, h^o)$  is repelling (see section B.3 in the appendix). The condition for this being the case is:

$$2\nu \frac{\phi_x \vartheta_y - (1 - c_2) u^o}{\phi_r \vartheta_y} > (1 - h^o) u^o \kappa_{rw} \quad (\mathbf{R2})$$

We can identify the parameters contributing to the satisfaction of the above restriction by examining how the parameters affect the difference ( $D$ ) between the two sides of the inequality **R2**:

$$D = 2\nu \frac{\phi_x \vartheta_y - (1 - c_2) u^o}{\phi_r \vartheta_y} - (1 - h^o) u^o \kappa_{rw}$$

The derivatives  $\partial D/\partial \vartheta_y$ ,  $\partial D/\partial \nu$  and  $\partial D/\partial \phi_x$  are all positive. These parameters measure the strength with which the past expectation structure affects the future expectation structure via investment (see equations **OG**, **IF'** and **SE**). This phenomenon can be understood as a version of Harrodian instability since it describes a feedback loop between expectations and investment (see Harrod (1939)). For example, if most firms expect a positive output gap ( $x > 0$ ), this can trigger investment (via eq. **IF'**), which as a component of aggregate demand drives up the output gap (via eq. **OG**), which, in turn, can make even more firms optimistic (via eq. **SE**). Conversely, expectations about a negative output gap can feed on themselves and can drive the output gap down. If the Harrodian instability is strong enough, endogenous business cycles occur in the model. The inequality **R2** is the analytic translation of this statement in the context of the current model.

The intuition behind the cyclical behavior of the model of the real sector runs as follows. During recessions, in the presence of a negative output gap, profitability increases since factor labor is willing to accept an increasingly lower real wage in order to become employed. These increases in profitability are corrected by an increasing output gap that puts factor labor in a better bargaining position. High profitability pushes the economy out of a recession by causing more investment. Conversely, low profitability causes the economy to go into a recession by bringing about less investment. The system is locally unstable because around the zero output gap the structure of the expectations is very heterogeneous and evidence in favor of a particular type of expectation makes the structure more homogeneous, which strongly affects aggregate investment. During expansions and contractions the system is dominated by self-reinforcing expectations. The system is globally stable since the degree of homogeneity of the structure of expectations is finite. Near the boundaries characterized by perfect homogeneity (all firms having the same expectation) profitability becomes the major driving force in the economy.

In Figure 2 (b) the model of the real sector has been simulated under a parametrization that allows it to roughly match properties of the filtered real world data presented in



Figure 2 (a). In particular, we see that, in the model, the output gap is lagging behind the profit share gap (which is defined as:  $h - h^o$ ). We observe the same regularity in the filtered data (see Figure 2 (a)). The simulated series also roughly match the average amplitude and period of the fluctuations of the filtered real world data. See section D of the appendix for the parameter values used in the simulation.

### 3. The financial sector

The financial sector we are going to introduce now comprises three components. First, we introduce the Taylor rule, with which the central bank attempts to control the borrowing conditions in the economy. We then derive a value of Tobin's  $Q$  based on the fundamentals from the real sector. The financial markets adjust towards this fundamental value of Tobin's  $Q$ . Finally, we introduce the endogenous risk premium and make it dependent on the performance of the firms in the real sector.

#### 3.1. The Taylor rule

The central bank's behavior is specified in terms of a difference equation as:

$$\frac{di}{dt} = \mu_i [i^o + \mu_\pi (\pi - \pi^*) + \mu_y y - i] \quad (\mathbf{TR})$$

Equation **TR** constitutes a standard Taylor rule with interest rate smoothing. Monetary policy reacts to inflation (diverging from its target) and to output gaps by changing the base interest rate ( $i$ ). The interest rate smoothing (captured here by the positive parameter  $\mu_i$ ) reflects the idea that the central bank faces uncertainty that might prompt it to change the base interest rate slowly. The lower  $\mu_i$  is, the slower the central bank's reaction is. The parameter  $i^o$  is the nominal base interest rate set by the central bank (in the limit, as time goes to infinity) in the absence of an output gap or a deviation of inflation from its target. We denote the time derivative of the base interest rate as  $di/dt$  to avoid placing an additional dot over the letter  $i$ .

#### 3.2. Tobin's $Q$

It is intuitive that the stock market can be affected, on the one hand, by the performance of the real sector and, on the other, by the interest rate environment, which constitutes the opportunity cost of capital in the economy. What follows is a translation of this intuition in terms of a mathematical specification.

We now derive a value of Tobin's  $Q$  that reflects the fundamentals in the model economy. The approach used in this section was introduced in the working paper by Franke and Ghonghadze (2014). Here we derive a more general case in which the firms can finance investment by issuing equity or debt (see Franke and Yanovski (2015)). The financing decisions are assumed not to be firm-specific. In the following derivation, we consider variables on the aggregate level.

The derivation involves a concept of a benchmark stock price inflation. It can be understood as the stock price inflation implied by the financing decisions of the firms and by the inflation in the real sector.<sup>16</sup> To derive this benchmark value of stock price inflation we can start from the definition of Tobin's  $Q$ :

$$q := \frac{p_e E + M}{p K}$$

Where  $p_e$  is the stock price index,  $p$  is the price level on the goods market,  $K$  is the stock of fixed capital,  $E$  is the number of shares outstanding and  $M$  is the firms' stock of debt. Tobin's  $Q$  is a measure of the value of a firm since it considers the value of a firm's equity relative to the replacement cost of the firm's capital stock and relative to the stock of debt. If we write this definition in terms of growth rates, we get:

$$\begin{aligned} \hat{q} &= \frac{q_e}{q} \hat{q}_e + \frac{m}{q} \hat{m}, \text{ or} \\ \hat{q} &= \frac{q_e}{q} (\hat{p}_e + g_e - \pi - g) + \frac{m}{q} \hat{m} \end{aligned} \quad (15)$$

Where  $m := M/pK$ ,  $q_e := p_e E/pK$ ,  $\hat{p}_e$  is the stock price inflation,  $g_e$  is the growth rate of the shares issued,  $g$  is the growth rate of the real capital stock and  $\pi$  is goods market inflation. By setting  $\hat{q} = 0$  we can solve for  $\hat{p}_e$  and get the benchmark stock price inflation:

$$\hat{p}_e^* := \hat{p}_e|_{\hat{q}=0} = \pi + g - g_e - \frac{m}{q_e} \hat{m} = \pi + g - g_e - \frac{m}{q_e} (\hat{M} - g - \pi) \quad (16)$$

Intuitively, setting  $\hat{q} = 0$  (and solving for  $\hat{p}_e$ ) amounts to enforcing that the equity price increases if retained profits (instead of equity or debt) are being used for financing investment. If  $(\pi + g - g_e - \frac{m}{q_e} \hat{m}) > 0$ , each share is now backed by more capital stock and thus the price of equity must go up.

Next, let us look at the definition of the net profit rate:

$$r^{net} = \frac{pY - W - \delta pK - (i + \xi_m) M}{pK} \quad (17)$$

Where  $i$  is the base interest rate set by the central bank, while  $i + \xi_m$  is rate at which the firms can actually borrow. The premium  $\xi_m$  is associated with risk related to the servicing of debt.  $W$  is the nominal wage bill,  $pY$  is total nominal output and  $\delta$  is the rate of capital depreciation. Next, consider the financing identity:

$$\begin{aligned} p I_n &= \sigma_f r^{net} p K + p_e \dot{E} + \dot{M}, \text{ which can be rewritten as} \\ p I_n &= \sigma_f r^{net} p K + p_e E g_e + M \hat{M} \end{aligned}$$

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<sup>16</sup>Since we have a one-good economy, the replacement value of a firm's capital stock is determined by the inflation rate in the real sector. Via this channel stock price inflation is affected by the inflation rate in the real sector.

Where  $I_n p$  denotes net nominal investment in fixed capital and  $\sigma_f$  and  $g_e$  are the retention rate and the growth rate of shares outstanding, respectively. The financing identity states that investment can be financed via retained profits (the first term) or via issuing equity or debt (the second and third terms, respectively). Solving for  $\sigma_f$  gives us:<sup>17</sup>

$$\sigma_f = \frac{g - q_e g_e - m(\hat{M})}{r^{net}} = \frac{g - q_e g_e - m(\hat{m} + g + \pi)}{r^{net}} \quad (18)$$

This is the retention rate needed to finance investment given the finance policy of the real sector (issuing equity, debt or retaining profits). The return on equity is given by:

$$r_e = Div + \hat{p}_e = \frac{(1 - \sigma_f) r^{net} p K}{p_e E} + \hat{p}_e$$

Where  $Div$  stands for dividends as a share of equity. The fundamental return on equity ( $r_e^f$ ) is the return on equity associated with the benchmark value of stock price inflation ( $\hat{p}_e = p_e^*$ , see eq. (16)), i.e.:

$$r_e^f = \frac{(1 - \sigma_f) r^{net} p K}{p_e E} + \pi + g - g_e - \frac{m}{q_e} \hat{m} \quad (19)$$

Inserting eq. (18) in (19) gives us:

$$r_e^f = \frac{r^{net} - g + m g + m \pi}{q - m} + \pi + g \quad (20)$$

Next, we can set up a no-arbitrage condition involving a comparison of the fundamental return on equity with the riskless base interest rate set by the central bank. When doing this, however, we should take into account that equity is not riskless. We can do this by adjusting the base interest rate by a risk premium ( $\xi_e$ ) associated with equity risk. The no-arbitrage condition equates the base rate (adjusted for equity risk) to the fundamental return on equity:

$$\frac{r^{net} - g + m g + m \pi}{q - m} + \pi + g = i + \xi_e \quad (21)$$

The fundamental value of Tobin's  $Q$  can be obtained by solving the no-arbitrage condition for  $q$ . We insert eq. (17) in (21) and solve for  $q$  to arrive at:

$$q^f = \frac{r - g + m(\xi_e - \xi_m)}{i + \xi_e - g - \pi} \quad (22)$$

Where  $r$  is the gross rate of profit as defined in **APR**. Of course, this specification of the fundamental value of Tobin's  $Q$  only makes sense if  $(i + \xi_e - g - \pi) > 0$ . We assume that this is always the case. The fundamental value of Tobin's  $Q$  is an expression of the value

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<sup>17</sup>We can also derive the fundamental value of Tobin's  $Q$  by solving the financing identity for other firm decision variable (e.g.  $g_e$ ). In this sense, in the context of this model, it does not matter which decision variable is chosen to be residually determined.

of the firm that satisfies the no-arbitrage condition (eq. (21)).

Recall that  $\xi_m$  is the premium associated with risk related to the servicing of debt. It can be argued that generally  $\xi_e > \xi_m$  because debt is a senior claim as compared to equity. In the absence of a hierarchy of claims we have  $\xi_m = \xi_e$ . Since  $m \geq 0$  and since  $\xi_e$  would fall in case there is no hierarchy of claims, the case in which  $\xi_m = \xi_e$  can be considered a lower bound of the fundamental value of Tobin's  $Q$ . For simplicity, we now assume that  $\xi_m = \xi_e$  in order to avoid having to model the dynamics of credit money. The uniform risk premium ( $\xi = \xi_m = \xi_e$ ) will be modeled instead to reflect the endogenous borrowing constraints suggested by Minsky (1986) (see next subsection). The final specification of the fundamental value of Tobin's  $Q$  thus becomes:<sup>18</sup>

$$q^f = \frac{r - g}{i + \xi - \pi - g} \quad (\mathbf{FQ})$$

The interpretation of **FQ** is straightforward. Given a particular investment rate ( $g$ ), the gross profit rate ( $r$ ) generated by the real sector over a particular period of time relative to the perceived real opportunity cost of capital over the same period ( $i + \xi - \pi$ ) determines the fundamental value of the real sector for that period. If the two rates are equal, i.e.  $r = (i + \xi - \pi)$ , the fundamental value of Tobin's  $Q$  would take on a value of one.

If we look at the fundamental value of Tobin's  $Q$  at the level of the individual firms, we see that the  $q^f$  that we have derived on the aggregate level is only approximately the average fundamental value of Tobin's  $Q$  across all firms:<sup>19</sup>

$$q^f = \frac{r - 1/N \sum_1^N g_j}{i + \xi - \pi - 1/N \sum_1^N g_j} \approx \frac{1}{N} \sum_1^N q_j^f := \frac{1}{N} \sum_1^N \frac{r - g_j}{i + \xi - \pi - g_j}$$

In the following we treat  $q^f$  as the average fundamental value of Tobin's  $Q$  across all firms.

The no-arbitrage condition in equation (21) is the driving force of the stock market in the model. We assume that the financial markets drive the equity price (which is in the numerator of  $q$ ) in a direction that restores the no-arbitrage condition. If  $r_e^f > i + \xi_e$ , there will be excess demand for equity driving the stock price index up and if  $r_e^f < i + \xi_e$ , excess supply of equity would bring the stock price index down. The actual value of Tobin's  $Q$  ( $q$ ) on the aggregate level is assumed to move towards the fundamental value ( $q^f$ ) because of arbitrage according to the adjustment equation:

$$\dot{q} = \varrho_q (q^f - q) \quad (\mathbf{AQ})$$

Where the parameter  $\varrho$  can take on values between 0 and 1. We assume that the adjustment process is not firm-specific. The dynamics of the stock market described above

<sup>18</sup>For a discussion on the fundamental value of Tobin's  $Q$  for  $\xi_m \neq \xi_e$  see Franke and Yanovski (2015).

<sup>19</sup>Recall that only the capital stocks of the firms are varying across firms ( $g_j$ ). In the determination of  $q^f$  the capital stock growth rate generally plays a very small role because it is present with the same sign both in the denominator and in the numerator of  $q^f$ .

can be considered in the tradition of the modeling approach of Taylor and O’Connell. “Wealthholders try to look through Wall Street to “fundamentals” on the production side” (see Semmler (1991, p. 7)).

Introducing momentum traders in the model of the stock market (like in Franke and Ghonghadze (2014), for example) does not change the behavior of the model in a qualitative way in terms of the dynamics of the stock market over the business cycle as long as the stock market remains ultimately driven by  $q^f$ .<sup>20</sup>

Finally, we assume that debt ( $M$ ) expands and contracts together with equity ( $p_e E$ ). In other words, we assume that the fraction  $p_e E/M$  is a constant. This assumption ensures that the equity part of Tobin’s  $Q$  ( $q_e$ ) is always proportional to  $q$ , which allows us to talk about the evolution of the stock market when referring to  $q$  in our model. In the data, stock price indices and measures of the value of a firm (like Tobin’s average  $Q$ ) behave very similarly at business cycle frequencies (see Figure D.8 at the end of the paper). We take this as a motivation to talk about the situation on the financial markets when referring to  $q$  as simulated by the model.

### 3.3. The endogenous risk premium

The interest rate environment in this model is specified as  $(i + \xi)$ . The central bank can attempt to alter the borrowing conditions in the economy by changing the base interest rate ( $i$ ), however, an endogenous risk premium ( $\xi$ ) might render these attempts futile.<sup>21</sup> The interest rate at which borrowing takes place depends on the perception of the lender about the related risk. This perception is obviously affected by the current or past performance of the borrower. As Minsky puts it, “success breeds a disregard of the possibilities of failure” (Minsky (1986, p. 237)). We take up this idea and specify an adjustment equation for the risk premium as a function of the observed performance of the firms in the real sector. In our model, we can proxy the observed performance by the term  $(r - i + \pi - \xi^c)$ . The parameter  $\xi^c$  stands for some benchmark performance measure that lenders use, such that  $(r - i + \pi - \xi^c) < 0$  indicates bad performance, while  $(r - i + \pi - \xi^c) > 0$  indicates good performance. The evolution of the risk premium is thus determined by the extent to which the actual profit rate ( $r$ ) is in excess of the real base interest rate ( $i - \pi$ ). The adjustment equation reads:

$$\dot{\xi} = -\varrho_{\xi}(r - i + \pi - \xi^c) \quad \text{(RP)}$$

Note that the adjustment equation for the risk premium does not depend on the capital structure in the economy. This is the case since the risk premium was assumed to be

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<sup>20</sup>In particular, such extensions can help us to deterministically model fluctuations in macroeconomic data that take place at a much higher frequency than the average business cycle frequency. However, such high frequency fluctuations do not exhibit a strong regularity in the data. It thus might be more useful to model high frequency fluctuations as shocks.

<sup>21</sup>An alternative approach to model endogenous borrowing constraints would be to endogenize the amount of credit that lenders in the economy would be willing to extend to the firms (see, for example, Ryoo (2010)). However, since we model the behavior of the central bank in terms of an interest rate rule, we endogenize the risk premium as opposed to the stock of firm debt for reasons of consistency.

uniform across equity and debt ( $\xi = \xi_e$ ).<sup>22</sup> The evolution of the risk premium has a dynamic specification to reflect the idea that the *level* of the risk premium depends on the *duration* of a particular excess performance. The intuition being that the lenders in the economy relax (tighten) the borrowing constraints as they observe good (bad) performance over a long time period.

#### 4. The integrated model of the real and financial sectors

We now put all the components of the real sector and the financial sector together. To integrate the two sectors we need to extend the investment function from the model of the real sector. The new investment function has the following specification:

$$g = g^o + \phi_r (r^n - i - \xi + \pi^c) + \phi_x x \quad (\mathbf{IF})$$

Instead of the constant parameter  $r^o$ , the firms now compare their normal profit rate ( $r^n$ ) with the borrowing conditions in the economy ( $i + \xi - \pi^c$ ).<sup>23</sup> The firms use their inflation expectations ( $\pi^c$ ) in order to assess the borrowing conditions. The integrated model of the

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<sup>22</sup>The idea that “success breeds a disregard of the possibilities of failure” (Minsky (1986, p.237)) can also be modeled by using the evolution of debt in the economy rather than the evolution of the risk premium (see, for example, Ryoo (2010)). The second key element of Minsky’s financial instability hypothesis is not present in this model. That is to say, unlike Ryoo (2010), this paper does not capture the fragility of economic systems with capital structures dominated by debt.

<sup>23</sup>We can also include Tobin’s  $Q$  ( $q$ ) in the investment function. However, most of the effect of Tobin’s  $Q$  on investment is already captured by the difference ( $r^n - i - \xi + \pi^c$ ) since  $q$  is driven by  $q^f = \frac{r - g}{i + \xi - \pi - g}$ . The actual rate of profit ( $r$ ) is very similar to  $r^n$ ,  $\pi^c$  is similar to  $\pi$  and  $g$  has a very small impact on  $q^f$  since it is present both in the denominator and in the numerator. Because of this we don’t include  $q$  in the investment function.

two sectors can thus be expressed as the following system of differential equations:

$$\begin{aligned}
\dot{x} &= \nu [(1-x) \exp(\vartheta_y y) - (1+x) \exp(-\vartheta_y y)] , & -1 \leq x \leq 1 & \quad \text{(SE)} \\
\dot{h} &= -(1-h) \kappa_{rw} y , & 0 < h < 1 & \quad \text{(PS)} \\
\dot{\pi}^c &= \alpha [\gamma \pi^* + (1-\gamma) \pi - \pi^c] & & \quad \text{(IC)} \\
\frac{di}{dt} &= \mu_i [i^o + \mu_\pi (\pi - \pi^*) + \mu_y y - i] & & \quad \text{(TR)} \\
\dot{q} &= \varrho_q (q^f - q) & & \quad \text{(AQ)} \\
\dot{\xi} &= -\varrho_\xi (r - i + \pi - \xi^c) & & \quad \text{(RP)} \\
g &= g^o + \phi_r (r^n - i - \xi + \pi^c) + \phi_x x & & \quad \text{(IF)} \\
y &= \frac{g - g^o}{(1 - c_2) u^o} & & \quad \text{(OG)} \\
\pi &= \pi^c + \kappa y & & \quad \text{(PC)} \\
r^n &= h u^o - \delta & & \quad \text{(NPR)} \\
r &= h u^o (1 + y) - \delta & & \quad \text{(APR)} \\
q^f &= \frac{r - g}{i + \xi - \pi - g} & & \quad \text{(FQ)} \\
i^o &= h^o u^o - \delta + \pi^* - \xi^c
\end{aligned}$$

We assume that in the absence of an output gap or deviations of inflation from its target, the central bank sets a nominal base rate  $i^o$  that allows the risk premium to come to rest ( $\dot{\xi} = 0$ ). The above system is no longer two dimensional and we explore its properties by means of simulations. The model is calibrated for measurements at a quarterly frequency.<sup>24</sup> In the simulations to come we are going to examine the endogenous fluctuations of the model economy around the point of rest characterized by:  $x = 0$ ,  $h = h^o$ ,  $\pi^c = \pi^*$ ,  $i = i^o$ ,  $q = 1$  and  $\xi = \xi^c$ .

The stabilizing and destabilizing forces (profitability and Harrodian instability) from the real sector are still present alongside two additional destabilizing forces and one additional stabilizing force. The inflation expectations ( $\pi^c$ ) are destabilizing since ceteris paribus deviations from  $\pi^*$  loosen (tighten) the borrowing conditions in real terms, which leads to a higher (lower) output gap, higher (lower) inflation and ultimately to expectations about even higher (lower) inflation (see equations **IF**, **OG**, **PC** and **IC**). The risk premium ( $\xi$ ) is also destabilizing since deviations from  $\xi^c$  loosen (tighten) the borrowing conditions for the firms, which ceteris paribus increases (decreases) the actual gross rate of profit ( $r$ ) leading to an even lower (higher) risk premium (see equations **IF**, **OG**, **APR** and **RP**). The central bank's reaction function (see eq. **TR**) constitutes the new stabilizing force. The central bank increases (decreases) the base nominal interest rate ( $i$ ) in reaction to a positive (negative) output gap or high (low) inflation, which ceteris paribus decreases (increases) investment, the output gap and consequently inflation (see equations **TR**, **IF**,

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<sup>24</sup>This means that parameters like  $u^o$ ,  $g^o$  or  $\pi^*$  are set to reflect their empirical counterparts at a quarterly frequency.

**OG** and **PC**).

Recall that the Harroddian instability is local in our model, while all other stabilizing and destabilizing forces are global. If all the destabilizing forces, excluding Harroddian instability, are weaker than all the stabilizing ones, and if we make Harroddian instability (which is a local force) strong enough, we can expect to get a sustained cyclical behavior of the system.<sup>25</sup> We are going to use parametrizations of the model producing such endogenous cycles to study the behavior of Tobin's  $Q$  over the business cycle while holding some of the properties of the system fixed. In all the explorations we try to keep the amplitudes, cycle period and relative positions of some of the variables fixed.<sup>26</sup> In particular, as with the calibration of the model of the real sector, the output gap and the profit share fluctuate with an amplitude of around  $\pm 3\%$  and  $\pm 1\%$ , respectively. Inflation fluctuates around the target of the central bank with an amplitude of around  $\pm 0.5\%$  and lags slightly behind the output gap.<sup>27</sup> We observe that the real base interest rate is also mostly pro-cyclical in the data, which implies that the central bank reacts strongly enough to compensate for inflation (see Figure 1). We thus set the reaction parameters  $\mu_{pi}$  and  $\mu_y$  in eq. **TR** high enough, to make the real base interest rate pro-cyclical. We keep the cycle period around 25 quarters by suitably adjusting parameters that have an effect on the cycle period (e.g. the parameter  $\nu$  in eq. **SE**).

## 5. The determinants of a pro-cyclical stock market

Equation **FQ** summarizes the driving forces of the stock market in the model since Tobin's  $Q$  ( $q$ ) adjusts towards  $q^f$  (see eq. **AQ**). The key determinants of  $q^f$  are the real base interest rate, the risk premium and the gross actual profit rate (see eq. **FQ**). Next, we explore how these determinants affect the behavior of the stock market over the business cycle.

### 5.1. The gross actual profit rate

If we look at eq. **APR**, we see that  $r$  is determined by the output gap ( $y$ ) and by the profit share ( $h$ ). We already know that the output gap is lagging behind the the profit share (see Figure 2 (b)), which implies that  $r$  is lagging behind  $h$  but leading  $y$ . Thus we can conclude that  $r$  is strongly pro-cyclical in the model. IF we look at eq. **FQ** again, we see that this allows the stock market to be pro-cyclical depending on the dynamics of the denominator of  $q^f$ .

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<sup>25</sup>We can make Harroddian instability strong enough by increasing one of the parameters involved in it (for example,  $\vartheta_y$ , see condition **R2**).

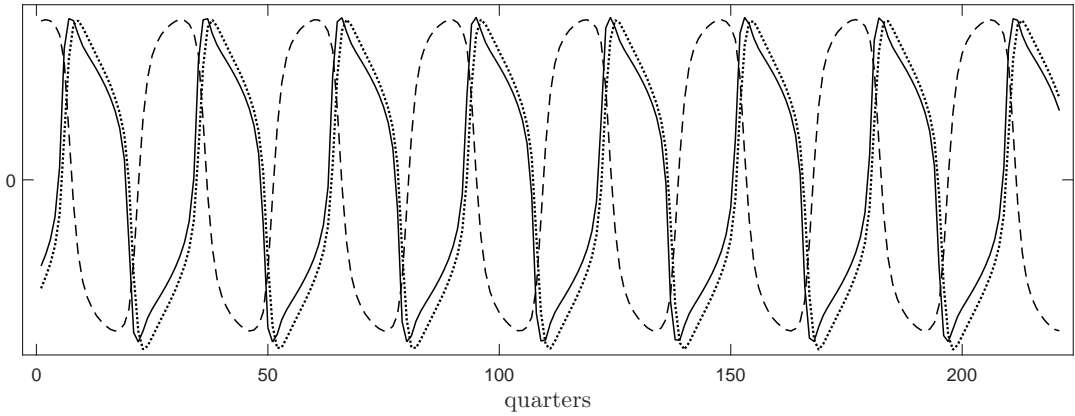
<sup>26</sup>Explorations with respect to the parametrization of the model indicate that the model seems to produce mostly *regular* cyclical behavior.

<sup>27</sup>The amplitude refers to inflation over a quarter. This implies an amplitude of the yearly inflation rate of  $\pm 2\%$ . Inflation is mostly pro-cyclical in the data with a slight lag behind the output gap.



### 5.2. Monetary policy

Generally, if the central bank is very aggressive in its reaction to high inflation and output gaps, it could prevent the stock market ( $q$ ) from being pro-cyclical. Intuitively, if we look at equation **FQ** we see that if, for example,  $i$  is high whenever  $r$  is high, this does not allow the stock market to be pro-cyclical. The strength of the reaction of  $i$  to  $r$  (via equations **TR** and **APR**) is one of the major determinants of the behavior of the financial markets in this model. A situation in which the model is calibrated such that the central bank reacts very strongly to  $y$  and  $\pi$  is shown in Figure 4, where the real base interest rate exhibits an amplitude of around  $\pm 0.9\%$ .<sup>28</sup> This involves setting high values for the parameters  $\mu_y$  and  $\mu_\pi$  in eq. **TR**. In reality, however, the central banks does not seem to be reacting very aggressively since the average amplitude of the fluctuations of the real base interest rate in the filtered data is around  $\pm 0.25\%$ . See section D of the appendix for the parameter values used in the simulation.



**Figure 4:** The fluctuations of the real base interest rate (dotted line), of real output (solid line) and of Tobin's  $Q$  (dashed line) generated by a simulation of the integrated model in which a very aggressive monetary policy is preventing the stock market from being pro-cyclical. The fluctuations have been re-scaled to make them comparable by means of visual inspection.

If the central bank reacts very slowly to  $y$  and  $\pi$ , this induces the real base interest rate to lag behind  $y$  (and thus behind  $r$ ) a lot, which allows the stock market to be more pro-cyclical (a higher correlation of  $q$  with the output gap is achieved).<sup>29</sup> However, this is something that we do not observe in the filtered data, where the real base interest rate usually does not lag a lot behind the output gap (see Figure 1).

### 5.3. The endogenous risk premium

The endogenous risk premium can offset monetary policy, since it loosens (tightens) the borrowing conditions, whenever the model economy is in a boom (recession). We can say

<sup>28</sup>As in Figure 1 the fluctuations have been re-scaled, so that the patterns can be recognized more easily.

<sup>29</sup>We can allow for the central bank to react very slowly by setting a value for the parameter  $\mu_i$  in the Taylor rule close to 0.

this since  $y$  and  $r$  move very closely together and since we know from eq. **APR** that the risk premium depends on  $r$ .<sup>30</sup> The endogenous risk premium can thus contribute to a pro-cyclical stock market. For instance, if an increase in the base interest rate due to a positive output gap takes place together with a fall in the risk premium due to a high actual gross profit rate, the denominator of  $q^f$  can remain roughly unchanged, so that the numerator is left as the key determinant of the behavior of  $q^f$ , and we already know that the numerator ( $r$ ) follows  $y$  very closely.

#### 5.4. The adjustment of the financial markets

Finally, the speed of adjustment of the stock market to changes in  $q^f$  also plays a role in determining the behavior of the stock market over the business cycle (see eq. **AQ**). Even if  $q^f$  is not strongly pro-cyclical, for example, because of the behavior of the central bank, a slow adjustment process (of  $q$  towards  $q^f$ ) can render the financial markets strongly pro-cyclical or even lagging behind the output gap.

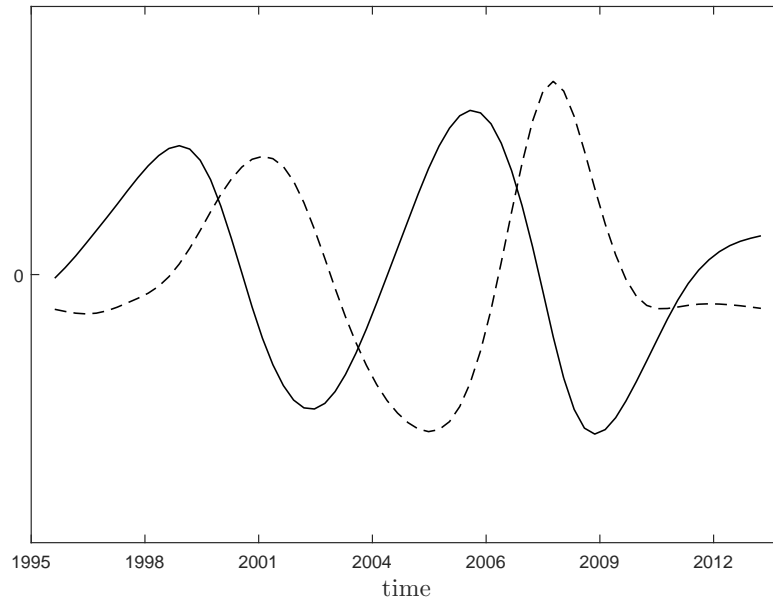
## 6. A final calibration of the integrated model

In this section, we are going to calibrate the integrated model such that it can fit some of the properties of the filtered data. In particular, we are going to be aiming for a pro-cyclical stock market given a pro-cyclical real base interest rate. Naturally, we are also going to attempt to match the average amplitude of the fluctuations of the respective variable observed in the filtered data.

Recall that we have already committed to reproducing certain regularities of inflation, the output gap and the profit share at the end of section 4. Given the amplitude and relative positions of inflation and the output gap we now set the parameter of the reaction function of the central bank such that the resulting real base interest rate is pro-cyclical, fluctuating with an amplitude of around  $\pm 0.25\%$ . The dynamic of the output gap and profit share to which we have committed determine the behavior of the gross actual profit rate ( $r$ ), which is the crucial component of the numerator of  $q^f$ . Finally,  $q^f$  is also affected by the risk premium ( $\xi$ ). The strength of the reaction of  $\xi$  (see eq. **RP**) to the performance of the real sector determines to what extent monetary policy is offset and thus has an effect on the behavior of  $q^f$  over the business cycle. We use this parameter ( $\varrho_\xi$ ) together with the adjustment speed ( $\varrho_q$ ) of the actual Tobin's  $Q$  (see eq. **AQ**) to allow the integrated model to yield a pro-cyclical stock market. In particular, we set the adjustment speed to  $\varrho_q = 0.3$ , which implies that per time step 30% of the distance between  $q$  and  $q^f$  is made good. The resulting amplitude of the  $q$  is roughly twice as large as the amplitude of the output gap ( $y$ ), i.e. roughly  $\pm 6\%$ . Filtered empirical measures of Tobin's  $Q$  yield a similar average amplitude.

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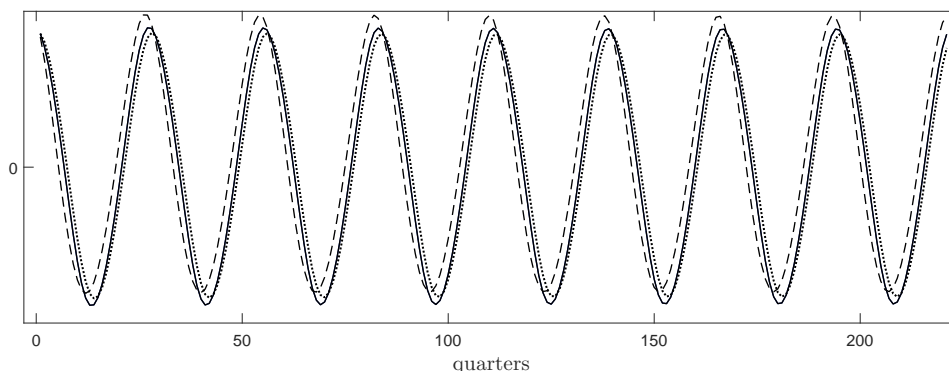
<sup>30</sup>For the endogenous risk premium to be able to offset monetary policy, the difference ( $r - i + \pi - \xi^c$ ) must be positive during a boom and negative during recessions (see eq. **APR**). We impose this to be the case by parametrizing the model such that the central bank is not being too aggressive in its reaction to  $y$  and  $\pi$ .



**Figure 5:** The filtered fluctuations of real gross value added of the non-financial sector (solid line) and of the BofA Merrill Lynch US High Yield Master II Option-Adjusted Spread (dashed line) around their respective long run trends for the US over the period (1983-2014). The fluctuations have been re-scaled to make them comparable by means of visual inspection.

The above calibration implies certain dynamics of the risk premium. In particular, under the above calibration the risk premium is counter-cyclical in the model. If we take the BofA Merrill Lynch US High Yield Master II Option-Adjusted Spread as a proxy for the risk premium in the US, we see that the risk premium seems to be counter-cyclical (see Figure 5).

Figure 6 shows the re-scaled dynamics of the real base interest rate, the output gap and Tobin's  $Q$  resulting from a simulation of the model using the final parametrization just discussed. The fluctuations are re-scaled in order to make the patterns easier to recognize and to make Figure 6 comparable to Figure 1.



**Figure 6:** The fluctuations of the real base interest rate (dotted line), of real output (solid line) and of Tobin's  $Q$  (dashed line) generated by a simulation of the integrated model in which both the real base interest rate and Tobin's  $Q$  are pro-cyclical. The fluctuations have been re-scaled to make them comparable by means of visual inspection.

Lag number at which maximum correlation with the output gap occurs:

	filtered smoothed data	model
profit share ( $h$ )	$-7$ (0.91)	$-7$ (0.98)
base interest rate ( $i$ )	$1$ (0.91)	$1$ (1)
inflation ( $\pi$ )	$0$ (0.63)	$0$ (0.99)
real base interest rate ( $i - \pi$ )	$1$ (0.84)	$1$ (0.99)
Tobin's $Q$ ( $q$ )	$-1$ (0.7)	$-1$ (0.98)
the negative of the risk premium ( $-\xi$ )	$5$ (0.92)	$1$ (0.99)

**Table 1:** The lag number at which maximum correlation between the output gap ( $y$ ) and the respective variables occurs in the data and in the model. We take the negative of the the risk premium since it is counter-cyclical. In brackets we see the correlation itself at the respective lag. All the data is quarterly, so the lags are in terms of quarters.

Empirical standard deviation of the respective variables:

	filtered smoothed data	model
output gap ( $y$ )	1.5	1.87
profit share ( $h$ )	0.95	0.61
base interest rate ( $i$ )	0.2	0.2
inflation ( $\pi$ )	0.088	0.085
real base interest rate ( $i - \pi$ )	0.15	0.11
Tobin's $Q$ ( $q$ )	5.6	9.8
risk premium ( $\xi$ )	0.30	0.45

**Table 2:** The empirical standard deviation of the respective variables (in terms of the number of percentage points) observed in the data and in the model. The standard deviation of the smoothed filtered data is generally smaller than that of just the filtered data.

Table 1 and 2 show the performance of the final calibration of the model in terms of some of the empirical moments exhibited by the filtered smoothed data during the last 25 years (1989–2014).

## 7. Conclusion

In this paper we have used an endogenous business cycle model to explore the factors contributing to a pro-cyclical stock market. Particularly during the last 25 years, the stock market has been strongly pro-cyclical in the US, which does not seem very intuitive given the counter-cyclical monetary policy conducted over that period. After all, if the central bank is able to set the opportunity cost of capital in the economy, then the high (low) profits during a boom (recession) would be discounted by a high (low) opportunity cost of capital, which should prevent the stock market from being strongly pro-cyclical.

In the context of the model, we identify several factors that contribute to a pro-cyclical stock market. Intuitively, the value of the firms in the real sector (Tobin's average  $Q$ ) is affected by the profits generated in it. A timid or ineffective monetary policy allows the stock market to be dominated by the fluctuations of the profits in the real sector. In particular, we modeled the potential ineffectiveness of monetary policy in terms of an endogenous risk premium. Finally, the adjustment speed of the financial markets to changes in the fundamentals of the model economy is also an important factor affecting the behavior of the stock market over the business cycle.

Using the above mentioned factors, we calibrate the model to fit key properties of the data from the last 25 years (1989–2014). In particular, the model can generate a pro-cyclical stock market in the presence of a counter-cyclical monetary policy.

The model of the real sector can be extended in many ways to study other issues in the context of endogenous business cycles. At its core lies the idea of endogenously emerging profit opportunities that are being subsequently exhausted by expansions of output.

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## Appendix A. The structure of demand expectations

Let  $N$ ,  $n^-$  and  $n^+$  be the total number of firms, the fraction of firms expecting a negative output gap and the fraction of firms expecting a positive output gap, respectively. Next, we define a measure of the structure of the expectations in the economy as  $x = n^+ - n^-$ . The index can only take on values between  $-1$  and  $1$  ( $-1 \leq x \leq 1$ ). A value of  $x$  close to  $1$  would indicate that most of the firms expect a positive output gap, while a value close to  $-1$  would mean that the majority expects a negative output gap. Let us consider next an arbitrary period  $\Delta t$  and assume that each firm can switch its attitude only once within this period. In addition, let  $p^{-+}$  be the probability *per unit of time* (uniform across all firms) that a firm expecting a negative output gap will switch to expecting a positive one, and  $p^{+-}$  be the probability of the opposite change happening. Thus,  $\Delta t p^{-+}$  and  $\Delta t p^{+-}$  constitute the probabilities that these switches occur within the time interval  $[t, t + \Delta t)$ .

The population shares of the positive output gap expectation and of the negative output gap expectation can be expressed in terms of  $x$  as  $n^+ = (1 + x)/2$  and  $n^- = (1 - x)/2$ ,

respectively.<sup>31</sup> A specification of the evolution of these shares over time can be obtained by using the aforementioned transition probabilities ( $p^{+-}$  and  $p^{-+}$ ). Over the period of time  $\Delta t$  the population share of the positive output gap expectation decreases exactly by  $\Delta t p^{+-} (1+x)/2$  and increases exactly by  $\Delta t p^{-+} (1-x)/2$  if we assume that the number of firms ( $N$ ) tends to infinity. This is assumed here. With signs reversed, the same holds true for the evolution of the population share of the negative output gap expectation over  $\Delta t$ . The net effect of these share dynamics on  $x$  can be summarized in the following differential equation in continuous time (if we let  $\Delta t$  approach zero)

$$\dot{x} = (1-x)p^{-+} - (1+x)p^{+-} \quad (\text{A.1})$$

This probabilistic approach allows for the individual agents (the firms) to form distinct expectations due to some firm idiosyncrasy. New information affects the probability of switches across the different expectations. In our case, the switching probabilities are a function of the observed output gap ( $y$ ). The following specification of the transition probabilities was taken from the book by Weidlich and Haag (1983, p. 41):

$$p^{-+} = \nu \exp(\vartheta_y y), \quad p^{+-} = \nu \exp(-\vartheta_y y) \quad (\text{A.2})$$

The specification assumes that the percentage change in the respective transition probability is linear in the absolute change of  $y$ .<sup>32</sup> The observed output gap affects both transition probabilities ( $p^{-+}$  and  $p^{+-}$ ) in a symmetric fashion.

Inserting (A.2) into (A.1) yields the complete specification of the dynamics of the structure of the demand expectations:

$$\dot{x} = \nu [(1-x) \exp(s) - (1+x) \exp(-s)] \quad (\text{SE})$$

## Appendix B. Stability analysis of the model of the real sector

There is a technicality concerning the boundaries of eq. **PS**. We have to define what happens as the system approaches the boundaries  $h = 1$  or  $h = 0$ . We assume that as the system approaches  $h = 1$  or  $h = 0$  at some very small distance  $\epsilon$  from the boundaries  $\dot{h}$  becomes 0. In the following, when we refer to points of rest involving  $h = 1$  or  $h = 0$  we will actually be referring to points of rest involving  $h = 1 - \epsilon$  or  $h = 0 + \epsilon$ . We skip the  $\epsilon$  to make the notation less cumbersome. The model of the real sector has 4 isoclines.  $IS_{h1}$ ,  $IS_{h2}$  and  $IS_{h3}$  stand for the isoclines that give rise to  $\dot{h} = 0$ , while  $IS_x$  is the isocline

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<sup>31</sup>Consider the following algebraic manipulations:  $n^+ = n^+/2 + n^+/2 = (1 - n^-)/2 + n^+/2 = (1 + n^+ - n^-)/2 = (1+x)/2$ . The second relationship follows analogously.

<sup>32</sup>Formally, the assumption reads  $dp^{-+}/p^{-+} = \alpha dy$  for some constant  $\alpha$ .

associated with  $\dot{x} = 0$ .<sup>33</sup>

$$h = 1 \tag{IS_{h1}}$$

$$h = 0 \tag{IS_{h2}}$$

$$h = h^o - \frac{\phi_x}{\phi_r u^o} x \tag{IS_{h3}}$$

$$h = h^o + \frac{\ln[(1+x)/(1-x)]}{2u^o B} - \frac{\phi_x}{\phi_r u^o} x \tag{IS_x}$$

Define the composite parameters  $A = \frac{\phi_x \vartheta_y}{(1-c_2)u^o} > 0$  and  $B = \frac{\phi_r \vartheta_y}{(1-c_2)u^o} > 0$ . First, note that the isocline  $IS_x$  lies always above the isocline  $IS_{h3}$  for  $x > 0$  and is below it for  $x < 0$ . This is because the isoclines are identical but for  $\frac{\ln[(1+x)/(1-x)]}{2u^o B}$ , which is negative for  $x < 0$  and positive for  $x > 0$ . This implies that  $IS_x$  and  $IS_{h3}$  can cross each other only once, i.e. at  $(0, h^o)$ . Next, we exclude potentially attractive points of rest involving the the corner solutions  $h = 1$  and  $h = 0$ .

*Appendix B.1. Excluding the potentially attractive points of rest involving  $h = 1$*

The isoclines  $IS_x$  and  $IS_{h1}$  can potentially cross three times resulting in points of rest involving  $h = 1$ . The two such points of rest located below the  $IS_{h3}$  isocline can be potentially attractive because below the  $IS_{h3}$  isocline the profit share is increasing (see Figure B.7 (c)). Obviously, by definition,  $h = 1$  can only be approached from below. We exclude any potentially attractive points of rest involving  $h = 1$  by making sure that at the lowest possible value of  $x$  (i.e.  $x = -1$ ) the isoclines  $IS_{h3}$  takes on a value below 1. In other words, we do not allow  $IS_{h3}$  and  $IS_{h1}$  to intersect. In this way, we can guarantee that  $IS_x$  and  $IS_{h1}$  never intersect below the  $IS_{h3}$  isocline, since  $IS_x$  lies below  $IS_{h3}$  for  $x < 0$ . In  $IS_{h3}$  we thus impose  $h^o - \frac{\phi_x}{\phi_r u^o} (-1) < 1$  which leads to the restriction:

$$\phi_x/\phi_r < (1 - h^o) u^o \tag{B.1}$$

The point of rest involving  $h = 1$  that is located above the  $IS_{h3}$  isocline and results from a potential intersection of  $IS_x$  with  $IS_{h1}$  is always repelling and is thus not seen as a concern (see Figure B.7 (b)).<sup>34</sup>

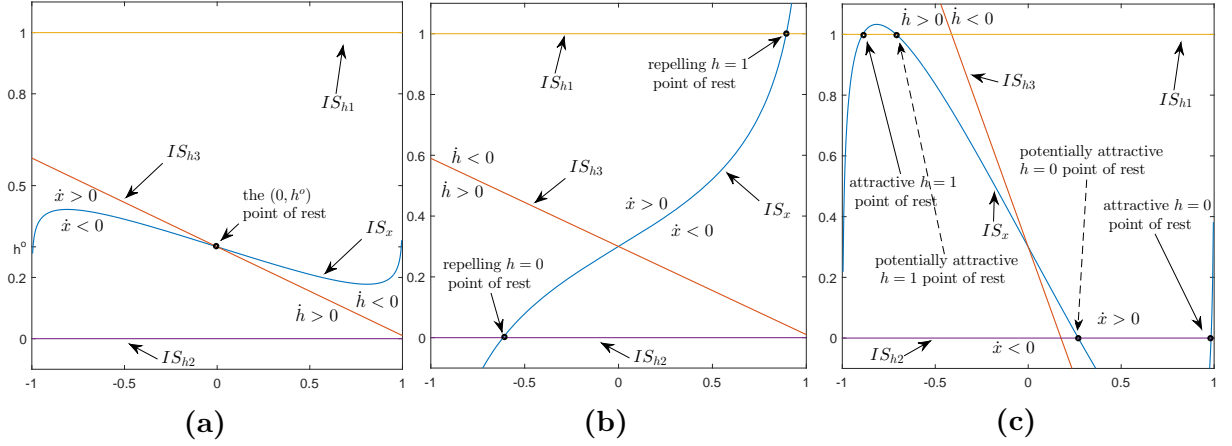
*Appendix B.2. Excluding the potentially attractive points of rest involving  $h = 0$*

Symmetrically, isoclines  $IS_x$  and  $IS_{h2}$  can also potentially cross three times resulting in points of rest involving  $h = 0$ . The two such points of rest located above the  $IS_{h3}$  isocline can be potentially attractive because above the  $IS_{h3}$  isocline the profit share is decreasing (see Figure B.7 (c)). Obviously, by definition,  $h = 0$  can only be approached from above. We exclude any potentially attractive points of rest involving  $h = 0$  by making sure that at the highest possible value of  $x$  (i.e.  $x = 1$ ) the isocline  $IS_{h3}$  takes on a value above 0.

<sup>33</sup>The eq. **SE** can be rewritten as  $\dot{x} = 2\nu [\tanh(\vartheta_y y) - x] \cosh(\vartheta_y y)$ . When solving for  $h$  to obtain the isocline  $IS_x$ , the identity  $\operatorname{arctanh}(x) = 0.5 \ln[(1+x)/(1-x)]$  was used.

<sup>34</sup>This point of rest is repelling since  $h$  is decreasing above the  $IS_{h3}$  isocline.





**Figure B.7:** (a) – The position of the isoclines in the standard case in which there is only one point of rest  $(0, h^o)$ ; (b) – The position of the isoclines in the case in which there is only one *attractive* point of rest  $(0, h^o)$ ; (c) – The position of the isoclines in the case in which there exist *attractive* points of rest associated with the corner solutions  $h = 1$  and  $h = 0$ .

In other words, we do not allow  $IS_{h3}$  and  $IS_{h2}$  to intersect. In this way, we can guarantee that  $IS_x$  and  $IS_{h2}$  never intersect above the  $IS_{h3}$  isocline, since  $IS_x$  lies above  $IS_{h3}$  for  $x > 0$ . In  $IS_{h3}$  we thus impose  $h^o - \frac{\phi_x}{\phi_r} u^o (1) > 0$  which leads to the restriction:

$$\phi_x / \phi_r < h^o u^o \quad (\text{B.2})$$

The point of rest involving  $h = 0$  that is located below the  $IS_{h3}$  isocline and results from a potential intersection of  $IS_x$  with  $IS_{h2}$  is always repelling and is thus not seen as a concern (see Figure B.7 (b)).<sup>35</sup>

Combing the inequalities B.1 and B.2 result in the restriction:

$$\phi_x / \phi_r < (0.5 - |h^o - 0.5|) u^o \quad (\text{R1})$$

This restriction excludes the existence of potentially attractive points of rest involving the corner solutions  $h = 1$  and  $h = 0$ .<sup>36</sup> In Figure B.7 (a) we see the standard constellation of the isoclines under parametrizations presented in this paper.

### Appendix B.3. Sustained cyclical behavior

The system is obviously globally stable since it is only defined for  $-1 \leq x \leq 1$  and  $0 < h < 1$ . In addition, under the restriction **R1**, we can say that all trajectories starting from within these boundaries must eventually enter, and then cannot leave, the rectangle given

<sup>35</sup>This point of rest is repelling since  $h$  is increasing below the  $IS_{h3}$  isocline.

<sup>36</sup>We can actually have a laxer restriction on the parameters by deriving a restriction preventing the intersections  $IS_x$  with  $IS_{h1}$  and  $IS_x$  with  $IS_{h2}$ , but still allowing for intersections  $IS_{h3}$  with  $IS_{h1}$  and  $IS_{h3}$  with  $IS_{h2}$ . However, this restriction turns out to be very cumbersome and uninformative. Because of this, we use a stronger restriction than what is sufficient.

by the 4 points:  $(-1, IS_{h3}(-1))$ ,  $(1, IS_{h3}(-1))$ ,  $(-1, IS_{h3}(1))$  and  $(1, IS_{h3}(1))$ .<sup>37</sup> This is the case because above the  $IS_{h3}$  isocline  $h$  is falling, while below it  $h$  is increasing. The conditions of the Poincaré-Bendixson Theorem are thus satisfied. Under the restriction **R1** sustained cyclical behavior is thus given if the point of rest  $(0, h^o)$  is repelling (see Figure B. (a)).

To derive the condition for that we can look at the Jacobian matrix around  $(0, h^o)$ . We can rewrite the system as:

$$\dot{x} = F_x(x, h) \quad \text{and} \quad \dot{h} = F_h(x, h)$$

We use the following type of notation for the partial derivatives:  $F_{xx} = \partial F_x / \partial x$ . The Jacobian matrix then becomes:

$$J = \begin{bmatrix} F_{xx} & F_{xh} \\ F_{hx} & F_{hh} \end{bmatrix} = \begin{bmatrix} \frac{2\nu [A - \cosh^2(\vartheta_y y)]}{\cosh^2(\vartheta_y y)} & \frac{2\nu B u^o}{\cosh^2(\vartheta_y y)} \\ -(1-h)\kappa_{rw}A & \kappa_{rw}Ax + \kappa_{rw}B(2hu^o - \delta - r^o - u^o) \end{bmatrix}$$

Around the point of rest  $(0, h^o)$  it reads:

$$J = \begin{bmatrix} 2\nu(A-1) & 2\nu B u^o \\ -(1-h^o)\kappa_{rw}A & -(1-h^o)u^o\kappa_{rw}B \end{bmatrix} = \begin{bmatrix} ? & + \\ - & - \end{bmatrix}$$

Only the upper-diagonal entry of the matrix can change its sign depending on the magnitude of the composite parameter  $A$ . The point of rest  $(0, h^o)$  is repelling for  $\text{trace}(J) > 0$  and  $\det(J) > 0$ . The determinant of  $J$  is bigger than 0 for any  $A$ . The trace of  $J$  is bigger than 0 if:

$$2\nu(A-1) > (1-h^o)u^o\kappa_{rw}B$$

After inserting the identities of  $A$  and  $B$  we get:

$$2\nu \frac{\phi_x \vartheta_y - (1-c_2)u^o}{\phi_r \vartheta_y} > (1-h^o)u^o\kappa_{rw} \quad (\mathbf{R2})$$

Thus the model of the real sector exhibits sustained cyclical behavior under the conditions **R1** and **R2**.

## Appendix C. Data filtering and sources

The cyclical components of all empirical series presented in this paper have been obtained using the following procedure. First, the raw data (or the natural logarithm of it, whenever an exponential trend is present) is detrended by using the Hodrick-Prescott filter with a smoothing parameter of 1600 (see Hodrick and Prescott (1997)). As a second step, the

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<sup>37</sup>Of course, this only makes sense if this rectangle is smaller than than the one given by the space over which the system is defined, i.e.  $(-1, 1)$ ,  $(1, 1)$ ,  $(-1, 0)$  and  $(1, 0)$ . The restriction described by the inequality **R1** guarantees that.

resulting cyclical component is smoothed additionally by applying a moving average over 11 quarters (over 5 lags, the current observation and 5 leads). The second step emphasizes the persistent movements in the data and is done to make the patterns in the series more recognizable.

What follows is a list of the empirical series mentioned in the paper together with their respective sources:

- The real gross value added of the non-financial sector is provided by the US. Bureau of Economic Analysis.
- The profit share in output is computed by using the National Income and Product Accounts (NIPA) Table 1.15 provided by the US. Bureau of Economic Analysis. We divide the “Profit Per Unit of Real Gross Value Added of Nonfinancial Domestic Corporate Business (after tax with IVA and CCAdj)” by the “Price per unit of real gross value added of nonfinancial corporate business”. Computing the profit share without considering taxes does not make a marked difference in terms of the dynamics of the profit share, only the level around which the profit share fluctuates changes.

Using data from the (NIPA) Table 1.14 yields very similar results. The profit share was computed in this way, for example, in the paper by Zipperer and Skott (2011).

- Different measures of Tobin’s  $Q$  and various stock market indices have very similar dynamics over the business cycle. The historical data for the S&P 500 index was taken from the “Political calculations” blog (<http://politicalcalculations.blogspot.com/2013/02/quarterly-data-for-s-500-since-1871.html>).

Tobin’s  $Q$  was computed by using data from the Federal Reserve Z.1 Statistical Release, section B.102 Balance Sheet of Non-farm Non-financial Corporate Business. Specifically, it is the ratio of Line 39 (Corporate Equities; Liability) divided by Line 36 (Corporate Business; Net Worth). Using other measure of Tobin’s  $Q$  does not impact the overall dynamics much, since, in all cases, the most powerful source of variation is the price of equity (the stock prices). This is the intuition behind the very similar dynamics of stock price indices and measures of Tobin’s  $Q$  over the business cycle.

- The real base interest rate – the quarterly series of the yearly “Effective Federal Funds Rate” divided by 4. It is taken from the Federal Reserve Economic Data (FRED).
- Inflation – the quarterly series of the yearly CPI inflation rate provided by the US. Bureau of Labor Statistics divided by 4.
- The risk premium – We take the BofA Merrill Lynch US High Yield Master II Option-Adjusted Spread as a proxy for the risk premium. The series are provided by BofA Merrill Lynch and are available only post 1996: <https://research.stlouisfed.org/fred2/series/BAMLH0A0HYM2>.

## Appendix D. The parameter values used in the simulations

All simulations were done using a built-in numerical solver for systems of first-order of differential equations in Matlab. We always display or measure the simulation results after the first 200 simulation quarters, when adjustments coming from the initial conditions have faded away. The model is calibrated for measurements at a quarterly frequency. This means that parameters like  $u^o$ ,  $g^o$ ,  $\delta$  or  $\pi^*$  are set to reflect their empirical counterparts at a quarterly frequency.

The simulation producing the dynamics shown in Figure 2 (b) are based on the following parametrization of the model of the real sector:<sup>38</sup>

$\nu$	$\phi_x$	$\phi_r$	$c_2$	$h^o$	$\vartheta_y$	$\kappa_{rw}$	$\delta$	$g^o$	$u^o$
0.3	0.008	0.5	0.3	0.3	23	0.1	0.025	0.005	0.25

The simulation producing the dynamics shown in Figure 4 are based on the following parametrization of the model of the real sector:

$\nu$	$\phi_x$	$\phi_r$	$c_2$	$h^o$	$\vartheta_y$	$\kappa_{rw}$	$\kappa$	$\alpha$	$\gamma$
0.4	0.008	0.45	0.3	0.3	60	0.15	0.03	0.4	0.8

$\pi^*$	$\mu_i$	$\mu_y$	$\mu_\pi$	$\varrho_q$	$\varrho_\xi$	$\xi^c$	$\delta$	$g^o$	$u^o$
0.005	1	0.3	0.3	1	0	0.015	0.025	0.005	0.25

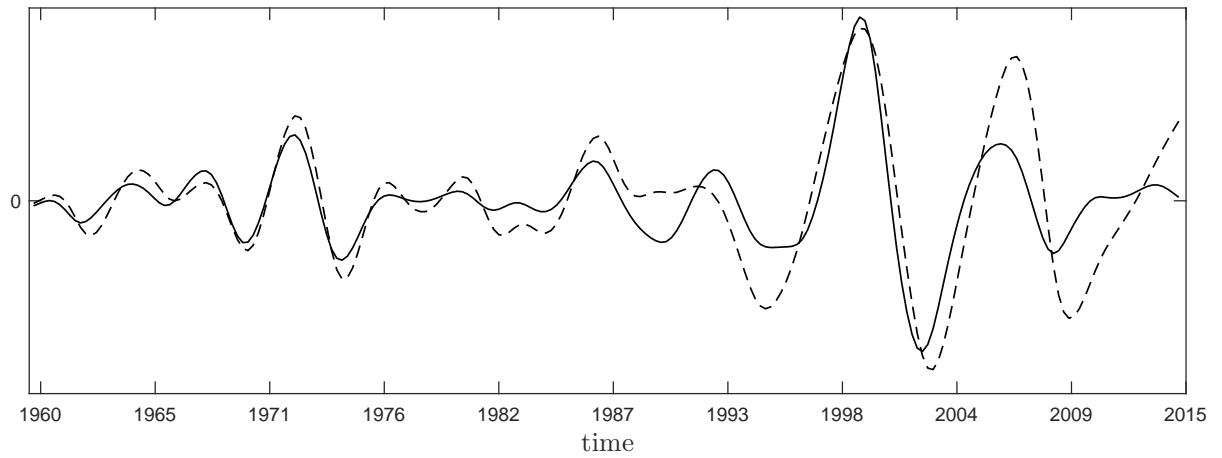
The values of the parameters underlying the final parametrization of the integrated model (see Section 6 and Figure 6) read as follows:

$\nu$	$\phi_x$	$\phi_r$	$c_2$	$h^o$	$\vartheta_y$	$\kappa_{rw}$	$\kappa$	$\alpha$	$\gamma$
0.2	0.008	0.5	0.3	0.3	18.7	0.15	0.04	0.4	0.8

$\pi^*$	$\mu_i$	$\mu_y$	$\mu_\pi$	$\varrho_q$	$\varrho_\xi$	$\xi^c$	$\delta$	$g^o$	$u^o$
0.005	1	0.1	0.1	0.3	0.3	0.015	0.025	0.005	0.25

The two parametrizations differ in terms of the values of the parameters  $\nu$ ,  $\phi_r$ ,  $\vartheta_y$ ,  $\mu_y$ ,  $\mu_\pi$ ,  $\varrho_q$  and  $\varrho_\xi$ . The values of the parameters  $\mu_y$  and  $\mu_\pi$  are higher under the first parametrization to reflect the more aggressive monetary policy, while the values of the parameters  $\varrho_\xi$  and  $\varrho_q$  reflect the introduction of an endogenous risk premium and an adjustment process for Tobin's  $Q$ . Adjustments in the parameters  $\nu$ ,  $\phi_r$  and  $\vartheta_y$  are needed to preserve the endogenous cycles produced by the model and to keep the desired amplitudes and cycle period of the variables fixed.

<sup>38</sup>The “normal” output capital ratio might appear too high ( $u^o = 0.25$ ). It should be noted that this ratio pertains to the non-financial sector and not to the whole economy. For the non-financial sector the average output capital ratio is higher than for the whole economy. See Franke (2015a, Section 5 and the Appendix) for more on this issue.



**Figure D.8:** The filtered fluctuations of the stock price index S&P 500 (dashed line) and of a measure of Tobin's Q (solid line) around their respective long run trends for the US over the period (1960-2014). The fluctuations have been re-scaled to make them comparable by means of visual inspection.